# Algebra, Geometry and Robots

## The two-way relationship between mathematics and robotics, with a focus on modern algebra and geometry.

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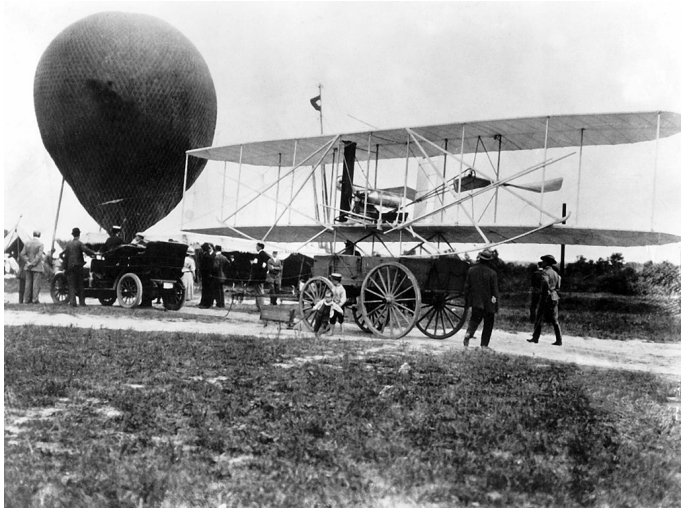
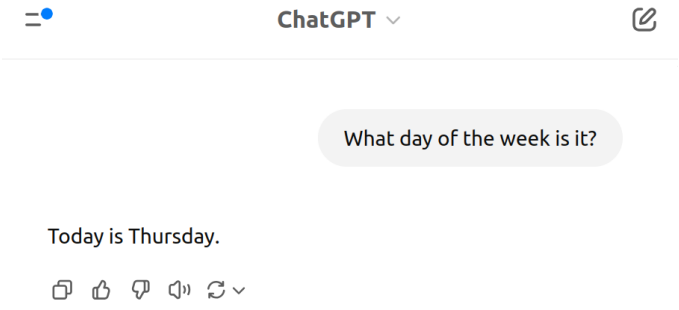
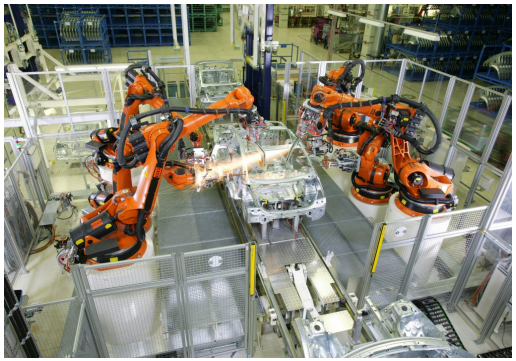
University of Sydney

# The Story of Humanity



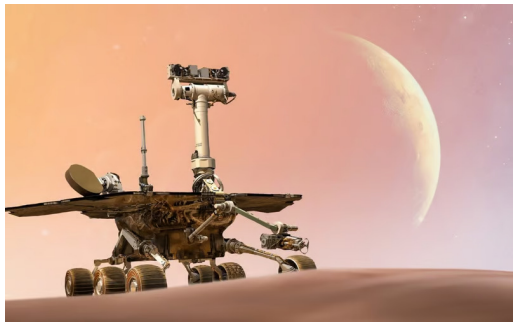


# Humanity's Next Tool: Robots

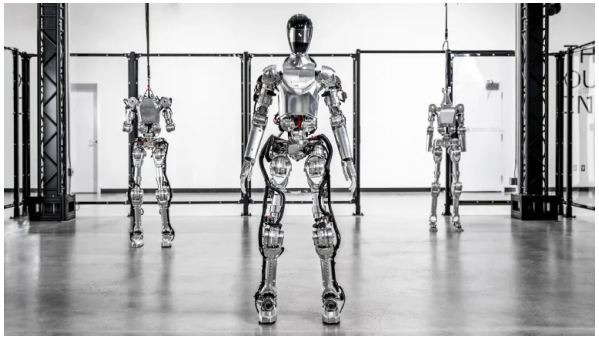


KUKA - Robotic Arms

Waymo - Self-Driving Cars

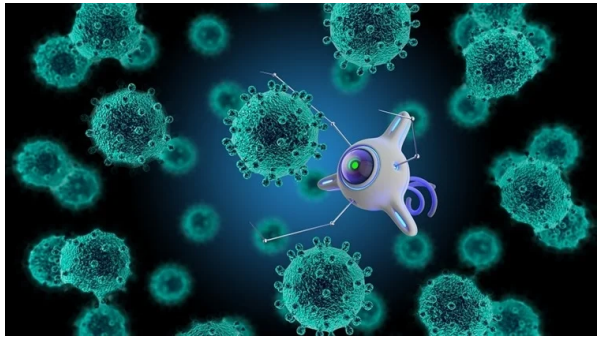


NASA - Martian Rovers



BMW - Humanoid Robots

Boston Dynamics - Dog Robots



Nanorobots

# What is Robotics?

## Common Definition of Robotics

$$R(t) = \text{Robotics at time } t$$

$$A(t) := \text{What's Theoretically Possible at time } t$$

$$P(t) := \text{What's Already Possible at time } t$$

$$R(t) := A(t) - P(t)$$

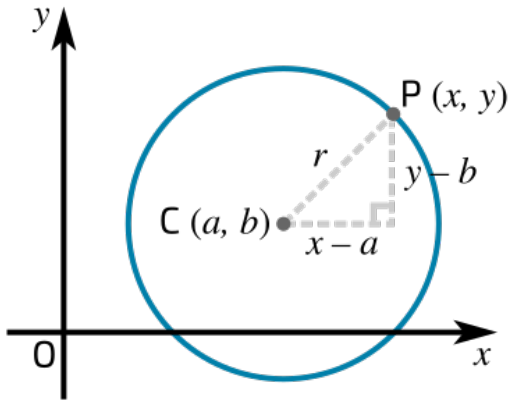
## Goal of Robotics

$$\lim_{t \to \infty} R(t) = 0$$

# The Story of Mathematics

$$1 + 2 = 3$$

$$ax^2 + bx + c = 0$$




$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$\mathrm{GL}_n(\mathbb{R})$$

$$\mathrm{Grp} \to \mathrm{Set}$$

# The Story of Mathematics: Today's Mathematics

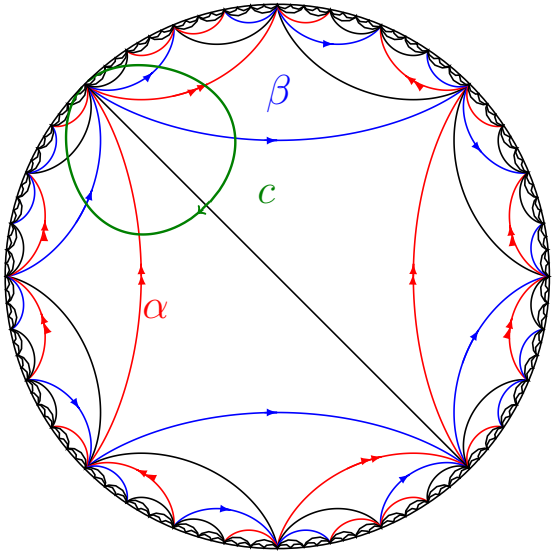
Algebra Analysis

Topology Geometry

# Overview


1. Humanity
2. Mathematics
3. Who am I?
4. Kempe's Universality Theorem
5. Gröbner Basis
6. Lie Theory
7. Other Applications

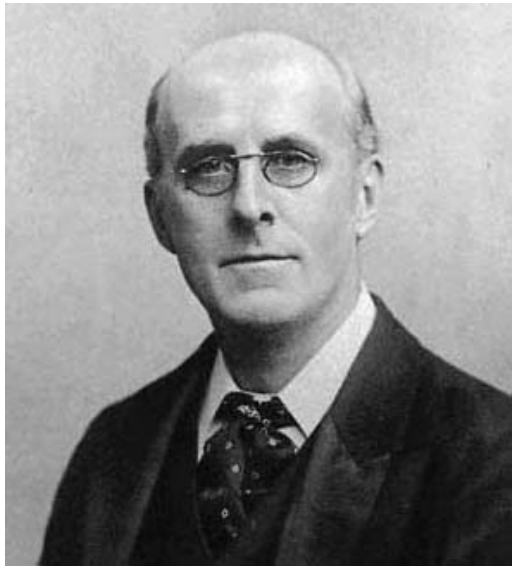
# Who am I?




Recent Master's in Mathematics (Geometric Topology)

Started PhD with ACFR and CSIRO's Robotic Perception and Autonomy Group

# Kempe - Steam Engines



Alfred Kempe
1849-1922

Steam Engine

# Kempe - Tracing Curves

https://www.youtube.com/watch?v=9NbwE4PMeyQ

# Kempe - Curves

## Curves in the Plane

$$P(x, y) = \sum_{ij} c_{ij} x^i y^j$$

A curve $\mathcal{C} \subseteq \mathbb{R}^2$ is defined as the set of solutions to the equation $P(x, y) = 0$.

# Gröbner Basis

# Lie Theory

# Other Applications of Mathematics