Machine Learning (HWS22)
Assignment 2: Logistic Regression

The archive provided to you contains this assignment description, a dataset in a binary format, as well as Python code fragments for you to complete. Comments and documentation in the code provide further information.

It suffices to fill out the "holes" that are marked in the code fragments provided to you, but feel free to modify the code to your liking. You need to stick with Python though.

Provide a single ZIP archive with name `ml22-<assignment number>-<your ILIAS login>.zip`. The archive needs to contain:

- A **single PDF report** that contains answers to the tasks specified in the assignment. Do not simply convert your Jupyter notebook to a PDF! Write a separate document, stay focused and brief. **Use at most 10 single-column pages.**
- All the **code** that you created and used in its original format.
- Optional: A PDF document that renders your Jupyter notebook with all figures. (If you don't use Jupyter, then you obviously do not need to provide this.)

You need to adhere to the above guidelines in your submission, otherwise we may grade your solution as a FAIL.

Generally, your report should

- include a high-level description of your approach and helpful figures,
- be self-explanatory (i.e., refer to code *only* for implementation-only tasks),
- follow standard scientific practice,
- include appropriate references if you used additional sources or material,
- not include any hand-written notes,
- label all figures/tables and refer to figures/tables via their labels,
- use one section per task and one subsection per subtask, each numbered with the (sub)task numbers from the assignment sheet.

Your report will be downgraded if you do not follow these points (e.g., you can't get EXCELLENT).

Hand-in your solution via ILIAS until the date specified there. This is a hard deadline.

Preliminaries: Spambase Dataset

In this assignment, we are using a dataset on email spam detection, provided in the file `data/spamData.mat`. You can find more information about this dataset at `https://archive.ics.uci.edu/ml/datasets/spambase`.

The dataset is derived from 4601 emails, each being labeled as no-spam (0) or spam (1). Each example has 57 features:

- 48 word features, each indicating the frequency percentage of a word in the email (e.g., "business", "free", "george").
- 6 character features, each indicating the frequency percentage of a character in the email (for `{[}!$#`).
- 3 features on length statistics of consecutive upper case letters (e.g., "HELLO" gives 5), one for minimum length, one for maximum length, and one for average length.

The dataset is split into a training set (3065 examples) and a test set (1536 examples). We provide code to load the data and provide all feature names.

1 Dataset Statistics

Explore and preprocess the dataset.

a) Look at the kernel density plot (code provided) of all features and discuss what you see (or don't see).

b) Normalize the data using z-scores, i.e., normalize each feature to mean 0 and variance 1. Normalize both training and test data. In particular, think about how test data should be normalized.

   Make sure to stick to the variable names provided in the code fragements. From now one, **we will exclusively work with the normalized data**.

c) Redo the kernel density plot on the normalized data. What changed? Is there anything that "sticks out"?

2 Maximum Likelihood Estimation

a) Show analytically that if we use a bias term, rescaling (multiply by constant) and shifting (add a constant) features leads to ML estimates with the same likelihood. Why do you think we computed z-scores then?

b) Complete the methods for computing the log-likelihood and gradient of the log-likelihood for logistic regression. We **do not use a bias term** throughout.

c) Implement gradient descent using the framework provided to you.

Optional. Can you implement each gradient descent epoch using only vectorized operations (no loops)?

d) Implement stochastic gradient descent.

e) Explore the behavior of both methods for the parameters provided to you. Discuss!

3 Prediction

Complete the `predict` and `classify` methods for the predicted spam probability and predicted class label, respectively. Explore the models that you fit in the previous task and discuss. Study the composition of the weight vector: which features are important, which are not? Is this intuitive?

4 Maximum Aposteriori Estimation

a) Implement gradient descent for MAP estimation of logistic regression with a Gaussian prior / L2 regularization with hyperparameter λ. You can reuse the methods of your solution for MLE.

b) Study the effect of the prior on the result by varying the value of λ. Consider at least the training data log-likelihood, the test data log-likelihood, and the prediction accuracy. Are these results surprising to you?

c) Study the composition of the weight vector for varying choices of λ (try very large values). Try to explain what you saw in the task above.

5 Optional: Exploration

Explore variants of preprocessing and logistic regression further. Suggestions include:

- Try gradient descent on the original data without using z-scores.
- Add a bias feature (make sure that you do not scale it).
- Try to reduce the training set size and compare MLE and MAP estimation (ideally using cross-validation).
- Run a logistic regression method from some existing library. Do you get the same results?
- Experiment with different gradient-based optimizers. To do so, we provide a PyTorch implementation, which allows you to quickly change the optimizer. For a list of optimizers, see here.