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## The game of life as a species model

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Conway's classic game of life is a two-dimensional cellular automaton in which each cell is alive or dead and evolves according to simple rules that depend solely on the number of live cells in its immediate neighborhood. The emergence of complex multi-cellular objects provides a fascinating vehicle for exploration. A variant of the classic game of life is presented, the generalized semi-classical game of life, in which each cell contains a qubit that evolves by repeated application of birth, death, and survival operators. Species are characterized by just two parameters: a preferred neighborhood liveness representing the tendency to herd and a resilience parameter representing species' vulnerability to environmental changes. This generalized model provides the opportunity to model the fortune of species and to compare to available data. The model is shown to mimic environmental catastrophes and is illustrated by the model's prediction of a return to the pre-hunting level of the global whale population by 2140. A student-designed predator-prey model is shown to qualitatively describe the fate of strongly- and weakly coupled predator-prey systems (snowshoe hare/lynx and rabbit/fox, respectively) and sudden and slow predatory impact (dodo and diprotodon, respectively). © 2023 Author(s). All article content, except where otherwise noted, is licensed under a Creative Commons Attribution (CC BY) license (<http://creativecommons.org/licenses/by/4.0/>).

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### I. INTRODUCTION

Conway's classic game of life was developed in the late 1960s and became the most popular cellular automaton in history.<sup>1-3</sup> Each cell of a two-dimensional matrix is either 1 (alive) or 0 (dead) and may change value at each time step according to a set of simple evolutionary rules that depend solely on the number of live cells in the eight cells of its immediate neighborhood, known as the Moore neighborhood.<sup>4</sup> The rules dictate for each cell whether a birth ( $0 \rightarrow 1$ ), death ( $1 \rightarrow 0$ ), or survival ( $0 \rightarrow 0$ ,  $1 \rightarrow 1$ ) event takes place.

Conway's rules are designed to mimic the evolution of a society of organisms in which fortunes are based on the local environment. If the number of live cells in the neighborhood is too high or too low, the cell dies due to overcrowding or loneliness, but flourishes if the surrounding liveness is optimum. Repeated application of the evolutionary rules to each cell at each time step (called a generation) can lead to an extraordinary myriad of complex patterns of live and dead cells.<sup>5,6</sup> Flammenkamp has catalogued 100 static and dynamic objects referred to as "ash," but which are more optimistically referred to as "lifeforms." These objects supply a fascinating example of the development of complexity through simple evolutionary rules.

Conway's original game of life has been the subject of many studies with most of the works published in prominent physics journals. Common themes include statistics and statistical mechanics,<sup>7,8</sup> nonlinear dynamics,<sup>9</sup> and critical behavior.<sup>8,10</sup> There have been numerous adaptations of Conway's

original game, including the introduction of a thermodynamic variable,<sup>7</sup> a three-dimensional version,<sup>11</sup> extension to Turing machines,<sup>12</sup> modification to a continuous space,<sup>13</sup> and probabilistic adaptations.<sup>14</sup>

Of interest here is the quantization of Conway's game introduced and developed by Flitney and Abbott where each live-or-dead cell is replaced by a qubit.<sup>15-17</sup> The underlying principles of Conway's original game are retained with a modified set of evolutionary rules in which qubits evolve via the application of birth, death, and survival operators. The game's creators termed the game "semi-quantum" because systems do not evolve time-reversibly. An educational illustration of the qubit is provided by López-Incera and Dür.<sup>18</sup>

The quantization of the game of life moves the game toward biological applications. The emerging discipline of quantum biology studies the evolution of quantum objects influenced by the local environment. General quantum aspects of life are discussed in Ref. 19. The semi-quantum game of life has a parallel in quantum biology where a qubit may represent an exciton produced by a photon in a protein complex during photosynthesis.<sup>20</sup> The exciton interacts with its environment to influence the energy transfer dynamics.<sup>21</sup>

A simplified version of the semi-quantum game of life has been introduced in which the cell qubit components are restricted to real numbers. This classical variant is denoted as the semi-classical game of life (SCGOL) to distinguish it from the semi-quantum game of life and Conway's classic game. Intriguing new phenomena were discovered during student project work, including chaotic effects and new

lifeforms composed of live cells, dead cells and part-live cells, plus objects that mimic seeds.<sup>22</sup>

Cellular automata, such as the SCGOL, are ideal for undergraduate projects that are designed to extend computational skills to matrix-based programming languages such as MATLAB.<sup>23</sup> Moore neighborhood calculations and application of boundary conditions are nontrivial problems that can be efficiently coded. Graphical presentation is well developed and quickly mastered by students. Once the fundamental rules are implemented, a wide range of different physical phenomena can be explored. Mathematical game-based cellular automata have been shown to be effective at educating students in the emergence of complexity<sup>24</sup> and logical thinking.<sup>25</sup> All MATLAB software used in this article is provided in the supplementary material.<sup>26</sup>

In the following, we generalize the SCGOL model to model the evolution of a species by choosing two flexibly chosen parameters: one representing the tendency of a species to herd and a resilience parameter representing vulnerability to environmental changes. Section II describes the generalized semi-classical game of life (gSCGOL) model and its evolutionary rules. Section III presents case studies of the catastrophic decline and subsequent recovery of a species, and examples of predator–prey behavior. We summarize our results in Sec. IV and conclude that gSCGOL provides exciting opportunities for original undergraduate research. Suggestions for new models are made in Sec. V.

## II. THE GENERALIZED SEMI-CLASSICAL GAME OF LIFE

The rules of the gSCGOL are based on a semi-quantum adaptation of Conway's game proposed by Flitney and Abbott.<sup>15</sup> Each cell of a two-dimensional grid is described by a normalized qubit  $|\psi\rangle$ , which is a superposition of alive  $|1\rangle$  and dead  $|0\rangle$  states such that

$$|\psi\rangle = a|1\rangle + b|0\rangle = a\begin{pmatrix} 1 \\ 0 \end{pmatrix} + b\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}, \quad (1)$$

where  $a$  and  $b$  represent the “liveness” or “deadness” of a cell, respectively. In the gSCGOL,  $a$  and  $b$  are both restricted to real numbers between 0 and 1. Thus, a cell may be “part live” or, in the spirit of Schrödinger's cat, a superposition of both dead and alive. In practice, the gSCGOL model merely converts the 0 or 1 dead-or-alive structure of Conway's classic game to one with a liveness density determined by the value of  $a$  at each cell. No claim to quantum-ness is made nor justified, even though the gSCGOL is underpinned by quantum principles.

The qubit at each cell location interacts with its Moore neighborhood liveness  $A$  and is modified according to a set of evolutionary rules. The Moore neighborhood liveness is defined as

$$A = \sum_{k=1}^8 a_k, \quad (2)$$

where  $a_k$  is the liveness of the  $k$ th cell in the Moore neighborhood surrounding a cell and periodic boundary conditions are used. The operators of birth ( $\hat{B}$ ), death ( $\hat{D}$ ), and survival ( $\hat{S}$ ) are given by<sup>15</sup>

$$\hat{B} = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \quad \hat{D} = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix} \quad \hat{S} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \quad (3)$$

Each cell evolves at each generation following the application of a cell-dependent generation operator  $\hat{G}$  according to the algorithm by Flitney and Abbott designed to mimic Conway's classic game.<sup>15</sup> The generation operator  $\hat{G}$  applicable to a cell is constructed from the birth, death, and survival operators, the Moore liveness  $A$  for the cell, and the two parameters,  $A_0$  and  $R$ . The rules that define  $\hat{G}$  are provided in Table I.

The gSCGOL is parameterized by  $A_0$  and  $R$ .  $A_0$  is the optimum, or the preferred Moore neighborhood. In Conway's game,  $A_0 = 3$  because a cell with exactly three neighbors becomes alive if dead and stays alive otherwise. In contrast, the gSCGOL allows any real number in the range  $1 \leq A_0 \leq 7$ . The resilience  $R$  defines when rule changes take place in terms of a change in  $A$  as seen in the first column of Table I. In Conway's integer-based games,  $R = 1$ , because the evolutionary rules may change at each integer value of  $A$  and each  $A$  differs by 1 unit. The gSCGOL accommodates any choice of  $R$  provided that  $A_0 - R \geq 0$  and  $A_0 + R \leq 8$ .

The next generation qubit  $|\psi'\rangle$  is obtained by applying the generation operation  $\hat{G}$  to each cell in the system

$$\begin{pmatrix} a' \\ b' \end{pmatrix} = \hat{G} \begin{pmatrix} a \\ b \end{pmatrix}. \quad (4)$$

The operation is not unitary and so the new state is normalized to ensure that  $|a'|^2 + |b'|^2 = 1$ . The system evolves by the repeated application of  $\hat{G}$  plus normalization to each cell at each generation. The extraordinary complexity of the various games of life is due to the interaction of each cell with its neighborhood at each generation.

## III. SPECIES MODELS

Games of life are so-called due to the emergence of aggregations of live cells that mimic multi-cellular organisms. These complex lifeforms arise from an apparent soup of live and dead cells purely by repeated application of simple rules. With one exception (the stationary block object shown in Fig. 2), these lifeforms are not of interest here – it is the rules which allow the gSCGOL to act as a novel species model that we exploit. Because the parameters  $A_0$  and  $R$  can take on many possible values, the gSCGOL provides many more opportunities for exploring species dynamics.

The optimum liveness  $A_0$  is the preferred Moore liveness and is a measure of the tendency of a species to herd, or equivalently, its sociability or gregariousness. Herding animals possess a large  $A_0$  and solitary creatures, such as bumble bees and snow leopards, a small value. The resilience parameter  $R$  represents a measure of the sensitivity of the species to deviations of Moore liveness from its optimum.

Table I. The  $\hat{G}$  operator for the gSCGOL game is defined for different ranges of the Moore liveness  $A$ . The optimum Moore liveness is  $A_0$ , and the resilience parameter is  $R$ .

| $A$                         | $\hat{G}$  |
|-----------------------------|--|
| $A \leq A_0 - 2R$           | $\hat{D}$  |
| $A_0 - 2R < A \leq A_0 - R$ | $(\sqrt{2} + 1)[(A_0 - R) - A]\hat{D} + [A - (A_0 - 2R)]\hat{S}$ |
| $A_0 - R < A \leq A_0$      | $(\sqrt{2} + 1)[A_0 - A]\hat{S} + [A - (A_0 - R)]\hat{B}$        |
| $A_0 < A \leq A_0 + R$      | $(\sqrt{2} + 1)[(A_0 + R) - A]\hat{B} + [A - A_0]\hat{D}$        |
| $A \geq A_0 + R$            | $\hat{D}$  |

A species such as coral, which is highly sensitive to sea temperature, has a small value of  $R$ , whereas a more robust species, such as ants, have a large value of  $R$ . The gSCGOL characterizes a species by the values of the parameter pair  $(A_0, R)$ .

A simulation proceeds by first defining a species by choosing the values of  $(A_0, R)$ . Qubits with random liveness  $a$  and normalized to unity are assigned to each cell of a two-dimensional grid. The gSCGOL rules presented in Table I are applied, and the mean liveness  $\langle a \rangle$  of the system is recorded. The process is repeated until  $\langle a \rangle$  fluctuates about a mean value indicating a steady state liveness has been achieved. This step is akin to achieving energy equilibrium in molecular dynamics. The system with the parameter pair  $(A_0, R)$  is now ready for the simulation proper to start.

The environment can be controlled through the resilience  $R$  and deteriorating environmental conditions can be modeled by decreasing  $R$ . Examples are presented in Sec. III A. Alternatively, two species can interact by first characterizing each species by their values of  $(A_0, R)$  and then introducing predation rules. The two systems run in parallel through the execution of the gSCGOL simulation while simultaneously interacting via the predation rules. Examples are presented in Sec. III B.

### A. An environmental catastrophe model

More than 37,000 species are threatened with extinction.<sup>27</sup> The biggest driving force of species extinction is human activity, primarily through loss of habitat,<sup>28</sup> pollution, climate change,<sup>29,30</sup> and unsustainable hunting and harvesting. The second most significant cause is due to invasive species, leading to predation or the introduction of infectious diseases.<sup>31–33</sup>

The decline of a species is modeled by a reduction of the resilience. The optimum liveness  $A_0$  remains constant and the resilience  $R$  is reduced at each generation. The mean liveness  $\langle a \rangle$  of the system is also recorded at each generation.

The results for the mean liveness for the parameter sets  $(3.0, R)$  and  $(2.5, R)$  are shown in Fig. 1. As anticipated,  $\langle a \rangle$  declines steadily as the resilience  $R$  drops from  $R = 2.0$  until a tipping point is reached whereupon the mean liveness drops sharply toward zero. Liveness density maps are displayed for systems with  $R = 0.93$  and  $R = 2.0$  for  $A_0 = 3.0$ .

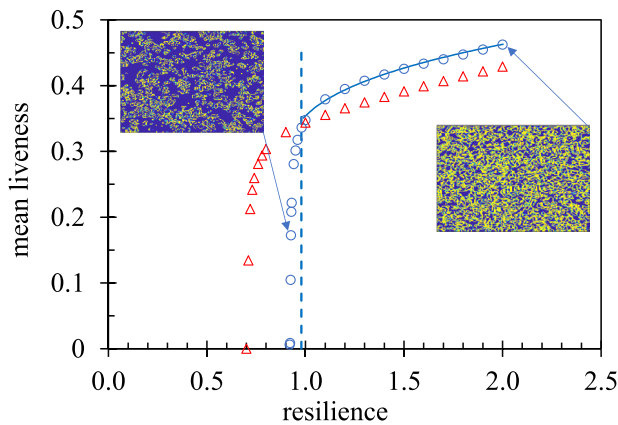


Fig. 1. The mean liveness for systems with optimum Moore neighborhood  $A_0 = 3.0$  (blue circles) and  $A_0 = 2.5$  (red triangles) is presented as a function of the resilience  $R$ . The solid line is fit using Eq. (5) as described in the text. The tipping point is indicated by the dashed line. The insets show population densities for the data points shown.

The sudden fall in liveness associated with a small reduction in  $R$  is characteristic of catastrophe theories. Catastrophe theory was developed by Thom in the 1960s, but came into vogue in the 1970s when applied to the biological and social sciences.<sup>34–36</sup> The critique by Sussman and Zahler provides the most accessible description of catastrophe theory and its applications.<sup>35</sup> Catastrophe theory predicts that<sup>35</sup>

$$x^3 - x = \alpha. \quad (5)$$

Equation (5) was solved for  $x$  (representing the mean liveness) for  $\alpha$  (representing resilience) ranging from  $-0.21$  to  $-0.09$  in steps of  $0.001$  to establish the extent to which the gSCGOL modelling matches the predictions of the purely mathematical description of an extinction process. The curve was scaled and offset to match the gSCGOL data for  $A_0 = 3.0$ . The result is shown by the solid line for  $R \geq 1.0$  in Fig. 1. The decline in mean liveness as the resilience is reduced is matched by Eq. (5) but only until  $R \approx 1$  below which Eq. (5) offers no solution.

The dashed line represents the “catastrophe limit” or “tipping point” at which point the catastrophe theory predicts  $\langle a \rangle$  drops instantaneously to zero. The purpose of the comparison is to illustrate the differences in predictions between catastrophe theory and the gSCGOL model for  $R \leq 1.0$ . For  $R \leq 1.0$ , the gSCGOL decline is steep but not infinite and  $\langle a \rangle$  does not drop to zero. Both differences are important in the analysis that follows and illustrates that the gSCGOL offers a more realistic description of the decline of species than does catastrophe theory.

We now explore the ability of a species to recover from severe population decline. The premise is that once the mean liveness  $\langle a \rangle$  drops below a pre-chosen threshold, humans will act to restore the environment to its original state or environmental conditions naturally improve. The system is first subject to a deteriorating environment modeled by incrementally reducing the resilience  $R$ . Then, once the pre-chosen threshold liveness density has been breached,  $R$  rises incrementally.

Figure 2 presents the mean liveness as a function of generation displayed in years for three systems, each with the parameter set  $(3.0, 1.0)$ . A constant mean liveness  $\langle a \rangle$  is first

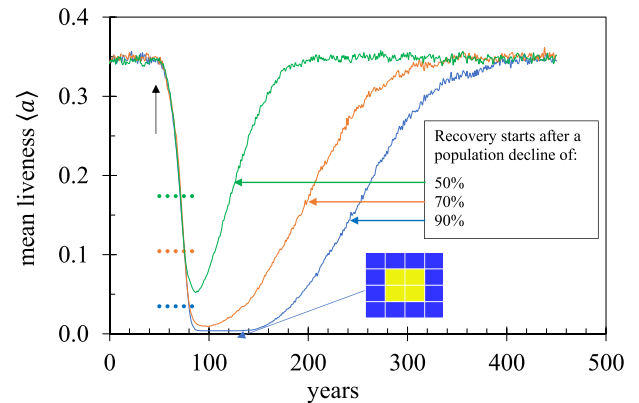


Fig. 2. The solid lines display three simulations with the mean liveness presented as a function of generation (year) at the optimum liveness  $A_0 = 3.0$ . The resilience  $R = 1.0$  for the first 50 yr, after which  $R$  is reduced by  $0.01$  per annum (indicated by the arrow).  $R$  is then increased by  $0.01$  per annum after  $\langle a \rangle$  has dropped by 50% (green), 70% (orange), and 90% (blue) as indicated by dotted lines. The inset shows a “block” object (see the text).



established. After 50 yr (marked by the arrow), the resilience is decreased by 0.01 a year reflecting a gradual deterioration in environmental conditions. At this stage, the population density declines identically in each system. In one system, the resilience decline is reversed once the population density has declined by 50%. The other two systems follow suit once  $\langle a \rangle$  has dropped by 70% and by 90%, respectively.

The horizontal dotted lines indicate the population declines of 50%, 70%, and 90%. These act as trigger points for the improvement of environmental conditions and a reversal of the decline in  $R$ . At these points,  $R$  increases at the same rate of 0.01 each year. All populations are found to continue to decline after the trigger point when the environmental conditions improve. The drop in  $R$  of 0.01 per year is too rapid to allow equilibrium liveness to be attained in a single year, and it takes time for populations to recover. Encouragingly, all populations eventually recover.

We first apply the gSCGOL model to qualitatively explore the fortune of whales. Whales have been a human resource for millennia, but whaling started to increase in the 18th century and accelerated significantly in the 19th century with the development of harpoon technology.<sup>37</sup> The influence of American whaling spread worldwide and became a multi-million-dollar industry by the beginning of the 20th century. It is thought by some scientists “that more whales were hunted in the early 1900s than in the previous four centuries combined.”<sup>37</sup> Despite the reduction in whaling after its peak in the early 1900s, the number of whales in the ocean continued to decline. In 1946 the International Whaling Commission was established to oversee whaling quotas, but whale numbers continued to decline. In 1982, the International Whaling Commission called for a moratorium on commercial whaling, and the whale population started to recover.

The results of a gSCGOL model designed to mimic the population of whales in response to human predation are shown in Fig. 3. The annual decline commences in 1760 with a reduction in resilience of 0.001 per annum. The peak of the slaughter is presumed to occur in about 1900 when the model whale population had declined by 70%. This decline acts as the trigger point for an improvement in resilience reflecting reductions in hunting. From 1900, the resilience is increased by 0.001 per annum. The whale population continues to decline and reaches a minimum at about 1960 reflecting a model reduction of more than 90% from the pre-1700 norm.

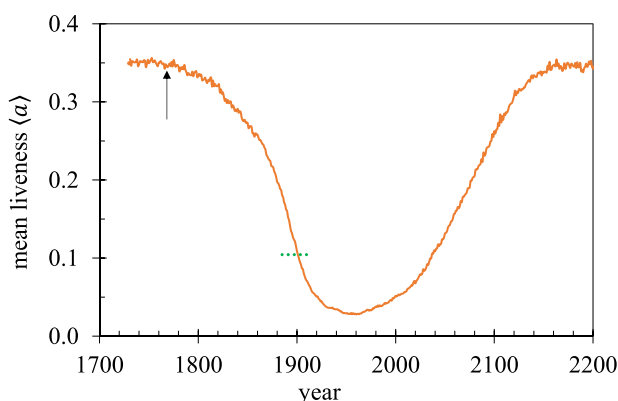


Fig. 3. A gSCGOL model simulation of the whale population in response to human predation with  $A_0 = 3.0$  and resilience  $R = 1.0$ . In 1760 (arrow),  $R$  is reduced by 0.001 per annum.  $R$  is increased by 0.001 per annum after the population declined by 70% (dotted line).

After 1960, the whale population starts to recover, and by 2020, the population is approximately 2.5 times its 1960 low, but is still 80% below the pre-1700 level. The model predicts that the whale population will fully recover by about 2140.

Of course, the decline and recovery of the whale population cannot reliably be quantitatively matched by a model that establishes a smooth decline of the species resilience followed by an equally smooth recovery. In addition, reliable historical population data for the actual worldwide whale population are not available. Nonetheless, there are some striking qualitative similarities between the limited data available and the general features of the gSCGOL model presented in Fig. 3.

We now return to Figs. 1 and 2 to explore extinction. In Fig. 1, the mean liveness with  $A_0 = 2.5$  drops to zero as the resilience  $R$  is reduced and the species becomes extinct. In this case the gSCGOL has mimicked the fate of the sabretooth tiger, the woolly mammoth, the dodo, the Tasmanian tiger, and countless other extinct species. In contrast, for  $A_0 = 3.0$  liveness does not decline to zero as can be seen in Figs. 1 and 2.

An object (a local persistent configuration) is created that is resistant to reduction in resilience. This object is illustrated in the inset of Fig. 2 and comprises four fully live cells (liveness  $a = 1$ ). This object appears in Conway's classic game as the second most common lifeform called the “block.”<sup>6</sup> In the gSCGOL, the surrounding cells are nearly, but not quite, dead. Their liveness is small such that each live cell has a Moore neighborhood fractionally more than 3 such that the evolutionary rules in Table I enable survival to the next generation.

The liveness of the cells surrounding the block approaches zero as the resilience is reduced. The local environment improves as  $R$  increases, and the liveness of the surrounding cells increases and eventually the block evolves into new objects and the liveness of the system increases. The block object therefore acts as a seed. It is able to lie dormant until the environmental conditions change so that the seed can evolve. The concept of a seed was introduced in Ref. 22 in the context of a SCGOL with a lifeform denoted by “qutub” containing part-live cells. The qutub remains dormant until certain conditions are met, and then evolves when the conditions are breached. An analogy is with plant seeds, but certain specialized animal species may exhibit a sudden blossoming of life after a dramatic environment-improving event such as rainfall.<sup>38,39</sup>

In the context of the gSCGOL species model, the appearance of the block object whose liveness deteriorates slowly as environmental conditions worsen reflects a population that has plummeted but has not quite become extinct. A few surviving lifeforms remain. These remnants are able to increase their numbers once the environment improves. There are numerous examples of the rediscovery of species thought to be extinct. Two or three species per annum previously considered extinct have been found,<sup>40</sup> most recently the Madagascan dusky tetraka.<sup>41</sup> Their numbers fall so low they are difficult to find.

## B. Predator–prey modelling

A predator–prey relationship is a set of complex interactions between a predator and a prey influenced by effects such as climate, habitat, and competing predators. The general features of the outcome of such interactions may be

modeled using the gSCGOL. The starting point is the strong predator–prey interdependence illustrated by the lynx and the snowshoe hare. In this case the predator exhibits a strong reliance on a specific prey for its food, and suffers if its food source is diminished. The fortunes of the predator and prey with less impactful interactions are then explored and illustrated with examples.

The gSCGOL model starts with two independent species characterized by the parameter sets  $(A_0, R) = (4.0, 1.50)$  for the prey and  $(1.5, 0.49)$  for the predator. Simulations are initially executed with no interaction between the species to establish the steady state liveness of each species. The mean liveness is  $\langle a \rangle_{\text{eq,prey}} = 0.483$  for the prey in the absence of predation and  $\langle a \rangle_{\text{eq,pred}} = 0.202$  for the predator with an abundance of food; the subscript denotes the steady state independent liveness density.

After 20 generations, the predator–prey interactions are switched on. The resilience of a species is a measure of its vulnerability to changes to its environment and the predator–prey interaction changes the resilience parameters  $R_{i,\text{prey}}$  and  $R_{i,\text{pred}}$  at each generation  $i$ . A generation does not necessarily correspond to a single year. We adopt the relations

$$R_{i,\text{prey}} = R_{\min,\text{prey}} + f_{i,\text{pred}}(R_{\text{prey}} - R_{\min,\text{prey}}) \quad (6)$$

$$f_{i,\text{pred}} = 1 - \frac{\langle a \rangle_{i,\text{pred}}}{\langle a \rangle_{\text{eq,pred}}} \quad (7)$$

$$R_{i,\text{pred}} = R_{\text{pred}} - f_{i,\text{prey}}(R_{\text{pred}} - R_{\min,\text{pred}}) \quad (8)$$

$$f_{i,\text{prey}} = 1 - \frac{\langle a \rangle_{i,\text{prey}}}{\langle a \rangle_{\text{eq,prey}}} \quad (9)$$

These relations recognize that the resilience of the prey diminishes as the predator liveness increases and the resilience of predators falls if there is less food. The new parameters  $R_{\min,\text{prey}}$  and  $R_{\min,\text{pred}}$  describe the strength of the interaction between the two species. All the simulation results presented in Fig. 4 are for  $R_{\min,\text{pred}} = 0.39$ , which is the minimum value of the resilience that can possibly lead to the survival of an isolated predator system with optimum liveness  $A_0 = 1.5$ . Changes of the single parameter  $R_{\min,\text{prey}}$  therefore allows outcomes due to different interaction strengths and types to be explored.

Figure 4 displays four sets of results. Two sets show both the predator and prey population densities as a function of generation. The first pair is labeled as lynx and hare and provides a classic predator–prey outcome with strong interaction. The lynx relies heavily on the snowshoe hare for its nutritional needs.<sup>42</sup> The gSCGOL is executed with  $R_{\min,\text{prey}} = 1.15$  with the interaction activated at generation 20. The population density of the prey drops rapidly. The resilience of the predator then declines according to Eq. (9) due to the scarcity of food, which in turn provides an opportunity for the prey to recover. Unlike idealized predator–prey distributions modeled by coupled differential equations, the magnitude of oscillations diminishes significantly after the first fall and rise. The first predator peak occurs 67 generations after the first peak of the prey population. After the first oscillations, fluctuations are clearly visible in both populations beyond 700 generations and a correlation analysis confirms predator peaks continue to be separated by about 70 generations. The distribution of prey liveness is uniform at

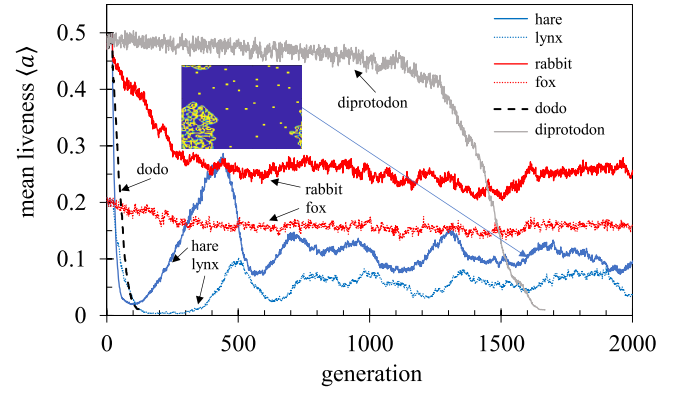


Fig. 4. The mean liveness as a function of generation for four predator–prey systems. The hare/lynx and rabbit/fox pairs represent strong and weak interactions, respectively. The dodo (rapid decline) and diprotodon (slow decline) represent gSCGOL models of species that have become extinct at least partially due to predation. The inset is the model representation of a community of snowshoe hares.

the start of the simulation, but the reduced population density is not uniform. The prey clusters with empty space between them is shown by the single cluster in the snapshot in Fig. 4. This spatial configuration arises to ensure that the prey achieves the optimum Moore neighborhood liveness despite a loss of numbers. We see that the gSCGOL model mimics nature where populations gather in regions with plentiful food resources.

In Canada, the snowshoe hare population cycles approximately every 10 yr with dramatic changes in population.<sup>43</sup> This behavior is only partially reproduced by the gSCGOL model where peaks and troughs may differ by a factor of three at most. A gSCGOL cycle repeats every 300 generations corresponding to 10 yr for the real lynx–hare system. The gSCGOL predator cycle lags by about 70 generations, or two years, in agreement with observation.<sup>43</sup> Additional factors contribute to the rapid fluctuations of the snowshoe hare population. The habitat cannot support high hare densities and so starvation and increased predation exacerbate population change. These factors could be included in a modified gSCGOL model.

The second example represents the fox, a more diverse predator, and the rabbit, its primary prey. Here the predator is omnivorous and has a broad diet including rodents, birds, and frogs as well as berries and scavenging. The weaker predator–prey interaction is executed with  $R_{\min,\text{prey}} = 1.36$ . The rabbit population declines without oscillations to about 50% of its predator-free equilibrium. The fox population drops by just over 20%.

The third (dodo) and fourth (diprotodon) systems in Fig. 4 show the fortunes of a prey in circumstances where the predator–prey interaction is one-way. In each case, the predator is unaffected by changes in the liveness density of the prey so that the resilience of the predator  $R_{i,\text{pred}} = R_{\text{pred}}$  is unchanged at each generation. The dodo is modeled by an instantaneous reduction of the resilience from 1.5 to 1.0. The slower demise of the diprotodon is modeled by the reduction of resilience of 0.0001 per generation.

The dodo was a large flightless bird found only on Mauritius in the Indian Ocean. It is believed that it became flightless due to the abundance of resources and lack of predators on the island. The first recorded account of the dodo

was in 1598 by Dutch sailors who established a small island trading settlement. There is scant evidence of impactful human predation, but the introduction of non-native species, especially pigs and crab-eating macaques, led to the destruction of the ground nests and competition for food.<sup>44</sup> Dodo sightings were rare by the 1660s and the best estimate of its extinction is 1693.<sup>45</sup> The dodo is modeled by a single instantaneous drop in resilience to 1.29 in Fig. 4. Assuming one generation corresponds to one year, the dodo population drops by 90% after 60 yr and is extinct within 99 yr, matching the observations well.

The fourth example cites the diprotodon. The diprotodon was a large marsupial native of Australia and one of many that became extinct about 46,000 yr ago.<sup>46</sup> The decline of large mammals in Australia over a period of a few thousand years is striking: “All Australian land mammals, reptiles, and birds weighing more than 100 kg, and six of the seven genera with a body mass of 45–100 kg, perished in the late Quaternary.”<sup>46</sup> The cause of the Australian megafauna extinctions could be due to climatic change, human predation, and/or loss of habitat due to human activity.<sup>46–48</sup> Because many of these species survived for millions of years and numerous ice ages before the arrival of humans in Australia, it is tempting to point the finger at humans. A kill every few months could lead to long-term population declines for animals with a slow rate of reproduction.

The gSCGOL model for the diprotodon decrements  $R_{i,\text{prey}}$  by 0.0001 at each generation. The result is presented in Fig. 4. The population decline is slow, dropping by just 15% after 1200 yr. Thereafter, the rate of population decline increases mirroring the catastrophe curves seen in Fig. 1. In this model, the diprotodon survives little more than 1500 yr. This example highlights the difficulties of conservation. The adverse impact of humans on a species may not be apparent until its tipping point is reached.

#### IV. SUMMARY

Cellular automata are widely used in undergraduate physics projects and in courses designed to develop students’ skills, especially in matrix-based programming languages, data management, and graphical presentation. Fascinating complex phenomena can be explored on the basis of simple algorithmic rules allowing individualized research projects in species ecology. Skills developed include learning a programming language, data management using matrices, application of boundary conditions widely used in simulations, and data interpretation and analysis in the context of often very limited literature data. In addition, the gSCGOL model increases student understanding of qubits and operators as taught in quantum mechanics courses.

We used the gSCGOL to describe a species through just two parameters: the preferred neighborhood liveness  $A_0$  describing the tendency of a species to herd and a resilience parameter  $R$  representing sensitivity to changes in the local environment. The gSCGOL model is shown to mimic environmental catastrophes in which a species becomes extinct but, more optimistically, also demonstrates that a population can survive in pockets and recover if the local environment improves. The decline and recovery of the global whale population was modeled using limited qualitative population data as a guide and predicted a return to pre-hunting whale populations by about year 2140.

Predator–prey modeling requires two interacting species each with distinct  $(A_0, R)$  values. Equations of interaction were given, and the results were presented for the snowshoe hare–lynx and rabbit–fox systems. The dodo and diprotodon were modeled in the context of systems in which the resilience of the predator is unaffected by the presence or absence of a specific prey. In all cases, qualitative agreement with limited data is obtained.

The Matlab files supplied in the supplementary material<sup>26</sup> may be used as a basis to tackle some of the suggested problems below.

#### V. PROBLEMS

The gSCGOL requires two primary parameters: the preferred neighborhood liveness  $A_0$  and the resilience  $R$ . A herding species typically has  $A_0 \geq 4$  and the gSCGOL will then yield population distributions that mimic herds. A resilience  $R \approx 1$  is a good starting value. Suitable changes in  $R$  at each time step are required for some studies and are guided by species data. Normally, a time step corresponds to one year (representing the natural breeding cycle) but corals, for instance, deteriorate and recover at a much faster rate. Example values for predator–prey models may be found in Sec. III B.

Problem 1. The gSCGOL opens up numerous possibilities for undergraduate projects where students can study the demise and recovery of a species of their choice. Many species have more reliable data available than the whale population studied in Sec. III A. Changes in the Indian elephant population due to forest defragmentation<sup>49,50</sup> and loss of African elephant populations due to the ivory trade<sup>51</sup> are examples.

Problem 2. Corals are worthy of exploration because hazards faced are many and varied. Kayal *et al.*<sup>52</sup> provided an excellent summary of hazards with sources. The Palmyra atoll was newsworthy in July 2022 because of a rapid recovery from bleaching as reported by Khen *et al.*<sup>53</sup> Hughes *et al.* presented the relation between global warming and bleaching in Ref. 54.

Problem 3. The woolly mammoth constitutes an obvious extinction study, but useful quantitative data are lacking. A useful starting point is Ref. 55, which discusses the demise of the woolly mammoth due to climate change and human impact.

Problem 4. Numerous near-extinct species are slowly recovering, and many others are in the process of decline. Examples for study exploiting the properties of the block object discussed in Sec. III A include the Lord Howe Island stick-insect, which was considered extinct until a climber discovered fresh remains in 1960 and was rediscovered in 2001 on a cliff,<sup>56</sup> and a presumed extinct nocturnal wolf spider spotted on a United Kingdom military base for the first time in 27 yr.<sup>57</sup>

Problem 5. More optimistically, the block object discussed in Sec. III A can be exploited as a “seed” to model species reintroductions. The dramatic success of the red kite in the United Kingdom is a prime example of a successful reintroduction with available data.<sup>58</sup>

Problem 6. A related and interesting predator–prey study on red kites and rabbits is possible using data on red kite populations located within high density rabbit areas in Spain.<sup>59</sup>



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This work is dedicated to John Horton Conway, the creator of the Game of Life, who made many contributions to mathematics and who sadly died after contracting COVID-19 in April 2020.<sup>60</sup>

## AUTHOR DECLARATIONS

### Conflict of Interest

The authors have no conflicts of interest to disclose.

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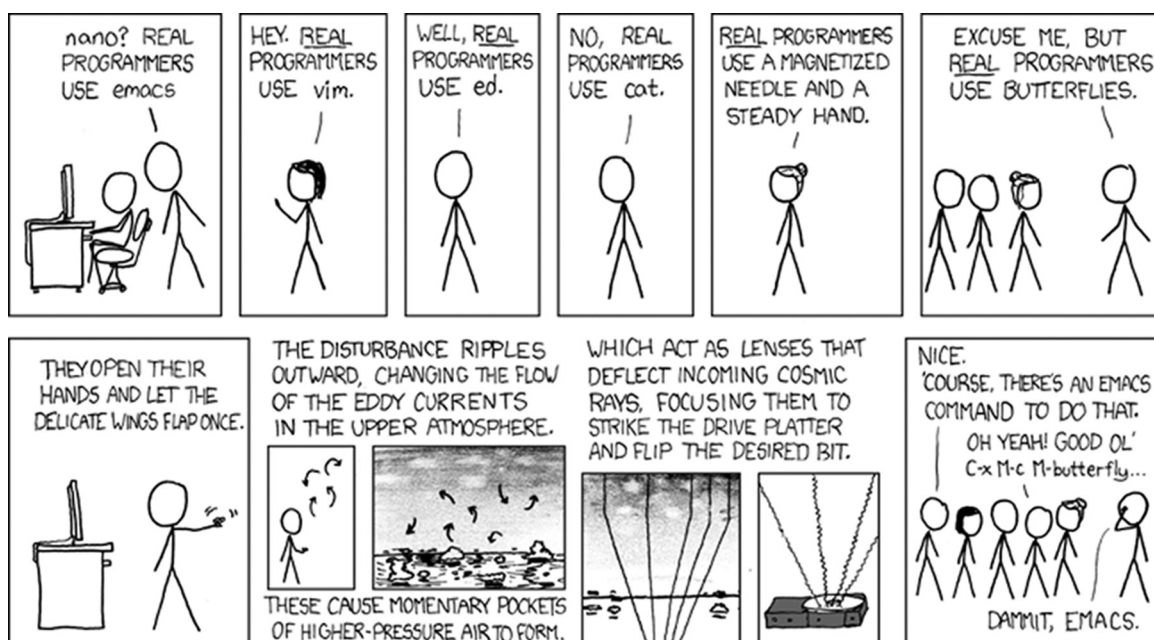
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Real programmers set the universal constants at the start such that the universe evolves to contain the disk with the data they want. (Source: <https://xkcd.com/378>)

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