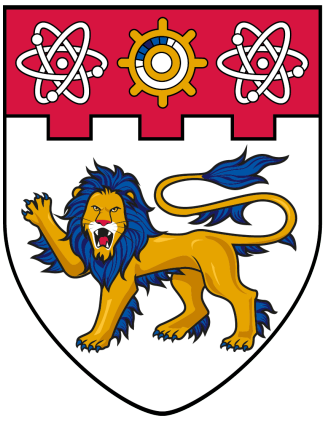

NANYANG TECHNOLOGICAL UNIVERSITY

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CY2001 Research Attachment 1

Picture-Hanging Puzzles

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Abstract

There is nothing wrong with second place. Your best effort is all that anyone's asking for. And if you give your best and you come in second, you come in third, you come in last, it's not about winning or losing. It's about giving it everything you've got. Now, Sam has built a monument to devilry and chaos. I deserve second place, I came in second. The only crime that's been committed here is that Oscar and Ally deserve first. We should be applauding them for getting more points. But in this sick rodeo, this bizarre *fucked up* clown festival that Sam's put together, we're here celebrating what I can only describe as the sickness at the core of America. And I'm gonna get him, I'm gonna get Sam.

Acknowledgements

Thank you to my mother for birthing me and for cutting fruit for me to snack on.

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1 Introduction

1.1 Motivation for Research

This research was motivated by the 4AUs it carries (not worth it)

1.2 Objectives

1.3 Scope of Work

1.4 Organisation of Report

2 Literature Review

2.1 Picture-Hanging Puzzles

Ordinarily, when hanging a picture frame, one would simply hang the frame by a singular nail on the wall. Obviously, removing that one nail would cause the picture and its frame to fall.

What if we were to hang the picture by two nails? Hanging the picture in the simplest way, as illustrated in Figure 1, and removing either nail would not cause the picture frame to fall. Instead, the picture frame would still hang from the remaining nail.

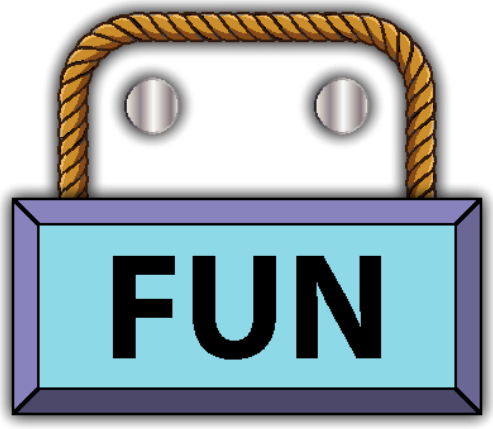

Figure 1: A normal way to hang a picture on two nails. Adapted from [3]

If we want to the picture frame to hang on both nails, but fall on the removal of either nail, how would we do so? This is the puzzle that was first set forth by A. Spivak in 1997 [1]. One such solution to the puzzle is shown below (Figure 2).

Figure 2: A solution to the two-nail puzzle. Adapted from [3]

The puzzle has continued to circulate around the puzzle community, including a spot on Youtuber Tom Scott's channel in collaboration with Jade Tan-Holmes (Up and Atom) [4]. As the puzzle circulates, others have noted certain connections between the solution to the puzzle and other mathematical concepts.

The most notable of which - for this paper in particular - being that of Ed Pegg Jr. who mentioned a connection between the solution and Borromean Rings and Brunnian Links, a formalisation that will be discussed further in Section 2.1.2 [2].

First, we will discusss the formalisation using Free Group Theory.

2.1.1 Connection to (Free) Group Theory

This section describes a general framework to study the wrapping of the puzzle.

2.1.2 Connection to Borromean and Brunnian Links

2.2 Research Gaps

3 Research Methodology

As is the case with pure mathematical research, the research is conducted analytically.

Firstly, an extensive literature review on the topic of picture-hanging puzzles is conducted to gain pre-requisite knowledge in the field of interest. This review also serves to build an understanding of particularly interesting or impactful open problems that are deserving of greater attention. Having obtained deeper intimacy with the topic, we then apply the theorems and results obtained within the papers reviewed to develop further theories in the field of picture-hanging puzzles that, for example, allow the spectator the most efficient algorithm to remove the fewest nails to fell the picture. We also draw upon the ideas used in proofs to guide our own thoughts.

In the attempt to solve an open problem, we will put forward various approaches and work through each avenue to see if any of them will crack open the problem.

3.1 Independent Parameters

3.2 Dependent Parameters

3.3 Scope of Investigation

4 Model Setup

4.1 Heading

5 Results

5.1 Overview of Results

6 Conclusions and Recommendations

6.1 Conclusions

6.2 Recommendations for Future Work

References

[1] A. Spivak, "Brainteasers b 201: Strange painting," *Quantum: The Magazine of Math and Science*, p. 13, May 1997, Brainteaser problem.

[2] T. Sillke, *Strange painting*, Online, http://www.mathematik.uni-bielefeld.de/~sillke/PUZZLES/quantum/B201, 2001. [Online]. Available: http://www.mathematik.uni-bielefeld.de/~sillke/PUZZLES/quantum/B201

[3] E. D. Demaine, M. L. Demaine, Y. N. Minsky, J. S. B. Mitchell, R. L. Rivest, and M. Pătraşcu, "Picture-hanging puzzles," *Theory of Computing Systems*, vol. 54, no. 4, pp. 531–550, Sep. 2013, ISSN: 1433-0490. DOI: 10.1007/s00224-013-9501-0 [Online]. Available: http://dx.doi.org/10.1007/s00224-013-9501-0

[4] Jade Tan-Holmes (Up and Atom) and Tom Scott, *How Knot To Hang A Painting*, Tom Scott Youtube Channel, Jan. 2019. Accessed: Jun. 30, 2025. [Online]. Available: https://youtu.be/-eVd2Ugk9BU?si=rC9jkem8syheqB1c

Appendix A - Heading

Appendix B - Heading