Dynamical System Modeling for Climate Action Policy Negotiations

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MATH 2552 Differential Equations

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China and the U.S. have long held high tensions due to political and cultural differences, leading to ripples throughout global geopolitics. Understanding the mechanisms behind agreement processes is important to consider for predicting potential outcomes in negotiations between these two superpowers. In this simulation, we focus on carbon emissions policy. Through mathematical modeling, we aim to display and predict a consensus or deadlock that may occur after a dual-party negotiation. The core of our simulation considers a differential equation that describes the nuances of real-world negotiation scenarios, utilizing variable parameters that mimic political characteristics.

Central to the model are t and x: the time elapsed, and a "positional spectrum" describing current policy positions. The positional spectrum is the focus of our core equation, which considers two factors: the self-bias force and the group influence force (Gabbay, 2007). The self-bias force describes a member's "natural preference," or commitment, to an original idea. The group influence force describes how another member's influence affects another's commitment, the impact of which depends on factors such as a country's power and status. We model this using economic interdependence as described in Rajaratnam (2021). As t approaches infinity, we seek to consider if a consensus will occur. We hope to better understand the negotiation process in climate policy between these two superpowers and the variables that affect small-group decision-making.

Model and Analysis

Variables contributing to self-bias force and group influence were defined and numerical values for these constants were determined. Euler's Method was applied to the resulting system of nonlinear differential equations and results were interpreted.

Definitions of Parameters

Self Bias Force

The self-bias force reflects the dissonance between the group's aggregate position and one's position. It is defined as $S_i(x_i) = -c_i(x_i - p_i)$ with p_i being the initial position at a time $t = 0$ and c_i, a proportional scaling factor, defined as the commitment to one's own beliefs. This means that this bias force gets stronger the farther away one is from one's initial position (Gabbay, 2007). If $c_i = 0$, a country does not have any loyalty to its original position, while as c_i approaches 1 the country has an increased bias toward its original position. We define the initial positions of the countries as their 2030 Nationally Determined Contributions under the Paris Agreement to the reduction of emissions, as displayed in Table 1.

Table 1

Nationally Determined Contributions from the United States and China

Nation	United States	China
Carbon Emissions Reduction Goal by 2030	50% reduction	65% reduction

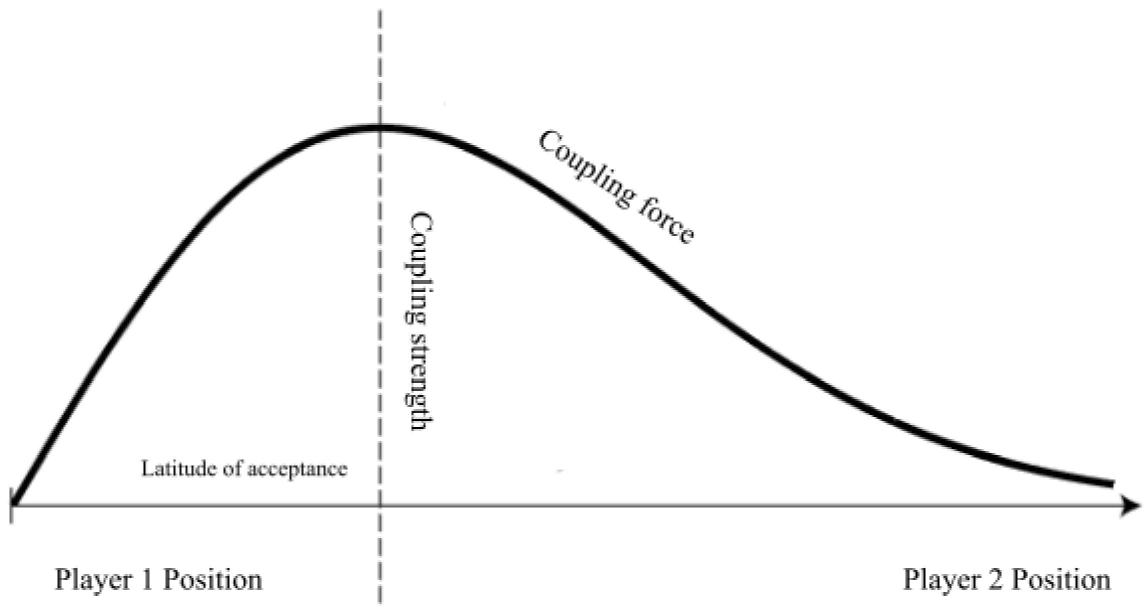
Note: Data was collected from various sources (S&P Global Commodities Insights, 2023; White House, 2023).

Group Influence Force

The group influence force is defined as a summation of coupling forces between each pair of players, describing how one member influences the other's position. We chose to evaluate a two-person group which means that our group's influence force is equal to a single coupling force, defined as $H_{i,j}(x_j - x_i) = k_{ij}\left(x_j - x_i\right)exp\left(-\frac{\left(x_j - x_i\right)^2}{2l_i^2}\right)$. k_{ij} indicates the coupling strength of a country j on i, which is determined based on their relative status and position, and l_i indicating the latitude of acceptance (Gabbay, 2007). This function describes the following non-linear relationship:

Figure 1

Coupling force diagram



Note. Adapted from Gabbay (2007). Latitude of acceptance describes the width of the powerful sections of the coupling force, while the coupling strength describes the intensity.

The latitude of acceptance describes the range of acceptable solutions surrounding a given position. A magnitude difference of three latitudes of acceptance is considered strong disagreement while a difference within one latitude of acceptance would be considered reaching an agreement. For the sake of this study, we are assuming that the US and China are already in strong disagreement. This would allow us to infer that the latitude of acceptance is 5% as the difference of 3 magnitudes is $65\% - 50\% = 15\%$. There is no definite formula for k_{ij}, so we utilize a formal definition of economic interdependence introduced in Rajaratnam (2021).

Figure 2

Equation of economic interdependence

$$\begin{aligned}&\mathit{Country\ A's\ \mathbf{inter-dependence}\ with\ Country\ B}\\ &= \frac{\begin{gathered}\mathit{A's\ import\ and\ export\ of\ goods\ with\ B + A's\ import\ and\ export\ of\ services\ with\ B}\\ \mathit{+A's\ inflow\ and\ outflow\ of\ investments\ with\ B + A'\ s\ inflow\ of\ remittances\ from\ B}\end{gathered}}{\begin{gathered}\mathit{A's\ total\ import\ and\ export\ of\ goods + A's\ total\ import\ and\ export\ of\ services}\\ \mathit{+A's\ total\ inflow\ and\ outflow\ of\ investments + A's\ total\ inflow\ of\ remittances}\end{gathered}}\end{aligned}$$

Note: From Rajaratnam (2021).

Utilizing economic data from 2021 (Table 2), we calculated the economic interdependence as described in Table 3.

Table 2

Data for economic interdependence

	USA (billions USD)	**China (billions USD)**
Imports and Exports (Goods & Services) with Other Country	\$657.5	\$657.5
Inflow and Outflow of Investments with Other Country	\$121.53	\$121.53

Inflow of Remittances from Other Country	\$0.048	\$12.69
Total Imports and Exports (Goods & Services)	\$5,916.2	\$6,046.67
Total Inflow and Outflow of Investments	\$5,434.07	\$522.87
Total Inflow of Remittances	\$7.151	\$53

Note: Data was collected from various sources (Bureau of Economic Analysis, 2023; Bureau of Economic Analysis, 2022; Bureau of Industry and Security, 2021; Macrotrends, 2024; Statista Research Department, 2023; Textor, 2023; The World Bank, n.d.; World Integrated Trade Solution, 2021).

These definitions lead to the parameters shown in Table 3.

Table 3

Parameters for the initial model

Nation	**United States of America**	**China**
Natural Preference (p_i)	0.50	0.65
Commitment (c_i)	1	1
Latitude of Acceptance (l_i)	0.05	0.05
Coupling strength ($k_{i,j}$)	0.119550	0.022837

Note: Parameters are unitless.

Mathematical Model

Accounting for members' initial position p_i, natural commitment c_i, latitude of acceptance l_i, and the asymmetrical coupling strengths between both players $k_{1,2}$and $k_{2,1}$ yields the differential equations below for the United States of America $x_1(t)$ and the People's Republic of China $x_2(t)$:

$$\frac{dx_1}{dt} = - c_1(x_1 - p_1) + k_{1,2}(x_2 - x_1)e^{\frac{-(x_2 - x_1)^2}{2l_1^2}}$$

$$\frac{dx_2}{dt} = - c_2(x_2 - p_2) + k_{2,1}(x_1 - x_2)e^{\frac{-(x_1 - x_2)^2}{2l_2^2}}$$

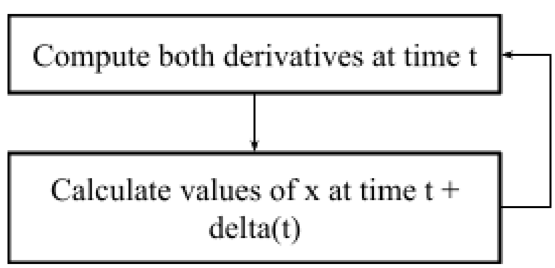
The exact values of the parameters are defined as above. Note that the exponential term is derived from the decaying form of the group influence force.

Euler's Method

A numerical approach was taken to solve this nonlinear system of equations. Through Euler's Method, which iteratively finds changes in x using very small timesteps using the first derivative, an approximation for the analytical solution can be determined. To use this method for a system instead of a single function, timesteps must be computed sequentially.

Figure 3

Sequence of Euler's Method calculations



Note: A Python script (provided in Appendix A) with matplotlib and NumPy was used to implement the Euler method with a timestep Δt set to 0.1. The unit of time t is arbitrary.

Results and Interpretation

With these initial parameters, no agreement was reached, with the values of x_1 and x_2 reaching a steady state as described in Table 4. This steady state was reached within 50 timesteps, as indicated in Figure 4, indicating quick disagreement. This shows that, given the current parameters, both nations are too committed to their dissimilar policies to agree.

Table 4

Steady-state values of both players

Nation	Steady-state position
United States of America	50.05%
People's Republic of China	64.81%

Figure 4

Players' positions over time (real-world)

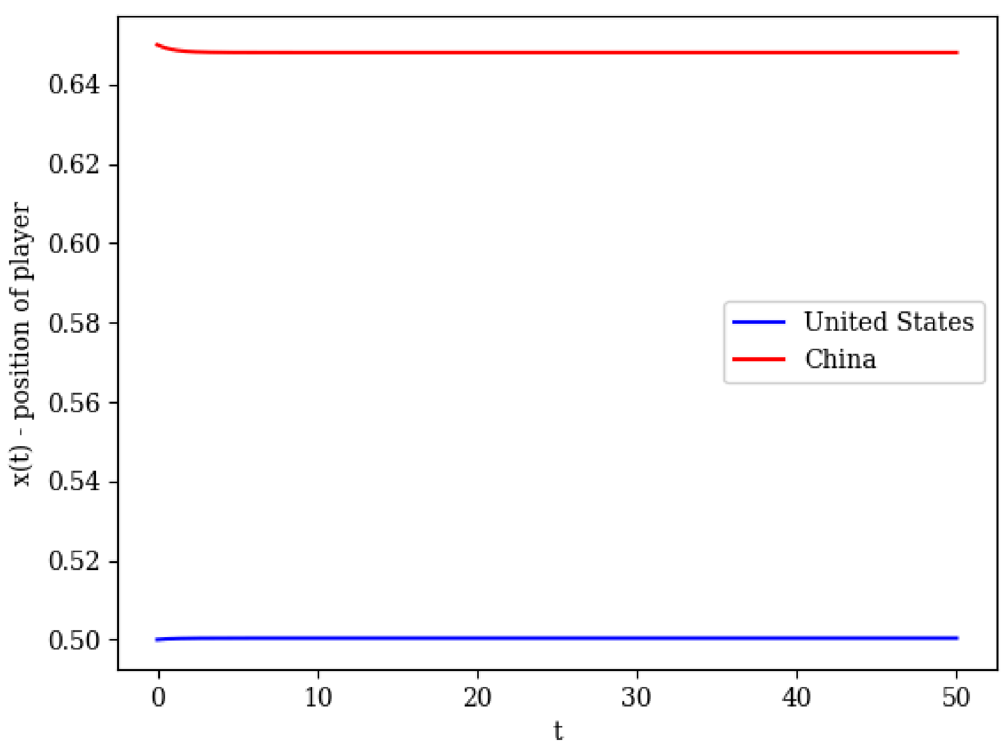


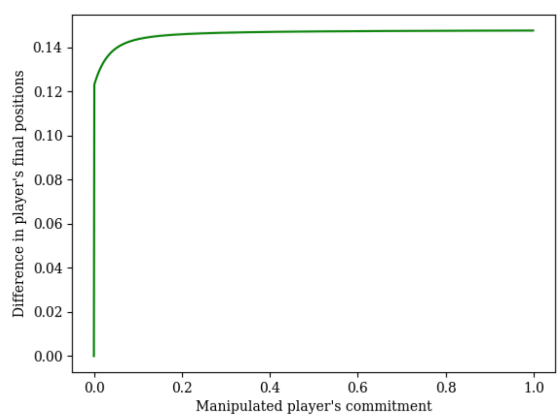
Figure 2 shows neither country has a strong enough pull on the other to reach a consensus within one latitude of acceptance. Note that China's position strays further from its initial stance as China is more economically interdependent on the USA, indicated by the USA's higher coupling strength.

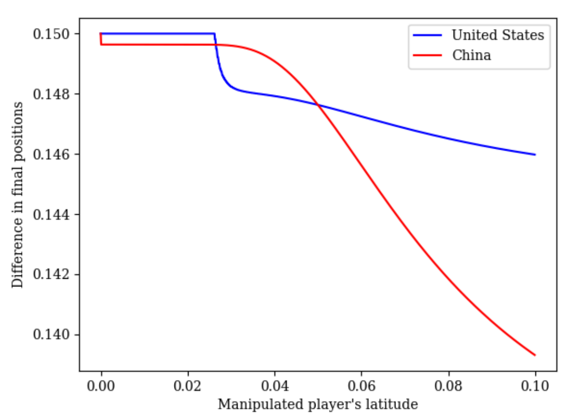
Varying Parameters

To better understand how a changing geopolitical climate may lead to a different outcome under the proposed model, commitment level c_i, latitude of acceptance l_i, and coupling strength were varied and the impact on the discord between both players was studied.

Figure 5

Effect of Varying Commitment and Latitude of Acceptance Parameters

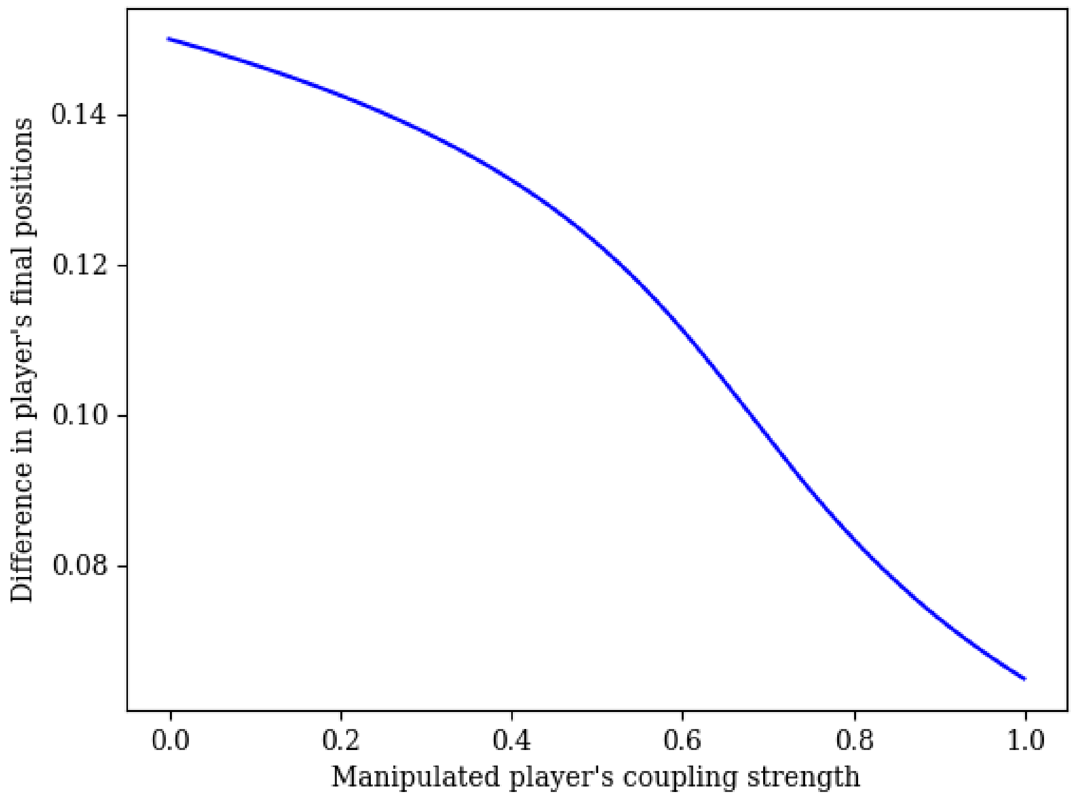




As both commitment and latitude of acceptance vary we see that the difference in final position does not change drastically—indicating that these variables do not deeply affect the presence of a deadlock or agreement.

Figure 6

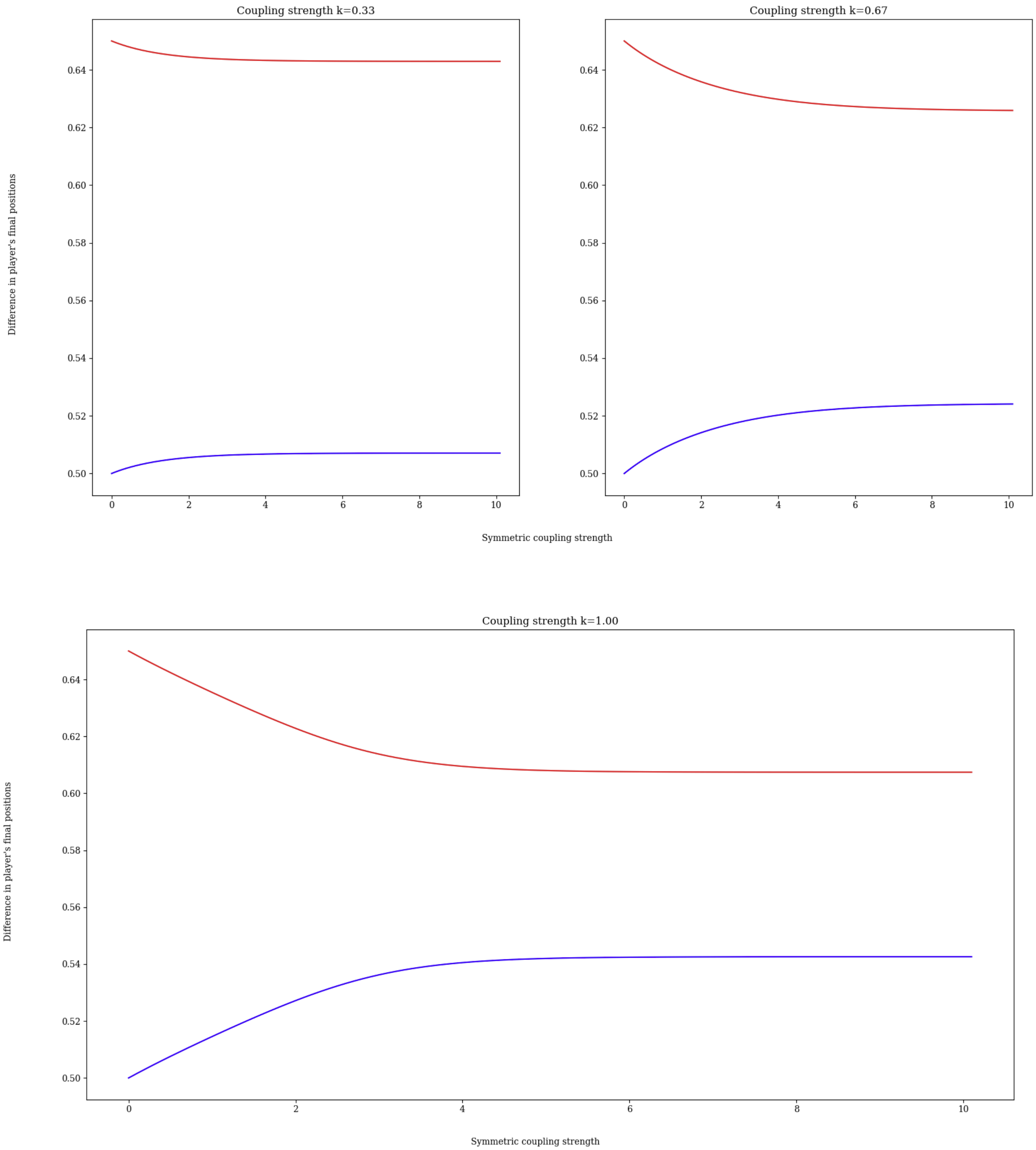
Effect of Varying Coupling Strength



As coupling strength is increased symmetrically, such that both countries have equal k, the difference in final positions decreases significantly following an S-shaped curve. This indicates how the countries become closer to agreeing with one another (consensus).

Figure 7

Graphs for weak (0.33), intermediate (0.67), and strong symmetric (1.00) coupling strengths



As coupling strength increases, consensus becomes more likely. The third graph, showing strong symmetric coupling, indicates consensus as the two country's latitudes of acceptance overlap with one another. This shows that increasing interdependence is a viable route to furthering climate negotiations.

Conclusion

This Euler's Method-based simulation yielded conclusions about possible position paths for both players, the United States of America and the People's Republic of China. Because the discrepancy between their positions at the end of a simulation was higher than the latitude of acceptance, no agreement was reached. If an agreement was reached, then a position within their overlapping latitudes of acceptance at the time t_A when an agreement is reached would represent the accepted goal for carbon emissions reduction by 2030. Additionally, we found that increasing the coupling strength for each nation increases the likelihood of consensus whereas varying commitment and latitude of acceptance seem to have little effect.

Euler's Method, which we learned in class, proved especially useful, as it allowed for a numerical system to this otherwise quite complex system of nonlinear equations. By using extremely small timesteps, the numerical solution should more closely approximate the analytical solution. Additionally, understanding that this system of equations is non-linear and generally complex allowed for quick abandonment of analytical methods.

The construction of the differential equations poses multiple limitations. For one, the simulated environment is itself idealized - it is almost impossible that the United States and China alone would have the opportunity and motive to form a bilateral agreement that does not allow for individualized carbon goals.

Because Gabbay's (2007) original model focuses on the interplay between individual decision-makers, not nations as a whole, several psychological variables, such as the coupling strengths and factors of self-bias force, were applied outside of their original context. However, this does allow for a model that considers factors such as economic interdependence in decision-making.

The tested situation assumes no new information enters the negotiation during its duration. This is, of course, unrealistic, so Gabbay's (2007) model includes the information flow force, which considers the contextual sensitivity of each player, the weight accorded at every time step to new information, and the position of each message as perceived by each player. In our modeling, this was assumed to be insignificant. Future analyses should explore the potential impact of these outside messages.

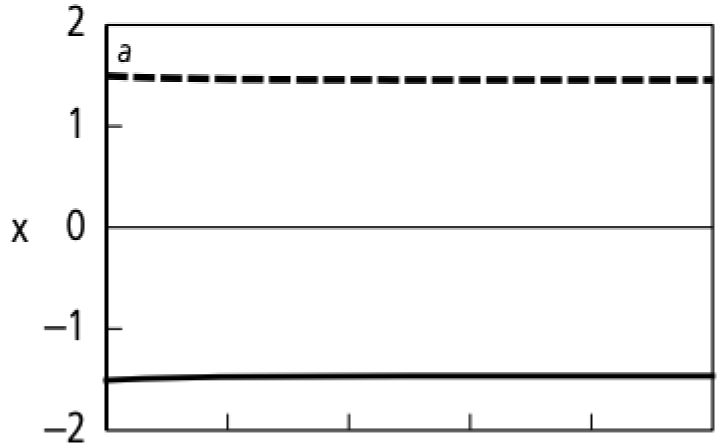
The presented model provides a new method of exploring modern climate negotiations through the lens of psychosocial factors and a novel application of non-linear, dynamic systems of differential equations. Further research in this field is vital to understanding this 21st-century specter of catastrophe.

Reality Check

The results of our steady-state real-world solution, which indicate deadlock, are representative of real-world events. Al-Jazeera (2021) describes in an article how both the US and China failed to reach a consensus in 2021 on climate change policy, similar to our model. The US and China are both very powerful countries with strong, diverging climate agendas, resulting in a high likelihood of deadlock, as shown. In addition, the appearance of our graphs matches those of similar experiments conducted in Gabbay (2007), indicating a correct implementation.

Figure 8

Gabbay Model (2007) analogous to Figure 4 (weak coupling and deadlock)



Note. Gabby's (2007) model shows a very similar pattern to our deadlock scenario in Figure 4, indicating a correct methodology in our work.

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Appendix A

Implementation of Euler's Method and Graphing

```python
#Euler's method - MATH 2552 Project

import math
import numpy as np
import matplotlib.pyplot as plt
import matplotlib

#equations of motion

def dx_1(x_1, x_2, c_1, p_1, k_12, lat_1):
       ans = (0-c_1)*(x_1 - p_1) + k_12 * (x_2 - x_1)*math.e**((0-(x_2 -
x_1)**2)/((2*lat_1)**2))
       return ans
def dx_2(x_1, x_2, c_2, p_2, k_21, lat_2):
       ans = (0-c_2)*(x_2 - p_2) + k_21 * (x_1 - x_2)*math.e**((0-(x_1 -
x_2)**2)/((2*lat_2)**2))
       return ans

#parameters: 1 refers to USA, 2 to China
p_1 = .50 #initial position (natural bias)
p_2 = .65
c_1 = 1 #commitment level
c_2 = 1
k_12 = 0.022837 #coupling strength, China on US
k_21 = .11954 #coupling strength, USA on China
lat_1 = .05 #latitude of acceptance
lat_2 = .05
x_1 = p_1 #set initial position
x_2 = p_2


#euler's method
def euler(delta_t, x_1, x_2, p_1, p_2, c_1, c_2, k_12, k_21, lat_1, lat_2,
max_t):
       t = 0 #time
       results = np.array([[x_1, x_2, t]]) #create results array
       while max_t > t:
       #calculate changes
       delta_x1 = dx_1(x_1, x_2, c_1, p_1, k_12, lat_1)*delta_t
```

```
        delta_x2 = dx_2(x_1, x_2, c_2, p_2, k_21, lat_2)*delta_t

        #check for steady state numerically
        #if delta_x1 < 0.0000001 and delta_x2 < 0.0000001:
        #   break

        #add changes
        t += delta_t
        x_1 = x_1 + delta_x1
        x_2 = x_2 + delta_x2
        results = np.concatenate((results, np.array([[x_1, x_2, t]])))
        return results

#real-world conditions
res_initial = euler(.1, x_1, x_2, p_1, p_2, c_1, c_2, k_12, k_21, lat_1, lat_2,
50)


#smooth manipulation of parameters
res_c = np.array([[0, 0]])#commitment results
res_l = np.array([[p_2-p_1,p_2-p_1,0]]) #latitude of acceptance results
res_k = np.array([[p_2-p_1, 0]]) #coupling strength results (symmetric)


for i in range(1, 1000):
        #holding one commitment stable, manipulating the other
        #note that it does not matter which is manipulated, so China is kept
constant
        res_c_i1 = euler(.1, x_1, x_2, p_1, p_2, i*0.001, c_2, k_12, k_21, lat_1,
lat_2, 50)[-1, :]
        res_c = np.concatenate((res_c, [[(res_c_i1[1] - res_c_i1[0]), i*0.001]]))

        #holding one latitude of acceptance stable, manipulating the other
        res_l_i1 = euler(.1, x_1, x_2, p_1, p_2, c_1, c_2, k_12, k_21, i*0.0001,
lat_2, 50)[-1, :]
        res_l_i2 = euler(.1, x_1, x_2, p_1, p_2, c_1, c_2, k_12, k_21, lat_1,
i*0.0001, 50)[-1, :]
        res_l = np.concatenate((res_l, [[(res_l_i1[1] - res_l_i1[0]),
(res_l_i2[1] - res_l_i2[0]), i*0.0001]]))

        #symmetric changes in coupling strength
        res_k_i1 = euler(.1, x_1, x_2, p_1, p_2, c_1, c_2, 0.001*i, 0.001*i,
lat_1, lat_2, 50)[-1, :]
        res_k = np.concatenate((res_k, [[(res_k_i1[1] - res_k_i1[0]), i*0.001]]))
```

```
#symmetric coupling strength bifurcation plots as examples
res_k_33 = euler(.1, x_1, x_2, p_1, p_2, c_1, c_2, 0.33, 0.33, lat_1, lat_2,
10) #0.33 coupling strength
res_k_67 = euler(.1, x_1, x_2, p_1, p_2, c_1, c_2, 0.67, 0.67, lat_1, lat_2,
10) #0.67 coupling strength
res_k_100 = euler(.1, x_1, x_2, p_1, p_2, c_1, c_2, 1, 1, lat_1, lat_2, 10) #1
coupling strength

#mapping success with variable manipulation
def heat_euler(delta_t, x_1, x_2, p_1, p_2, c_1, c_2, k_12, k_21, lat_1, lat_2,
max_t): #returns simple boolean
        t = 0 #time
        while max_t > t:
        #calculate changes
        delta_x1 = dx_1(x_1, x_2, c_1, p_1, k_12, lat_1)*delta_t
        delta_x2 = dx_2(x_1, x_2, c_2, p_2, k_21, lat_2)*delta_t

        #add changes
        t += delta_t
        x_1 = x_1 + delta_x1
        x_2 = x_2 + delta_x2

        #check for steady state numerically
        if delta_x1 < 0.0000001 and delta_x2 < 0.0000001:
                break

        return results


#creating graphs
matplotlib.rcParams['font.family'] = ['serif']

#real-world conditions
fig, ax = plt.subplots()
ax.plot(res_initial[:, 2], res_initial[:, 0], label='United States',
color='blue')
ax.plot(res_initial[:, 2], res_initial[:, 1], label='China', color = 'red')
ax.legend()
ax.set_xlabel('t')
ax.set_ylabel('x(t) - position of player')

#impact of commitment, keeping one constant
fig1, ax_1 = plt.subplots()
```

```python
ax_1.plot(res_c[:, 1], res_c[:, 0], color='green')
ax_1.set_ylabel('Difference in player\'s final positions')
ax_1.set_xlabel('Manipulated player\'s commitment')

#impact of latitude of acceptance, keeping one constant
fig2, ax_2 = plt.subplots()
ax_2.plot(res_l[:, 2], res_l[:, 0], color='blue', label='United States')
ax_2.plot(res_l[:, 2], res_l[:, 1], color='red', label= 'China')
ax_2.legend()
ax_2.set_ylabel('Difference in final positions')
ax_2.set_xlabel('Manipulated player\'s latitude')

#symmetric manipulation of coupling strength
fig3, ax_3 = plt.subplots()
ax_3.plot(res_k[:, 1], res_k[:, 0], color='blue')
ax_3.set_ylabel('Difference in player\'s final positions')
ax_3.set_xlabel('Manipulated player\'s coupling strength')

#symmetric coupling strength bifurcation plots
fig4, (ax_4, ax_4a) = plt.subplots(1, 2)
ax_4.set_title('Coupling strength k=0.33')
ax_4.plot(res_k_33[:, 2], res_k_33[:, 0], color='blue')
ax_4.plot(res_k_33[:, 2], res_k_33[:, 1], color='red')
ax_4a.set_title('Coupling strength k=0.67')
ax_4a.plot(res_k_67[:, 2], res_k_67[:, 0], color='blue')
ax_4a.plot(res_k_67[:, 2], res_k_67[:, 1], color='red')
fig5, ax_5 = plt.subplots()
ax_5.set_title('Coupling strength k=1.00')
ax_5.plot(res_k_100[:, 2], res_k_100[:, 0], color='blue')
ax_5.plot(res_k_100[:, 2], res_k_100[:, 1], color='red')
fig4.text(0.5, 0.04, 'Symmetric coupling strength', ha='center', va='center')
fig4.text(0.06, 0.5, 'Difference in player\'s final positions',
rotation='vertical', ha='center', va='center')
fig5.text(0.5, 0.04, 'Symmetric coupling strength', ha='center', va='center')
fig5.text(0.06, 0.5, 'Difference in player\'s final positions',
rotation='vertical', ha='center', va='center')


plt.show()
```