Lab 4

Newton's 2^{nd} Law on a Linear Track with the Sonic Ranger

Experimental Objectives

In this experiment, we will assume that Newton's first law is true and focus on Newton's second law.

- By measuring
  - the velocity versus time for a cart being pulled down a track and
  - the applied force that is pulling it,

  we can plot the acceleration versus the force and verify the validity of Newton's second law of motion: $\vec{F}_{\text{net}} = m\vec{a}$.

Introduction to Forces

Forces are related to the natural motion of bodies, where one object can affect the motion of another object. That is, forces are interactions between objects affecting their motion. Although the famous Greek philosopher Aristotle claimed that a force was necessary to *maintain* any motion, careful analysis by Italian physicist Galileo Galilei in the mid-17^{th} century and by Sir Isaac Newton, a British mathematician and physicist (1642-1727), eventually distinguished the effects of friction and allowed Newton to create a mathematically consistent theory of motion. These concepts were published in Newton's book "Mathematical Principles of Natural Philosophy" in 1687, for which (among other accomplishments) Newton is regarded as one of the greatest scientists of all time.

All forces can be placed in one of two main categories. First, there are natural (or fundamental) forces like the gravitational force, the electromagnetic force, or the nuclear forces. The gravitational force is a force on a body by another body (like the Earth), this force is an interaction between their two masses. The electromagnetic force is an interaction between the charges of two bodies. These forces may act on an object without any direct physical contact between the two bodies. This type of force is sometimes called an "action at a distance" force. All other forces are in a second category called "contact forces."

Newton's First Law:

> If there are no forces acting, then objects will remain at rest or, if not at rest, will maintain their velocity.

If this is true, then we can study the forces acting on a body based on the motion of the body, specifically through the change in the velocity of an object.

Newton's Second Law: Not only is a force necessary to change the motion (to cause an acceleration), the amount of acceleration that a force causes is predictable and is inversely proportional to the mass. The same sized force causes a small mass to accelerate a lot and a large mass to accelerate a little. This is expressed by the equation:

$$\vec{F}_{\text{net}} = m\vec{a}.$$

The net force, $\vec{F}_{\text{net}}$, is the vector sum of all forces acting on an object. If we have an extended object (such as a weight hanging off of a table, but connected to a cart that is on the table), then we need only consider forces that are "external" to the system: So long as both objects accelerate at the same rate, we do not need to consider the "internal" tension that the string exerts between the connected bodies.

Newton's Third Law: Inherent in the description of a force is that it is an interaction between objects: there must always be two objects that interact. These objects exert equal and opposite forces on each other. That is,

> If there is a force exerted on object 1 by object 2, then there is necessarily and simultaneously a force exerted on object 2 by object 1 that is equal (in magnitude) and opposite (in direction) to the original force.

Remember that these two forces are on different objects and that the two bodies in direct contact exert forces on each other. Remember then that if there is contact between the object (any part of the system) and anything else then there is an outside force on the object (system) and that if there is no contact (the two bodies break contact) then there is no force.

4.1 Pre-Lab Considerations

- Based on your understanding of Subsection 4.2.1, draw a free-body force diagram for the cart and for the hanging mass.
- You should be prepared to derive an equation for the acceleration of the system, in terms of, the mass of the cart and the hanging mass, while assuming that the cart has no friction with the track. (Hint: There is only one force accelerating the system.)

4.2 Procedure

4.2.1 The Experimental Setup

Materials A low-friction linear track with a wheeled cart and a pulley at one end of the track. Weights that can ride in the cart without jostling. A string connecting the cart to a light-weight support for small masses, which sits over the pulley allowing the masses to fall vertically while pulling the cart horizontally. A "sonic ranger" that uses sonar to measure the position, velocity, and acceleration of the cart.

- A low-friction linear cart and track will be used, this reduces the friction between the cart and the track.
- A string will be connected to the cart and a known mass will be hanging from the end of the string (and over a pulley). The hanging mass will exert a constant horizontal force on the cart as the mass falls all the way to the floor. This gives a constant acceleration to the cart.
- The sonic motion sensor will be used to measure the position of the cart as a function of time.
- The carts and tracks need to be handled with care. Scratches can add friction to the system.

4.2.2 Procedure

- If the cart is given an initial push (without the hanging mass and string attached) then the cart should travel with a constant velocity down the horizontal track, if there are no other forces acting on the cart. Carry out a couple of constant velocity runs on the track, to check for the effects of friction and to see how level the track is. The track may need a level adjustment. Do runs in both directions. Maybe the track can be tilted so that the friction is countered by the tilt of the track.

  It might help to review Exercises 3.2.1, 3.2.2, and 3.2.3 from the Constant Acceleration lab.

- Connect a string to the cart and run it over a pulley. Measure the height of the string at both ends of the track, to ensure that the string is as level as the track.

Exercise 4.2.1. If the pulley has a very large wheel or is set so that the string is low at the cart, but high at the pulley, then the force pulling the cart is not horizontal. Draw the free-body diagram for the cart in this case. Comment on if this increases, decreases, or does not affect each of the other forces (F_{normal}, F_{gravity}, F_{friction}).

Hint. If the tension pulls up, then the normal force does not need to support all of the weight.

Hint. If the tension pulls slightly up, is there a convenient way to find then angle at which it pulls? Will that angle depend on how close the cart is to the pulley?

- The hanging mass should be much less than the mass of the cart. Use a small plastic cup to hold the hanging masses. Measure the mass of this cup. The total mass of the system must be kept constant for all parts of the experiment. The hanging mass and the mass of the cart should vary, but their total must be kept constant, by moving small mass amounts from the cart to the hanging cup. Record the mass of the cart, the hanging cup mass, and the extra masses which are to be transferred from the cart to the cup.

Exercise 4.2.2. If the hanging mass is not "much less" than the mass of the cart, then the acceleration will be very large and the cart will move too quickly.

Your total mass should include the mass of the string because it is also being accelerated. As an interesting thought experiment, you might notice that the amount of string that hangs off the pulley is contributing to the mass of the basket (the amount pulling the cart). But this changes as the cart moves! Without using calculus it is impossible to include this consideration, so we *hope* this is a small effect. Do you have a way of ensuring that this is a small effect?

Hint. How many significant digits do you have in the mass of the cart? Is the mass of the basket large enough to be a *significant* effect in the overall mass?

Hint. How many significant digits do you have in the mass of the cart? Is the mass of the string large enough to be a *significant* effect in the overall mass?

Hint. What percentage of the total mass of the system is the mass of the string? What percentage of the mass of the cart is the mass of the string?

Note The Pasco Capstone interface is also used in Lab 3, Lab 5, and Labs 7–8. You should start becoming familiar with the hardware and software.

- Take data with Capstone and the motion sensor as the cart travels with constant acceleration down the track. Determine the acceleration of the cart from a linear regression using the velocity vs time data (a linear fit line in Capstone). Record the acceleration value and its uncertainty.
- Collect 7 data runs, where about 2-5 grams is transferred each time from the cart to the hanging mass. Determine the acceleration of the cart (and the uncertainty for the acceleration) for each of these 7 runs.

4.3 Analysis

- In Excel, make a graph of the acceleration of the system (y-axis) versus the weight (mg) of the *hanging* body (x-axis). You should include at least 7 data points. Carry out a linear regression for this data set. Quote the slope and intercept values, their uncertainties, their p-values, and the R^2 value. Show a sample error bar (on the graph) for at least one of the points of this graph.
- Derive (show it completely) an equation for the acceleration of the system versus the weight of the hanging body. Plot this theory equation on your graph (as a second series, a line but no points).
- Compare your graph to the predicted theoretical equation, that is compare the values of the slopes and intercepts.

Exercise 4.3.1. Note: In almost every lab you will be comparing a theoretical equation to the equation of a line and interpreting what the slope and intercept mean.

What is the physical significance of the slope and of the intercept from the graph? That is, what physical quantity does the slope of this graph equal?

Hint. It should help to recall that Newton's second law looks very similar to the generic equation of a line: $y = mx + b$.

- In many mechanics experiments, there may be deviations from the expected or theoretical results because of the effects of friction. Frictional forces are sometimes difficult to take into consideration. If there are deviations between your results and the predicted theory then try to distinguish whether they are caused by a tilt of the track, friction between the cart and the track or the friction between the string and the pulley.

Exercise 4.3.2. What might be expected in the results from these different systematic effects? That is, would the slope be expected to increase or decrease slightly because of the effects of friction? Would the slope be expected to increase or decrease slightly because of the effects of an unlevel track?

- When designing experiments, it is important to keep control parameters; in this case a parameter which is kept constant.

Exercise 4.3.3. What parameter is held constant in this experiment? Is there an obvious reason for keeping this constant?

4.4 Questions

1. Why is it important to keep the total mass of the system constant? What would happen if the total mass of the system were not held constant? If one simply added mass to the hanger without keeping the system's mass constant, how would the data appear on the graph of the acceleration vs mg?

2. What would happen if the track was not level? If the beginning end of the track is higher, how would the acceleration of the system be affected?

Hint. This describes the cart going downhill during the experiment.

Hint. Would your measured acceleration be equal to, larger than, or smaller than expected?

3. If your group has a discrepancy between the results and the theory, could friction be used to explain why your results differ from what is expected? Explain how.

Hint. Would friction tend to make the measured acceleration larger then or smaller than the expected value?

Hint. Is your result larger than or smaller than the expected result?

Hint. Would tilting the track one way or the other help overcome (or counter-act) the effect of friction? If so, which way would you tilt the track to do so? Is there a way to gauge how much the track would need to tilt?

4. If your group has a discrepancy between the results and the theory, would a tilt in the track explain the discrepancy? Show this calculation.

Hint. Would you need the cart to be going uphill or downhill to explain whichever discrepancy you see?

5. How would the cart's acceleration change, if at all, if the cart was given an initial push?

Hint. Recall Exercise 3.3.5.

6. What are the two greatest sources of uncertainty in this experiment? Are they random or systematic errors? Be specific and quantify your answer.

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A PDF version might be found at Newton.pdf (291 kB)