Servo Motor Control: Mathematical Modeling and Simulation

1. System Description

A DC servo motor converts electrical input voltage into mechanical rotation.
The system includes electrical and mechanical components:

Variables and Parameters:

- $\theta(t)$: Angular position (rad)
- $\omega(t)$: Angular velocity (rad/s) = $d\theta/dt$
- $i(t)$: Armature current (A)
- $V(t)$: Input voltage (V)

- J : Rotor moment of inertia ($kg \cdot m^2$)
- b : Viscous friction coefficient ($N \cdot m \cdot s$)
- K_t : Motor torque constant ($N \cdot m/A$)
- K_e : Back EMF constant ($V \cdot s/rad$)
- R : Armature resistance (Ω)
- L : Armature inductance (H)

2. Electrical Dynamics (Armature Circuit)

The armature voltage equation (Kirchhoff's voltage law):

$$L \frac{di}{dt} + R i(t) + K_e \frac{d\theta}{dt} = V(t)$$

3. Mechanical Dynamics (Rotational Motion)

Newton's second law for rotation:

$$J \frac{d^2\theta}{dt^2} + b \frac{d\theta}{dt} = K_t i(t)$$

4. State-space Representation

Define state vector:

$$x = \begin{bmatrix} \theta \\ \omega \\ i \end{bmatrix} = \begin{bmatrix} \theta \\ \dot{\theta} \\ i \end{bmatrix}$$

The system dynamics:

$$\frac{d}{dt} \begin{bmatrix} \theta \\ \omega \\ i \end{bmatrix} = \begin{bmatrix} \omega \\ -\frac{b}{J} \omega + \frac{K_t}{J} i \\ -\frac{K_e}{L} \omega - \frac{R}{L} i + \frac{1}{L} V(t) \end{bmatrix}$$

5. Input Signal

Step input voltage to command motor movement:

$$V(t) = \begin{cases} 0, & t < 0 \\ V_0, & t \geq 0 \end{cases}$$