Exercise 1.3 (Learning From Data — Abu-Mostafa et al.)

Problem Rubric

For the perceptron update rule, each part should be answered by using the definition of misclassification, the weight update equation, and basic algebraic reasoning to analyze how the update affects the classification of the current training example.

The perceptron weight update rule is

$$w(t+1) = w(t) + y(t)x(t).$$

(a)

Since $x(t)$ is misclassified by $w(t)$, the sign of $w^T(t)x(t)$ is opposite to $y(t)$, and therefore

$$y(t)w^T(t)x(t) < 0.$$

(b)

Using the update rule,

$$y(t)w^T(t+1)x(t) = y(t)w^T(t)x(t) + x^T(t)x(t),$$

which implies

$$y(t)w^T(t+1)x(t) > y(t)w^T(t)x(t).$$

(c)

The update increases $y(t)w^T(t)x(t)$, moving the weight vector toward correctly classifying $x(t)$; hence, the update is a move in the right direction.