Lab 14

Scaling Laws Among Biological Specimens

Procedure You will be given many animal skulls. Measure some dimension that can be measured on all skulls. Verify graphically (make the plot of mass versus your measured dimension) if this measurement can be reliably correlated to the mass of the skull. We will assume that the mass of the skull is directly proportional to the overall size of the animal. This technique allows mammalogists to gauge the size of animals when they can only find partial skeletons.

Analysis When you fit this data with a trendline, the relationship might be the familiar linear relationship, or it might instead be something else. Some possibilities are:

- Linear $y = mx + b$. In this case, you should plot the skull mass versus your measurement, determine the slope and intercept and the relevant regression data. Discuss why the intercept might have the value that it does.
- Geometric $y = ax^b$, where the coefficient a and the exponent b are to be determined. In general, it is useful to fit your data this way first. If you find b is close to an integer, then try a second fit with the appropriate polynomial. For example, if $b \approx 1$ then use a linear fit.
- Exponential $y = ae^{bx}$, where the coefficient a and the base b are to be determined.

Excel can fit to any of these and provide you with the equation for the trendline. However, Excel cannot provide the uncertainties unless the graph is linear. If one uses logarithms, then we can make a power-fit trendline or an exponential-fit trendline plot as a line so that Excel can provide the slope and intercepts with uncertainties. This will allow us to create a reliable mathematical relationship between the variables. In general, it is true that linear regression is significantly easier than a general functional regression.

If your power fit is better then we can apply the logarithm to the geometric relation:

$$\begin{aligned}
\ln y &= \ln\left(ax^b\right) \\
\ln y &= \ln a + \ln x^b \\
\ln y &= \ln a + b \ln x \\
\ln y &= [b] \ln x + [\ln a]
\end{aligned}$$

In this case, it is useful to make a "log-log plot" of $\ln y$ versus $\ln x$, which should be a straight line. The slope of the log-log relation is b, the exponent, and the intercept of the log-log relation is $\ln a$, so that $e^{(\ln a)} = a$ the coefficient.

If your exponential fit is better then we can apply the logarithm to the exponential relation:

$$\begin{aligned}
\ln y &= \ln\left(ae^{bx}\right) \\
\ln y &= \ln a + \ln\left(e^{bx}\right) \\
\ln y &= \ln a + bx \ln(e) \\
\ln y &= [b \ln e]\, x + [\ln a]
\end{aligned}$$

In this case, it is useful to make a "semi-log plot" of $\ln y$ versus x, which should be a straight line. The slope of the semi-log relation is b (because $\ln e = 1$) and the intercept of the semi-log relation is $\ln a$, so that $e^{(\ln a)} = a$ the coefficient.

In biological relationships, it is common to find an exponential relationship with a base of $e = 2.717182\cdots$;

this is why we find it useful to use the "natural logarithm."

Once you determine if you have an exponential or a geometric relationship, decide if there are any "outliers," which animals those are, and justify why those data points might reasonably be excluded. Re-fit the data after excluding the outliers.

A PDF version might be found at scaling.pdf (59 kB)

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