Quantum Blackjack: Formalism and Rules

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Abstract

The aim of this document is to show the Quantum Blackjack rules. To do so, the rules of the *classical* Blackjack are explained. After this, the Quantum Mechanics of the new Quantum Cards is introduced: The state space, measurements and entanglement of states/quantum cards. Finally, the new rules of the Quantum Blackjack are presented, showing where they differ from the classical ones.

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1 Classical Game

The game is played with the standard 52 card deck, between the dealer and the player. The main **objective** for the player on each round is to get a count on the cards as close to 21 as possible, without going over 21 (and higher than the dealer) [1]. The values of each cards are the following:

- Pip cards: Their pip value
- Ace: 1 or 11, depending on what the player wants.
- Face cards (King, Queen, Jack): 10

1.1 Betting

The betting must be done before starting each round. The betting is done by using casino chips. After each hand, the possible outcomes of the bet are [2]:

- **Lose:** The player's bet is given to the dealer.
- **Win:** The player wins as much as he has bet. If the bet was of 10 €, the player keeps those 10 €, and also wins another 10 €.
- **Blackjack:** The player wins 1.5 times the bet. If the bet was of 10 €, the player keeps those 10 €, and also wins another 15 €.
- **Push:** There is a draw. The player keeps his bet (he doesn't lose any money).

1.2 Gameplay

First of all the dealer shuffles the cards. Afterwards, the dealer deals one card face up clockwise to each player (in our case to just one player), and then to himself face down. Then deals again, now all cards face up. In the end, players must have two cards face up, and the dealer one card faced up, and the other one down [1].

1.2.1 Natural/Blackjack

A **natural** or **blackjack** happens when after dealing cards one player has 10-valued card and an ace (the count will be 21). The player with said cards automatically wins (except if the dealer also has a blackjack), and wins 1.5 times the bet. When both player and dealer have a blackjack, then there is a Push [2].

1.2.2 Player Turn

After the initial dealing, the player can make the following decisions [1, 2]:

- **Hit:** Ask for another card, in order to get closer to 21.
- **Stand:** Not ask for another card.
- **Bust:** The sum of the cards is over 21. In this case the player loses.

For example, the player may ask for more cards (hit) until the hand is strong enough to go up against the dealer (and then stand), or goes bust.

Note. As seen in [1], there are other actions that the player can make. They can be implemented in future developments.

1.2.3 Dealers turn

After the player has played, the face down card is turned up. Now, the different options are:

- If the count is lower than 17, the dealer takes a card.
- If the count is 17 or more, the dealer must stand.
- If the count is 21 (a blackjack) after turning up the card, then the dealer wins, and every player without a blackjack loses.

1.2.4 Reshuffling

After every bets are settled, the dealer gets every card used in the round and places them at the side. Then the next round can begin. When there are no cards from the deck, the dealer shuffles again and the game continues.

1.2.5 End of the game

The game ends when the dealer or the player loses all their chips.

2 Quantum Game

2.1 Quantum Cards

Instead of classical cards, now the game is played with *Quantum Cards*. Cards will be elements of a state space, and will be denoted by a ket $|\psi\rangle$. This space is generated by a set of states, the *deck base*, which are characterized by the *suit* and the *rank*:

$$|\text{suit, rank}\rangle \qquad \text{with} \qquad \begin{cases} \text{suit} = \clubsuit, \diamondsuit, \heartsuit, \spadesuit \\ \text{rank} = \text{A}, 2, 3, 4, 5, 6, 7, 8, 9, 10, \text{J}, \text{Q}, \text{K} \end{cases} \tag{1}$$

These states are called *deck states*. For example, one can write $|\clubsuit, 9\rangle$. This states are the analogue of the classical cards, as in both of them the suit and the rank are known.

Now, each $|\psi\rangle$ can be written as a linear combination of this states, hence they are a **superposition** of states. For instance, one example of a state could be

$$|\psi\rangle = c_1 |\diamondsuit, 2\rangle + c_2 |\spadesuit, \text{K}\rangle \qquad |c_1|^2 + |c_2|^2 = 1 \tag{2}$$

2.1.1 Deck base measurements

As it can be seen, in general <u>on quantum cards the value of the suit and the rank are not know</u>, as opposed to the classical cards. To play blackjack, one needs to know the rank of the card. Therefore, <u>One must apply the postulates of Quantum Mechanics regarding measurement in order to arrive to a state of the deck base</u> [3].

To do this, one can define a **card operator**, whose eigenvectors are the deck states. In this way, the square modulus of the coefficients that accompany the deck states are the probabilities of measuring said card.

2.1.2 Entanglement of quantum cards

Consider two states, each one of them a linear combination of two deck states. By applying quantum gates, one can produce an entanglement between both states. This is better seen with an example:

Example 1. Consider the following general states (s and k mean the suit and the rank):

$$|\psi_1\rangle = c_1 |\mathrm{s}_1, \mathrm{r}_1\rangle + c_2 |\mathrm{s}_2, \mathrm{r}_2\rangle \qquad |\psi_2\rangle = c_3 |\mathrm{s}_3, \mathrm{r}_3\rangle + c_4 |\mathrm{s}_4, \mathrm{r}_4\rangle \tag{3}$$

To clarify the operations one can make this change of notation

$$|\mathrm{s}_i, \mathrm{r}_i\rangle = \begin{cases} |0\rangle & \text{if} \quad i = 1, 3 \\ |1\rangle & \text{if} \quad i = 2, 4 \end{cases} \tag{4}$$

Therefore, the state one takes into acount are

$$|\psi_1\rangle = c_1 |0\rangle + c_2 |1\rangle \qquad |\psi_2\rangle = c_3 |0\rangle + c_4 |1\rangle \tag{5}$$

Now, one can apply a CNOT gate, but to do so one has to work with $|\psi_1\rangle \otimes |\psi_2\rangle$

$$|\psi_1\rangle \otimes |\psi_2\rangle = c_1c_3 |00\rangle + c_1c_4 |01\rangle + c_2c_3 |10\rangle + c_2c_4 |11\rangle \tag{6}$$

and the outcome will be

$$|\psi\rangle = c_1c_3 |00\rangle + c_1c_4 |01\rangle + c_2c_4 |10\rangle + c_2c_3 |11\rangle \tag{7}$$

Except if $c_1 = 0$, $c_2 = 0$, or $|c_3|^2 = |c_4|^2$ the final state will be entangled. Therefore, if measurements are performed on the suit and rank of the first "card", the state after the measurement will affect the probabilities of the measurement on the other card.

2.2 New rules

Taking into account the new quantum nature of the cards, the classical rules explained in Section 1 must be modified. This section covers these changes. Whatever is not specified here will remain as in the classic rules.

2.2.1 Dealing process

During the dealing process, what is given to player are states $|\psi\rangle$ of the cards state space. These states must be a linear combination of two states (this is what is needed to produce the entanglement, as explained in Section 2.1.2). As the cards are turned up, the player knows the expression of said states.

An important remark is that in a state $|\psi\rangle$, in general the number of the card is not known. Here enters the process of **measurement**, which will be used later on.

Card's given to the dealer are states from de deck base.

2.2.2 Player turn

After the card/states are dealt, players will have the opportunity of making a measurement on each of their cards.

Example 2. Suppose that the player has been given the following cards

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}(|\spadesuit, \mathrm{Q}\rangle - |\diamondsuit, 4\rangle) \qquad |\psi_2\rangle = \frac{1}{2}(|\heartsuit, 4\rangle + \sqrt{3}\,|\clubsuit, \mathrm{A}\rangle) \tag{8}$$

Then the player knows that:

- On the first card, there is 50/50 probability of getting a Queen (value of 10) and a 4
- On the second card, there is a probability of 1/4 of obtaining a 4 and 3/4 of obtaining an ace.

Because of Quantum Mechanics, the player does not have the Blackjack guaranteed. Instead, there is a possibility of 3/8 of obtaining ait. Moreover, this is only one possible outcome.

With this one can see that the Quantum Blackjack adds another layer of probabilities into the game.

After receiving the two cards, the player now has two options, hit or stand. After standing, the measurement process will take place, or the entanglement, which is explained below.

2.2.3 Producing entanglements

Each player will have a number of tokens. With them, the player will be able to produce an entanglement between two cards. To do so, after standing the player will choose which cards will be entangled. A quantum circuit is applied to both of them, obtaining an entanglement between cards.

Following the entanglement the measurement process takes place. Now, the measurement of the first card will affect the result on the second one: the probabilities will change depending on the outcome on the first cards, enhancing the randomness of the game.

2.2.4 Reshufling

The cards/states that are taken to a side will be the ones that are left after the measurements, i.e., the states to which the state collapses after measurements. The rest of the states will go back to the deck, so that new superpositions can be made.

When there are no more states of the base on the deck, one can build again states with all the deck basis.

References

[1] *Learn to play Blackjack*. URL: https://bicyclecards.com/how-to-play/blackjack. (Last visited: 16-9-2025).

[2] *Official Blackjack Rules*. URL: https://officialgamerules.org/game-rules/blackjack/. (Last visited: 16-9-2025).

[3] Claude Cohen Tannoudji, Franck Laloë, and Bernard Diu. *Quantum Mechanics, Volume 1: Basic Concepts, Tools, and Applications*. 2nd ed. Weinheim: Wiley-Vch, 2019. ISBN: 978-3-527-34553-3.