Stock Market Fluctuations Model – Geometric Brownian Motion (GBM)

1. Model Overview

The Geometric Brownian Motion (GBM) is widely used to model stock price dynamics
It treats stock prices as a continuous-time stochastic process.

Let $S(t)$ denote the stock price at time t.

2. Stochastic Differential Equation (SDE)

The GBM is defined by the SDE:

$$dS(t) = \mu S(t) dt + \sigma S(t) dW(t)$$

Where:
- μ is the expected return (drift)
- σ is the volatility (diffusion)
- $W(t)$ is a standard Wiener process (Brownian motion)

3. Solution of the SDE

The explicit solution to the SDE with initial price S_0 is:

$$S(t) = S_0 \exp\left(\left(\mu - \frac{\sigma^2}{2} \right) t + \sigma W(t$$

This implies the logarithm of stock prices follows a Brownian motion with drift.

4. Interpretation

- The drift term μ represents the average rate of return.
- The volatility term σ models random fluctuations around the drift.
- Stock prices modeled by GBM have a log-normal distribution.

5. Applications

- Pricing options using the Black-Scholes formula.
- Simulating future stock price paths.
- Risk assessment and portfolio management.