Recursive Causal Discovery with Julia

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Overview

1. Causality
2. Causal discovery
3. Recursive causal discovery
4. Recursive causal discovery with Julia
5. Future direction

What is causality?

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Causation is **NOT** correlation!

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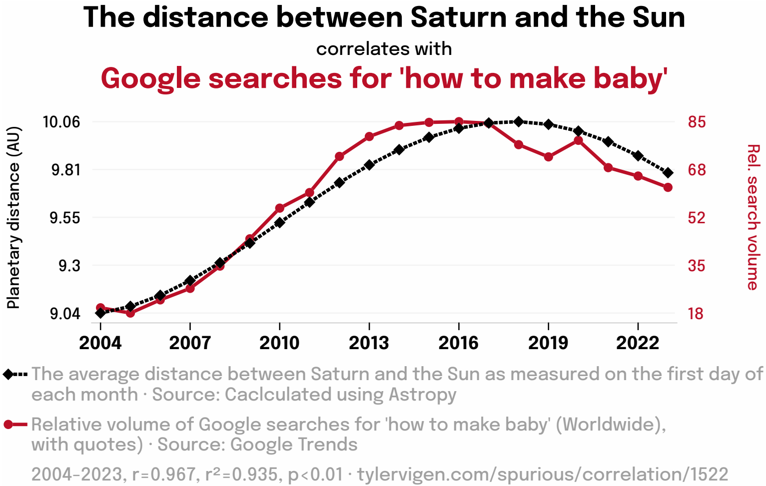

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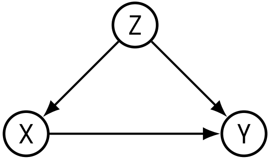

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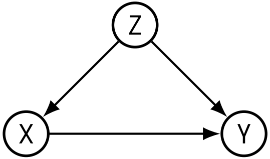

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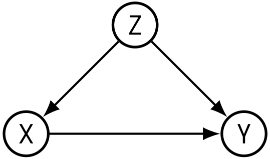

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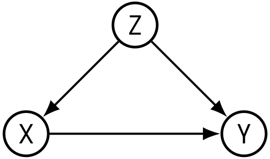

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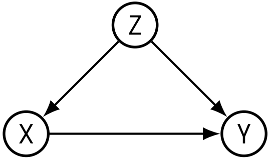

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- Policy making: Evaluating if tax incentives stimulate business growth.

Causal graphs

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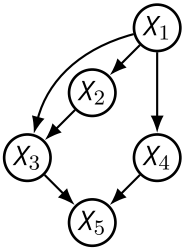

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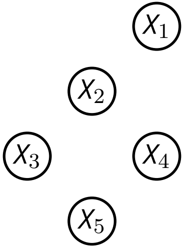

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What if we don't know the causal graph?

Causal discovery

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Common approach: PC algorithm

PC Algorithm [Spirtes et al., 2000]

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Problem with the PC Algorithm

PC has exponential complexity: PC requires potentially conditioning on every subset of variables: $O(n^2 2^n)$.

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RCD: example

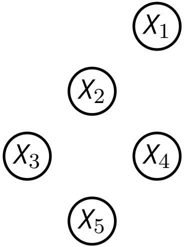

Figure: Remaining variables

Figure: Learned skeleton so far by RCD

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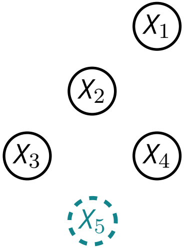

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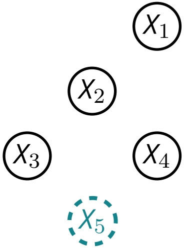

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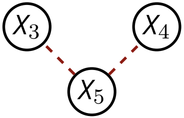

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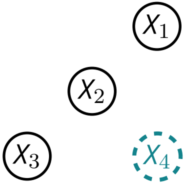

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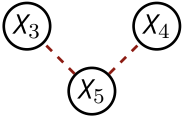

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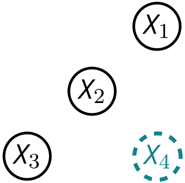

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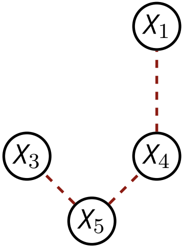

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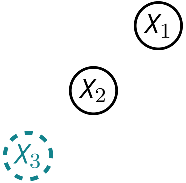

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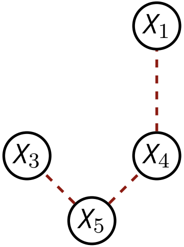

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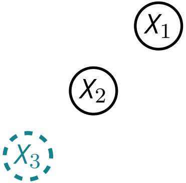

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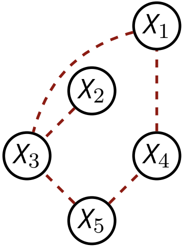

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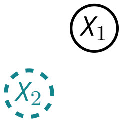

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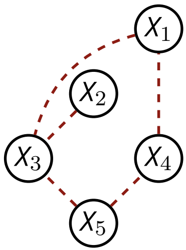

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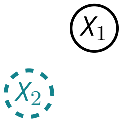

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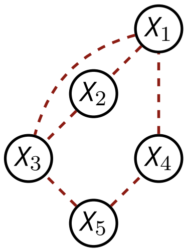

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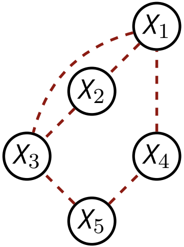

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RCD: algorithms

Algorithm	Completeness	#CI tests
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RSL	YES	$\mathcal{O}(n^2 + n\Delta_{in}^{m+1})$
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For more details, see our paper at go.epfl.ch/rcd.

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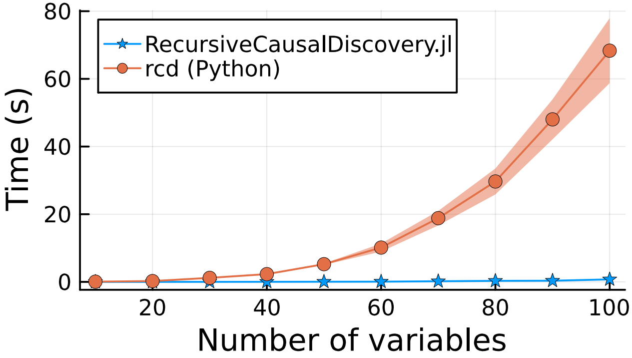

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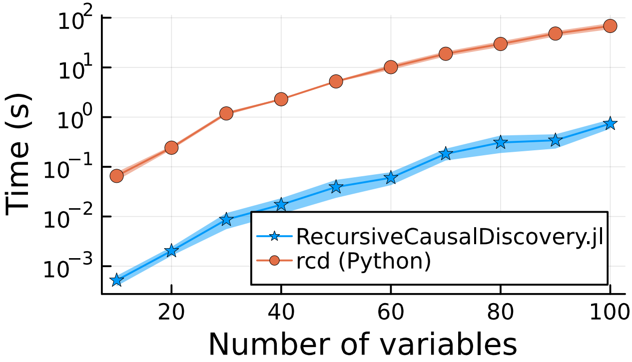

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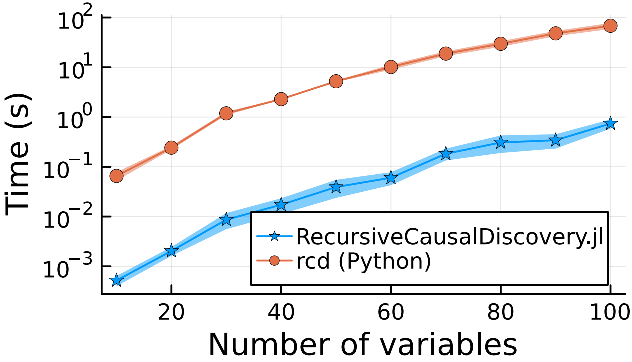

Julia is faster by a factor of 150!

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Installation:

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] add RecursiveCausalDiscovery
```

RCD in Julia

Simple demo:

```julia
using RecursiveCausalDiscovery
using CSV
using Tables

# load data (columns are variables and rows are samples)
data = CSV.read("data.csv", Tables.matrix)

# use a Gaussian conditional independence test
sig_level =  0.01
ci_test = (x, y, cond_vec, data) ->  fisher_z(x, y, cond_vec, data, sig_level)

# learn the skeleton of causal graph using RSL
learned_skeleton = learn_and_get_skeleton(data, ci_test)
```

RCD versus PC

Comparison: RCD vs. PC on learning graphs from synthetic data.

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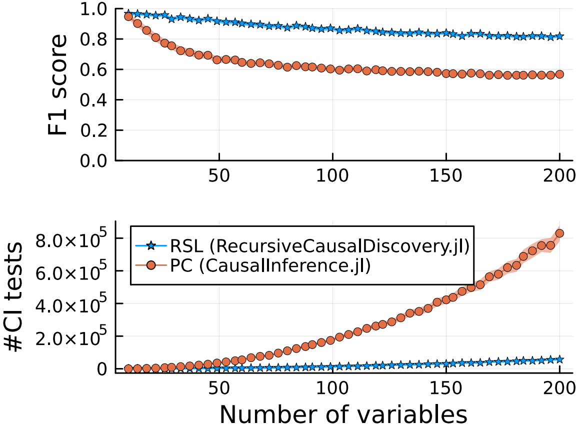

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