Chapter 2

Standard Deviation

Introduction

Suppose you are standing in front of a dart-board. You have a large number of darts and you throw the darts one at a time, trying to hit a 1 cm thick vertical line drawn on the dart-board. Since you have a very good aim, let us say that 45% of the darts hit the line. This then means that you miss the line 55% of the time; with a significant number of these misses being between .5 to 1.5 cm either to the left or to the right from the center of the line, and with a smaller number of misses being between 1.5 to 2.5 cm from the center line.

Is there some mathematical way of characterizing how good you are at this game? What, for example, is the probability of missing the line by 1 cm? The statistical analysis of random fluctuations in data can help answer these questions. The word "statistical" implies that a relatively large set of similar measurements of a given physical quantity is available. The random fluctuations in the data can be measured with the use of a mathematical term called the "standard deviation."

Suppose you collect data on a large number of throws, separating the data into categories (bins): The number of darts on the line, the number within .5 to 1.5 cm from the center, the number within 1.5 to 2.5 cm, etc. A plot of this data with the dart positions on the x-axis and the number of darts hit within each bin plotted on the y-axis is called a histogram (see Figure 2.0.1). The envelope of this graphical data set is bell shaped, and is called a Gaussian or a Normal distribution curve.

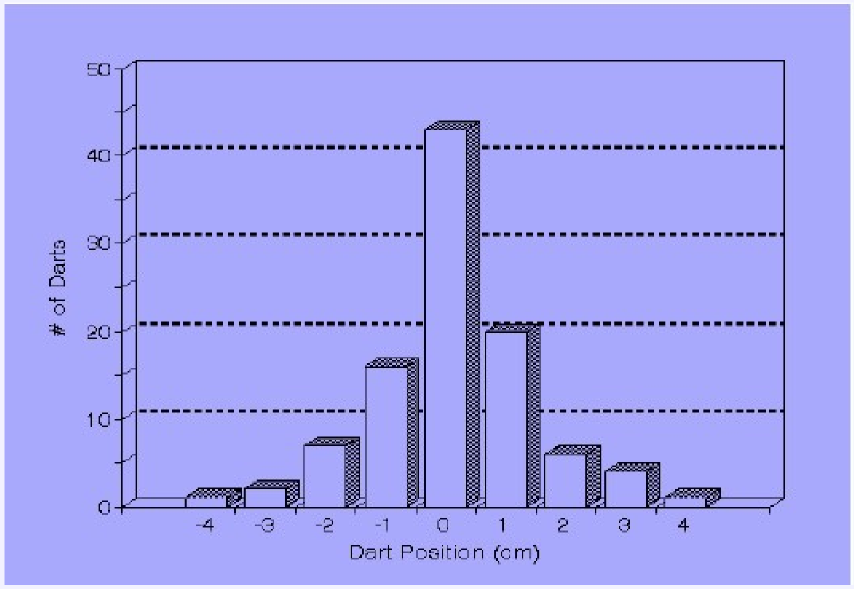

Figure 2.0.1: A sample histogram for the number of darts binned by distance from the centerline.

The "standard deviation" is a measure of the spread or width of the histogram data. A small standard deviation means that there is a small spread in the data about the central mean value and implies that the data cluster closely about one value. That is, there is a high degree of precision in the measurements.

The area beneath the curve, or below a part of the curve represents the probability of occurrence. For example, the area beneath the curve between plus and minus one standard deviation from the mean represents a 68% probability of your next throw falling within this range. The area beneath the curve between plus and minus two standard deviations from the mean represents a 95% probability of your next throw falling within this range.

In this experiment, you will study the use of the "standard deviation" in the statistical analysis and probability involved with flipping pennies.

Objectives: Experimental Objectives

The goal of the experiment is to determine how the mean, standard deviation, and standard deviation of the mean depend on the amount of data (number of samples) taken. You should also consider how well the data fit to a normal distribution.

2.1 Student Outcomes

In this experiment, you should learn

1. the formulas for and the roles of the mean, the standard deviation, and the standard deviation of the mean in the statistical analysis of data containing random errors,
2. how to create histograms and scatter plots,
3. how to include trendlines on scatter plots, and
4. why more data is always better.

2.2 Pre-Lab Work

- Define the following, both in a sentence and with a mathematical formula: mean, standard deviation (sometimes called the standard deviation of a single observation), and standard deviation of the mean. Be sure to describe the difference between these two terms.
- Define the following: histogram, probability, probability distribution (Normal distribution).
- What does the total area under the Normal distribution curve represent?
- How should the axes of the histogram of the data you will take for this lab be labeled? Give a very specific scale for the x-axis. (Hint: Read the procedure below.)
- Make a sketch of an inverse function, like $y = 1/x$.

2.3 Experimental Procedure

Each individual will receive 20 pennies and will then simultaneously toss all of the pennies. Count the number of heads, and repeat 25 times. Each individual will then have collected 25 pieces of data.

After doing this, you will 25 "counts" of the number of heads among that 20 coins. Your lab partner will also have 25 "counts". The two of you together have access to 50 "counts". You, your partner, and the pair of people near you will together have 100 "counts" among you. You will therefore be able to consider a histogram for your $N = 25$, another histogram for your $N = 50$, etc. After you consider each of these histograms, you will consider how the shape a histogram changes as N increases.

2.4 Analysis

- Calculate the mean, the standard deviation and the standard deviation of the mean for your first ten tosses and for all 25 tosses. Show your calculations.
- Make a histogram plot (by hand in your note book) for your 25 tosses. On this histogram superimpose a sketch of your best guess of the corresponding Normal distribution curve.
- How well does your data fit a normal distribution curve? Explain the reasons for any large discrepancies.
- Obtain histograms and calculations of the mean, the standard deviation and the standard deviation of the mean for the following data sets:

  1. your lab group,
  2. about 1/2 of the class, and
  3. the entire class.

  These histograms can be obtained with the use of a computer program, provided by the instructor.

- Draw graphs of: the mean, the standard deviation, the standard deviation of the mean, versus the number of data entries. Use the standard deviation of the mean as the error bars for both the mean and the standard deviation, draw these error bars on these two graphs.

- Describe how the values of the mean, the standard deviation, and the standard deviation of the mean, change as the number of data items in the set increases. What can you infer about the accuracy and about the precision of the data as more and more observations are made?

- Show that the standard deviation of the mean varies inversely as the square root of the number of data items in the sample. How well does the data in this lab agree with this prediction? (Make a graph of the standard deviation of the mean versus the square root of the number of data items in the sample.)

2.5 Questions

1. What is the probability of flipping the 20 pennies and getting 5 heads, or 8 heads, or 10 heads, or 15 heads? Answer this question by analyzing your data, the entire class' data and the normal distribution curve fit to your data. Explain any differences between these sets.

Hint. Consider the percentage of total throws that produce each number of heads.

Hint. The percentage of total throws that give 5 heads is the number of throws that gave 5 heads divided by the total number of throws.

Hint. You might consider this percentage (of throws that produces 5 heads) for $N = 25$ versus $N = 50$ versus other values of N to explore the stability of this percentage. (Remember that more data is better.)

2. What percentage of your individual readings fall within plus or minus one standard deviation, two standard deviations? Compare your answer to the theoretical answer from a normal distribution curve. What are the percentages for the class data?

Hint. Hint 2.5.1.2 explains finding a percent of throws.

Hint. If the mean is 10.12 and the standard deviation is 2.74, then "within plus or minus one standard deviation" implies that you should consider the percentage of times you get between 7.38 and 12.86. This is larger than 7 and smaller than 13. So, consider how many throws gave 8, 9, 10, 11, or 12 heads.

Hint. To compare to the theoretical answer, you can figure out the actual mathematical formula, but you can also use the "Gaussian Prediction" column in the Excel spreadsheet for the corresponding range.

3. Does the height of the histograms change as a function of the number of trials? If so, how?

4. How does the width at 1/2 the maximum height for the histograms change as a function of the number of trials? Label this width on your histograms. Is this width a reasonable estimate of the standard deviation?

Hint. To get the "full-width at half-maximum" on your histogram, find the mean on the horizontal axis and note the vertical value of that peak (read the vertical axis). Half this peak value and then find the smallest number of heads thrown that is at least this half-value and the largest number of heads thrown that is at least this half-value.

For example, if the mean of your data is 10.12, then perhaps you had 6 throws that resulted in 10 heads (the number of heads closest to the mean). Half of the 6 throws is 3 throws. So, you will likely have find that you have fewer than 3 throws for 6-or-fewer heads but more than 3 throws for 7-or-more heads. On the other end, you will likely find that you have fewer than 3 throws for 14-or-more heads but more than 3 throws for 13-or-fewer heads. This says that the "full-width at half-maximum" is the range from 7 heads to 13 heads, which is a width of 6-heads thrown. You should consider how this "full-width at half-maximum" compares to your standard deviation.

(Please remember that you not actually throwing heads, you are throwing coins. Do not decapitate your lab partner.)

5. Does the standard deviation of a single observation and/or does the standard deviation of the mean change significantly as the number of tosses increase? What can you infer about the accuracy and about the precision of the data as more and more observations are made?

2.6 References and Suggested Readings

[1] Meyer, *Data Analysis for Scientists and Engineers.* John Wiley, 1975; p.19-48, 223-253

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A PDF version might be found at StDev.pdf