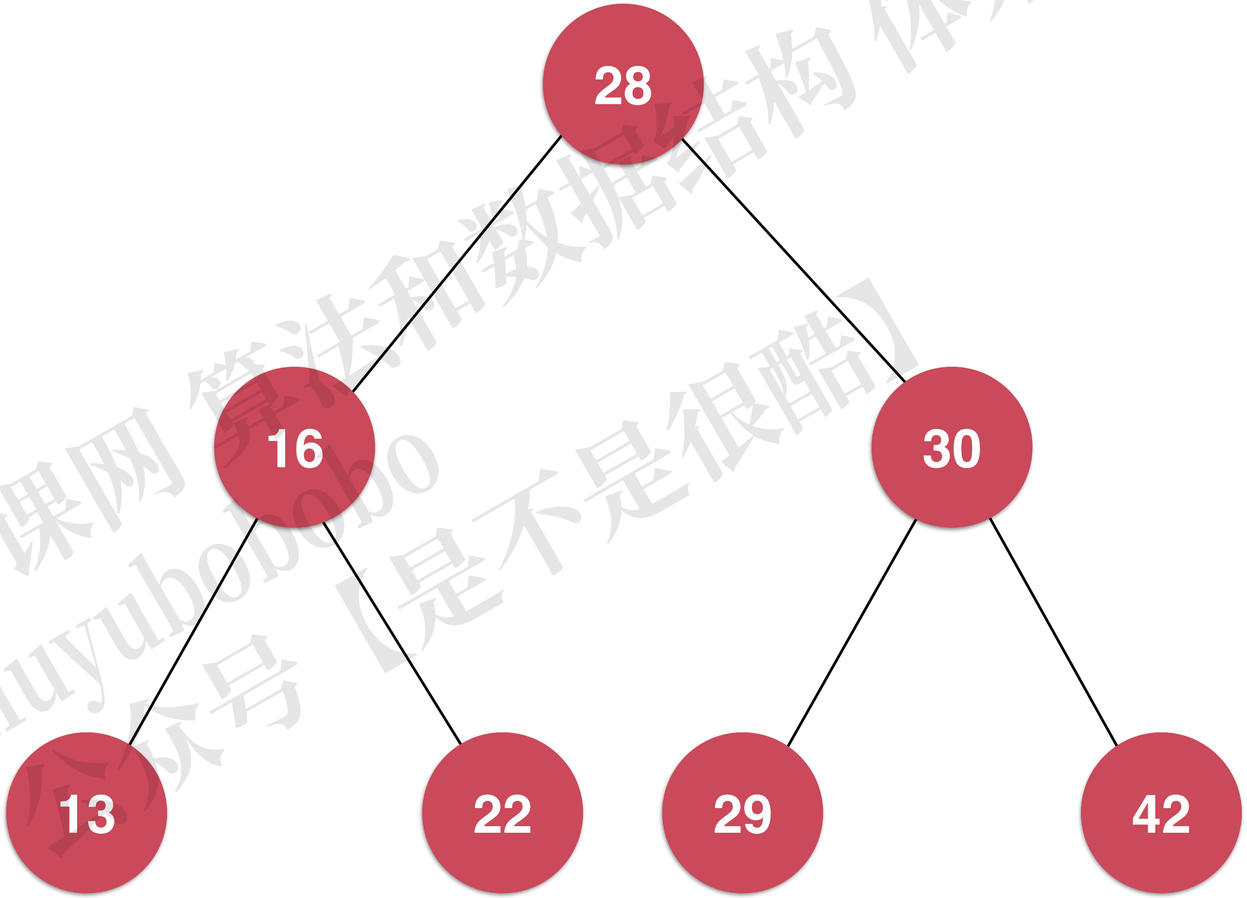
算法与数据结构体系课程

liuyubobobo

二分搜索树

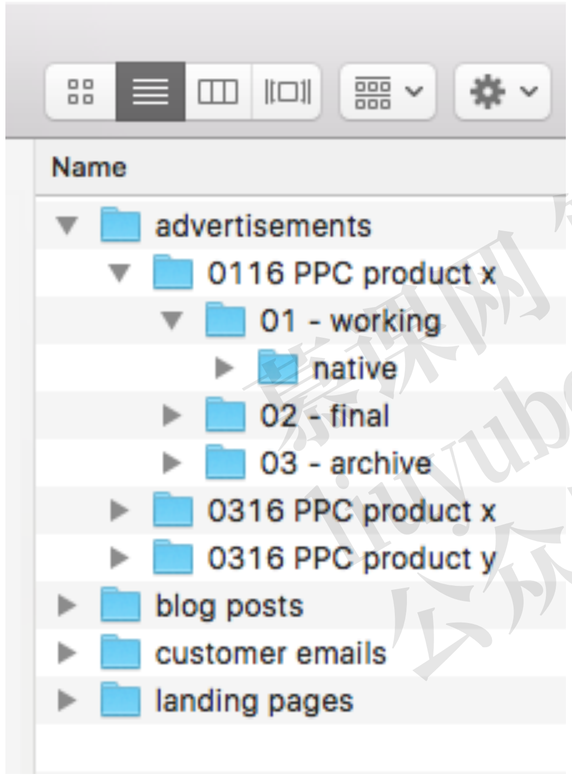
为什么要有（要学习）树结构

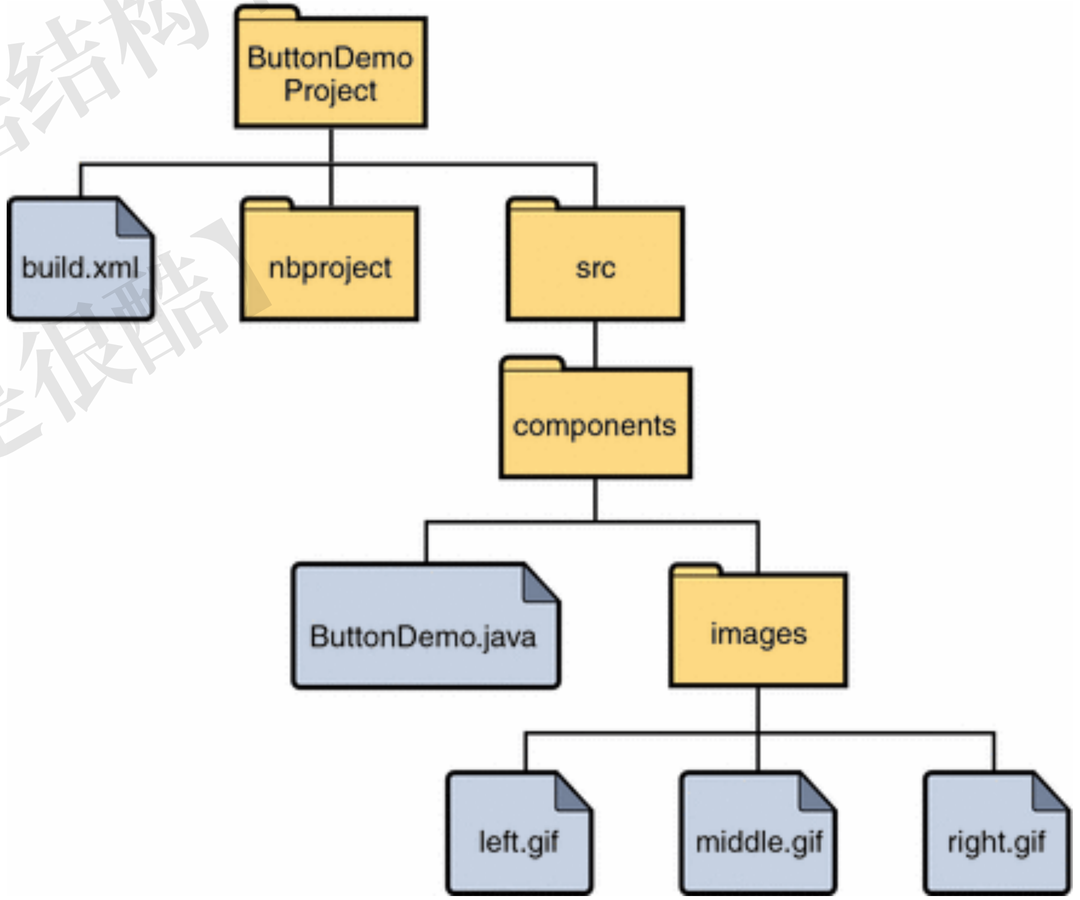
树结构



为什么要有树结构?

- 树结构本身是一种天然的组织结构





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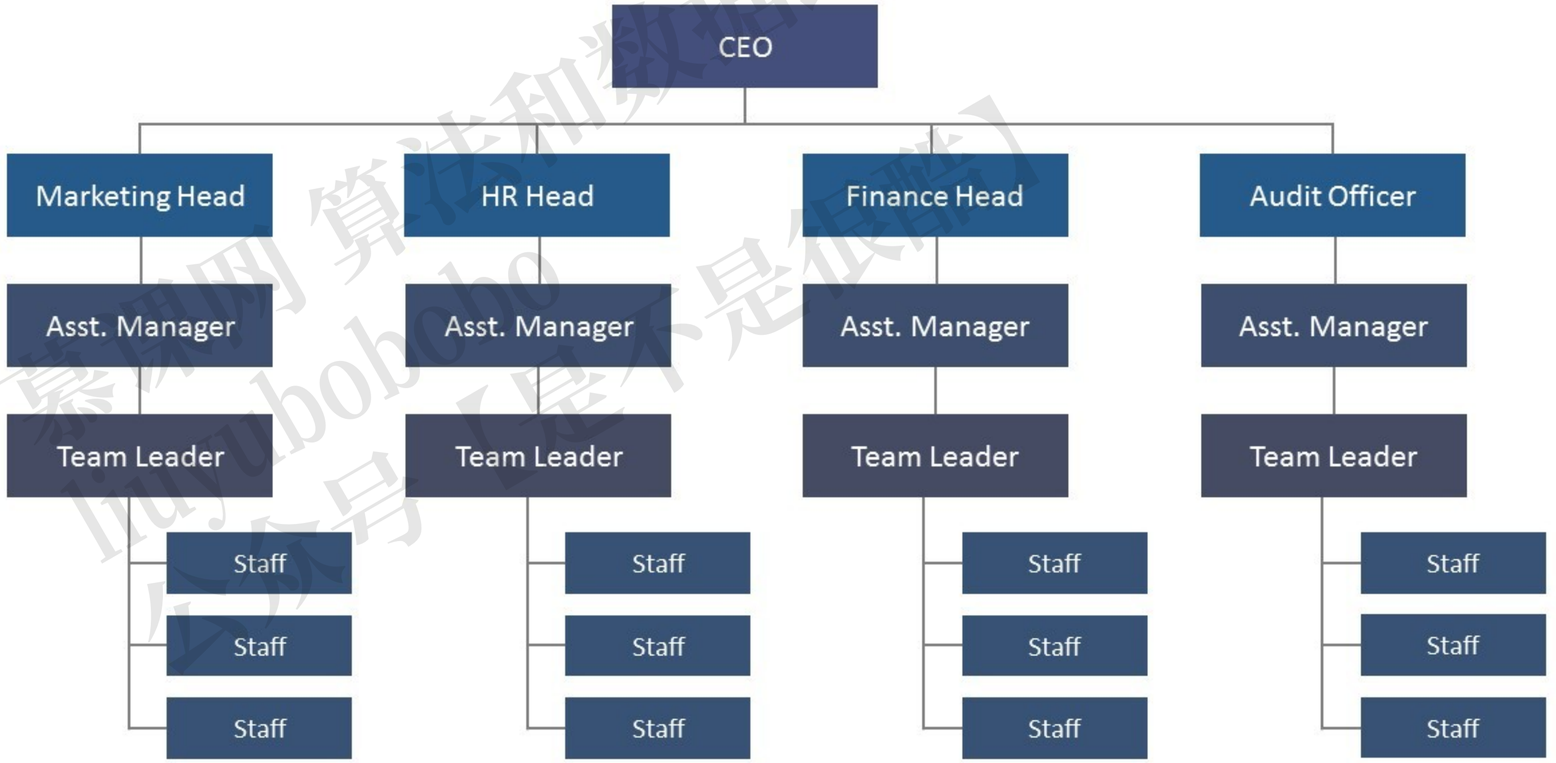
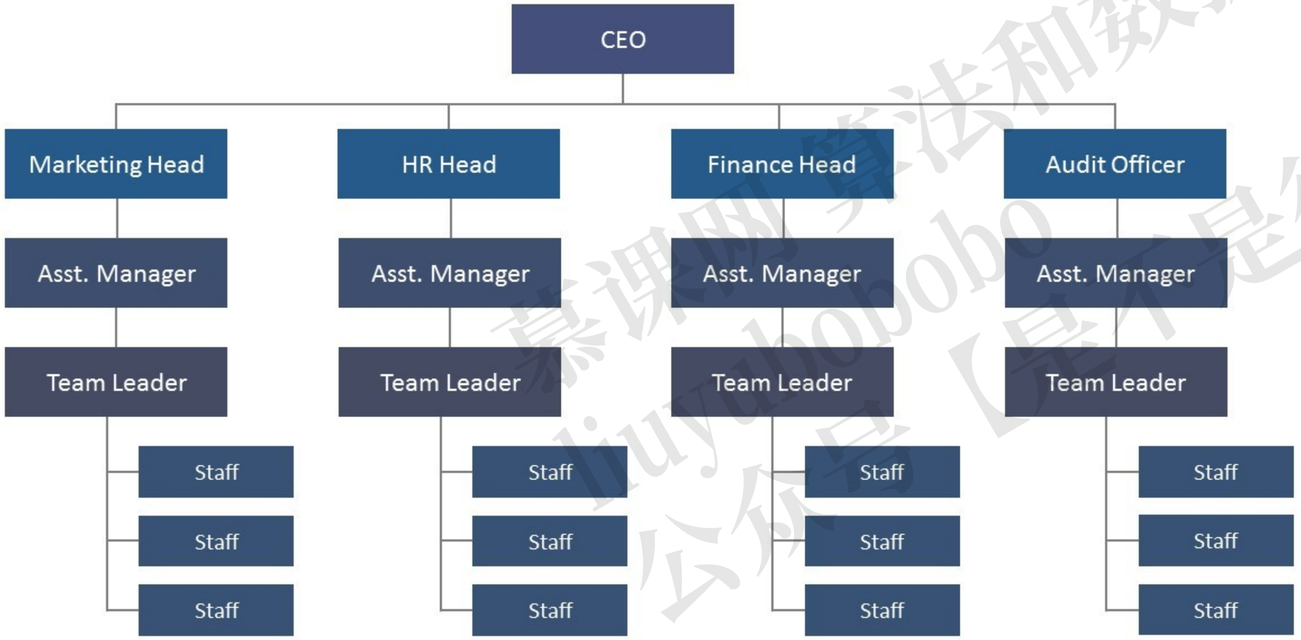
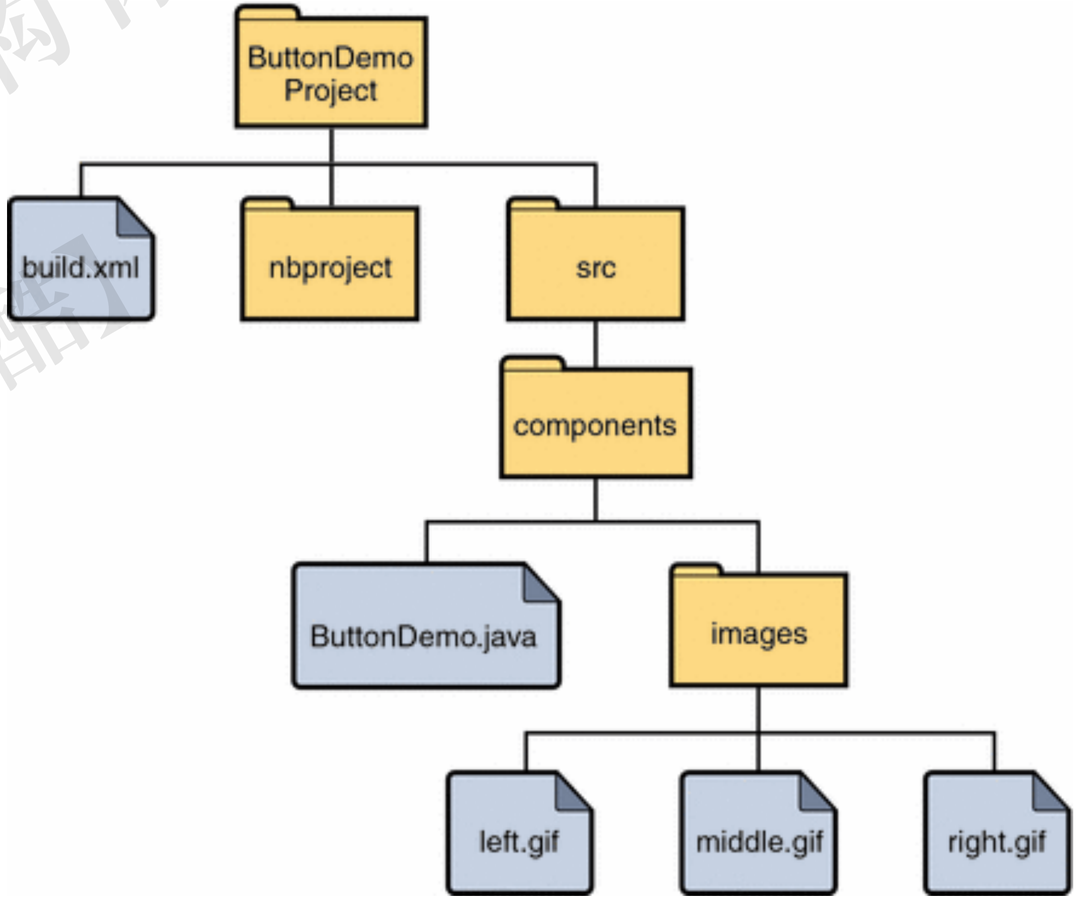


为什么要有树结构?

- 高效





为什么要有树结构?

- 将数据使用树结构存储后，出奇的高效

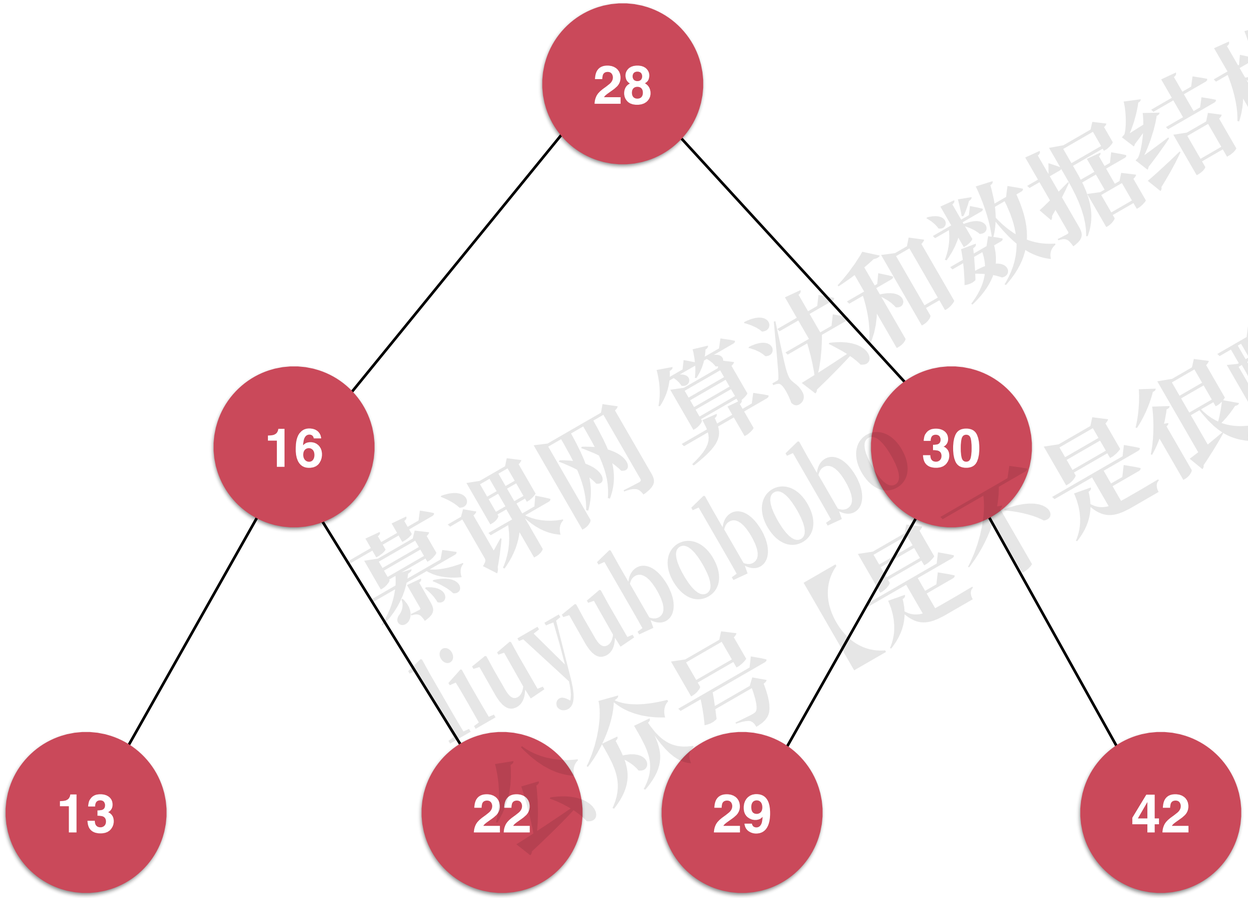
二分搜索树（Binary Search Tree）

平衡二叉树：AVL；红黑树

堆；并查集

线段树；Trie (字典树，前缀树)

二叉树

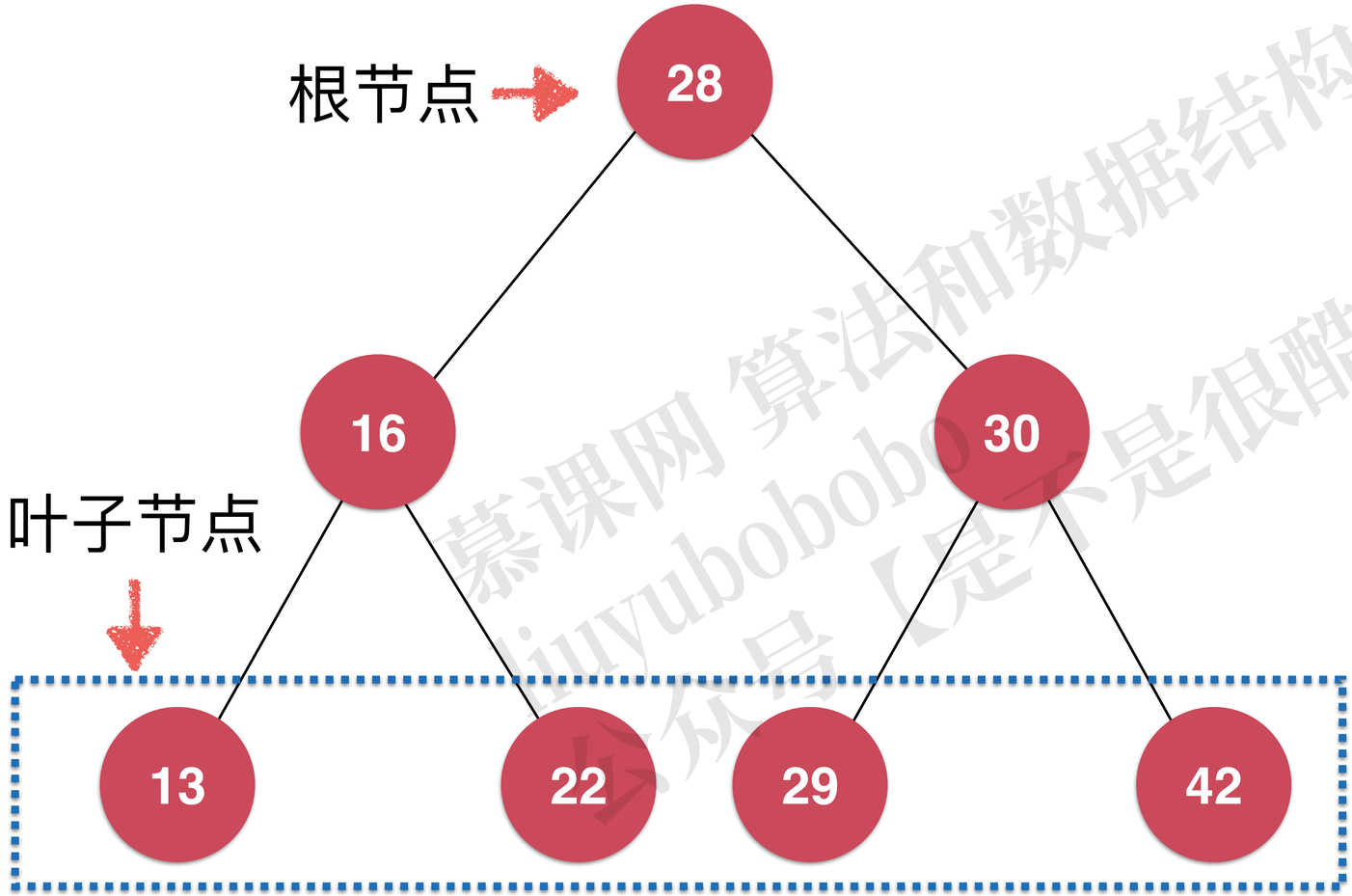


- 和链表一样，动态数据结构

```
class Node {
    E e;
    Node left;
    Node right;
}
```

- 二叉树（多叉树）

二叉树

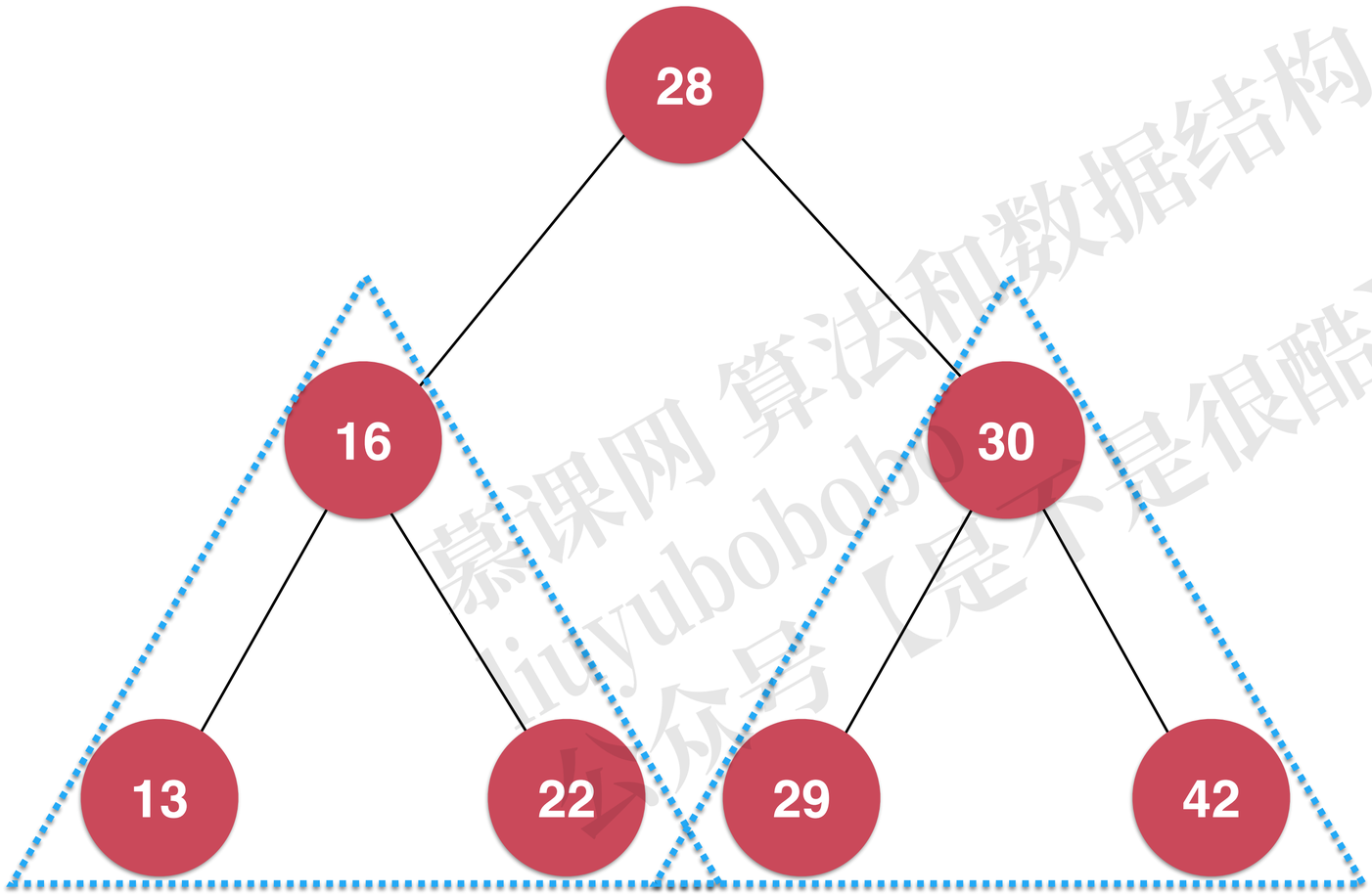


- 二叉树具有具有唯一根节点

```
class Node {
    E e;
    Node left;   左孩子
    Node right;  右孩子
}
```

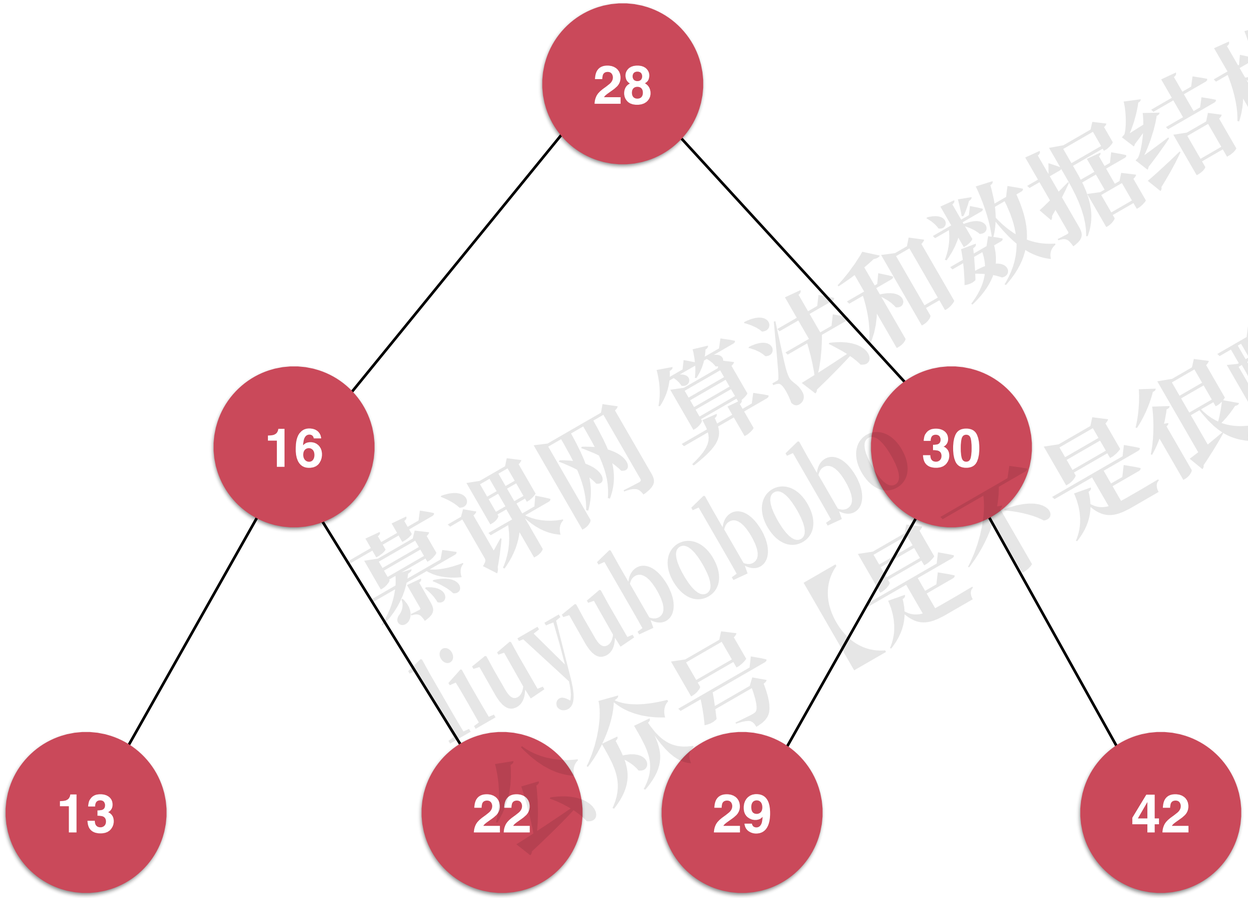
- 二叉树每个节点最多有两个孩子
- 二叉树每个节点最多有一个父亲

二叉树



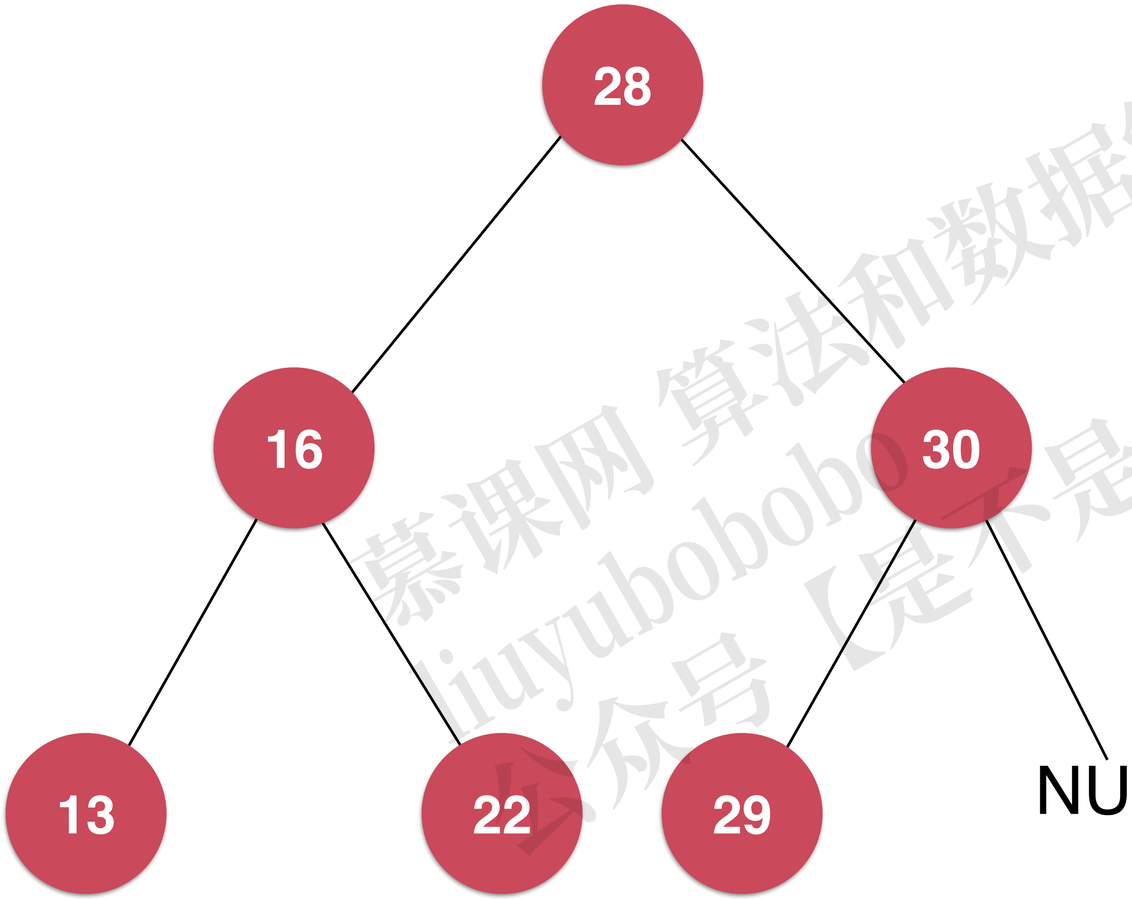
- 二叉树具有天然递归结构
  - 每个节点的左子树也是二叉树
  - 每个节点的右子树也是二叉树

二叉树



- 二叉树不一定是“满”的

二叉树



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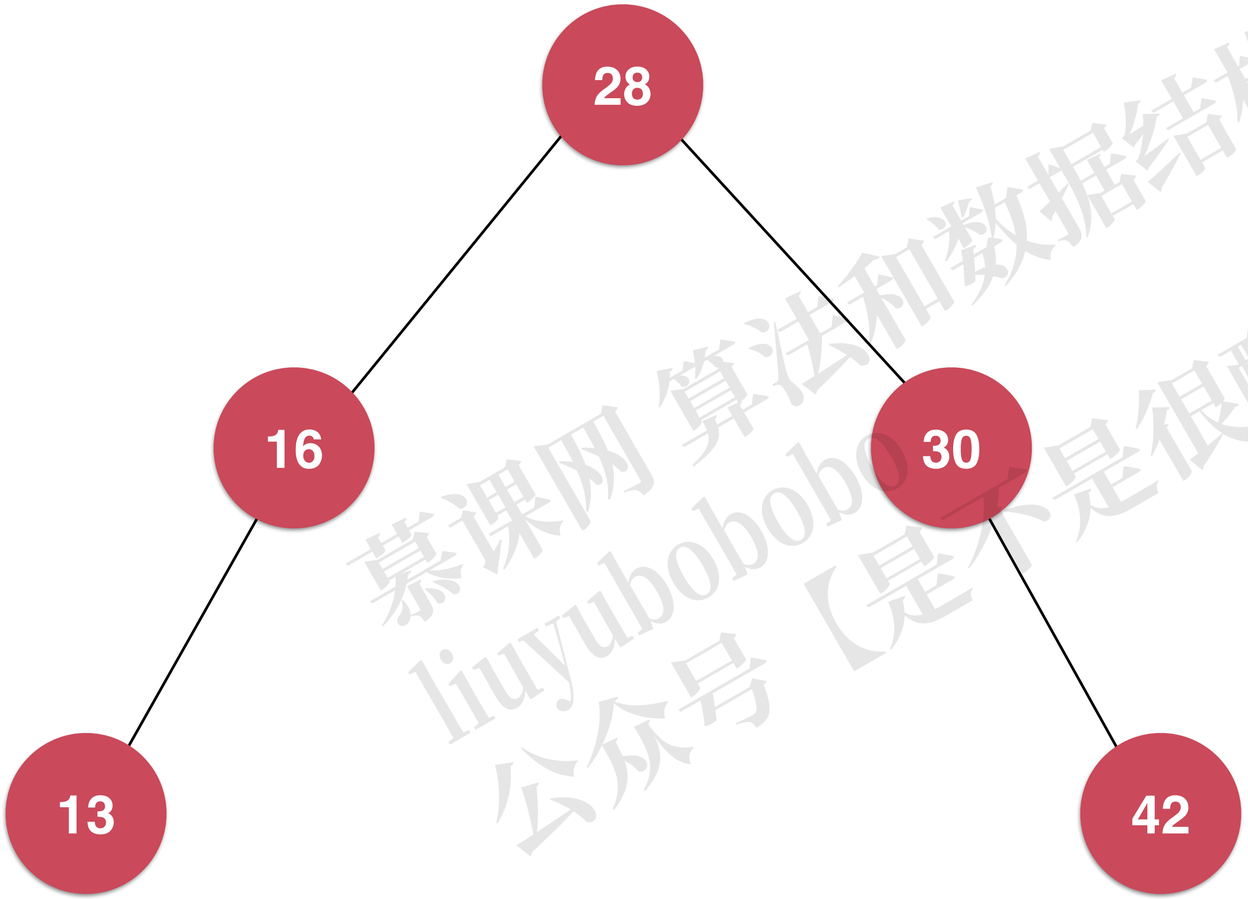
二叉树



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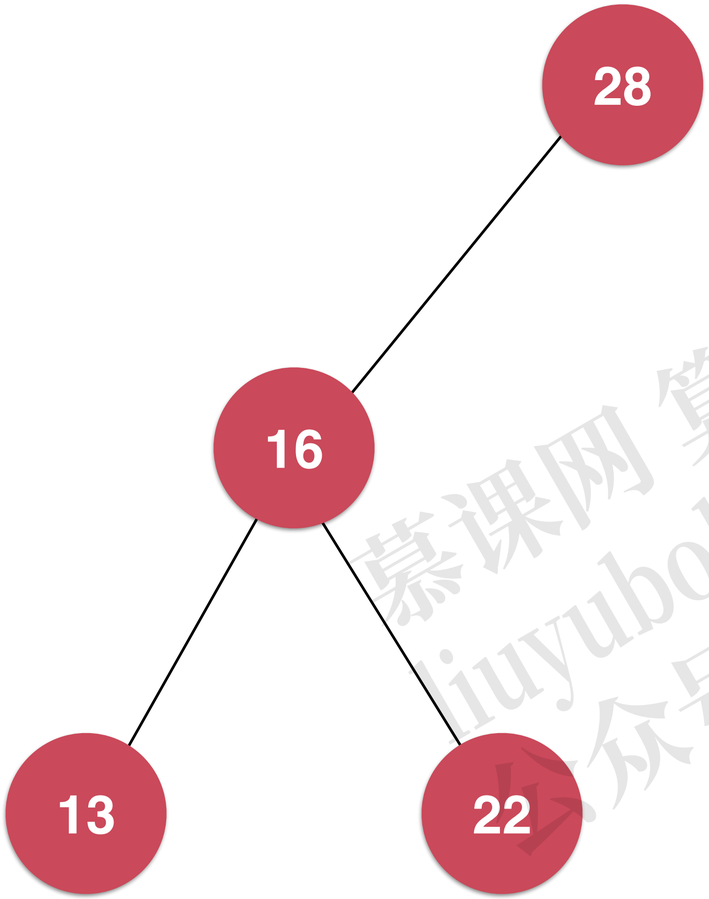
二叉树



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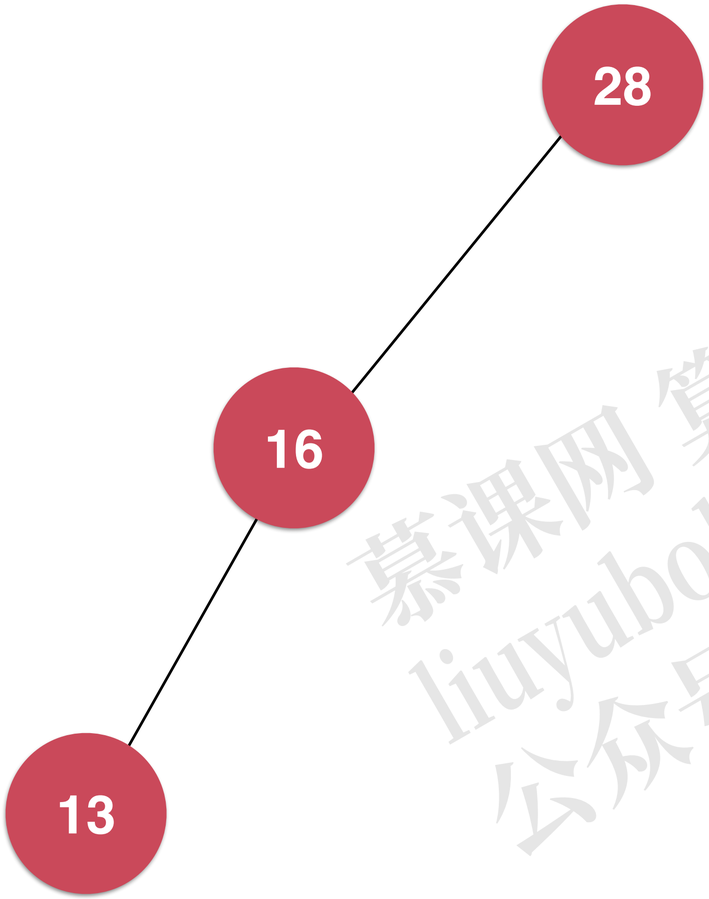
二叉树



- 二叉树不一定是“满”的

二叉树

- 二叉树不一定是“满”的

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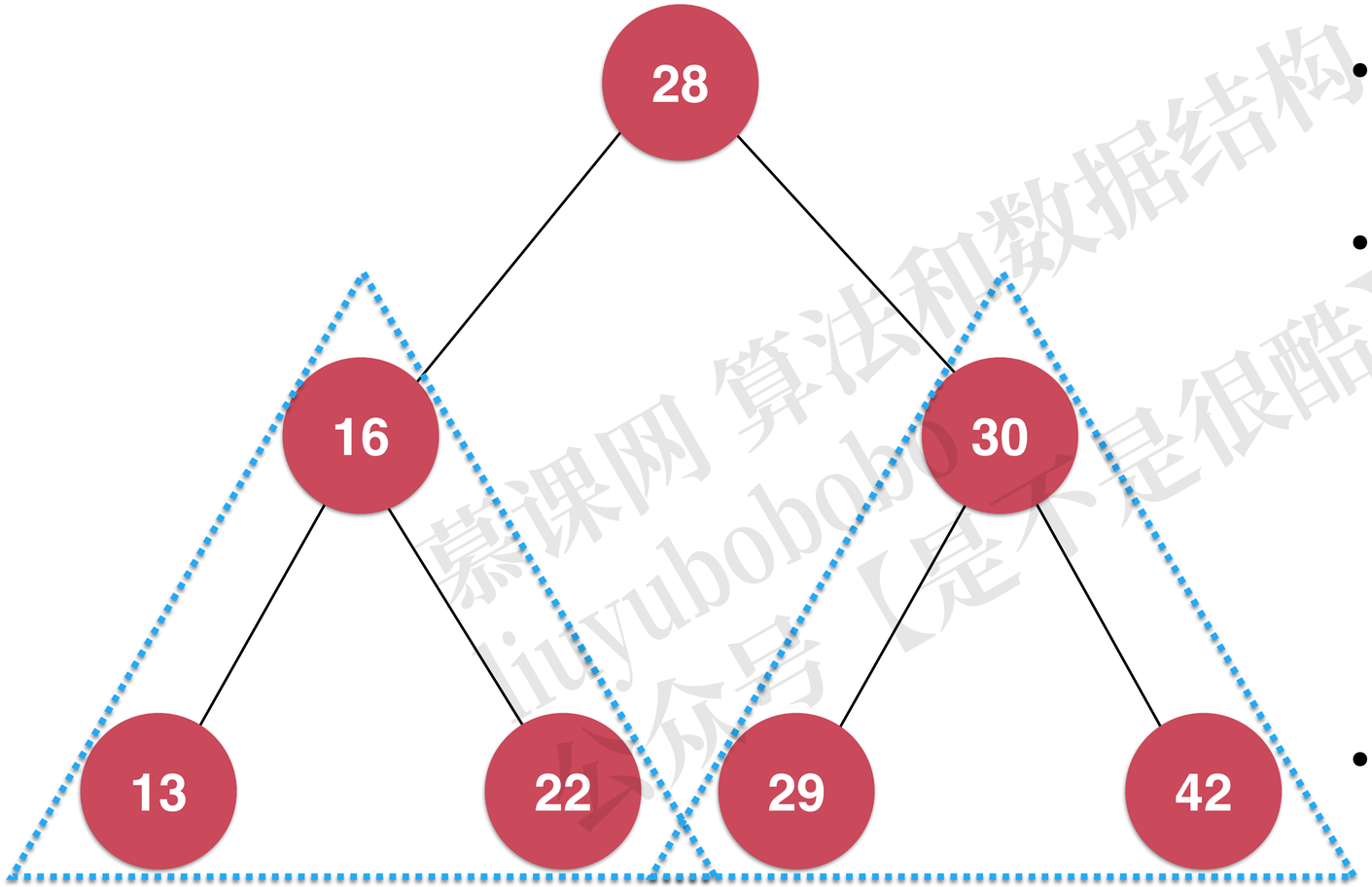
一个节点也是二叉树

NULL

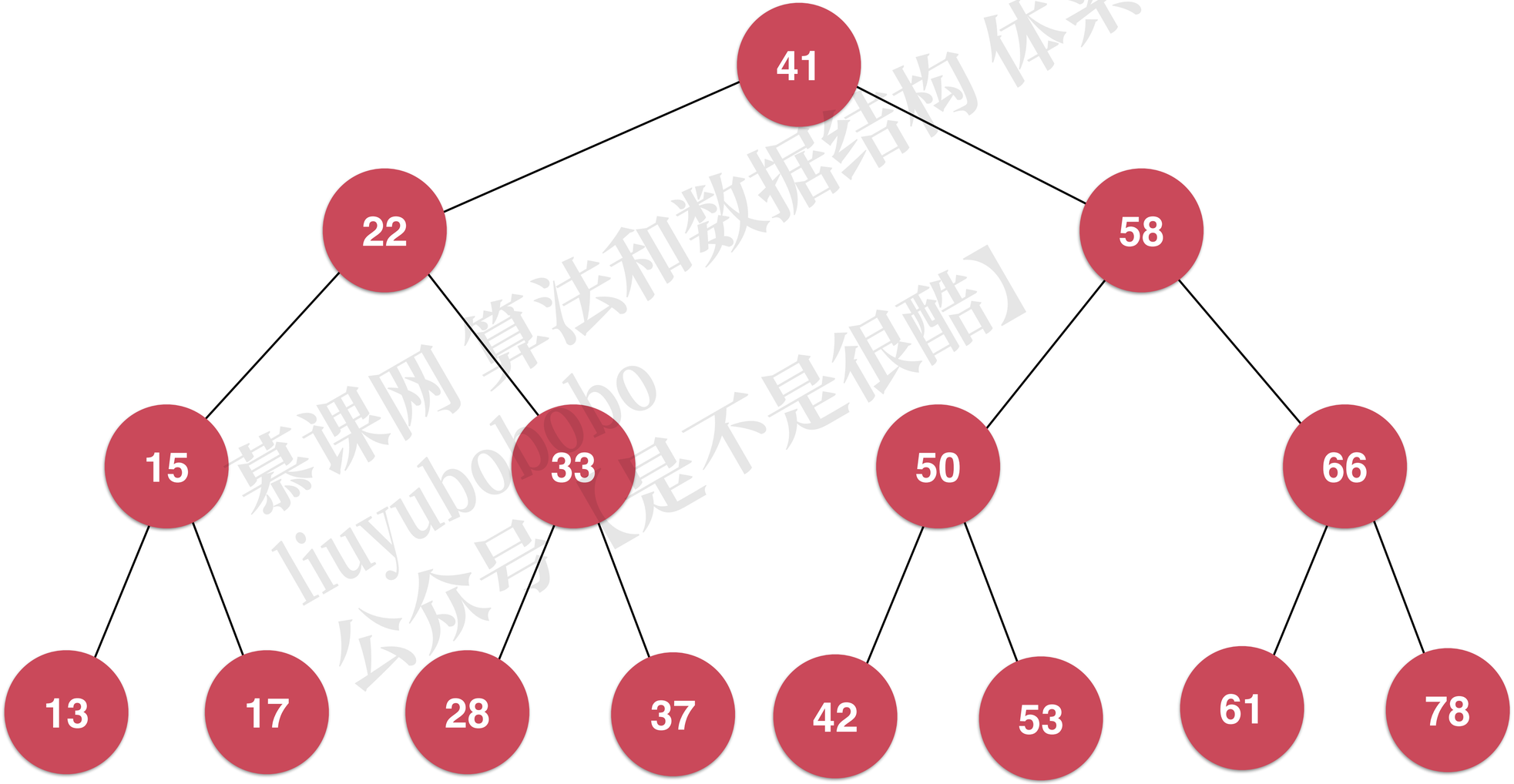
空也是二叉树

二分搜索树 Binary Search Tree

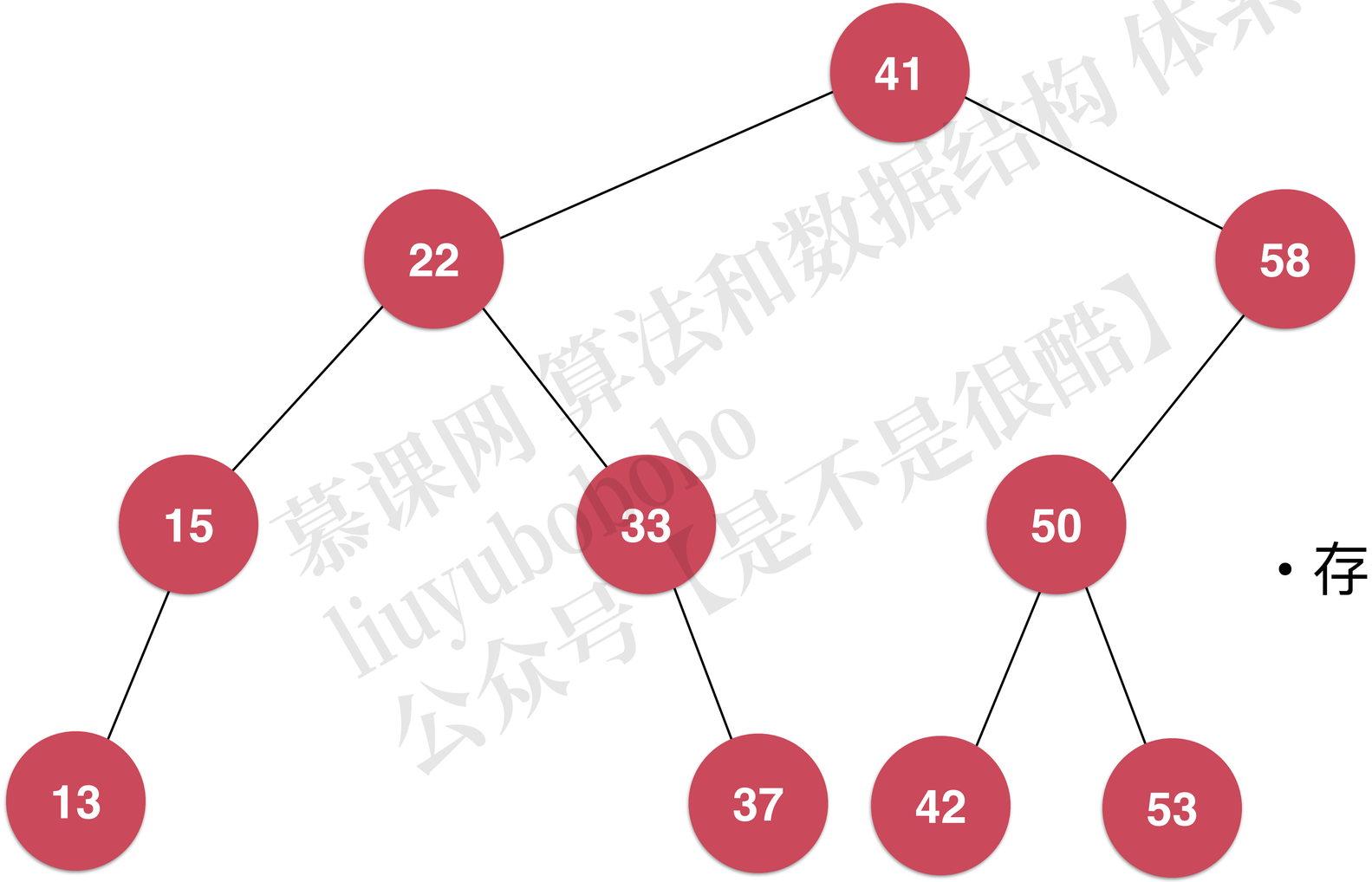


- 二分搜索树是二叉树
- 二分搜索树的每个节点的值：
  - 大于其左子树的所有节点的值
  - 小于其右子树的所有节点的值
- 每一棵子树也是二分搜索树

二分搜索树 Binary Search Tree



二分搜索树 Binary Search Tree



- 存储的元素必须有可比较性

实践：二分搜索树基础结构

二分搜索树添加新元素

二分搜索树添加新元素

NULL

二分搜索树添加新元素

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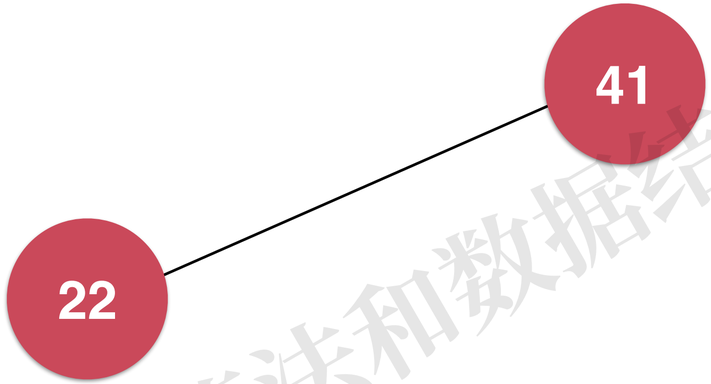
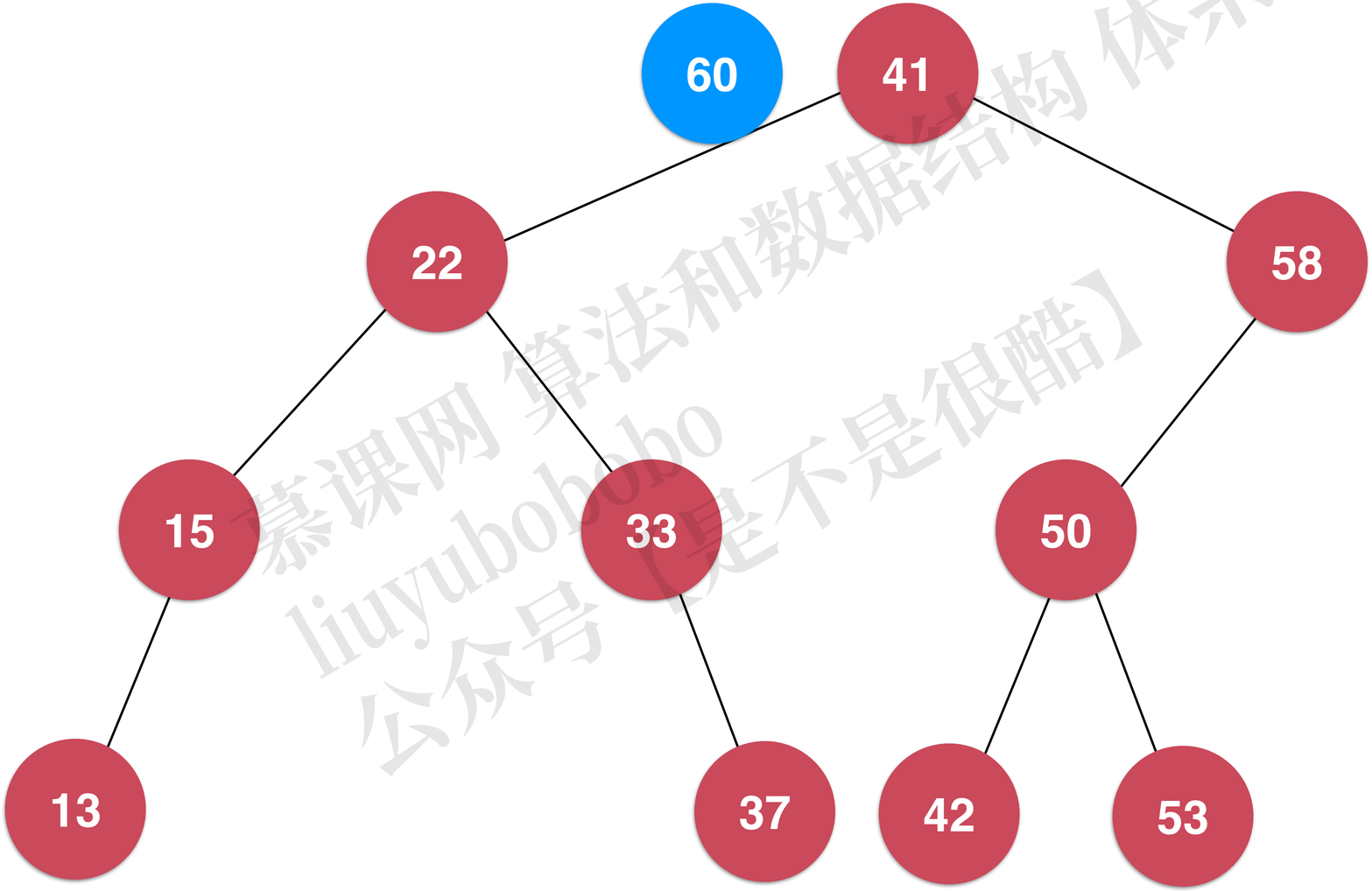
二分搜索树添加新元素

二分搜索树添加新元素

二分搜索树添加新元素



二分搜索树添加新元素



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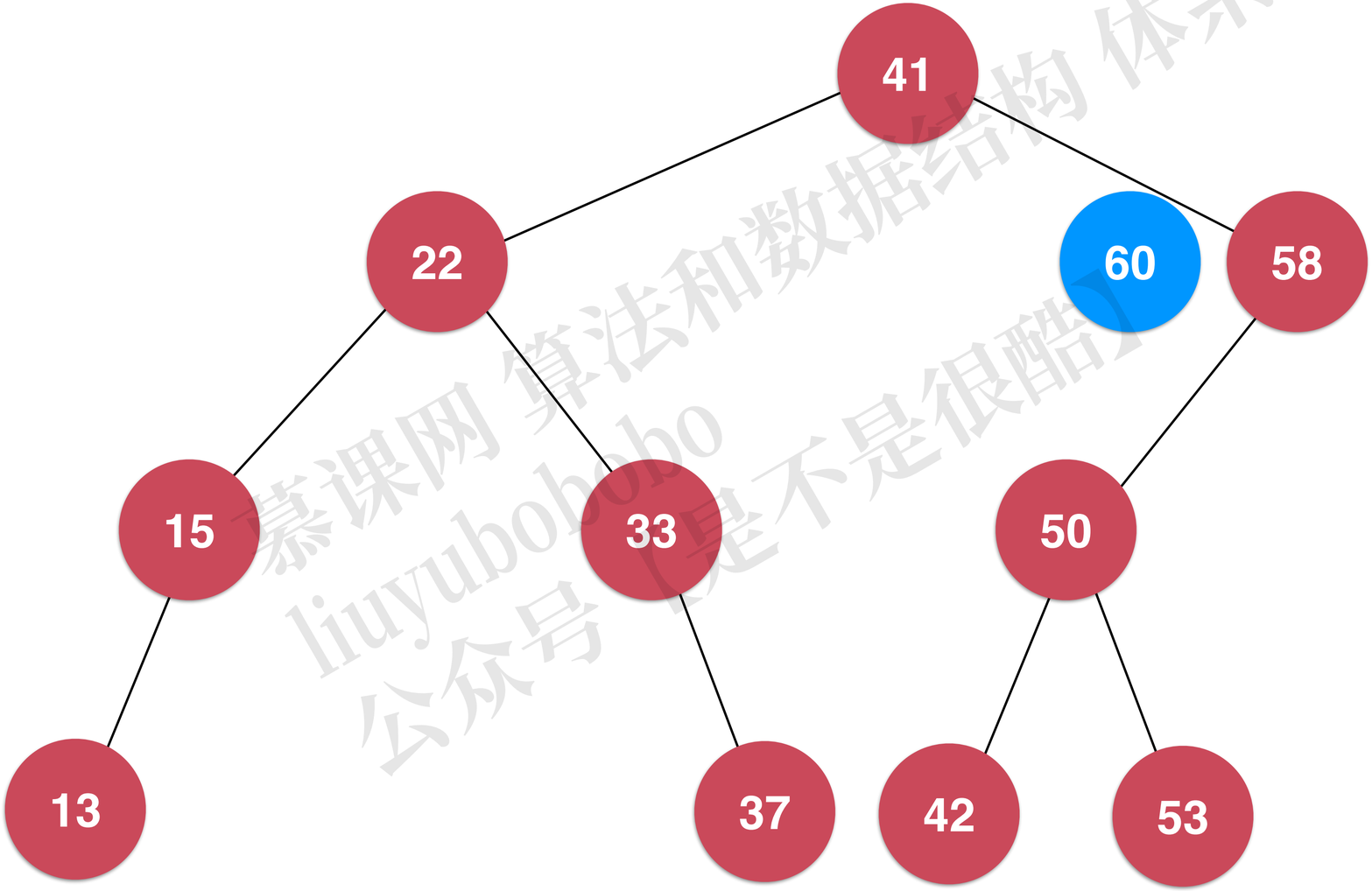

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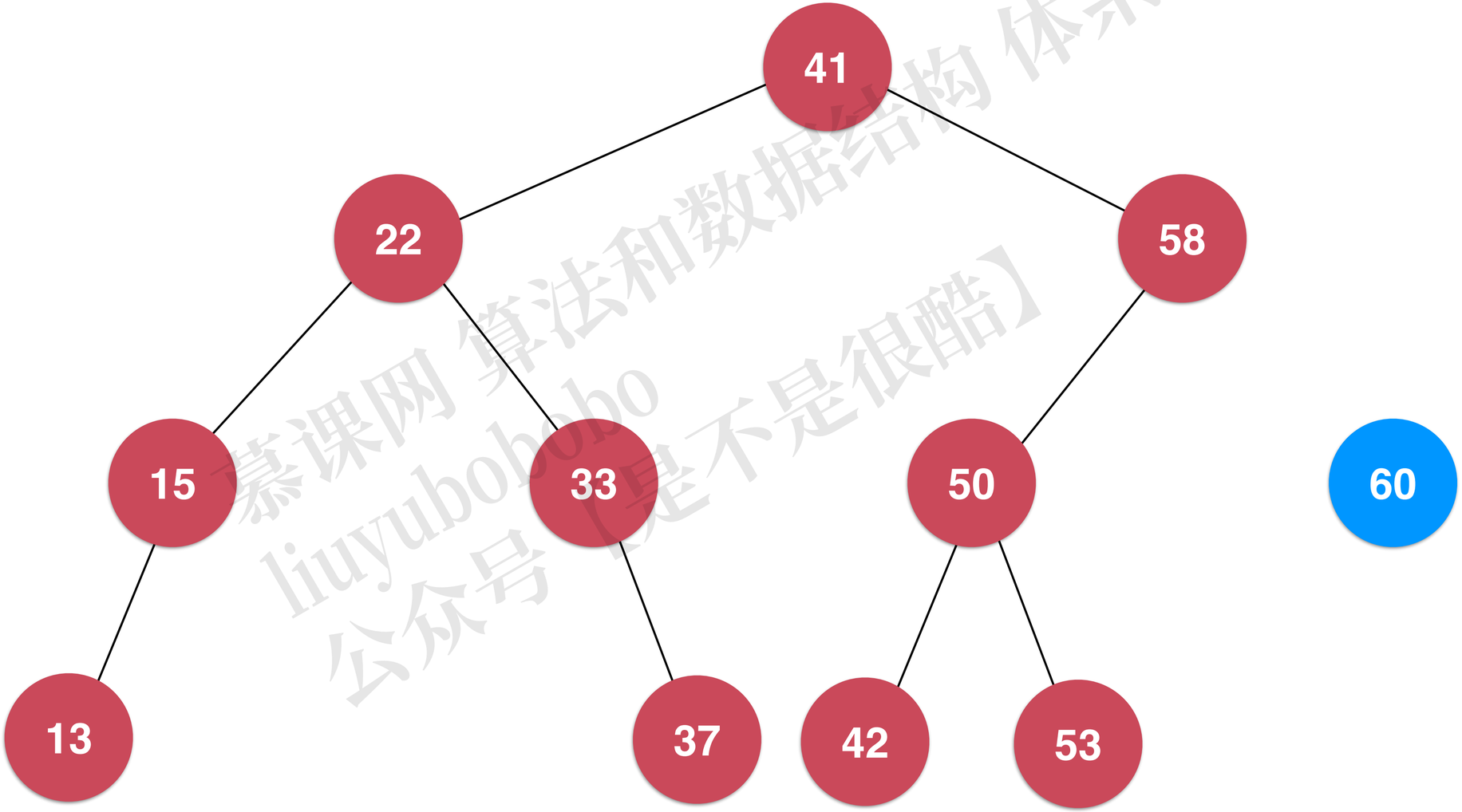

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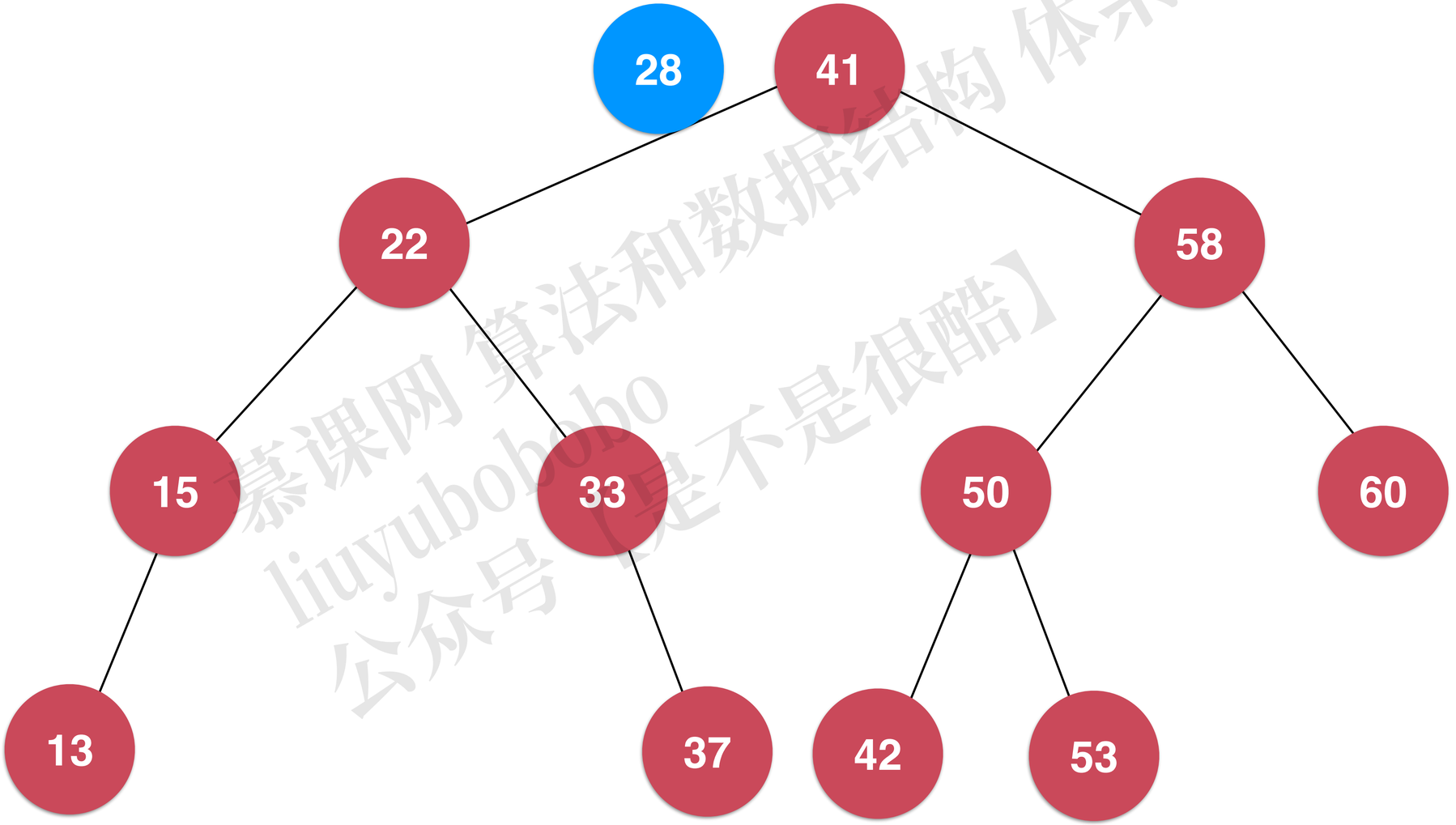

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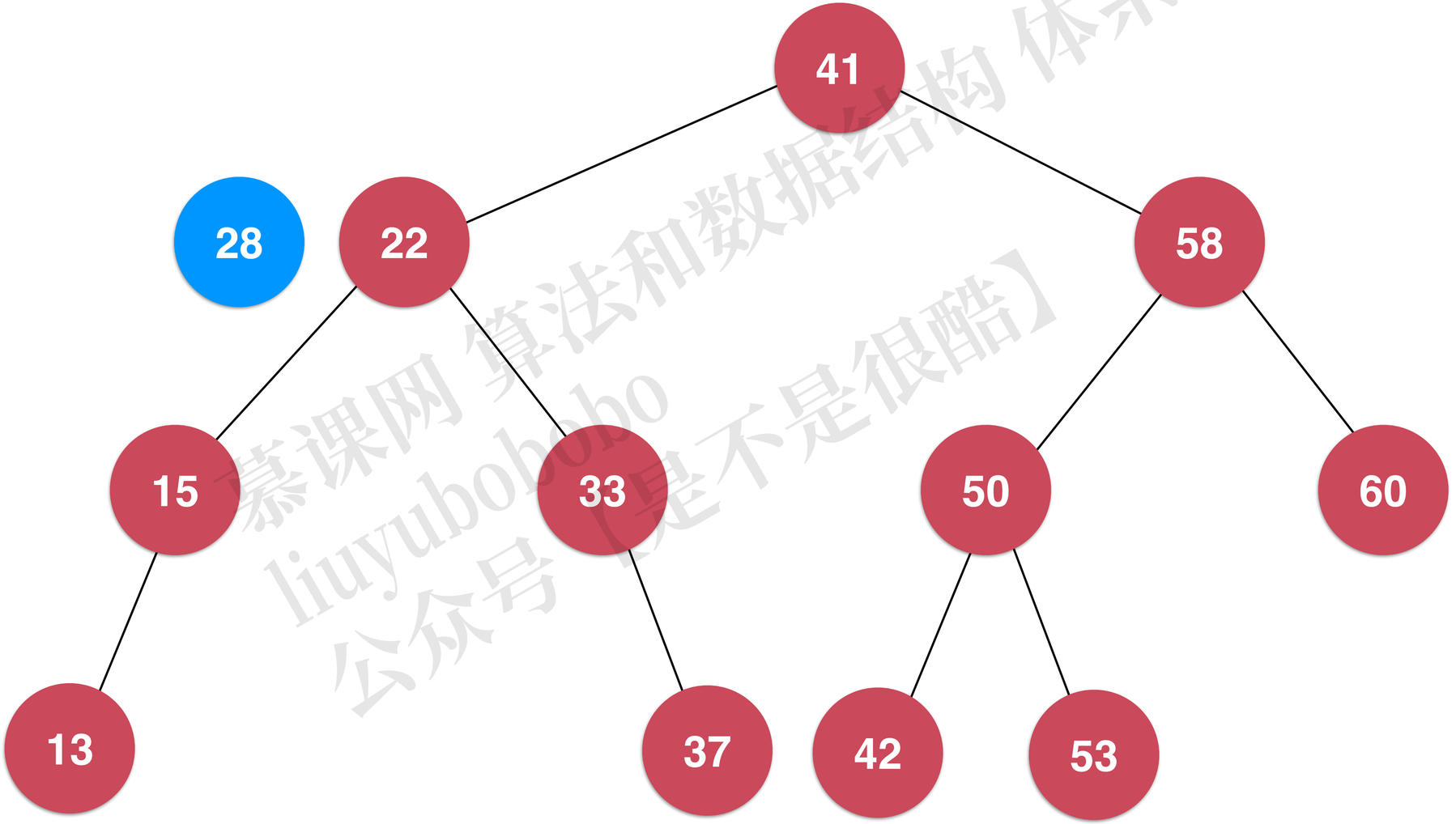

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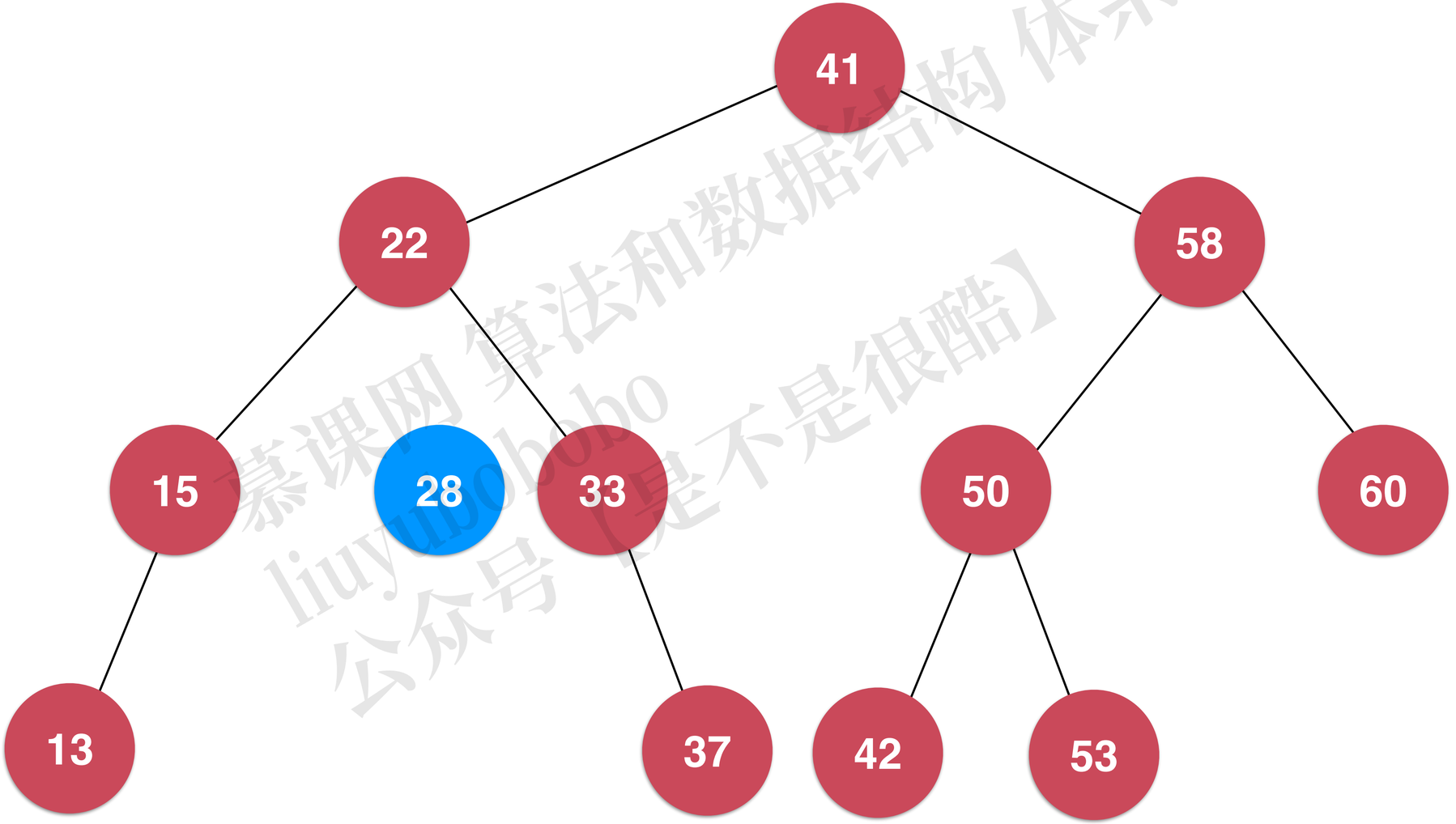

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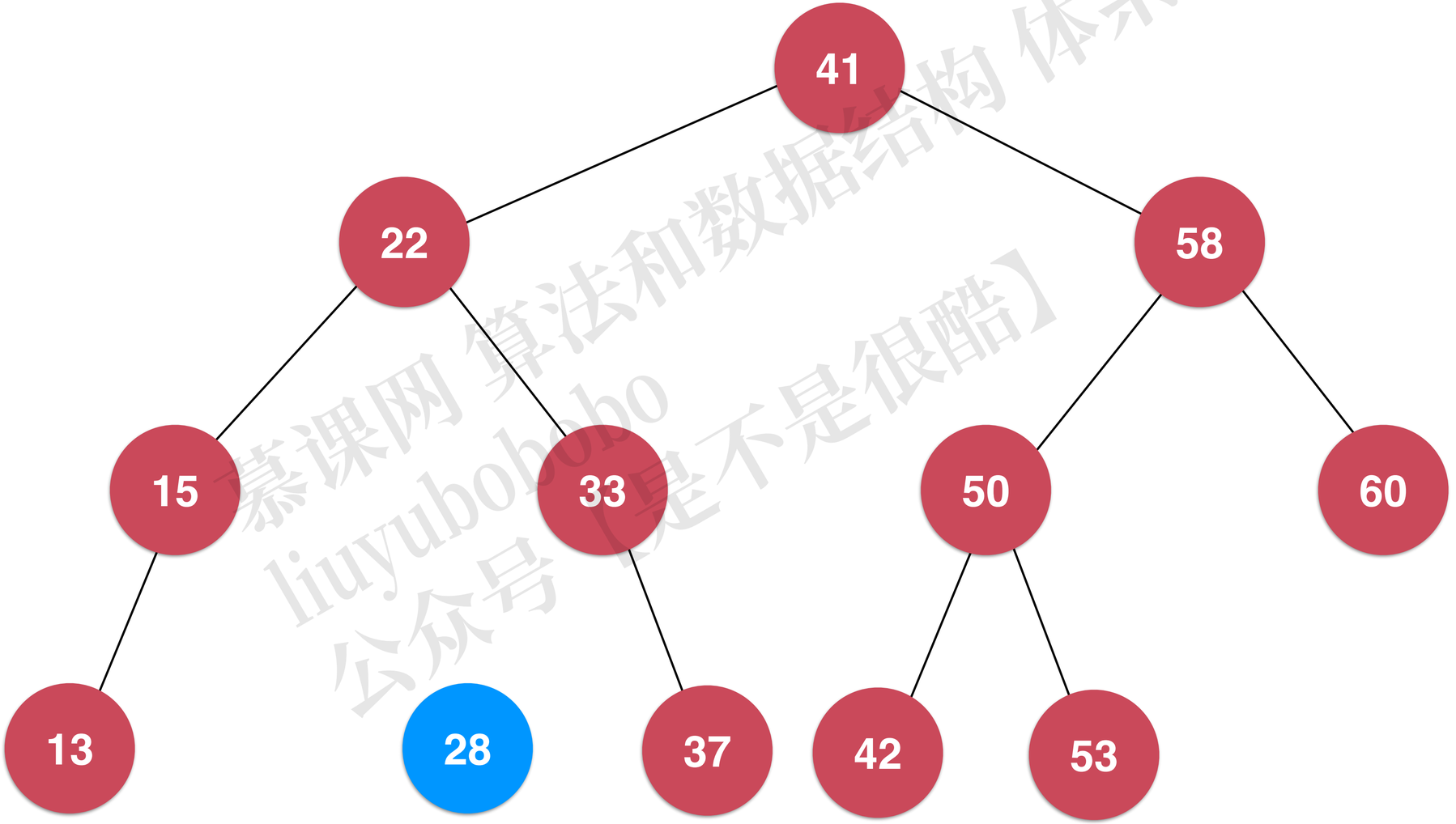
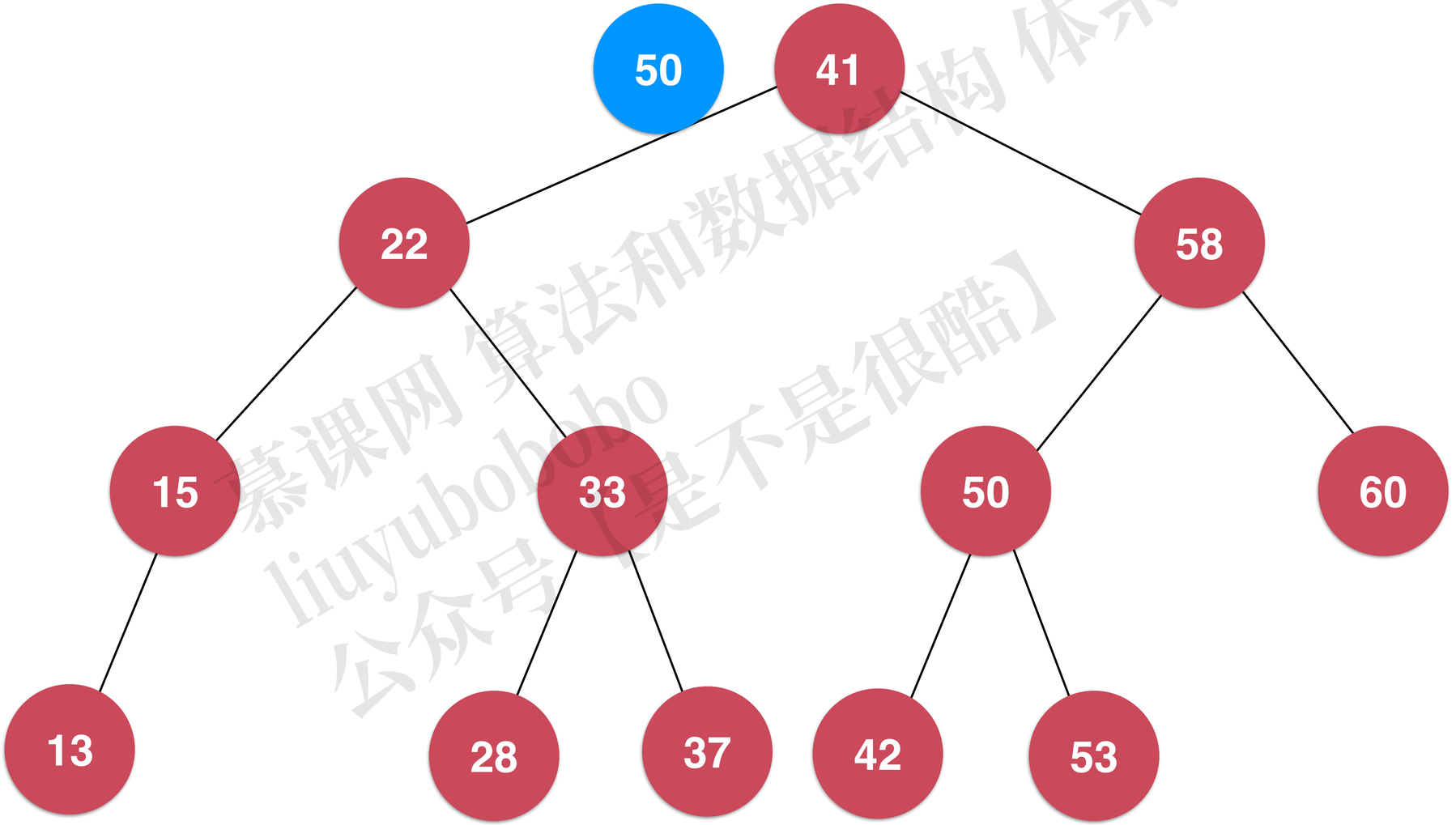


二分搜索树添加新元素



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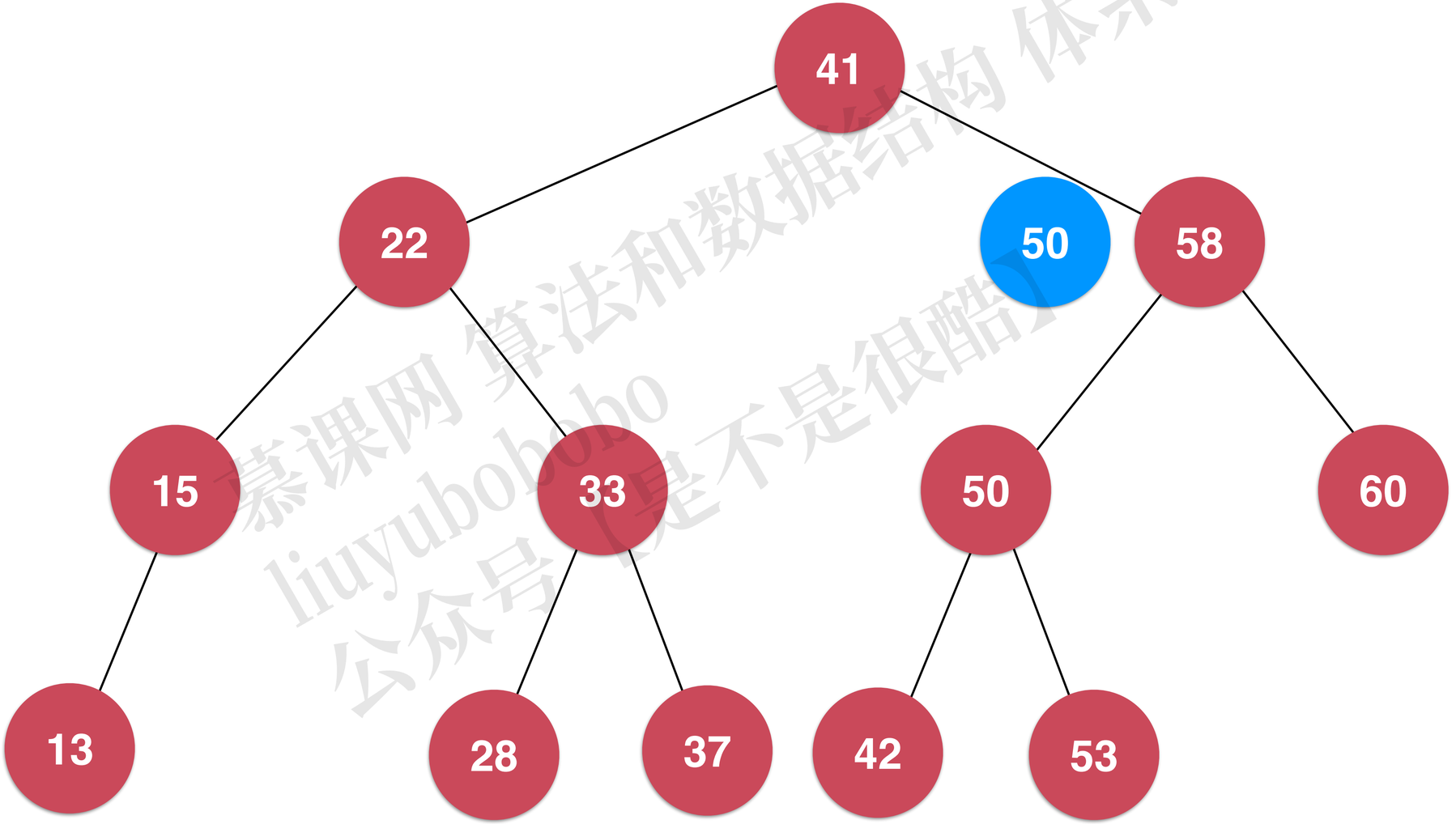

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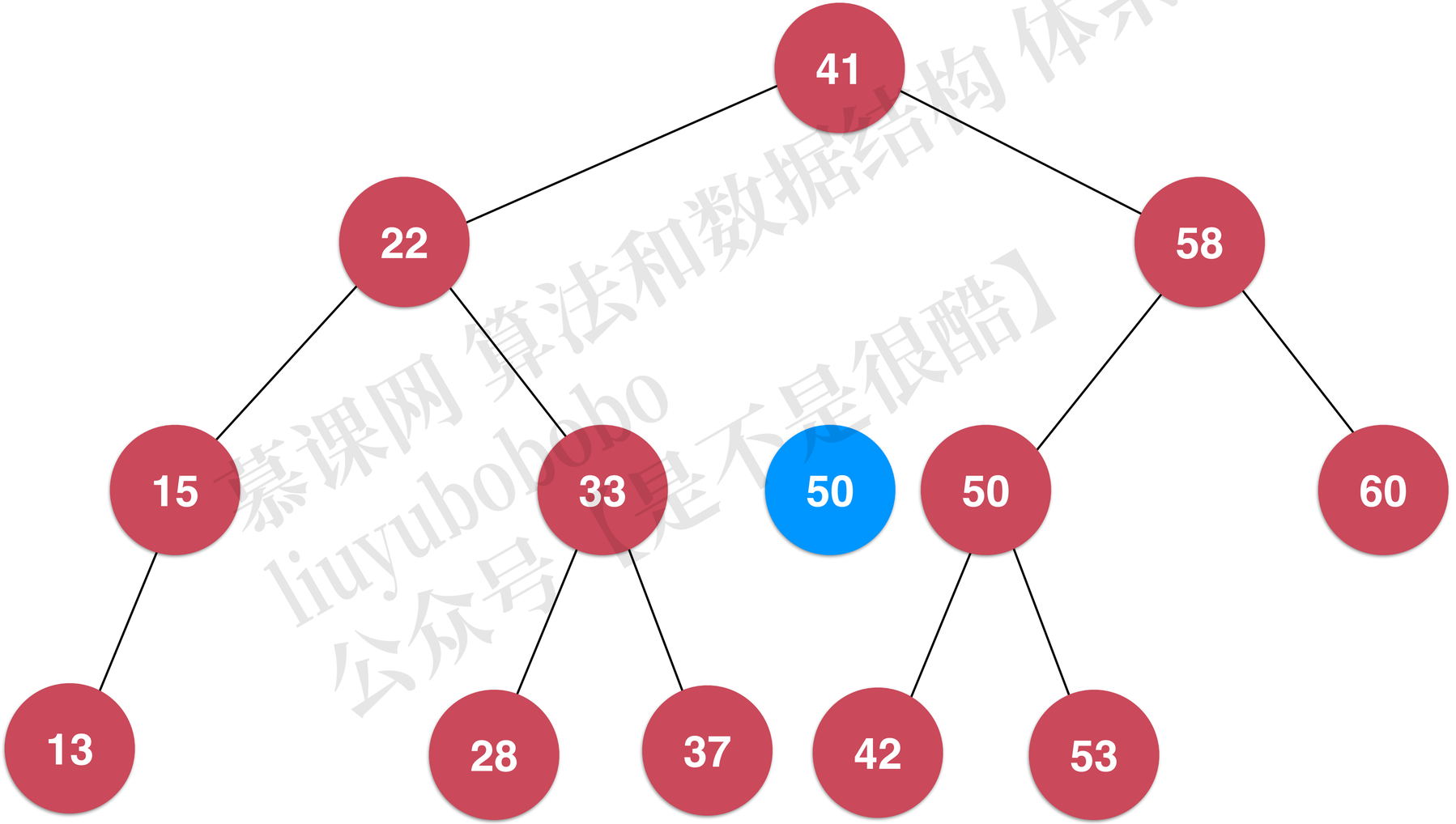
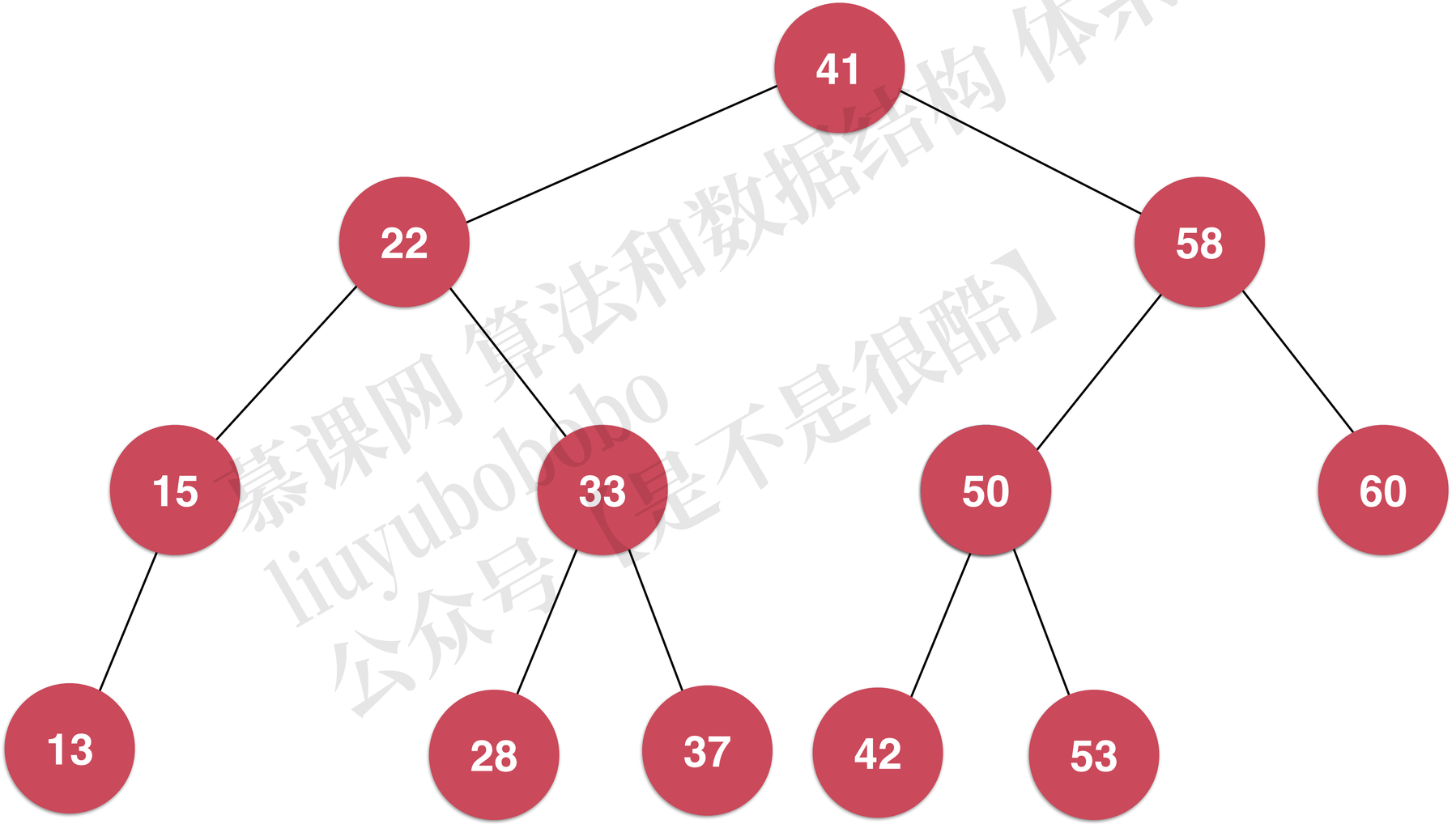


二分搜索树添加新元素



二分搜索树添加新元素

- 我们的二分搜索树不包含重复元素

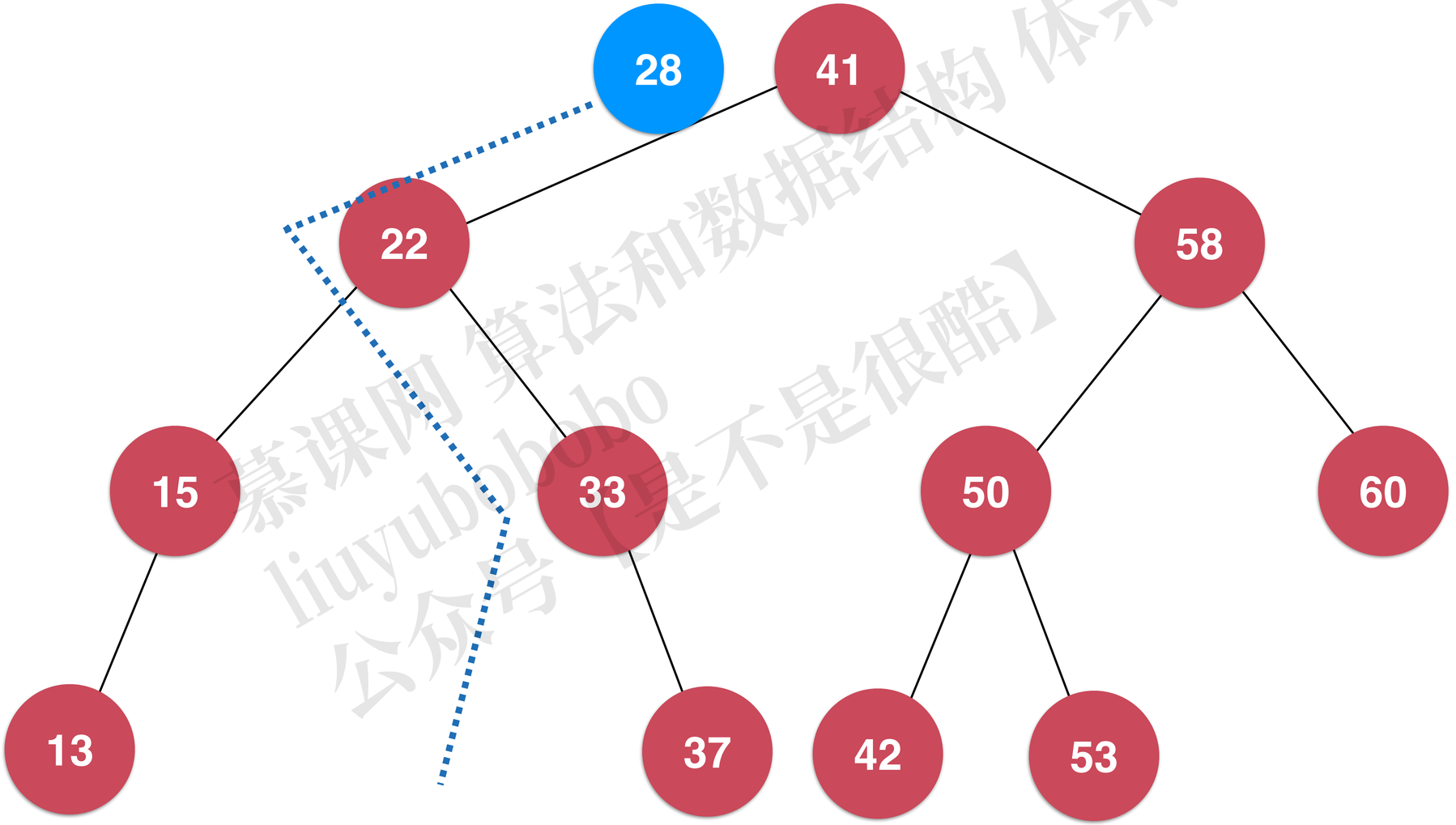
  如果想包含重复元素的话，只需要定义：

  左子树小于等于节点；或者右子树大于等于节点

  注意：我们之前讲的数组和链表，可以有重复元素

- 二分搜索树添加元素的非递归写法，和链表很像

二分搜索树添加新元素



二分搜索树添加新元素

- 二分搜索树添加元素的非递归写法，和链表很像
- 这个课程在二分搜索树方面的实现，关注递归实现
- 二分搜索树一些方法的非递归实现，留做练习
- 在二分搜索树方面，递归比非递归实现简单：）

实践：二分搜索树添加新元素

递归的终止条件

实践：另一种方法递归实现二分搜索树添加新元素

作业：再看递归

问题在哪里？

```
private void add(Node node, E e){
    if(node == null){
        size ++;
        node = new Node(e);
        return;
    }

    if(e.compareTo(node.e) < 0)
        add(node.left, e);
    else if(e.compareTo(node.e) > 0)
        add(node.right, e);
}
```

作业：尝试使用非递归书写二分搜索树添加元素

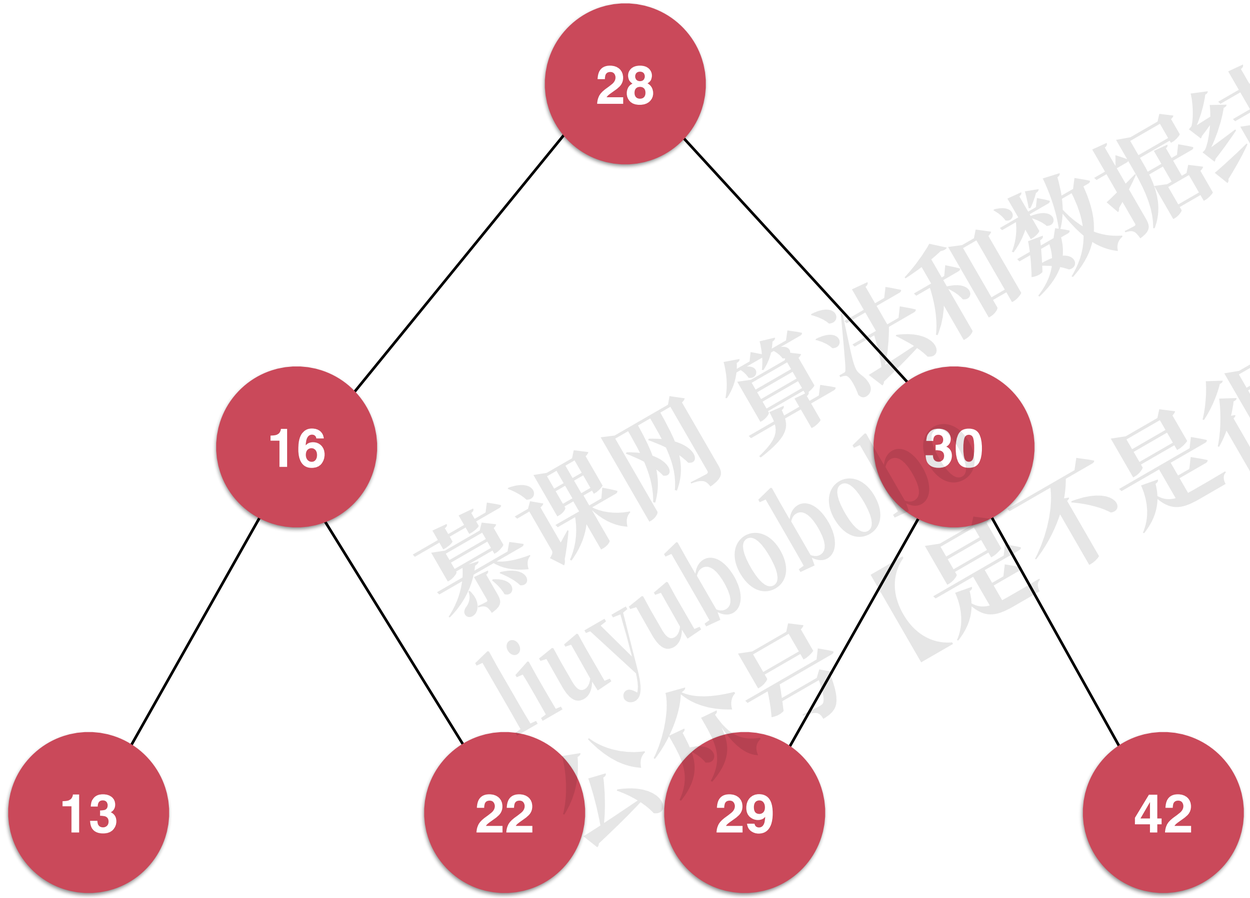
作业解析：尝试使用非递归书写二分搜索树添加元素

二分搜索树的查询

实践：二分搜索树的查询

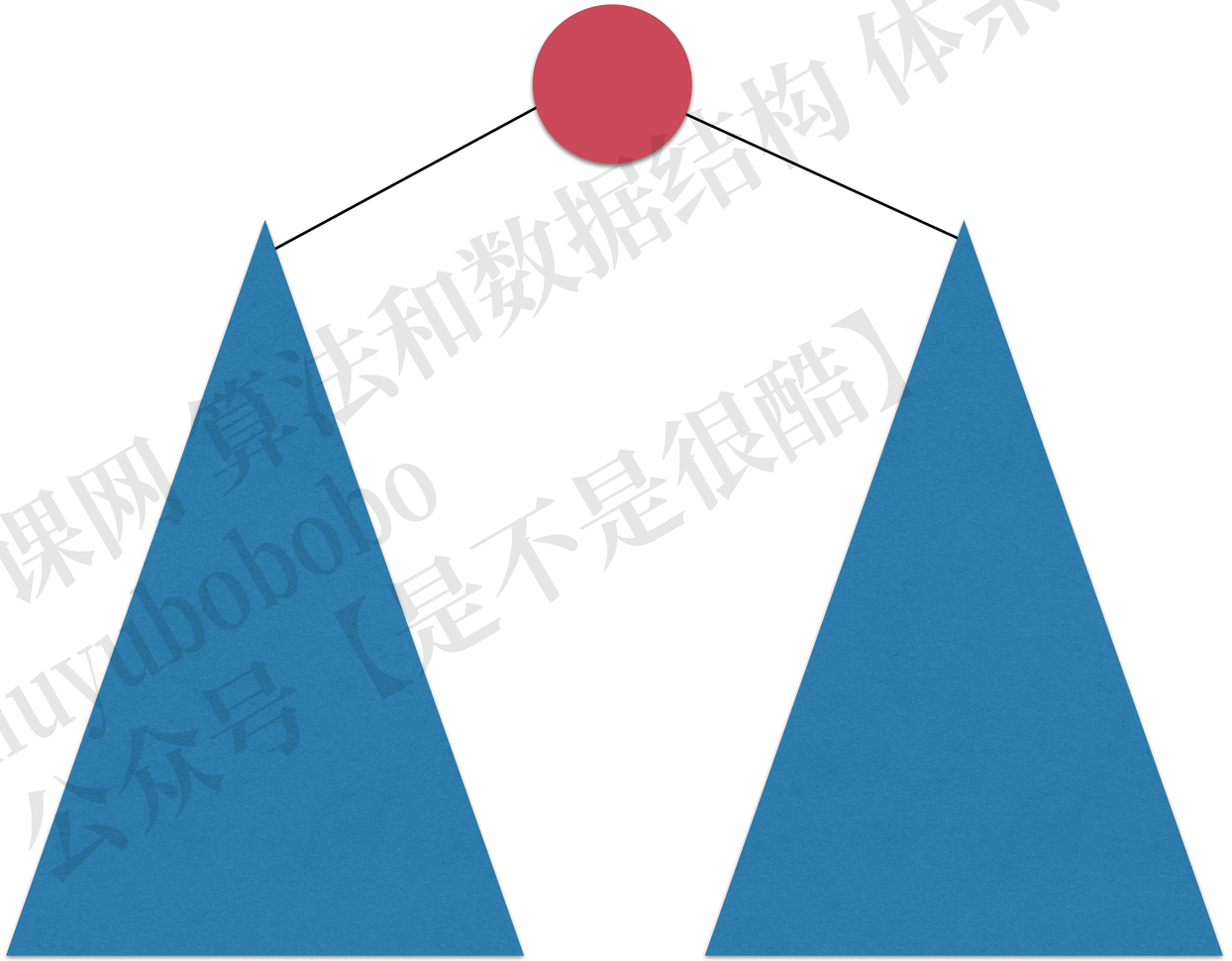
二分搜索树的遍历

什么是遍历操作

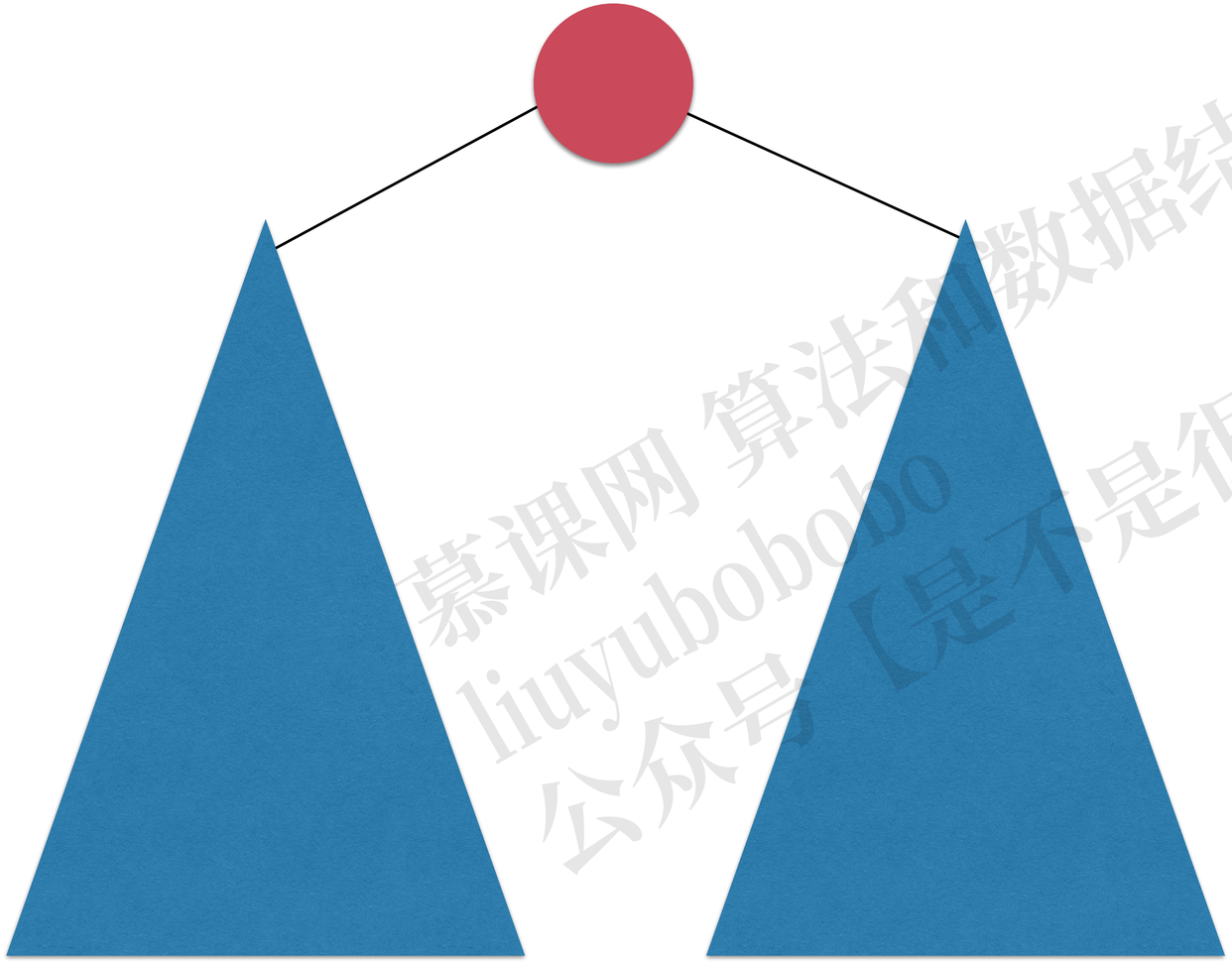


- 遍历操作就是把所有节点都访问一遍
- 访问的原因和业务相关
- 在线性结构下，遍历是极其容易的
- 在树结构下，也没那么难：）

二分搜索树的递归操作



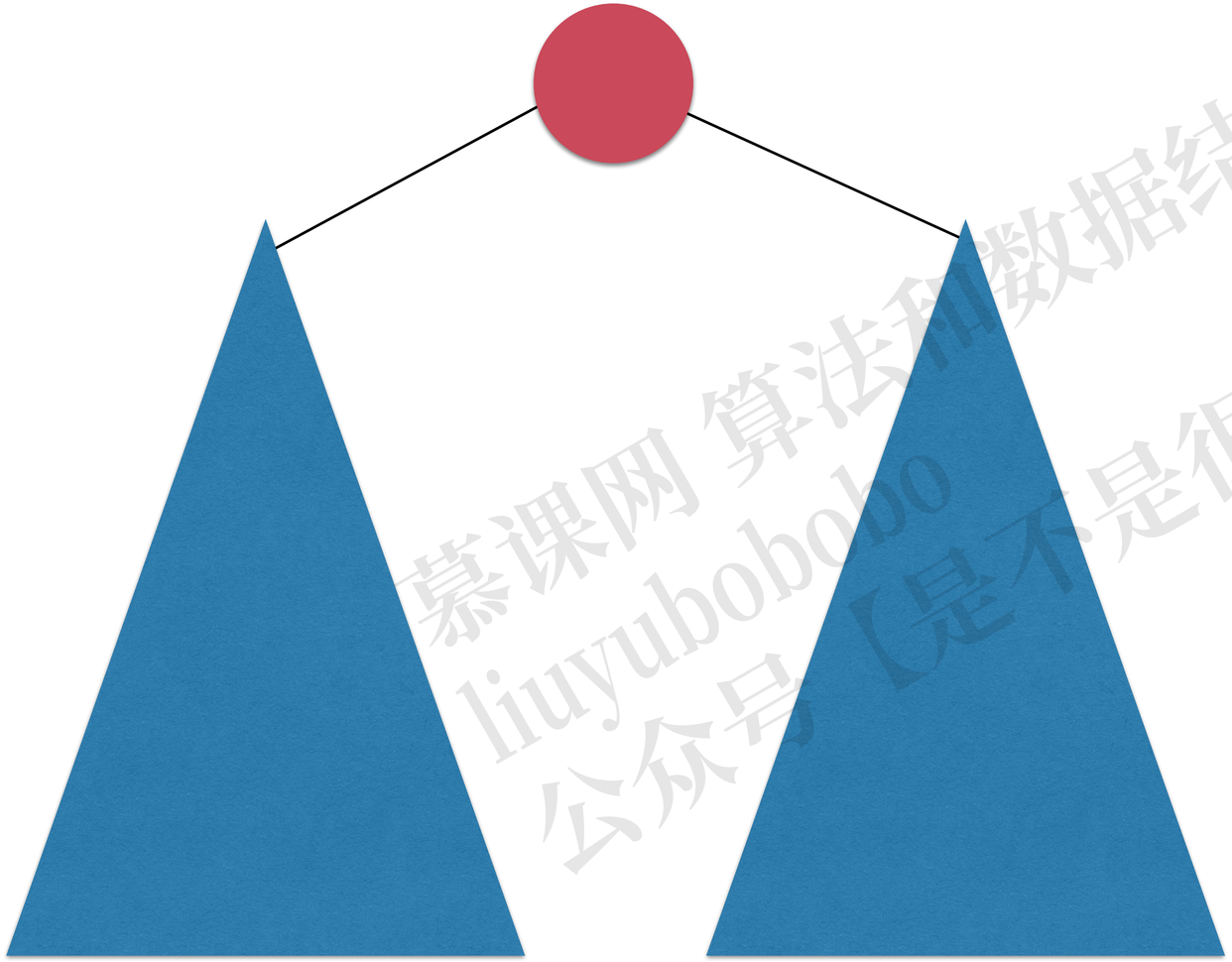
二分搜索树的递归操作



- 对于遍历操作，两棵子树都要顾及

```
function traverse(node):
    if(node == null)
        return;

访问该节点

traverse(node.left)
traverse(node.right)
```

二分搜索树的前序遍历



- 对于遍历操作，两棵子树都要顾及

```
function traverse(node):
    if(node == null)
        return;

    访问该节点

    traverse(node.left)
    traverse(node.right)
```

实践：二分搜索树的前序遍历

实践：二分搜索树的toString

二分搜索树的前中后序遍历

前序遍历

```
function traverse(node):
    if(node == null)
        return;

    访问该节点

    traverse(node.left)
    traverse(node.right)
```

前序遍历

```
function traverse(node):
    if(node == null)
        return;

    访问该节点
    traverse(node.left)
    traverse(node.right)
```

前序遍历

```
function traverse(node):
    if(node == null)
        return;

    访问该节点
    traverse(node.left)
    traverse(node.right)
```

- 最自然的遍历方式
- 最常用的遍历方式

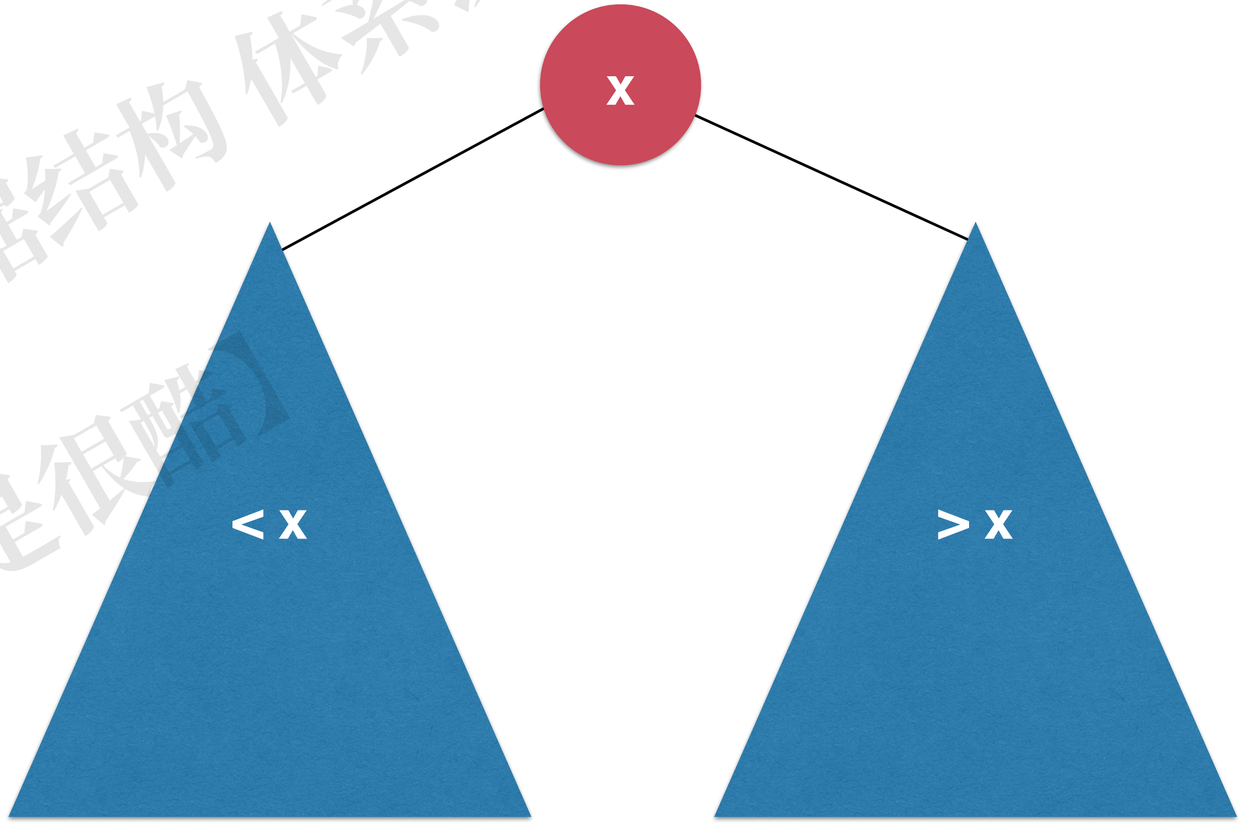
中序遍历

```
function traverse(node):
    if(node == null)
        return;

    traverse(node.left)
    访问该节点
    traverse(node.right)
```

实践：二分搜索树的中序遍历

中序遍历

```
function traverse(node):
    if(node == null)
        return;

    traverse(node.left)
    访问该节点
    traverse(node.right)
```



- 二分搜索树的中序遍历结果是顺序的

后序遍历

```
function traverse(node):
    if(node == null)
        return;

    traverse(node.left)
    traverse(node.right)
    访问该节点
```

实践：二分搜索树的后序遍历

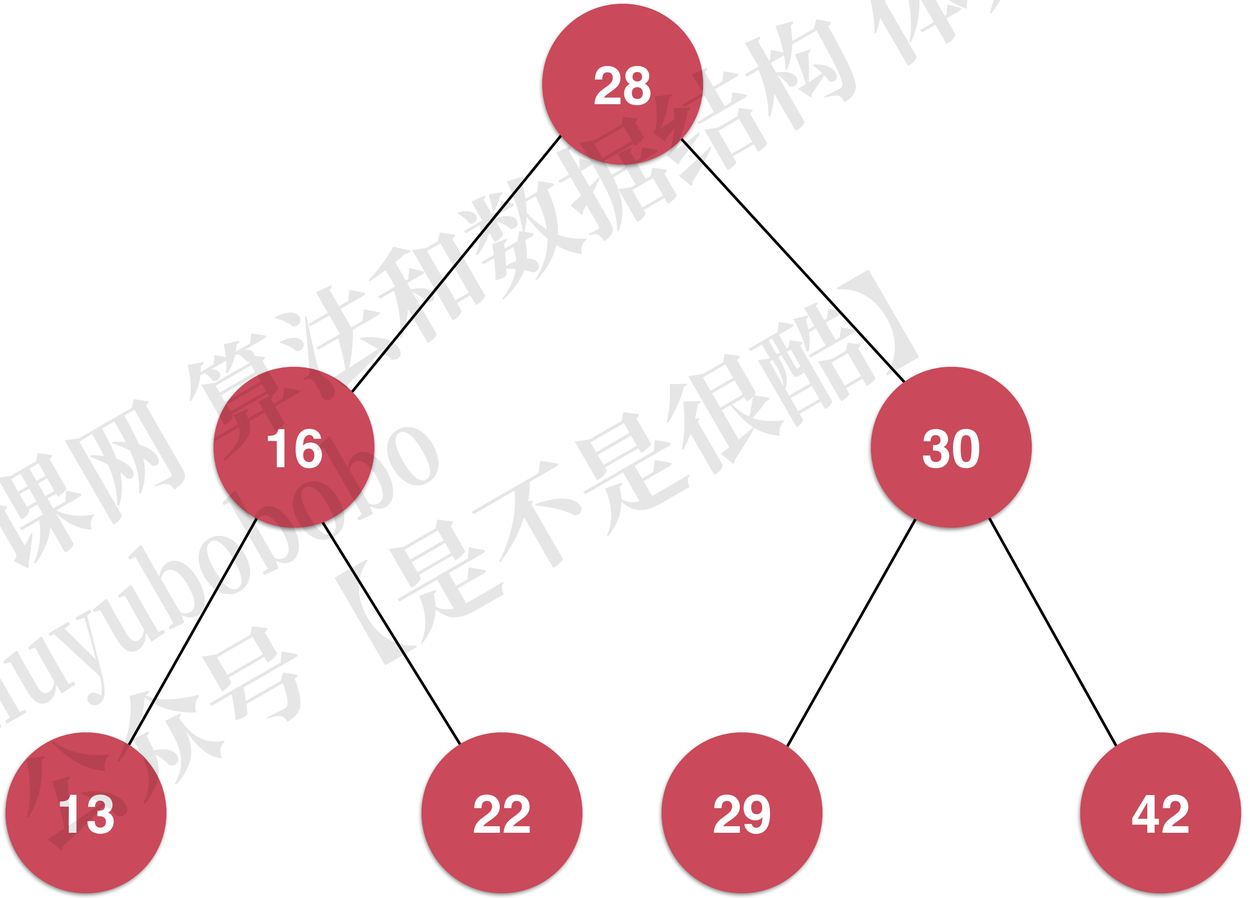
后序遍历

```
function traverse(node):
    if(node == null)
        return;

    traverse(node.left)
    traverse(node.right)
    访问该节点
```

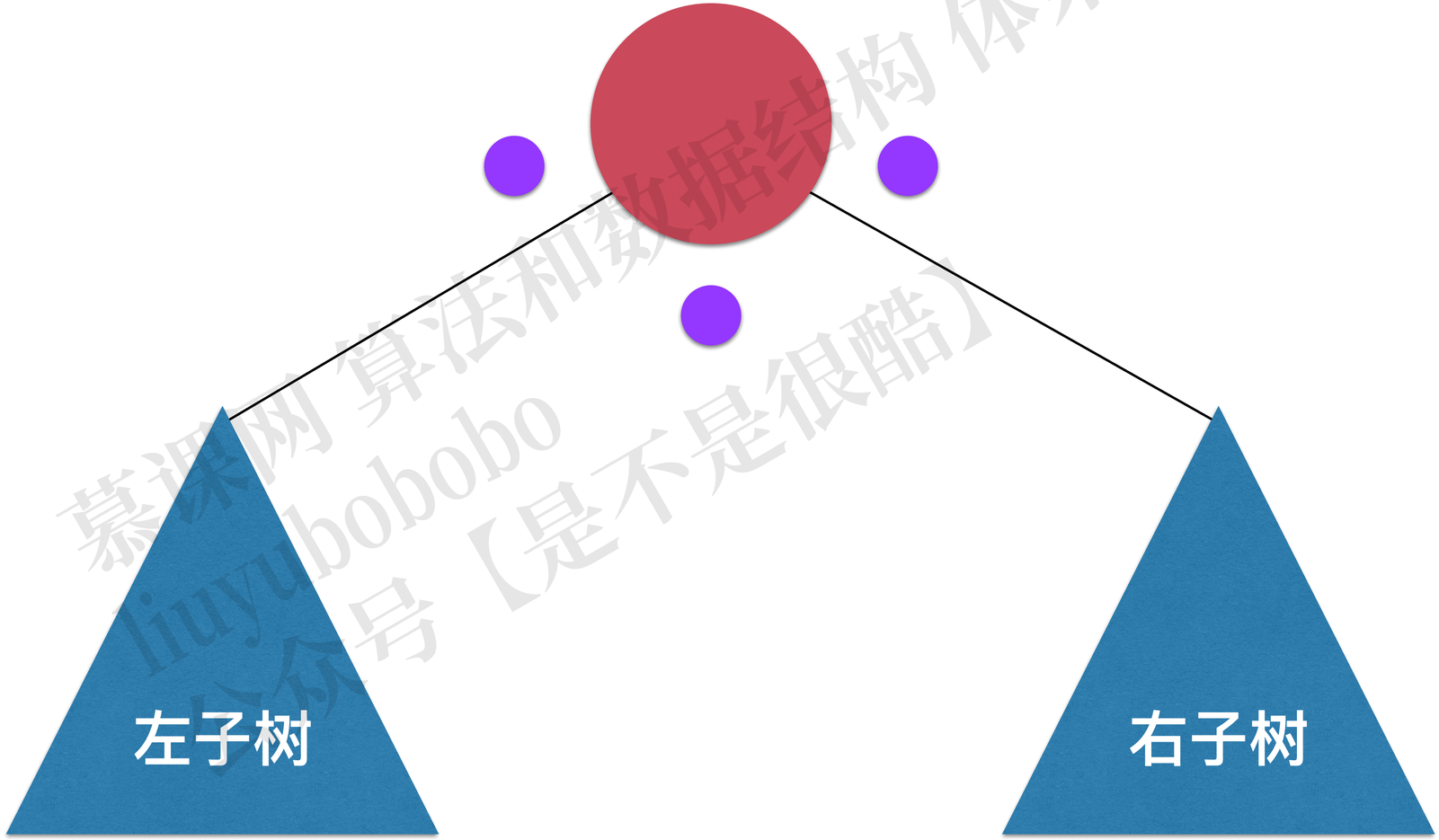
- 后序遍历的一个应用：
- 为二分搜索树释放内存

再看二分搜索树的遍历

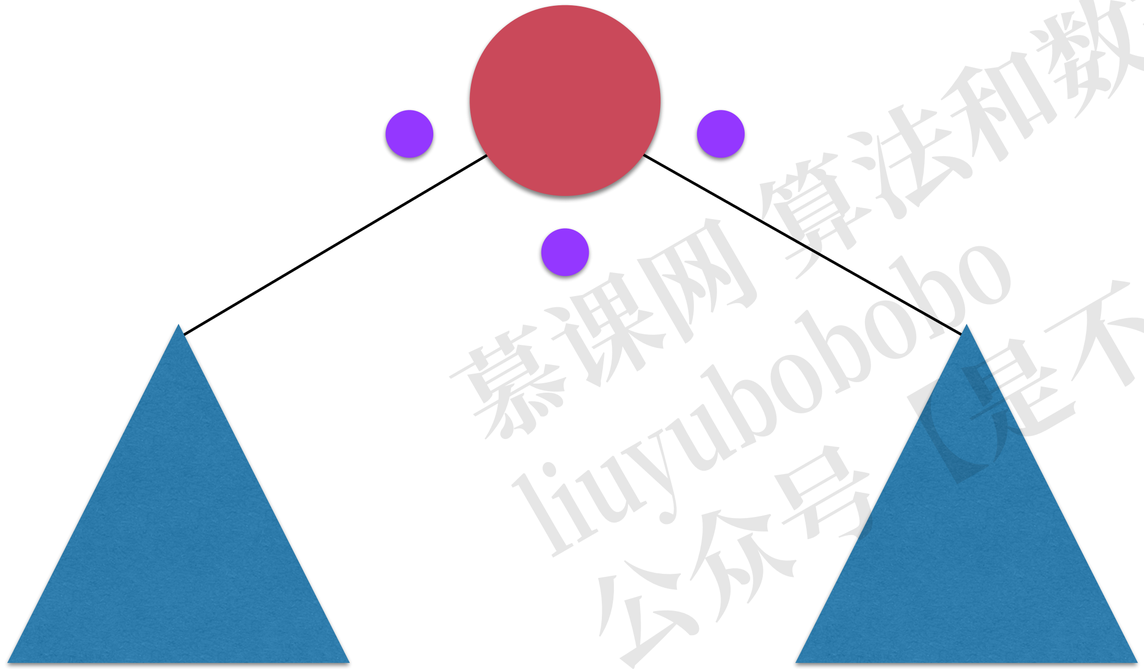
二分搜索树的遍历



二分搜索树的遍历



二分搜索树的遍历



```
function traverse(node):
    if(node == null) return;
```

- 访问该节点?

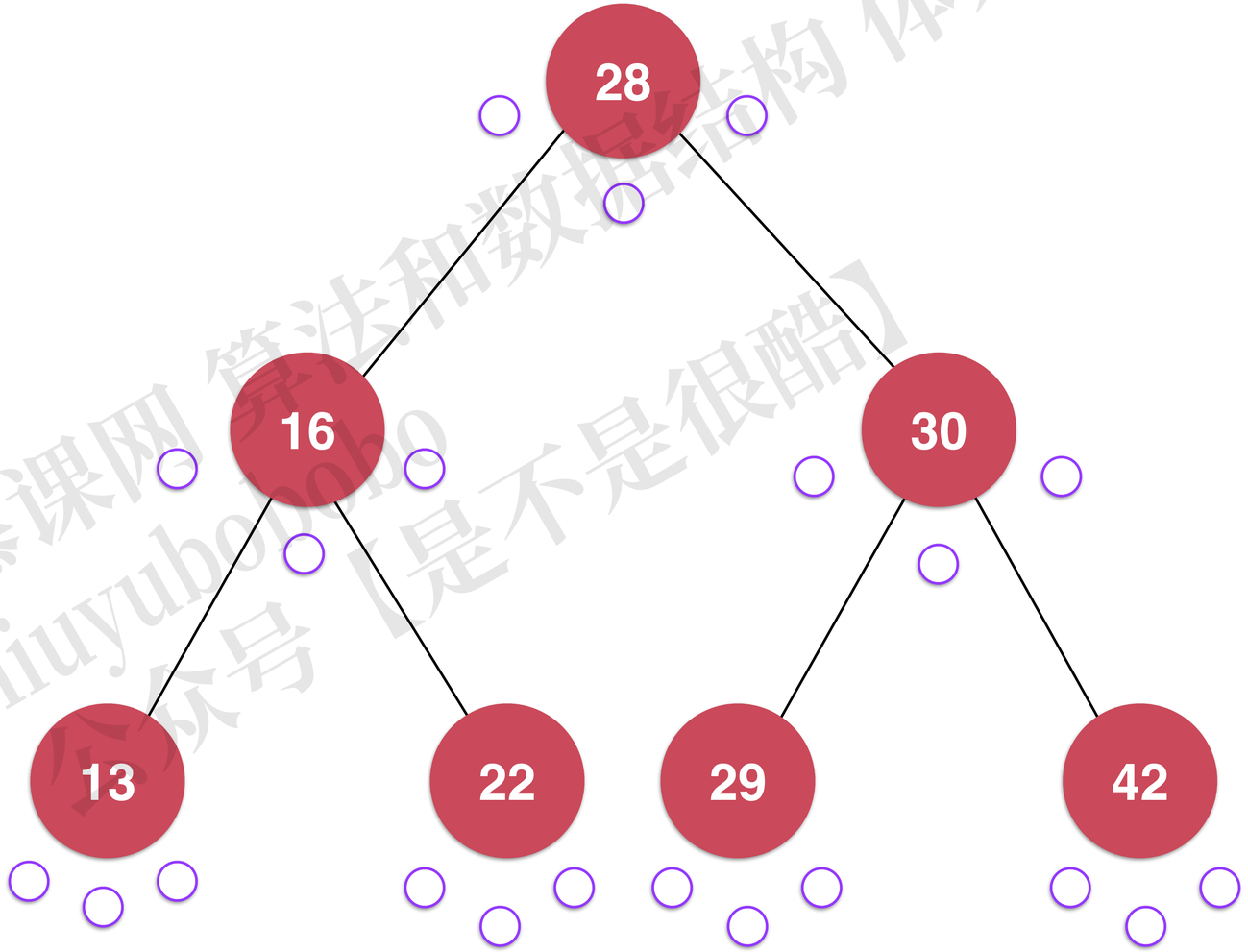
  traverse(node.left)

- 访问该节点?

  traverse(node.right)

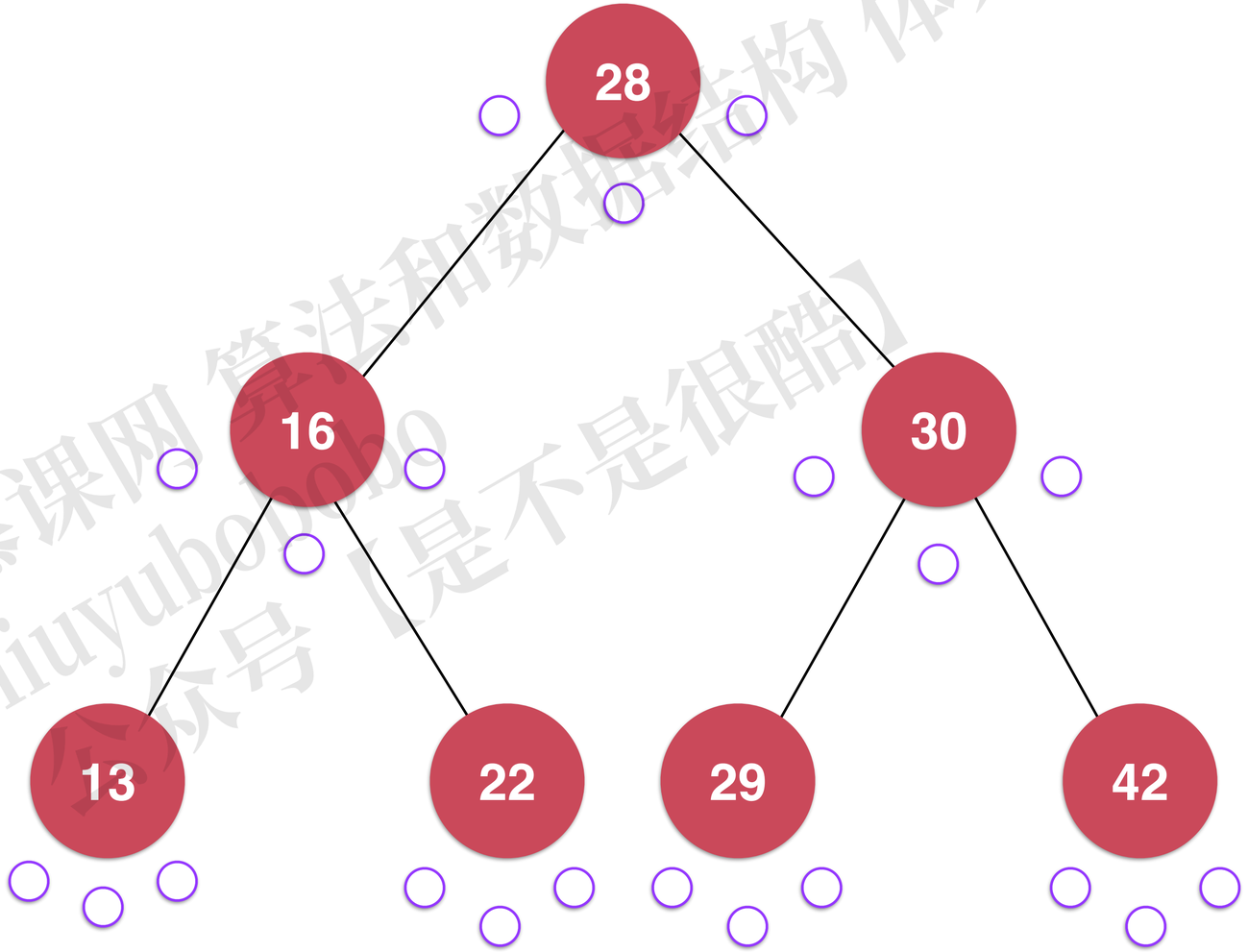
- 访问该节点?

二分搜索树的遍历

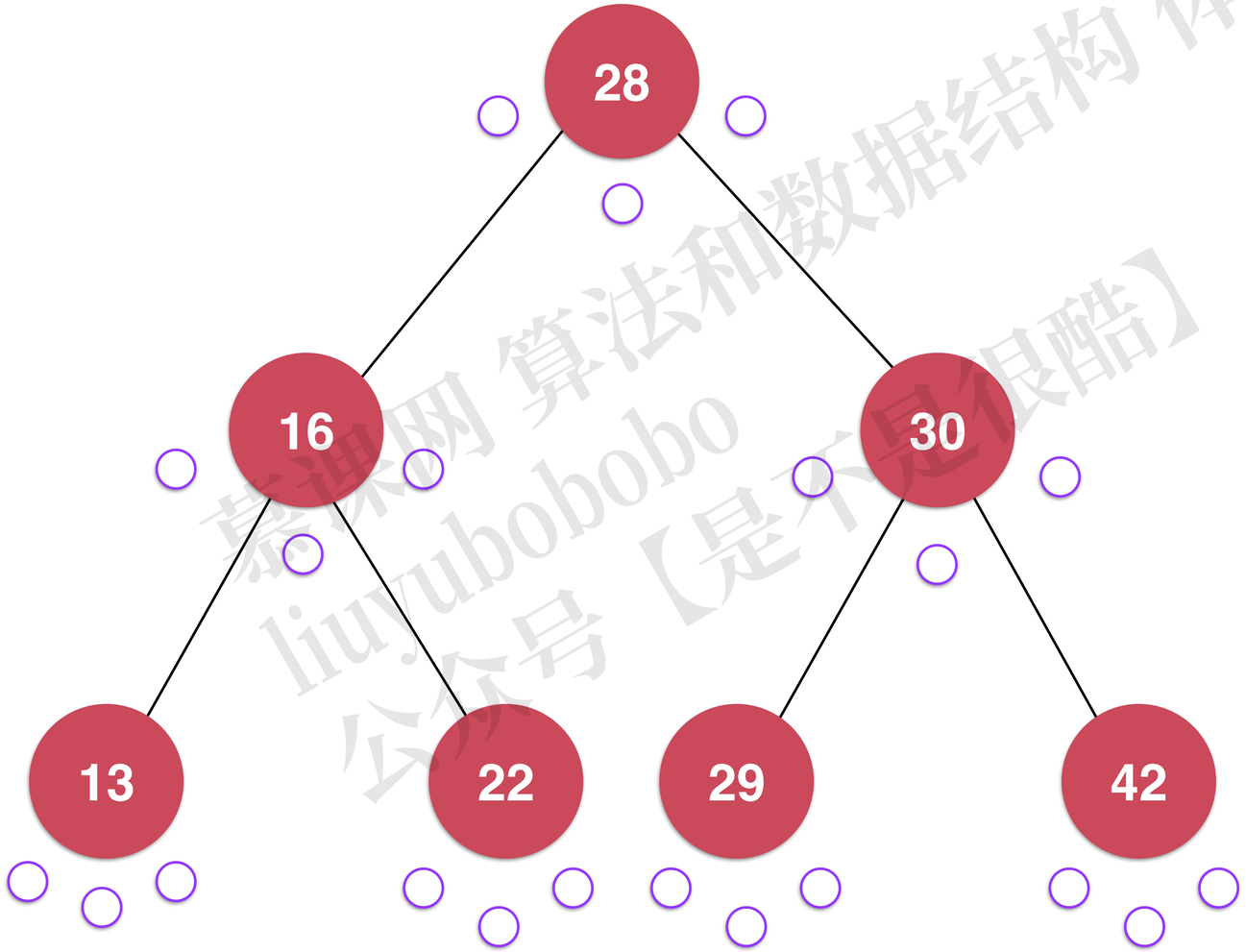


再看二分搜索树的前序遍历

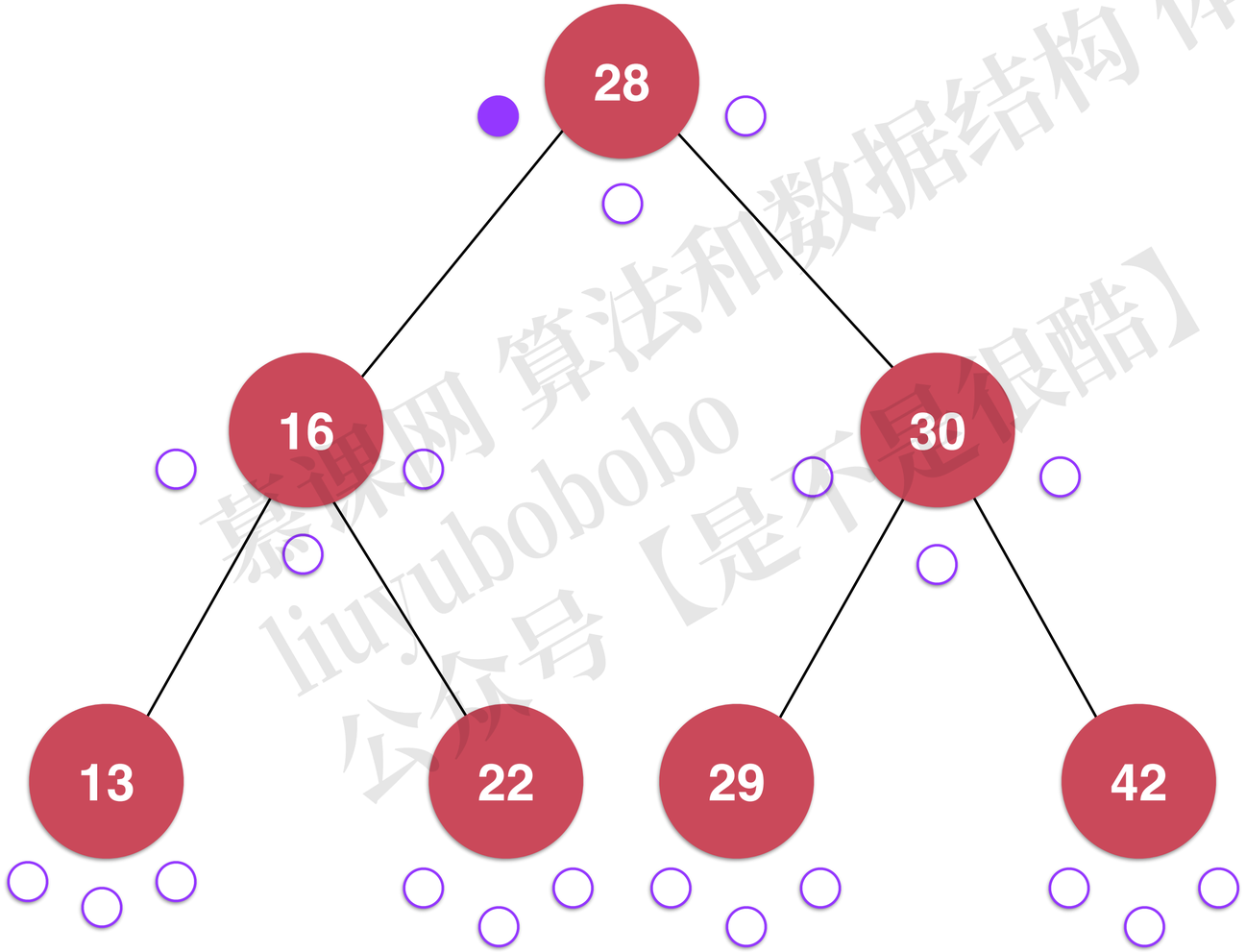
二分搜索树的遍历



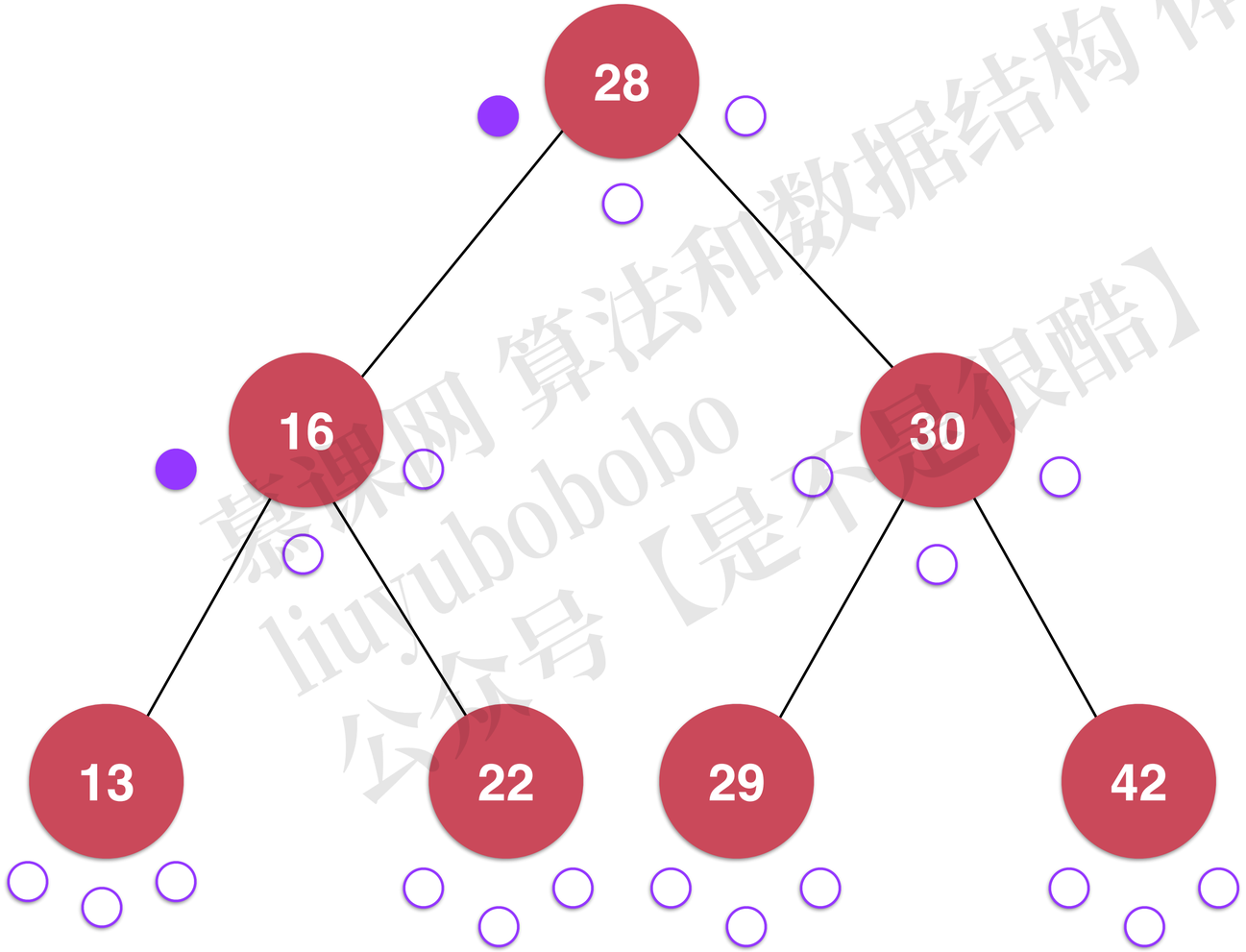
二分搜索树的前序遍历



二分搜索树的前序遍历



二分搜索树的前序遍历



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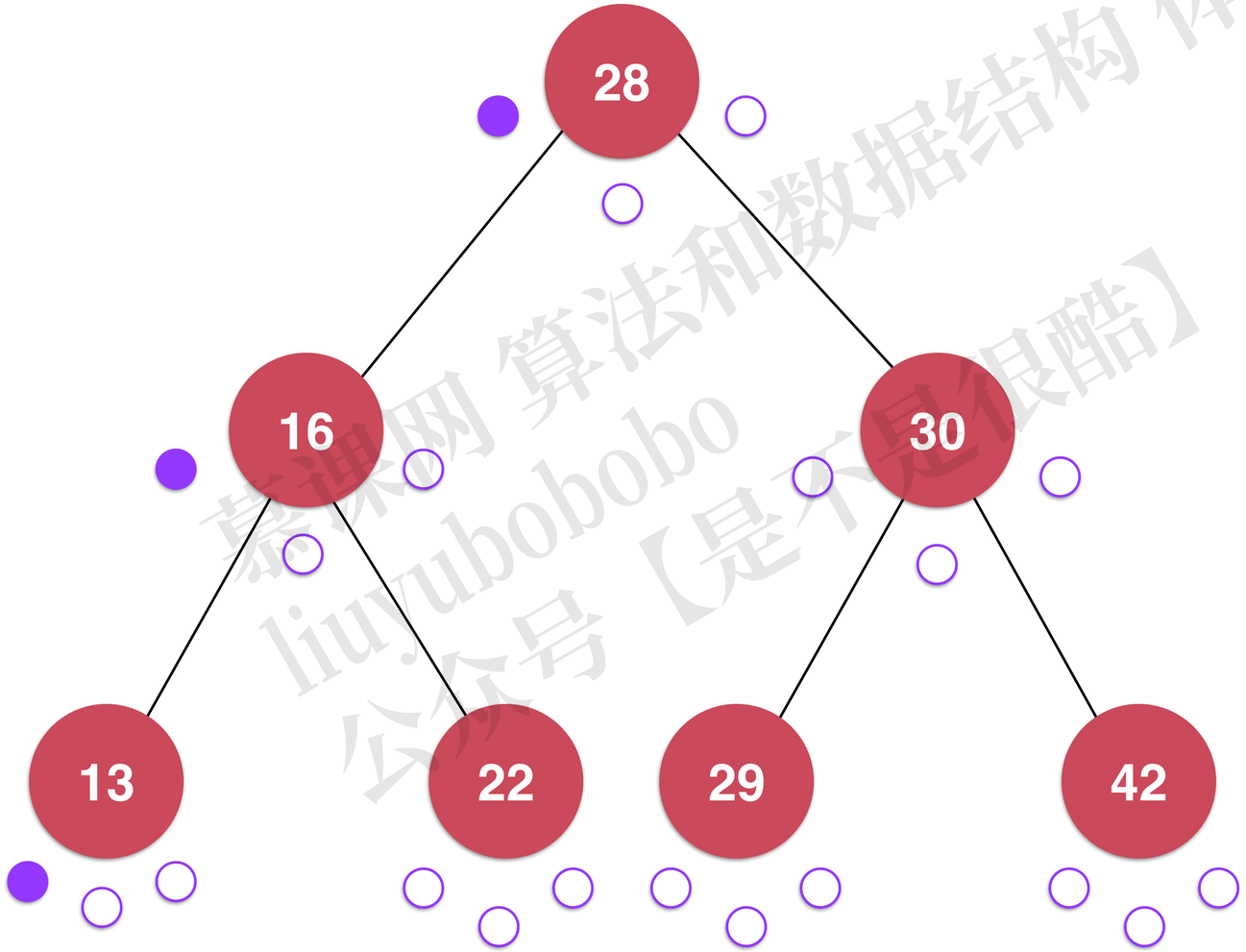
二分搜索树的前序遍历



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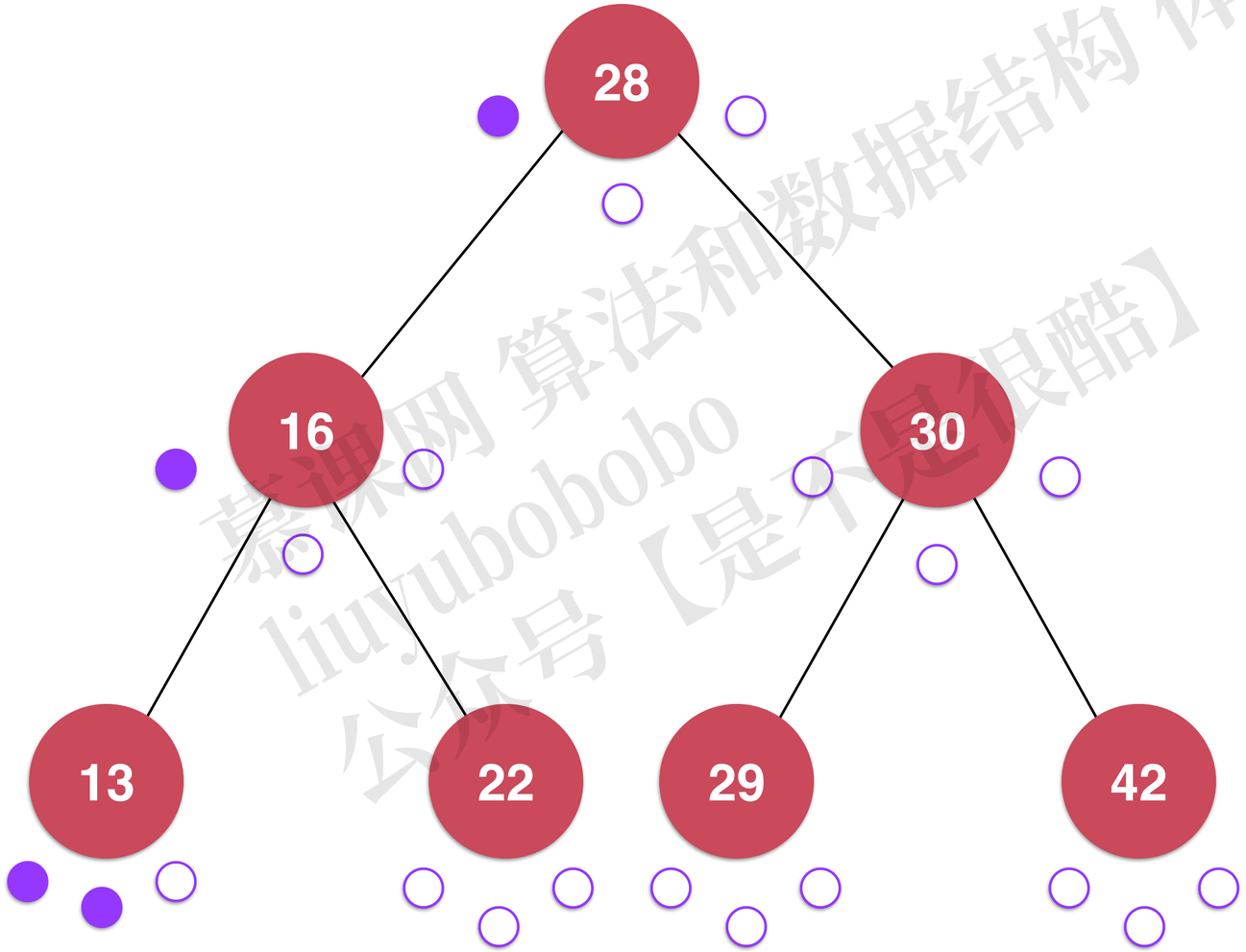
二分搜索树的前序遍历



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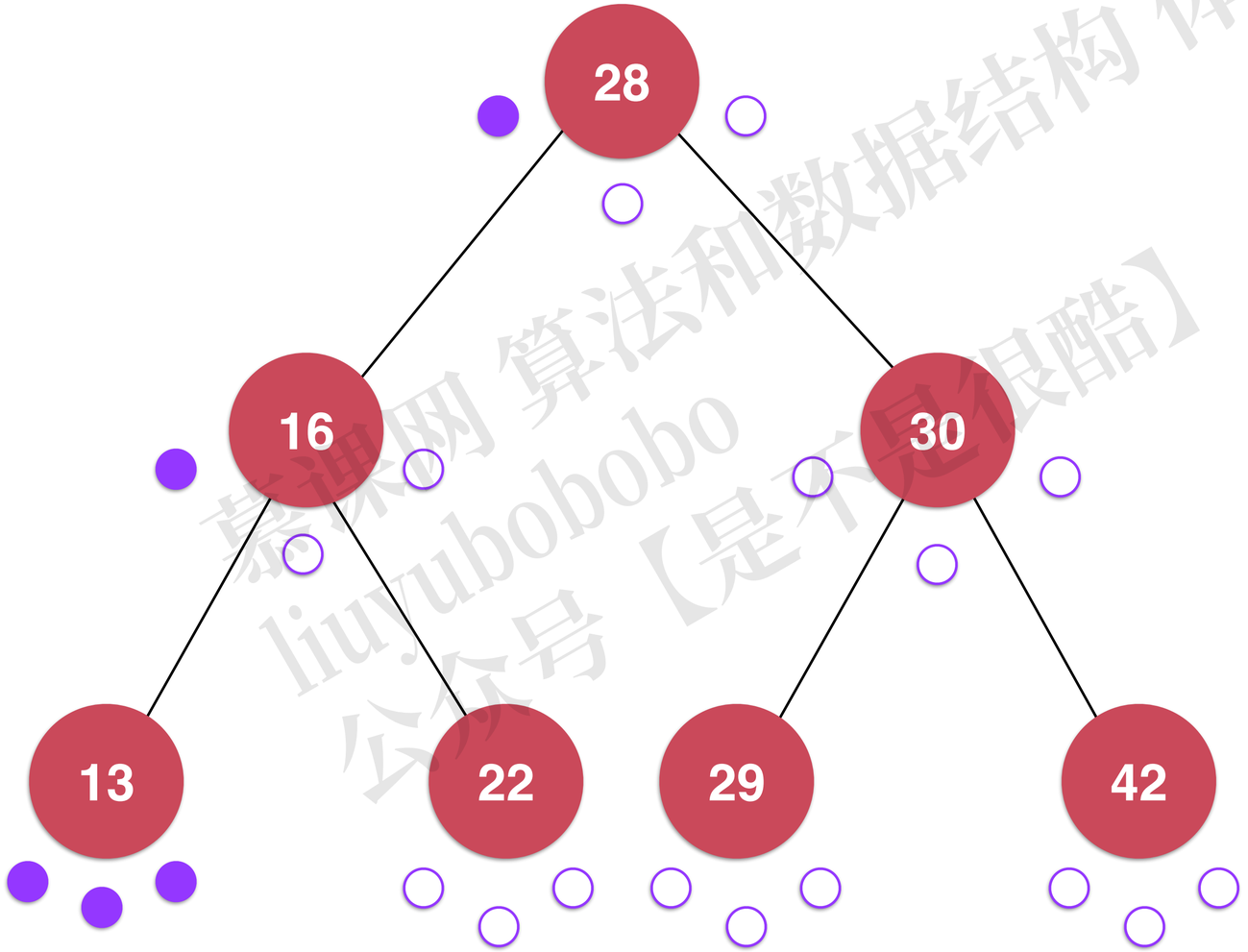
二分搜索树的前序遍历



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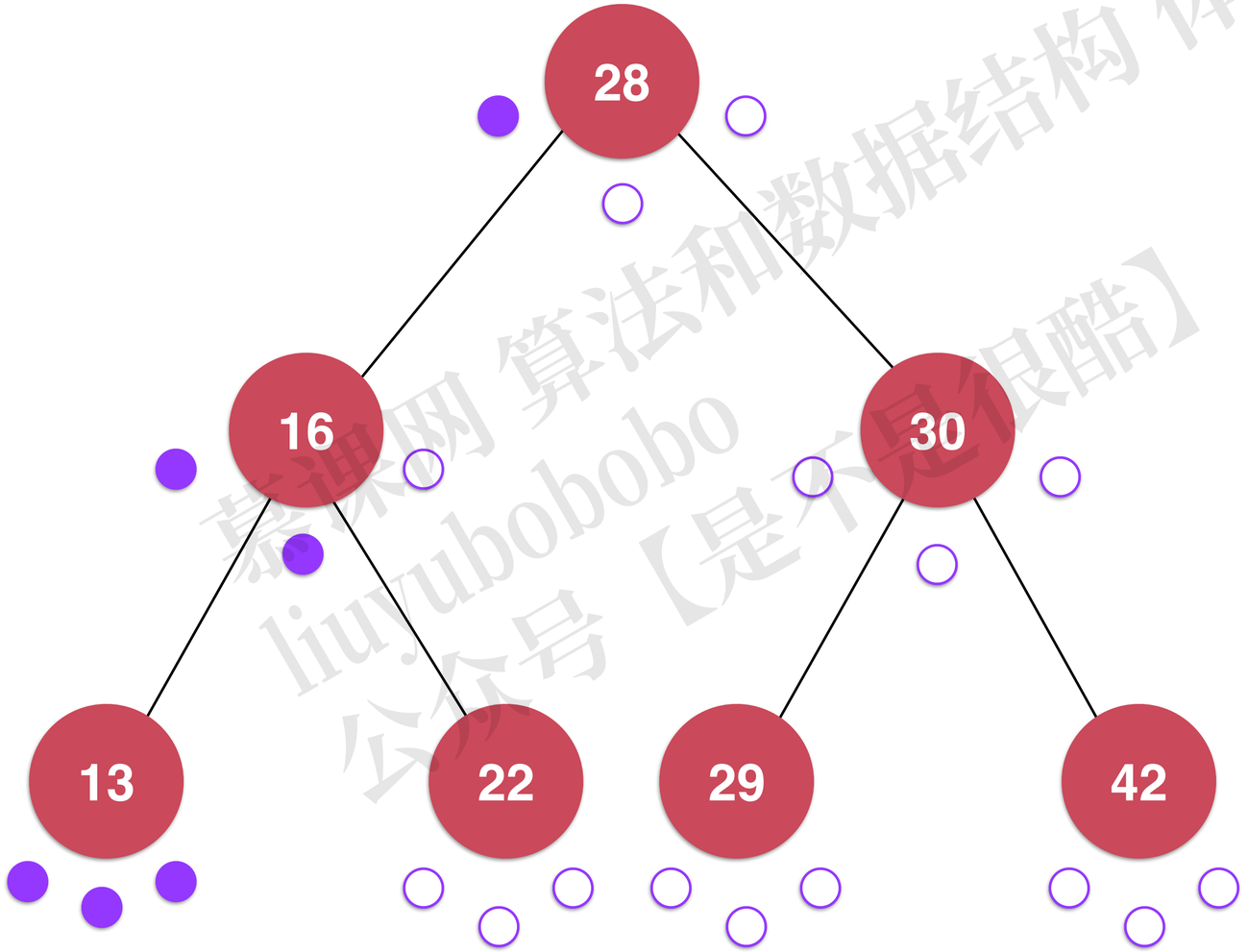
二分搜索树的前序遍历



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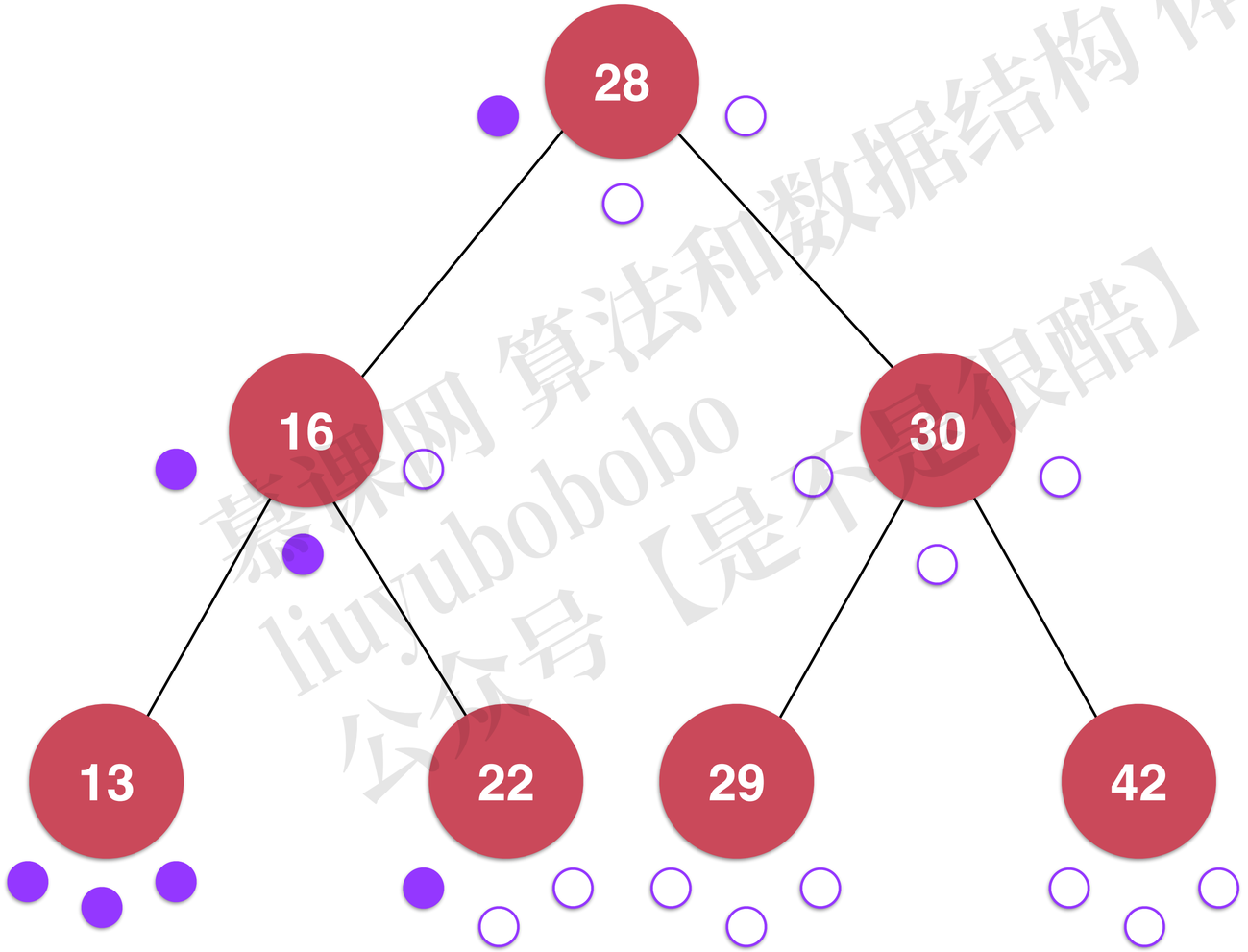
二分搜索树的前序遍历



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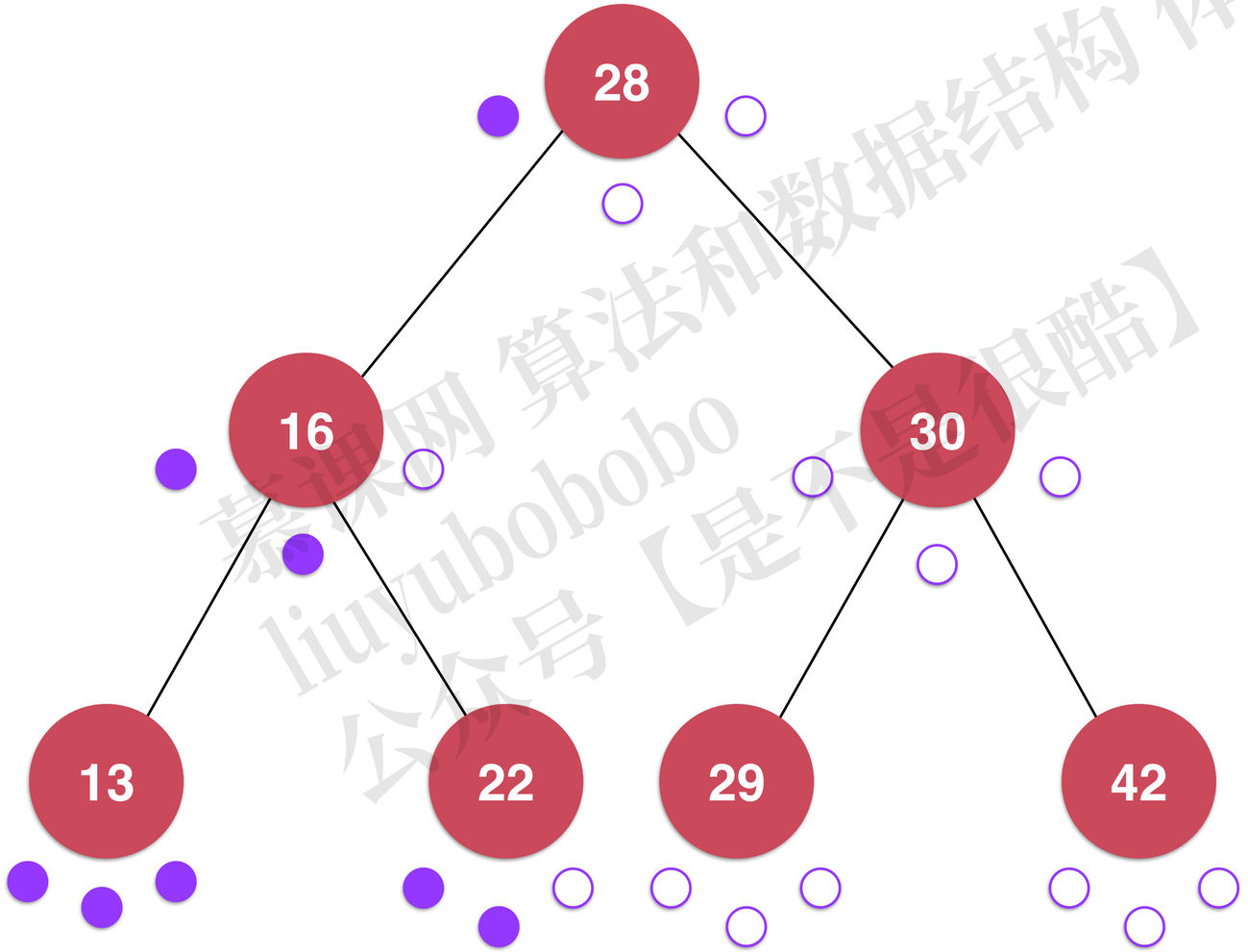
二分搜索树的前序遍历



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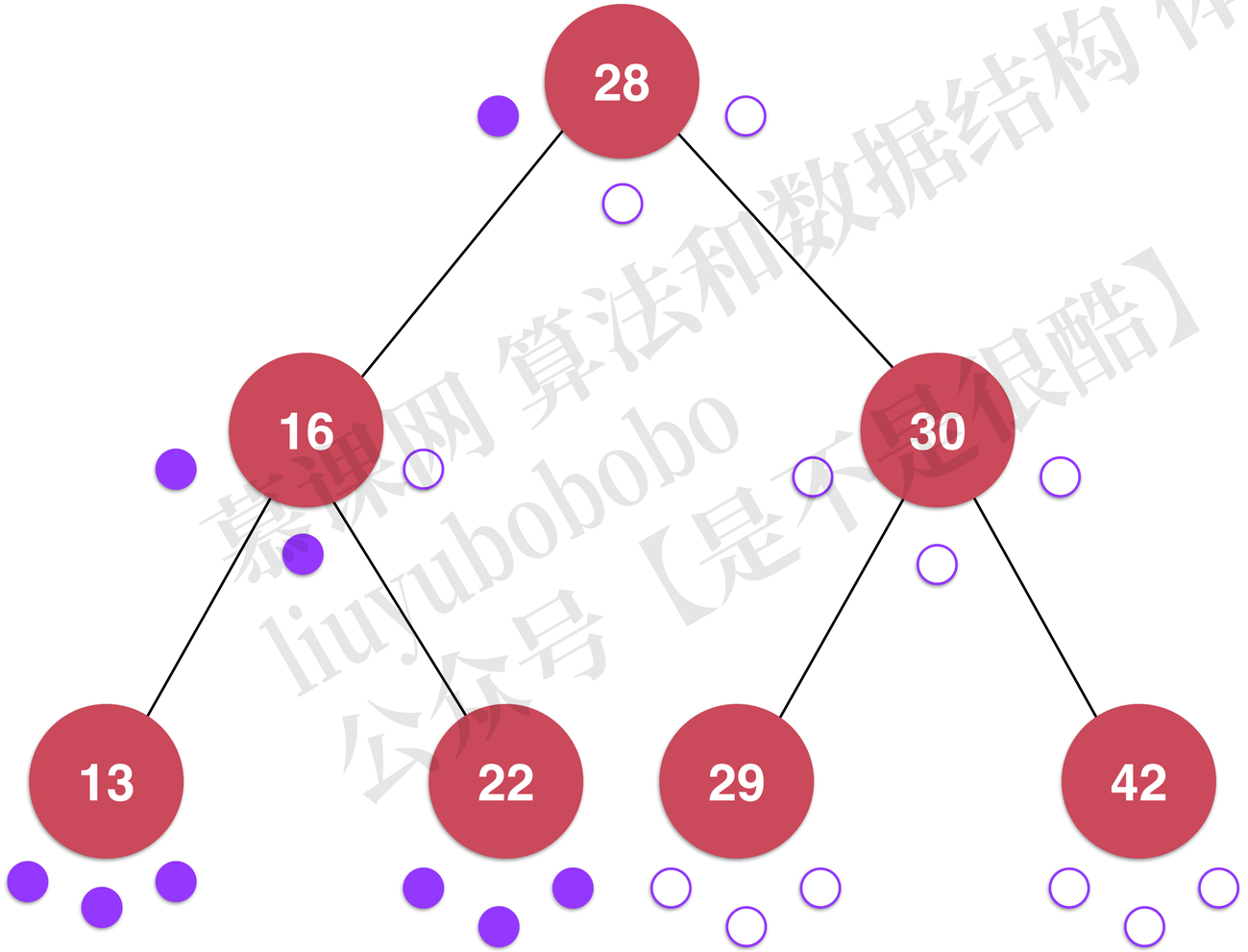
二分搜索树的前序遍历



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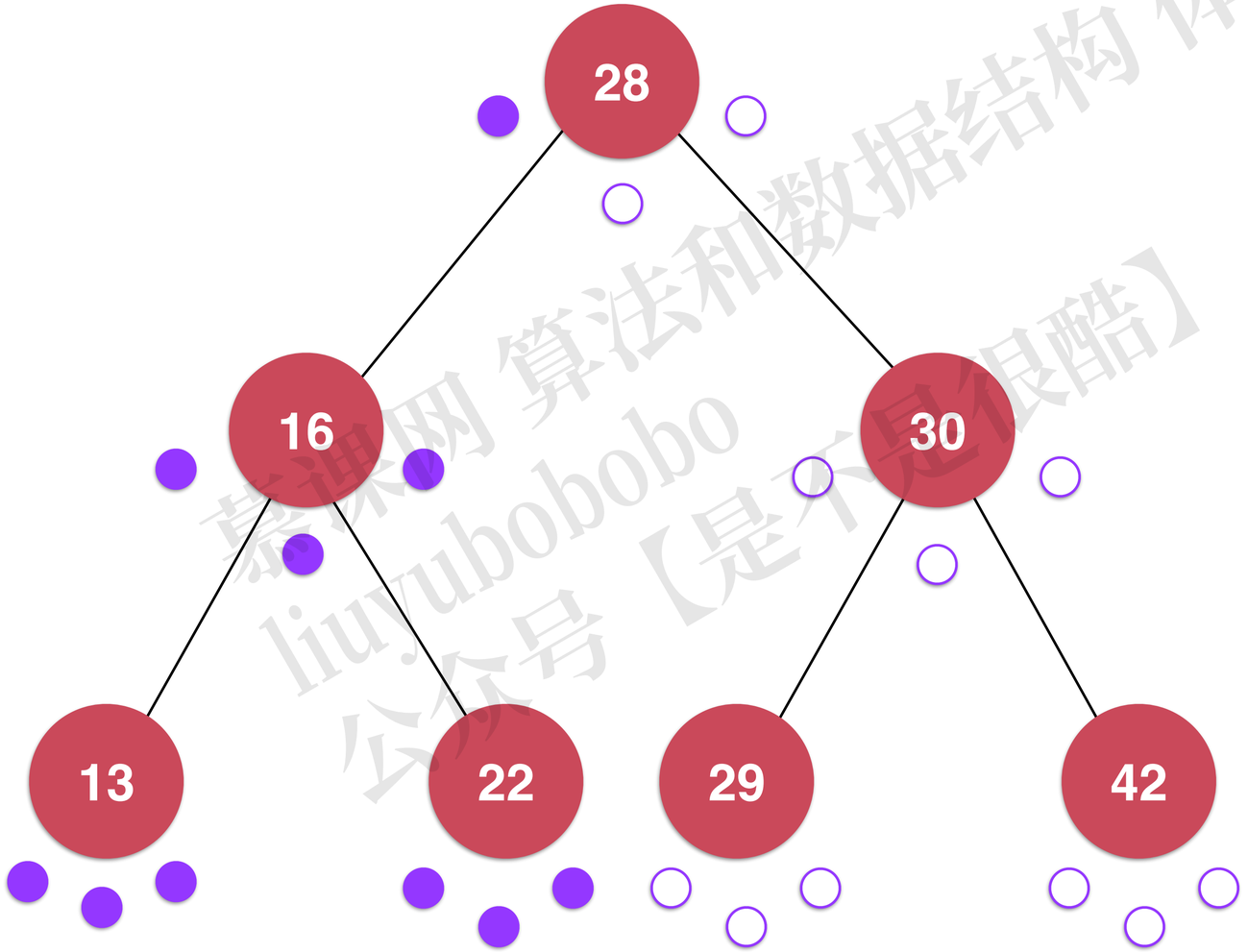
二分搜索树的前序遍历



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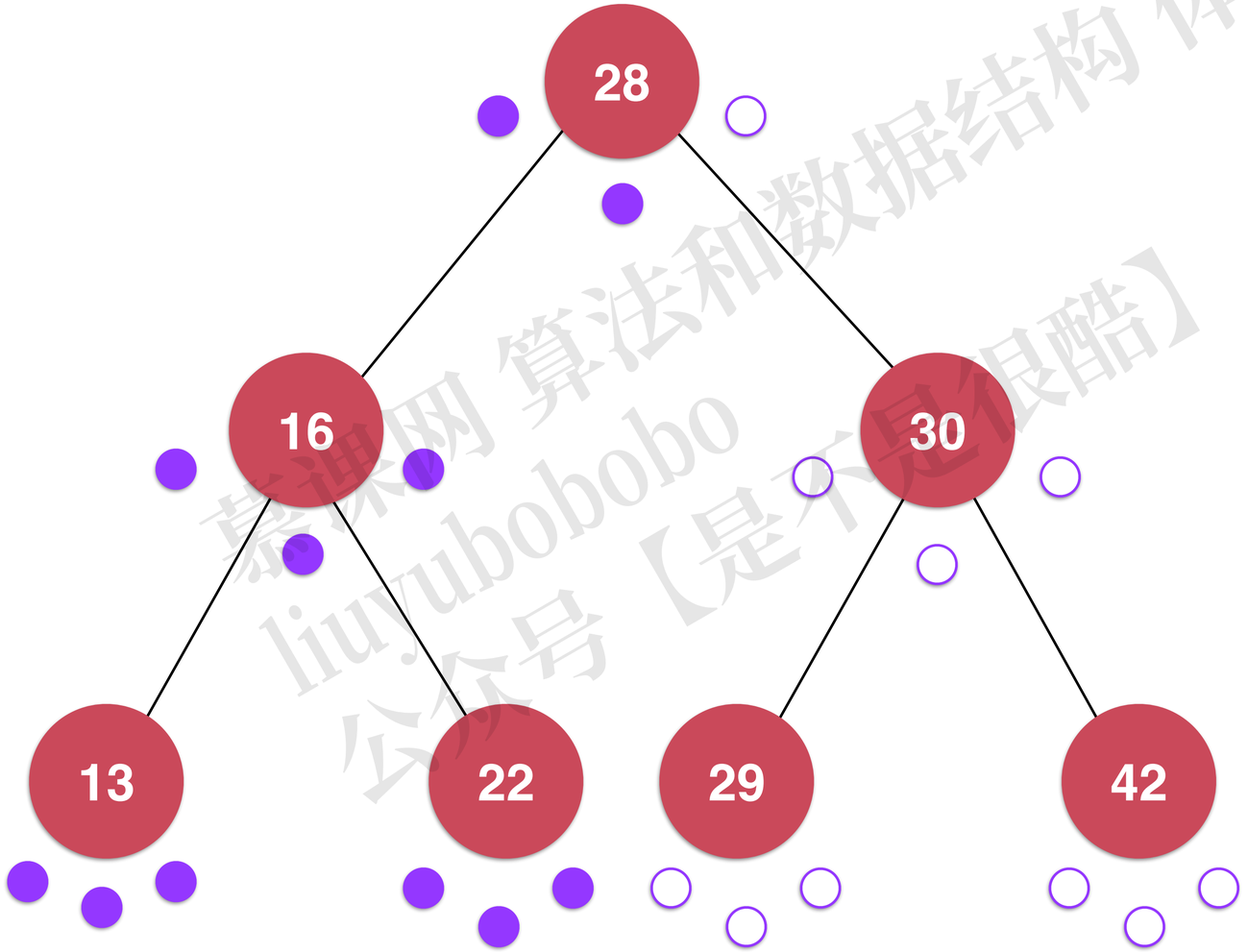
二分搜索树的前序遍历



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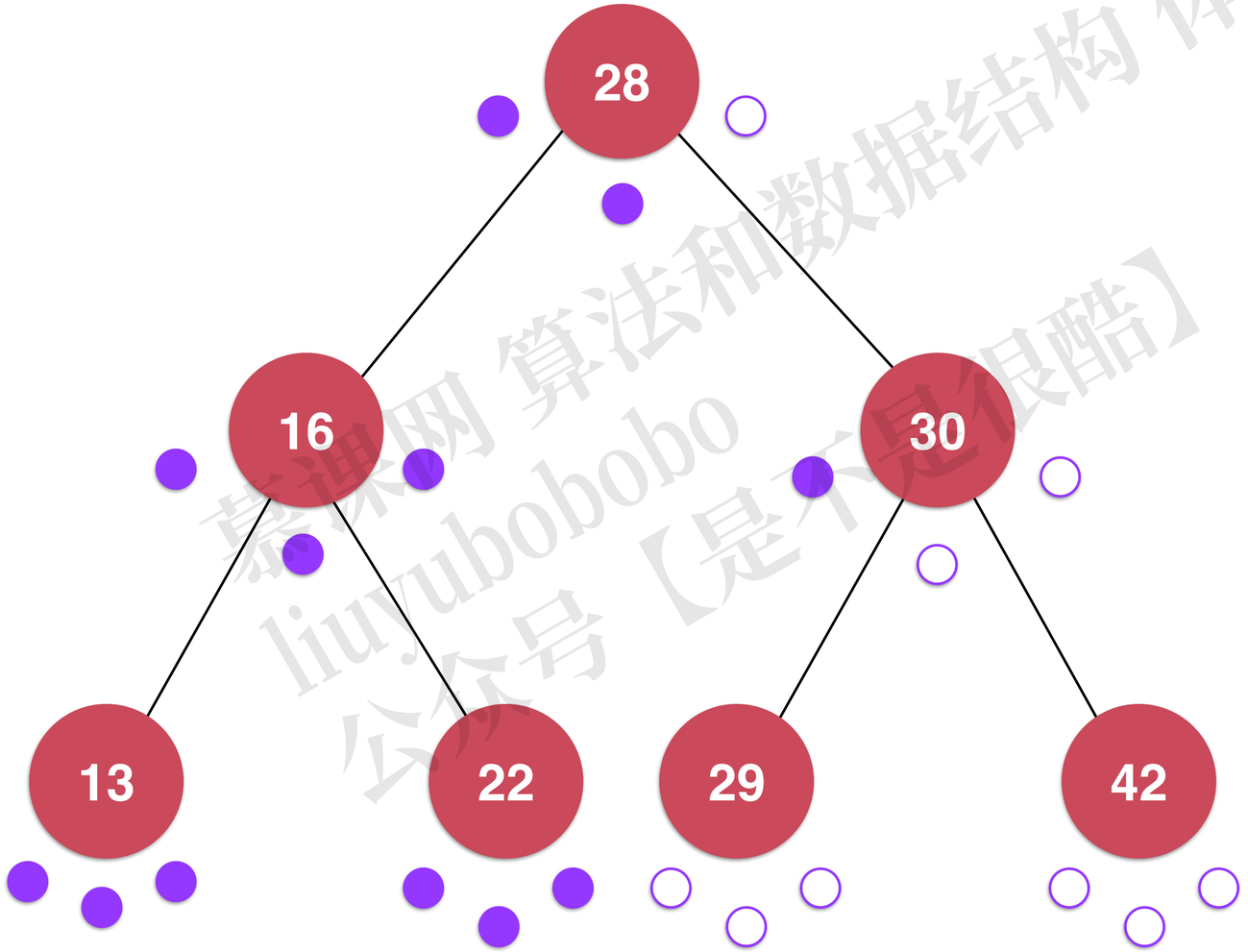
二分搜索树的前序遍历



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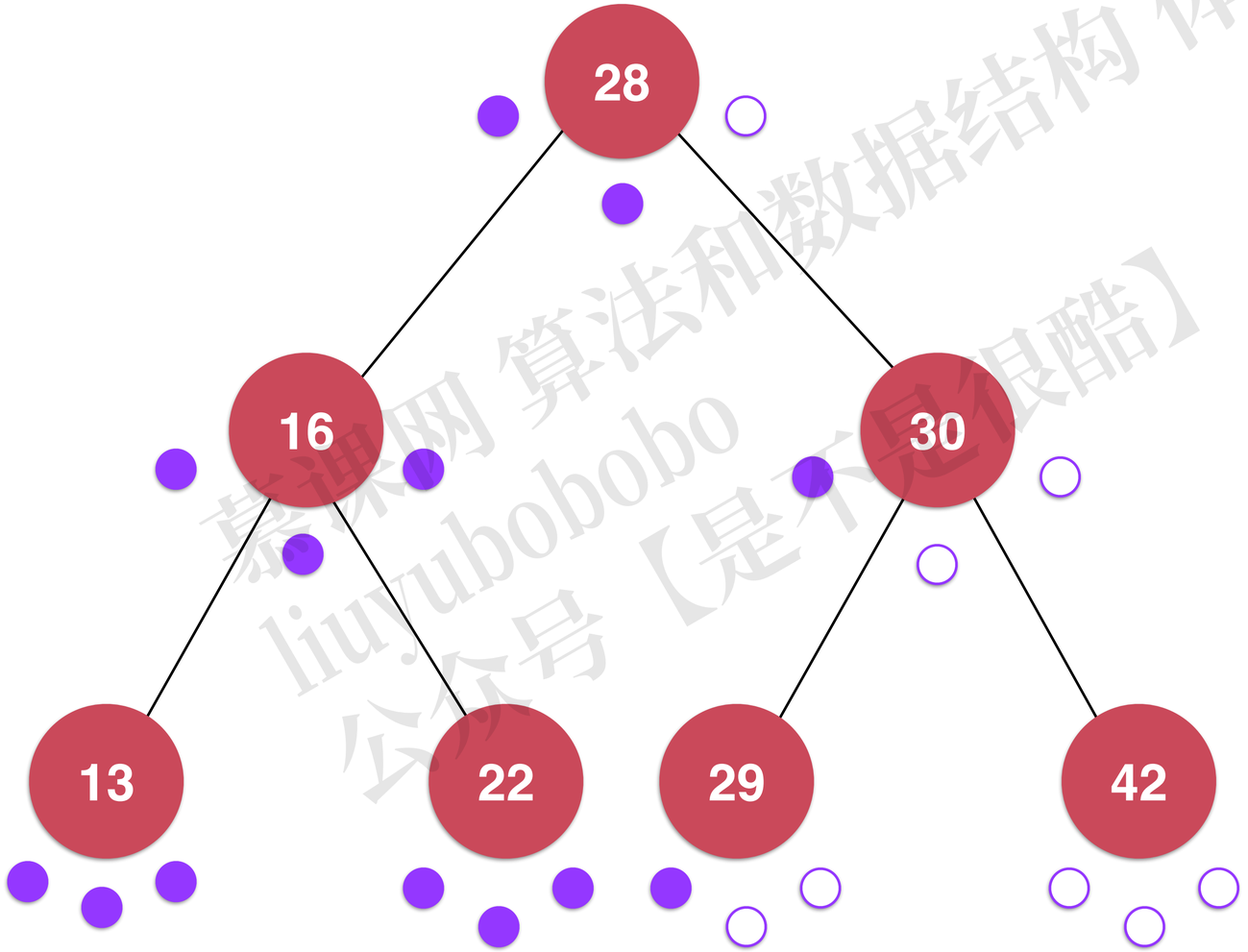
二分搜索树的前序遍历



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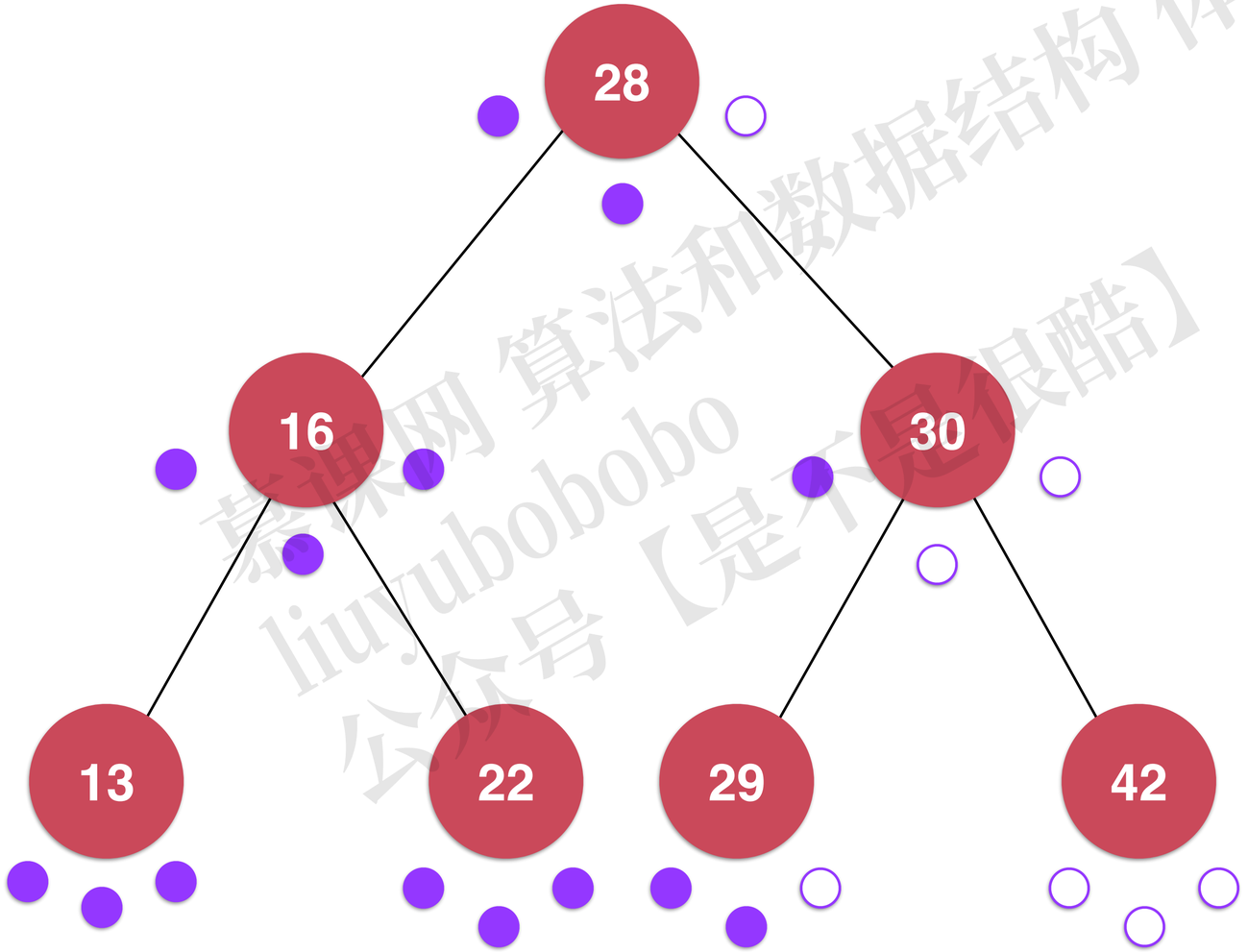
二分搜索树的前序遍历



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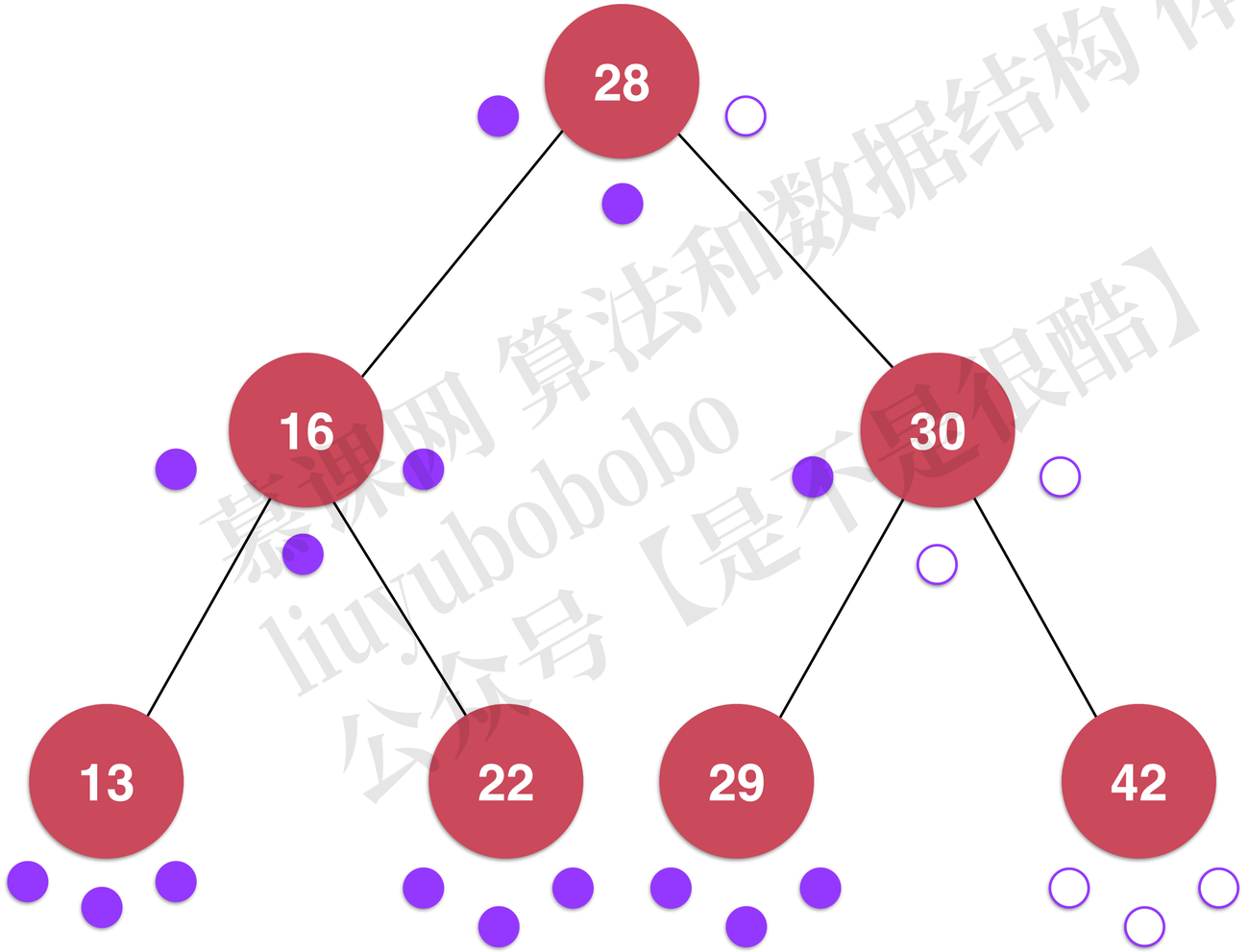
二分搜索树的前序遍历



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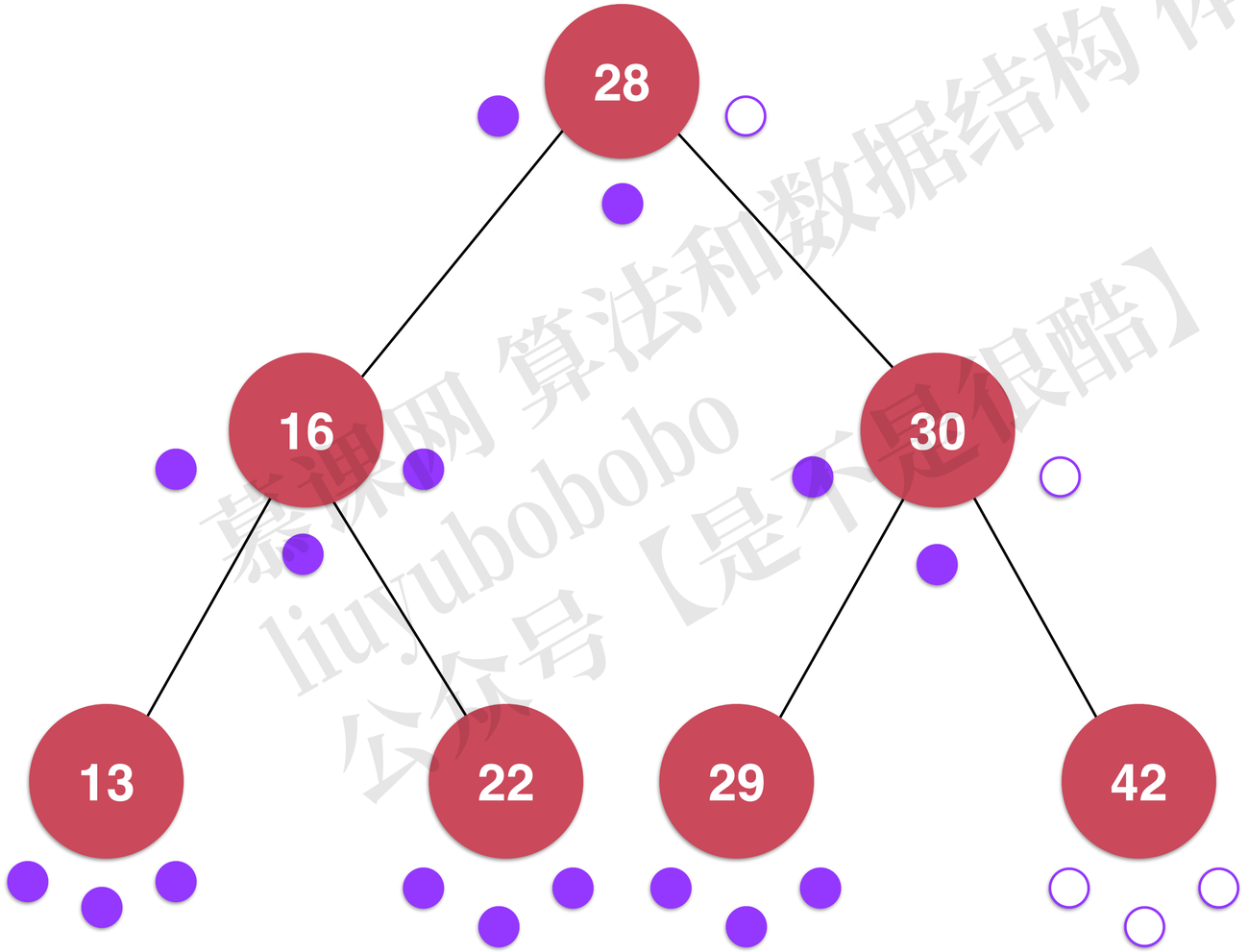
二分搜索树的前序遍历



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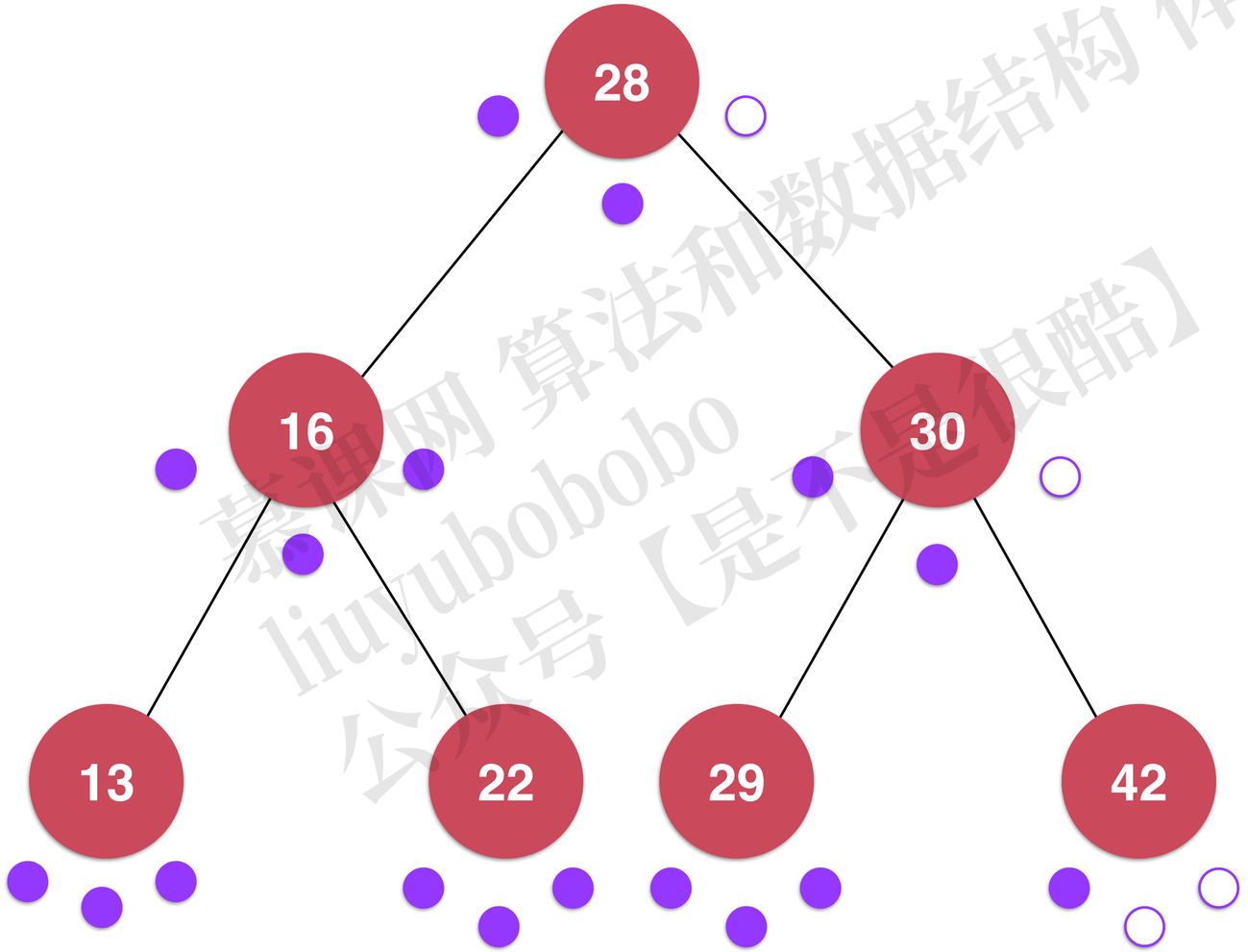
二分搜索树的前序遍历



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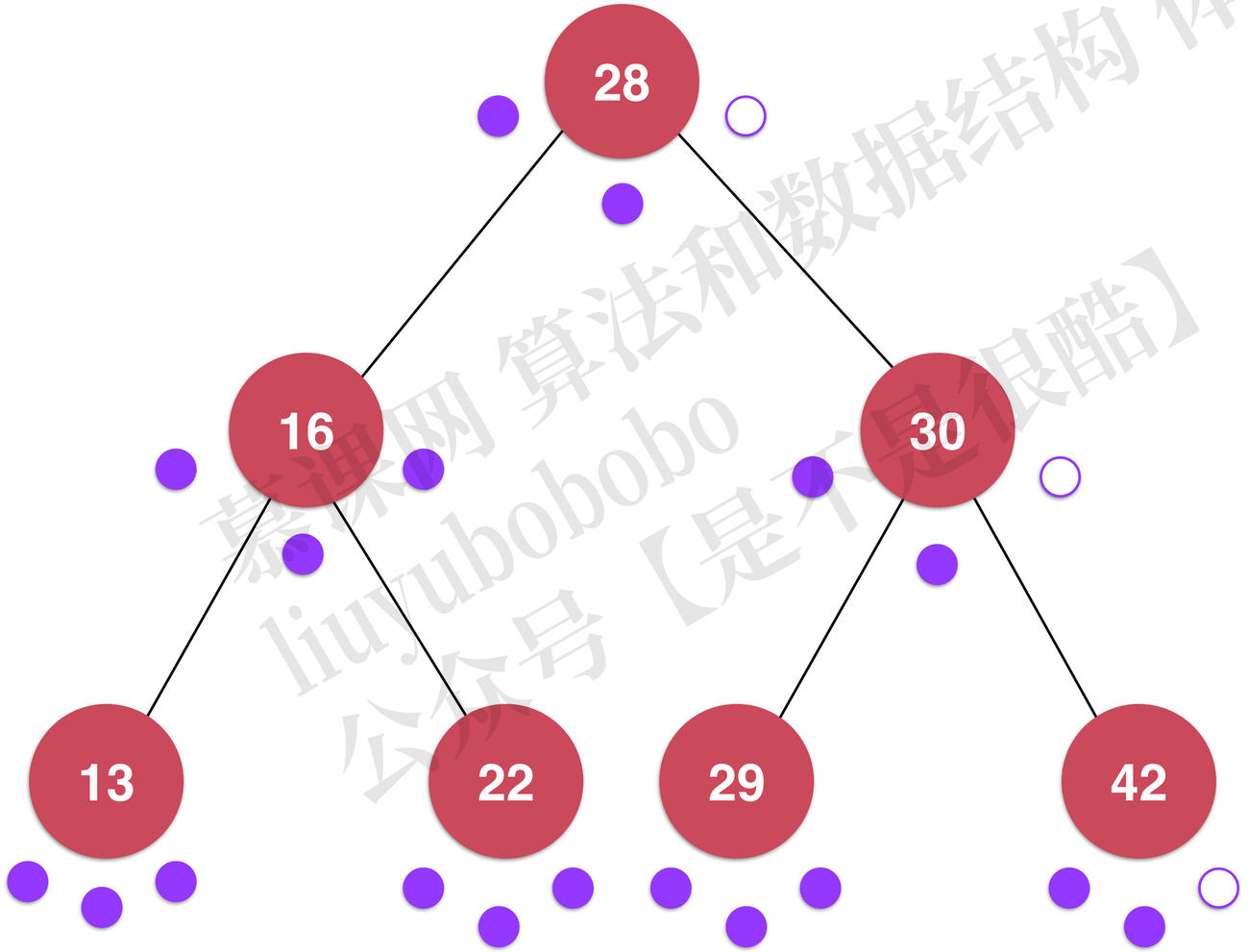
二分搜索树的前序遍历



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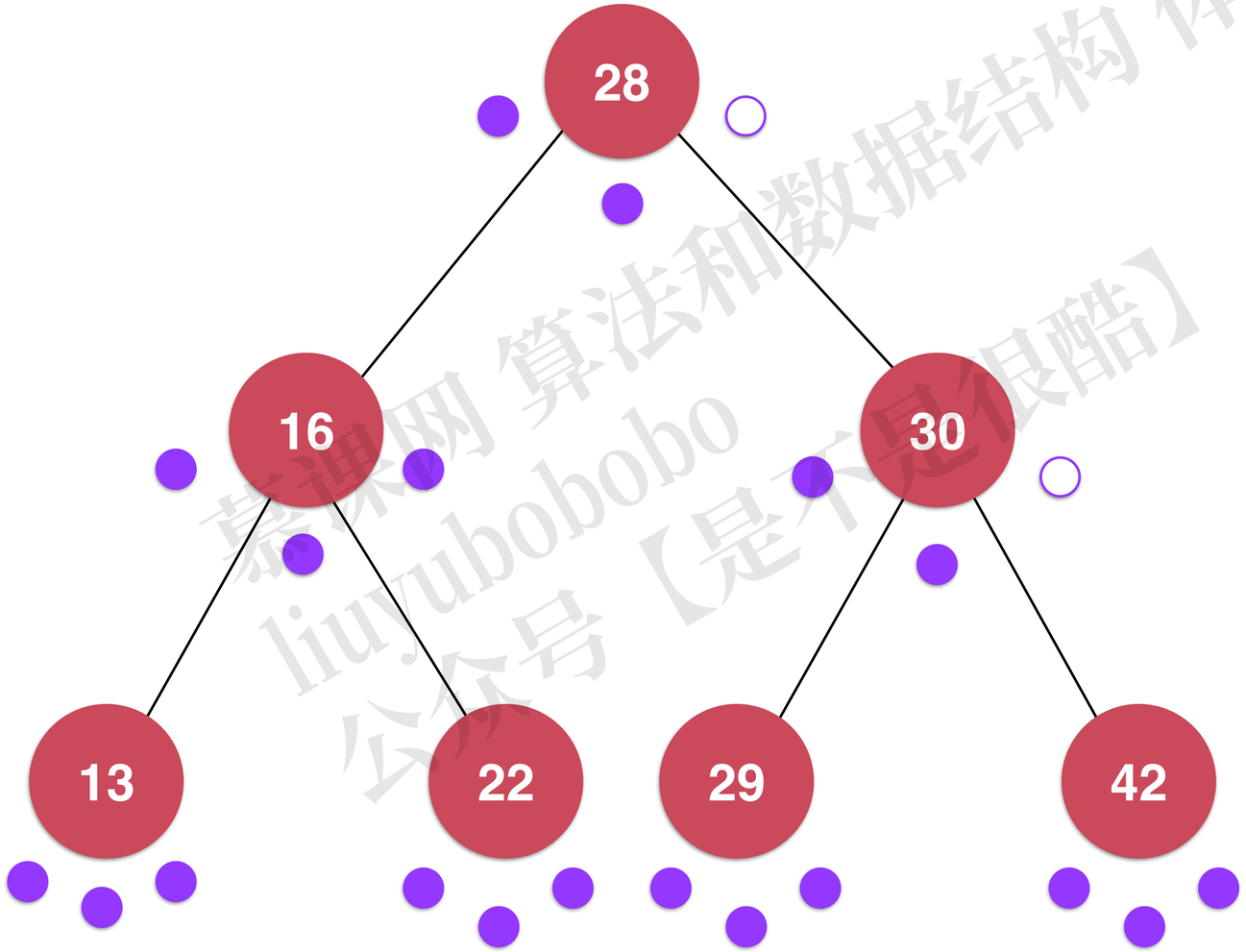
二分搜索树的前序遍历



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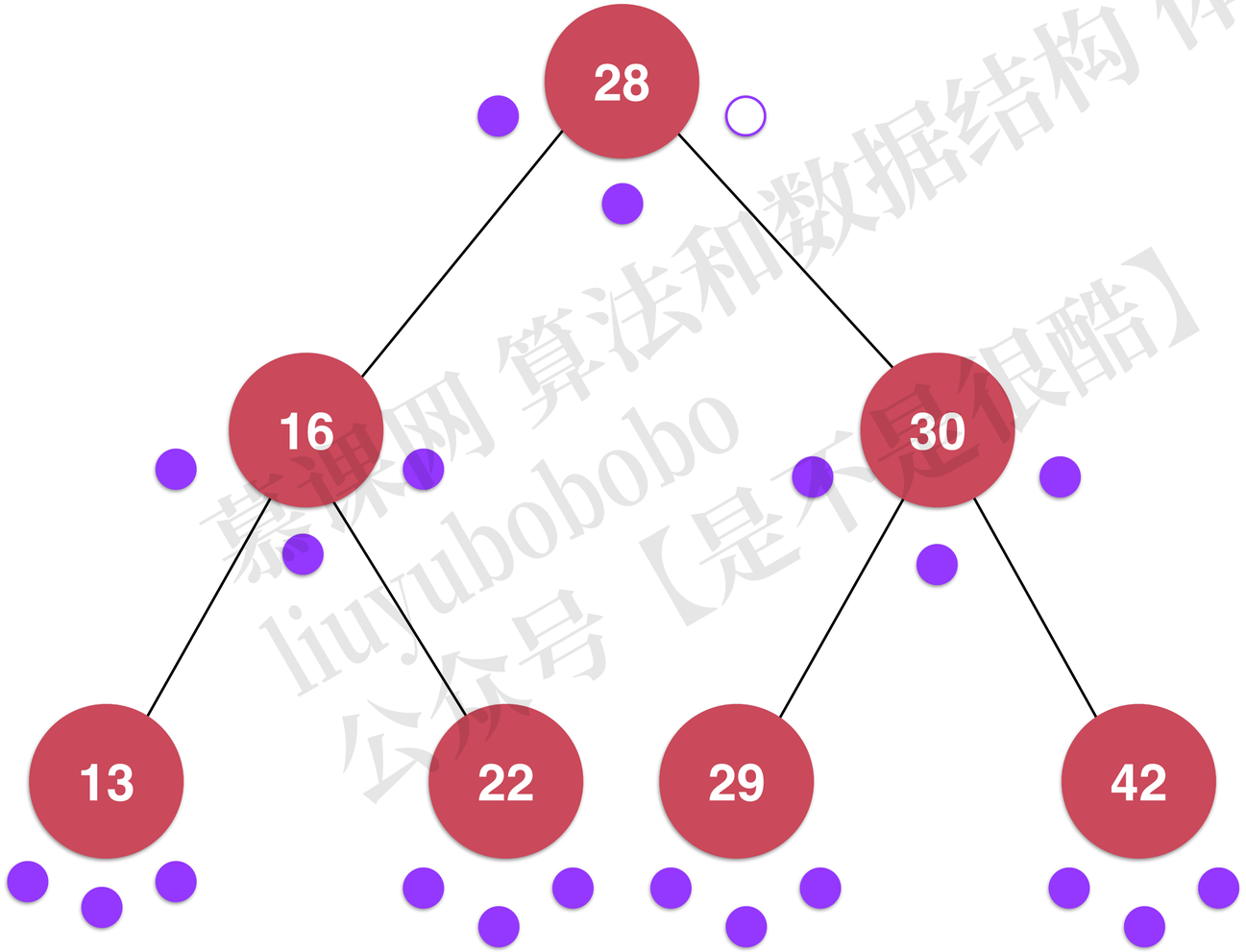
二分搜索树的前序遍历



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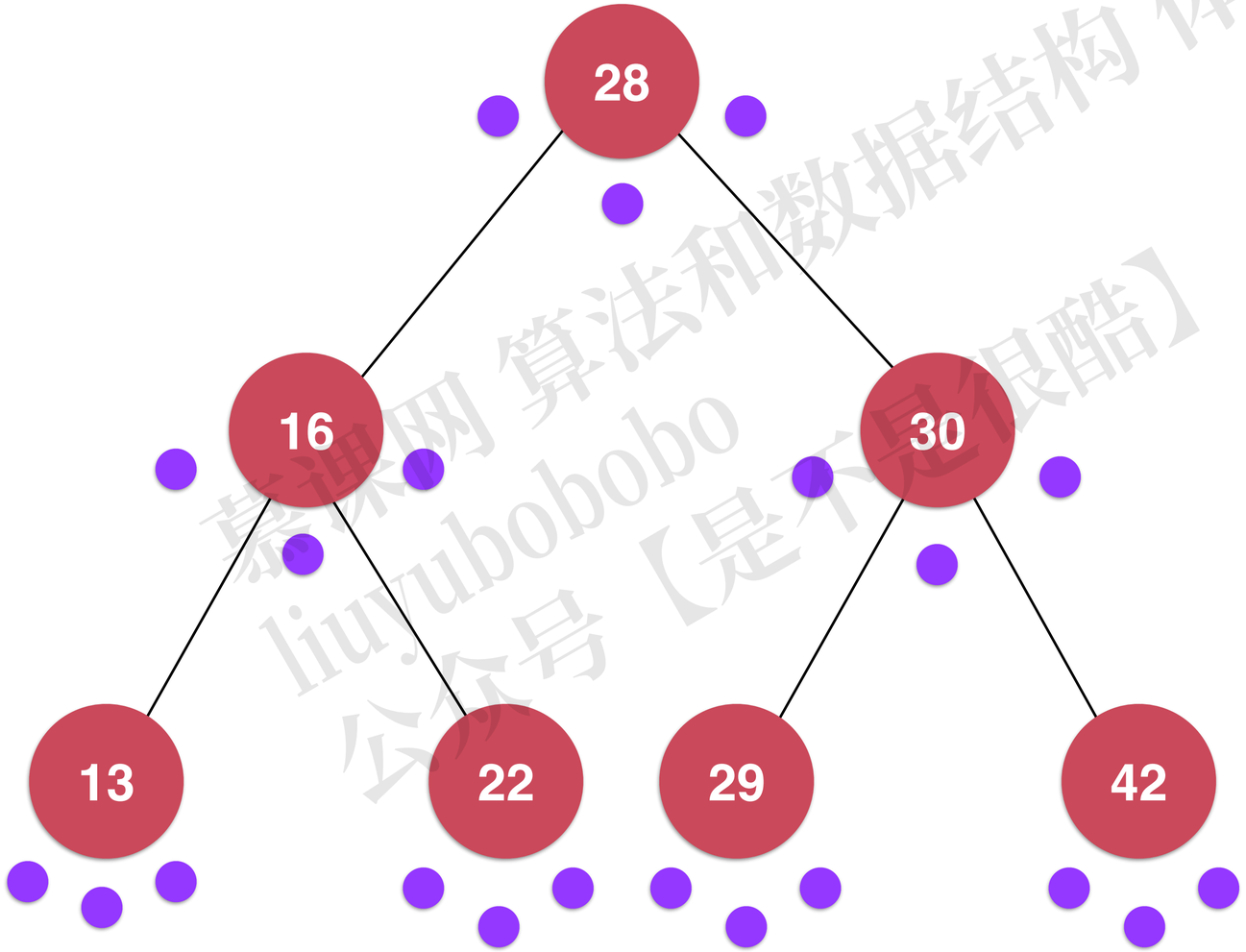
二分搜索树的前序遍历



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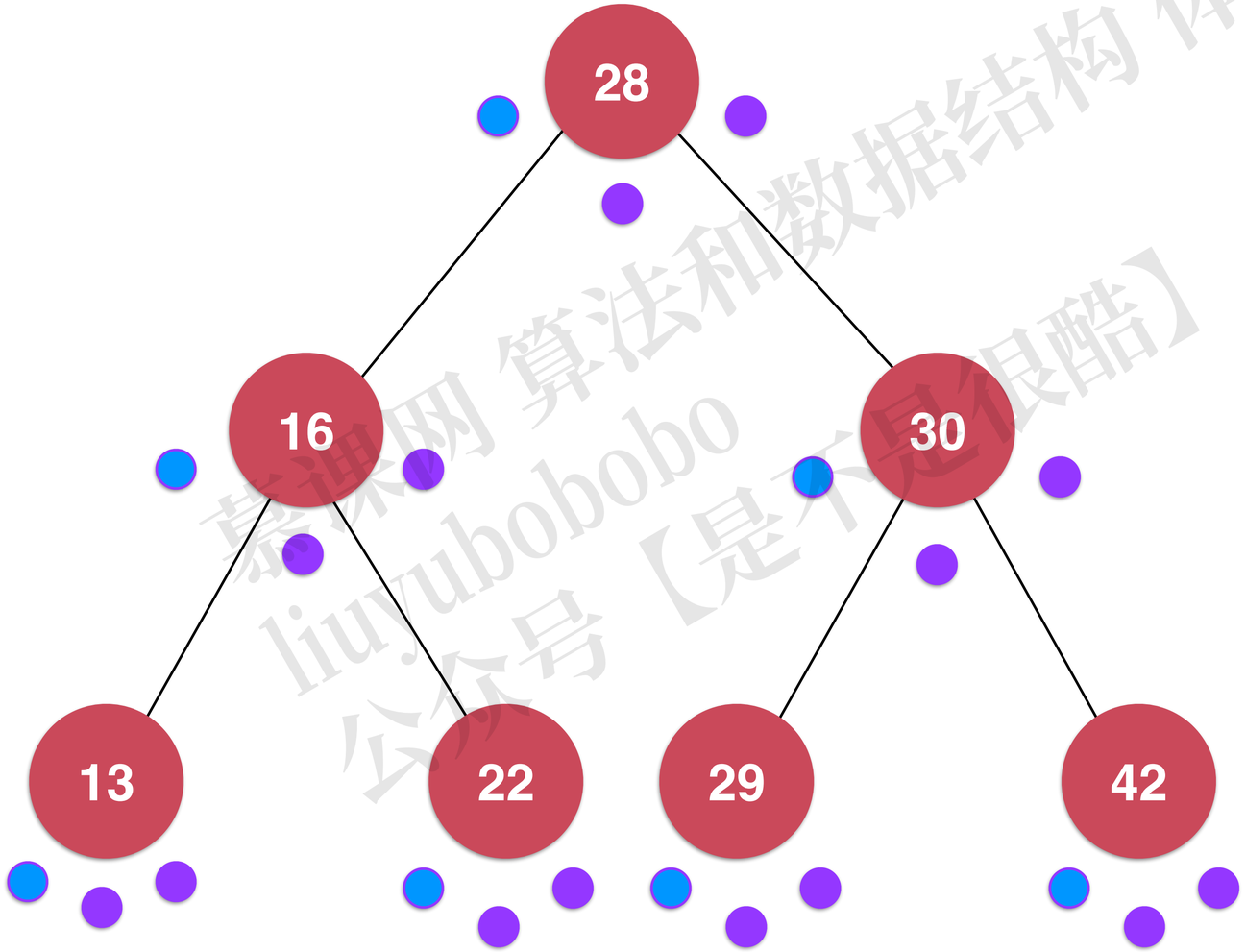
二分搜索树的前序遍历



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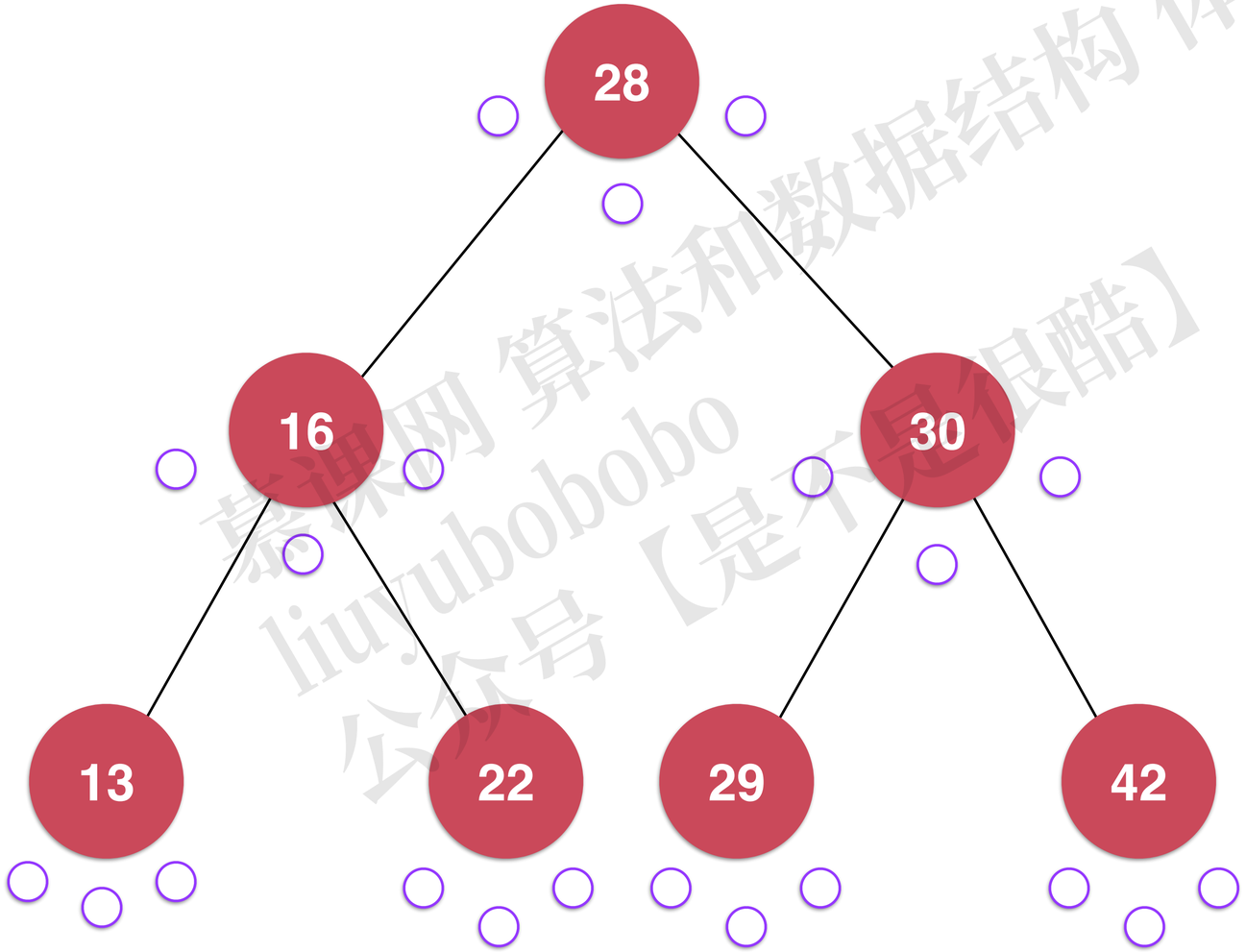
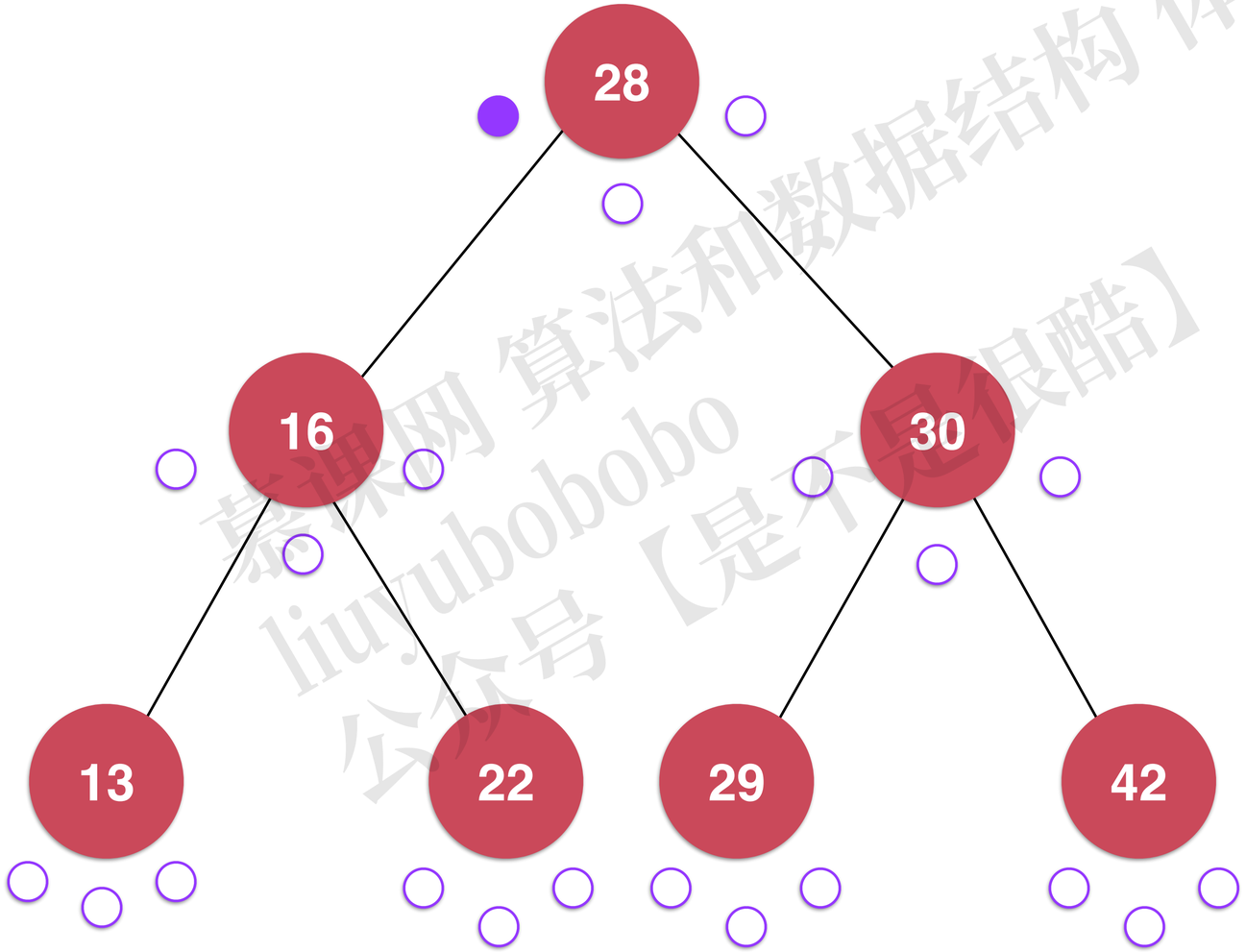
再看二分搜索树的中序遍历

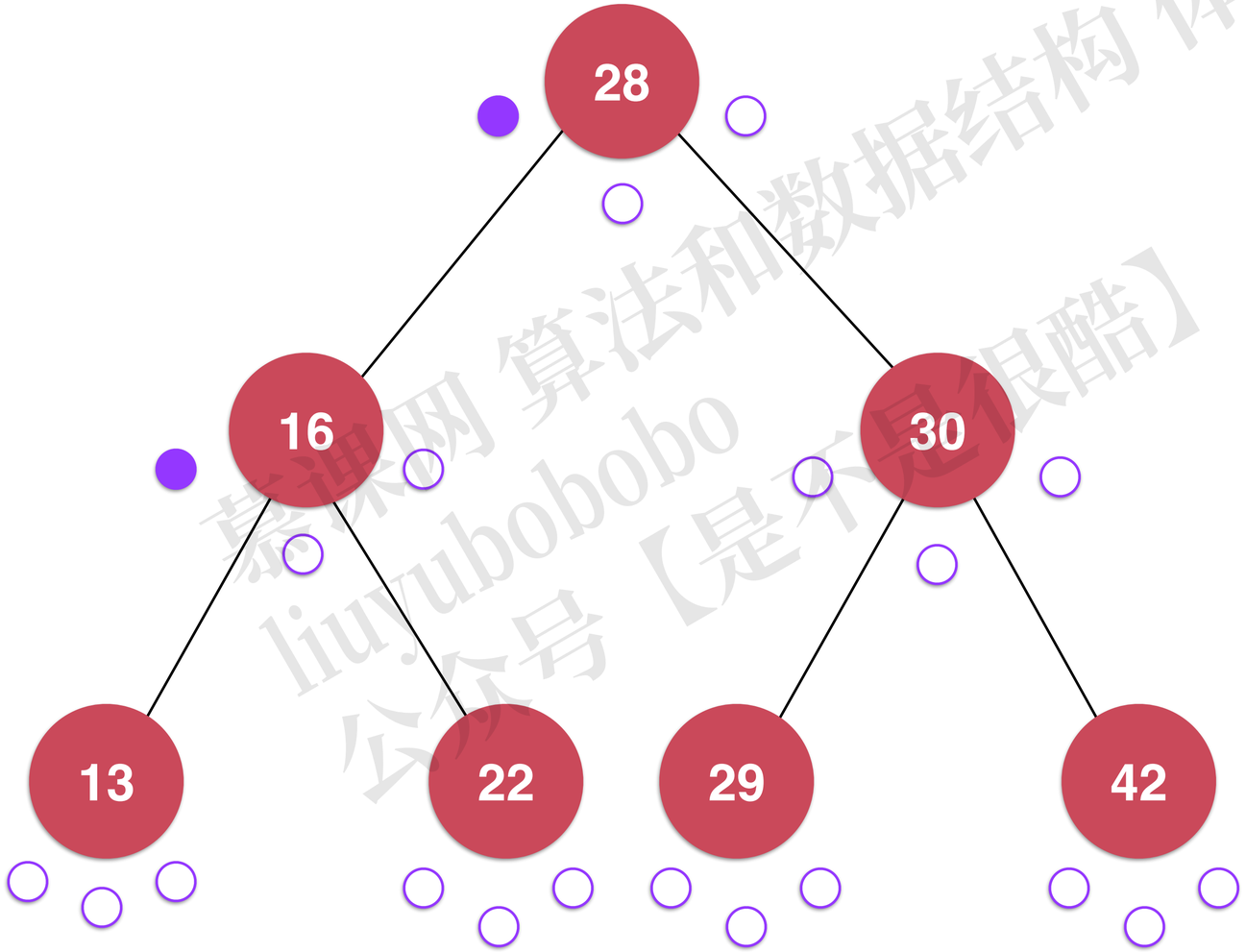
二分搜索树的中序遍历



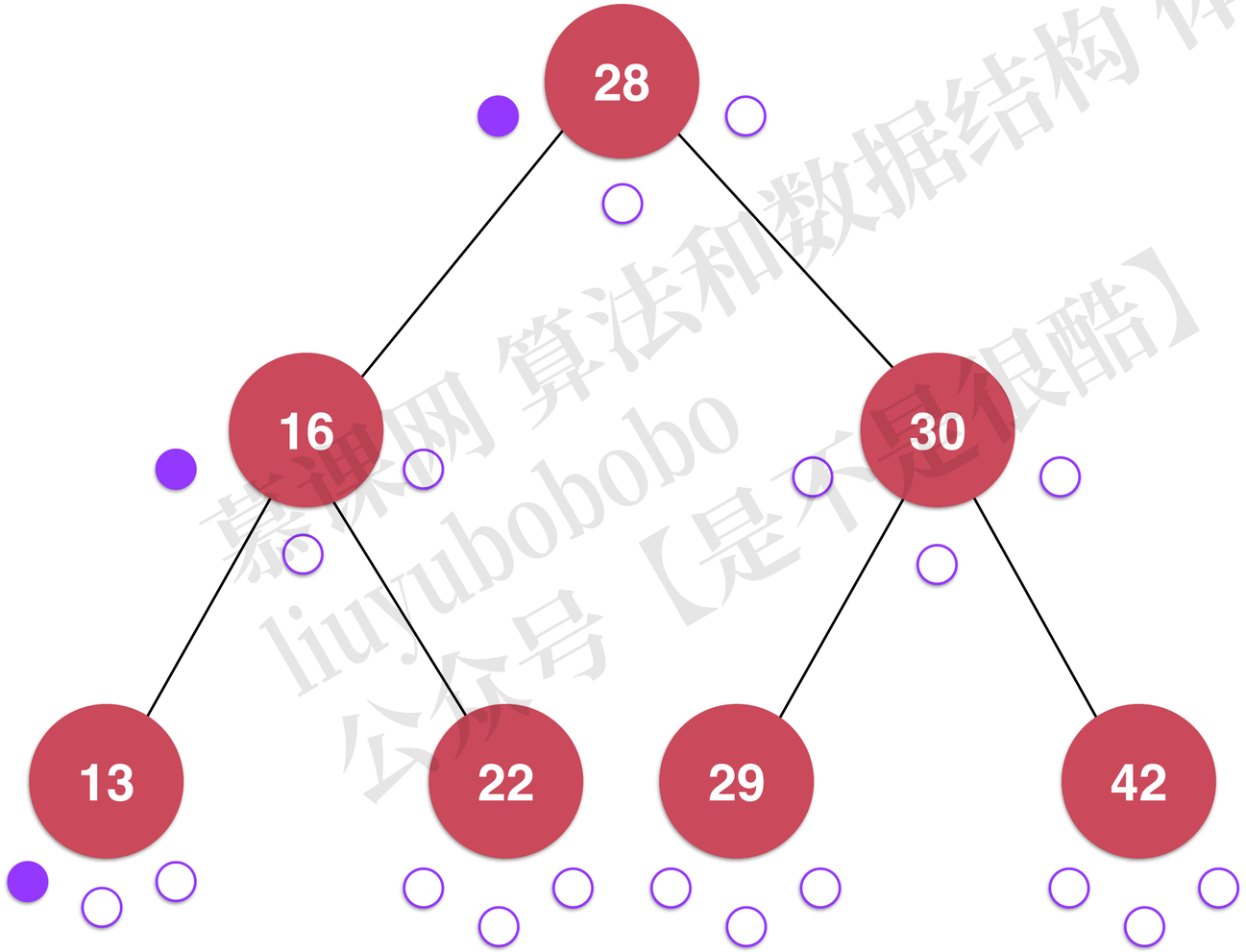
二分搜索树的中序遍历



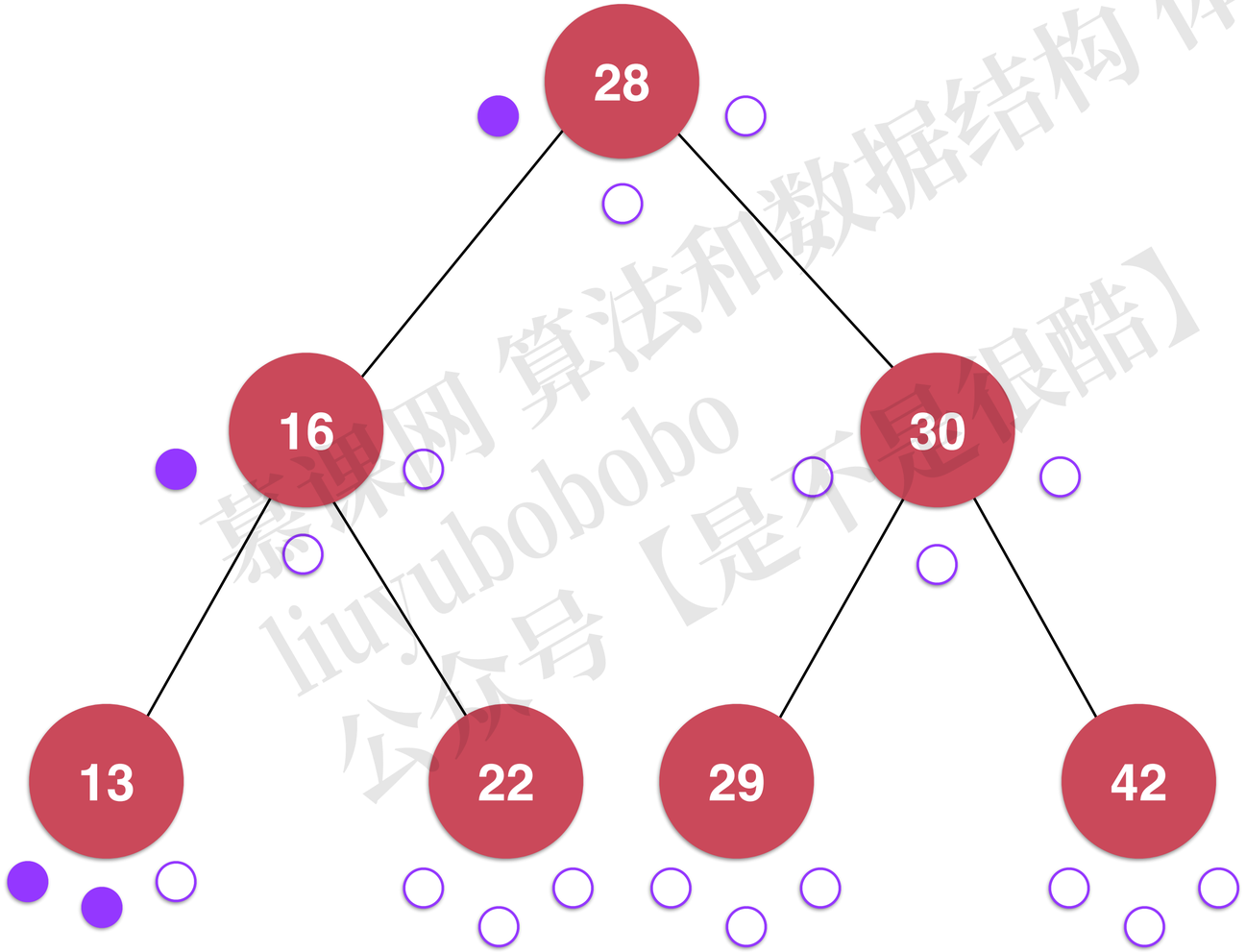
二分搜索树的中序遍历



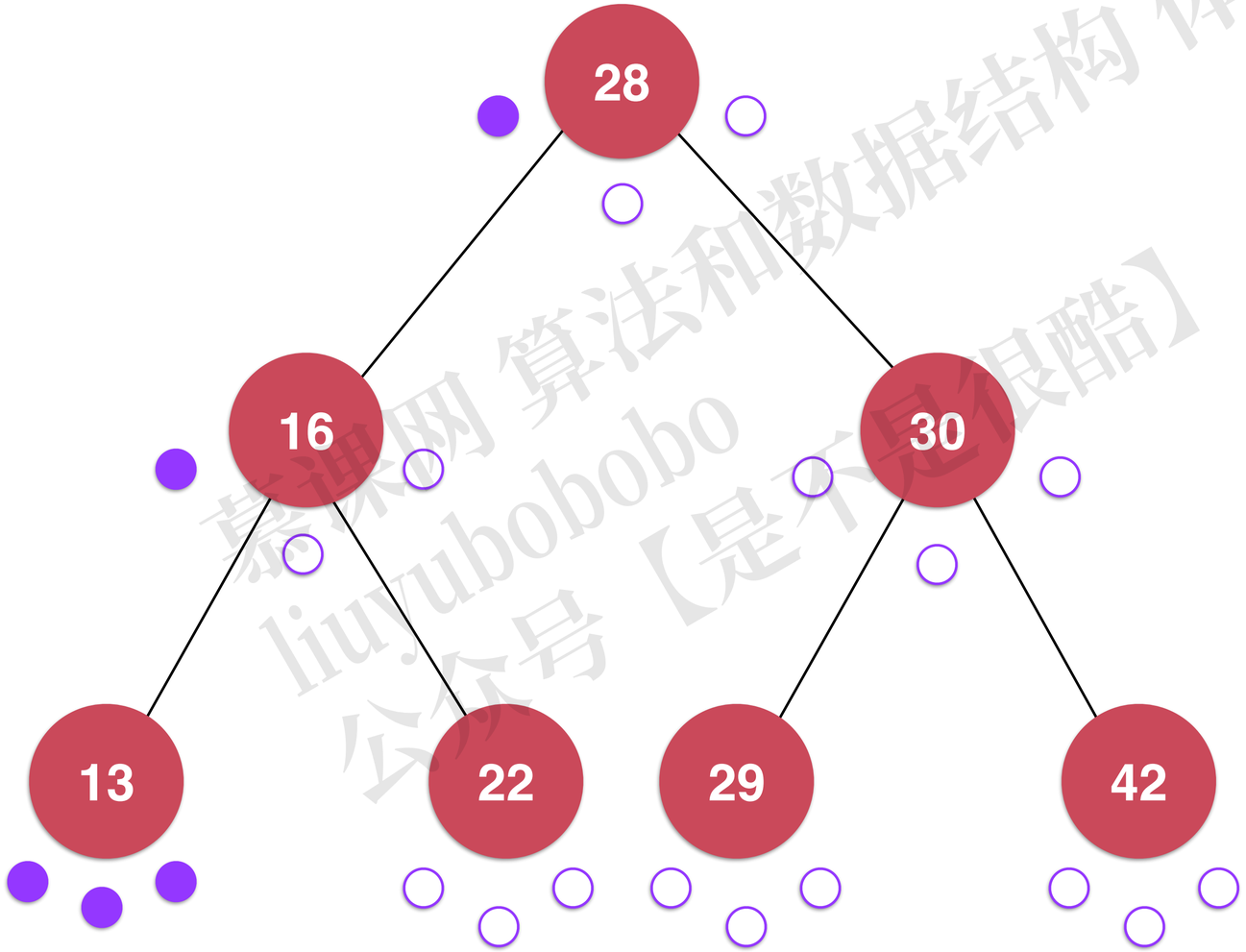
二分搜索树的中序遍历



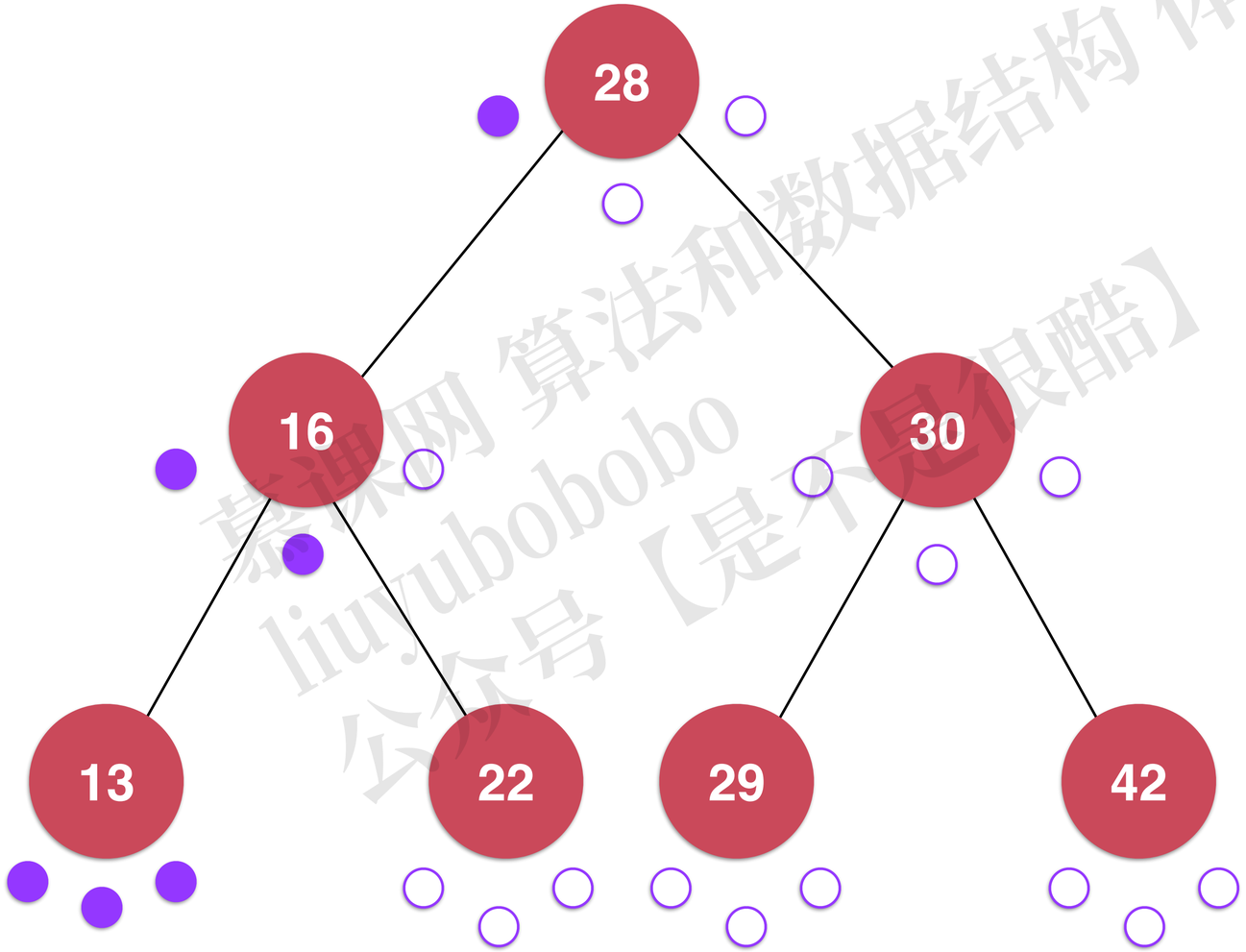
二分搜索树的中序遍历



二分搜索树的中序遍历



二分搜索树的中序遍历



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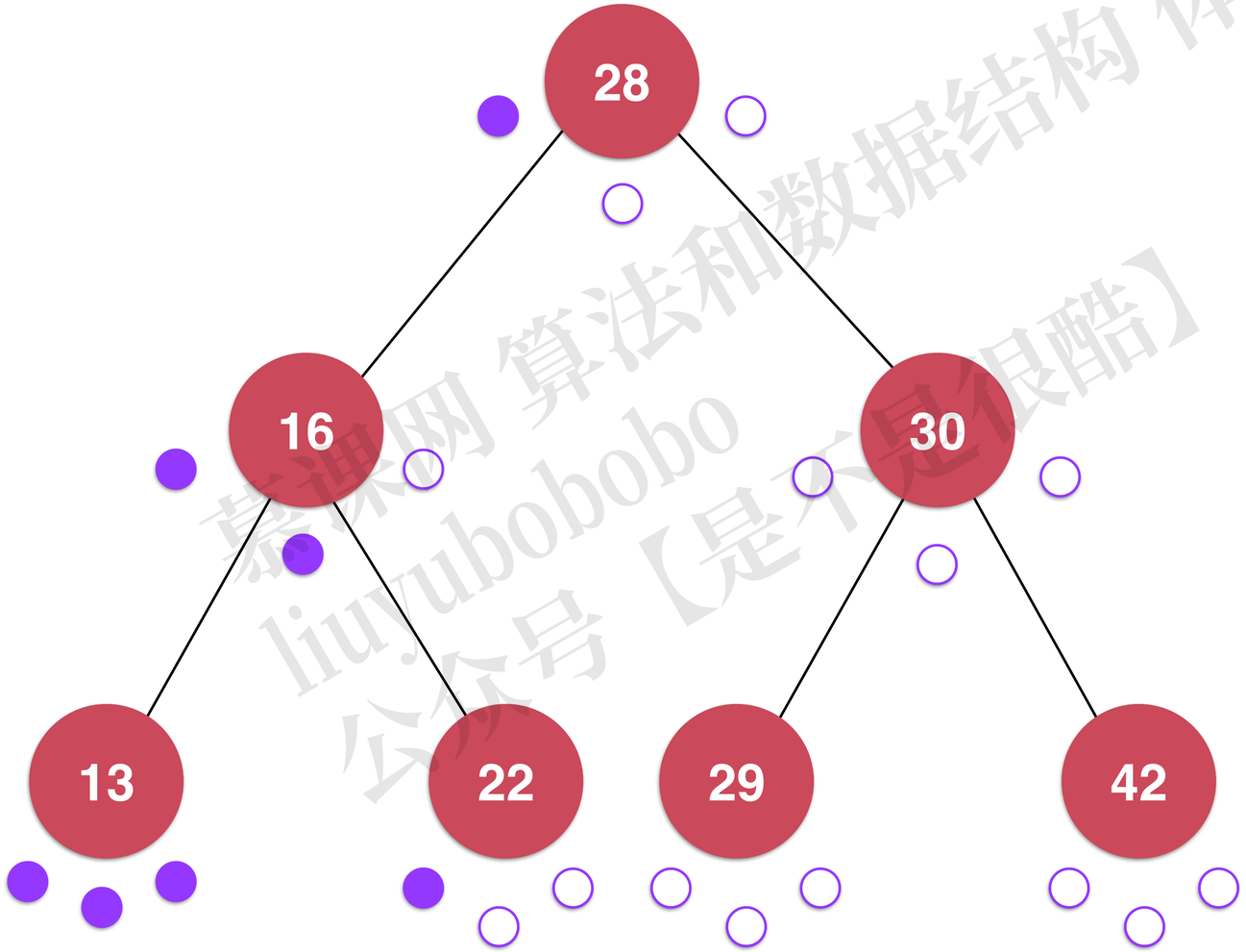
二分搜索树的中序遍历



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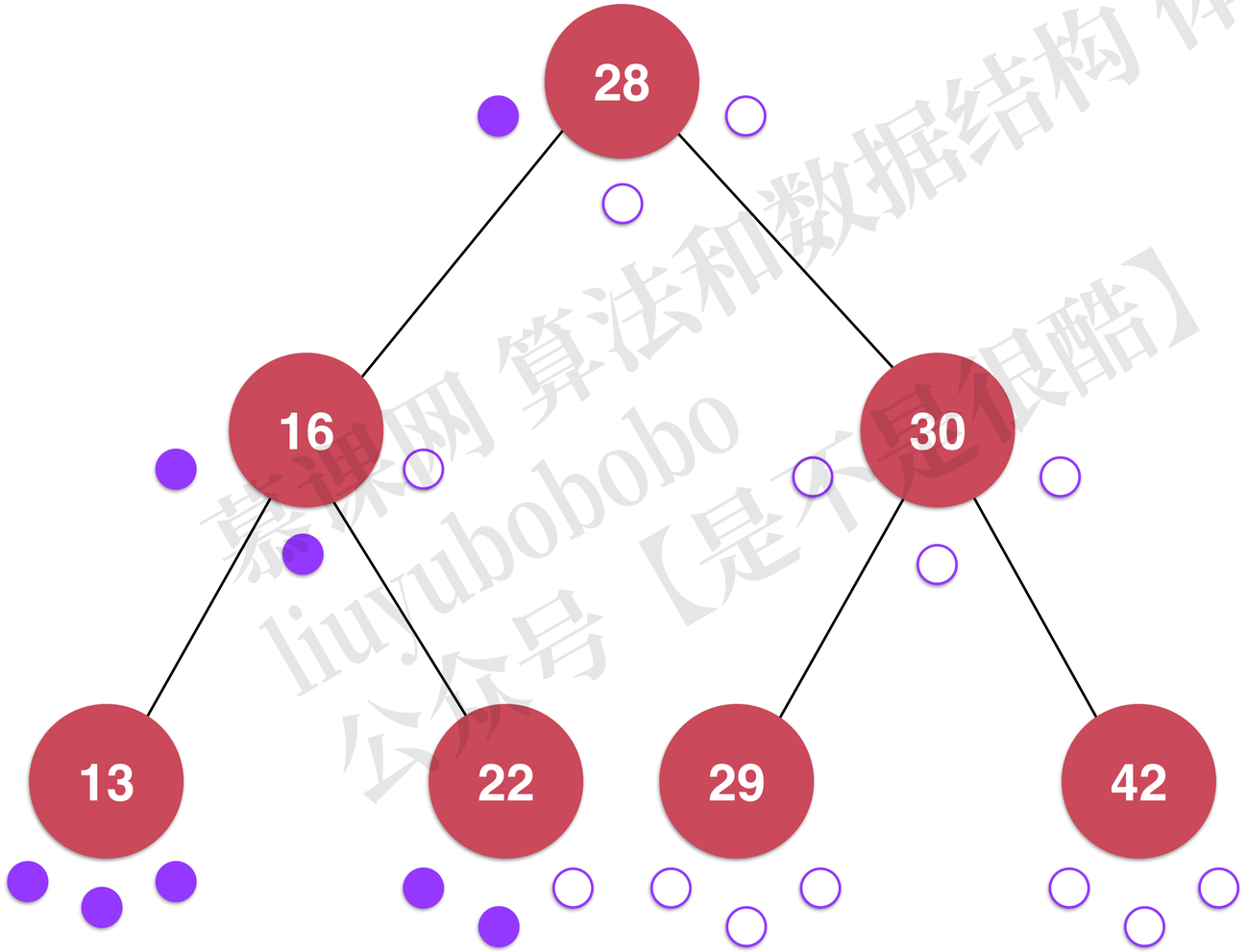
二分搜索树的中序遍历



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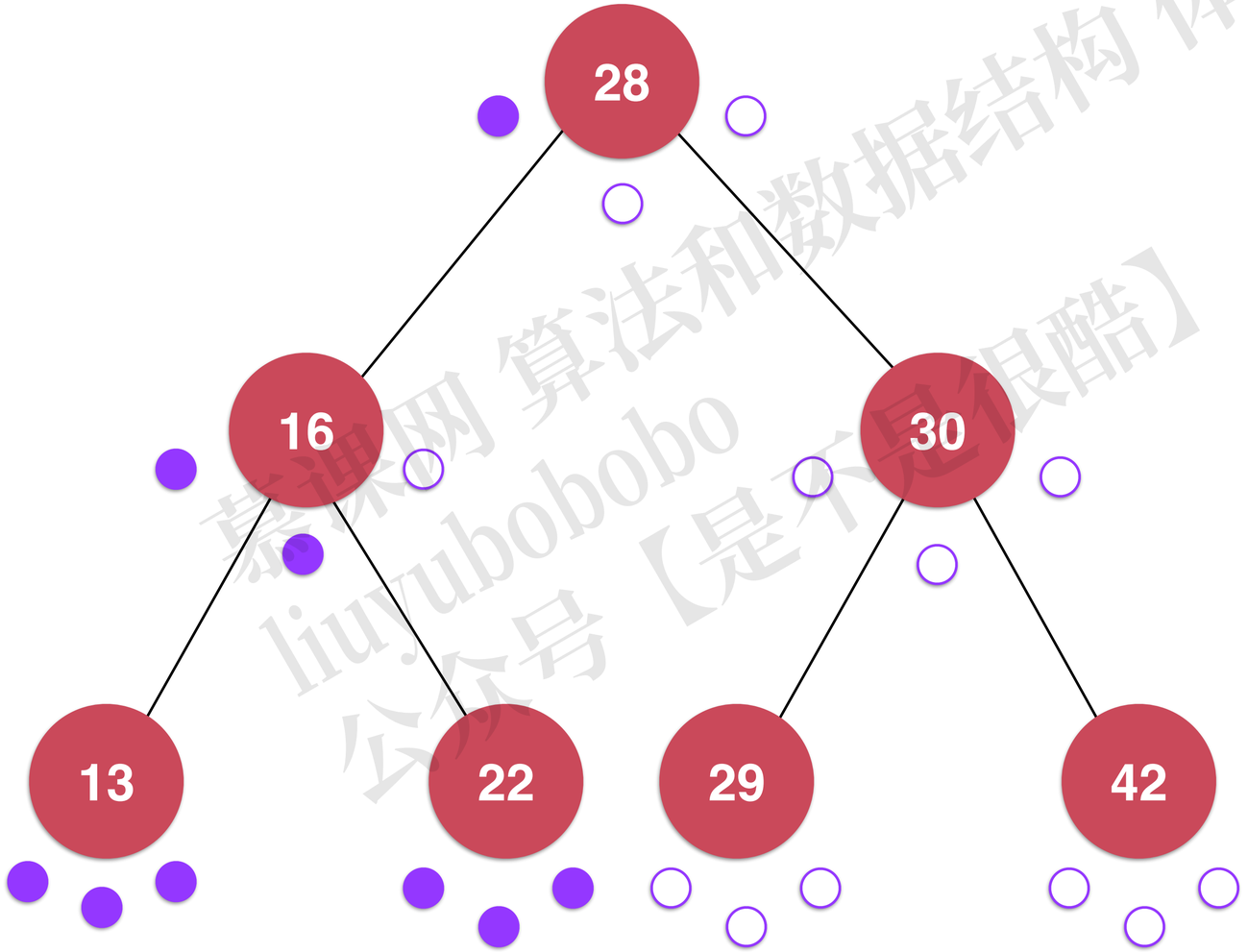
二分搜索树的中序遍历



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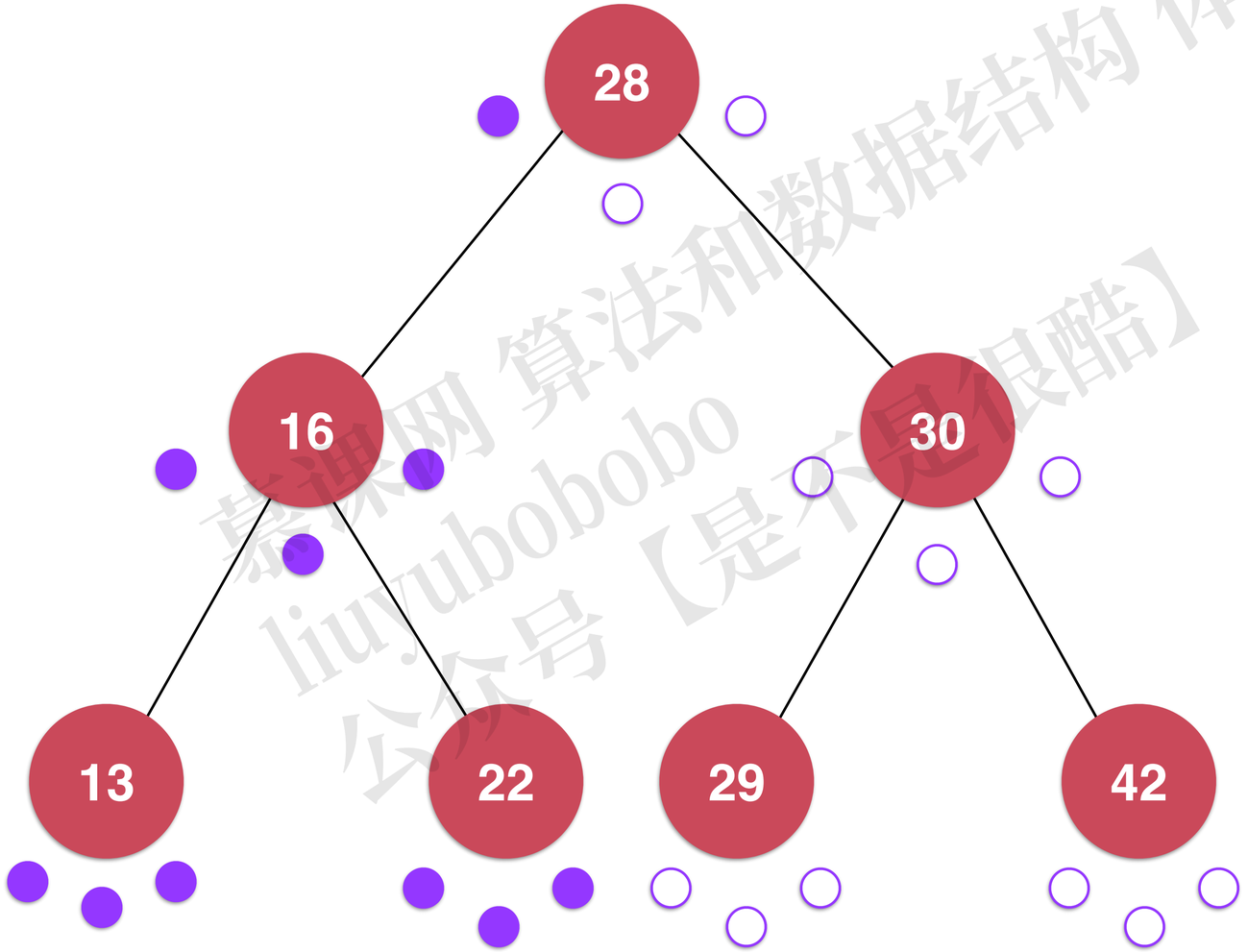
二分搜索树的中序遍历



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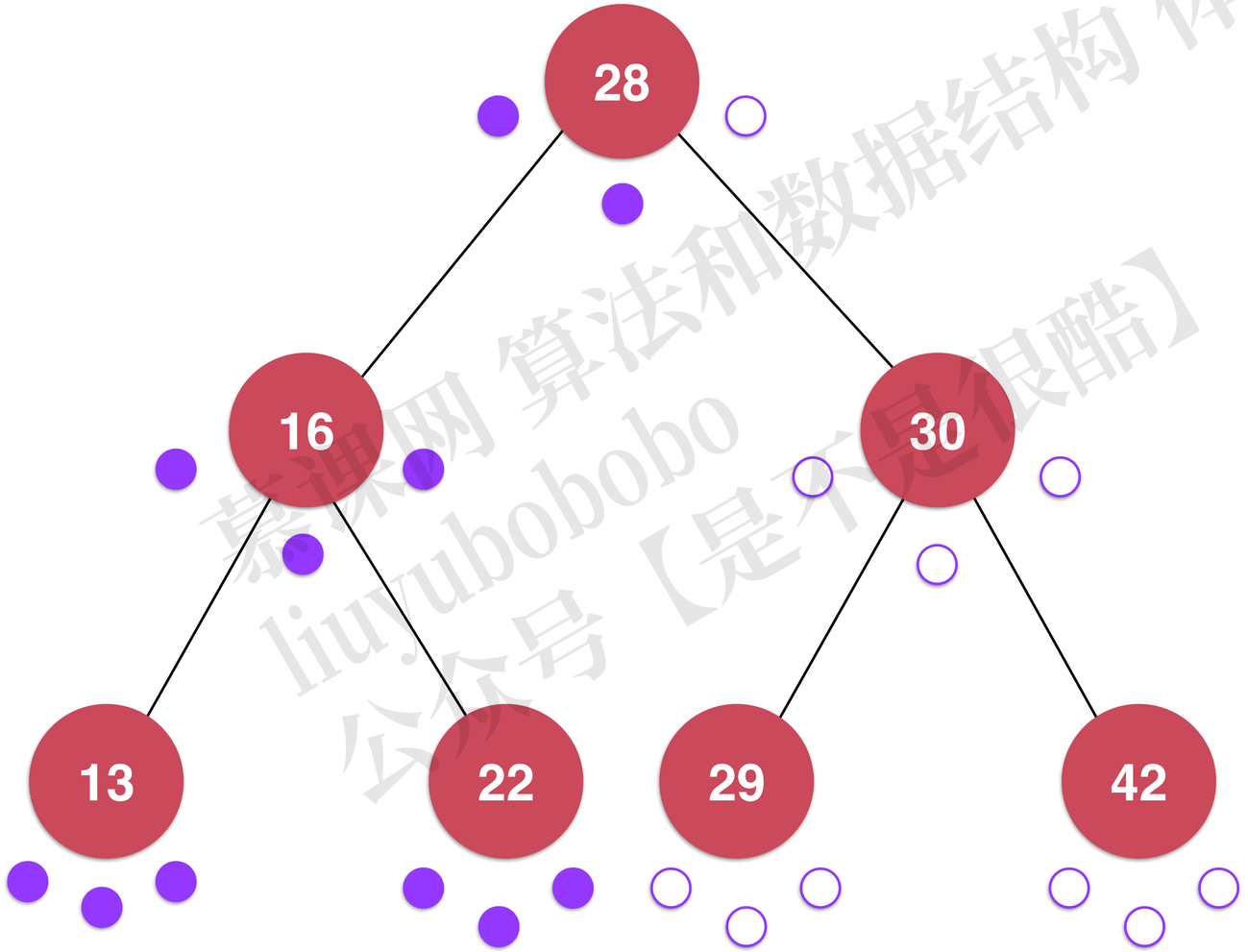
二分搜索树的中序遍历



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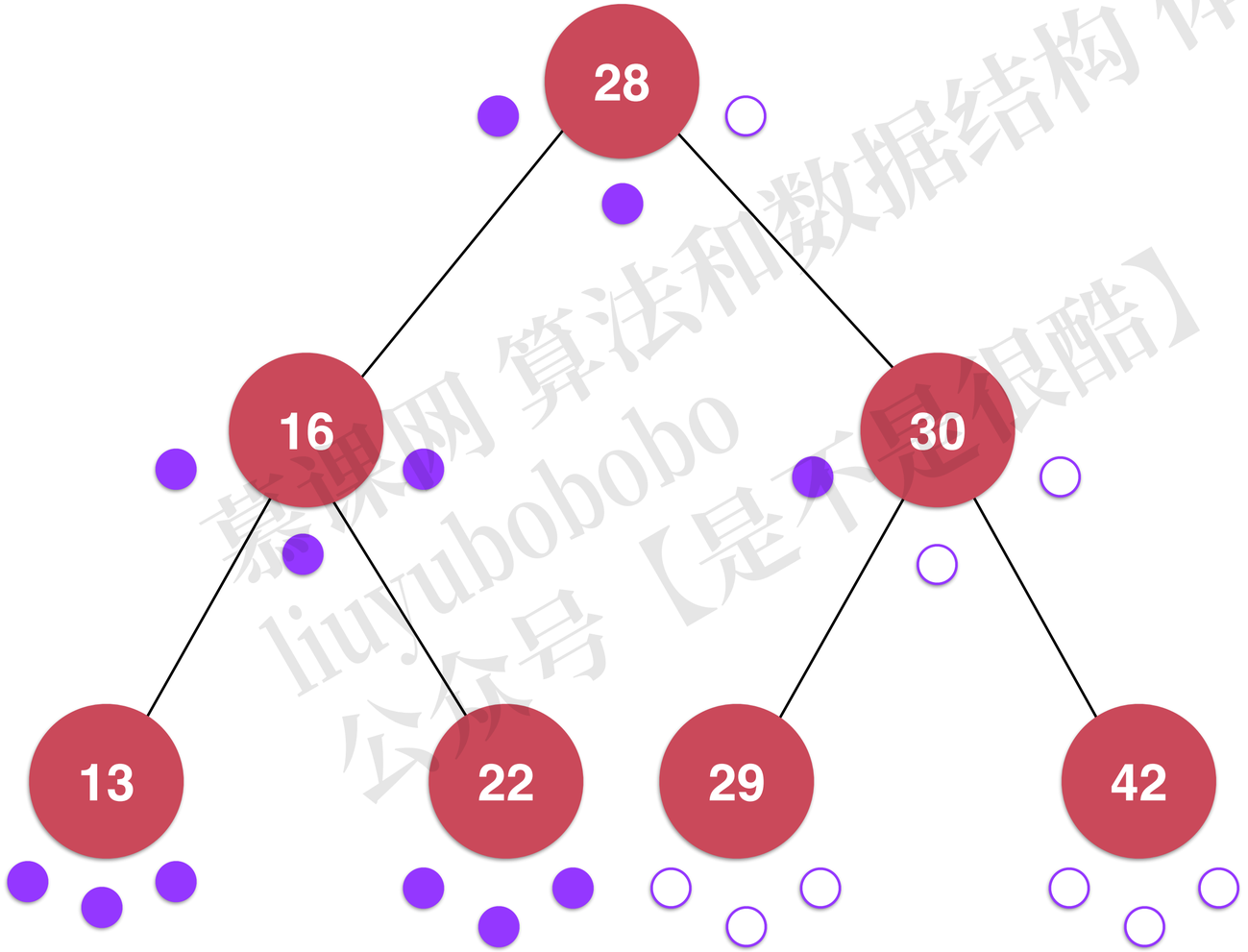
二分搜索树的中序遍历



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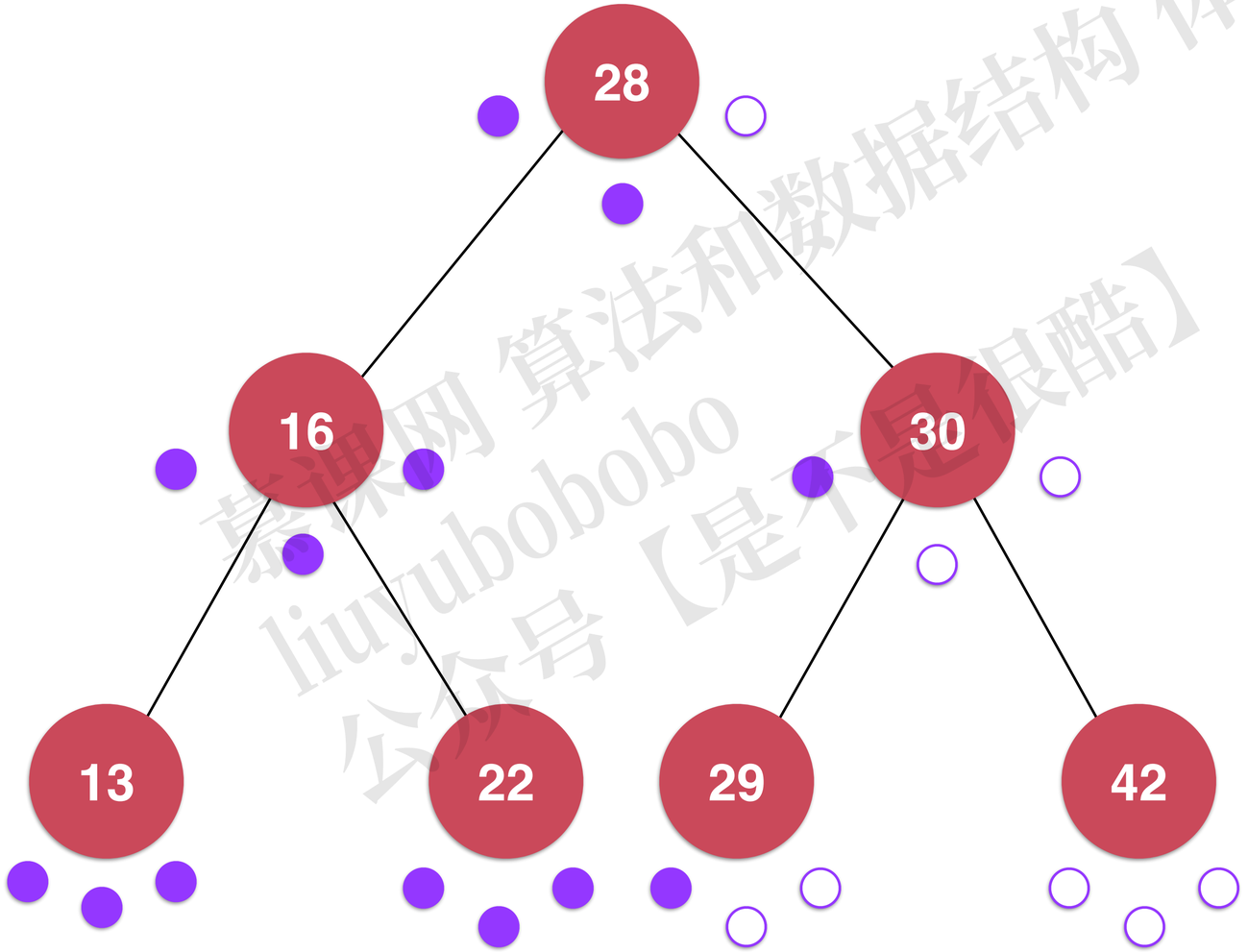
二分搜索树的中序遍历



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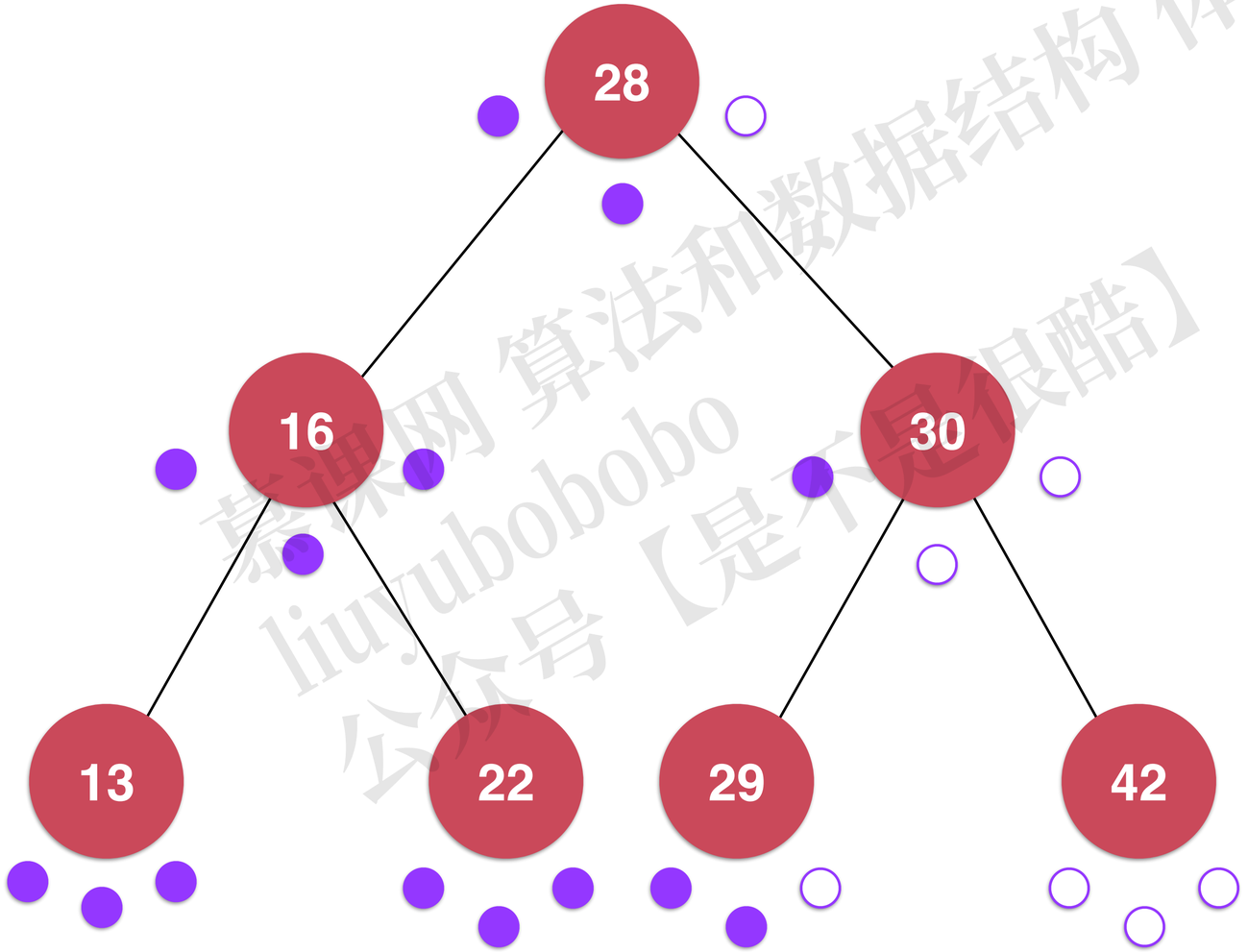
二分搜索树的中序遍历



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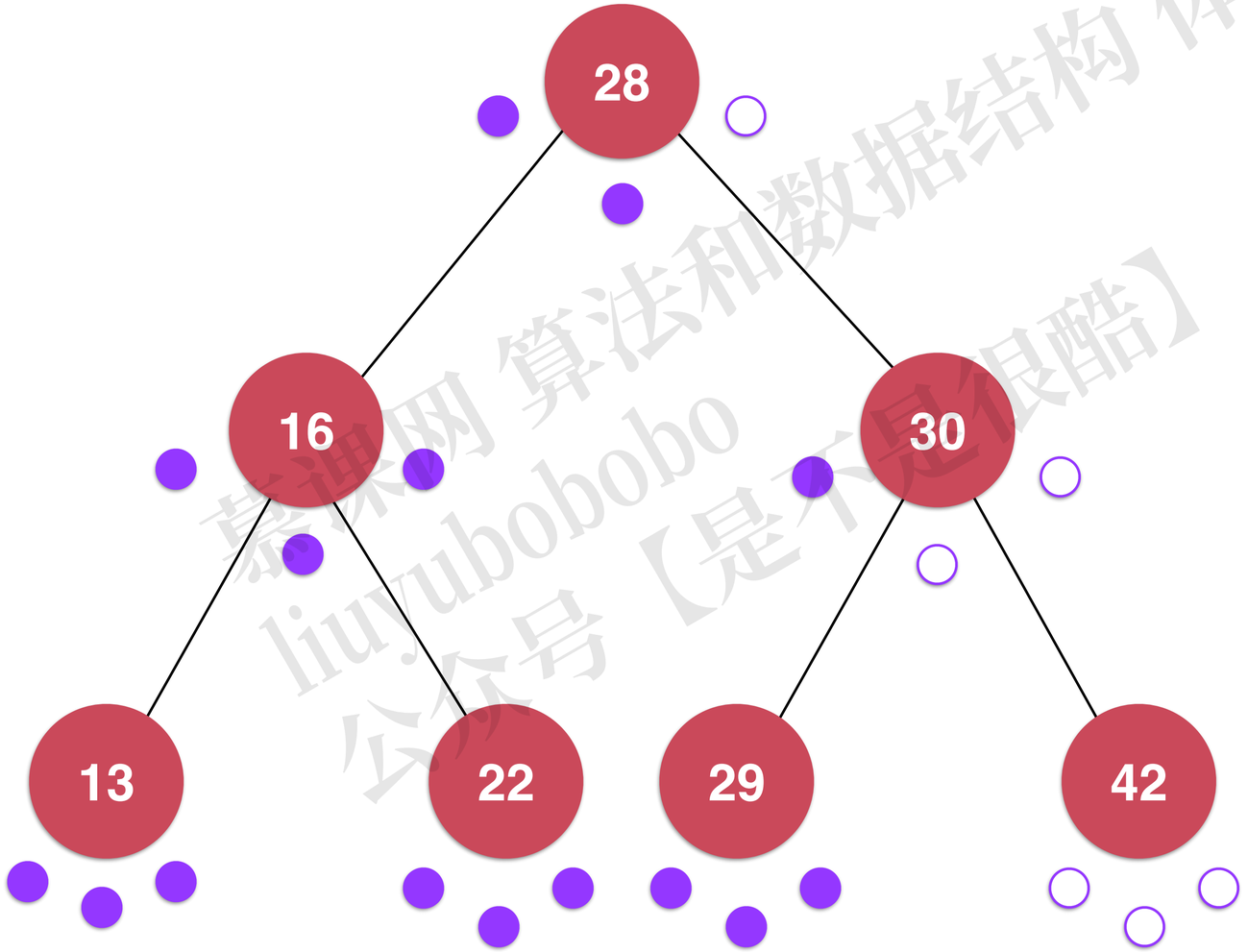
二分搜索树的中序遍历



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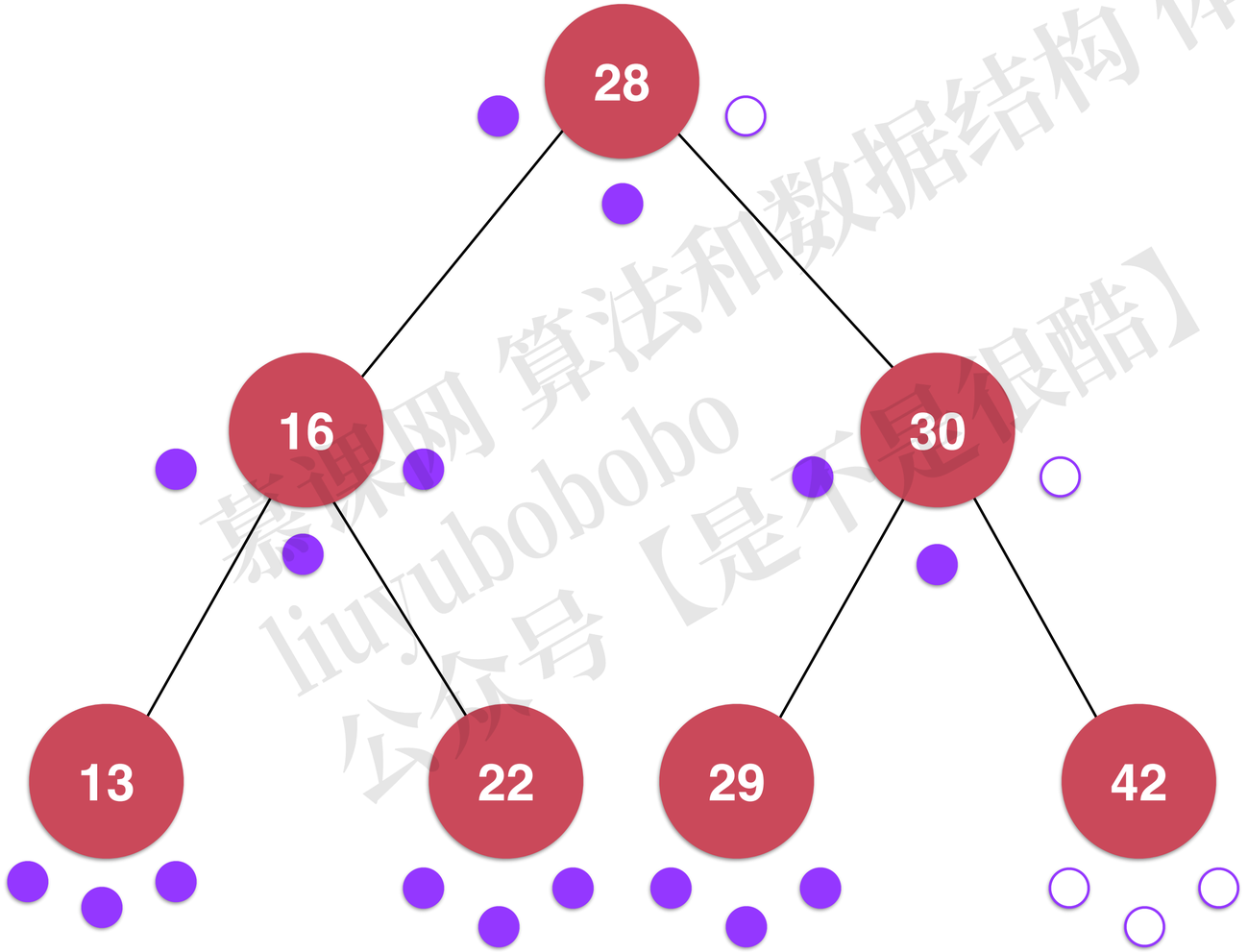
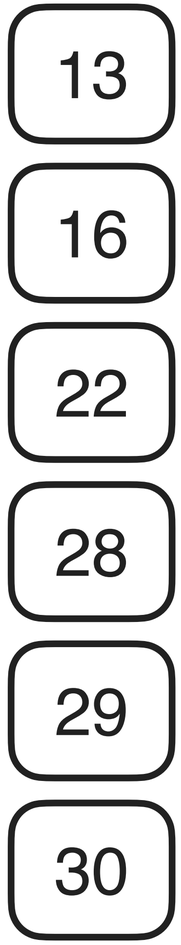
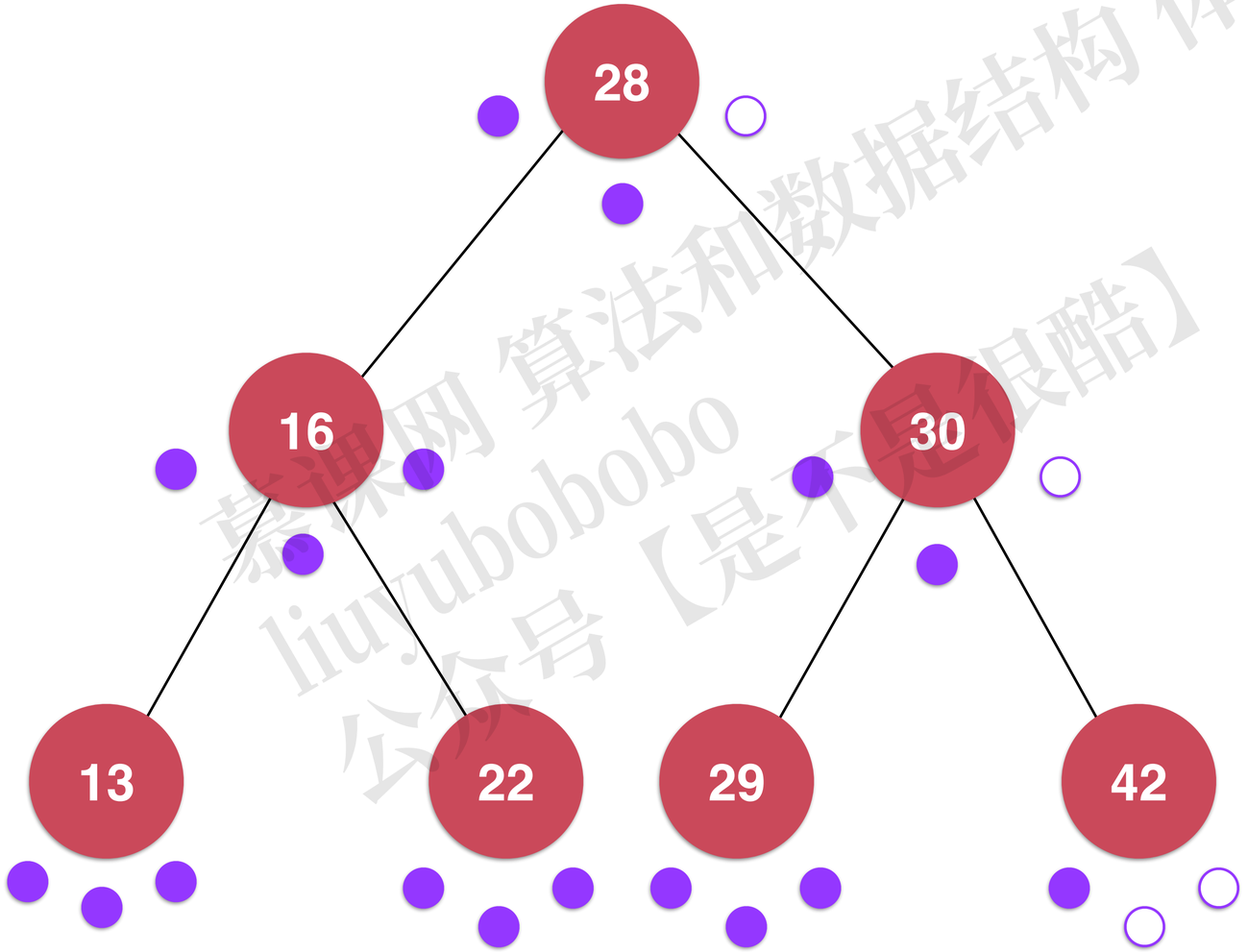
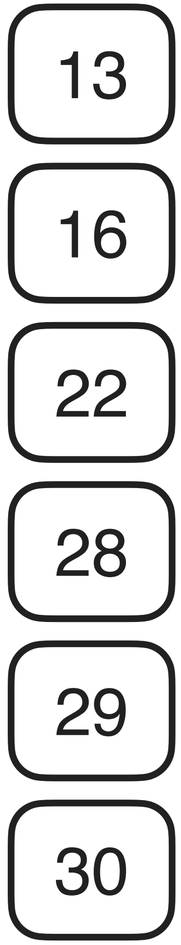
二分搜索树的中序遍历



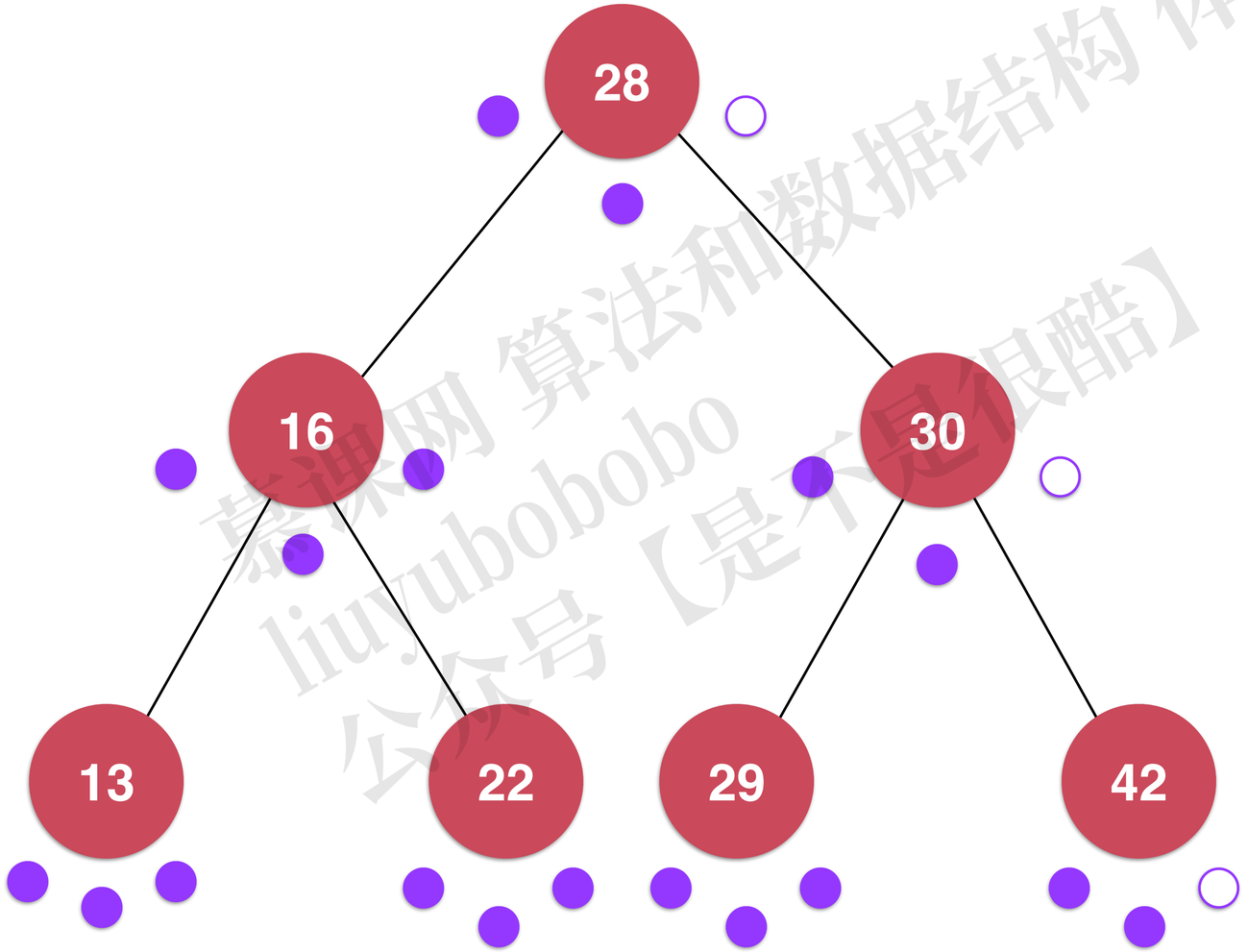


二分搜索树的中序遍历





二分搜索树的中序遍历



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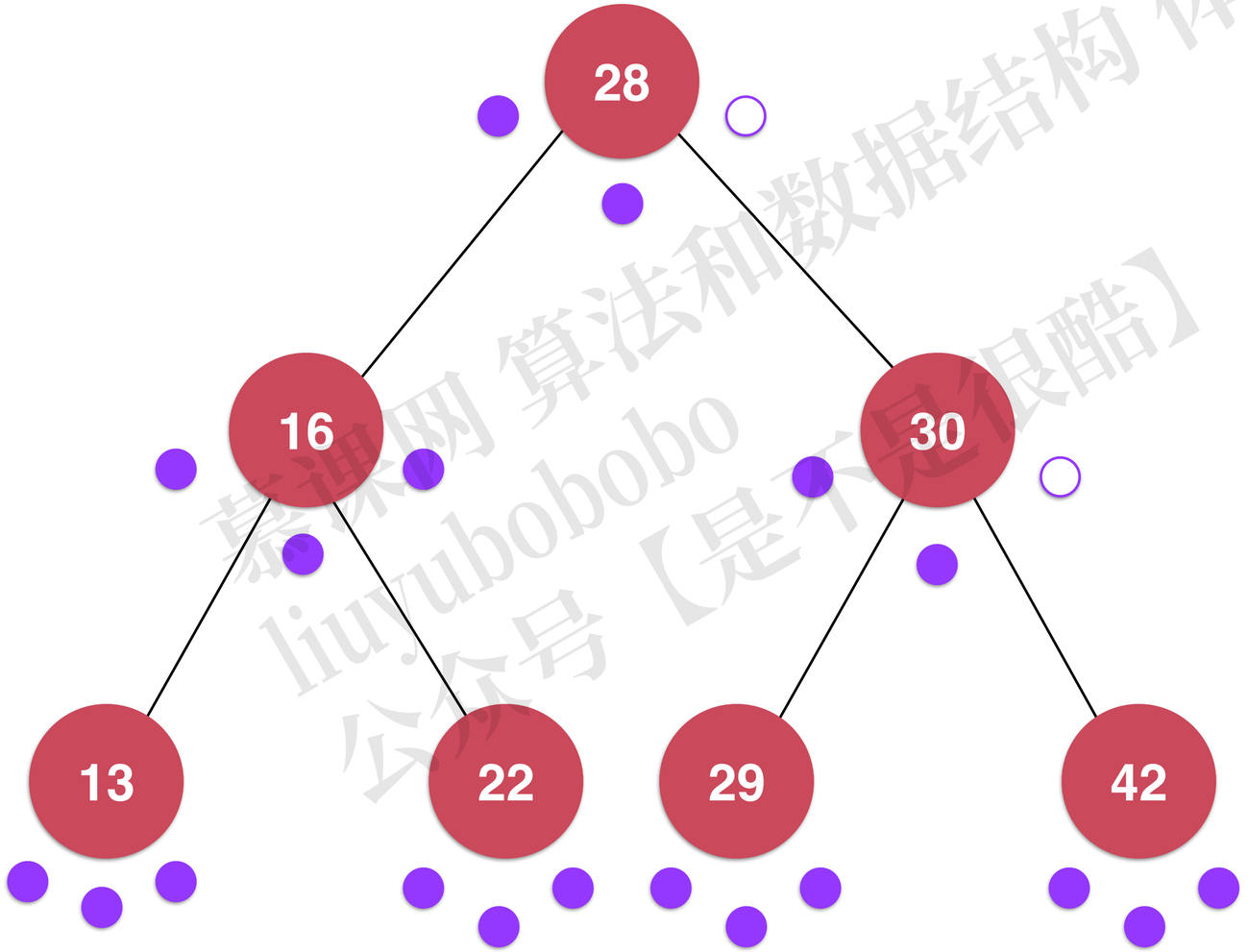
二分搜索树的中序遍历



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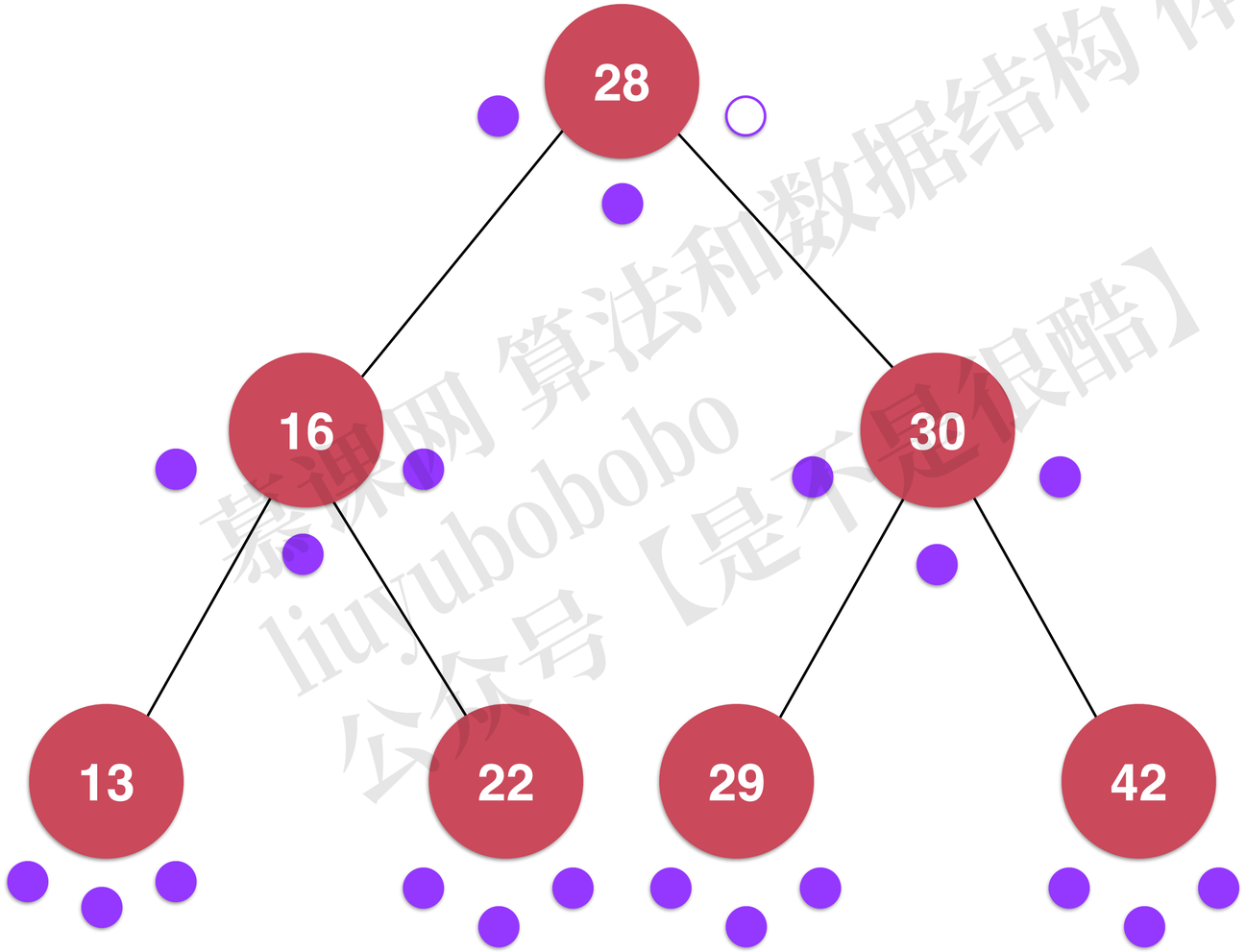
二分搜索树的中序遍历



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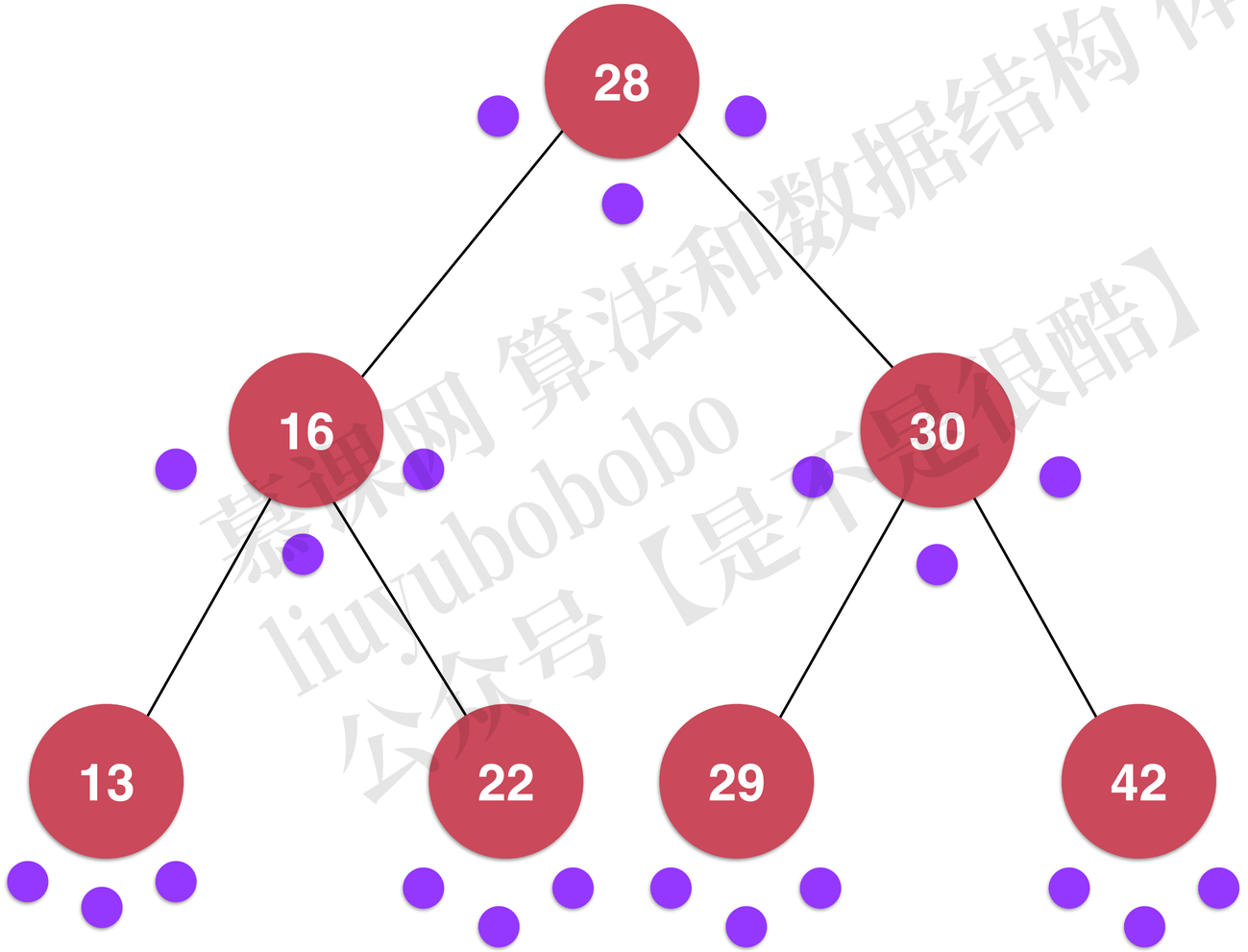
二分搜索树的中序遍历



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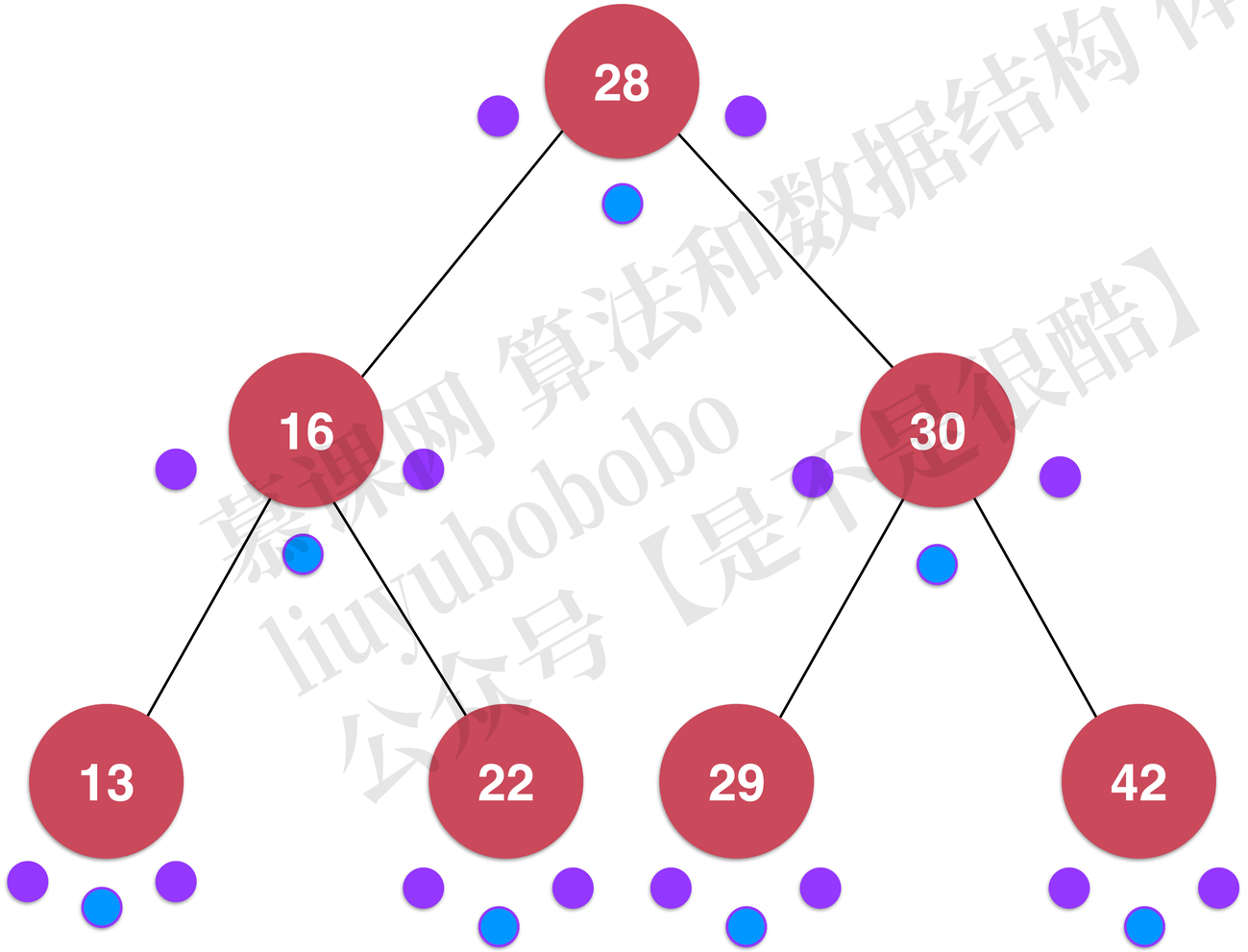
二分搜索树的中序遍历



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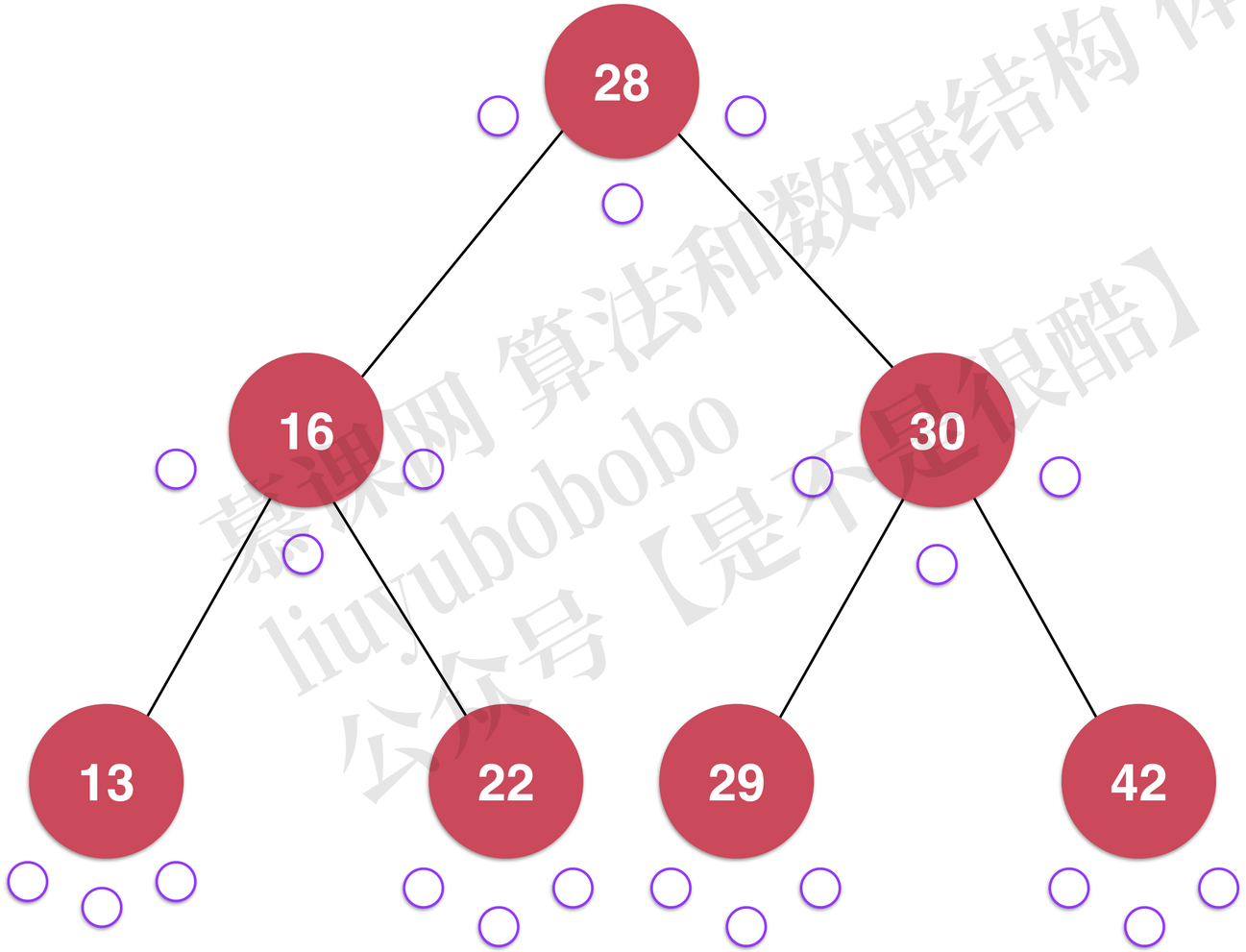
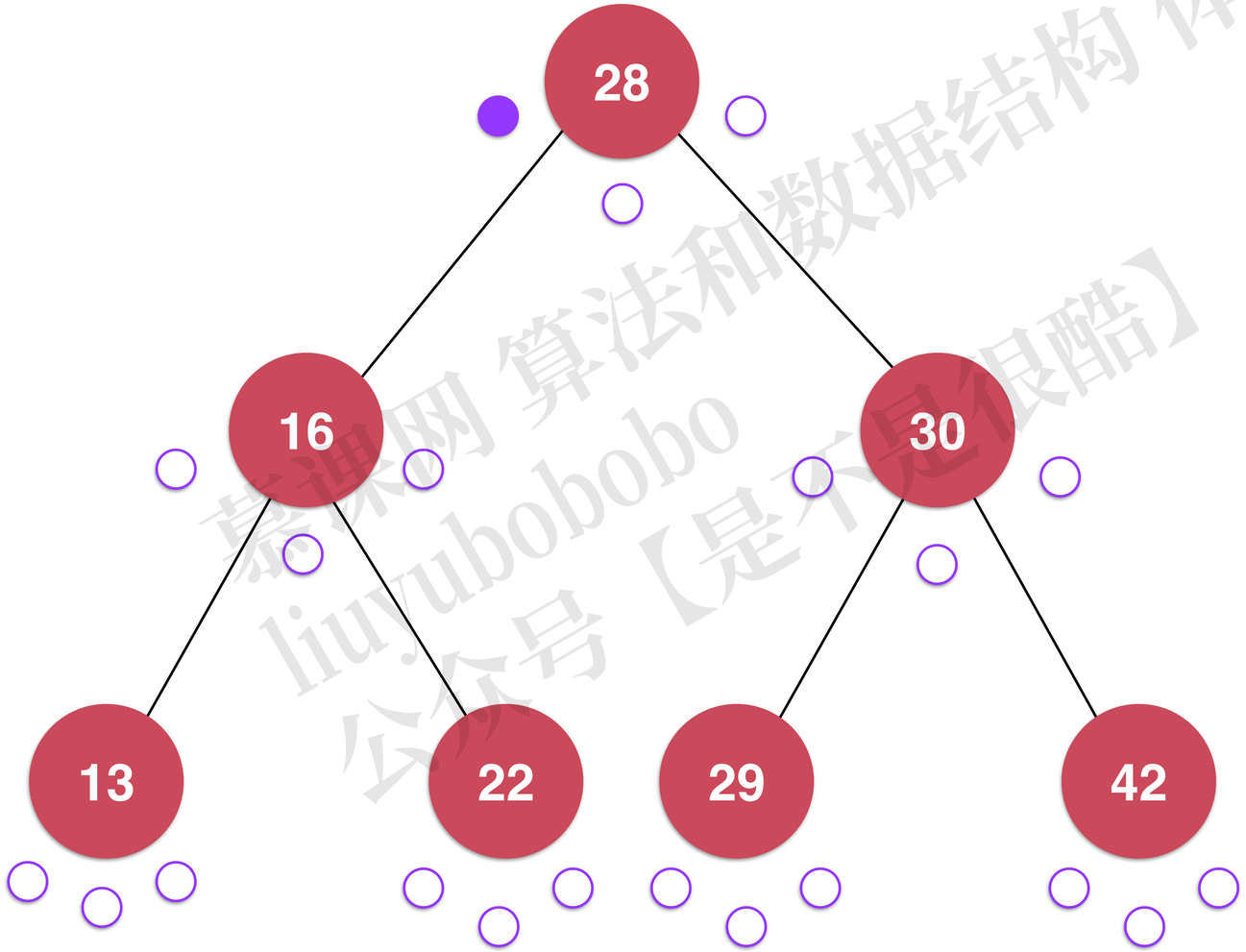
再看二分搜索树的后序遍历

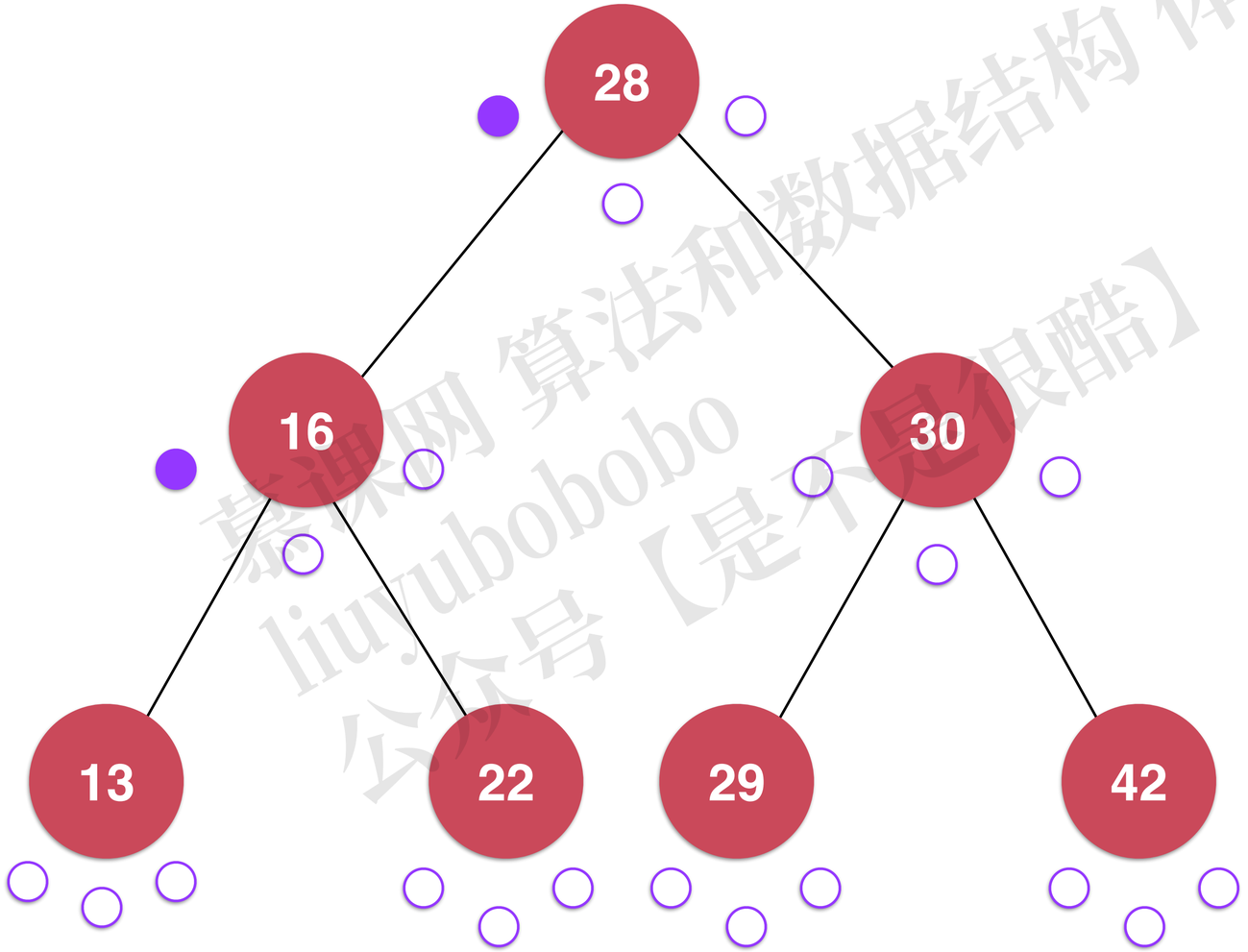
二分搜索树的后序遍历



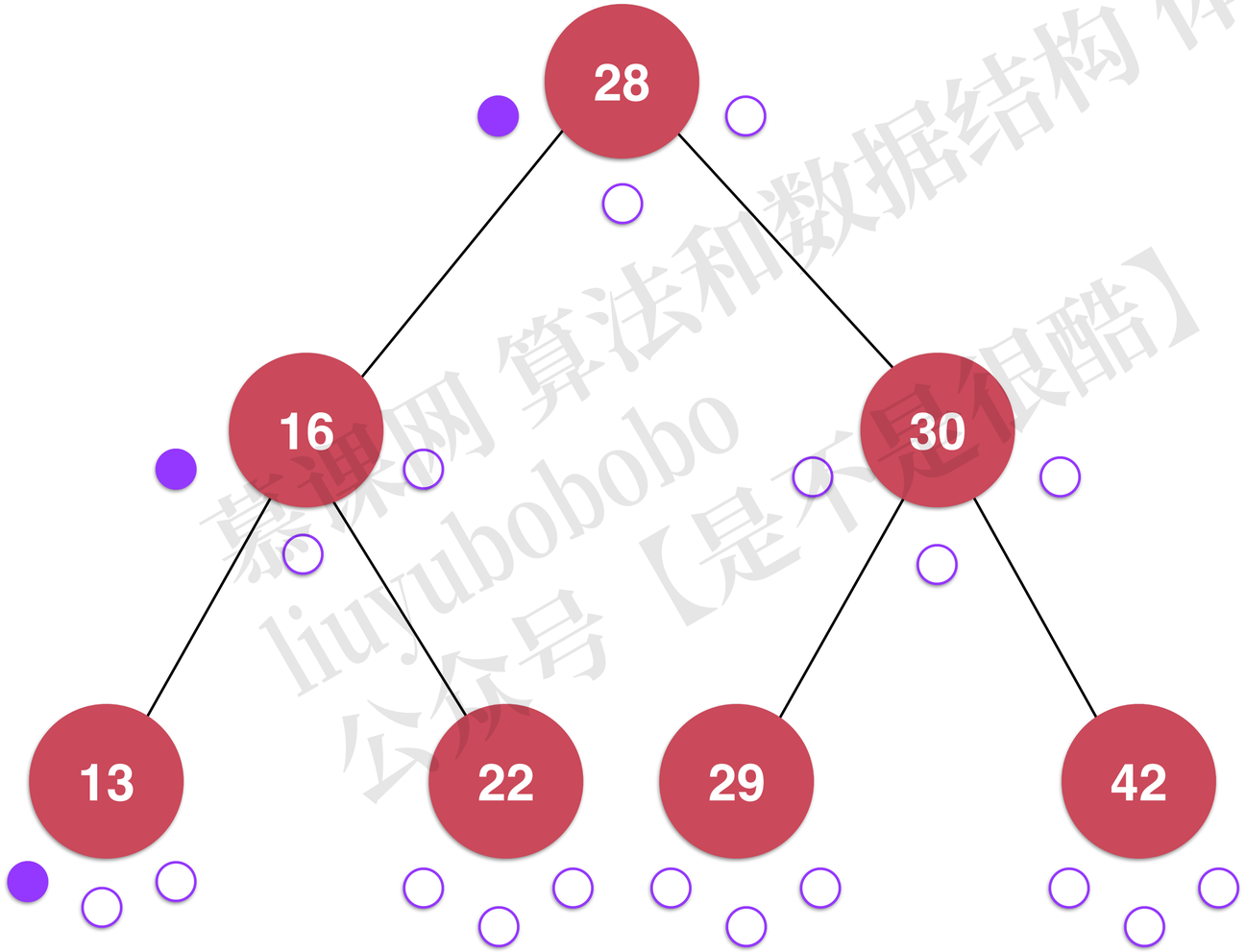
二分搜索树的后序遍历



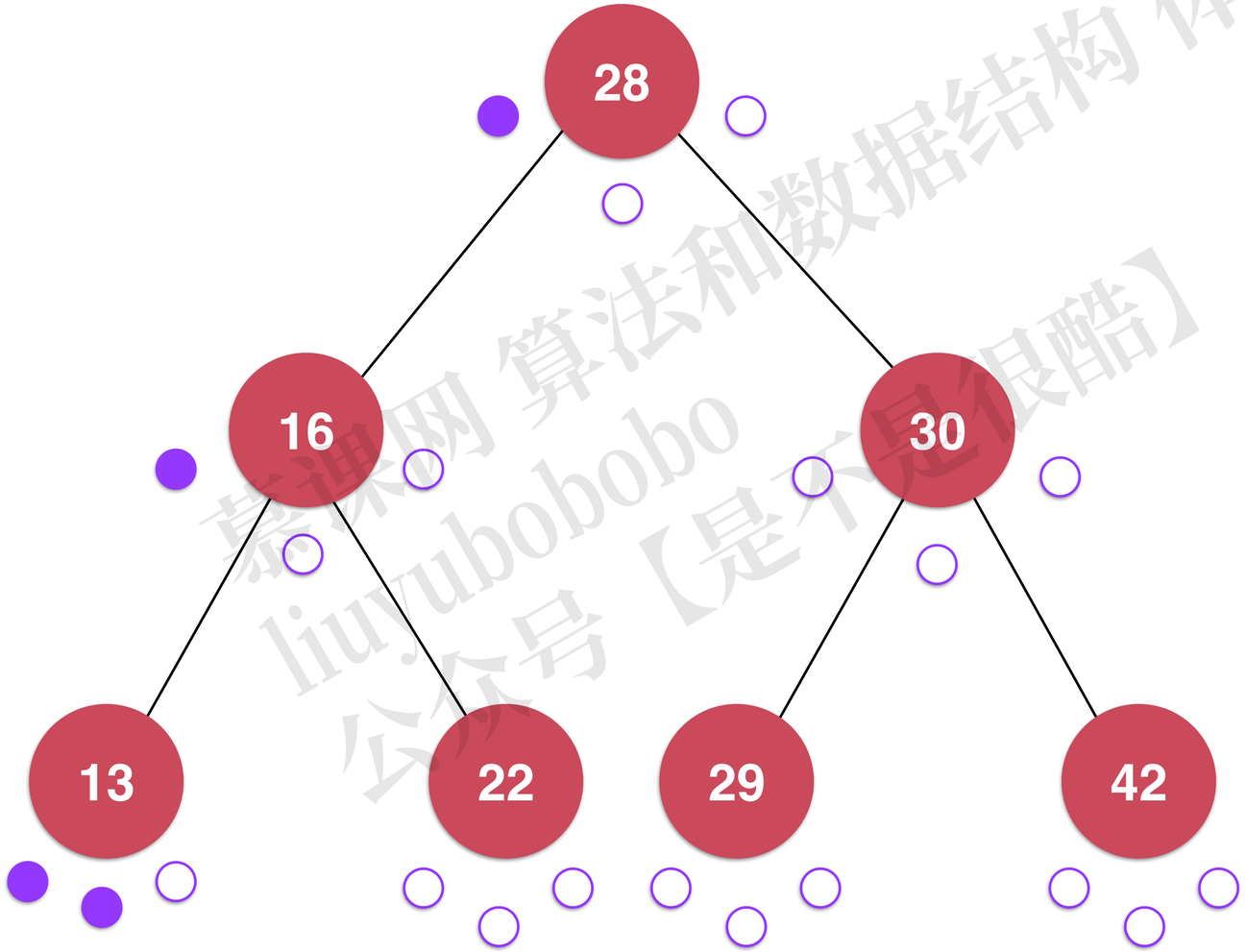
二分搜索树的后序遍历



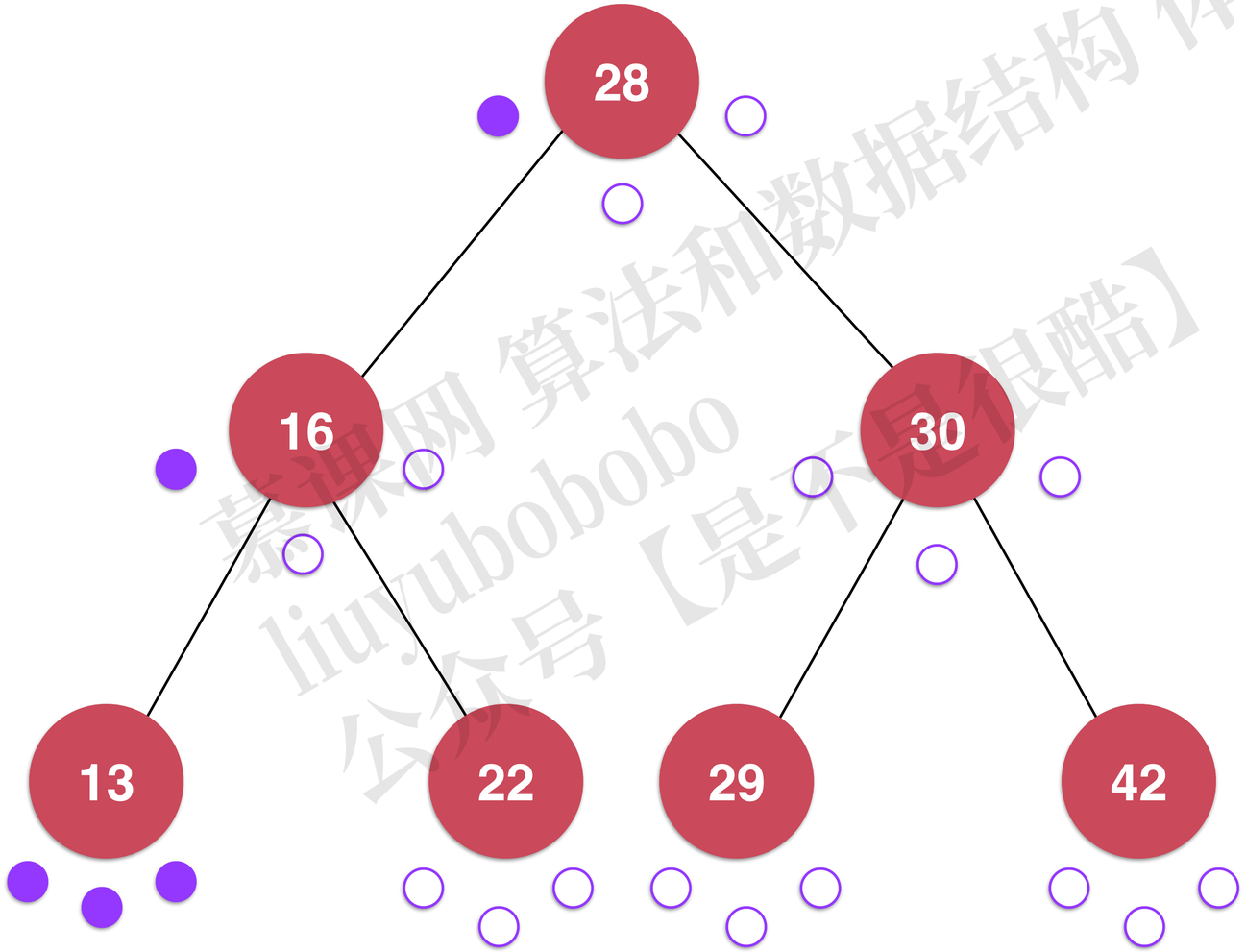
二分搜索树的后序遍历



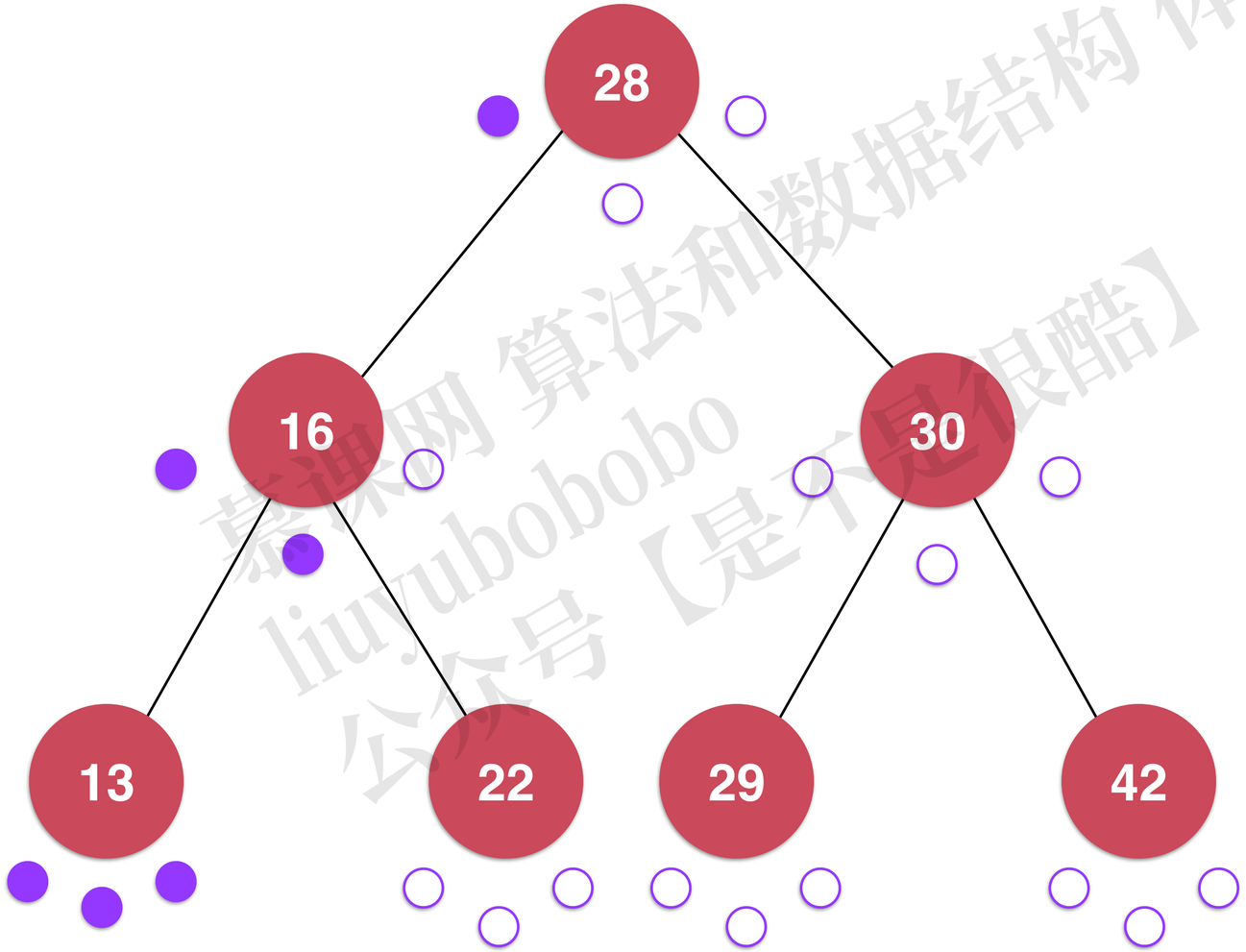
二分搜索树的后序遍历



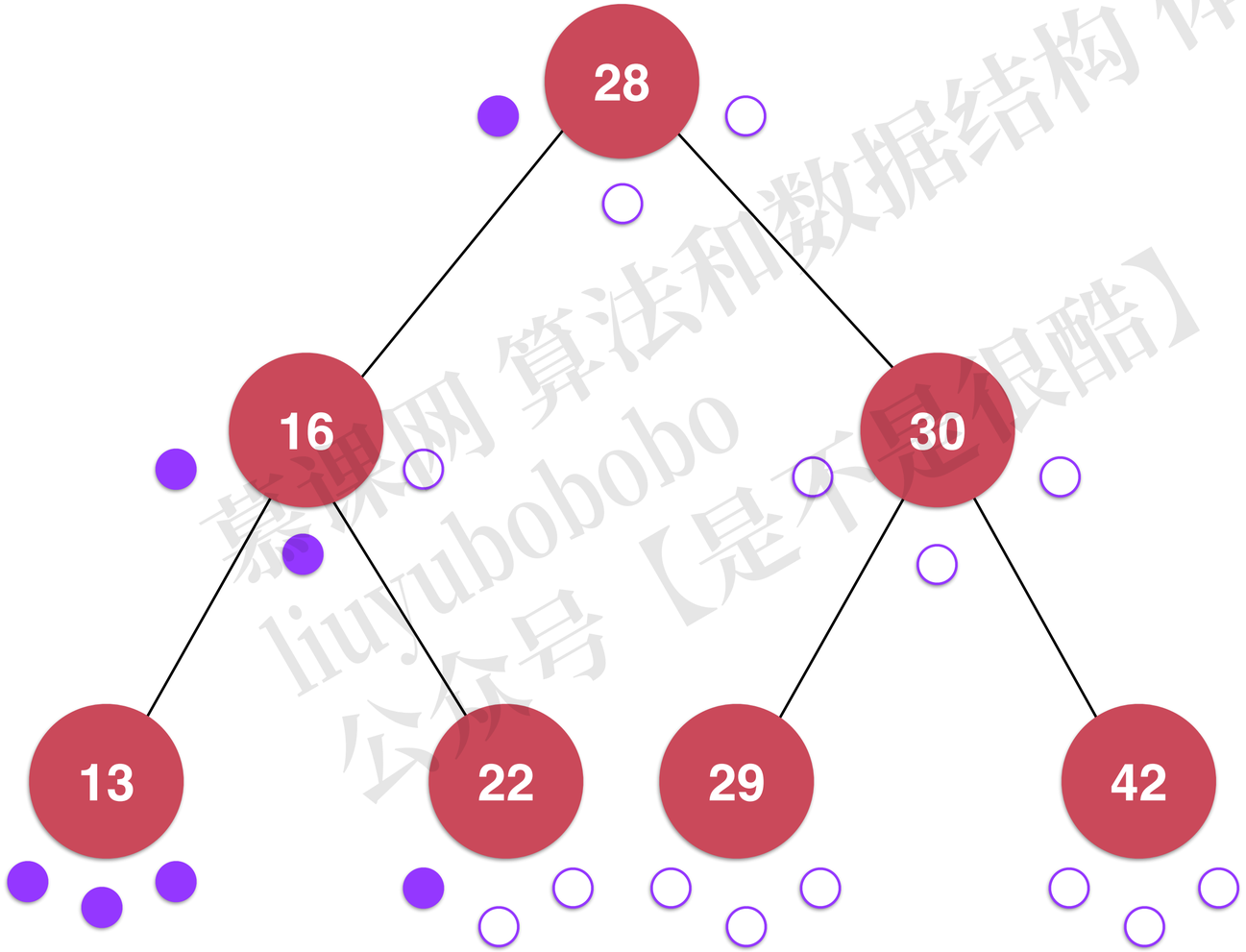
二分搜索树的后序遍历



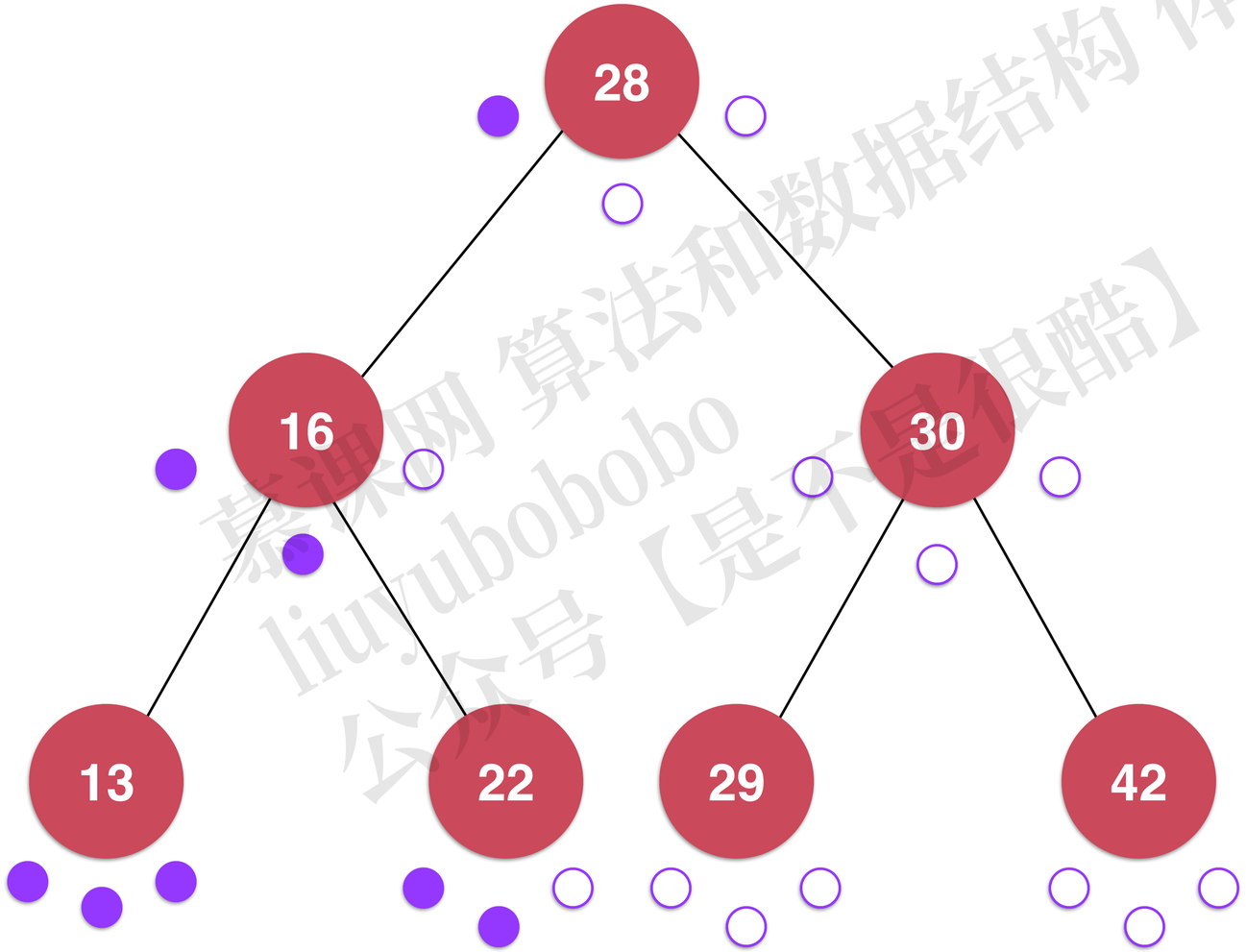
二分搜索树的后序遍历



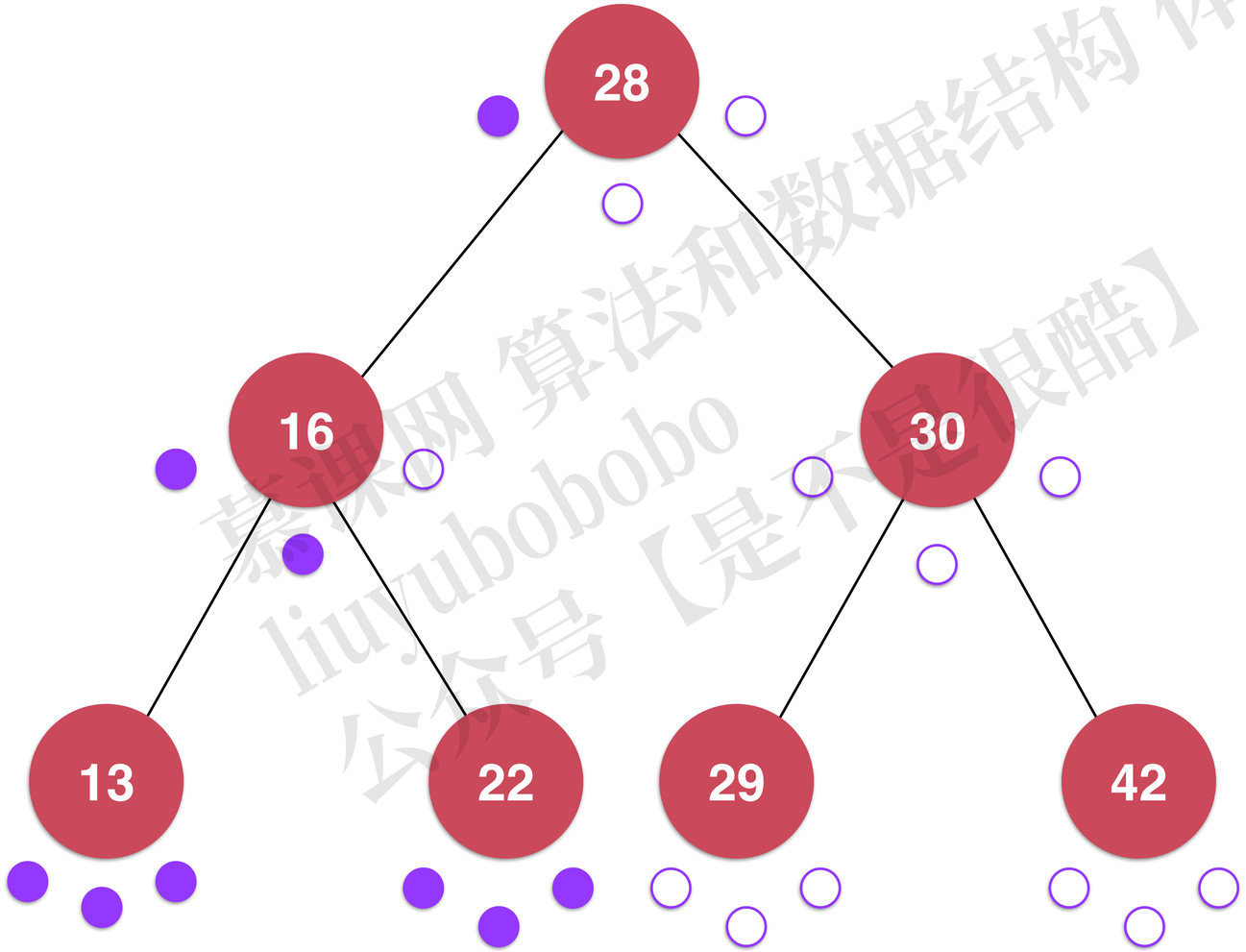
二分搜索树的后序遍历



二分搜索树的后序遍历



二分搜索树的后序遍历



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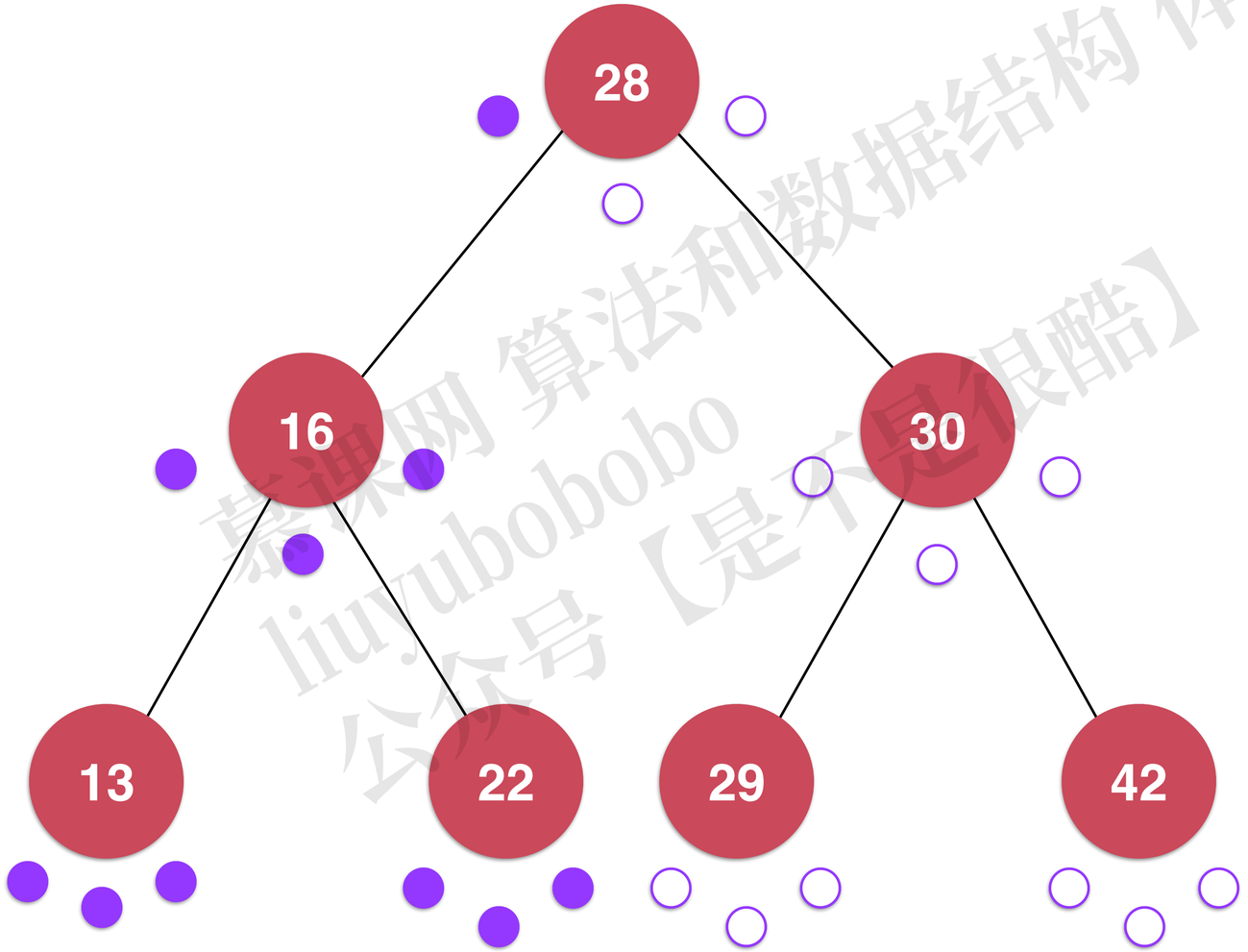
二分搜索树的后序遍历



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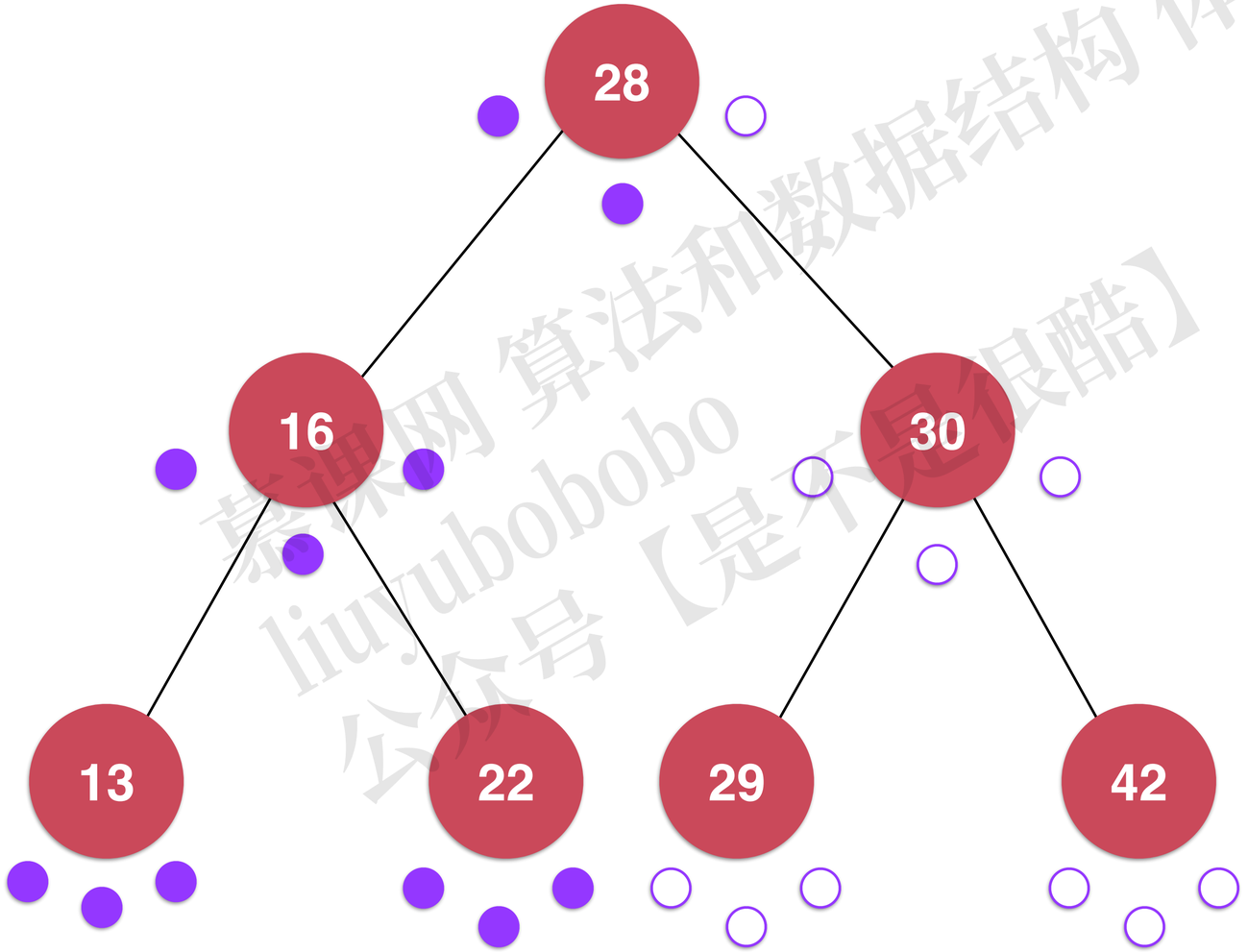
二分搜索树的后序遍历



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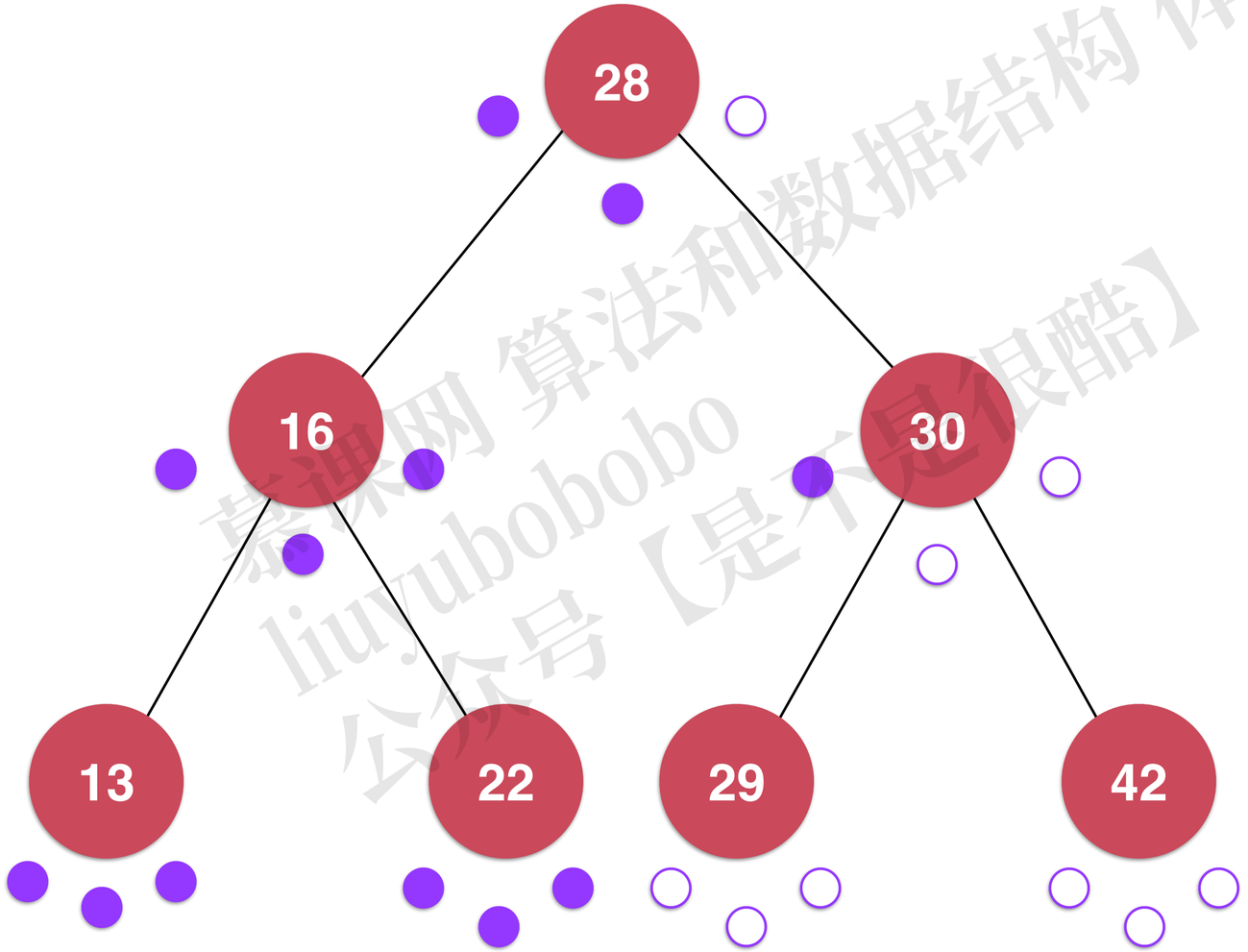
二分搜索树的后序遍历



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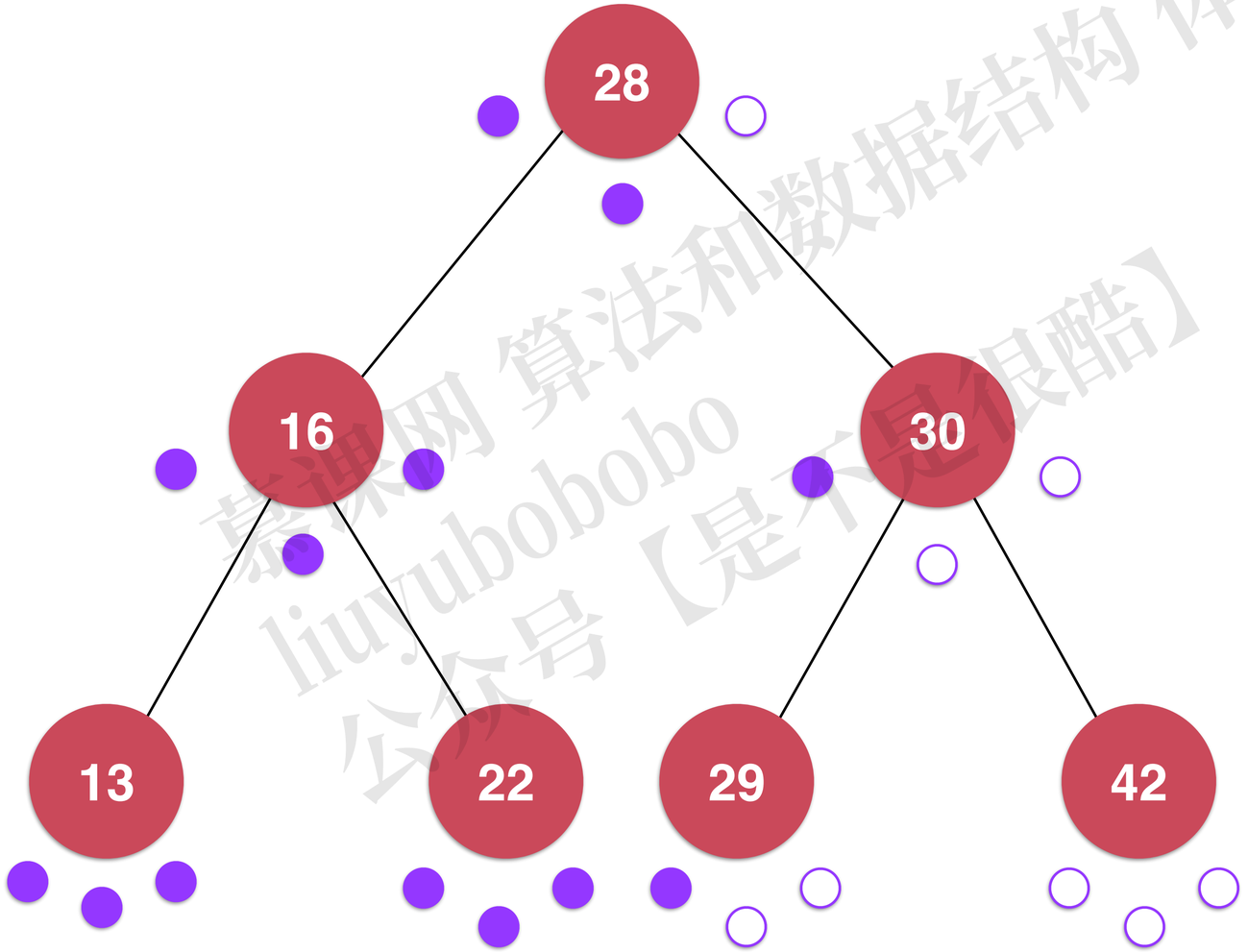
二分搜索树的后序遍历



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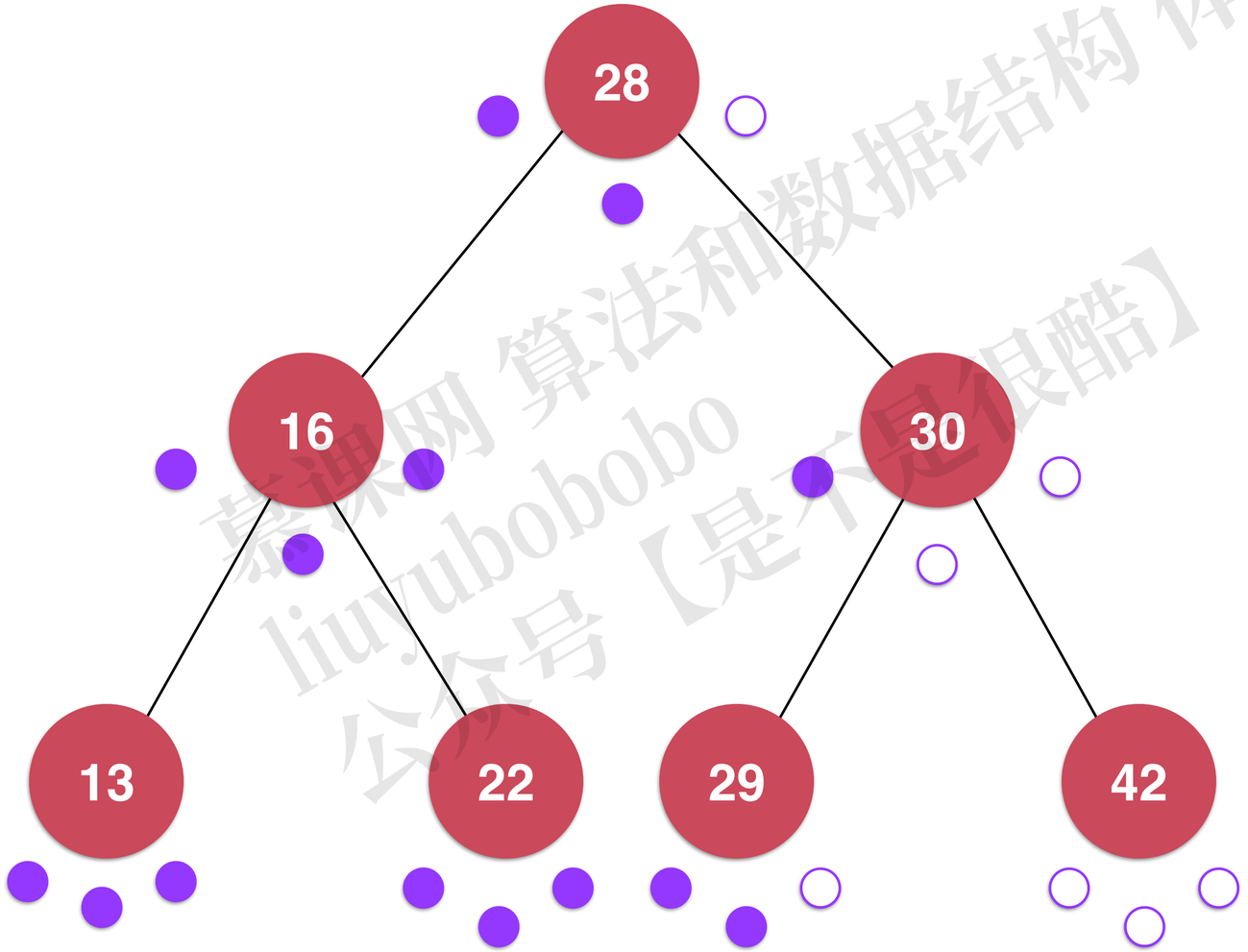
二分搜索树的后序遍历



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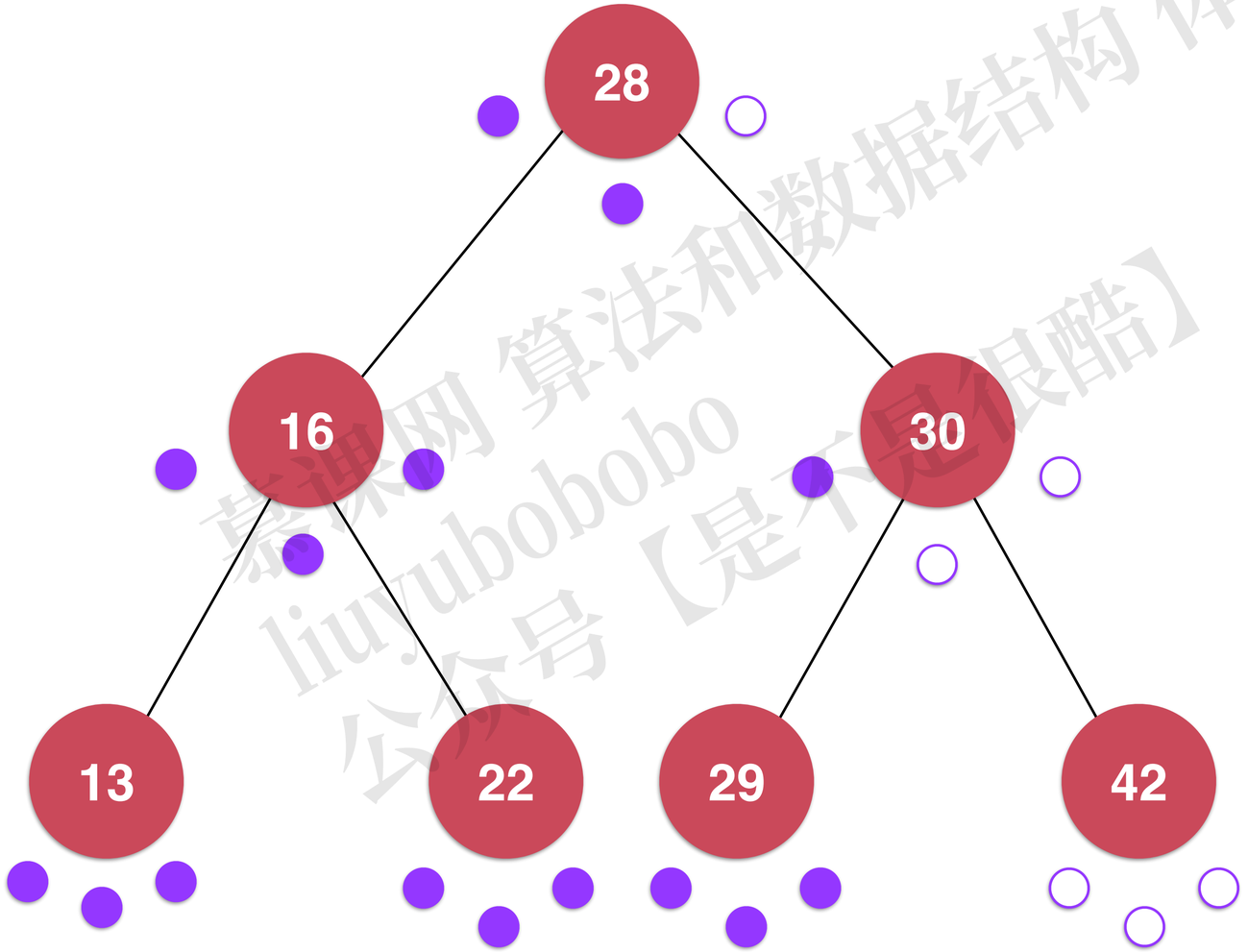
二分搜索树的后序遍历



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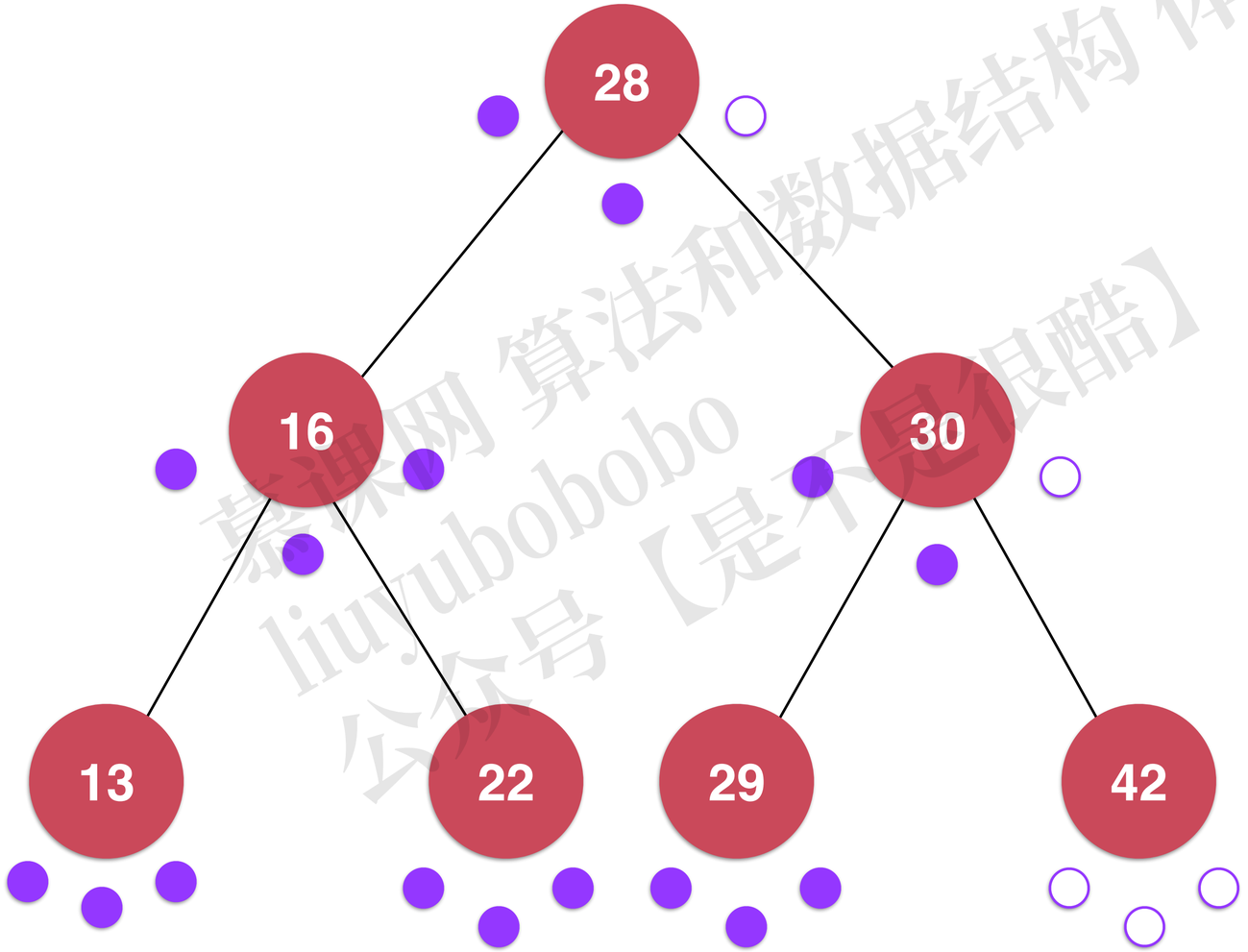
二分搜索树的后序遍历



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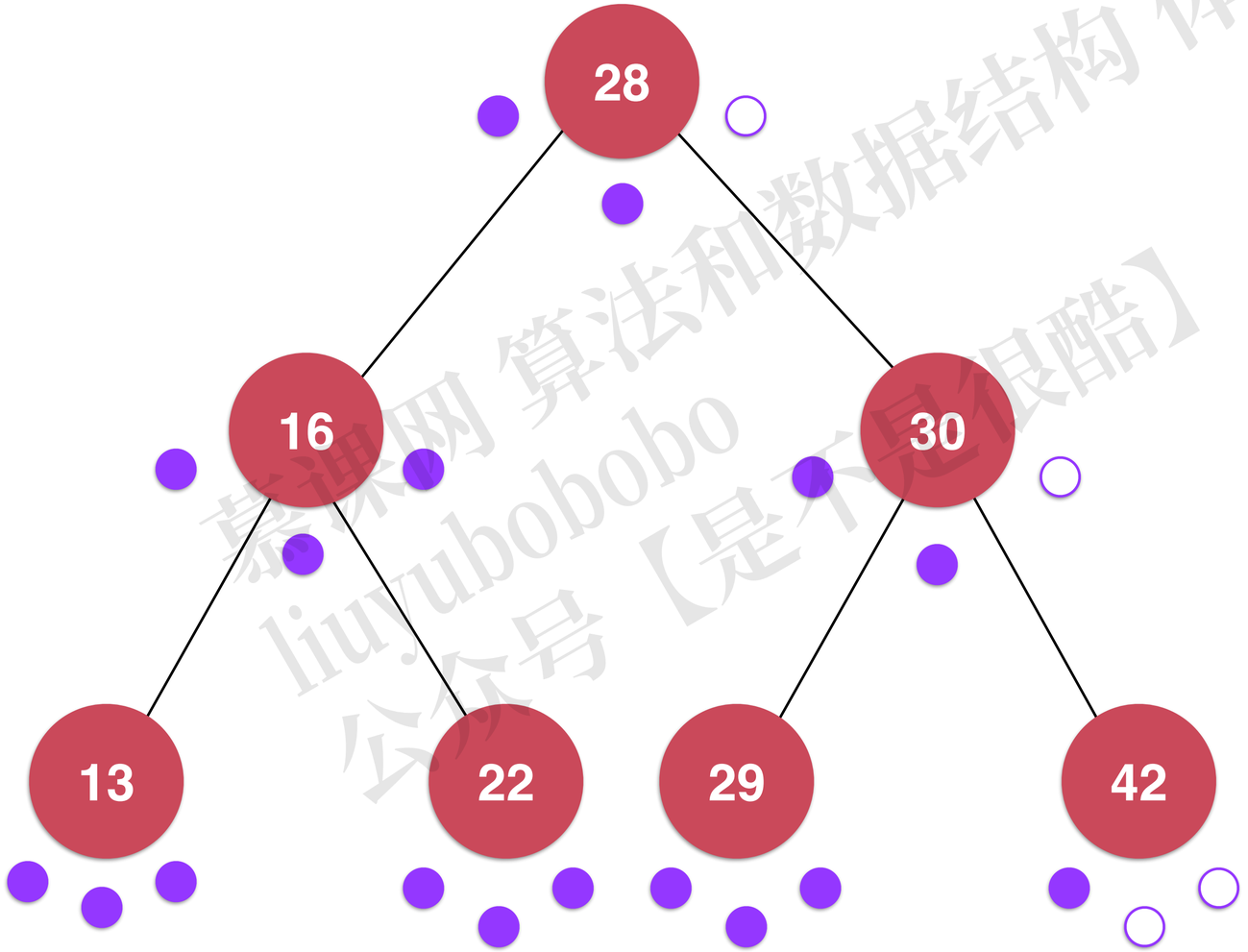
二分搜索树的后序遍历



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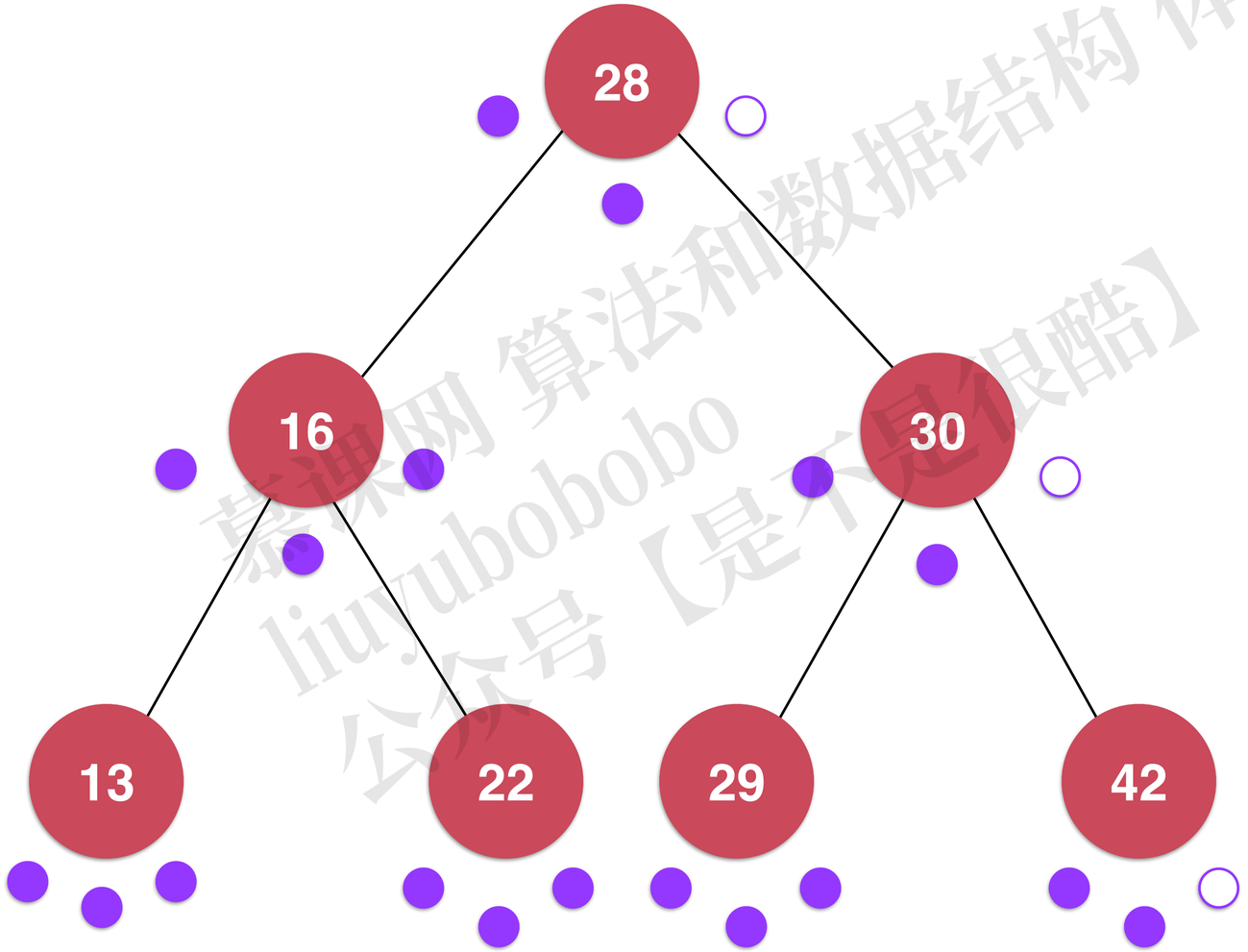
二分搜索树的后序遍历



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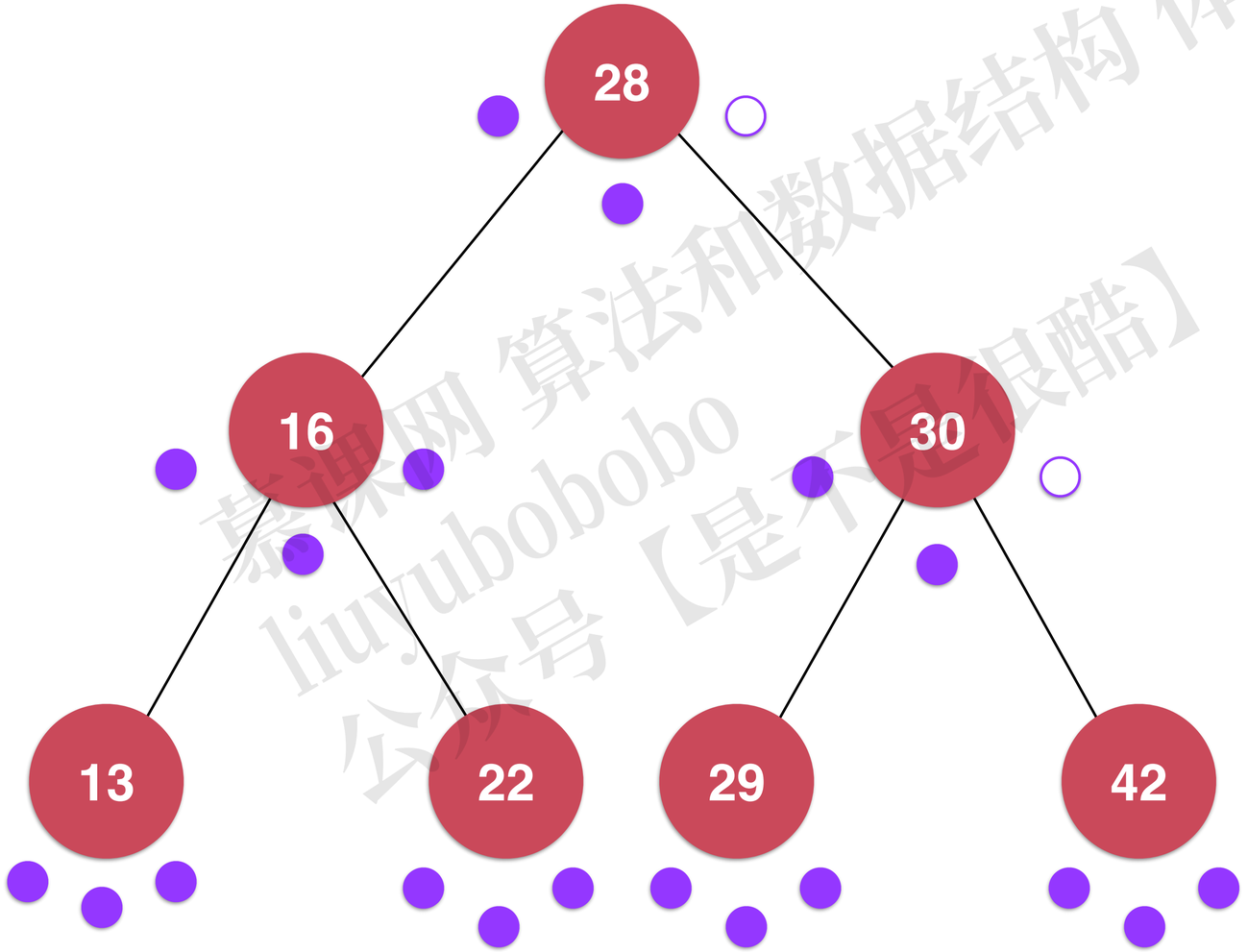
二分搜索树的后序遍历



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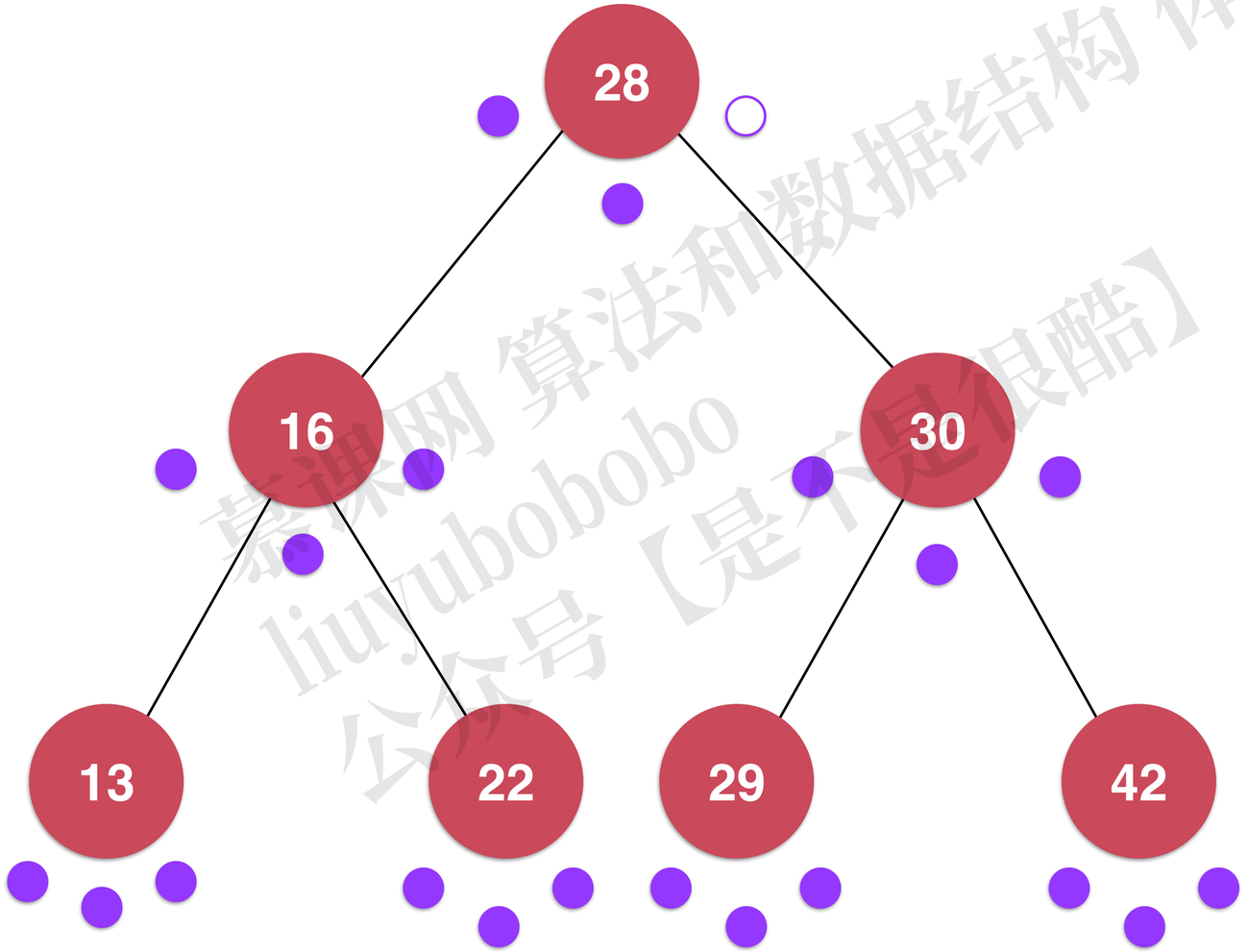
二分搜索树的后序遍历



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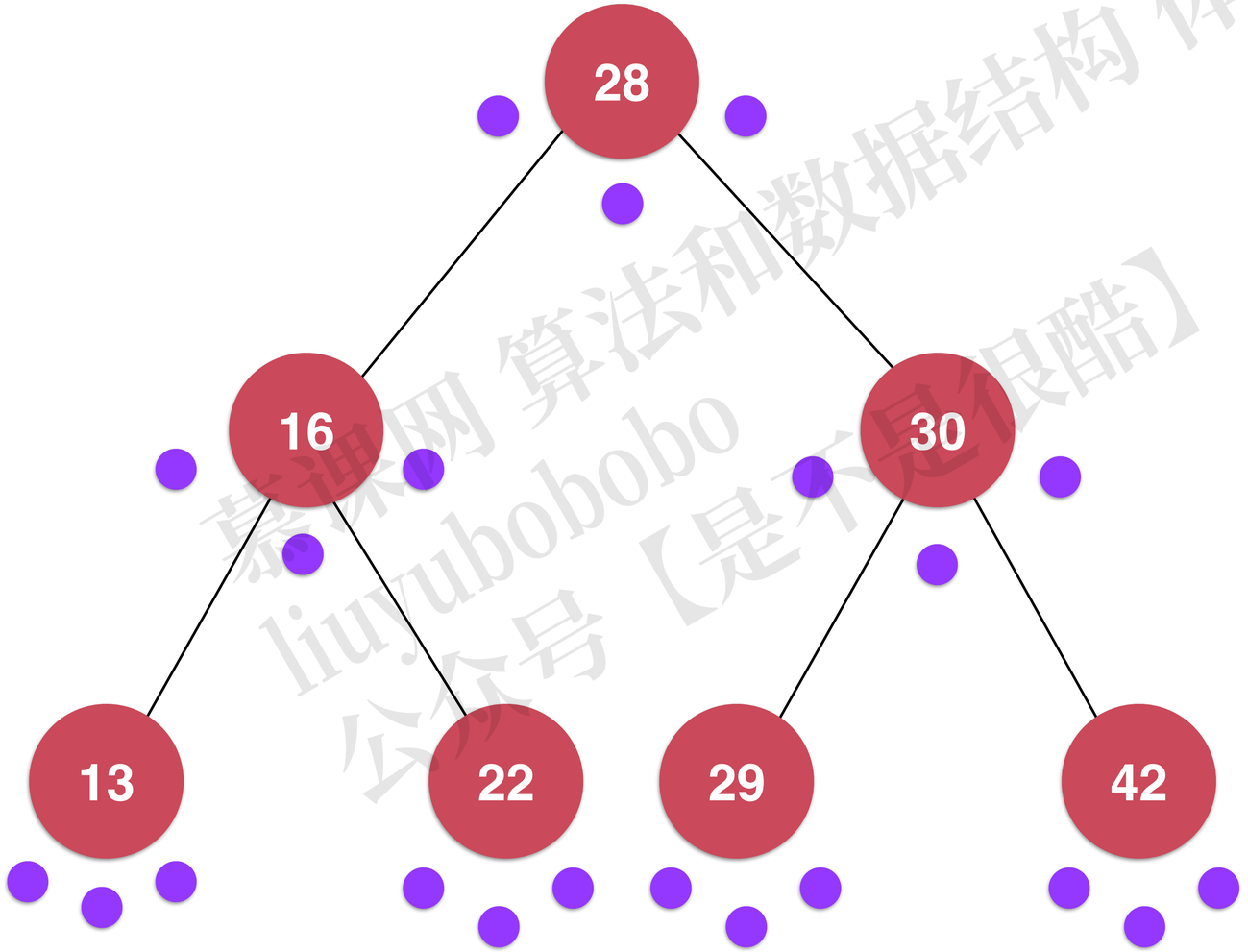
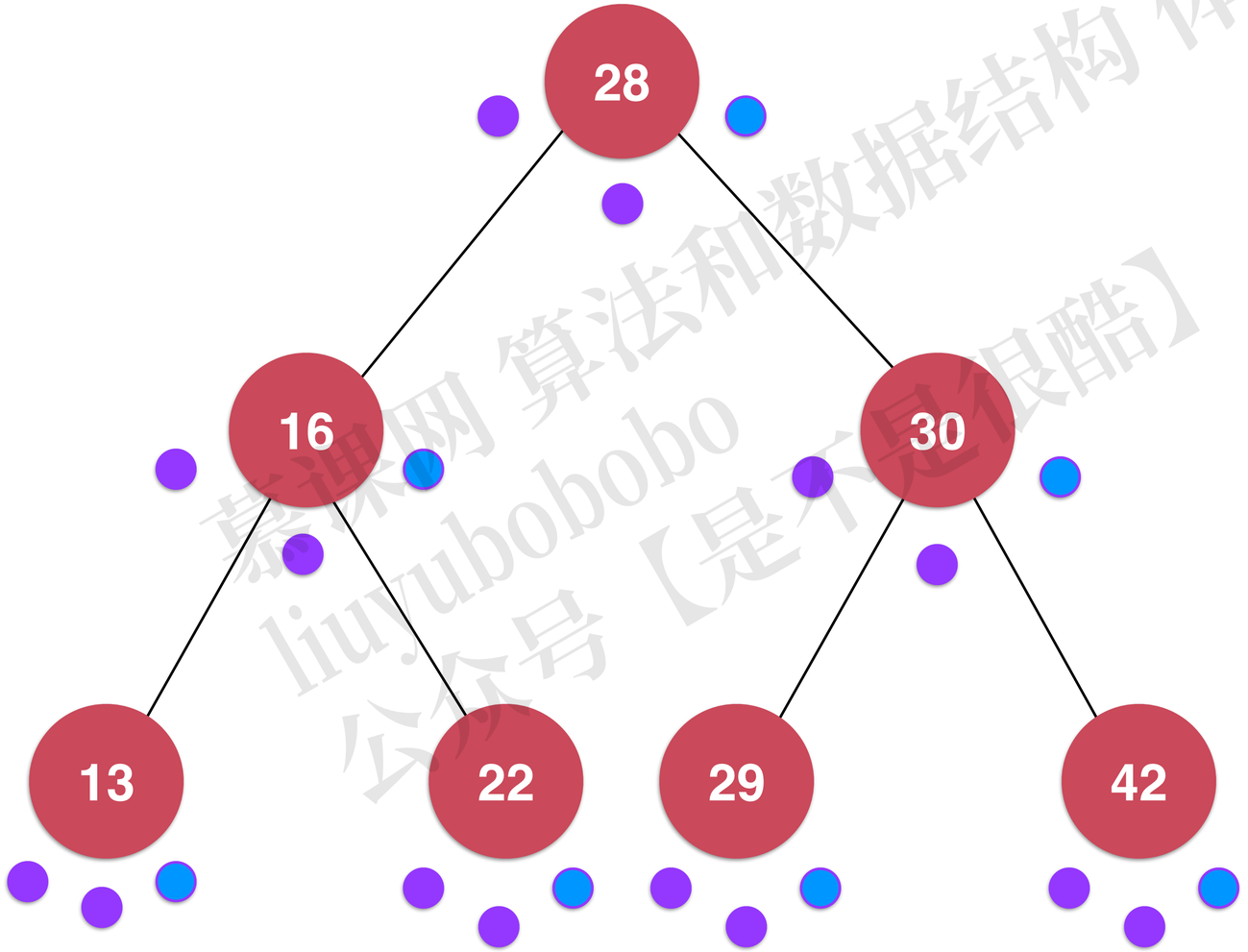
二分搜索树的后序遍历



二分搜索树的后序遍历

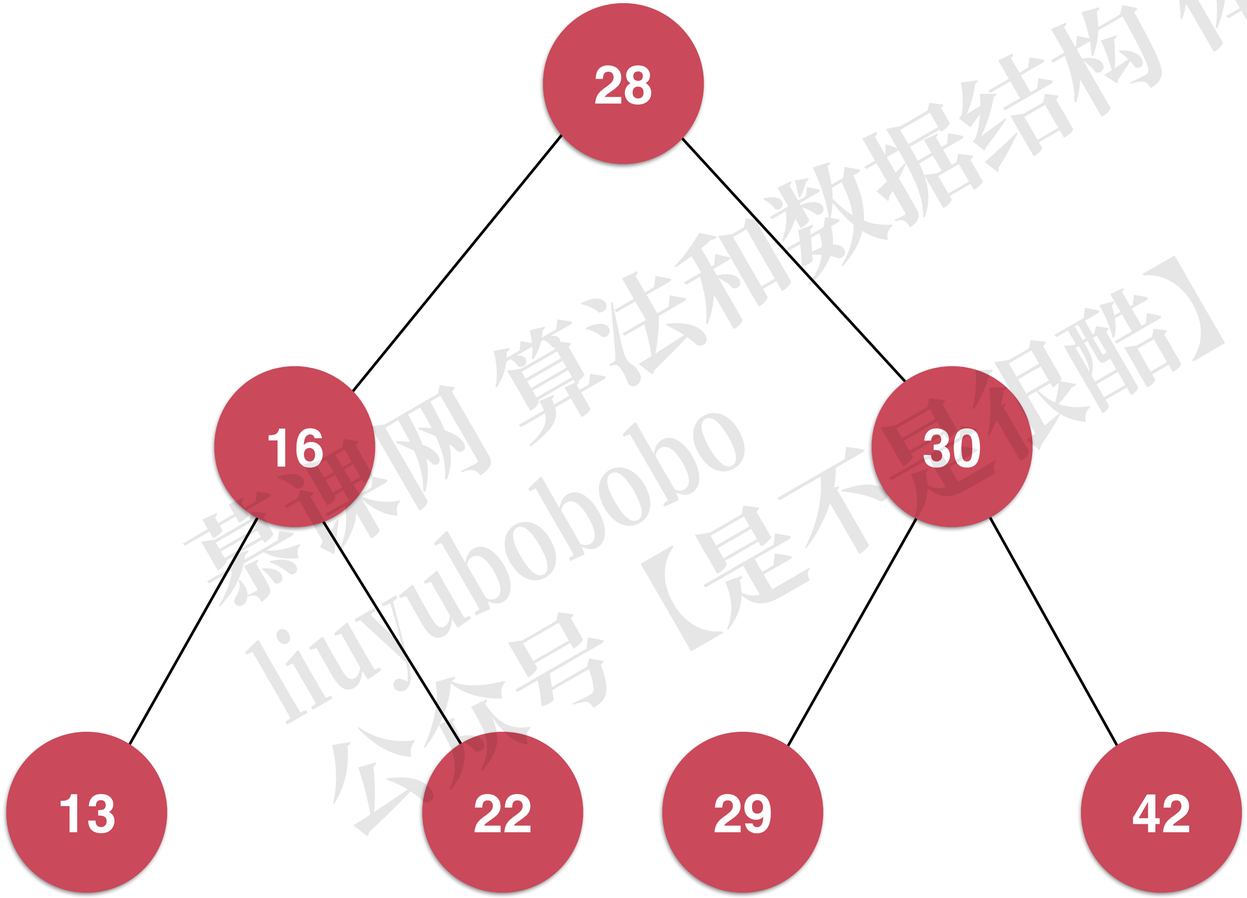


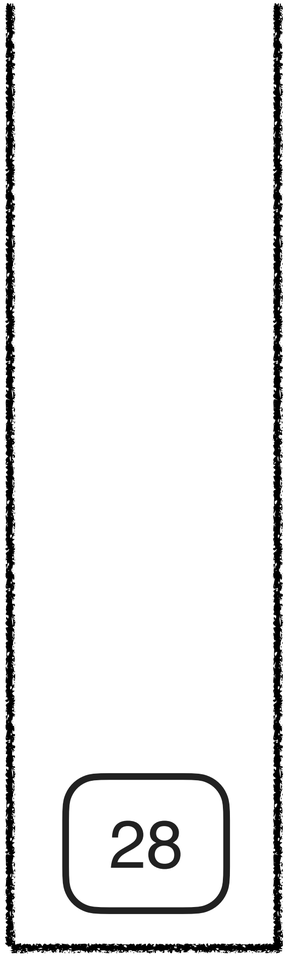
二分搜索树前序遍历的非递归写法

前序遍历

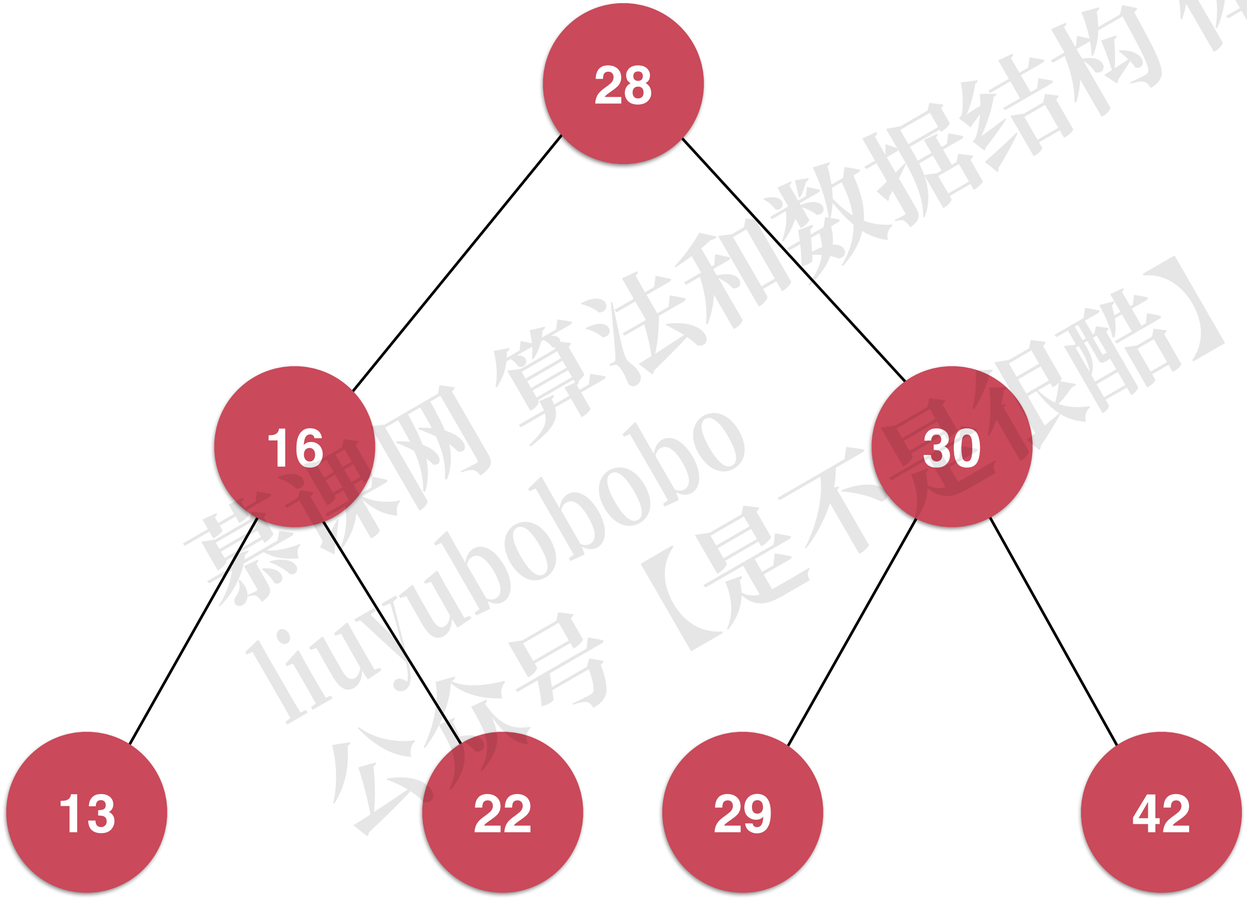
```
function traverse(node):
    if(node == null)
        return;

    访问该节点

    traverse(node.left)
    traverse(node.right)
```

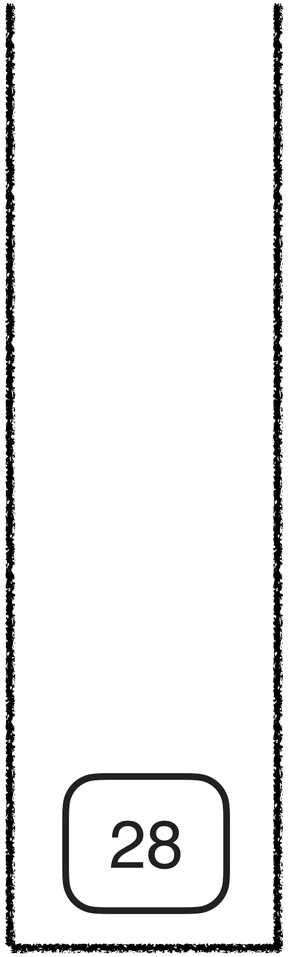
二分搜索树前序遍历的非递归写法



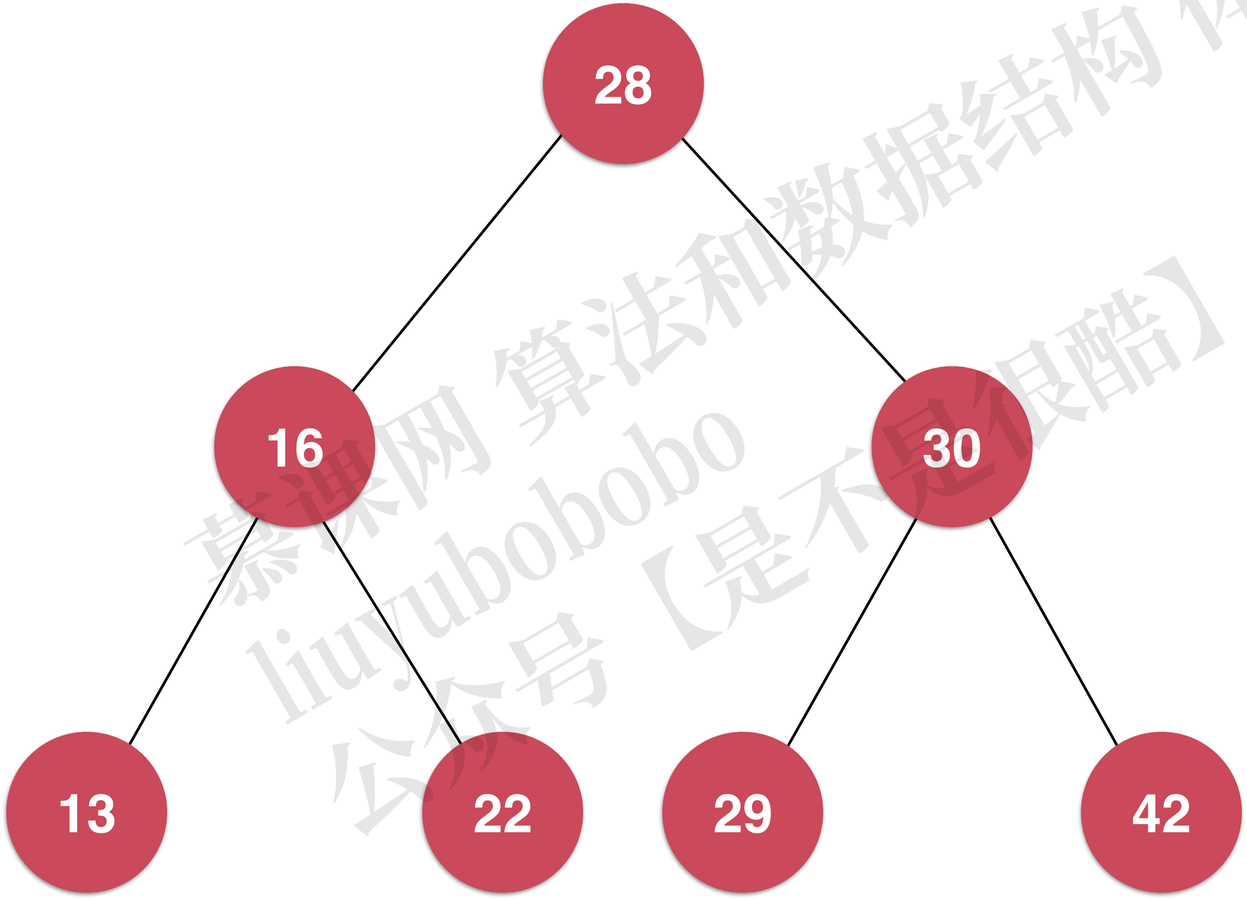


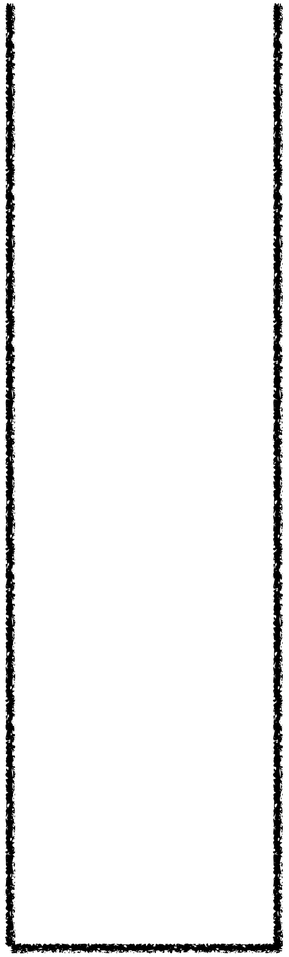
二分搜索树前序遍历的非递归写法



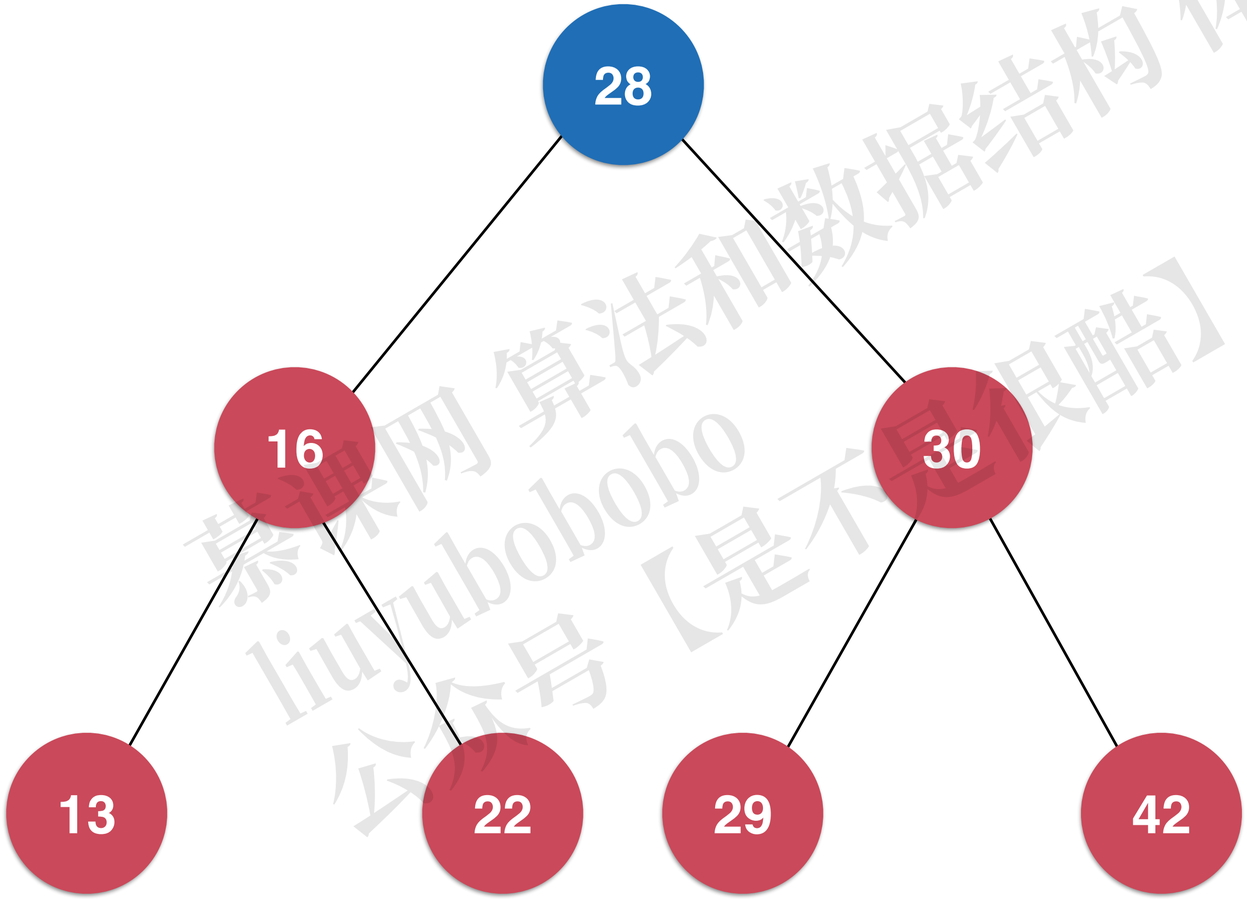


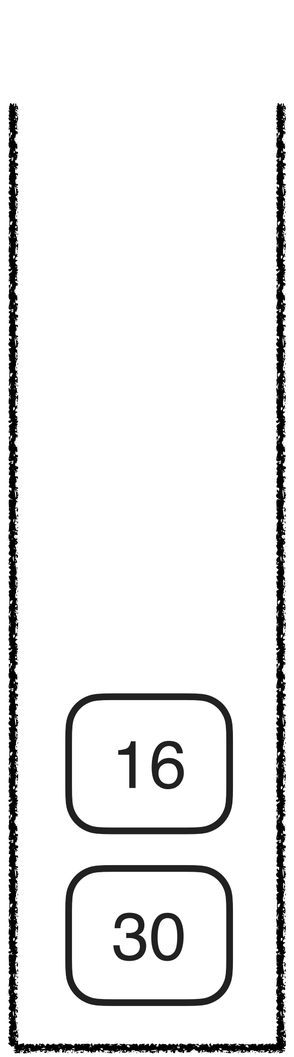
二分搜索树前序遍历的非递归写法





二分搜索树前序遍历的非递归写法





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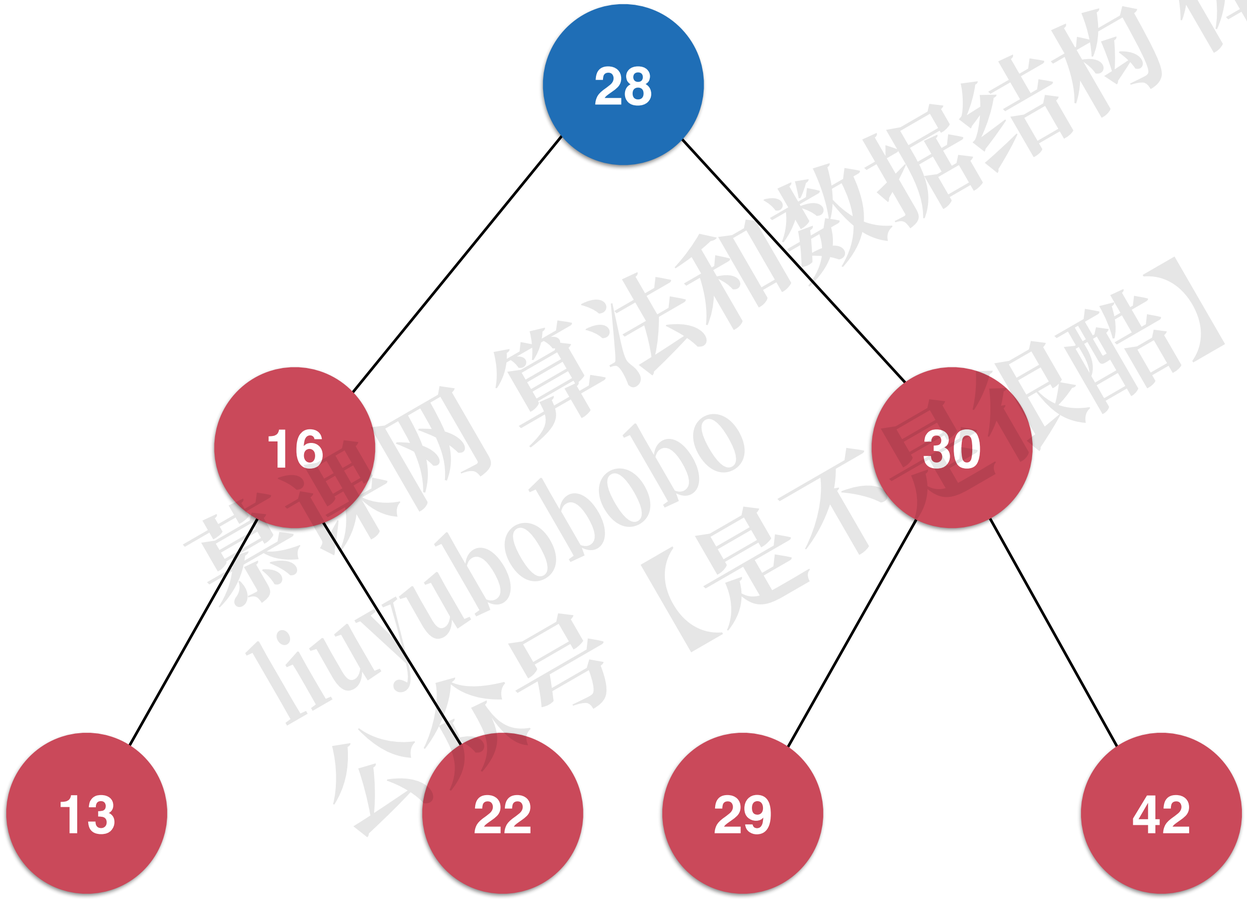
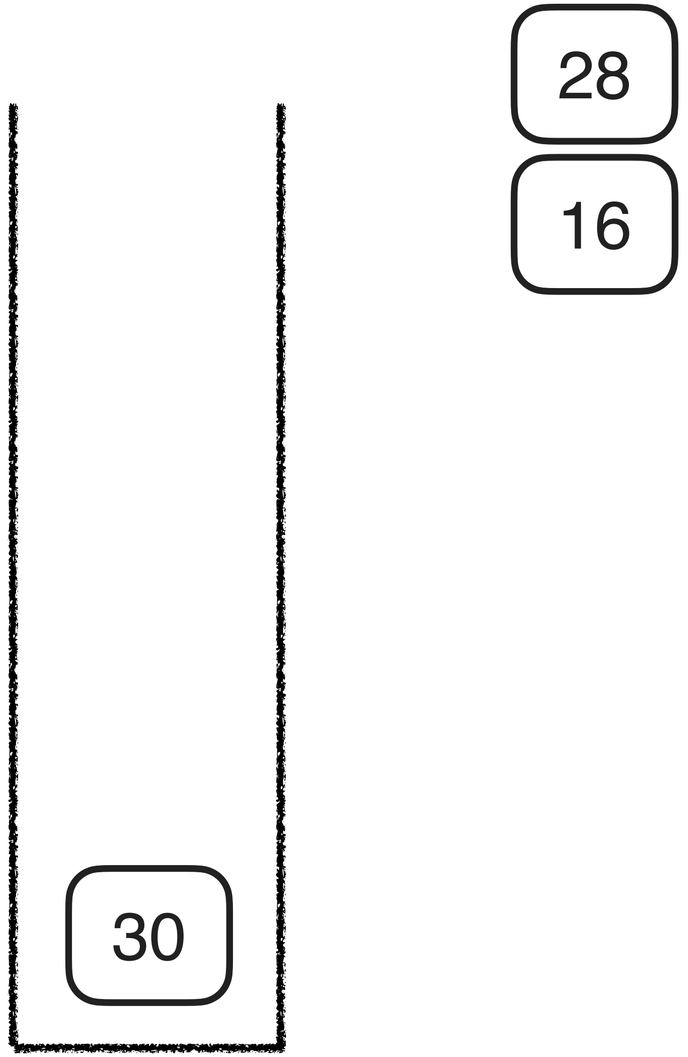
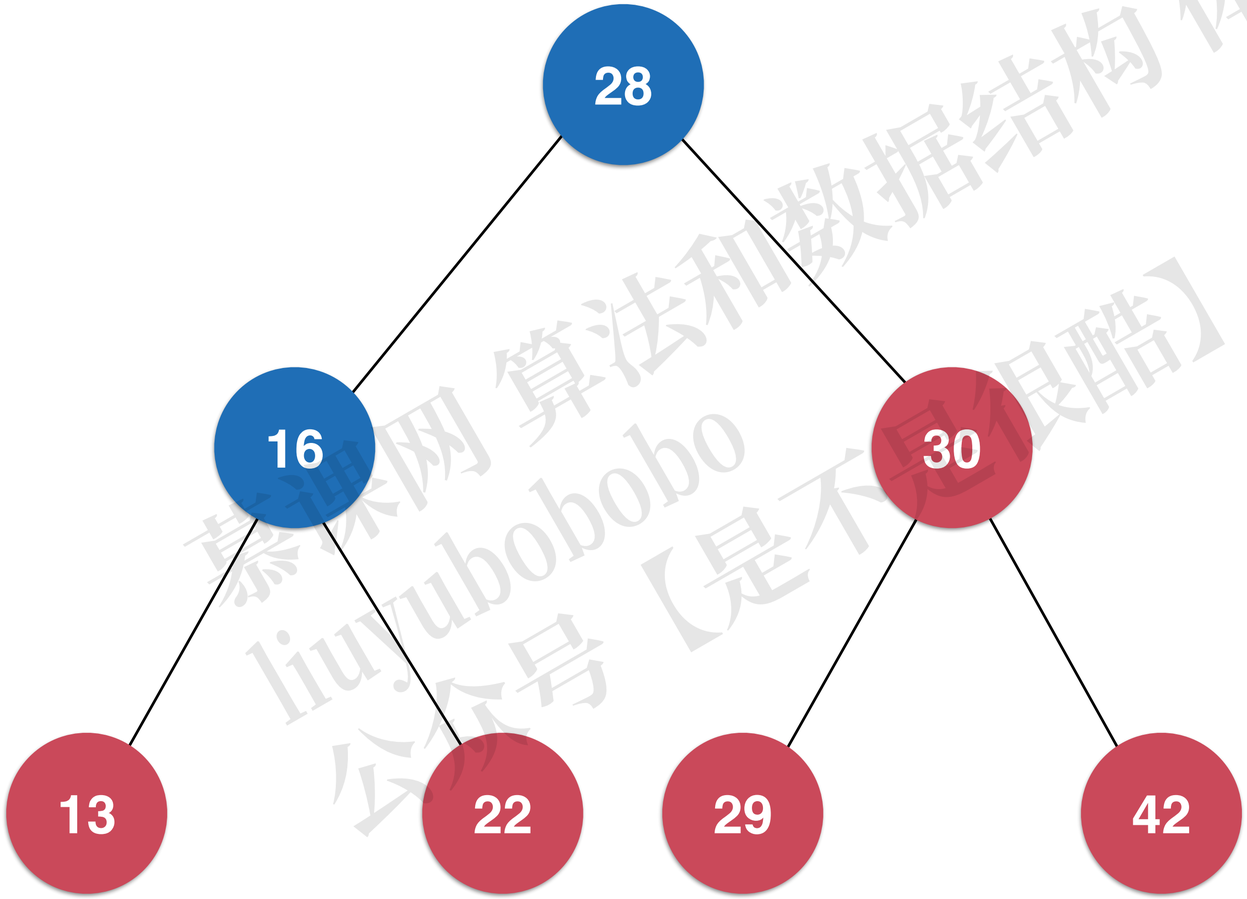
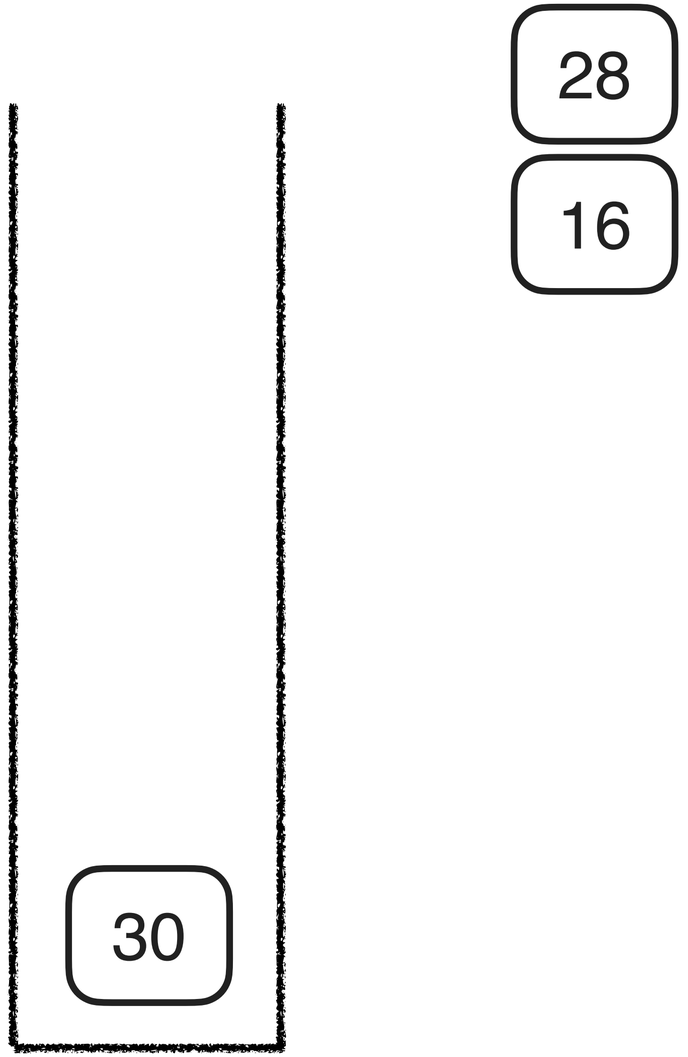
二分搜索树前序遍历的非递归写法



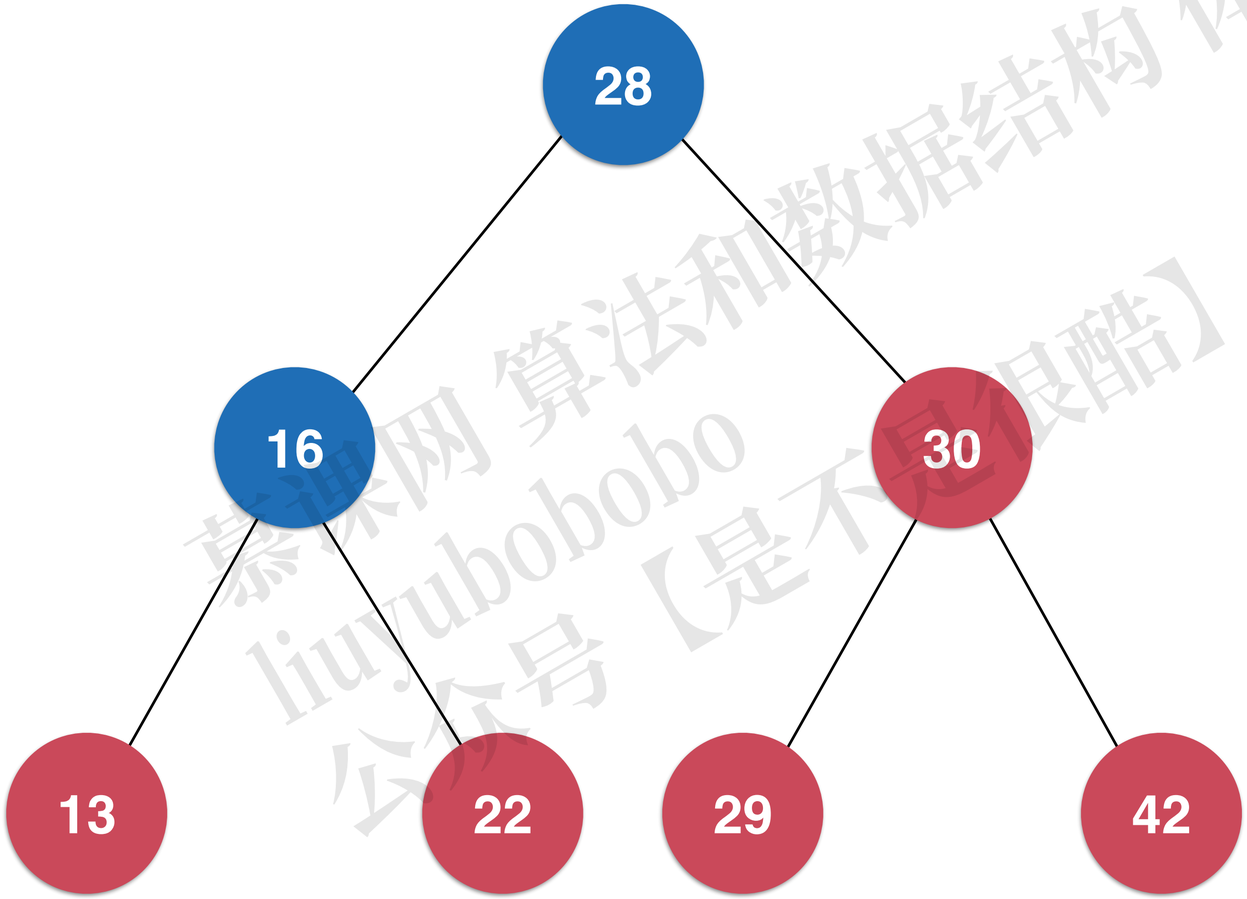


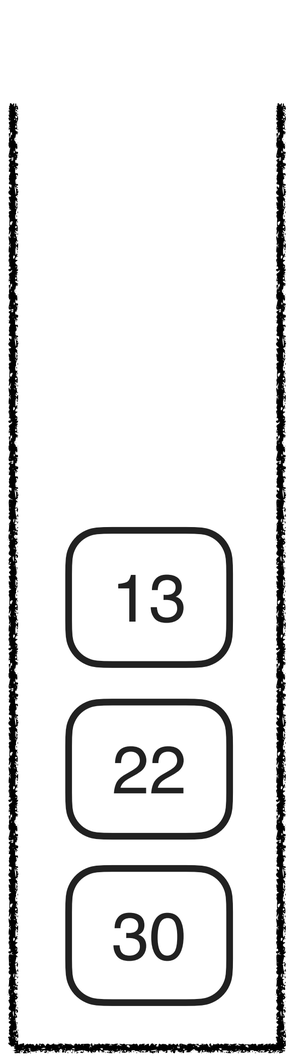
二分搜索树前序遍历的非递归写法





二分搜索树前序遍历的非递归写法





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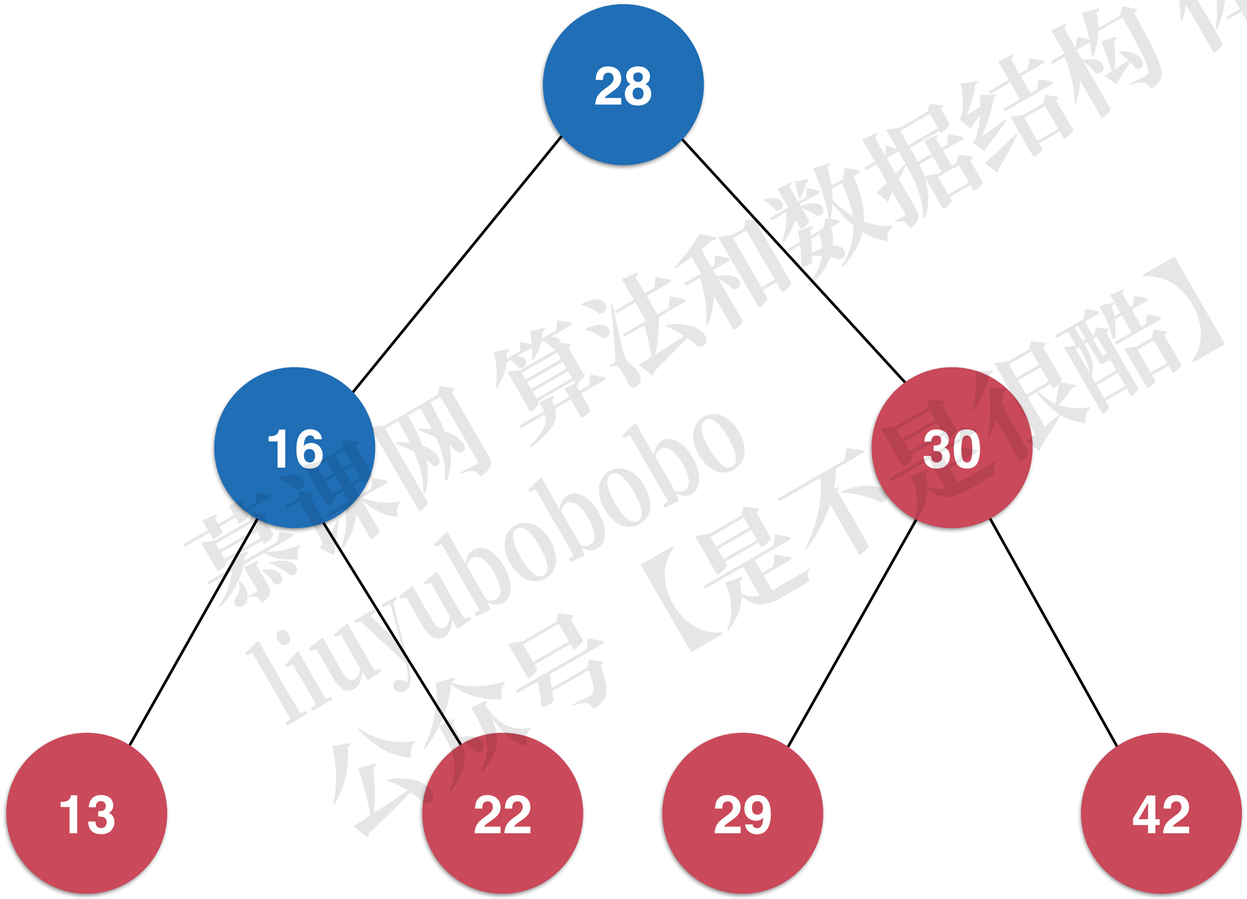
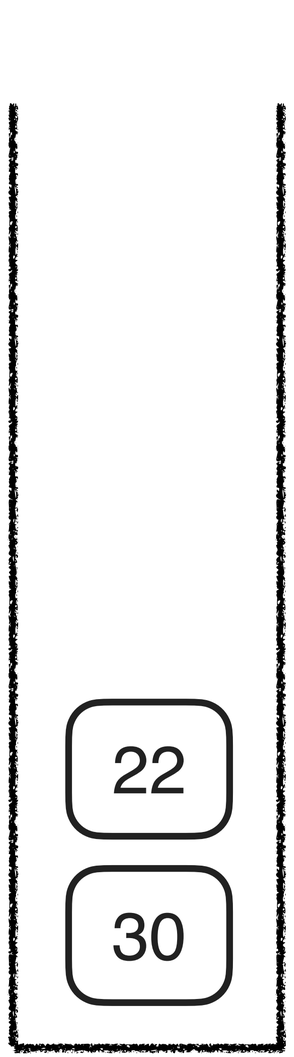
二分搜索树前序遍历的非递归写法





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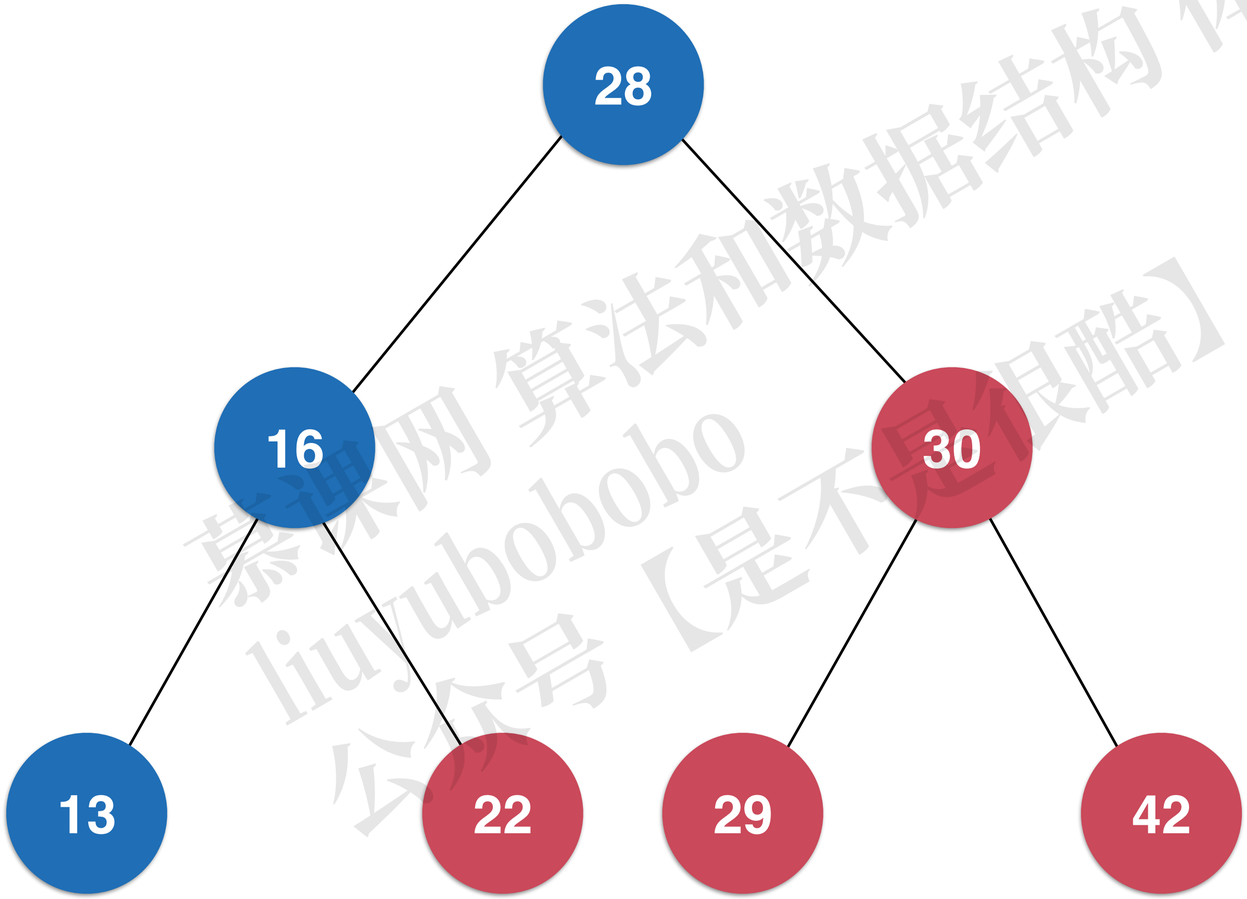
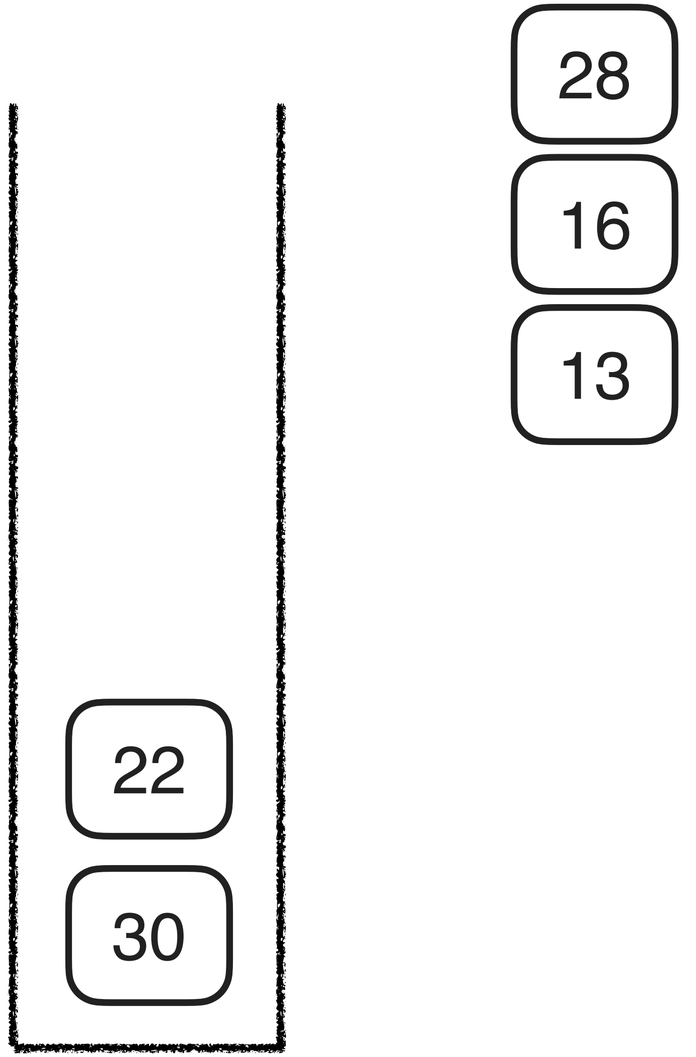
13

二分搜索树前序遍历的非递归写法





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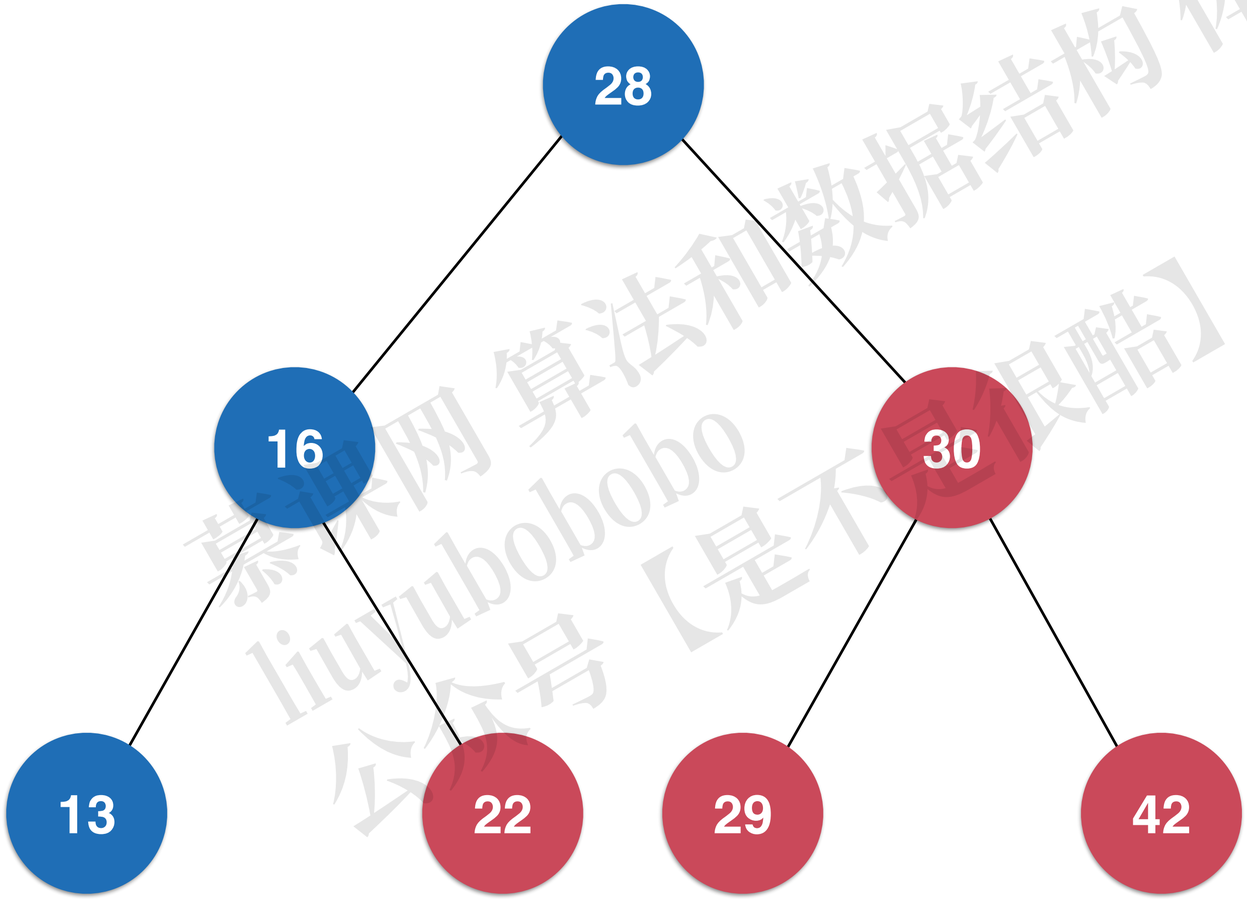
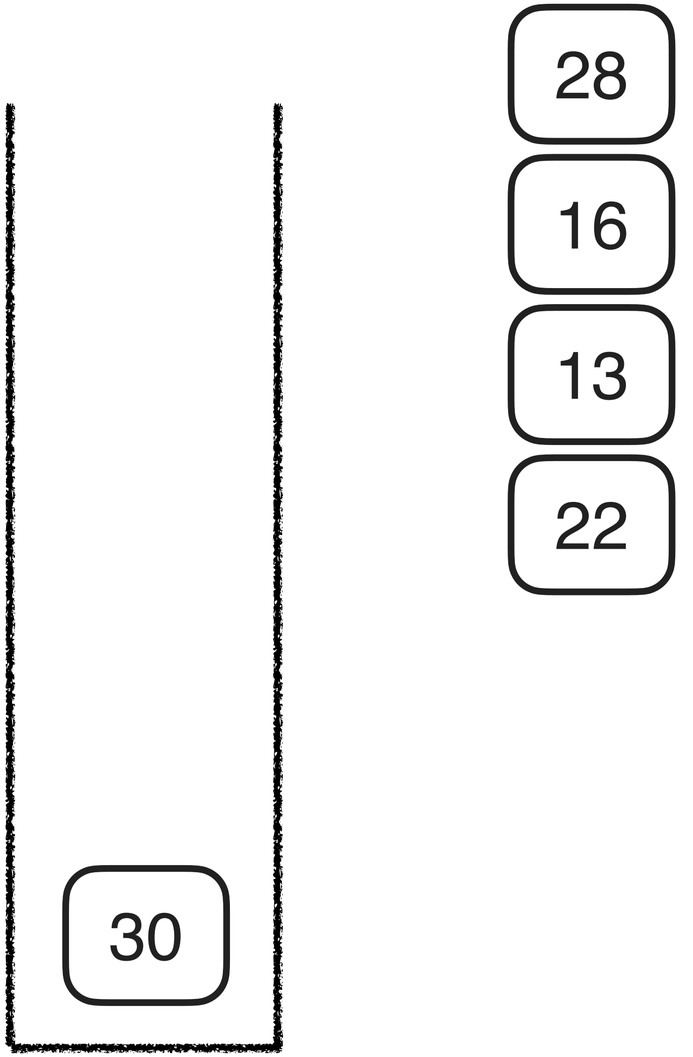
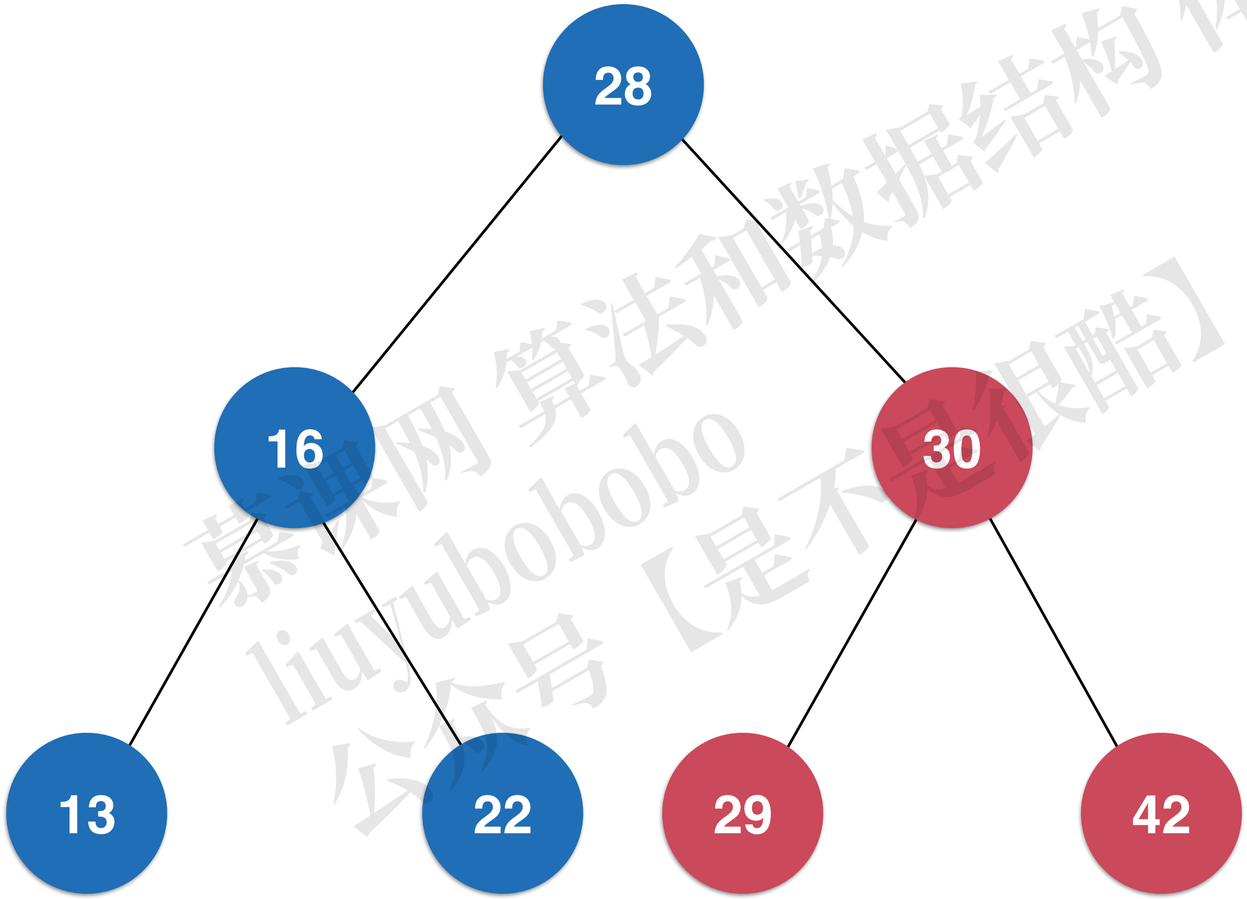
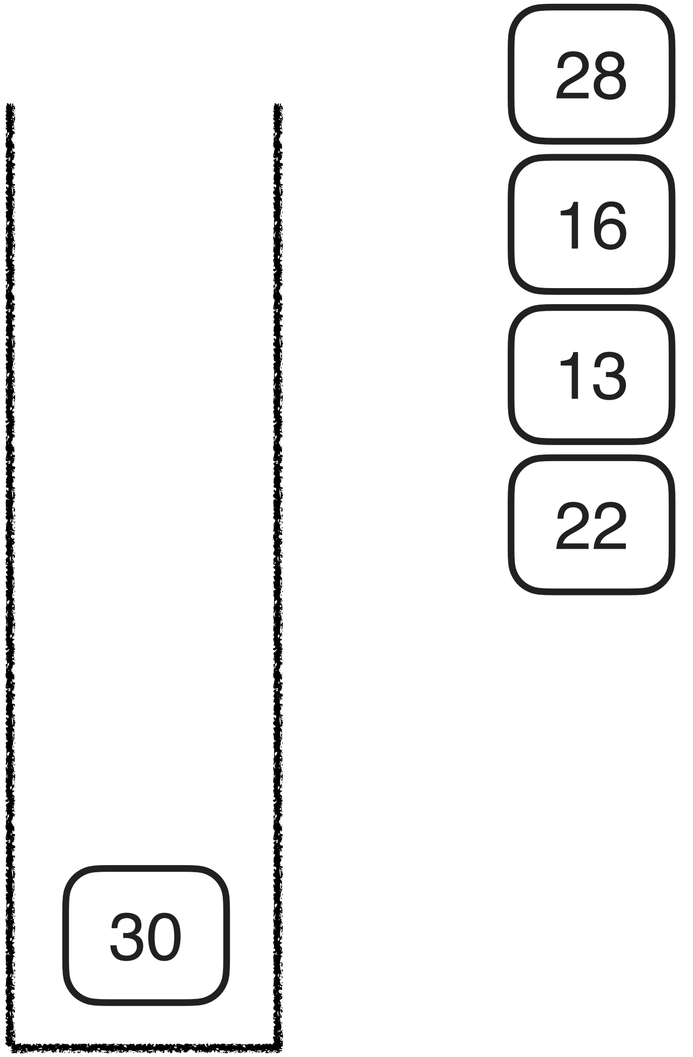




二分搜索树前序遍历的非递归写法





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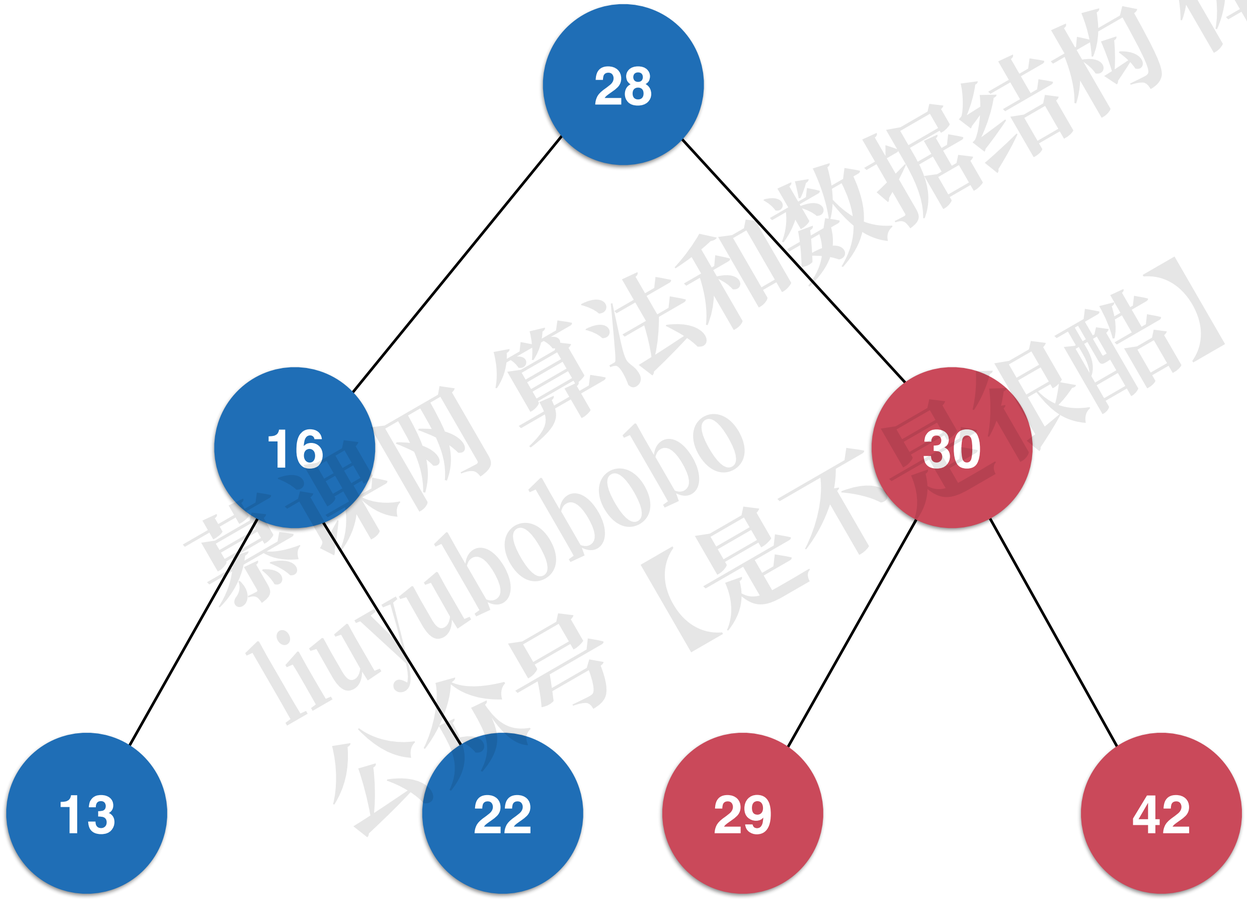
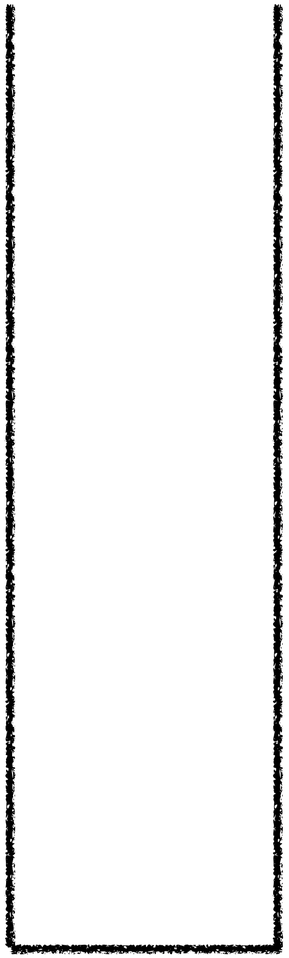
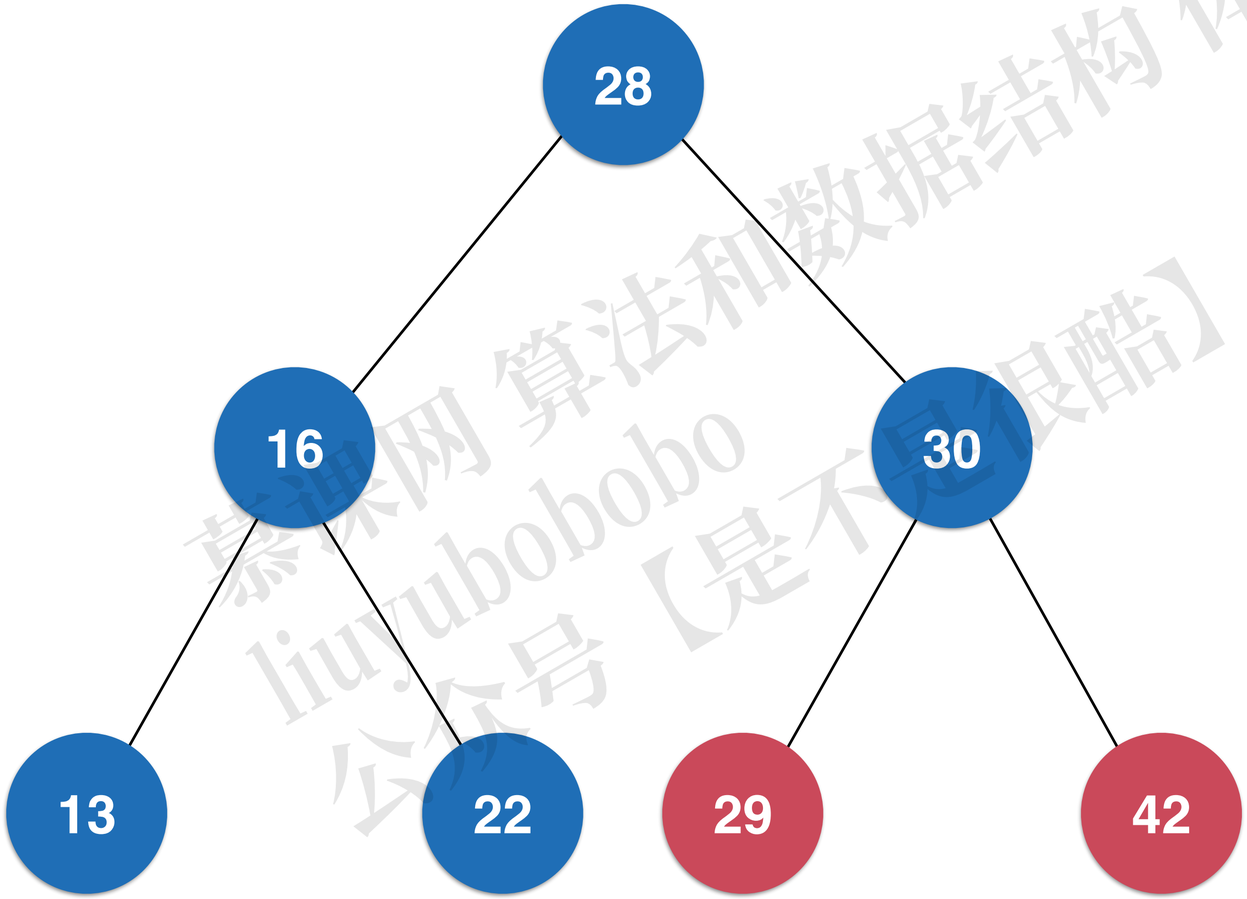
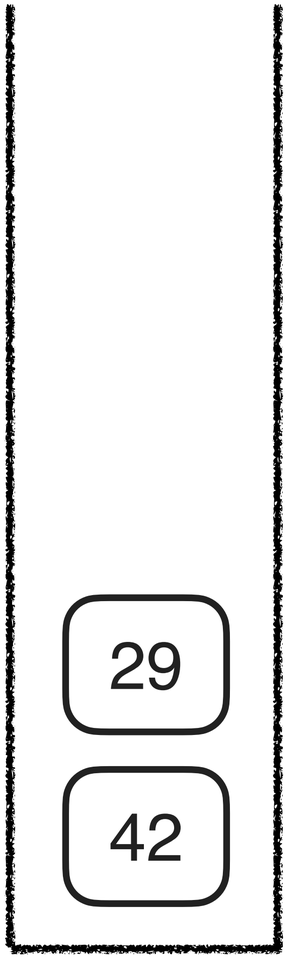




二分搜索树前序遍历的非递归写法





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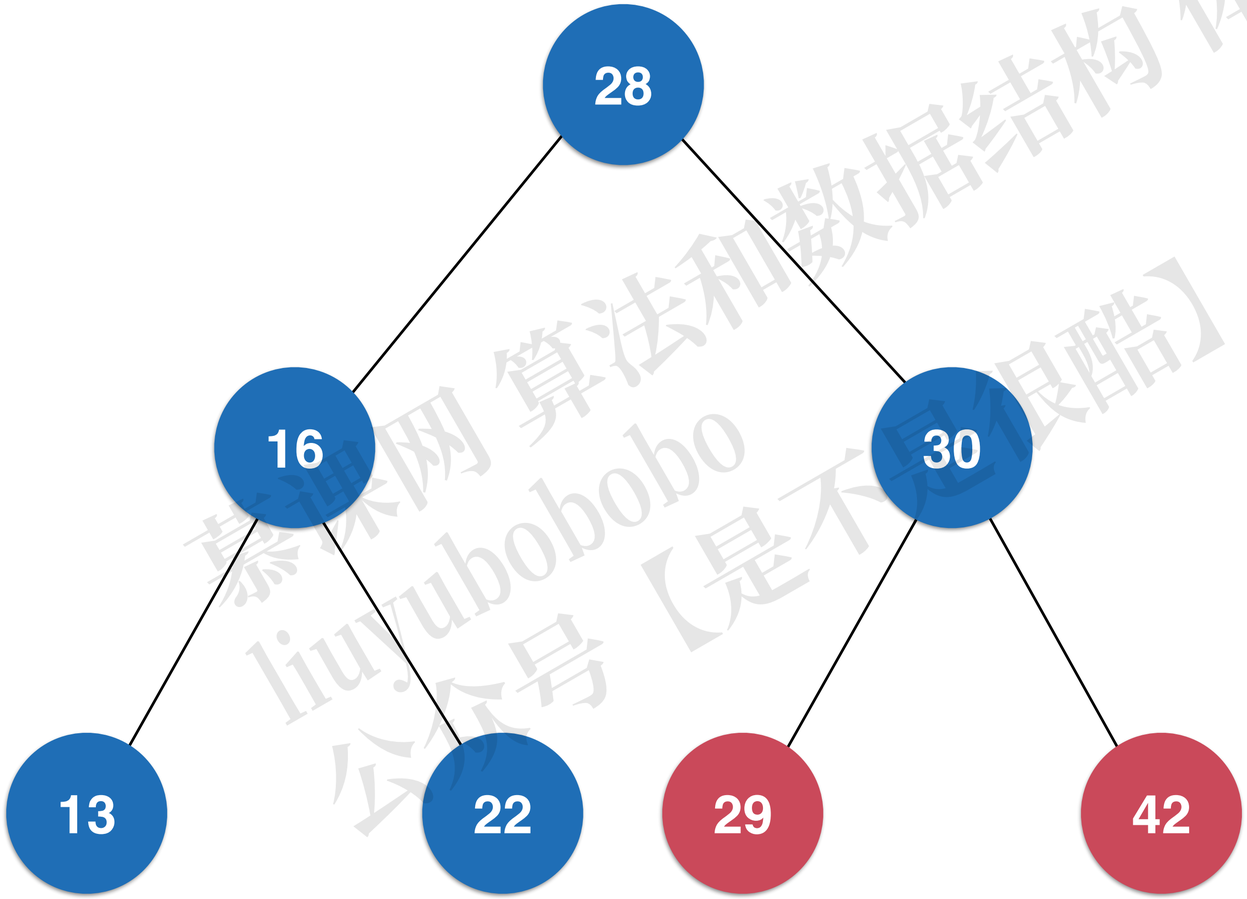

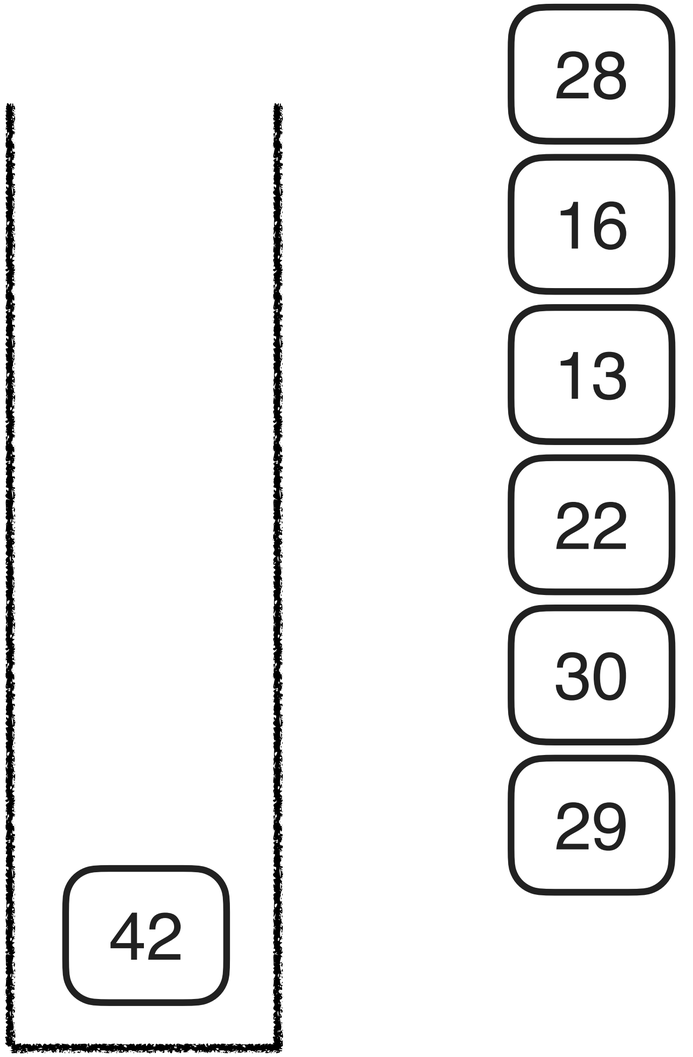

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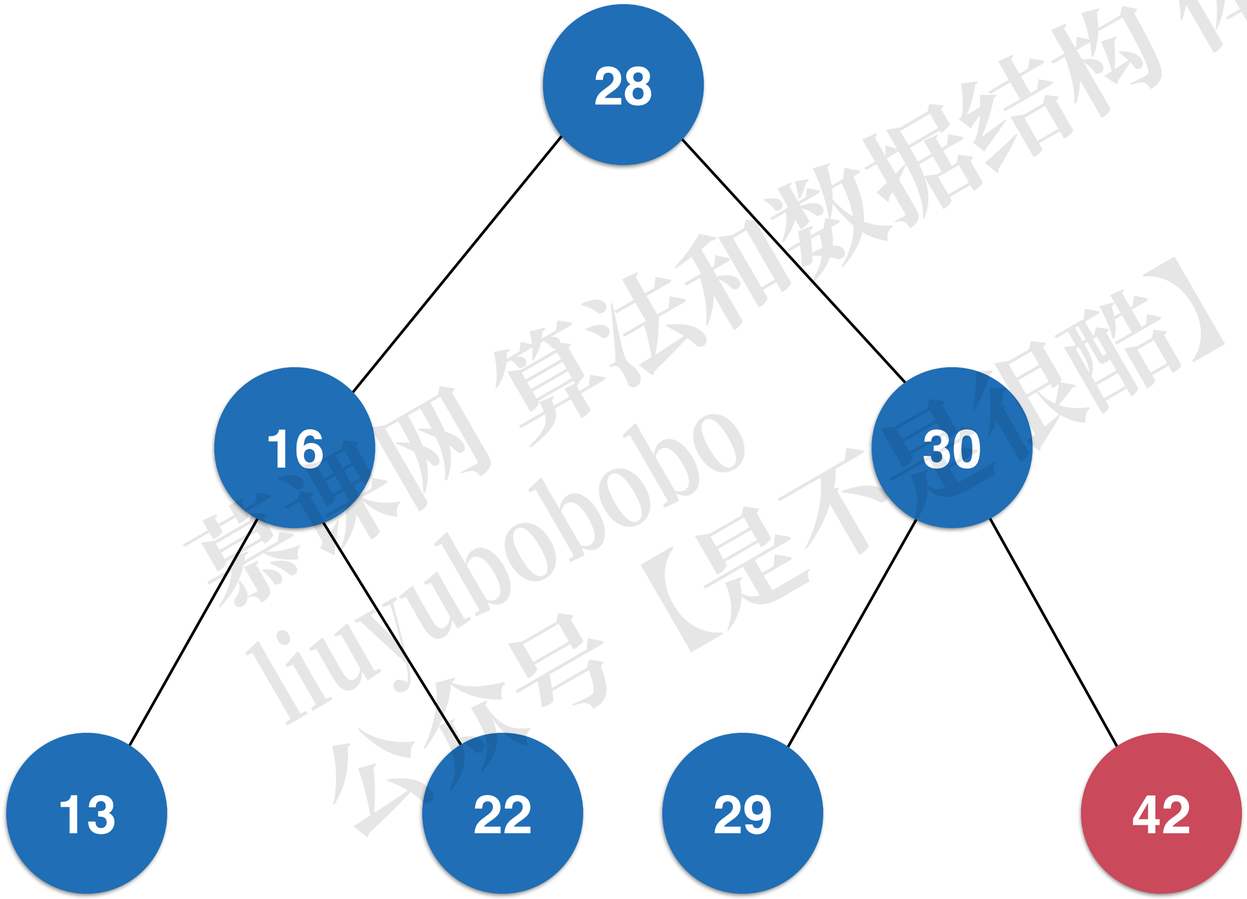

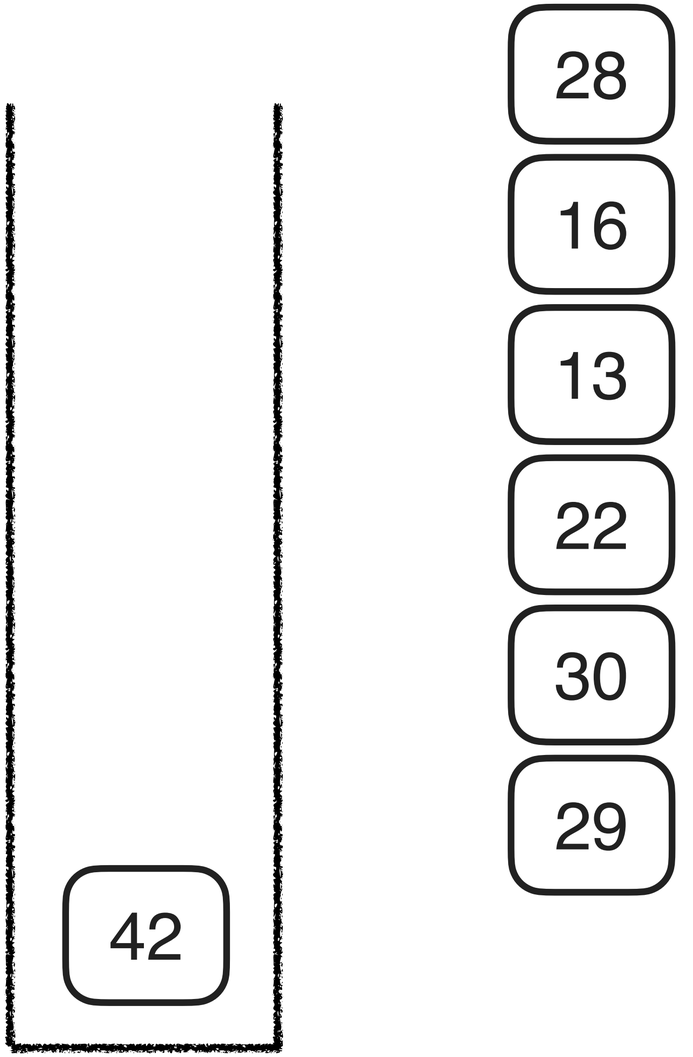

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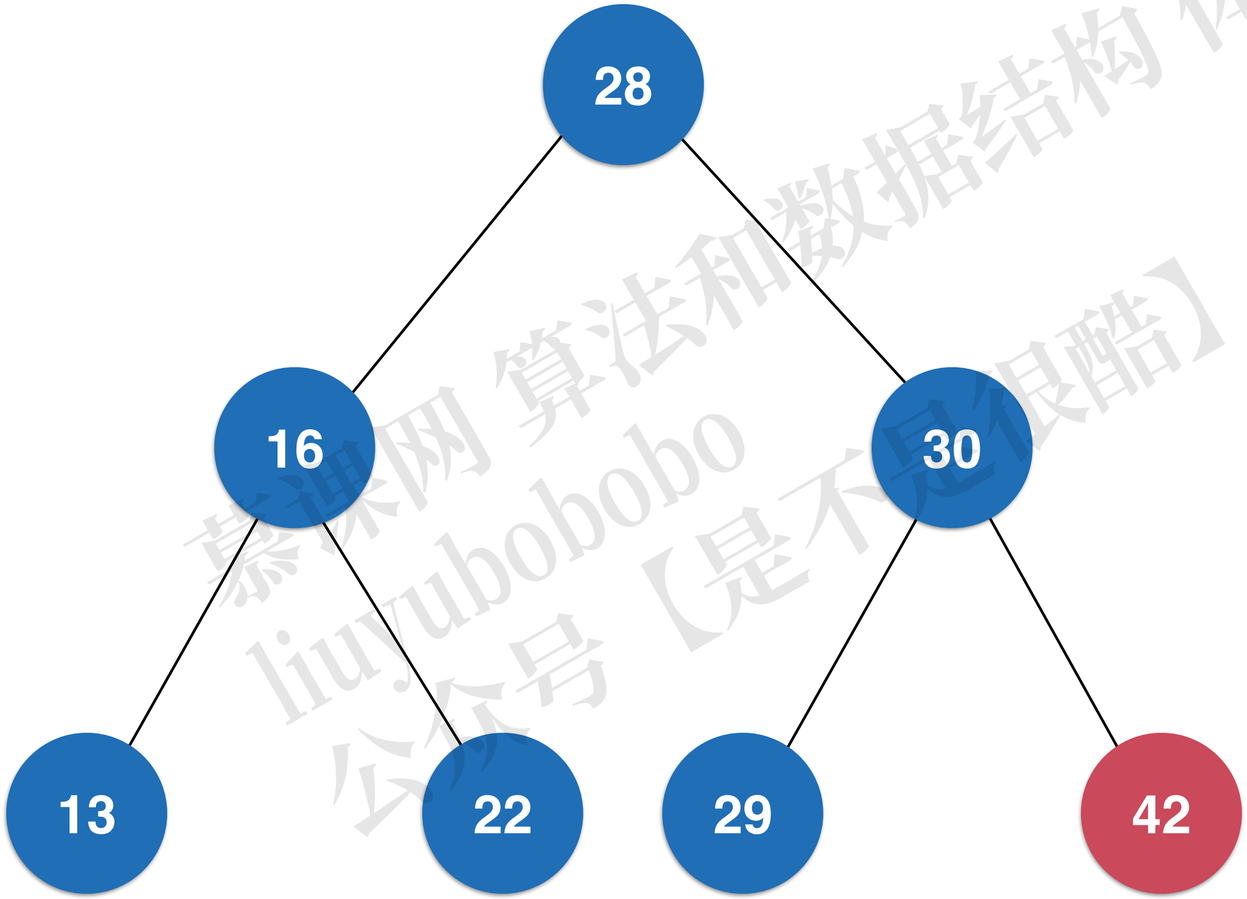
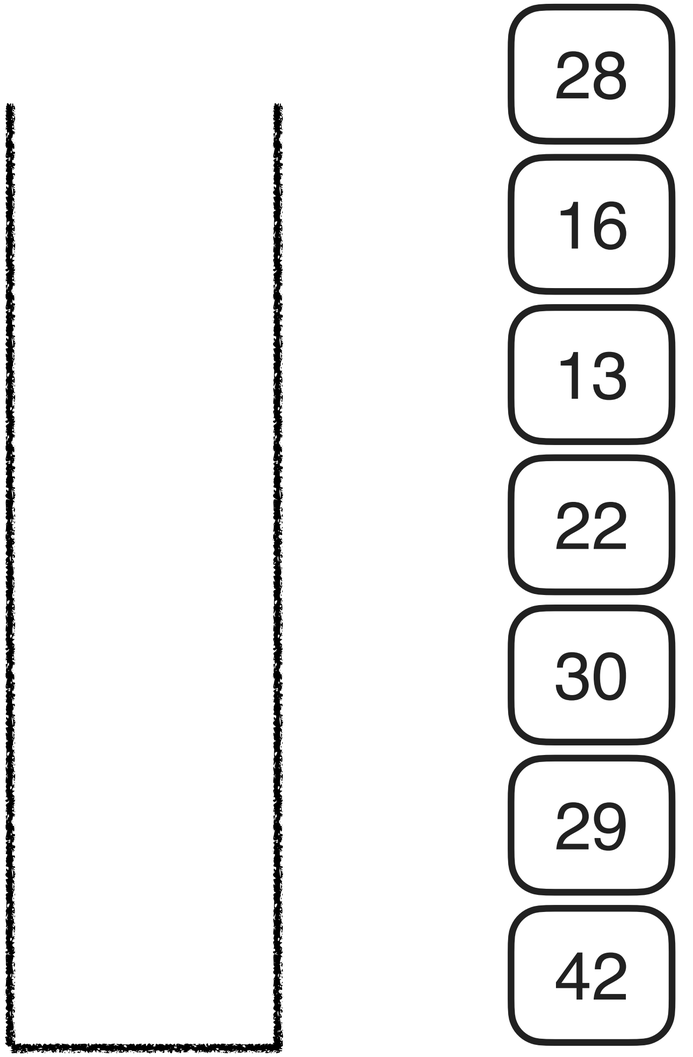
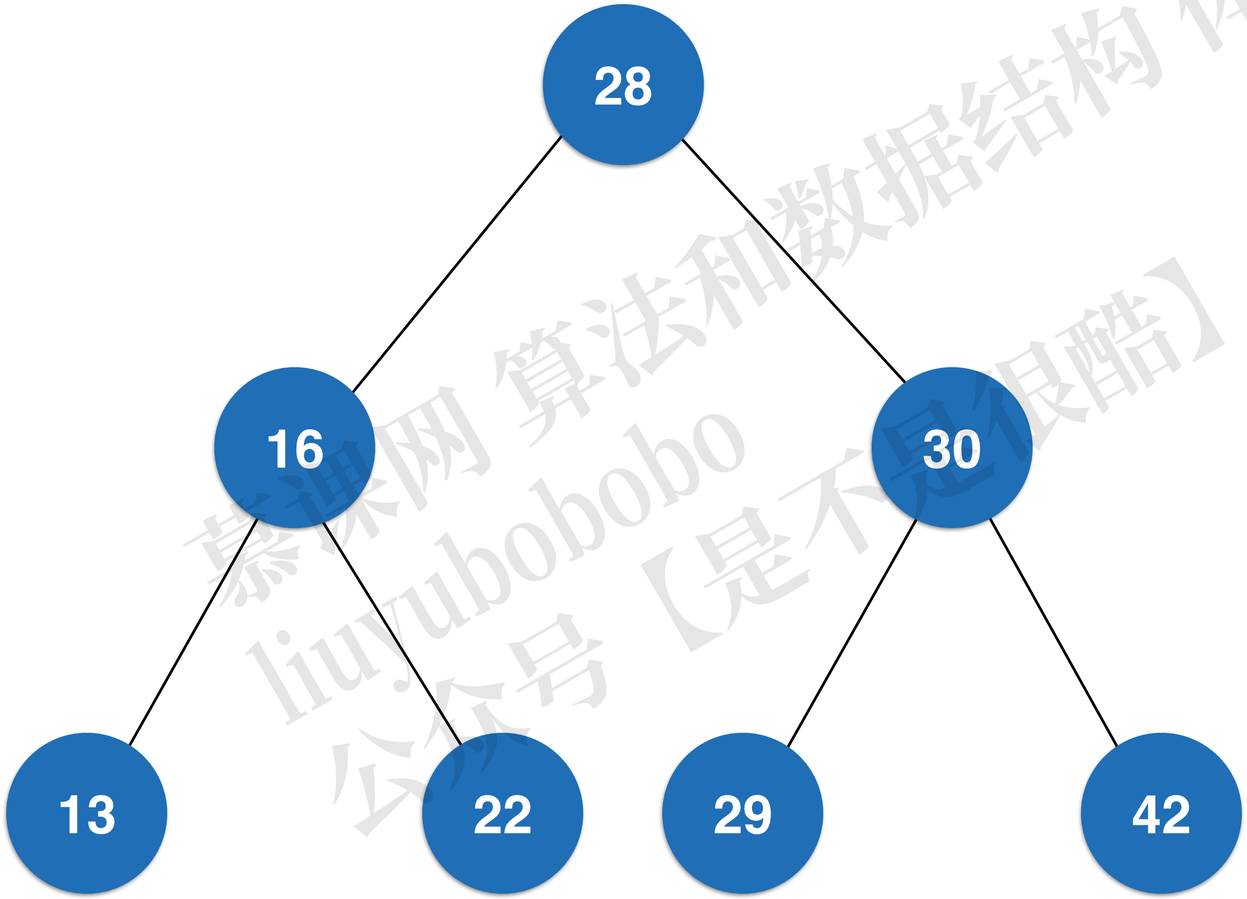
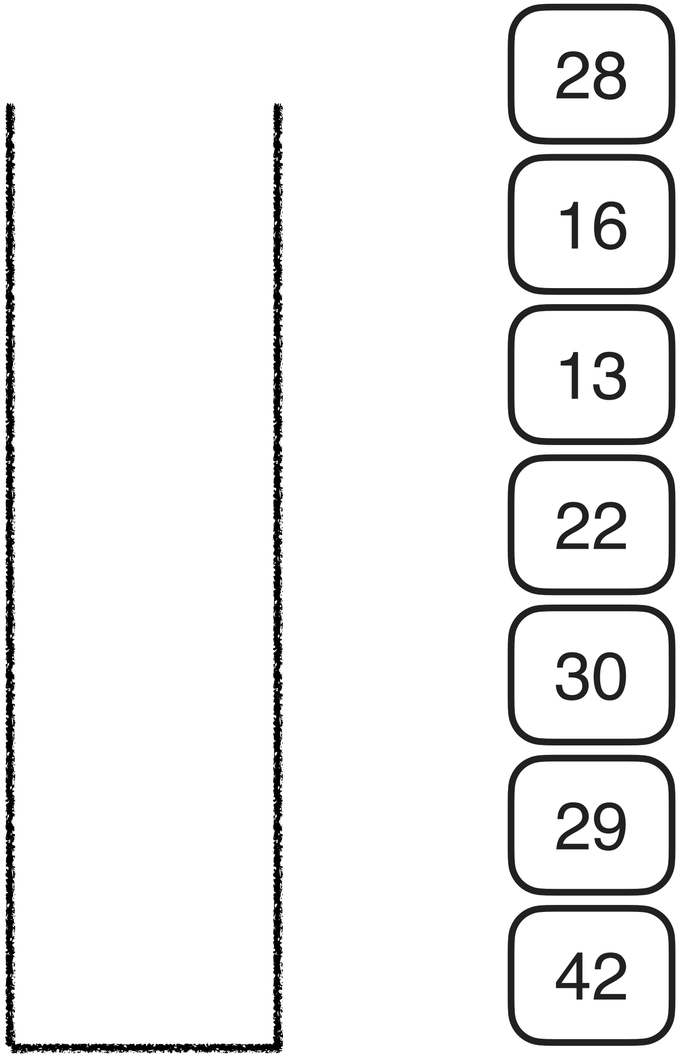




二分搜索树前序遍历的非递归写法





实践：二分搜索树前序遍历的非递归写法

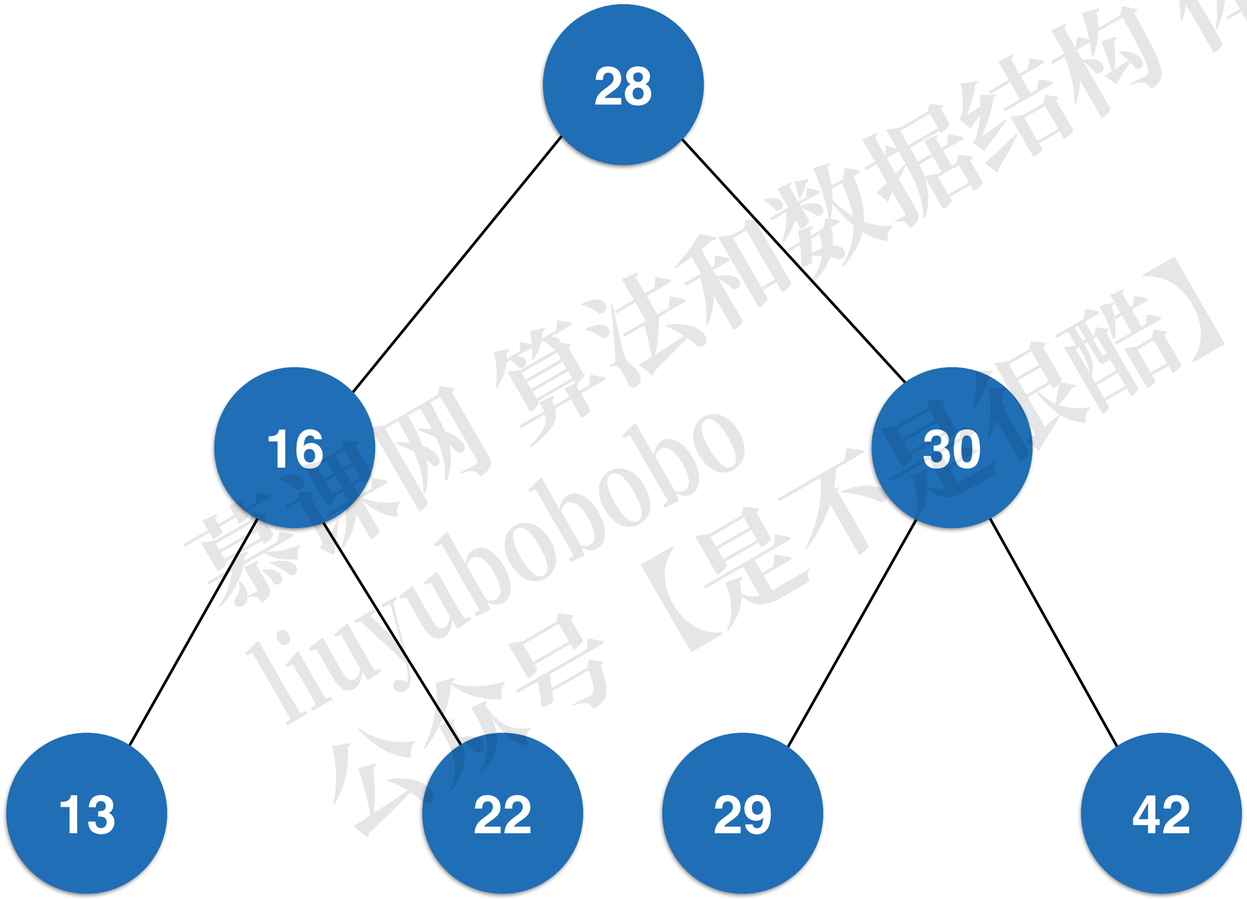
二分搜索树遍历的非递归实现

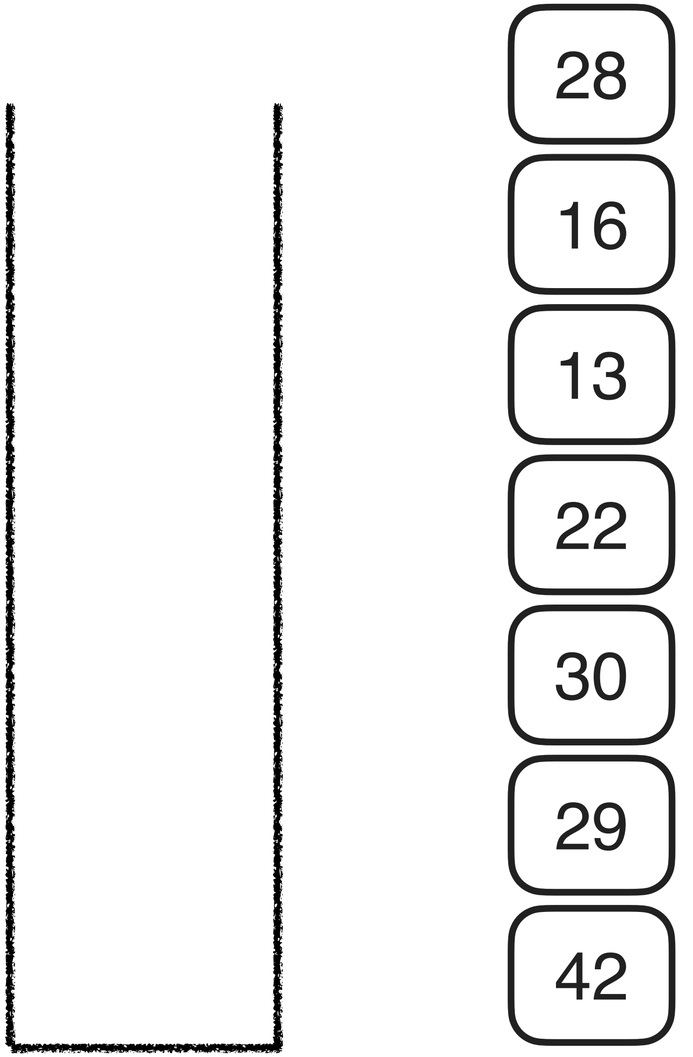
- 二分搜索树遍历的非递归实现，比递归实现复杂很多
- 中序遍历和后序遍历的非递归实现更复杂
- 中序遍历和后序遍历的非递归实现，实际应用不广
- 中序遍历和后序遍历的非递归实现留做练习

二分搜索树遍历的非递归实现

- 经典教科书的写法
- 《玩转算法面试》中完全模拟系统栈的写法

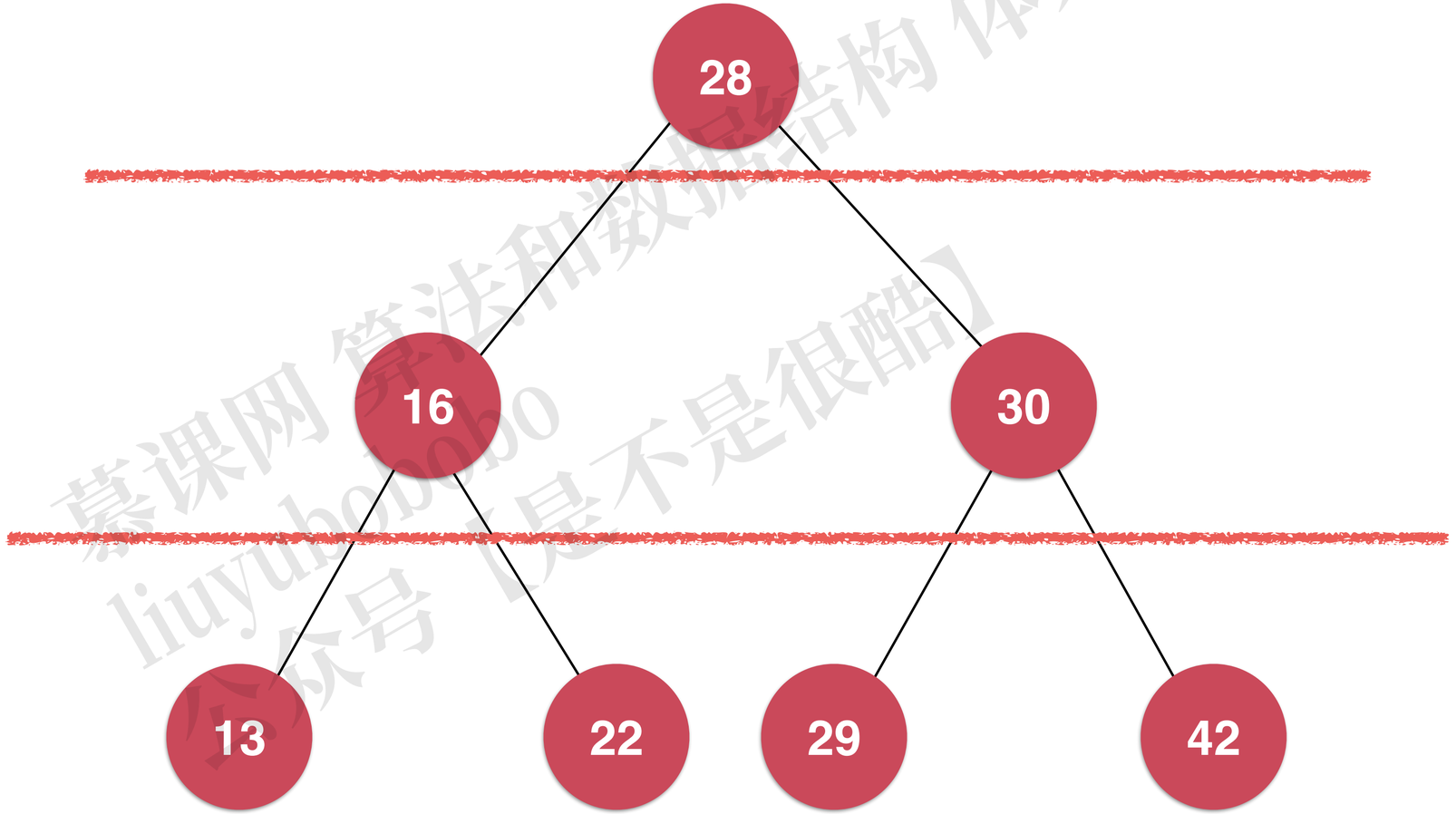
二分搜索树前序遍历的非递归写法



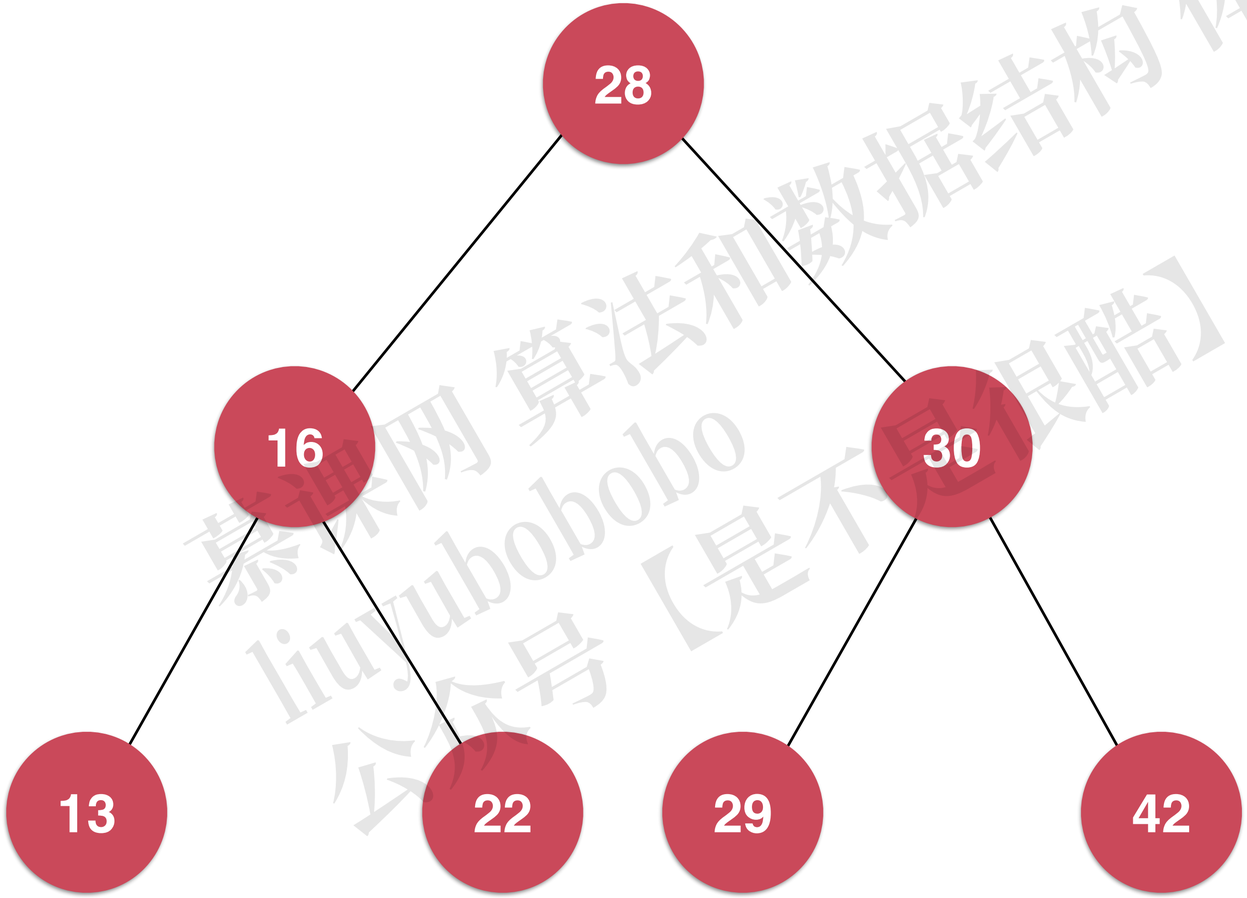


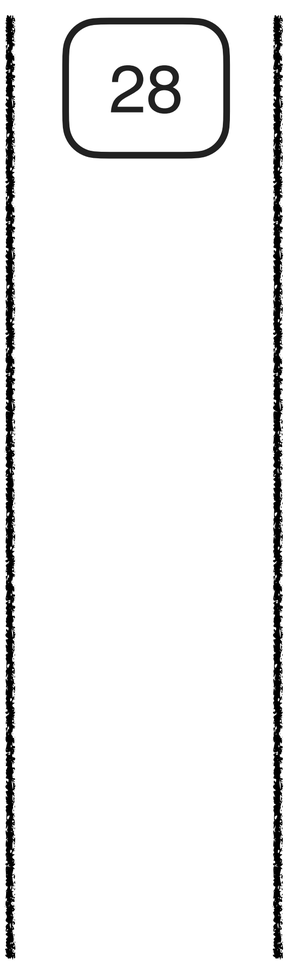
二分搜索树的层序遍历

二分搜索树的层序遍历

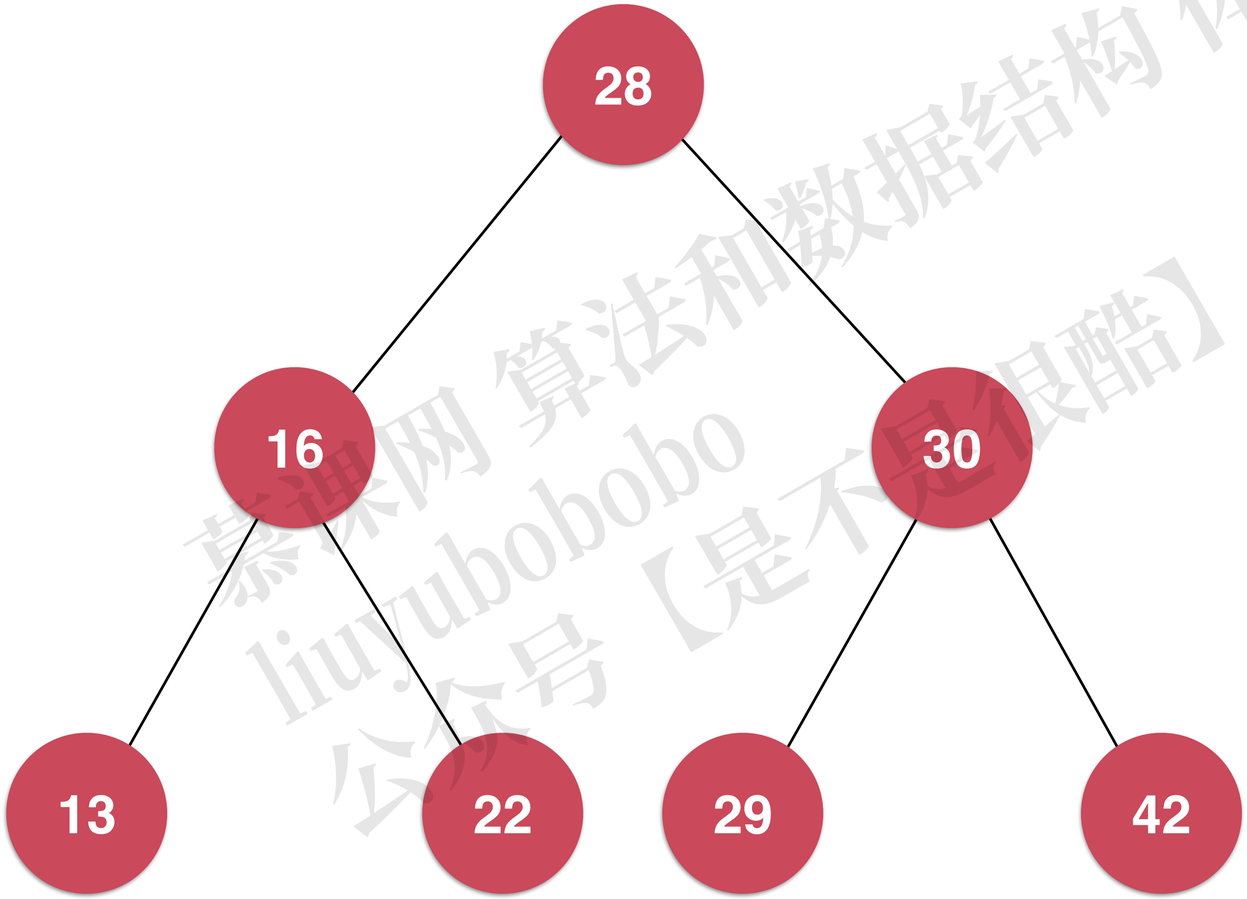


二分搜索树的层序遍历





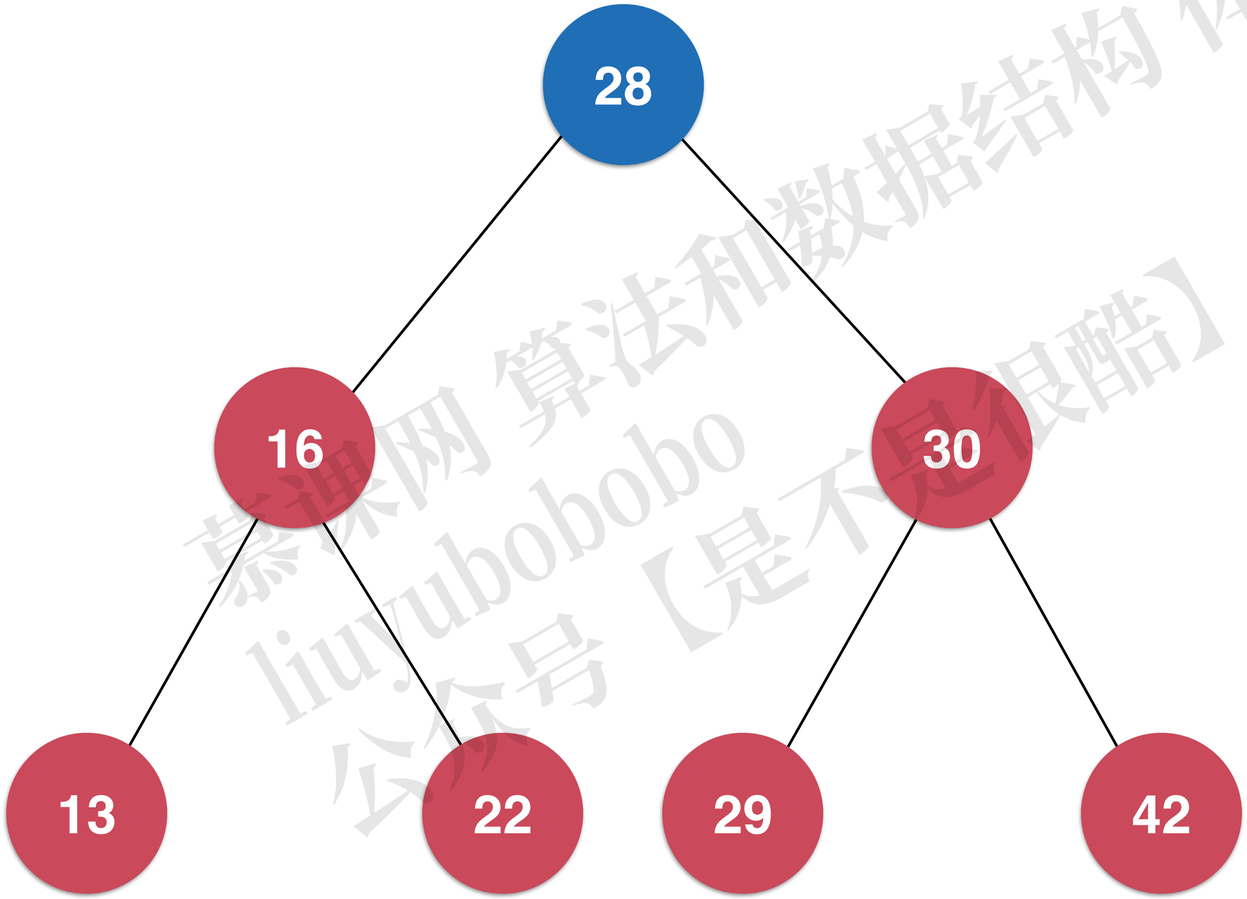
二分搜索树的层序遍历

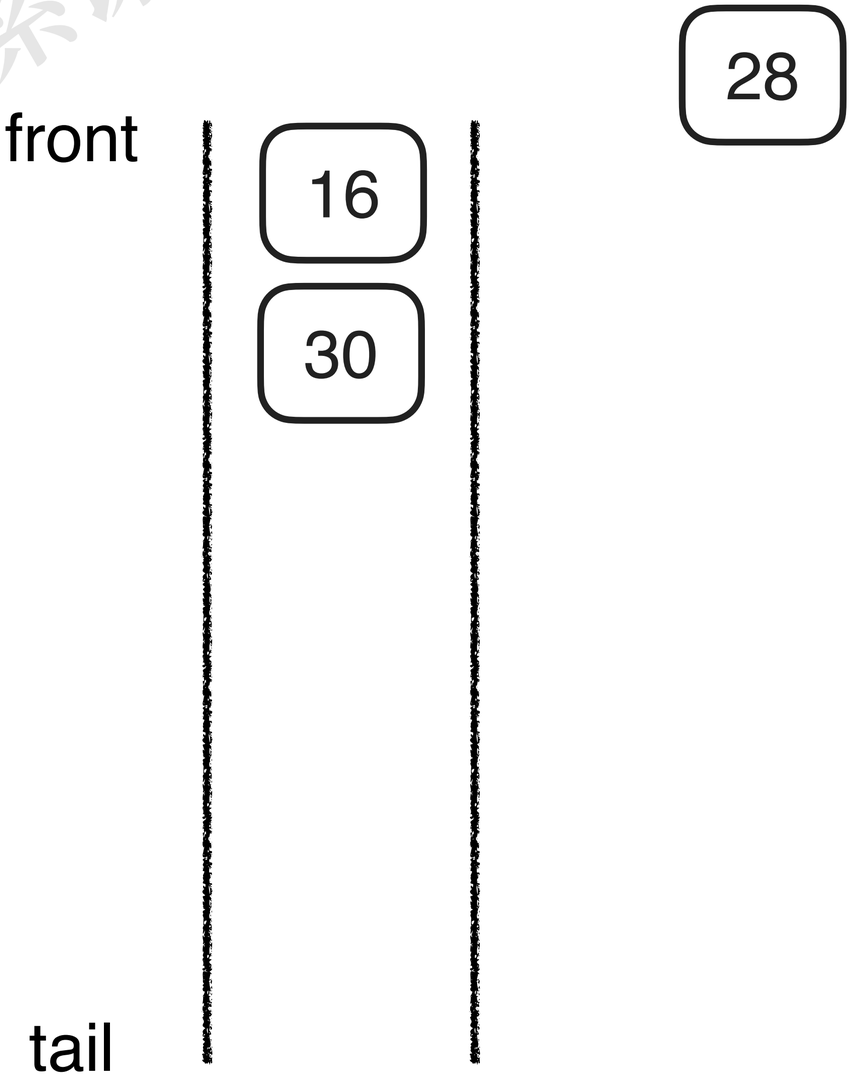


front

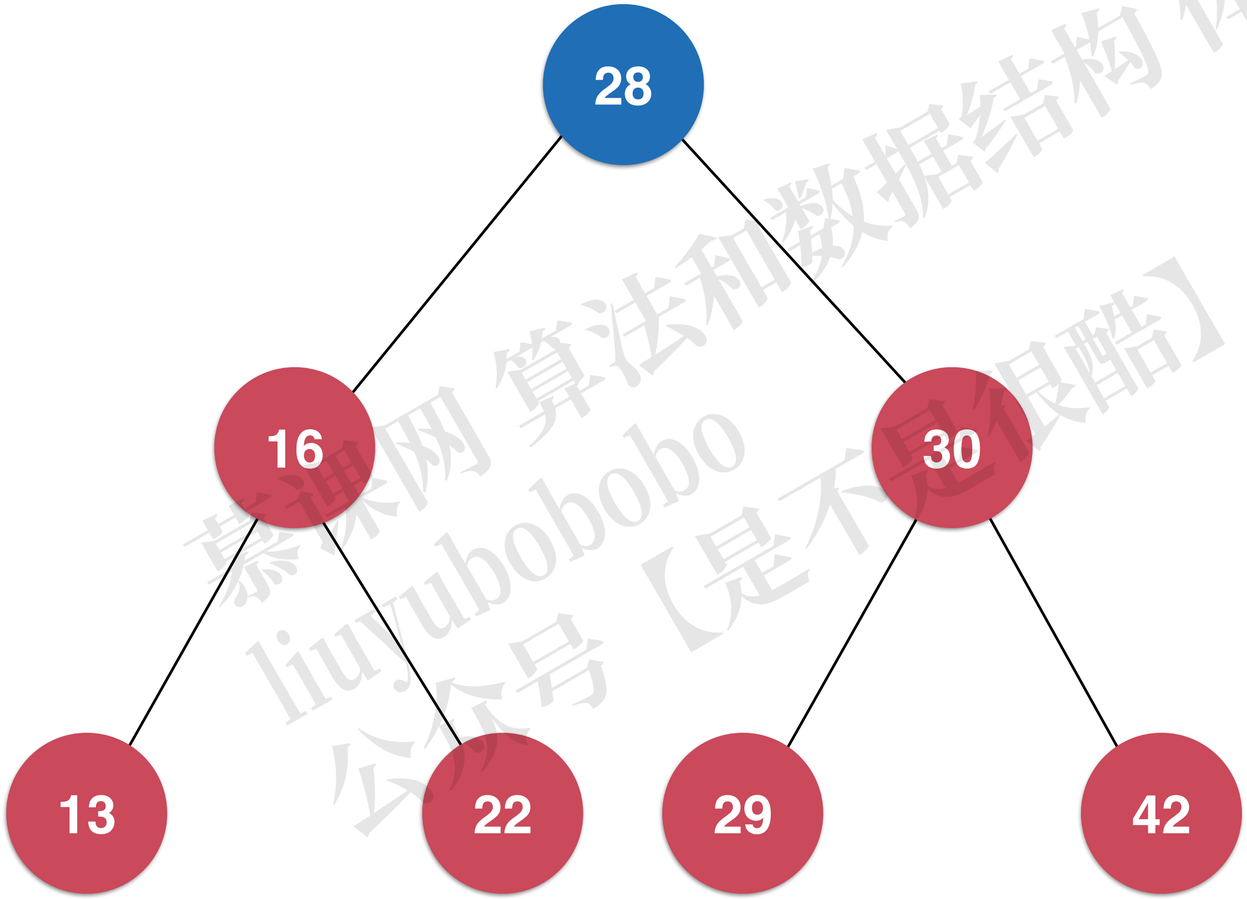
tail

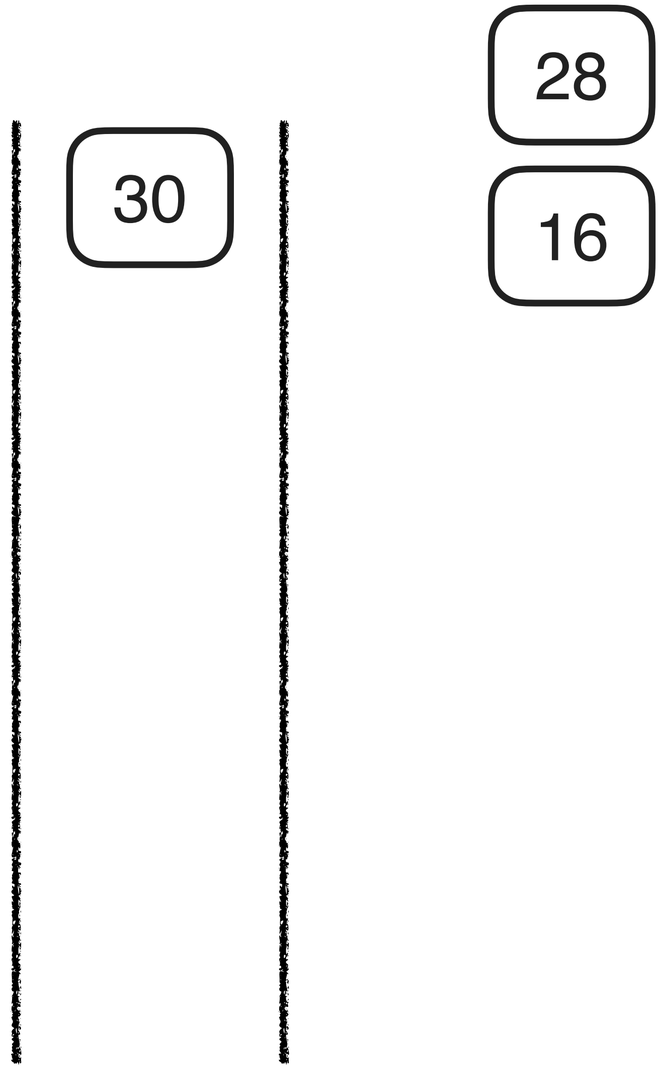
二分搜索树的层序遍历



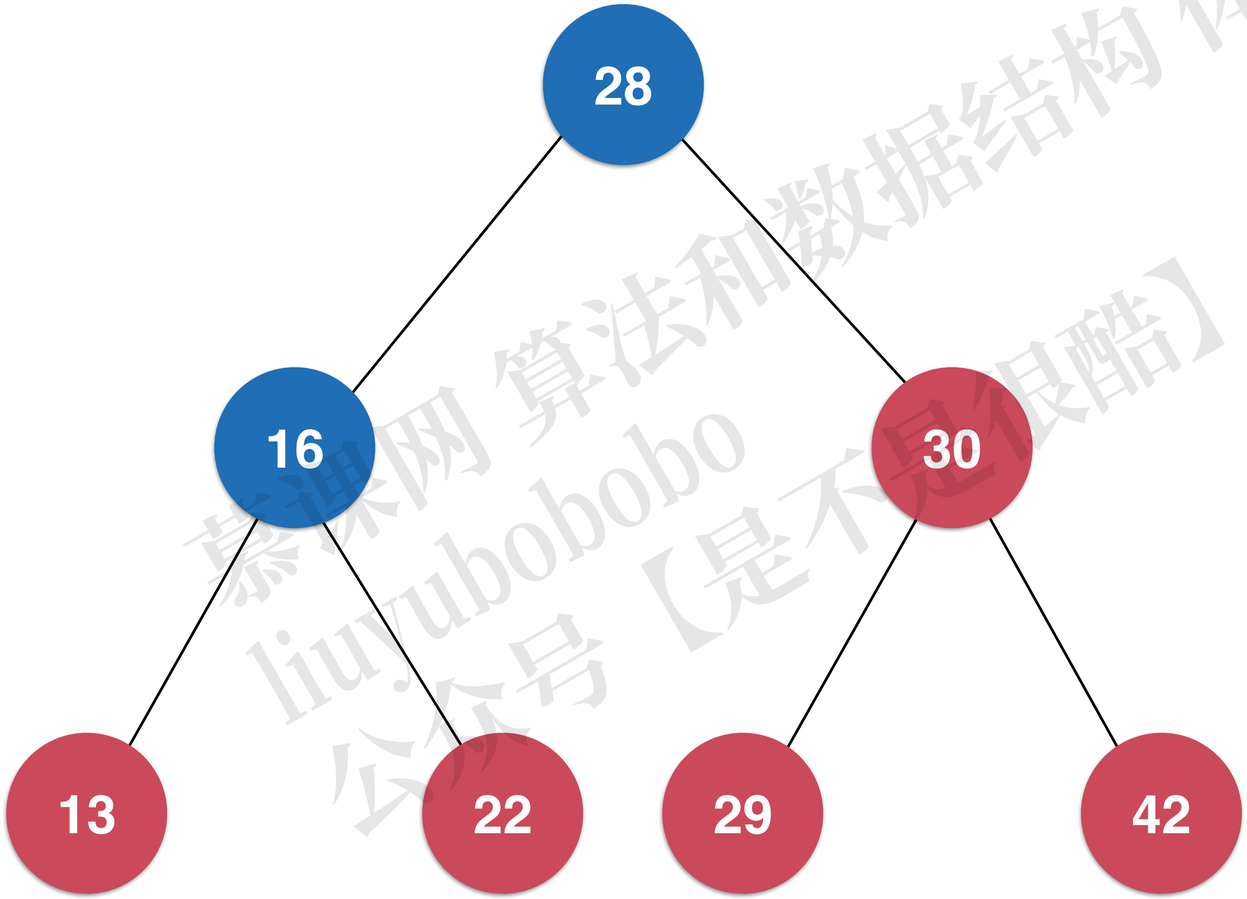


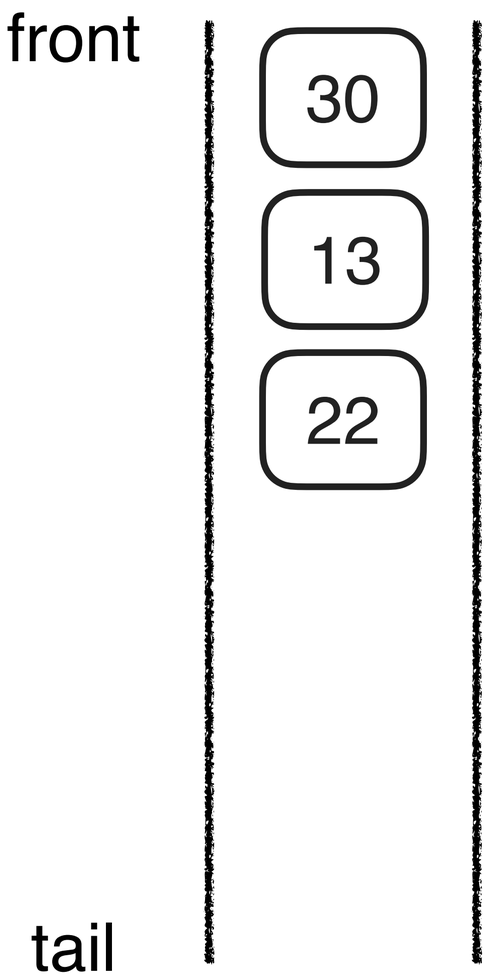
二分搜索树的层序遍历

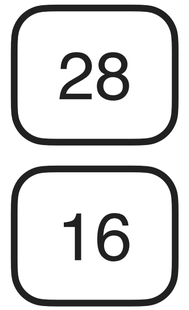




二分搜索树的层序遍历







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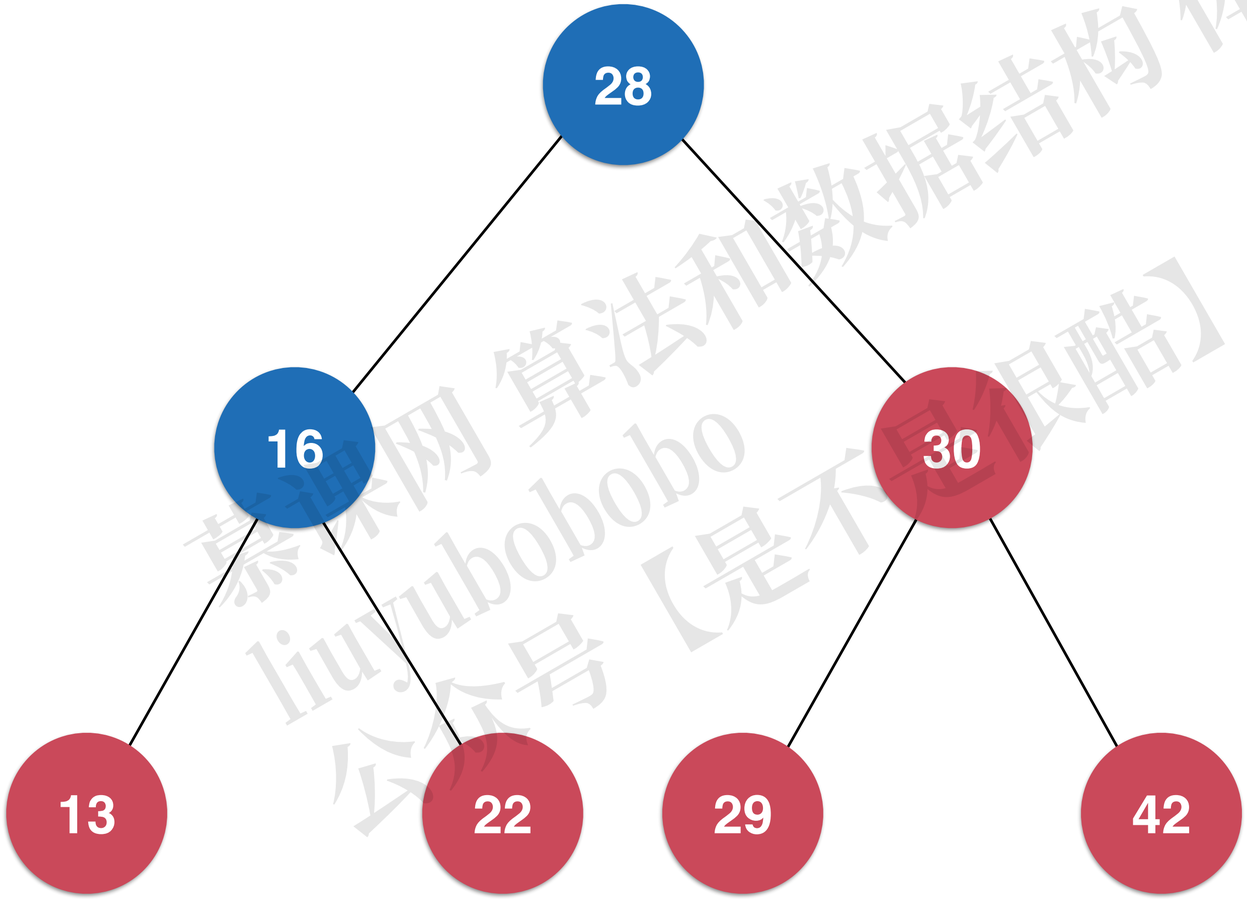

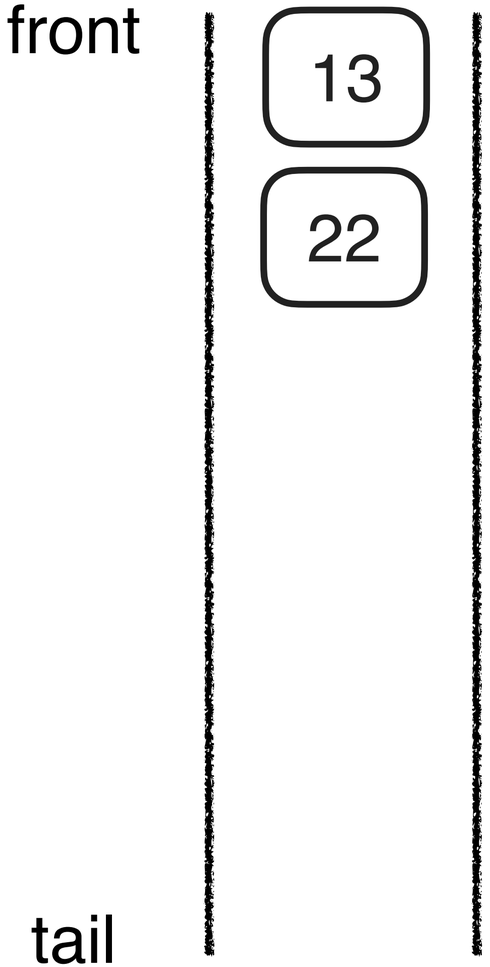

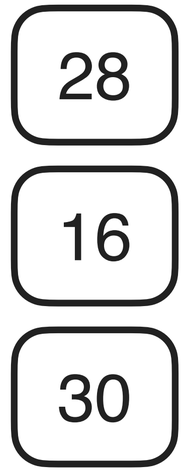

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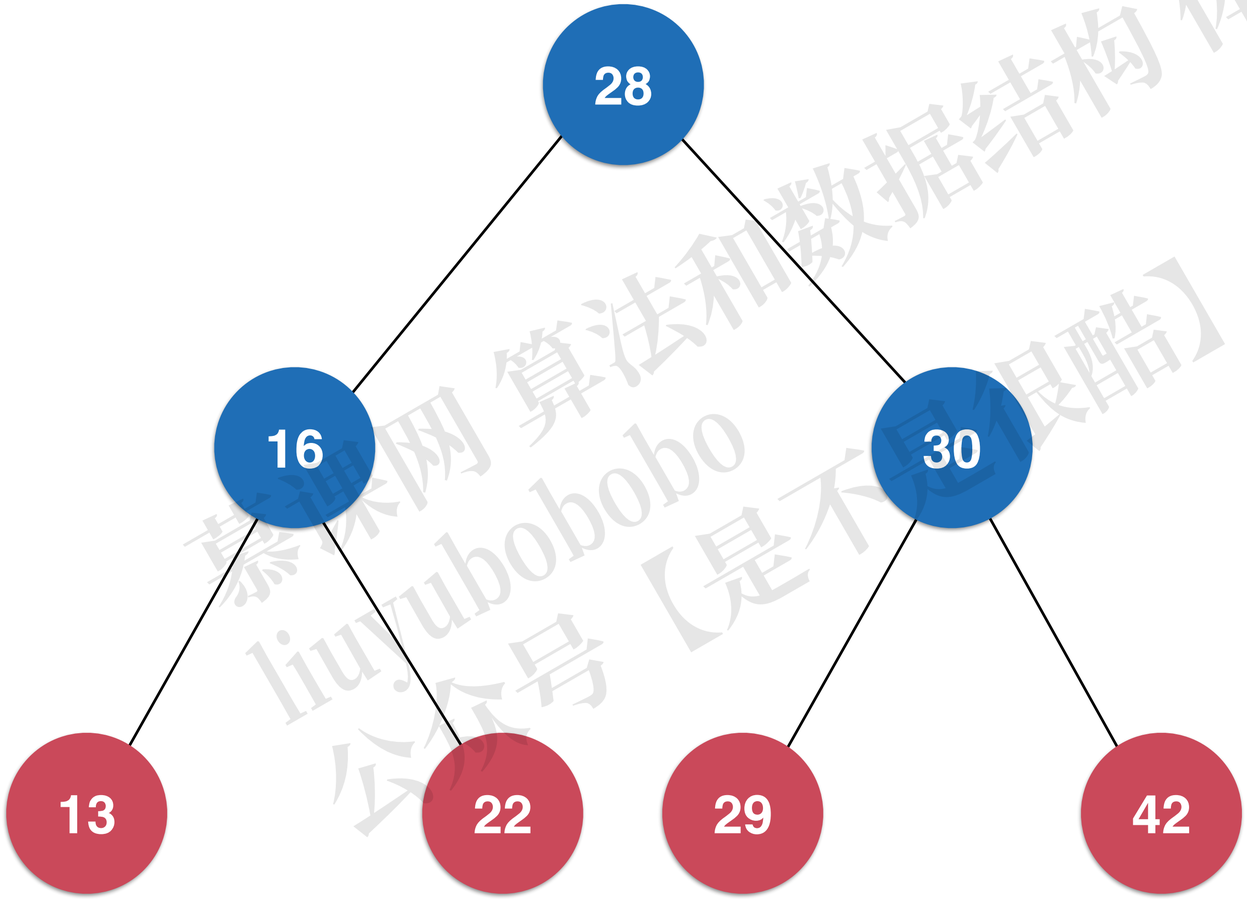

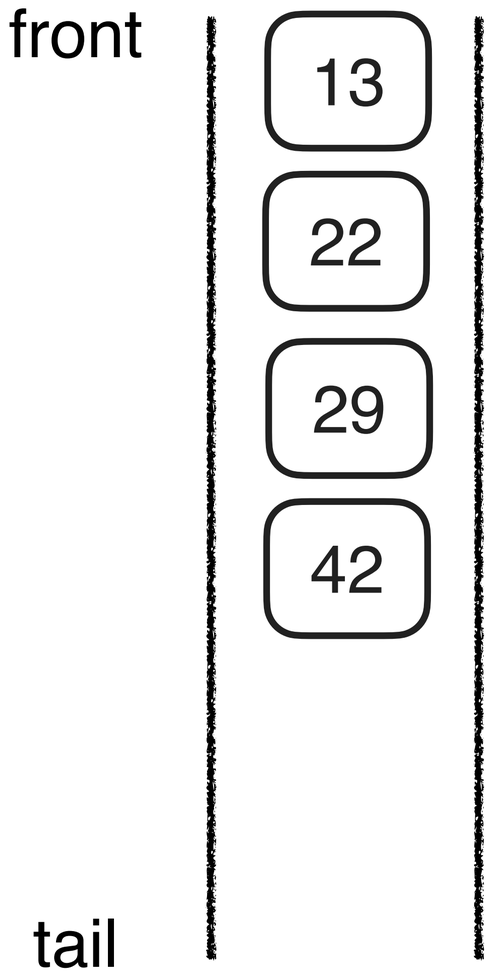

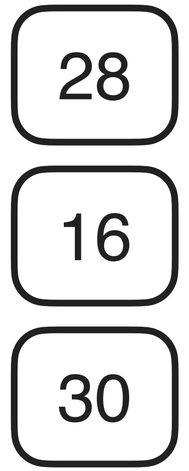

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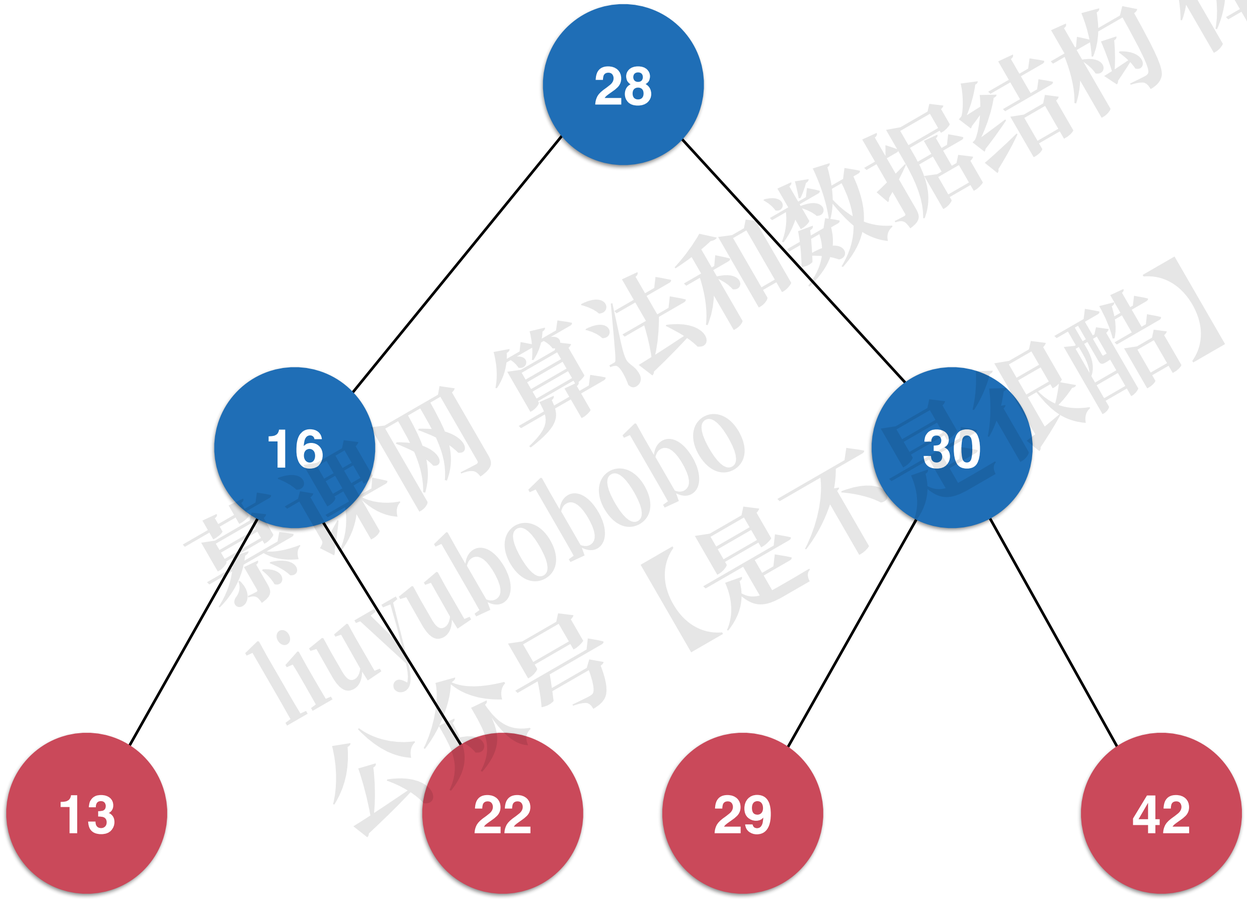

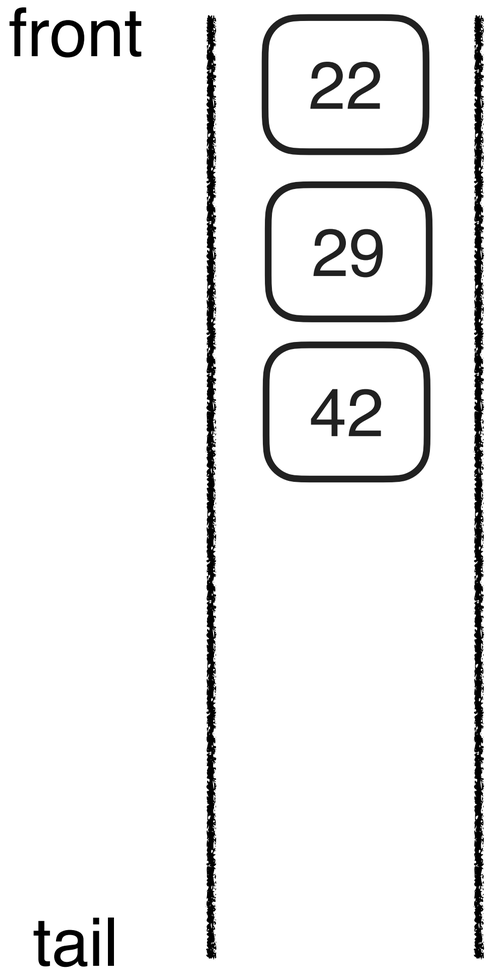

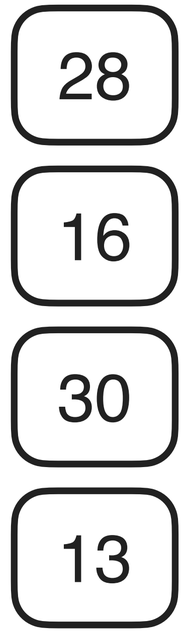

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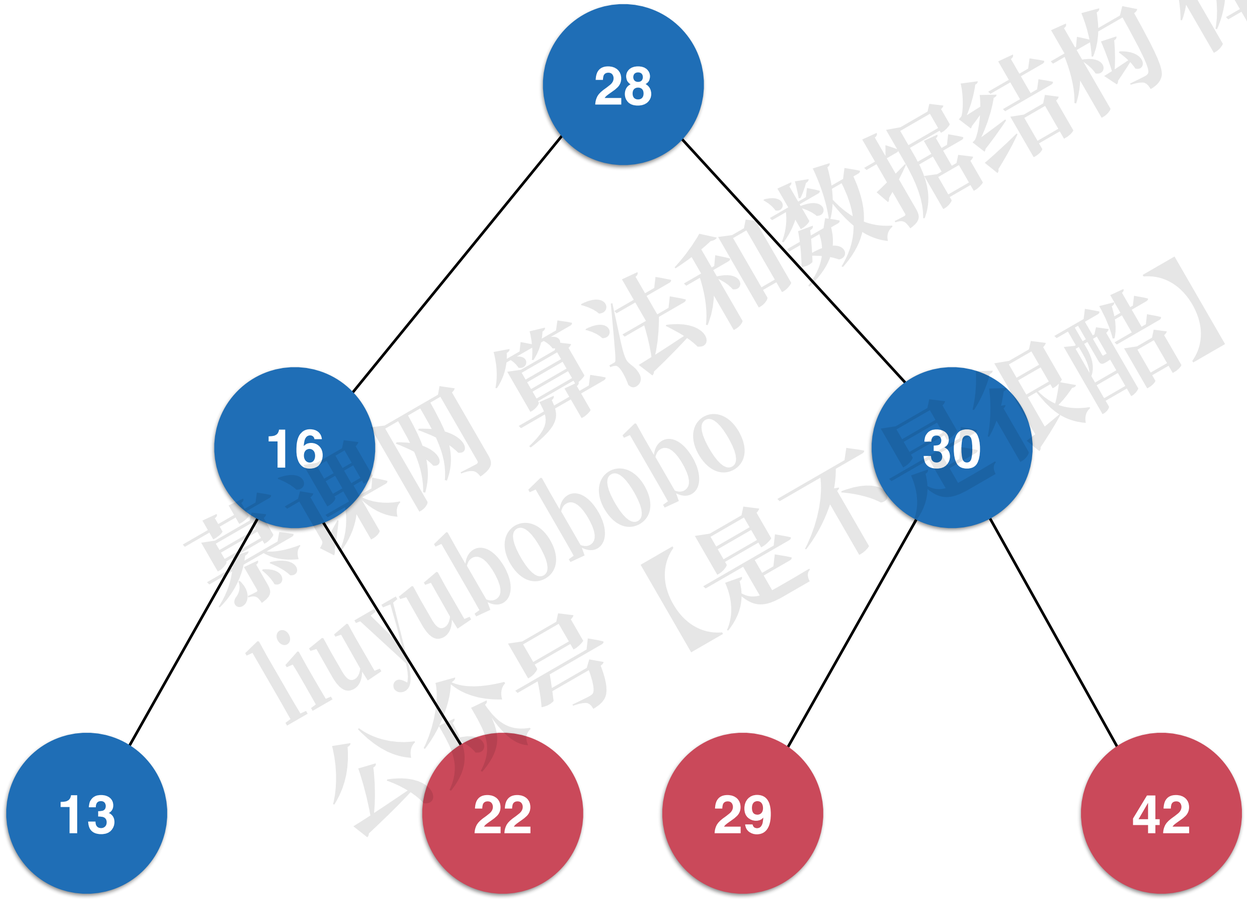
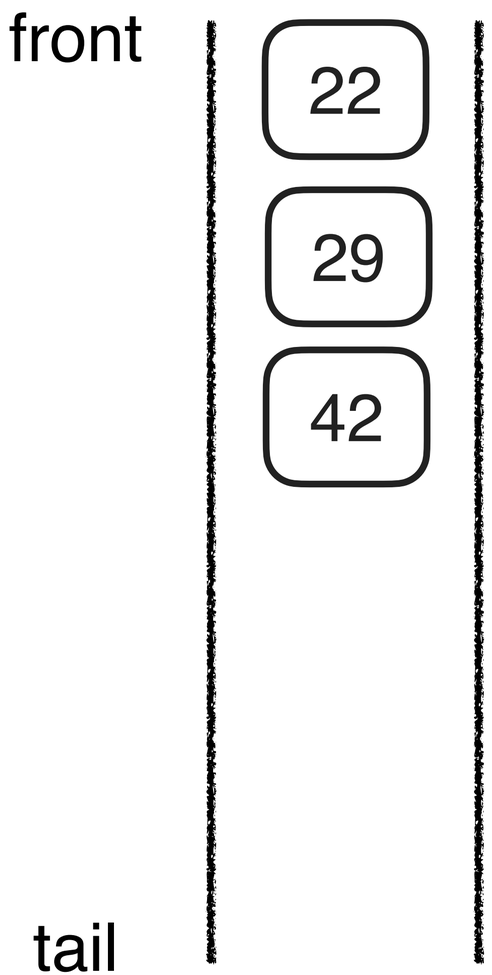
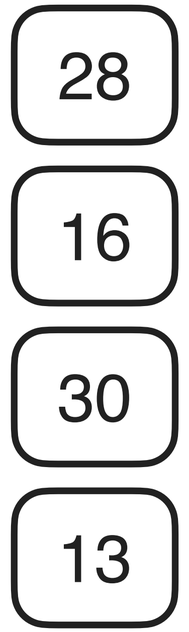
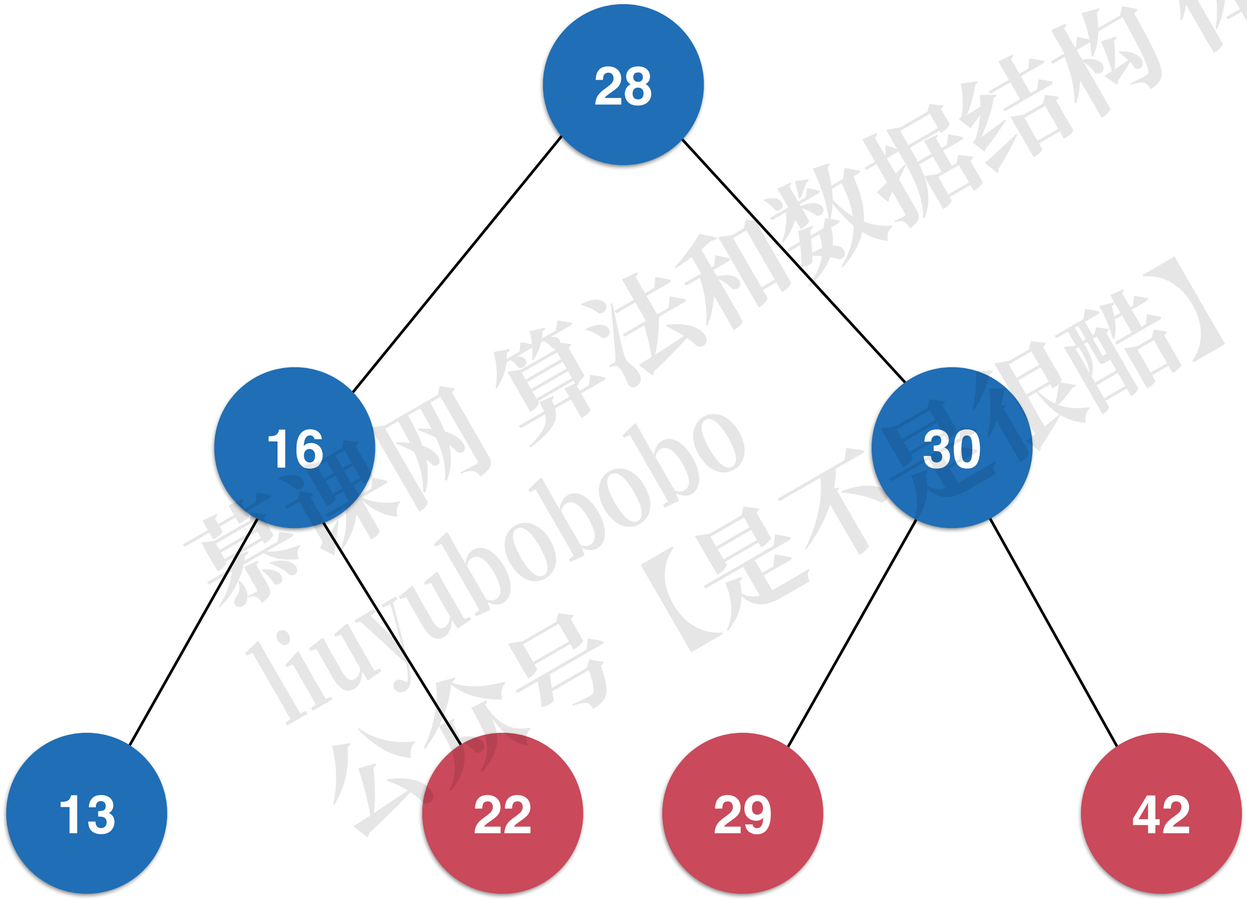
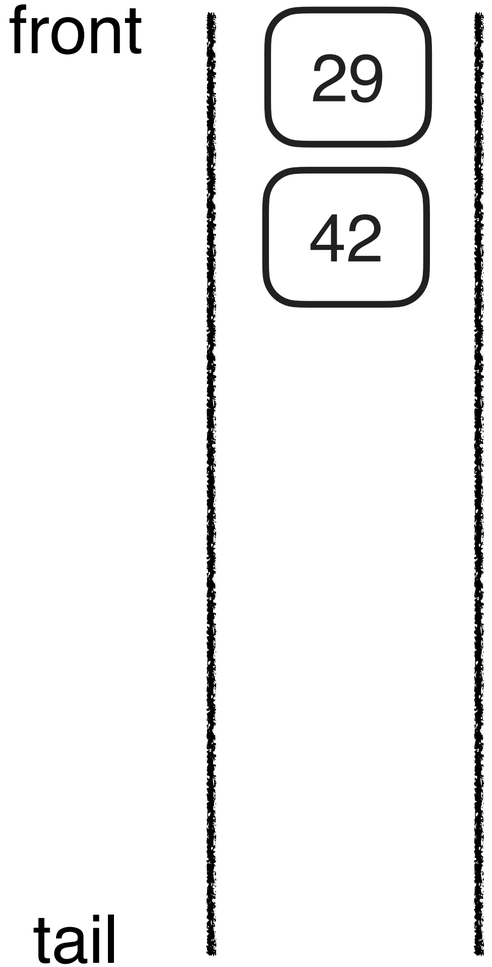


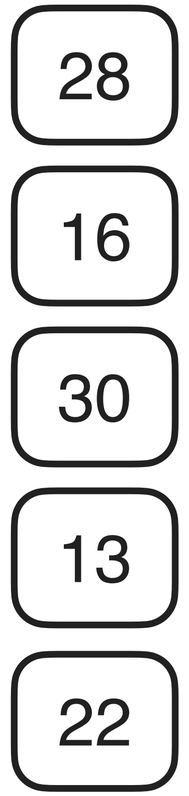




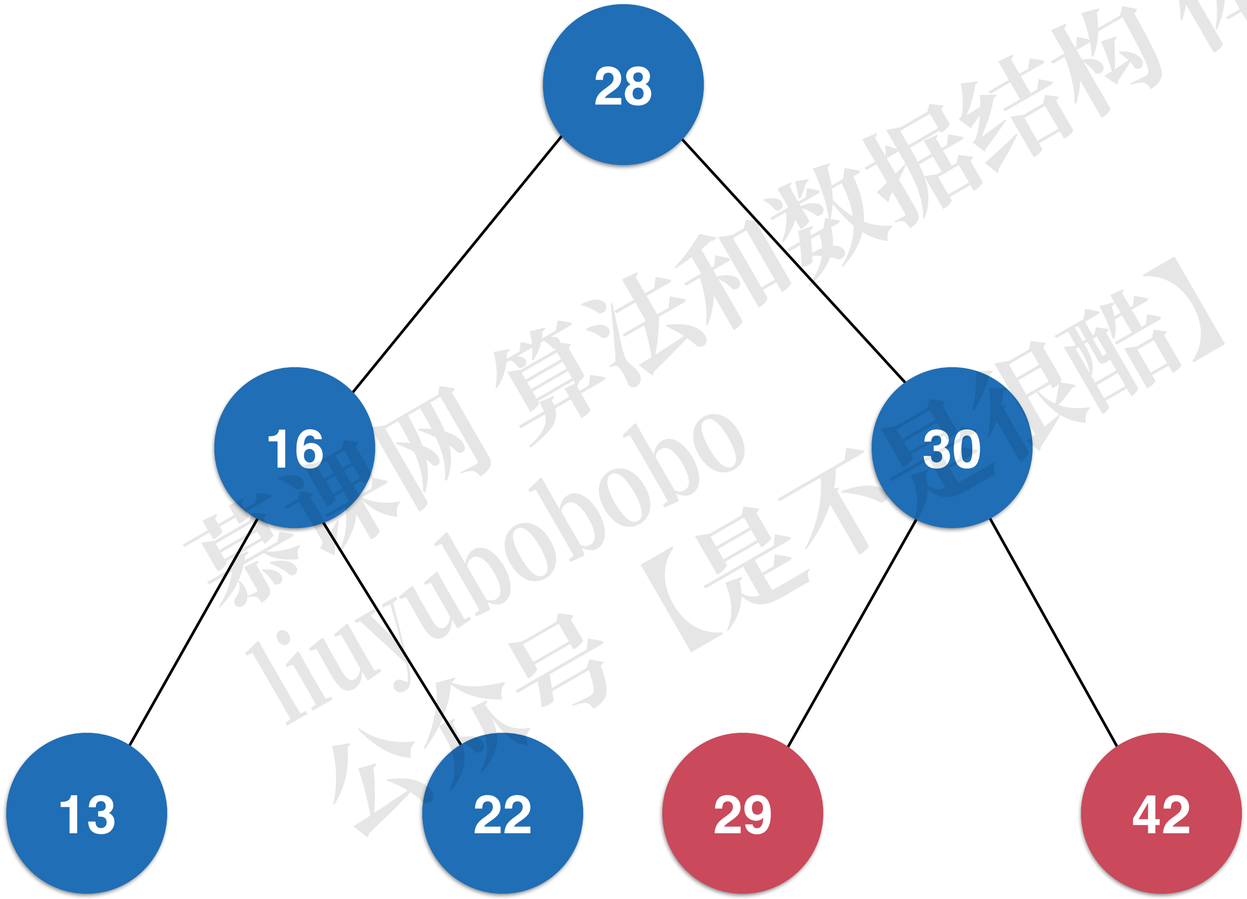
二分搜索树的层序遍历

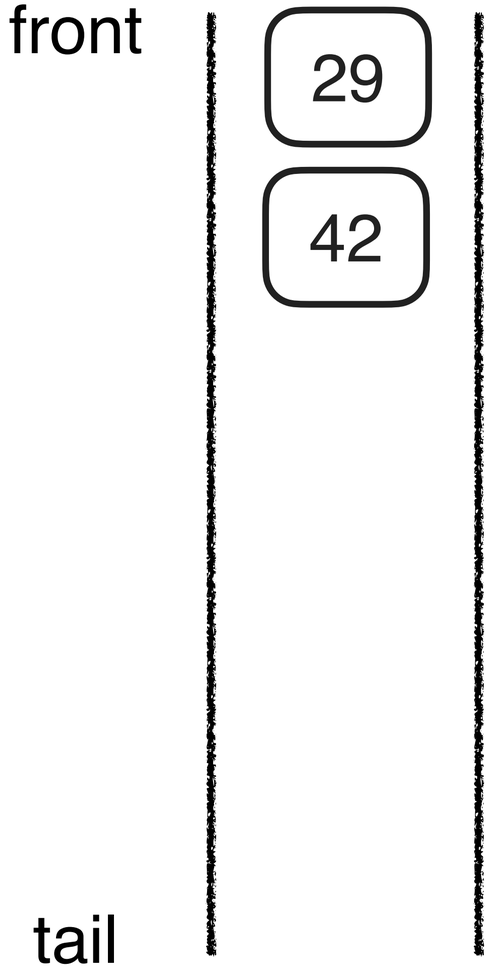


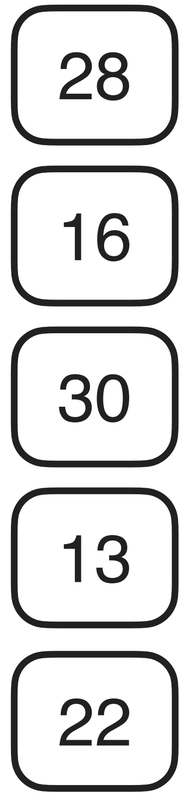




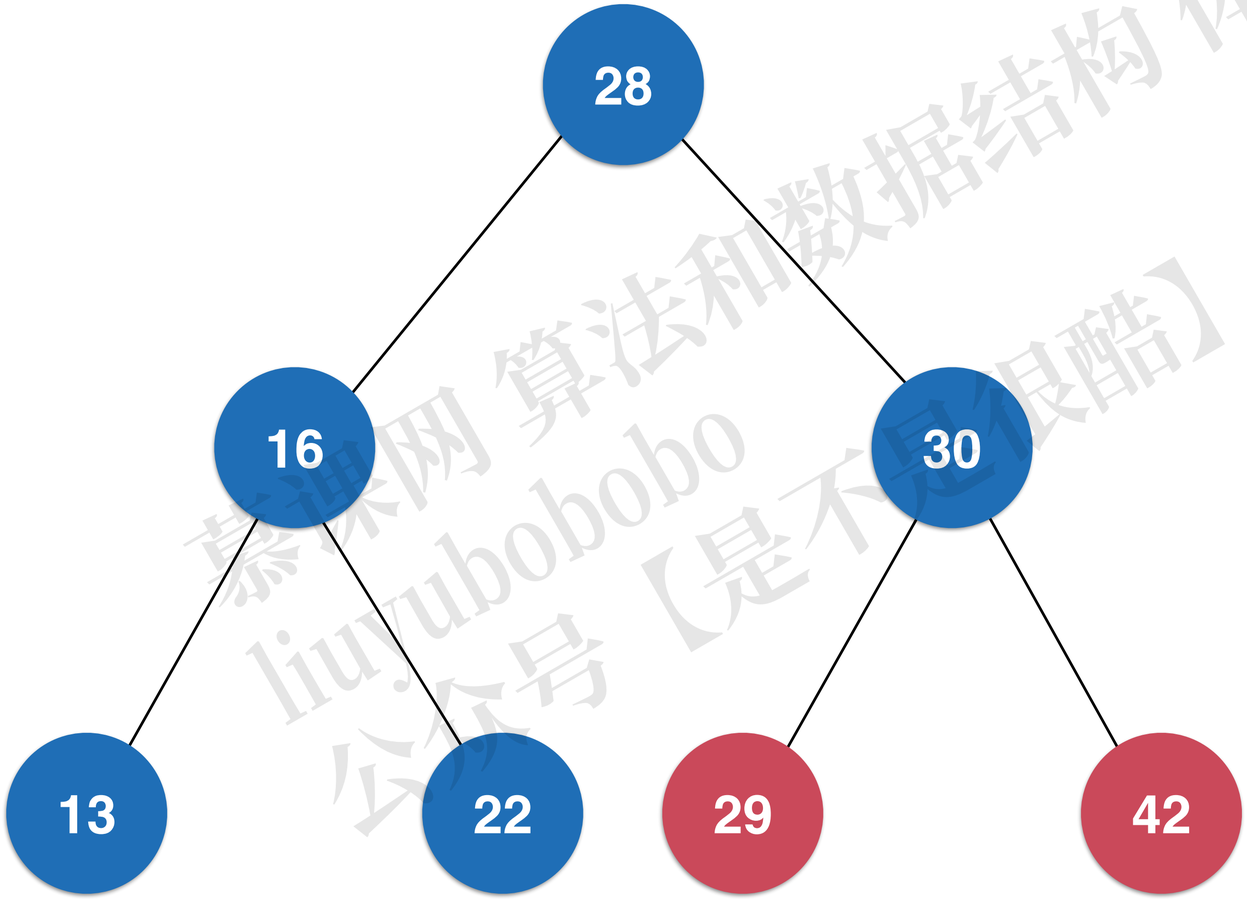
二分搜索树的层序遍历

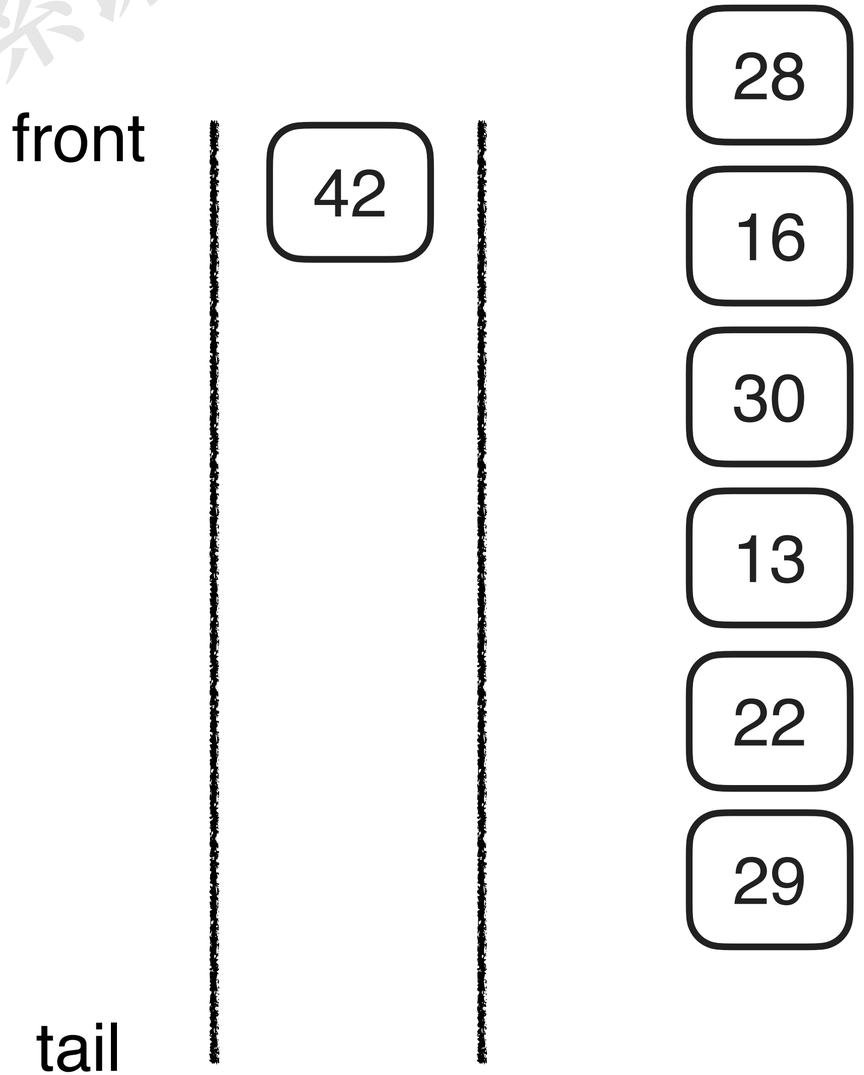




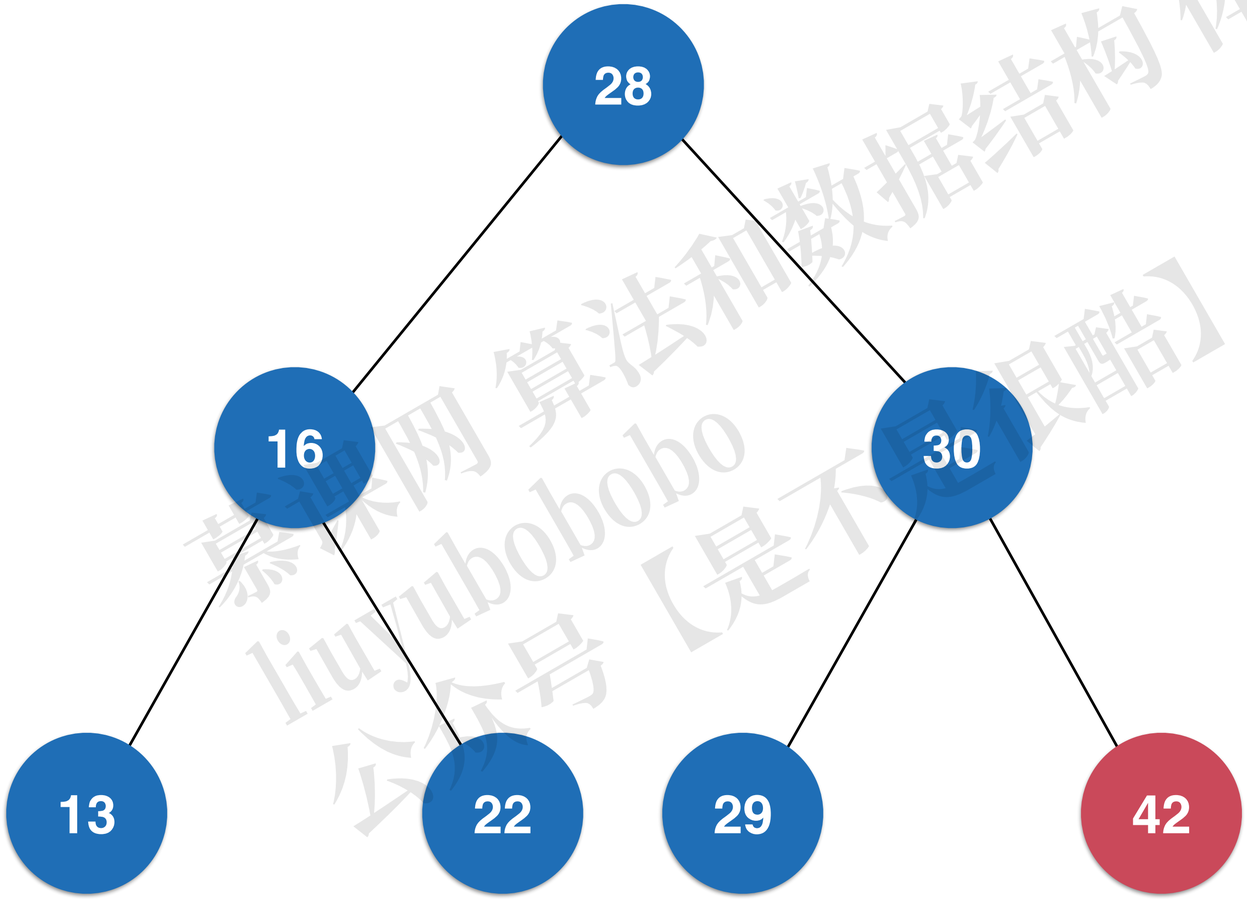


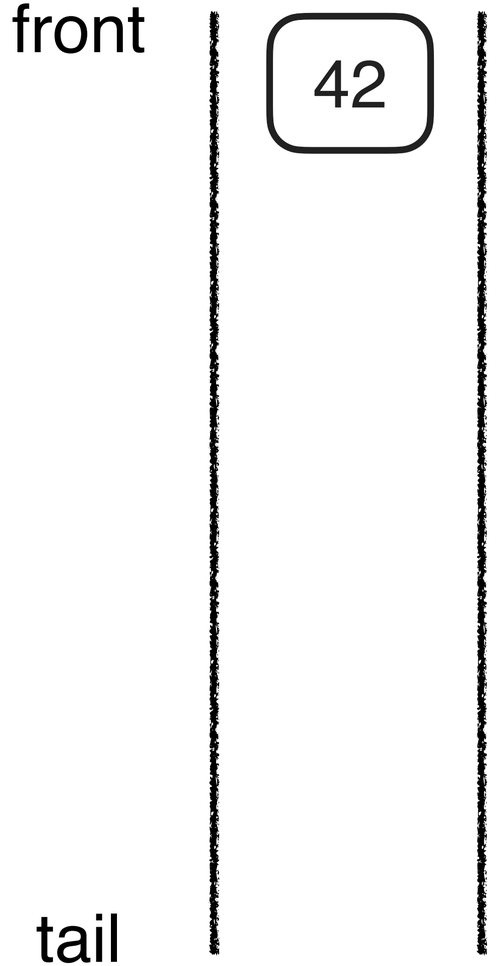
二分搜索树的层序遍历

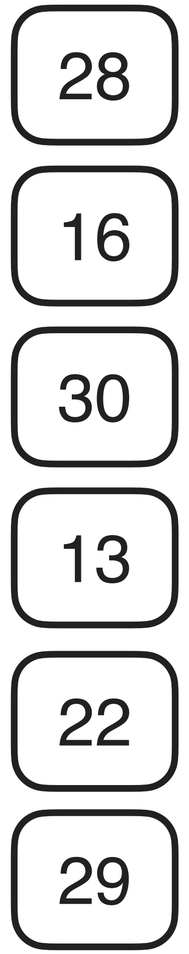




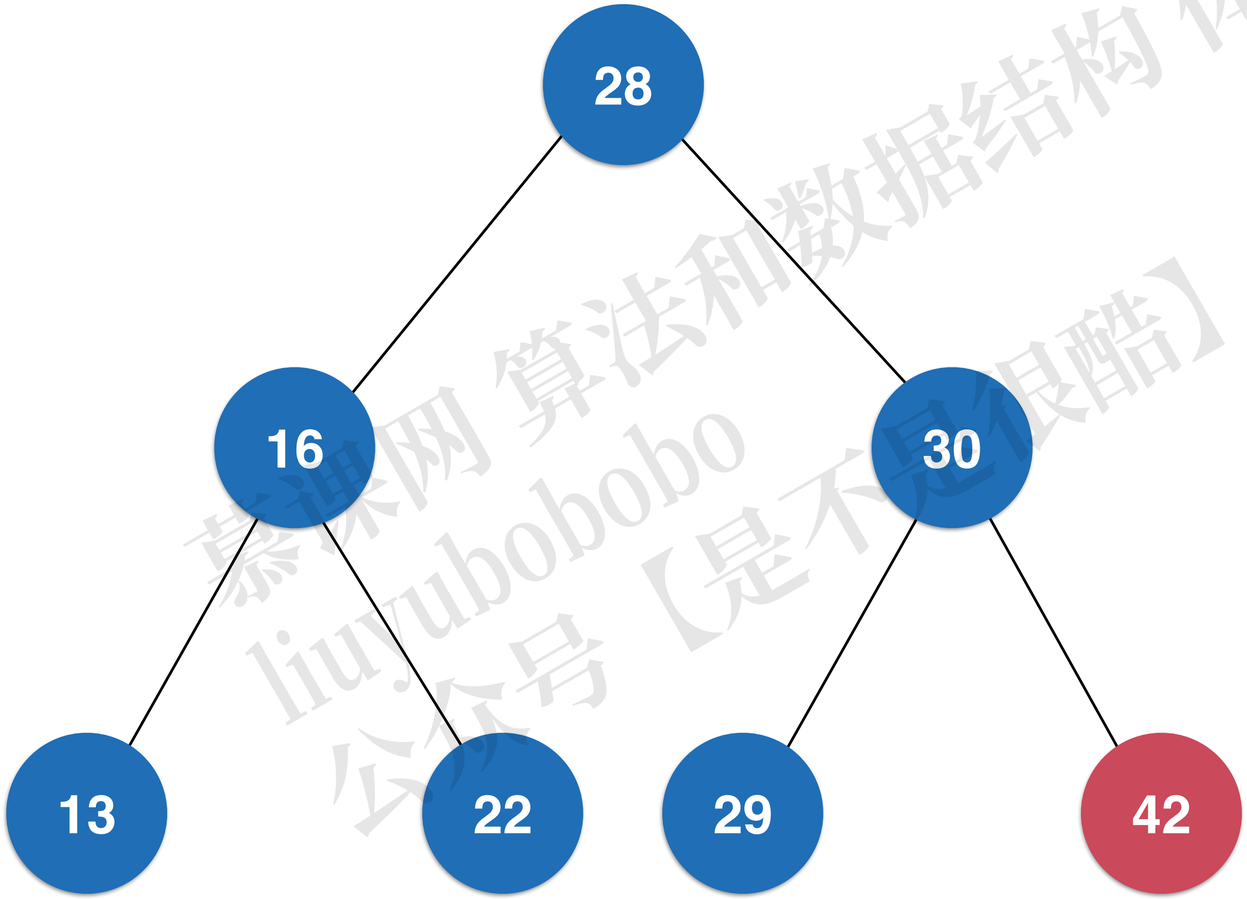
二分搜索树的层序遍历

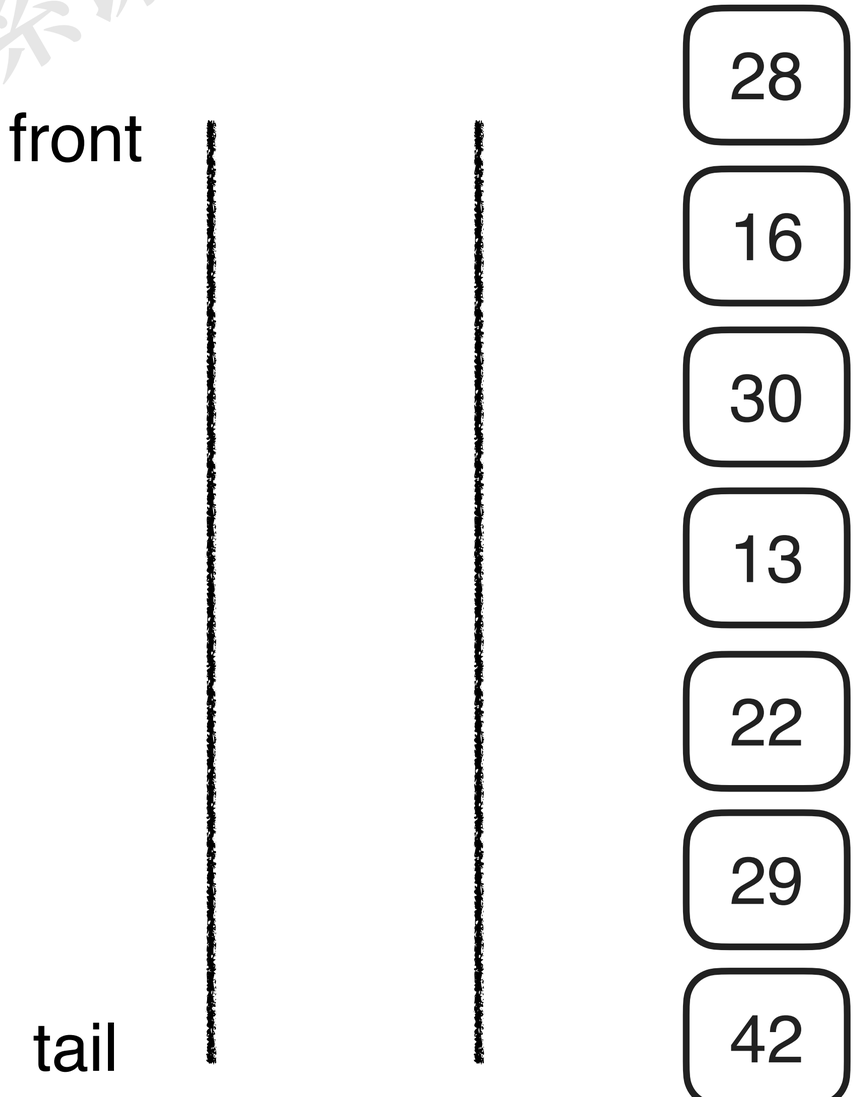




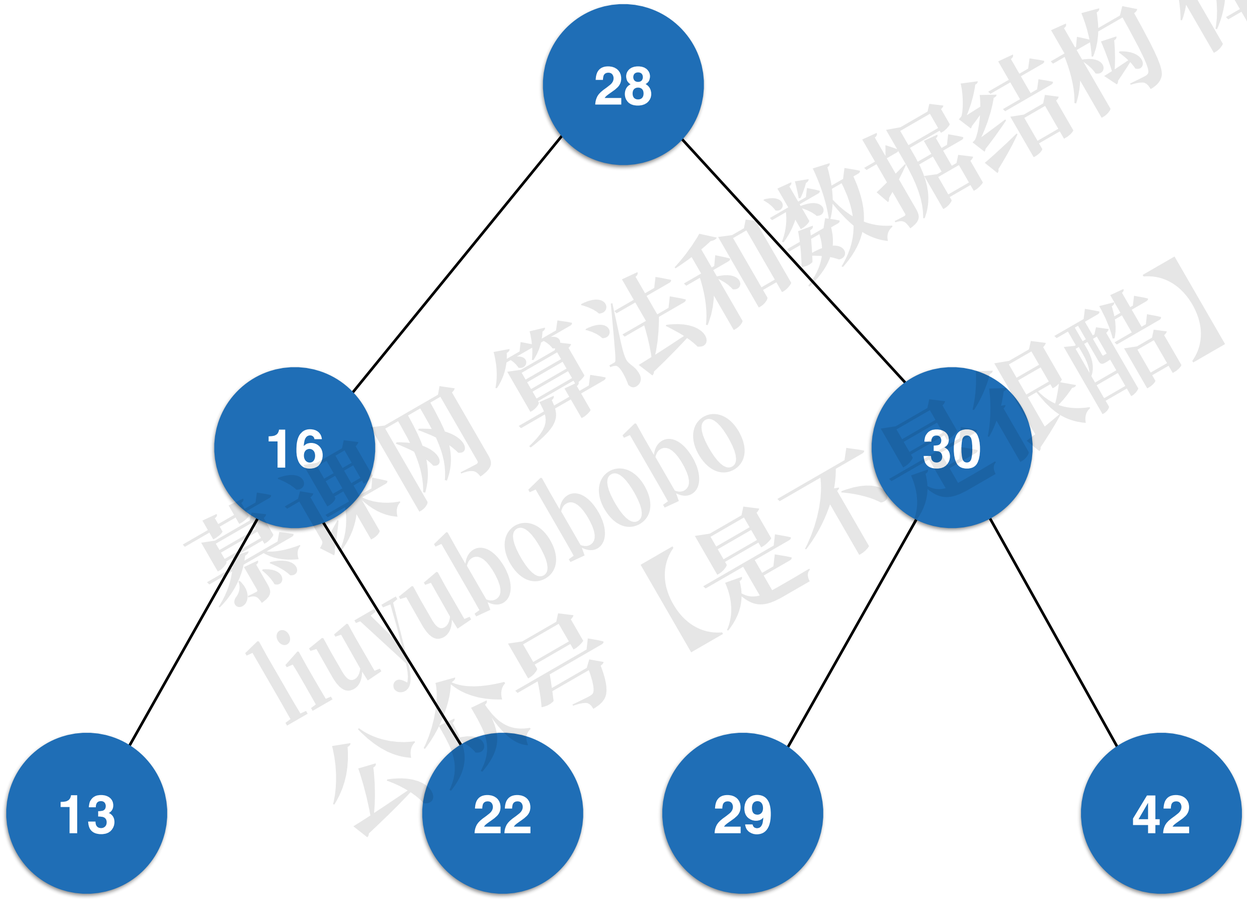


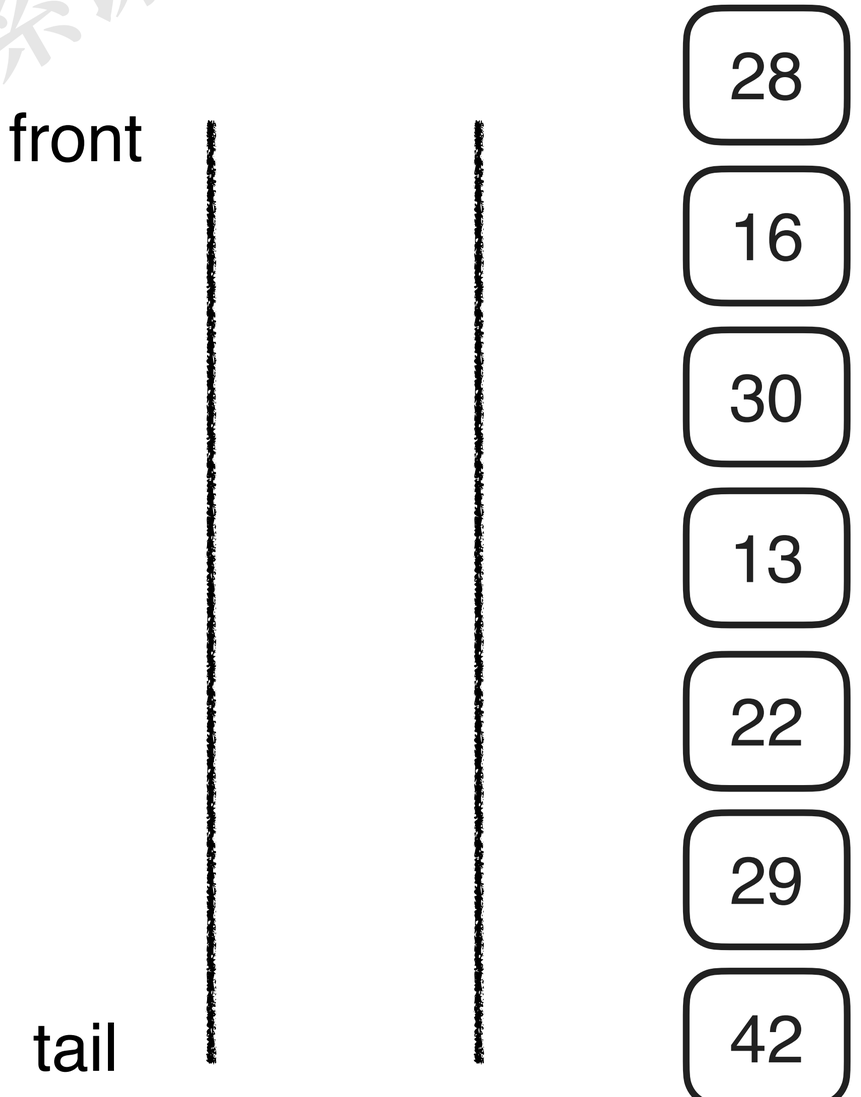
二分搜索树的层序遍历





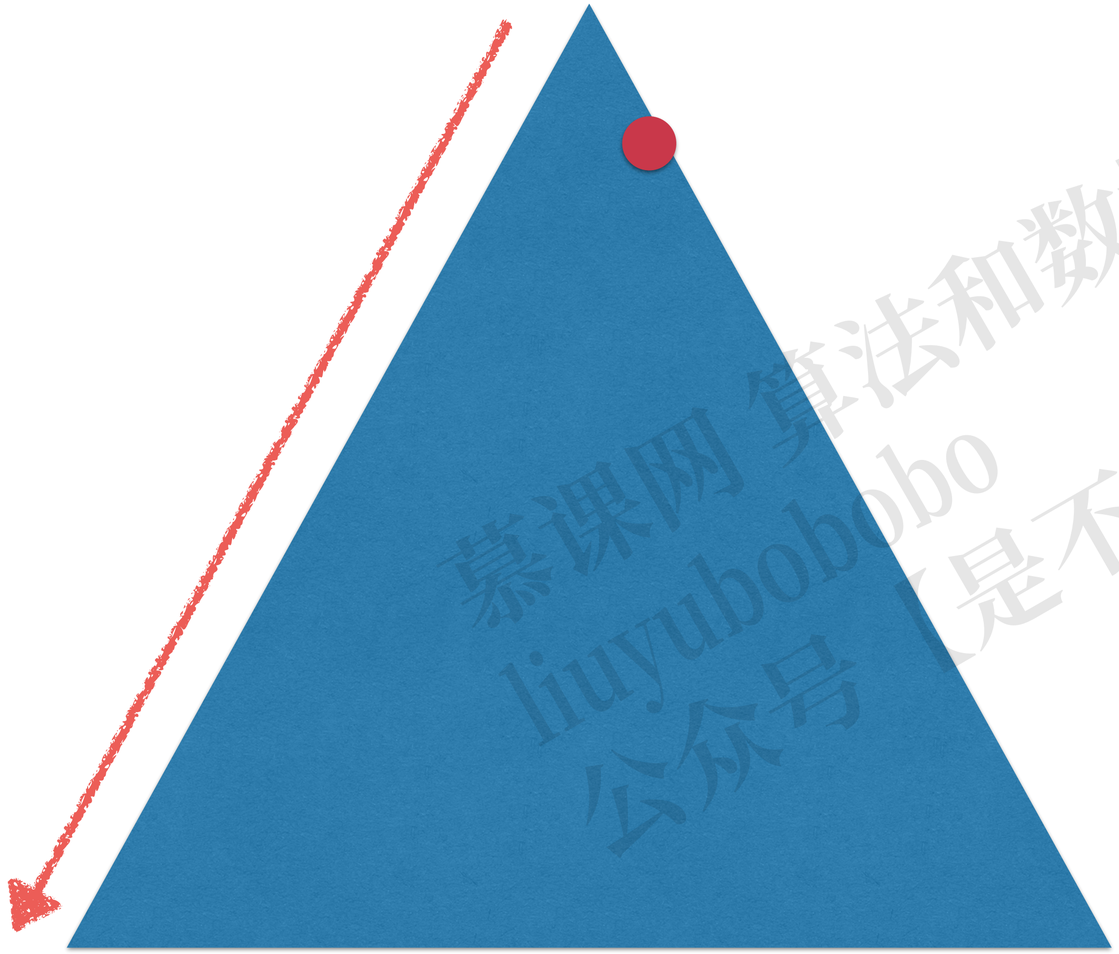
二分搜索树的层序遍历





实践：二分搜索树的层序遍历

广度优先遍历的意义

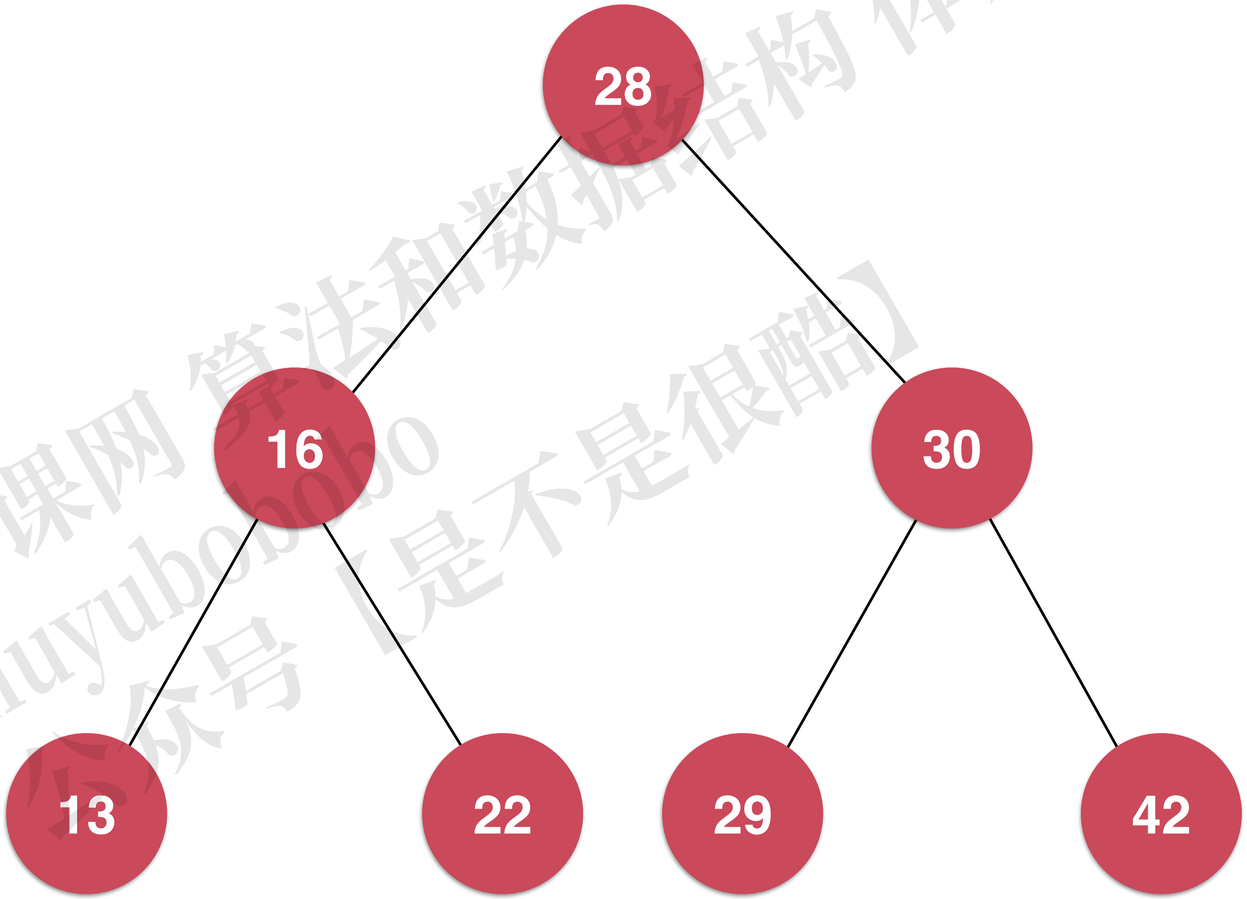


- 更快的找到问题的解
- 常用于算法设计中 - 最短路径
- 图中的深度优先遍历和广度优先遍历

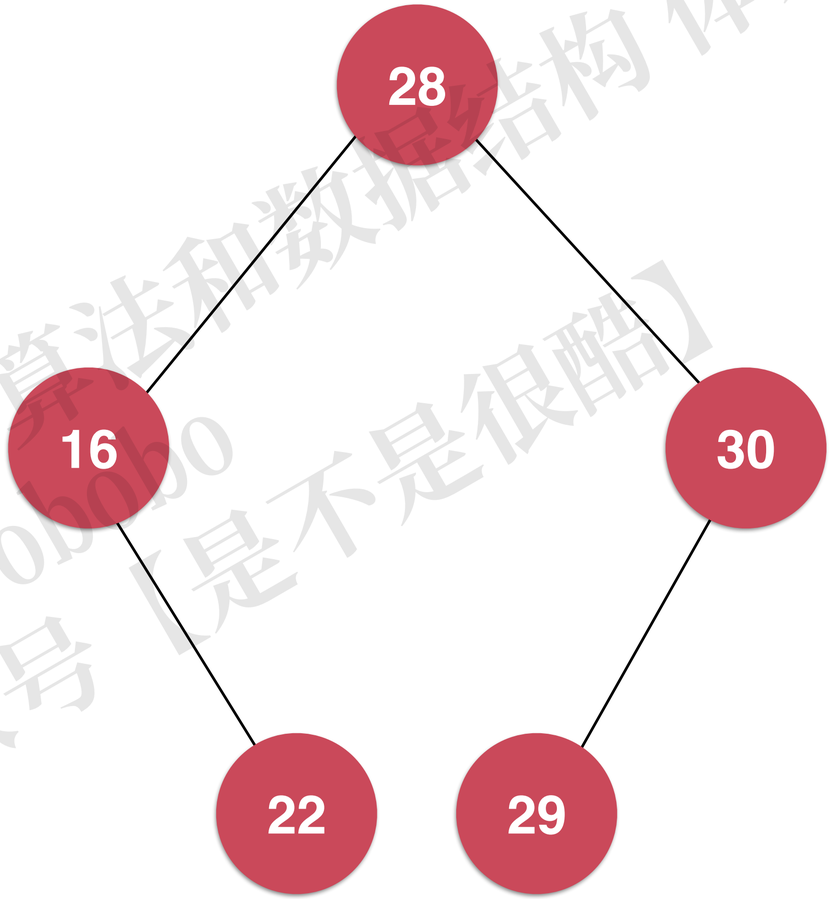
二分搜索树 删除节点

从最简单的，删除二分搜索树的最小值和最大值开始

二分搜索树的最小值和最大值

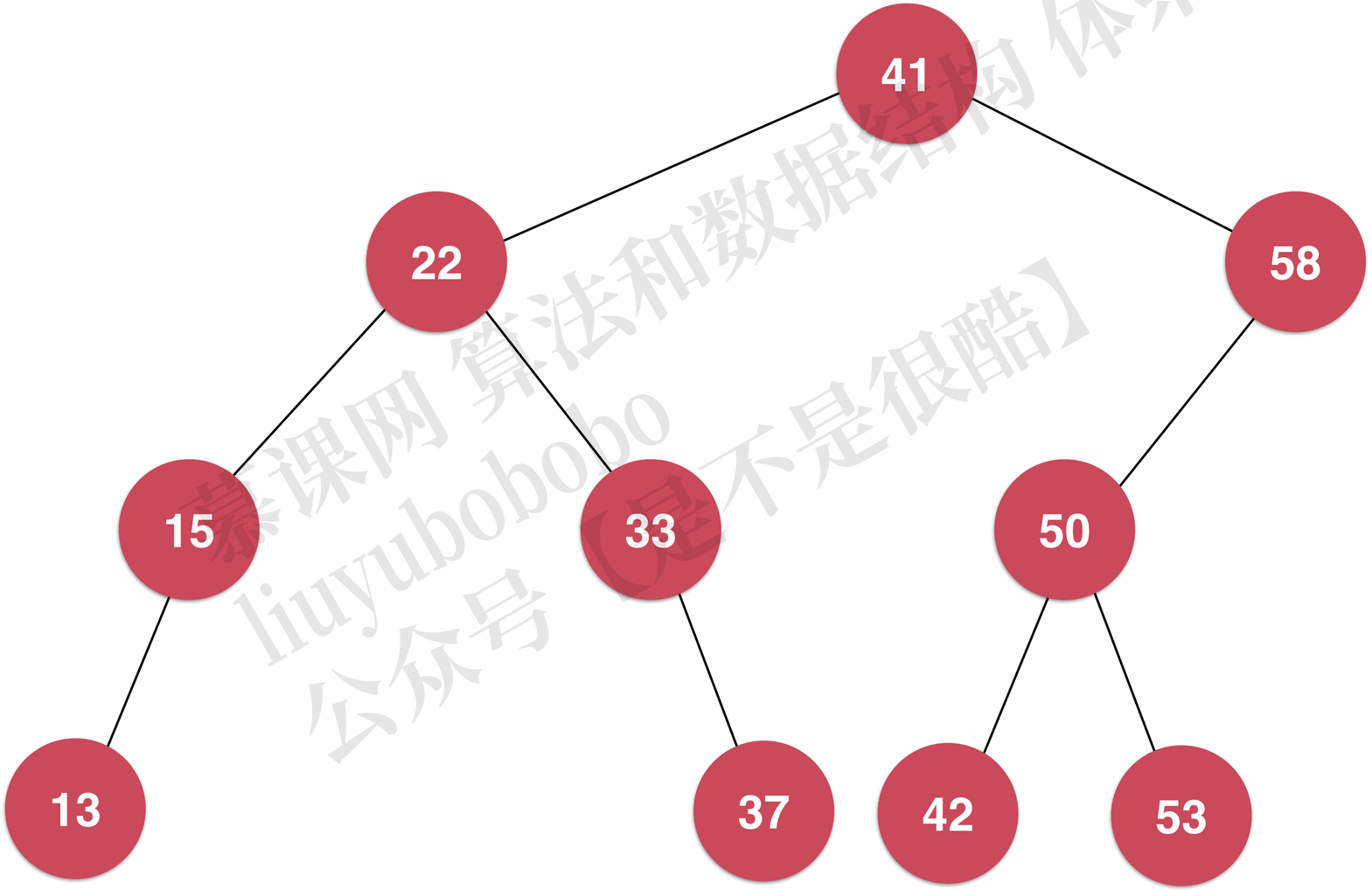


二分搜索树的最小值和最大值

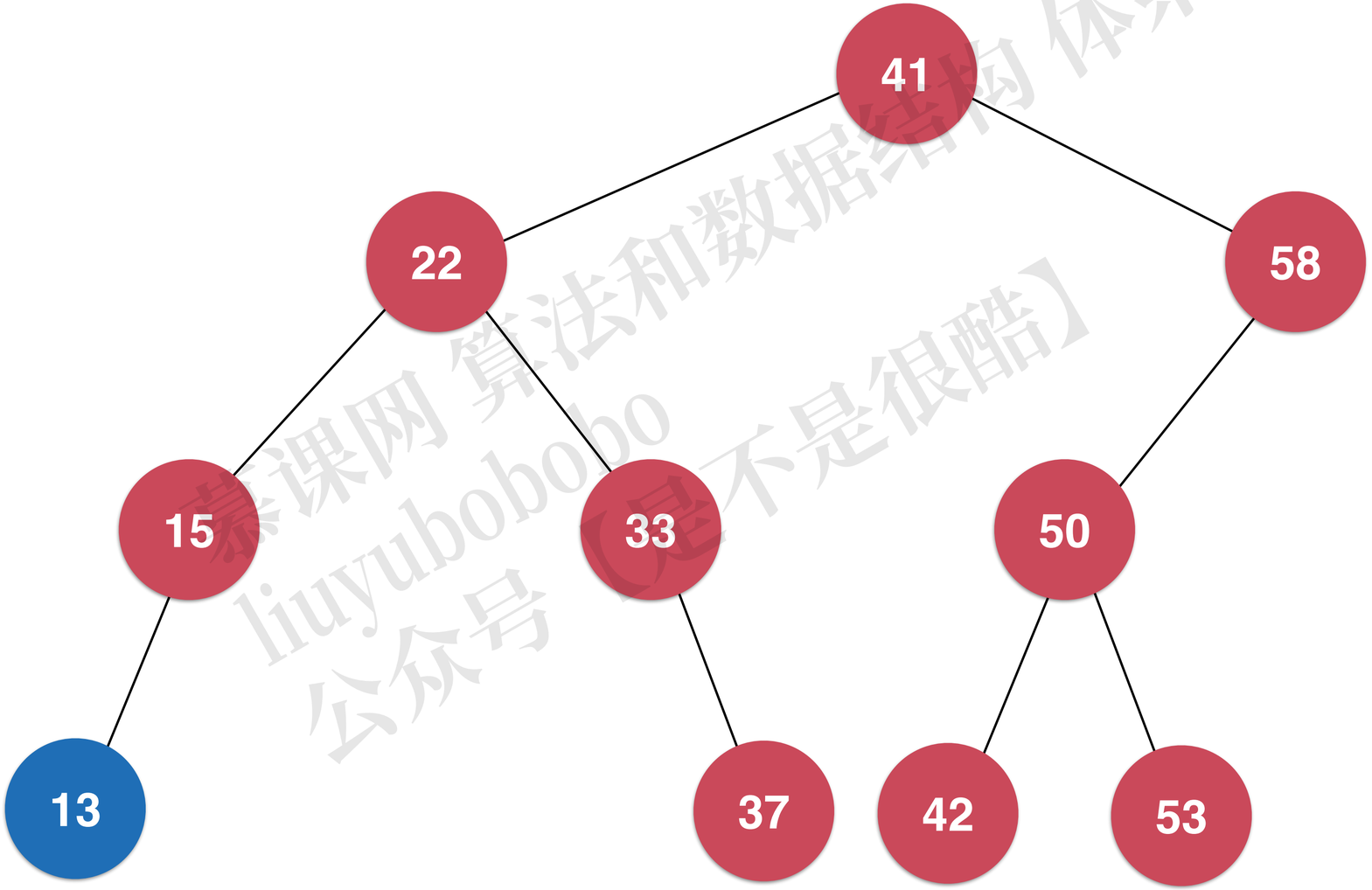


实践：求二分搜索树的最小值和最大值

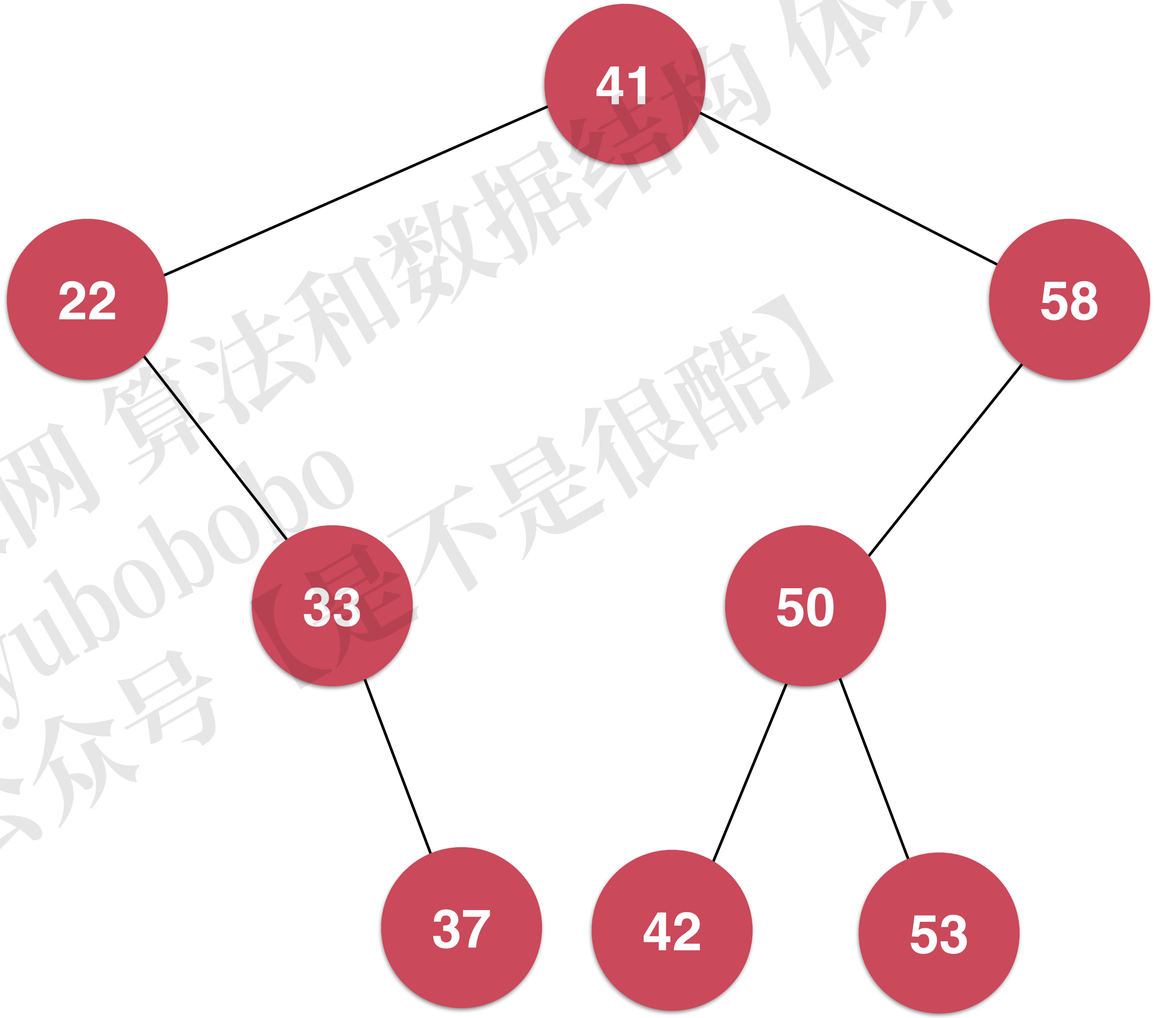
删除二分搜索树的最小值



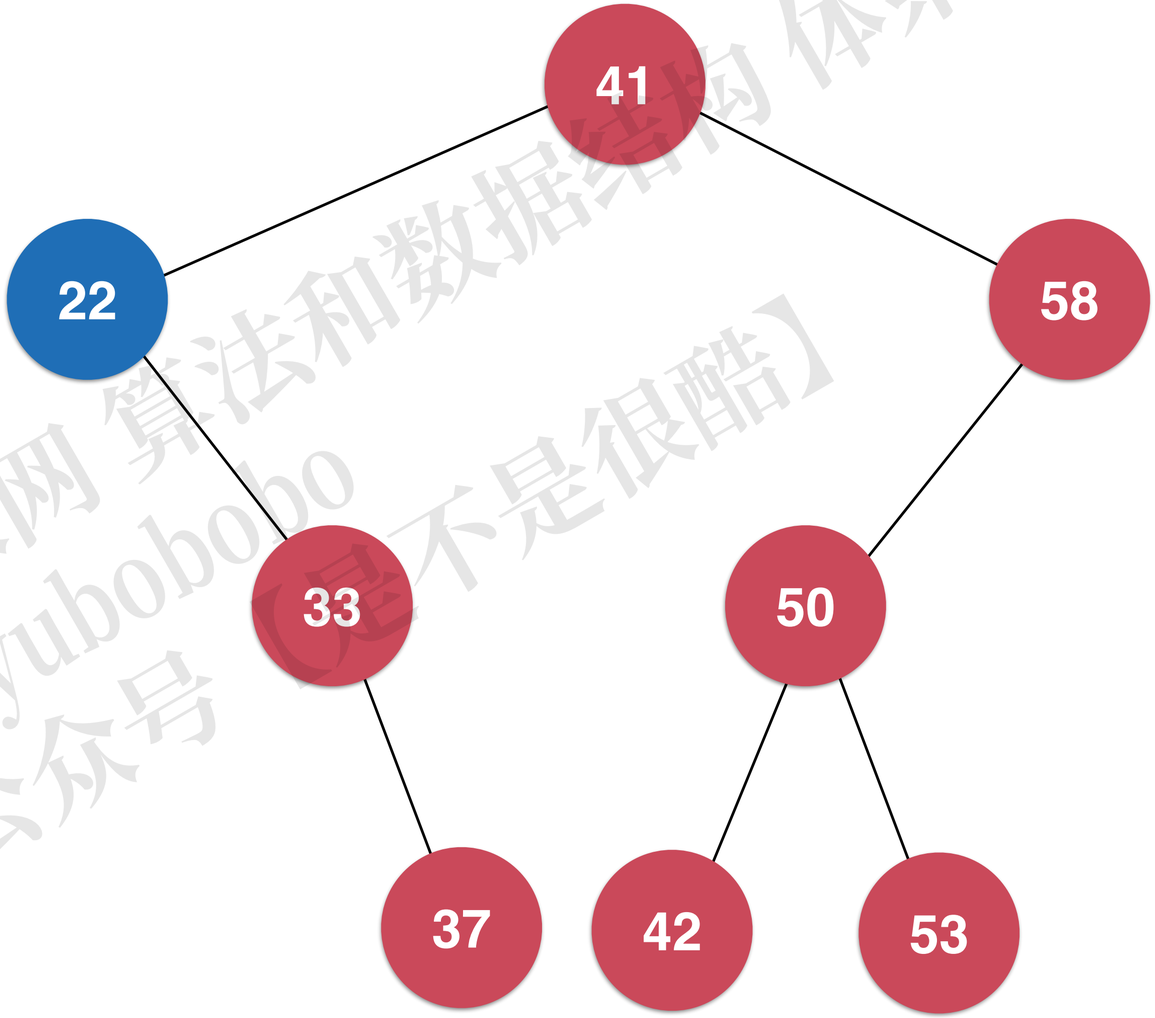
删除二分搜索树的最小值



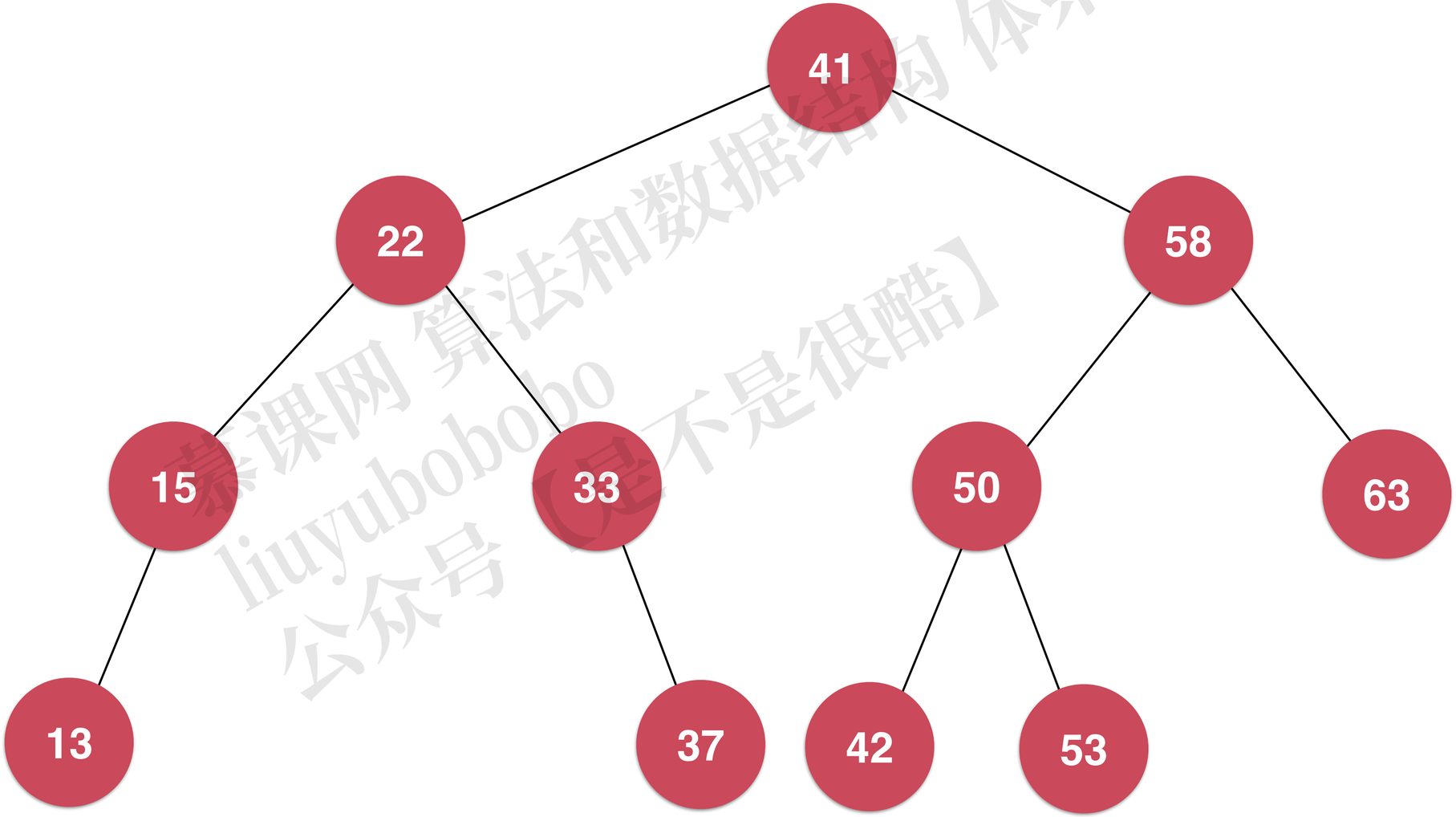
删除二分搜索树的最小值



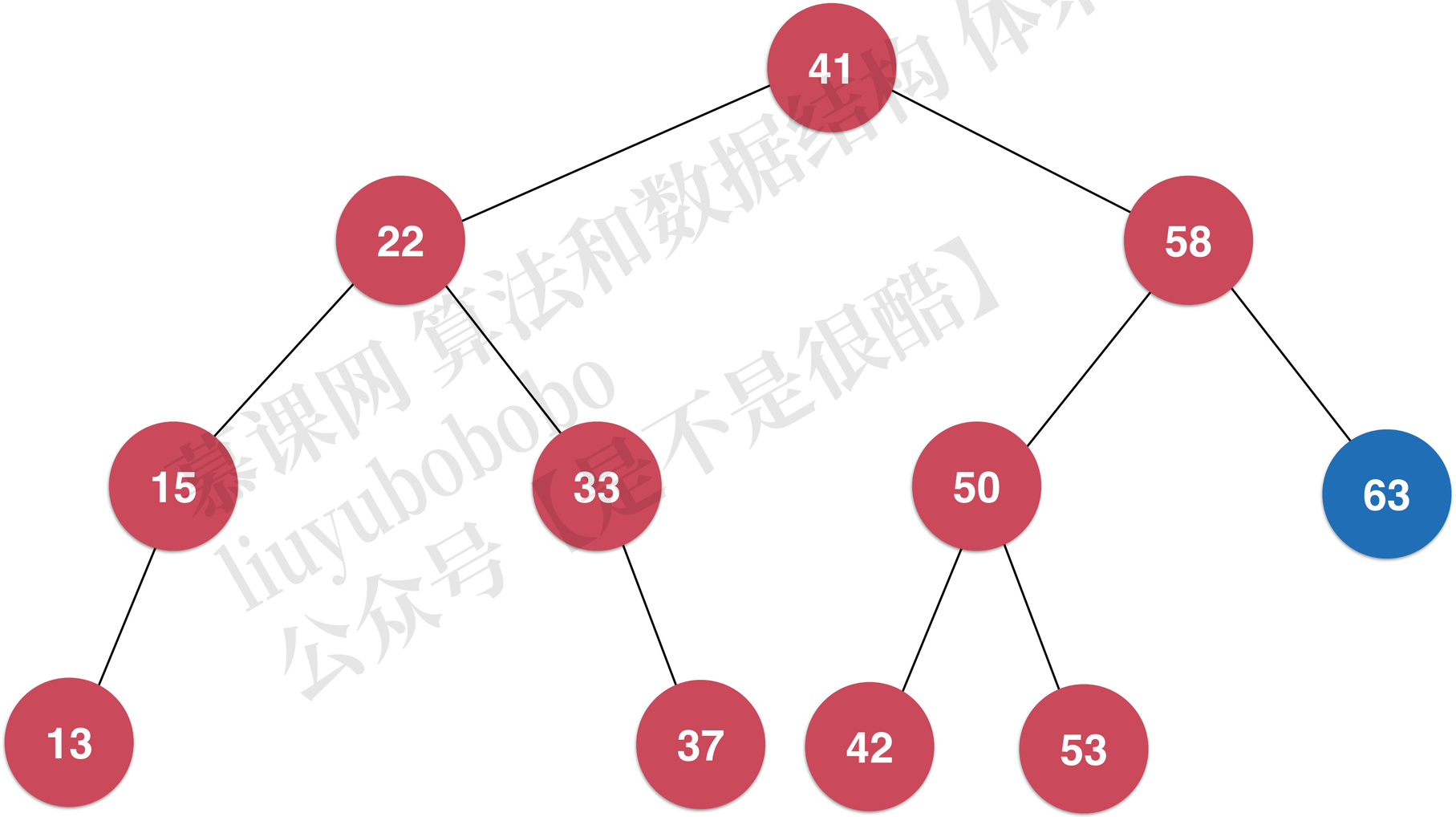
删除二分搜索树的最小值



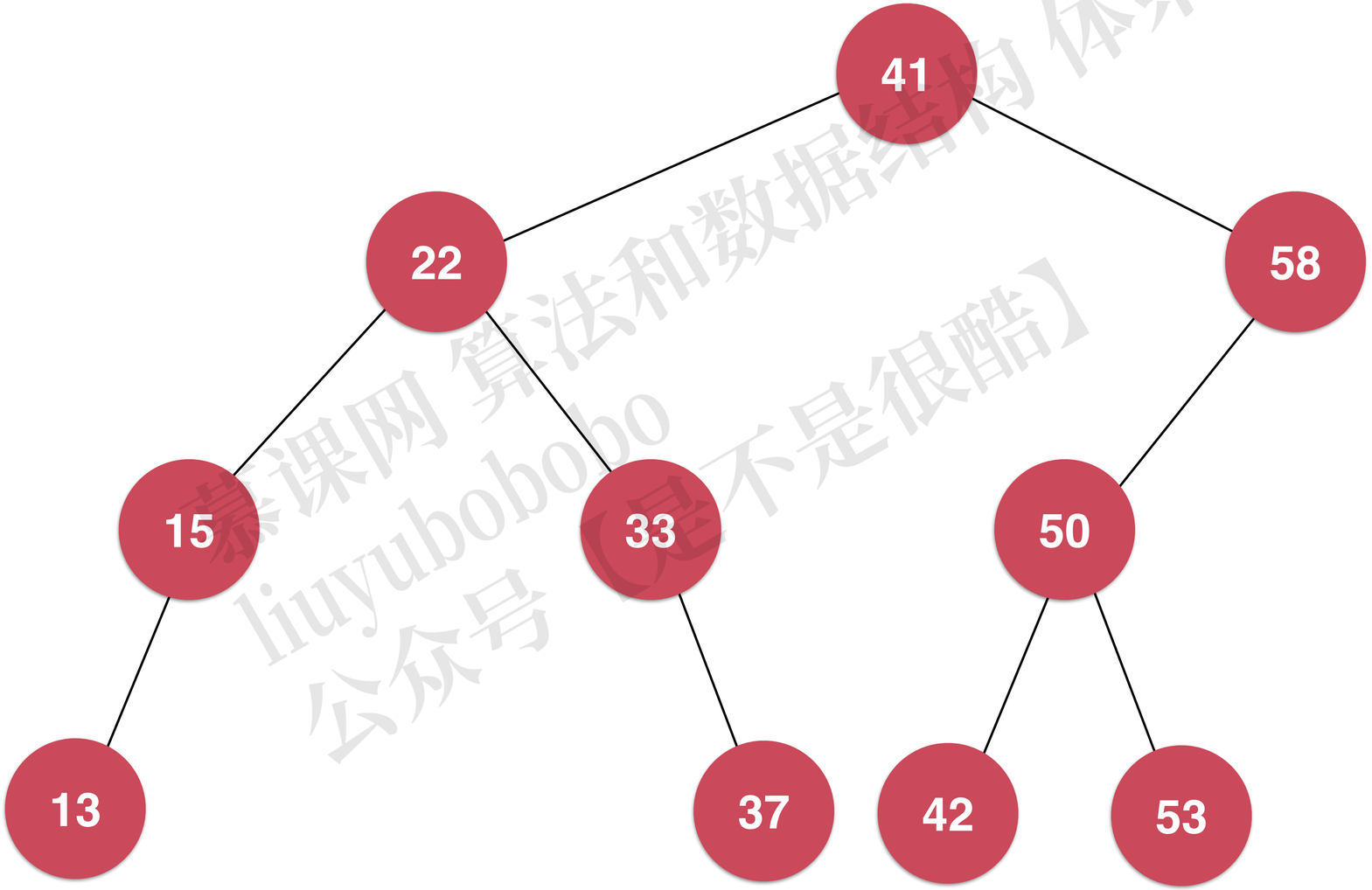
删除二分搜索树的最大值



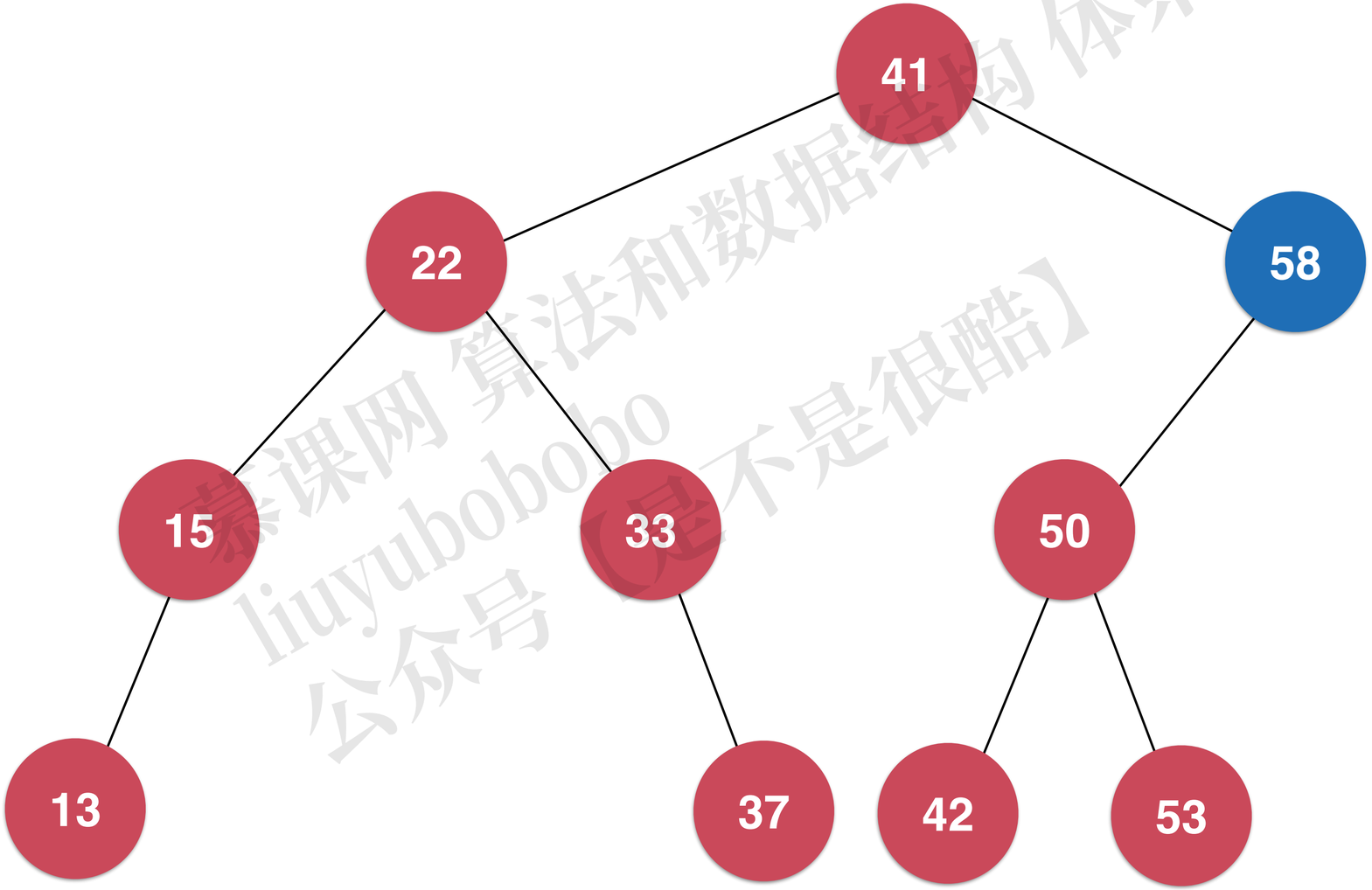
删除二分搜索树的最大值



删除二分搜索树的最大值



删除二分搜索树的最大值

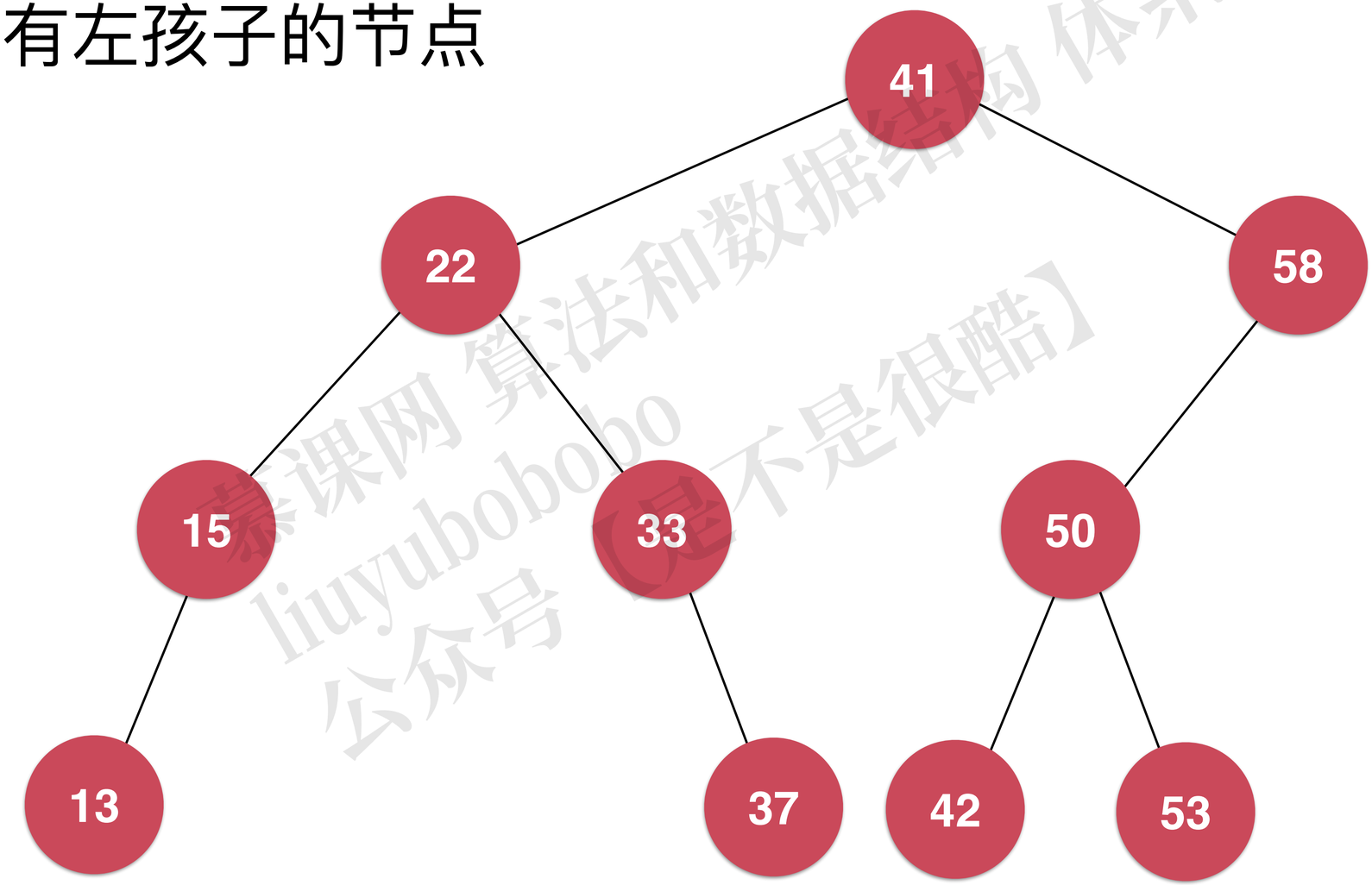


实践：删除二分搜索树的最小值和最大值

二分搜索树删除节点

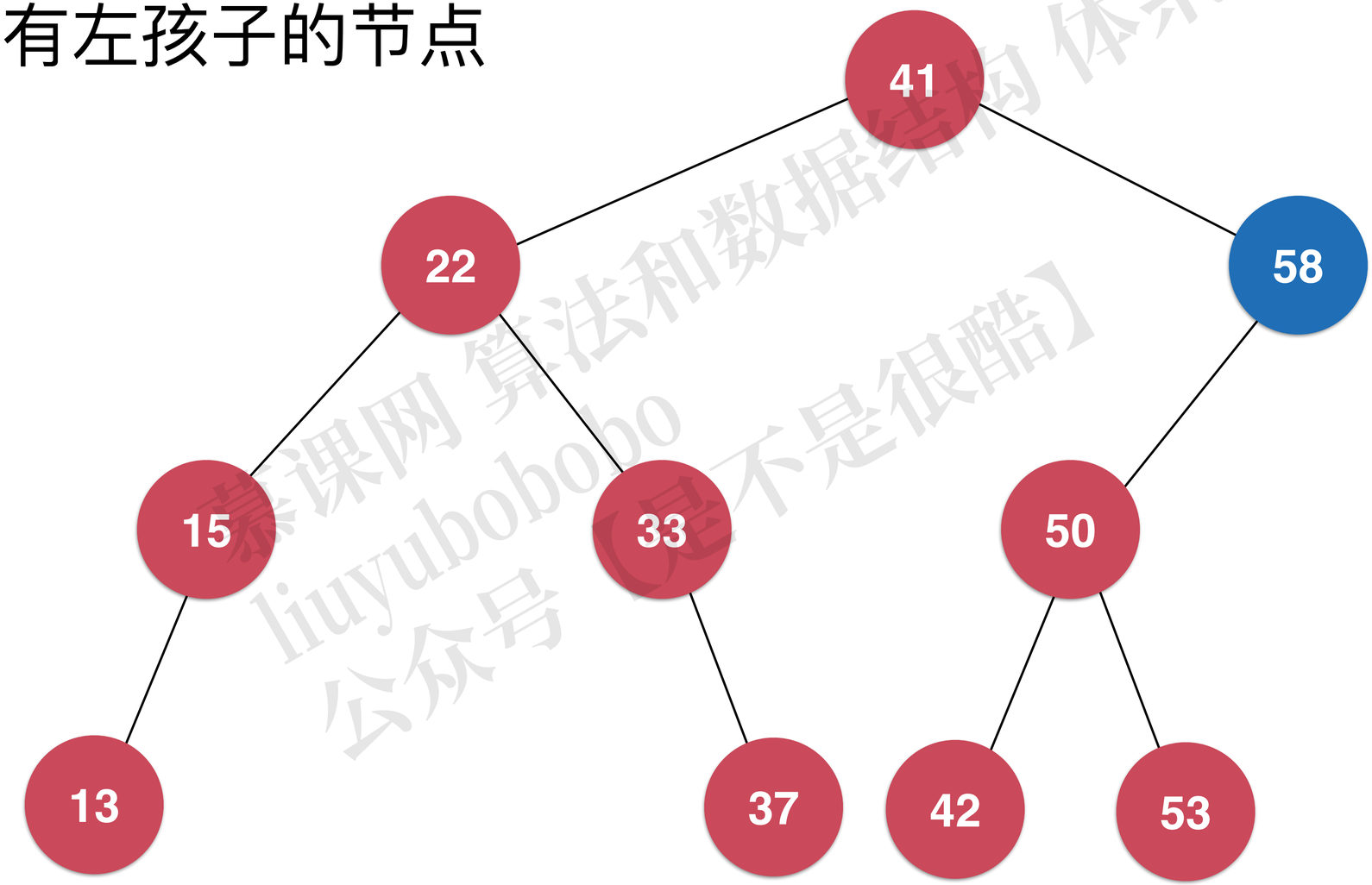
二分搜索树删除节点

删除只有左孩子的节点



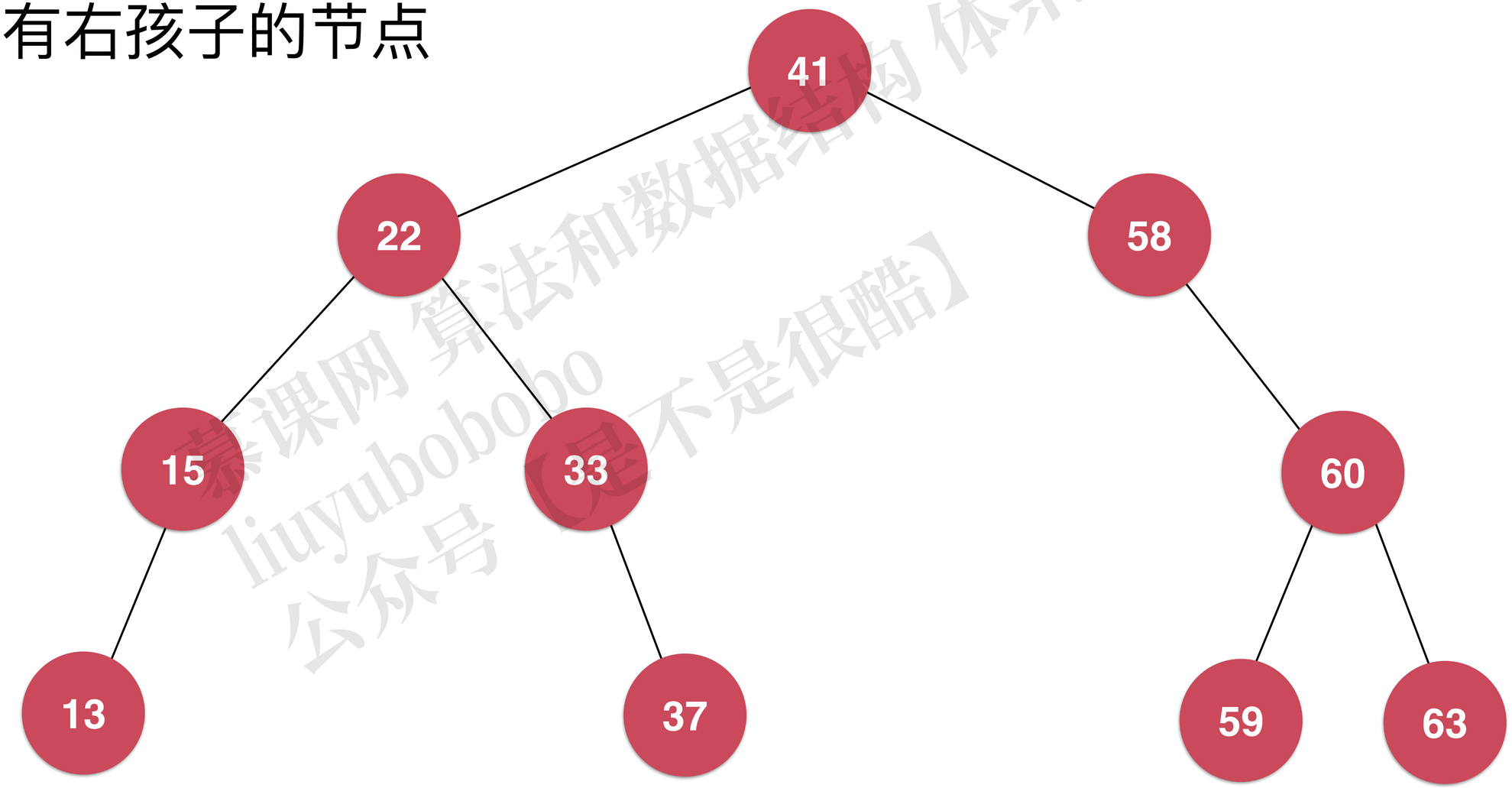
二分搜索树删除节点

删除只有左孩子的节点



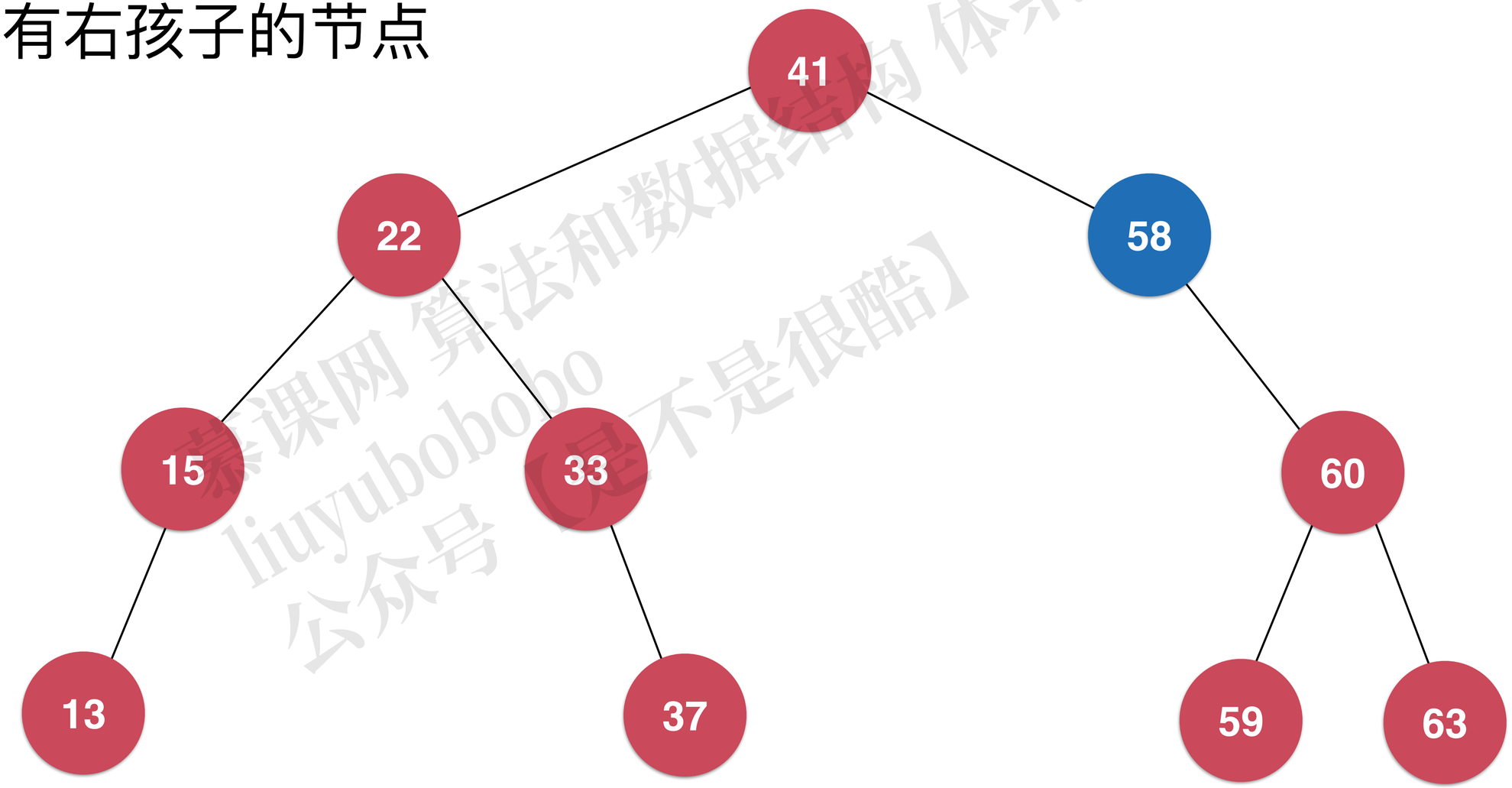
二分搜索树删除节点

删除只有右孩子的节点



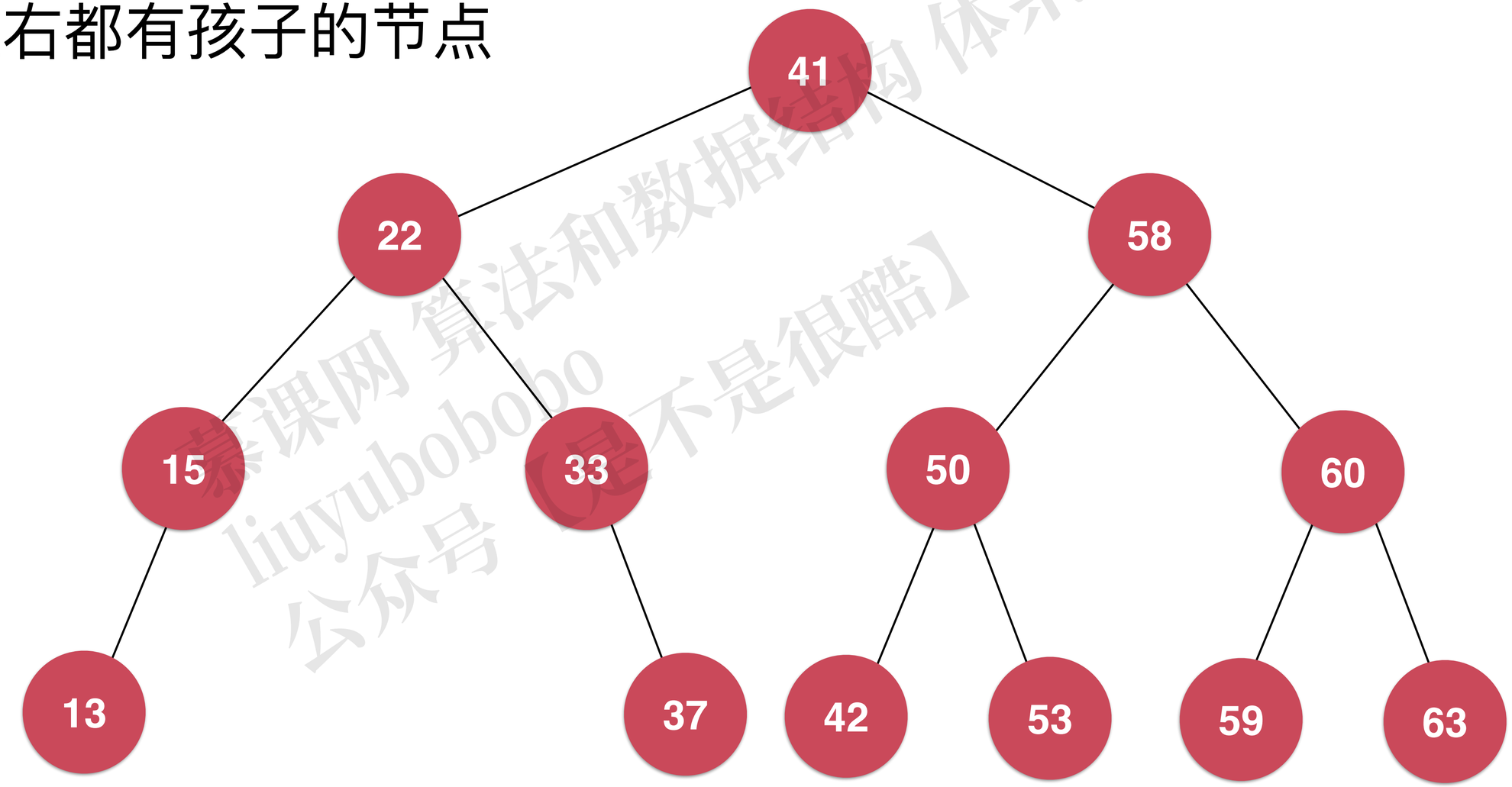
二分搜索树删除节点

删除只有右孩子的节点



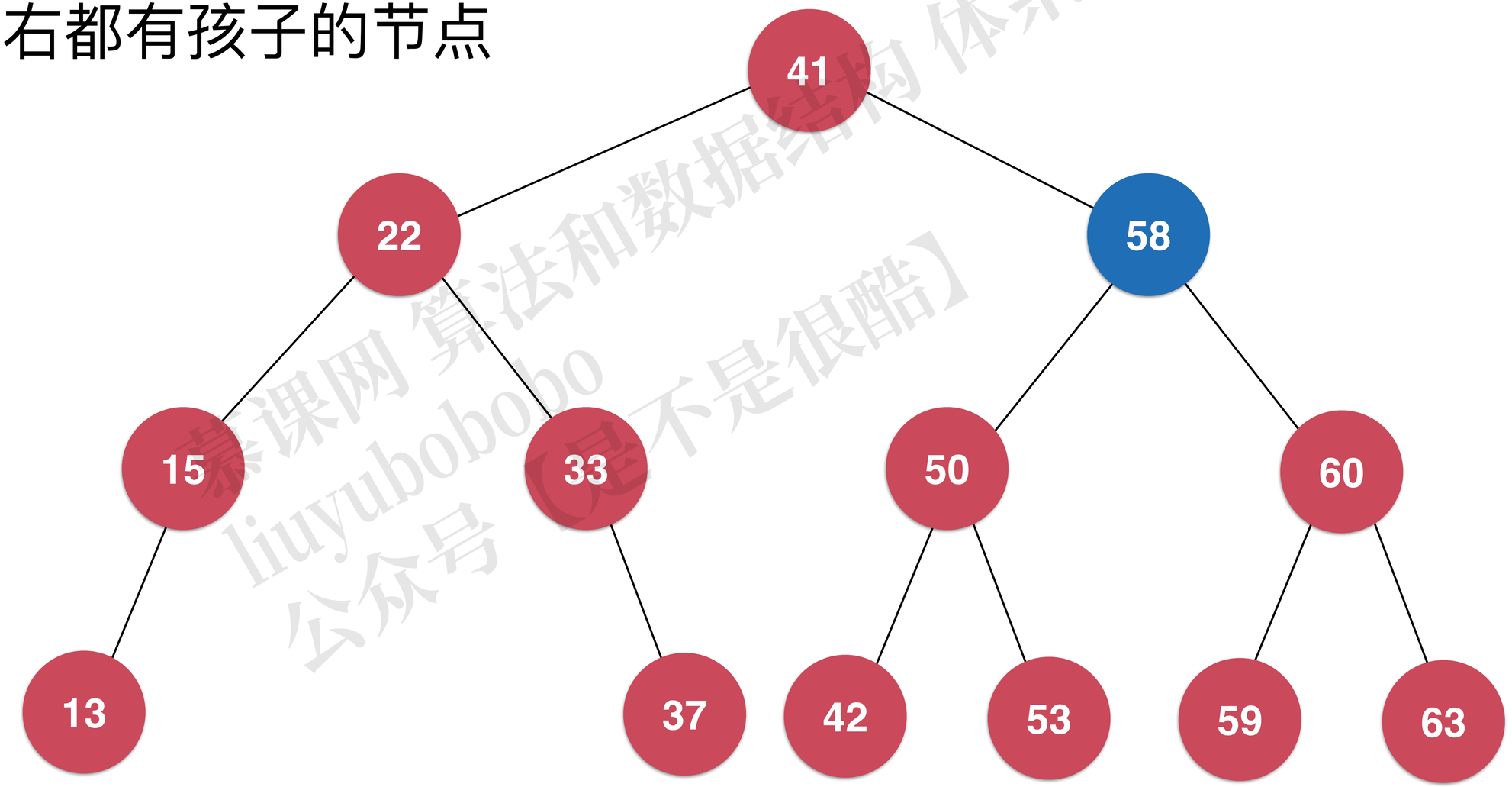
二分搜索树删除节点

删除左右都有孩子的节点



二分搜索树删除节点

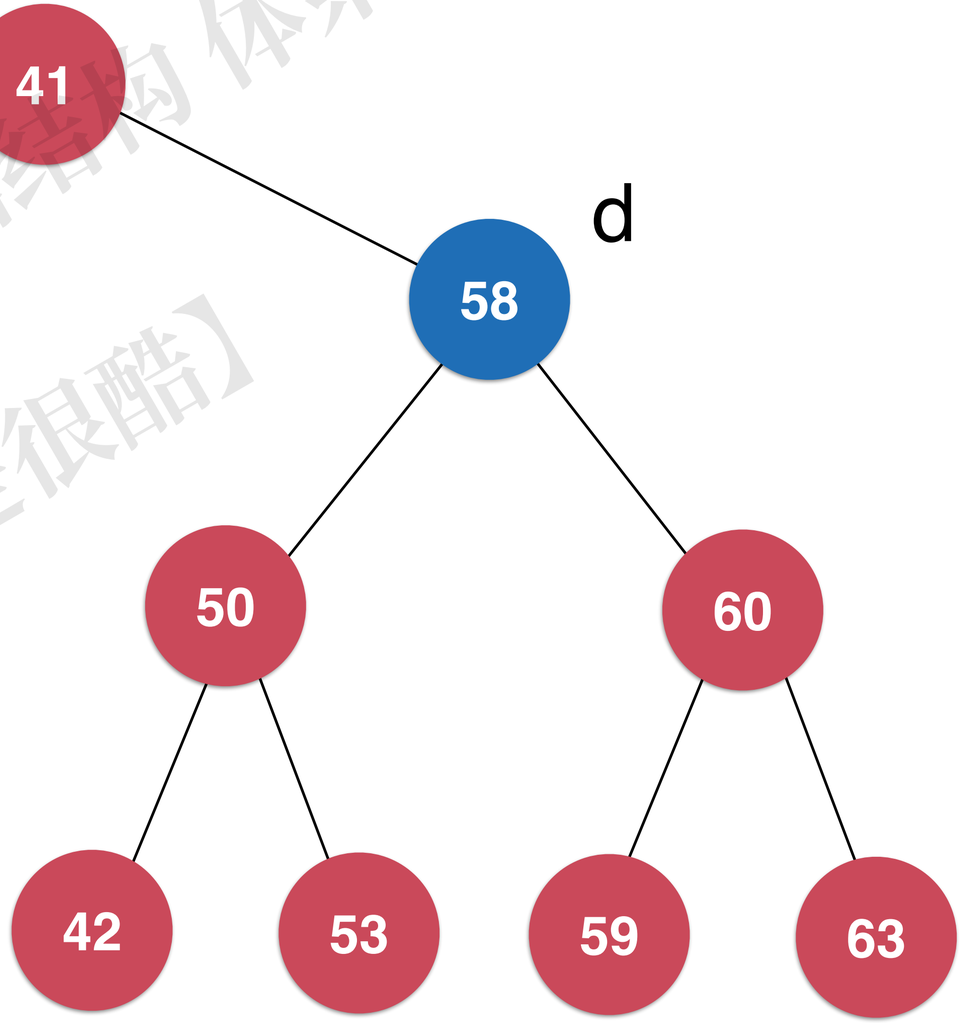
删除左右都有孩子的节点



1962年，Hibbard提出 - Hibbard Deletion

二分搜索树删除节点

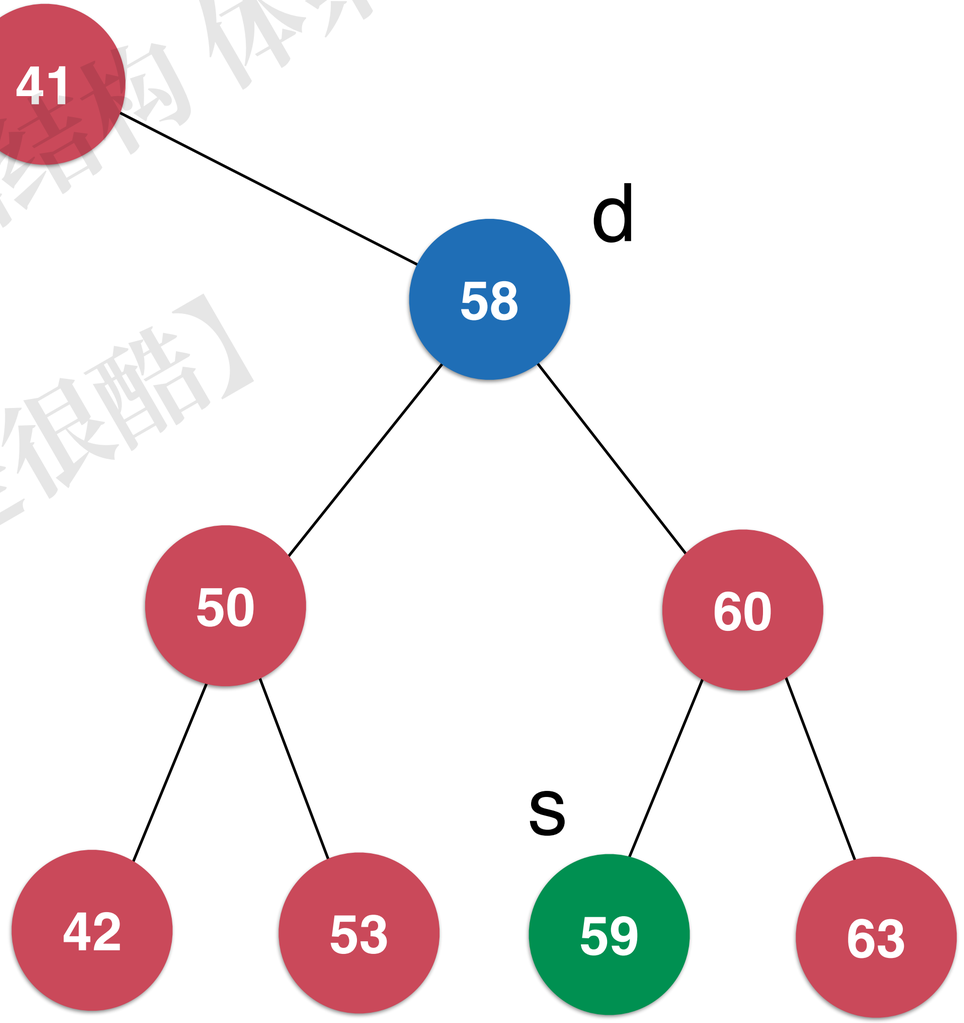
删除左右都有孩子的节点 d



二分搜索树删除节点

删除左右都有孩子的节点 d

找到 s = min(d->right)

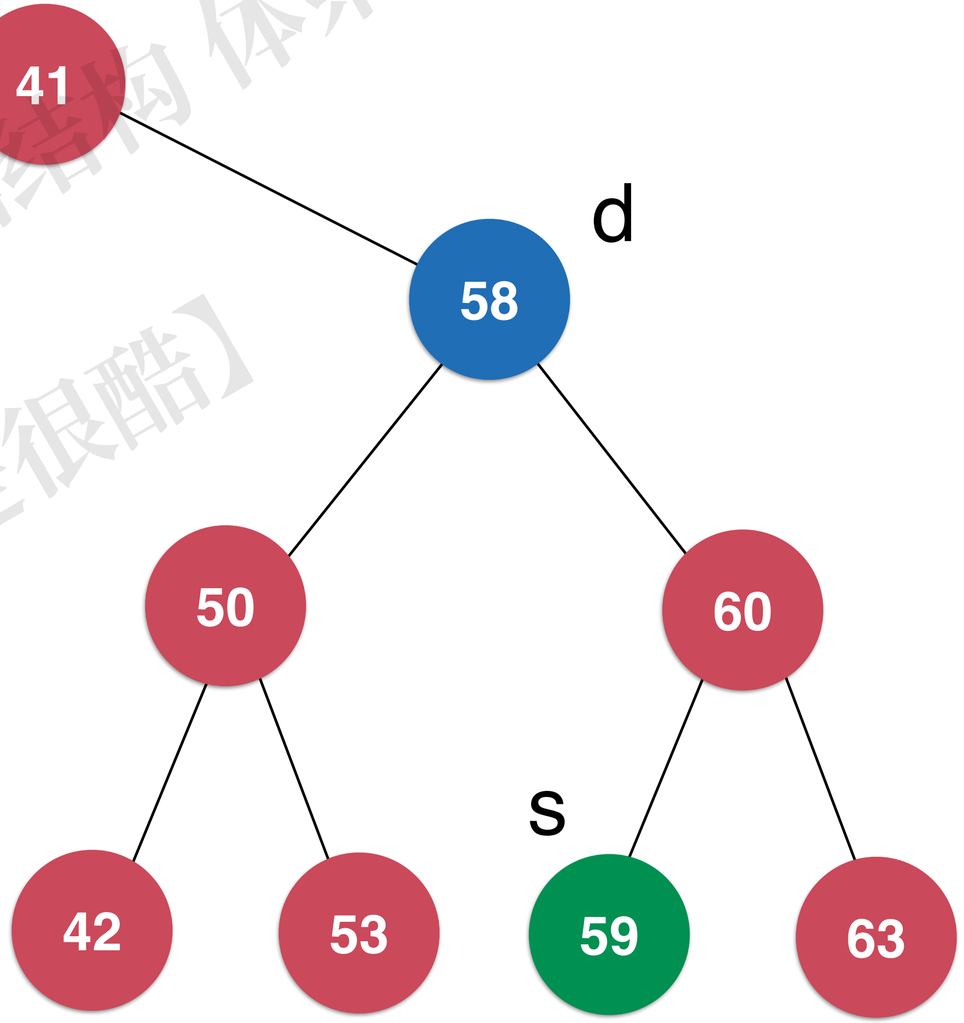


二分搜索树删除节点

删除左右都有孩子的节点 d

找到 s = min(d->right)

s 是 d 的后继



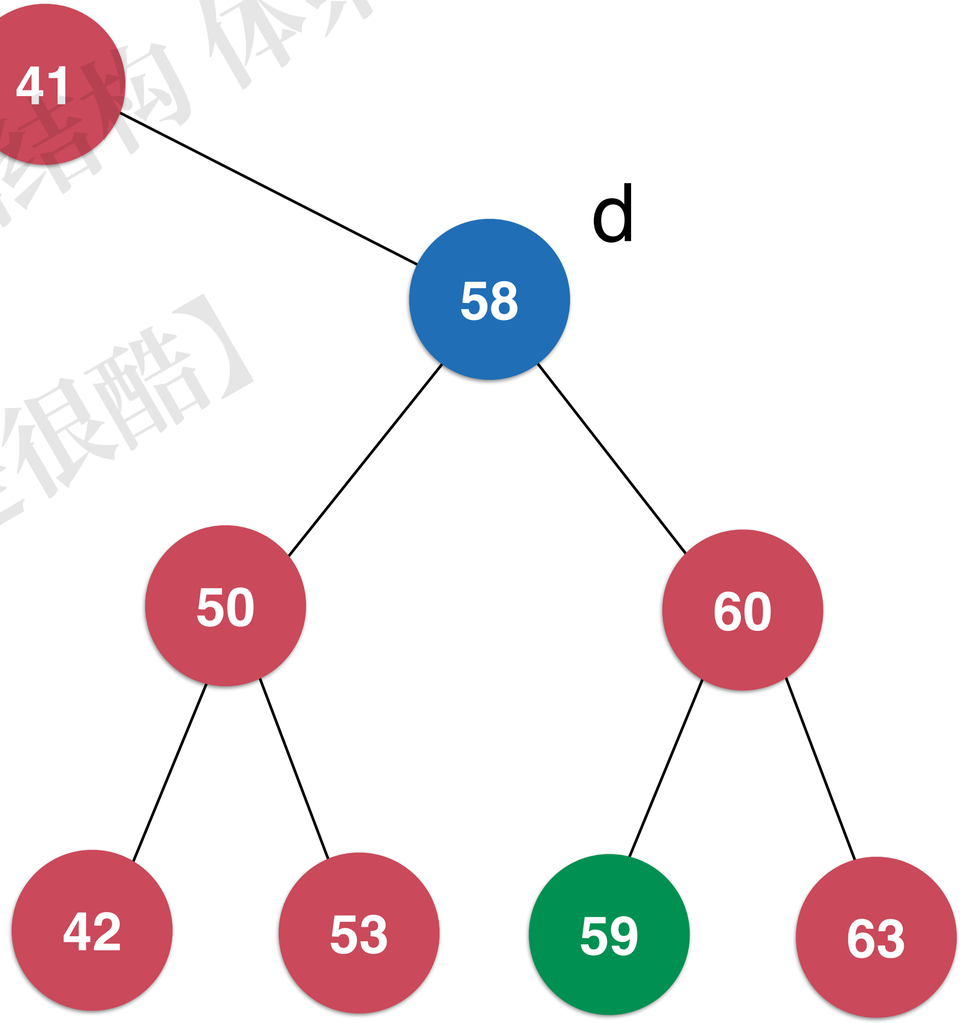
二分搜索树删除节点

删除左右都有孩子的节点 d

找到 s = min(d->right)

s 是 d 的后继

s->right = delMin(d->right)



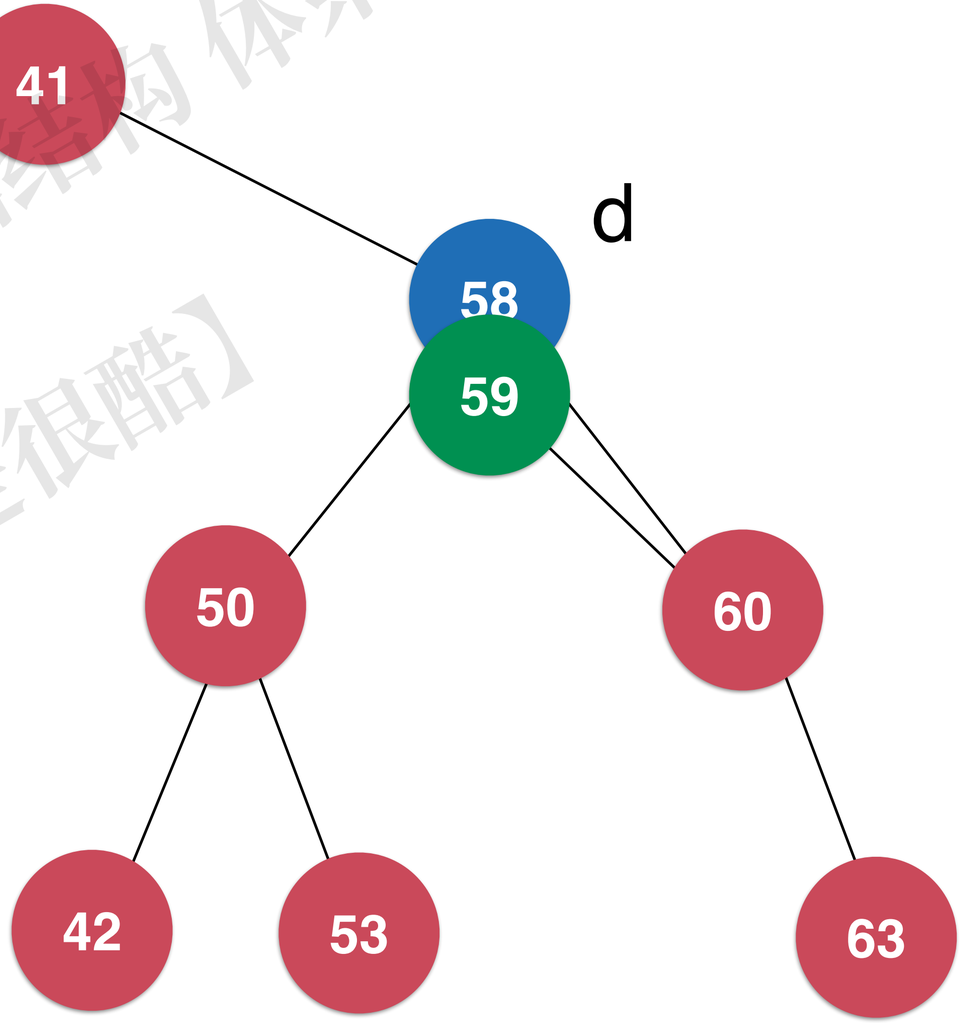
二分搜索树删除节点

删除左右都有孩子的节点 d

找到 s = min(d->right)

s 是 d 的后继

s->right = delMin(d->right)



二分搜索树删除节点

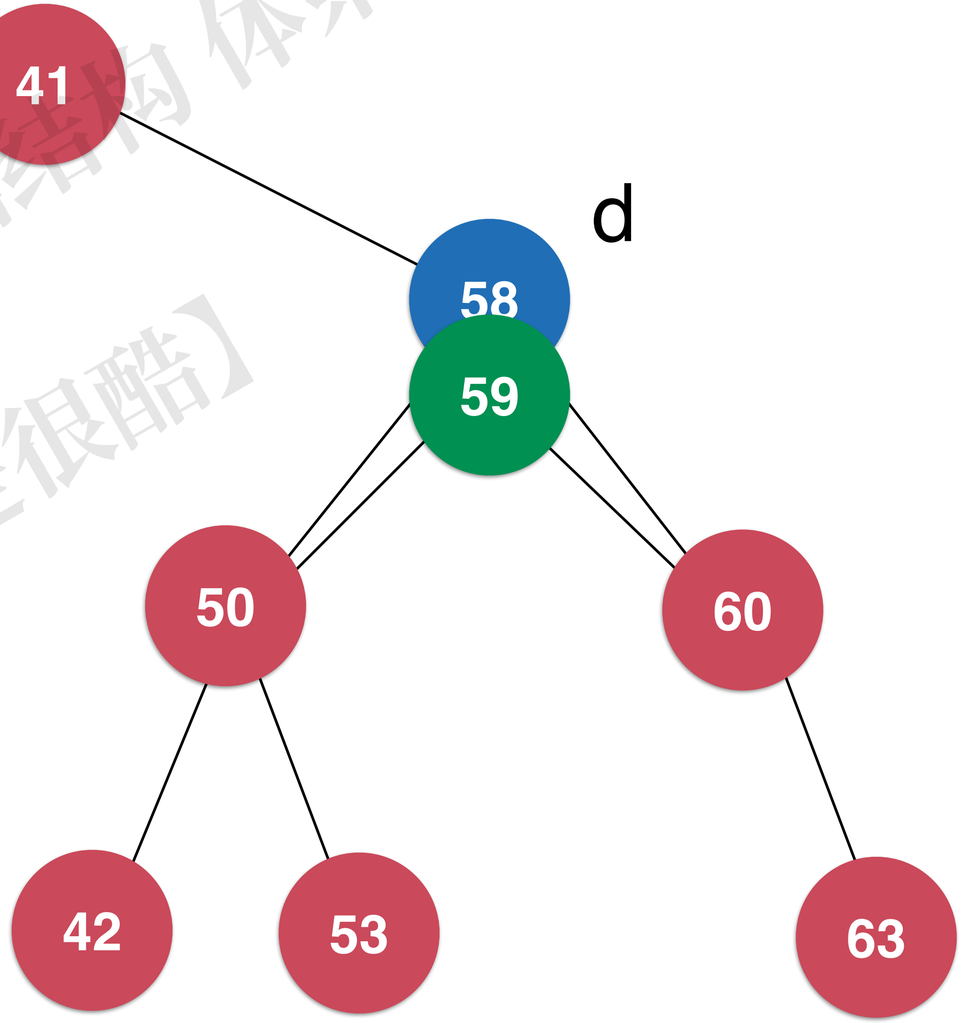
删除左右都有孩子的节点 d

找到 s = min(d->right)

s 是 d 的后继

s->right = delMin(d->right)

s->left = d->left



二分搜索树删除节点

删除左右都有孩子的节点 d

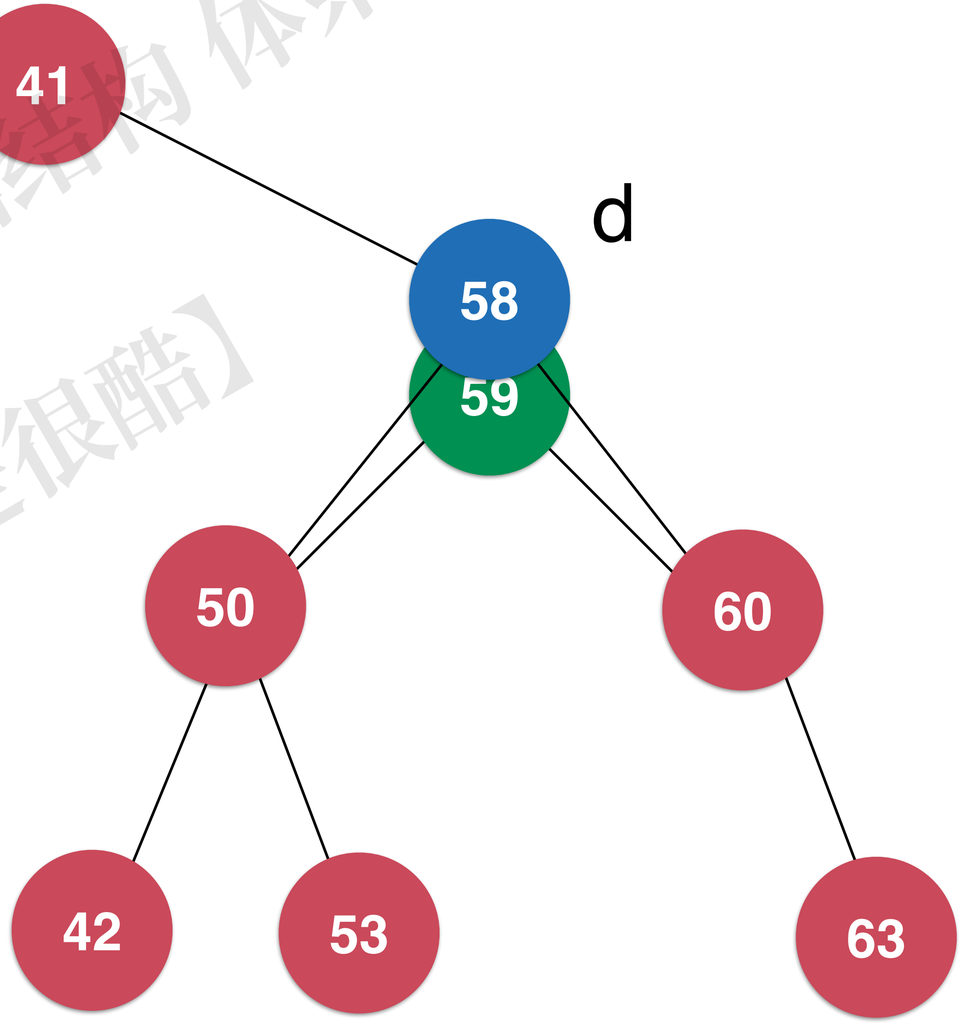
找到 s = min(d->right)

s 是 d 的后继

s->right = delMin(d->right)

s->left = d->left

删除d，s是新的子树的根



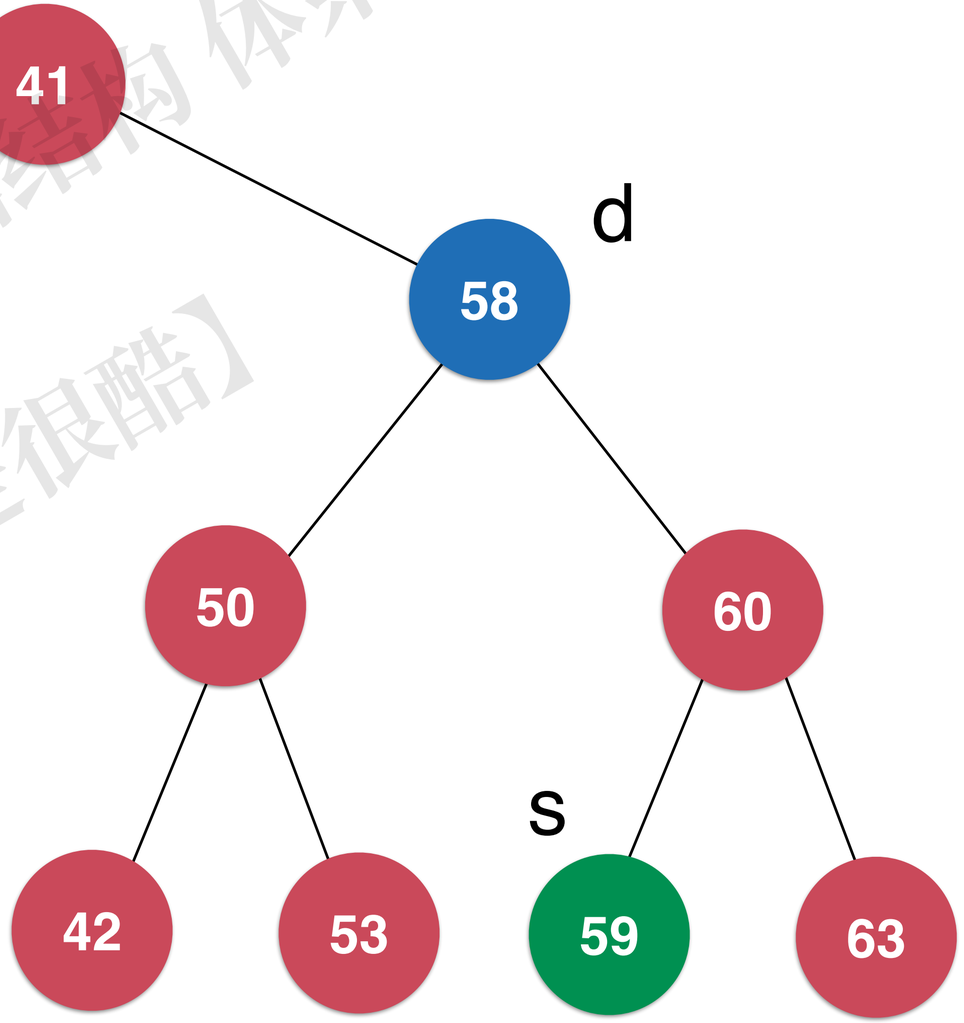
实践：删除二分搜索树的任意一个节点

二分搜索树删除节点

删除左右都有孩子的节点 d

找到 s = min(d->right)

s 是 d 的后继

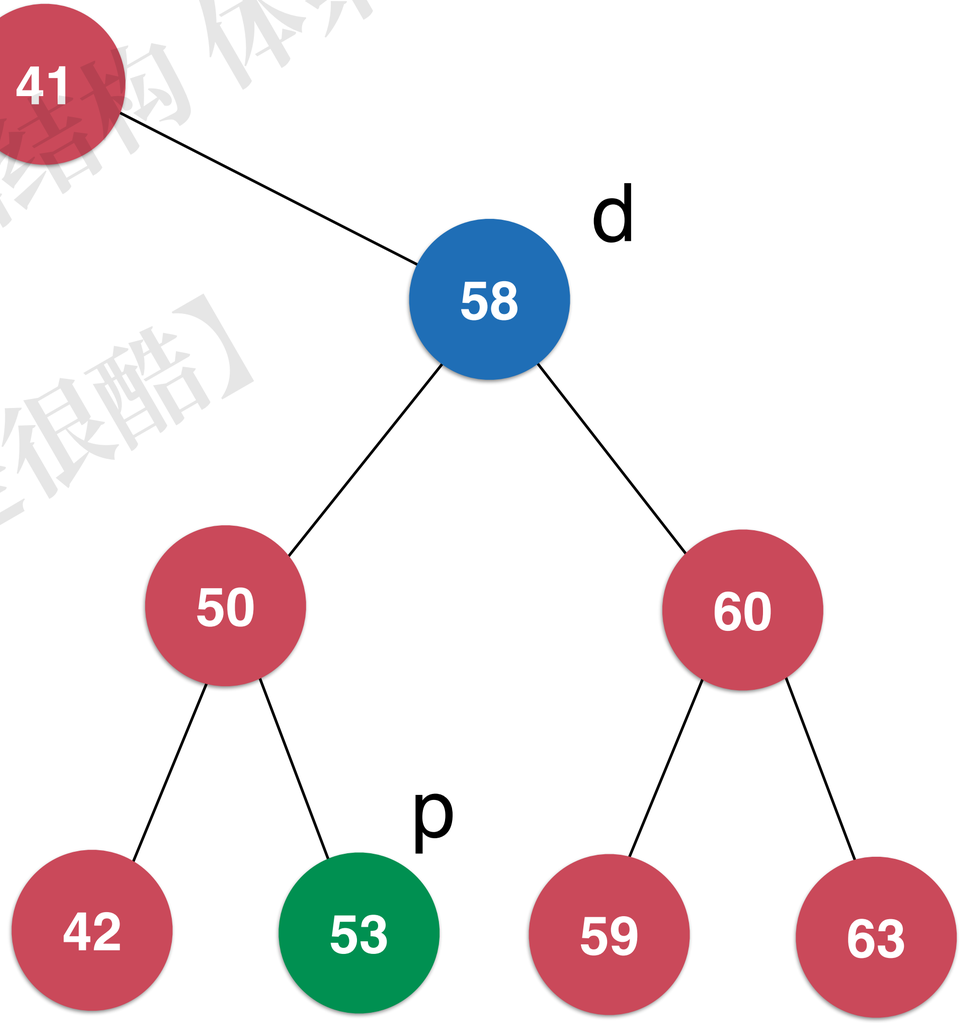


二分搜索树删除节点

删除左右都有孩子的节点 d

找到 p = max(d->left)

p 是 d 的前驱



更多二分搜索树相关的问题

二分搜索树的顺序性

二分搜索树的顺序性

minimum , maximum

二分搜索树的顺序性

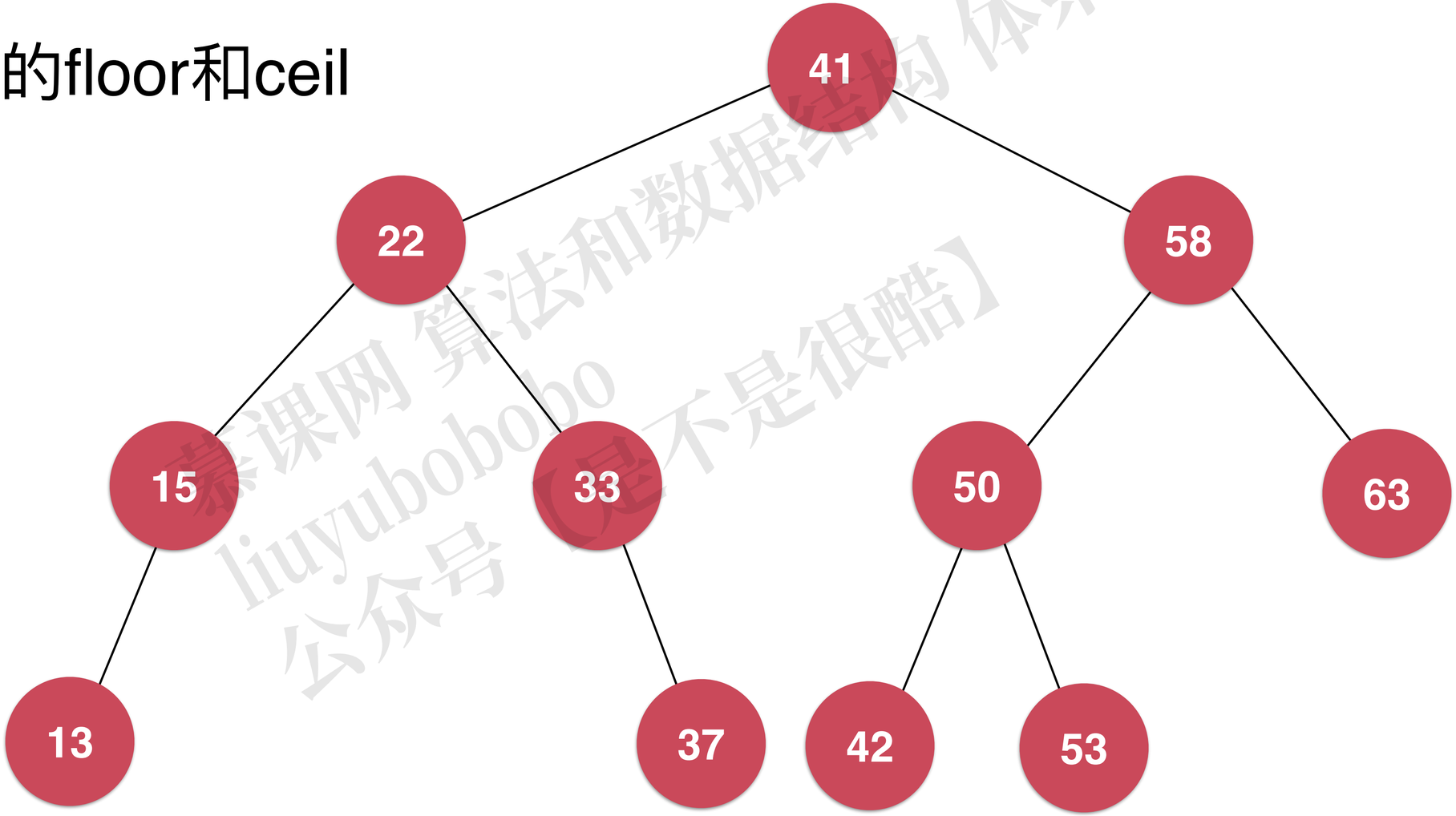
successor，predecessor

二分搜索树的顺序性

floor, ceil

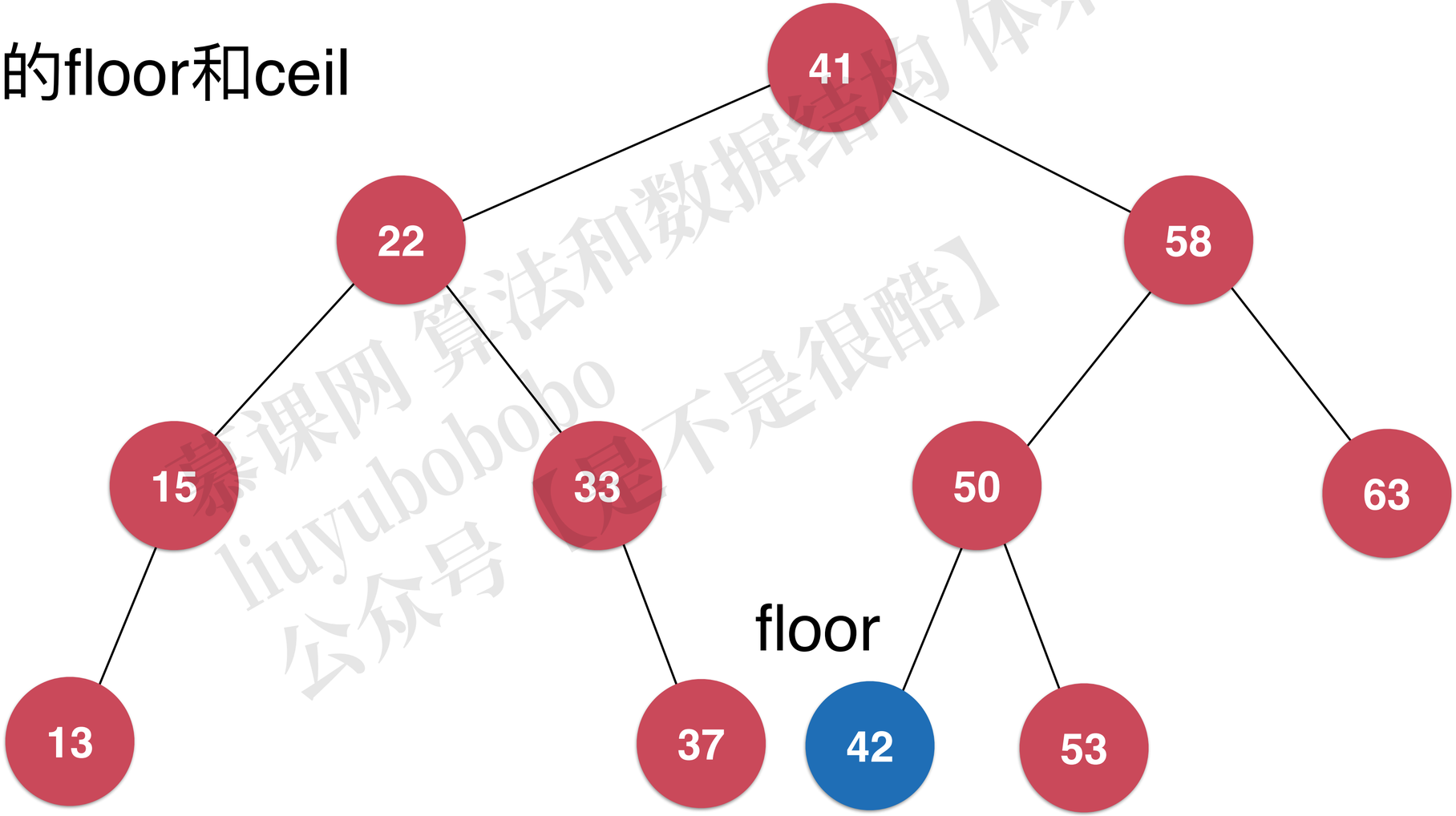
二分搜索树的floor和ceil

寻找45的floor和ceil



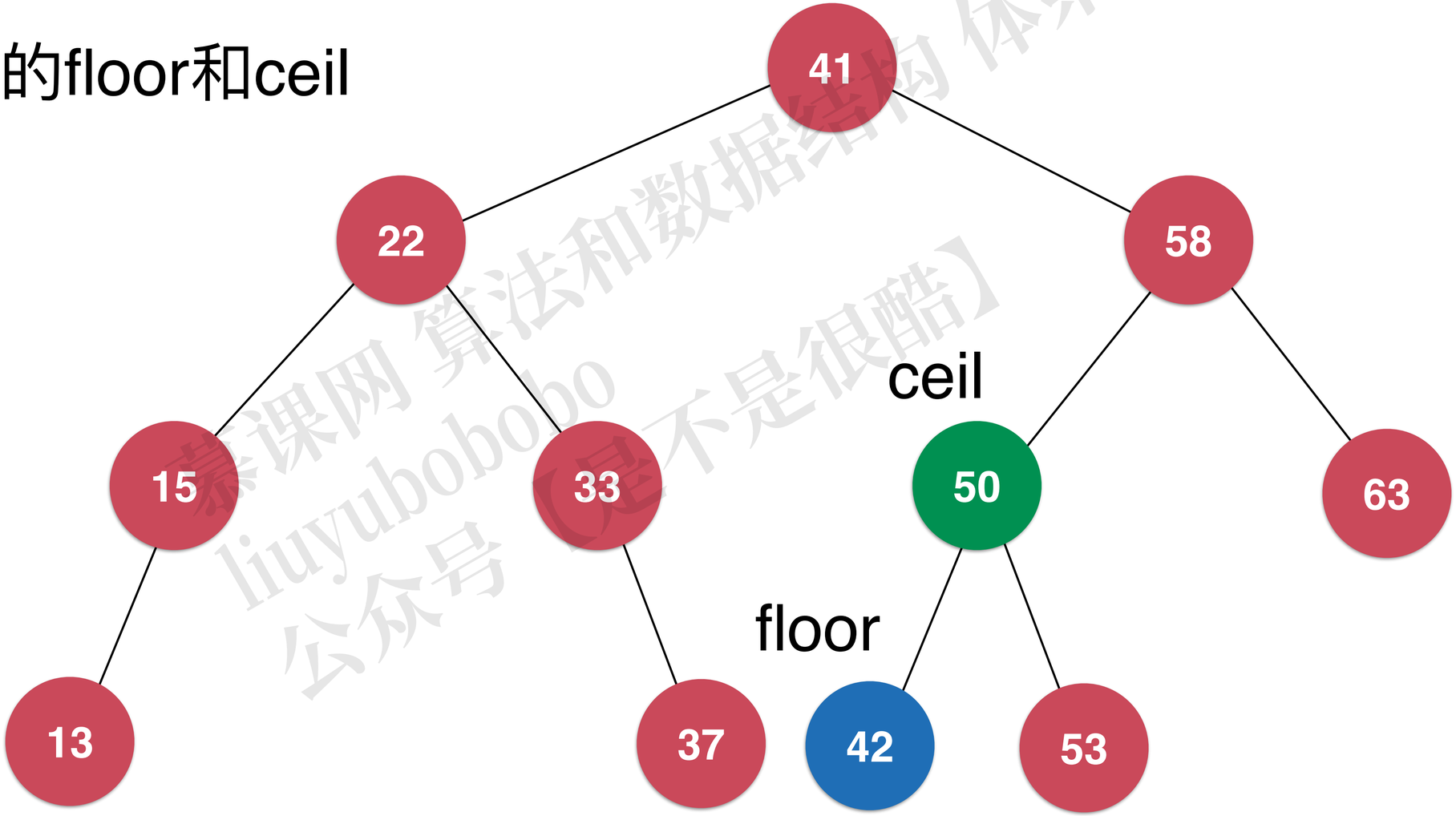
二分搜索树的floor和ceil

寻找45的floor和ceil



二分搜索树的floor和ceil

寻找45的floor和ceil

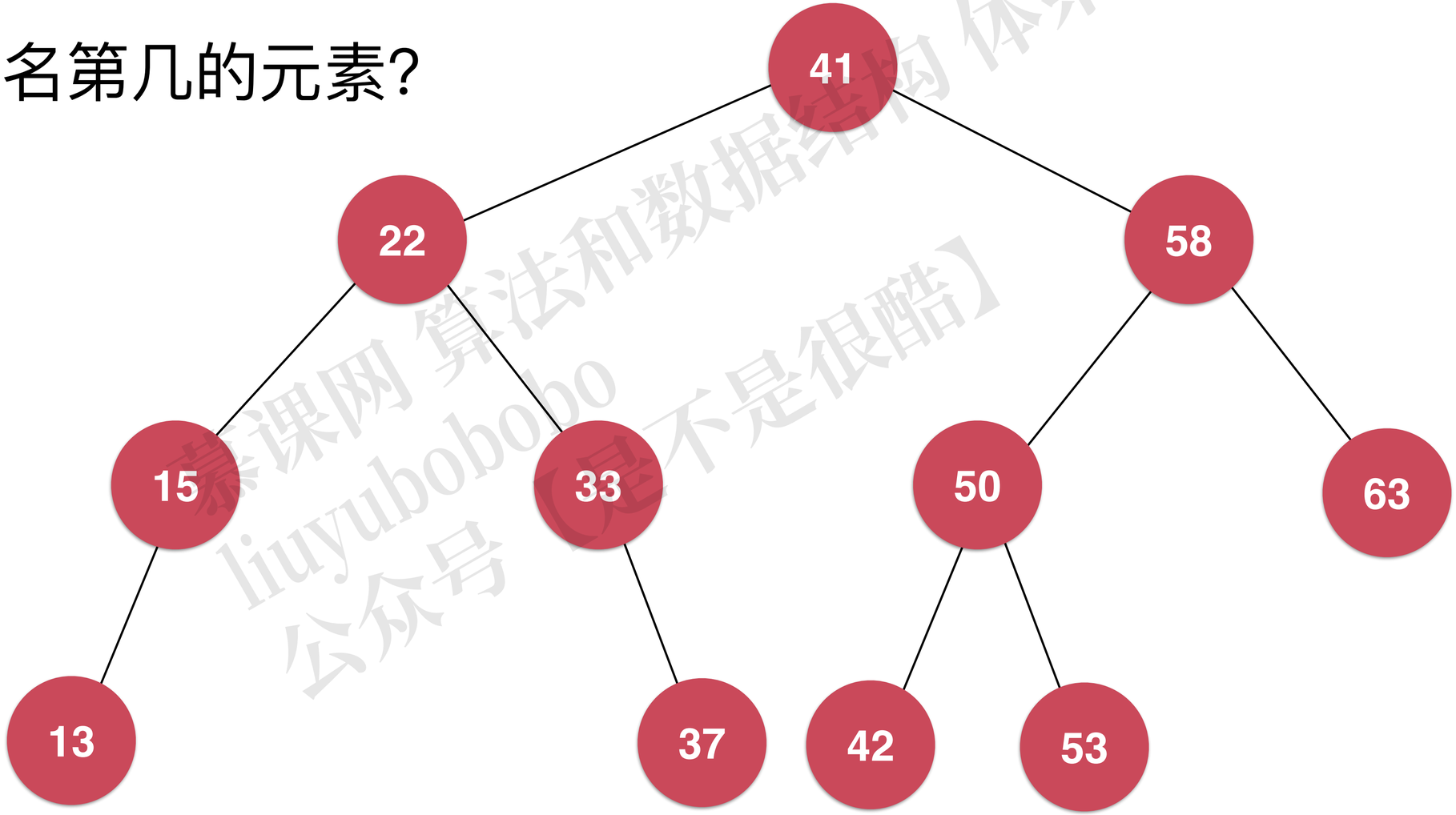


二分搜索树的顺序性

rank , select

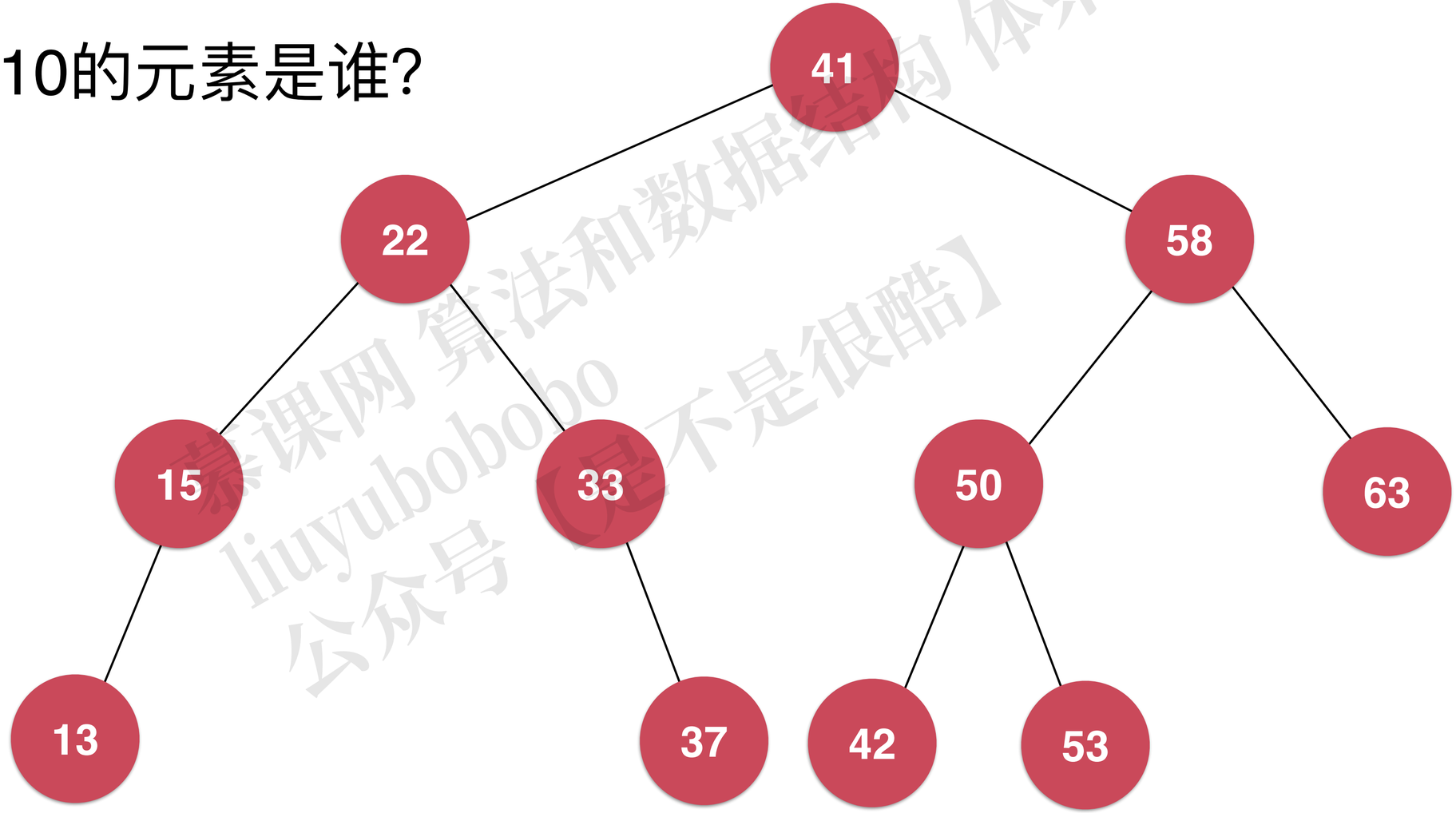
二分搜索树的rank

58是排名第几的元素?

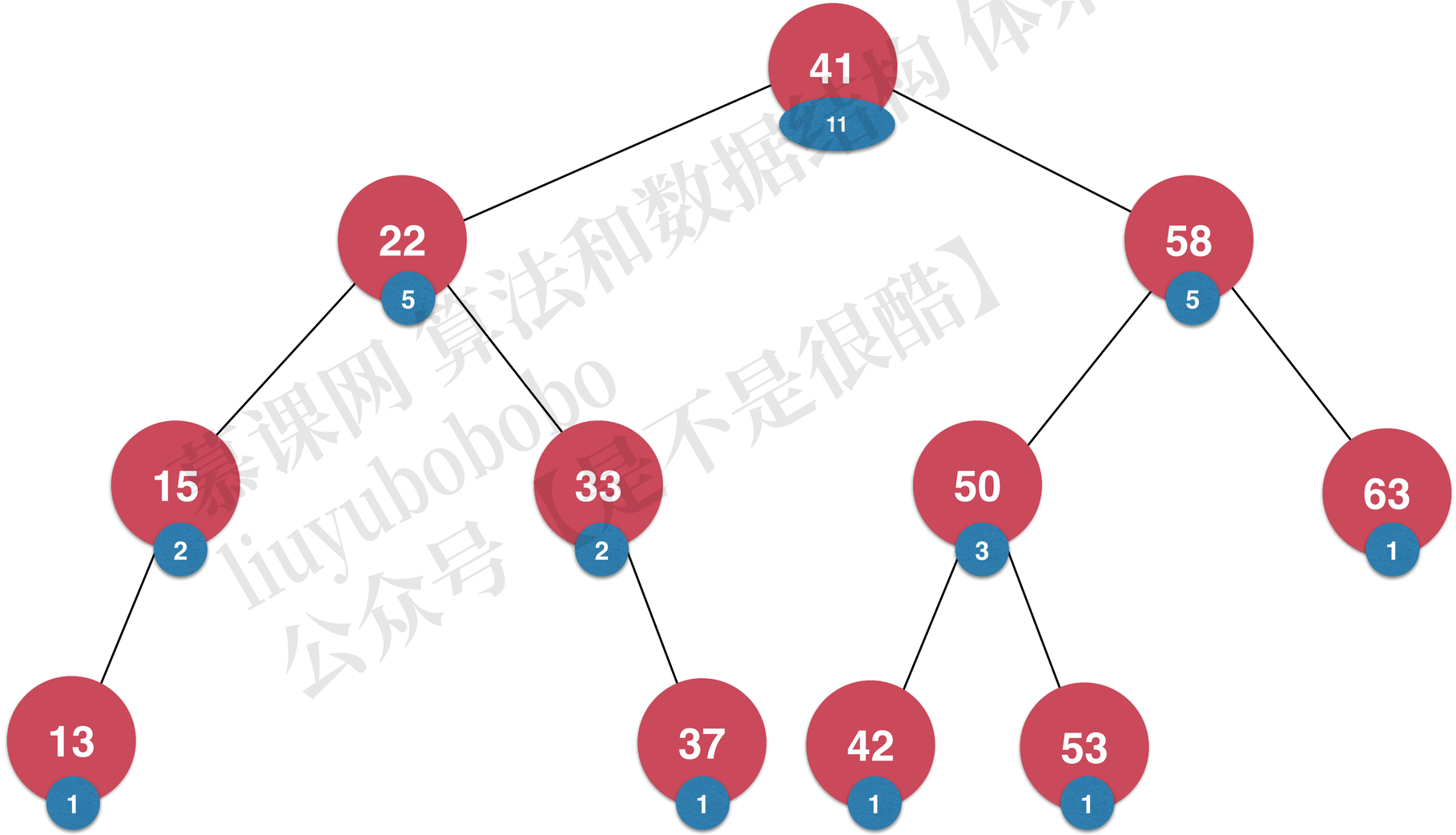


二分搜索树的select

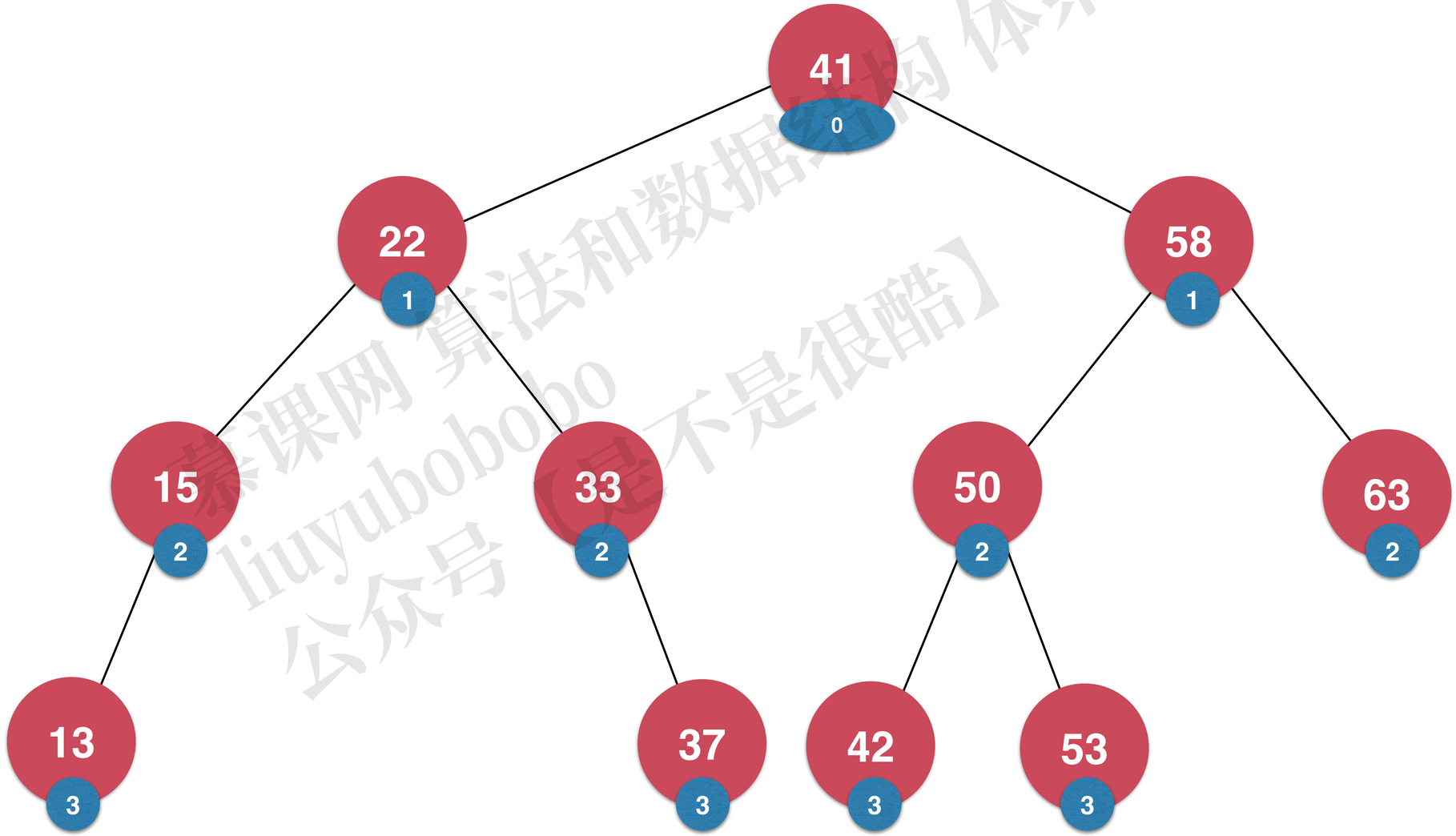
排名第10的元素是谁?



维护size的二分搜索树

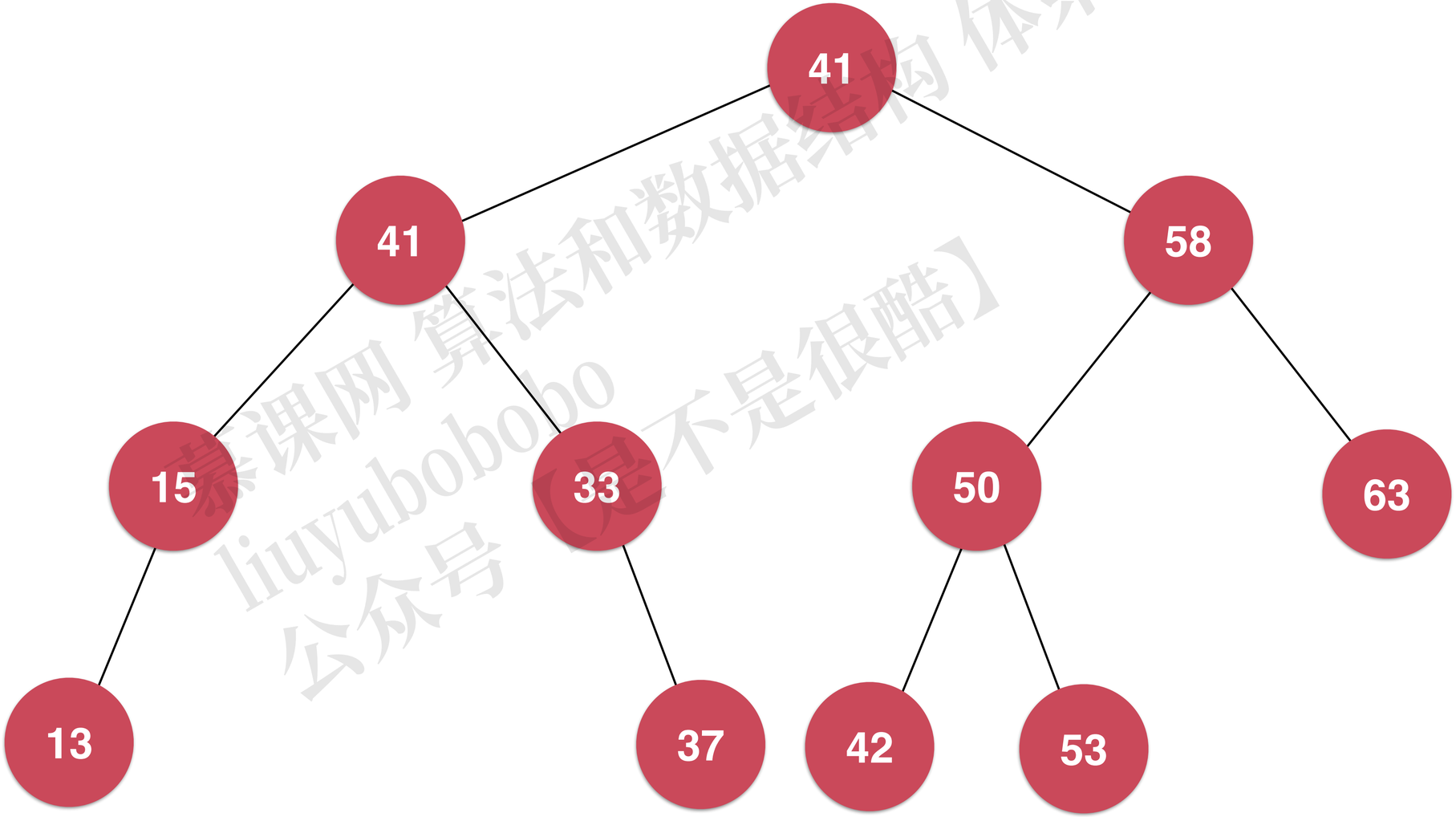


维护depth的二分搜索树

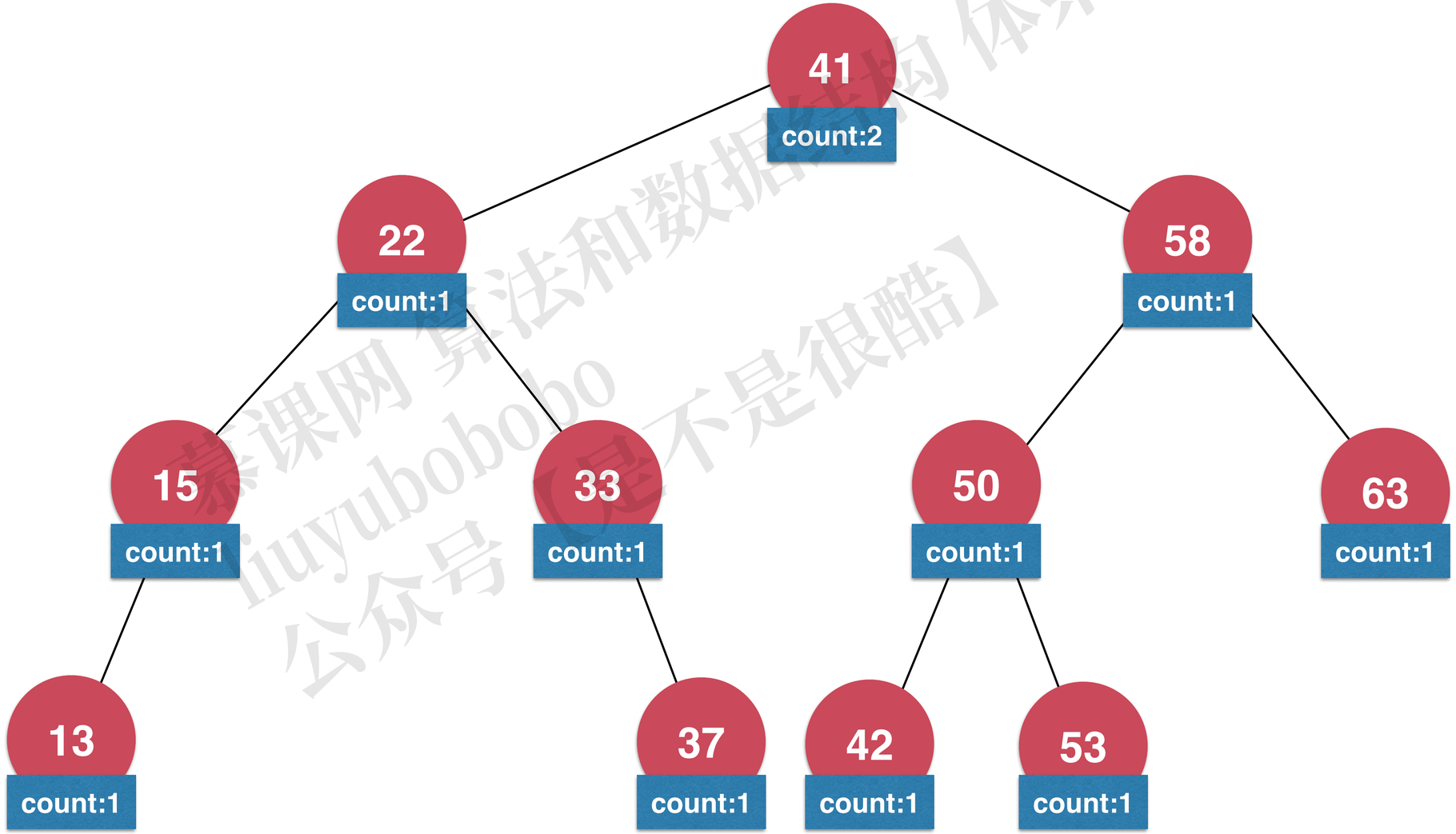


支持重复元素的二分搜索树

支持重复元素的二分搜索树



支持重复元素的二分搜索树



Leetcode上和二分搜索树相关的问题

二分搜索树 Binary Search Tree

其他

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