Elements of Physics Laboratory Manual

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Contents

A Experimental Uncertainties, Defining "Error"

Measurements are never exact. For example, if one apple is divided among three people, your calculator will tell you that each person has 0.3333333333 of an apple. A measurement of each slice will tell you two pieces of information: (1) how many 3s to keep and (2) how well you know the final 3. In this example, both 0.33 ± 0.01 and 0.33 ± 0.04 imply that the measurement is accurate to two decimal places, but the first implies that you trust the second 3 more than if you report it as the second number.

Some technical terms and their use in physics (which may differ from common use): **CAUTION: Because physicists "know what we mean", they are often sloppy with their language and use the words "error" and "uncertainty" interchangeably.**

accuracy: How close a number is to the true (but usually unknowable) result. This is usually expressed by the (absolute or relative) error

precision: How well you trust the measurement. This is vaguely expressed by the number of decimals, or clearly expressed by the size of the (absolute or relative) uncertainty

error: a number that expresses the accuracy. This can be expressed as the "absolute error", the "relative error", or the "percent error".

$$\text{absolute error} = |\text{true value} - \text{measured value}|$$

$$\text{relative error} = \left|\frac{(\text{true value} - \text{measured value})}{\text{true value}}\right|$$

$$\text{\%-error} = 100\% * \left|\frac{(\text{true value} - \text{measured value})}{\text{true value}}\right|$$

uncertainty: a number that expresses the precision of a measurement or of a computed result. This can be expressed as the "absolute uncertainty" (explained in Section A.2), the "relative uncertainty", or the "percent uncertainty".

$$\text{relative uncertainty} = \left|\frac{(\text{absolute uncertainty})}{\text{measured value}}\right|$$

$$\text{\%-uncertainty} = 100\% * \left|\frac{(\text{absolute uncetainty})}{\text{measured value}}\right|$$

difference: a number that expresses the consistency of a multiple measurements. This can be expressed as the "absolute difference", the "relative difference", or the "percent difference". You should notice that since we don't know *which* measurement to trust, we take the absolute difference relative to the average of the measurements (rather than choosing one measurement as "true").

$$\text{absolute difference} = (\text{measurement}_1 - \text{measurement}_2)$$

$$\text{relative difference} = \frac{(\text{measurement}_1 - \text{measurement}_2)}{\left[\frac{(\text{measurement}_1)+(\text{measurement}_2)}{2}\right]}$$

$$\text{\%-difference} = 100\% * \frac{(\text{measurement}_1 - \text{measurement}_2)}{\left[\frac{(\text{measurement}_1)+(\text{measurement}_2)}{2}\right]}$$

A.1 Writing an Analysis of Error

The conclusion of your lab report should be based on an analysis of the error in the experiment. The analysis of error is one of the most certain gauges available to the instructor by which the student's scientific insight can be evaluated. To be done well, this analysis calls for comments about the factors that impacted the extent to which the experimental results agree with the theoretical value (what factors impact the percent error), the limitations and restrictions of the instruments used (what factors impact the uncertainty), and the legitimacy of the assumptions.

Physicists usually use the phrase "sources of error" (or "sources of uncertainty") to describe how the limits of measurement propagate through a calculation (see Section A.3) to impact the uncertainty in the final result. This type of "error analysis" gives insight into the accuracy of the result. Section A.1.2 provides questions that can help you describe which of several measurements can most effectively improve the precision of the result so that you can gain insight into the accuracy of the result. The accuracy allows one to gauge the veracity (truth) of an underlying relationship, but precision allows you to gauge accuracy. Said another way, a small percent difference usually is used to imply a small percent error. Said another way, imprecise measurements always *seem* accurate.

A.1.1 Technically, Errors are not Mistakes

Your report should not list "human error" because most students misunderstand this term to mean "places I might have made a mistake" rather than "the limiting factor when using the equipment correctly." Section A.2 discusses measurement uncertainties as defined above.

In the example of the apple above, the fact that one person has 0.33 ± 0.04 of an apple does **not** reflect a "mistake" in the cutting, but rather reflects that the cutter is limited in their precision. What is important is to use the uncertainty to express how well one can repeated cut the apple into thirds. The absolute uncertainty of 0.04 is generally interpreted to say that most instances (roughly 68%, as explained in Section A.2.2) of the cutting of an apple in this way will result in having between 0.29 to 0.37 of an apple for any given slice.

When describing the cause of an error (difference from the theoretical value) or of an uncertainty (the extent you trust a number), you can usually categorize this source of error as a random error (a cause that could skew the result too large *or* too small) or as a systematic error (a cause that tends to skew the result in one particular direction).

Random Error: An environmental circumstances, generally uncontrollable, that sometimes makes the measured result too high and sometimes make it too low in an unpredictable fashion. Random errors may have a statistical origin – that is, they are due to chance. For example, if one hundred pennies are dumped on a table, on average we expect that fifty would land heads up. But we should not be surprised if fifty-three or forty-seven actually landed heads-up. This deviation is statistical in nature because the way in which a penny lands is due to chance. Random errors can sometimes be reduced by either collecting more data and averaging the readings, or by using instruments with greater precision.

Systematic Error: A systematic error can be ascribed to a factor which would tend to push the result in a certain direction away from the theory value. The error would make all of the results either systematically too high or systematically to low. One key idea here is that systematic errors can be eliminated or reduced if the factor causing the error can be eliminated or controlled. This is sometimes a big 'if', because not all factors can be controlled. Systematic errors can be caused by instruments which are not calibrated correctly, maybe a zero-point error (an error with the zero reading of the instrument). This type of error can usually be found and corrected. Systematic errors also often arise because the experimental setup is somehow different from that assumed in the theory. If the acceleration due to gravity was measured to be 9.52m/s^2 with an experimental uncertainty (precision) of 0.05m/s^2, rather than the textbook value of 9.81m/s^2, then we should be concerned with why the accuracy is not as good as the precision. This is most likely to mean that there is a significant systematic error in the experiment, where one of the initial assumptions may not be valid. The textbook value does not consider the effects of the air. The effects of the air may or may not be controllable, and the difference between the theory and the data may be (within appropriate limits or tests) considered a correction factor for the systematic error.

A.1.2 Considerations for the Error Analysis

In order to help you get started on your discussion of error, the following list of questions is provided. It is not an exhaustive list. You need not answer all of these questions in a single report.

1. Is the error large or small? Is it random or systematic? ¡ellipsis /¿ statistical? ¡ellipsis /¿ cumulative?
   (a) What accuracy (precision) was expected? Why? What accuracy (precision) was attained? If different, why?
   (b) Was the experimental technique sensitive enough? Was the effect masked by noise?
2. Is it possible to determine which measurements are responsible for greater percent error by checking items measured and reasoning from the physical principles, the nature of the measuring instrument, and using the rules for propagation of error?
   (a) Is the error partly attributable to the fact that the experimental set-up did not approximate the ideal that was required by the physical theory closely enough? How did it fail?
   (b) If a systematic error skews high (low), then is your result too high (low)? Is this a reasonable explanation? Is the size of the skew enough to explain the result?
   (c) What can be done to improve the equipment and eliminate error? How can the influence of environmental factors be diminished? Why is this so?
3. Is the error (deviation) in the experiment reasonable?

A.2 Finding the Precision of a Measurement

A.2.1 Uncertainty of a single measurement

A.2.2 Uncertainty of multiple, repeated measurements

Calculate or estimate the precision of a measurement by one or more of the following methods:

1. by the precision of the measuring instrument, and take into account any uncertainties that are intrinsic to the object itself;
2. by the range of values obtained, the minimum and/or maximum deviation (d);
$$d_i = |X_i - X_{\text{ave}}|$$
3. by the standard deviation, which is the square root of the sum of the squares of the individual deviations (d) divided by the number of readings (N) minus one;
$$\sigma = \sqrt{\frac{1}{(N-1)}\sum d^2} = \sqrt{\frac{1}{(N-1)}\sum_i |X_i - X_{\text{ave}}|^2}$$

4. by the standard deviation of the mean, which is the standard deviation divided by the square root of the number of readings;

5. by the square root of the number of readings ($\sqrt{N}$), if N is considered large;

If many data points were taken and plotted on a histogram, it would smooth out and approach the symmetrical graph typical of the binomial distribution (see the Figure 2.3 on page 15). This distribution and many others in statistics may be approximated by the gaussian distribution.

The standard deviation, σ, can be estimated from the above graph. It is a measure of the "width" of the distribution. For the case shown, the standard deviation has the value of five. The greater the standard deviation, the wider the distribution and the less likely that an individual reading will be close to the average value. About 68% of the individual readings fall within one standard deviation (between 45 and 55 in this case). About 96% of the readings fall within two standard deviations (between 40 and 60 in this case).

As more and more readings are taken, the effect of the random error is gradually eliminated. In the absence of systematic error, the average value of the readings should gradually approach the true value. The smooth curve above was drawn assuming that there was no systematic error. If there were, the graph would merely be displaced sideways. The average value for the number would then be say 55.

The distribution of many average (mean) readings is also gaussian in shape. Comparing this to the distribution for individual readings, it is much narrower. We would expect this, since each reading on this graph is an average of individual readings and has much less random error. By taking an average of readings, a considerable portion of the random error has been canceled. The standard deviation for this distribution is called the standard deviation of the mean (σ_m). For this distribution, 68% of the averages of the readings are within one standard deviation of the mean, and 98% of the average readings fall within two standard deviations of the mean.

The standard deviation of the mean tells how close a particular <u>average</u> of several readings is likely to be to an overall average when many readings are taken. The standard deviation tells how close an <u>individual</u> reading is likely to be to the average.

There is one case for which the standard deviation can be estimated from one reading. In counting experiments (radioactivity, for example), the distribution is a Poisson distribution. For this distribution, the standard deviation is just the square root of the average reading. One reading can give an estimate of the average, and therefore, give an estimate of-the standard deviation.

A.3 Propagation of Uncertainties

The previous sections discussed the uncertainties of directly measured quantities. Now we need to consider how these uncertainties affect the rest of the analysis. In most experiments, the analysis or final results are obtained by adding, subtracting, multiplying, or dividing the primary data. The uncertainty in the final result is therefore a combination of the errors in the primary data. The way in which the error propagates from the primary data through the calculations to the final result may be summarized as follows:

1. The error to be assigned to the sum or difference of two quantities is equal to the sum of their absolute errors.

2. Relative error is the ratio of the absolute error to the quantity itself. The relative error to be assigned to the product or quotient of two quantities is the sum of their relative errors.

3. The relative error to be assigned to the power of a quantity is the power times the relative error of the quantity itself.

These rules are not arbitrary, but rather they follow directly from the nature of the mathematical operations. These rules may be derived using calculus. These rules are written in mathematical terms, together with a few specific examples, in Appendix A.

Example:

The area and perimeter of a rectangular table are to be calculated. The table is measured to be 176.7 cm ± 0.2 cm along one side and 148.3 cm ± 0.3 cm along the other side. Because the perimeter is found by adding the sides, Rule 1 is used:

$$\begin{aligned} P &= (176.7\text{cm}) + (148.3\text{cm}) + (176.7\text{cm}) + (148.3\text{cm}) \\ P &= 650.0\text{cm} \end{aligned}$$

$$\begin{aligned} \Delta P &= (0.2\text{cm}) + (0.3\text{cm}) + (0.2\text{cm}) + (0.3\text{cm}) \\ \Delta P &= 1.0\text{cm} \end{aligned}$$

The perimeter is $\boxed{P = 650\text{cm} \pm 1\text{cm}}$. Because the area of the table is calculated using multiplication, Rule 2 is used to find the uncertainty. The area, then, is found to be (significant digits are underlined)

$$A = (176.7\text{cm}) \times (148.3\text{cm}) = \underline{2620}4.61\text{cm}^2$$

$$\begin{aligned} \Delta A &= \left(\frac{.2\text{cm}}{176.7\text{cm}}100\%\right) + \left(\frac{0.3\text{cm}}{148.3\text{cm}}100\%\right) \\ \Delta A &= (.11\%) + (.20\%) = (.31\%) \end{aligned}$$

Since $(.31\%) \times (\underline{2620}4.61\text{cm}^2) = 82.67\text{cm}^2$, we write the area in a variety of ways:

$$\begin{aligned} A &= (2.620 \times 10^4\text{cm}^2) \pm 0.3\% \\ &= (2.620 \times 10^4) \pm (0.008 \times 10^4)\text{cm}^2 \\ &= 2.620(8) \times 10^4\text{cm}^2 \end{aligned}$$

Please be aware that the reason some digits are called insignificant is that they are insignificant:

$$\begin{aligned} &(.31548\%) \times (26204.61) = 82.67 \\ &(.31\%) \times (26204.61) = 81.23 \\ &(.31\%) \times (26200) = 81.22 \\ &(.3\%) \times (26204.61) = 78.61 \\ &(.3\%) \times (26200) = 78.60 \end{aligned}$$

All of these round to 80cm^2, giving

$$\boxed{A = (2.620 \times 10^4) \pm (0.008 \times 10^4)\text{cm}^2}.$$

A.4 Significant Figures only *approximates* Uncertainty

The precision/accuracy of any measurement or number is approximated by writing the number with a convention called using *significant figures*. Every measuring instrument can be read with only so much precision and no more. For example, a meter stick can be used to measure the length of a small metal rod to one-tenth of a millimeter, whereas a micrometer can be used to measure the length to one-thousandth of a millimeter. When reporting these two measurements, the precision is indicated by the number of digits used to express the result. You should always record your data and results using the convention of significant figures. To give a specific example, suppose that the rod mentioned above was 52.430 mm long. When making this measurement with the meter stick, you would count off the total number of millimeters in the length of the rod and then add your best guess that the rod was four-tenths of a millimeter longer than that. Using the micrometer, you would count off the hundredths of a millimeter and then add your best guess of the number of thousandths of a millimeter, to complete the measurement. How would you communicate the fact that one measurement is more precise than the other? If you wrote both quantities in the same way, you could not tell which was which.

RULE 1: Significant figures include all certain digits plus the first of the doubtful digits.

The reading obtained from the meter stick would be written as 52.4 mm; all digits up to and including the first doubtful digit. The reading from the micrometer would be written as 52.430 mm. The first doubtful digit in the case of the meter stick is .4 mm. The first doubtful digit in the case of the micrometer is zero-thousandths of a millimeter. Note that the number of significant figures is related to the precision of the measuring instrument – it is not an abstraction about the number.

Sometimes the character zero is confusing. In the example above, the reading of the micrometer was given as 52.430 mm. The zero is a significant figure, it communicates the fact that the micrometer measurement is good to a thousandth of a millimeter. Zeroes used to the left of significant digits to position the decimal point are not significant. They are not communicating the precision of the measurement. For example, if the measurement from the meter stick were written as 0.0524 meter, the zeroes would not be significant digits. Because of the units (meters instead of millimeters), the decimal point had to be moved to the left. This measurement still has only three significant figures.

RULE 2: Zeroes to the right of the number are significant; zeroes on the left are not.

Suppose that you wished to give the meter stick measurement in terms of microns (μ) (1 micron = 1 millionth of a meter). We determined that the meter stick reading has 3 sig. figs., one good way to write this is to use scientific notation. Write the number as 5.24×10^4microns. The factor of 10^4 shows the order of magnitude, while the 5.24 retains the proper number of significant figures. Study the following examples:

5.24 cm	3 significant figures
52.4 mm	3 significant figures
0.0524 m	3 significant figures
5.24×10^4 μm	3 significant figures
52.430 mm	5 significant figures
0.052430 m	5 significant figures
5.2430×10^4 μm	5 significant figures
52430 μm	4 significant figures
52430. μm	5 significant figures

Because all measurements are limited in their precision, then all results derived from these measurements are also limited in their precision. Many students get carried away with the number of digits produced by a calculator and mislead the reader by reporting their results with more significant figures than their data permits. Form the habit of rejecting all figures which will have no influence in the final result and report the result with only the number of significant digits allowed by the data. The following rules will help you do this successfully.

RULE 3: Round the number, increasing by one the last digit retained if the following digit is greater than five.

RULE 4: In addition and subtraction, carry the result only up to the first doubtful decimal place of any of the starting numbers.

RULE 5: In multiplication and division, retain as many significant figures in the answer as there are in the starting number with the smallest number of significant figures.

When determining or estimating the experimental uncertainty, the precision of the measuring instrument is important, as shown in the above examples. But you must also be aware of other experimental factors. For example, a good stopwatch may have a precision of 0.01 seconds. Is this the total uncertainty of the measurement? You must remember that our physical reaction time maybe another 0.3 seconds. This is more than 10 times larger than the precision of the timer. This is very significant. Another example is trying to measure the diameter of a fuzzy cotton ball with a micrometer. Why is this not a very productive procedure? There are major uncertainties here that are intrinsic to the object itself and are unrelated to the measuring instrument. One must use common sense when estimating these uncertainties.

The *actual uncertainty* written in the units of the measurement, may not convey a sense of how good the precision is. A better measure of the precision is given by the relative uncertainty. This is defined as the actual uncertainty divided by the measurement itself and multiplied by 100, the *relative uncertainty* does not carry any units, just a %.

B Discovering Relationships - Graphical Analysis

The primary purpose of experimentation is to discover relationships between various physical quantities. This is usually best achieved with a graphical analysis. Graphs of data and/or graphs of other results can be very enlightening. We often try to choose to plot variables for a graph so that the resulting relationship is linear, whose slope and intercept may be of physical interest.

B.1 Linear Relationships

When two quantities are related, there are many, many possible mathematical relationships. The simplest relationship is the direct proportion. This relationship is represented on a graph by a straight line which goes through the origin. The linear relationship is similar, it however, may have an intercept with a coordinate axes. The general equation for a straight line is:

$$y = mx + b,$$

where x and y are the plotted quantities, m is the slope, b is the y-intercept. The slope and intercepts of a linear relationship often have physical significance. It is therefore very important to calculate the slope and intercept and then for you to interpret their meaning, and always give the units of the slope and intercept. The slope can be computed by choosing two places, (x_1, y_1) and (x_2, y_2), on the straight line. The slope is then given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

B.2 Linearizing Non-Linear graphs

Most relationships which are not linear, can be graphed so that the graph is a straight line. This process is called a linearization of the data. This does not change the fundamental relationship or what it represents, but it does change how the graph looks. The goal of this method is that it gives an excellent and easy method for obtaining a mathematical representation of the data.

For example, the equation

$$X \times Y = \text{constant}$$

represents an **inverse proportion** between X and Y. A graph of this equation is not a straight line with a negative slope. Think about this and sketch the curve for yourself. This relationship can be graphed in such a way so that the new graph is a straight line. This change is accomplished by choosing a new set of axes, and plotting new numbers which are related to the original set. In this case if we would plot $1/X$ on the x-axis instead of just X, this will yield a straight line graph. Try it.

There are many other possible relationships which are easy to linearize. These include: exponential function, trigonometric functions, and power functions (squares, square roots, etc.) A change of either the x or y-axis may linearize a function for you.

To linearize the **power relationship**

$$Y = Bx^M,$$

take the natural logarithms of both sides to obtain

$$\ln(Y) = \ln(B) + M \ln(X),$$

and a graph of this function yields a straight line graph.

To linearize the **exponential relationship**,

$$Y = Be^{MX},$$

take the logarithm of both sides to obtain

$$\ln(Y) = \ln(B) + MX,$$

and again a graph of this function yields a straight line graph.

C Using PASCO Capstone

For notational purposes, I will use (parentheses) to indicate clarifying comments, [square brackets] to indicate click-able selections on the computer, and {curly brackets} to indicate a phrase or the name of a region on the screen.

PASCO corporation has manufactured much of the equipment that will interface with the computer. This section is a primer for using that that interface. To begin, log in to the computer. If you use your TMC login information, then you will have access to your network drive where you can save data that will be accessible to you at any location on (and, if you know how, off) campus. Once you are logged in, find and click on the [Pasco Capstone] icon, which should be located on the desktop.

When this loads, your screen will be divided into multiple parts. (It is possible that some of these are missing. In the next paragraph we will talk about how to hide or reveal these parts.) We will refer to the middle of the screen (which starts white with 8 choices) as the {Main Workspace}. There is a region near the top-left which we will refer to as the {Menu}, from which you can perform actions such as [Save], [Edit], [Undo], etc. There are also "palettes" on the left, right, and bottom of the screen. These palettes can be hidden. On the left is the {Tools palette}. On the right is the {Display palette}. On the bottom is the {Controls palette}.

Each palette has several icons, each of which, when clicked, will expand into and cover up part of the {Main Workspace}. **To hide or reveal one of the palettes**, look to the {Menu} at the top. Click the menu item called [Workbook]. You will get a menu that allows you to checkbox (toggle on/off) each of the palettes individually or all of them together. With a checkmark, it will be displayed; without, it will be hidden.

C.1 Using the Motion Sensor

To begin, look to the left, on the {Tools palette}. Click [Hardware Setup]. (It will expand into the {Main Workspace}.) You should see an image of the Pasco interface box. Plug the motion Sensor into the slots indicated by your instructor.

1 Meaningful Measurements

1.1 Introduction

Physics is a science which is based on precise measurements of the seven fundamental physical quantities, three of which are: time (in seconds), length (in meters) and mass (in kilograms); and all of these measurements have an experimental uncertainty associated with them. It is very important for the experimenter to estimate these experimental uncertainties for every measurement taken. There are three factors that must be taken into account when estimating the uncertainty of a measurement:

1. statistical variations in the measurements,
2. using one-half of the smallest division on the measurement instrument,
3. any mechanical motions of the apparatus.

Physicists study the physical relationships between these defined fundamental quantities and usually give a name to the newly derived physical quantity. These derived physical quantities have units which are combinations of the units of the fundamental ones. For example, the product of the lengths (in meters (m)) of the three sides of a cube is called volume and has units of m^3. The ratio of mass to volume is called density and has units of $\mathrm{kg/m}^3$. The concepts of volume and density are therefore derived from the fundamental physical quantities, rather than fundamental themselves.

1.2 Experimental Objectives

Determine the material of the objects by calculating their density and matching it to the accepted values for various common materials.

1.3 Student Outcomes

In this exercise, students should learn how to to make precise and accurate length measurements with a meter stick and two types of calipers, how to read a vernier scale, and how to estimate uncertainty in a measurement. Students will make use of the relationship of the fundamental properties of mass and length to the derived concepts of volume and density.

1.4 Procedure

⇒ There are several measuring devices at your lab station, a metric ruler, a vernier caliper, and a micrometer caliper. Check these instruments for any zero-point errors. The use of the caliper and micrometer is outlined below.

⇒ There are also several solids available: a cylinder, a cube, and a sphere.

- Measure the dimensions of two of the objects with each of the following instruments: a metric ruler, a vernier caliper, and a micrometer caliper. Take all measurements minimizing any parallax errors.
- Estimate the experimental uncertainties of your measurements. Consider Question 1.
- Repeat the measurements at several positions and orientations around the object, compute the average and the relative uncertainty.

⇒ As outlined in the Analysis section, compute the volume and the density.

1.4.1 The Vernier Caliper

The vernier scale was invented by Pierre Vernier in 1631. This scale has the advantage of enabling the user to determine one additional significant figure of precision over that of a straight ruler. For example, this eliminates the need for estimating to the tenth of a millimeter on the metric ruler. The vernier caliper shown in Figure 1.1 can measure distances using three different parts of the caliper: outside diameters (large jaws), inside diameters (small jaws), and depths (probe). You should locate these three places on your caliper. The vernier device consists of the main scale and a movable vernier scale. The fraction of a millimeter can be read off the vernier scale by choosing the mark on the vernier scale which best aligns with a mark on the main scale. *The reading of the caliper from Fig. 1.1 is 1.35 mm ± .01mm because we can distinguish 1.35 from 1.34 and 1.36, but we cannot gauge the result any more precisely.* To move the vernier scale relative to the main scale press down on the thumb-lock, this releases the lock and then move the vernier scale. Do not try to move the vernier scale without releasing the lock.

1.4.2 The Micrometer Caliper

A micrometer caliper is shown in Figure 1.2. This instrument is used for the precise length measurement of a small object. The object is placed with care between the anvil and the rod. It is very important to not tighten down on the object with a vise-like grip. Tightening with force will decalibrate the micrometer. The rotating cylinder moves the rod, opening or closing the rod onto the object. There is a rachet, at the far end, for taking up the slack distance between the anvil, the object and the rod, so again do not over-tighten with the rotating cylinder. The linear dimension of the object can be read from the scale. Rotating the cylinder one revolution moves the rod 0.5 millimeters. The rotating cylinder has 50 marks on it. Read the mark on the rotating cylinder that aligns with the central line on the main scale.

The reading of the micrometer from Fig. 1.2 is 6.730 ± .005 mm. Since each mark corresponds to 0.01 mm and you can probably gauge a distance about half-way between the lines, the precision of this instrument is 0.005 mm.

1.5 Analysis

⇒ After finding the relevant dimensions of the object, calculate the volume of the object three times: once using the measurements from the ruler, once from the caliper, and once with the micrometer.

⇒ Using the rules of propagation of uncertainty, compute the uncertainty in the volume for each. Answer Question 2.

⇒ Measure the volume directly with the graduated cylinder. Consider Question 3.

⇒ Measure the mass and then, using the overall most precise measurement of volume, compute the density with its uncertainty.

⇒ Using your best density value, find the percent-error against the appropriate value given by the text, or the Handbook of Physics & Chemistry.

1.5.1 Questions

1. What were the zero-point errors for each of the measuring instrument? Estimate the parallax errors. How can these errors be corrected for? Are these random or systematic errors?

2. Which instrument is the most precise?

3. Are any of the volume measurements inconsistent? What can you infer about the accuracy of these instruments? Using the most precise indirect measurement of volume (those calculated from other measurements), calculate a percent-difference with the direct measurement of volume.

4. What would be the best method to measure the volume of an irregularly shaped object? Why?

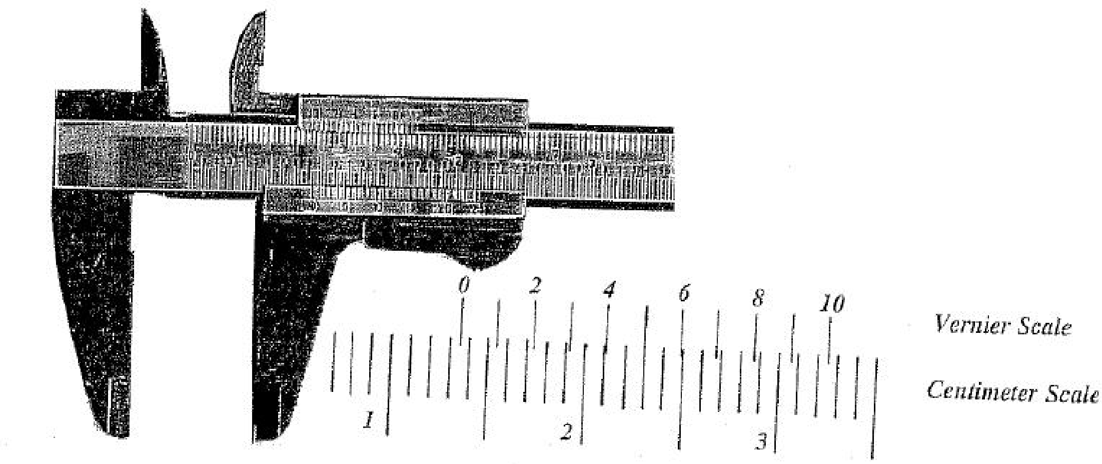

Figure 1.1: The Vernier Caliper: The location of the zero on the vernier scale tells you where to read the centimeter scale (1.3 cm). The vernier-scale line that lines up tells you the next digit (5). This picture measures $1.35 \pm 0.01\,\mathrm{cm}$.

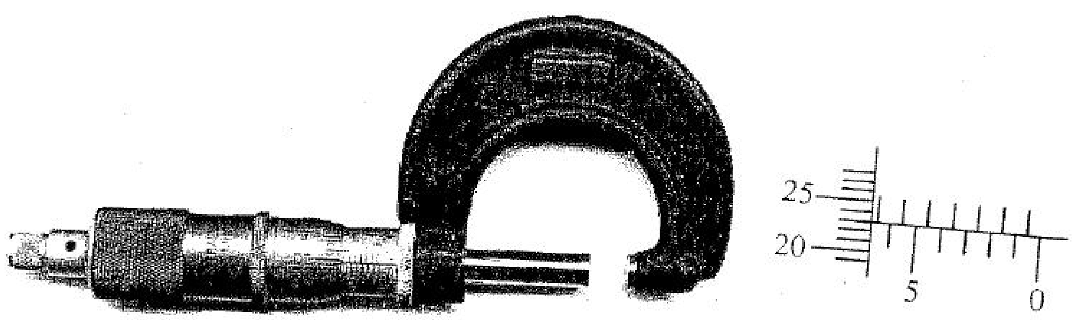

Figure 1.2: The Micrometer Caliper: Notice on the coarse scale, that the lower lines read (1, 2, 3, ¡ellipsis /¿ 6 in this picture) and the higher lines read the half-marks (0.5, 1.5, 2.5, ¡ellipsis /¿ 6.5 in this picture). The location of the turning dial tells you where to read the coarse scale (6.5 mm). The center line of the coarse scale tells you where to read the fine scale. This is 23.0 (in units of $\times 10^{-2}$ mm), but not 23.5 and not 22.5 so the precision is 0.5 (in these units). This measurement in mm reads $6.5\,\mathrm{mm} + 0.230\,\mathrm{mm} = 6.730 \pm 0.005\,\mathrm{mm}$.

2 Standard Deviation

2.1 Introduction

Suppose you are standing in front of a dart-board. You have a large number of darts and you throw the darts one at a time, trying to hit a 1 cm thick vertical line drawn on the dart-board. Since you have a very good aim, let us say that 45% of the darts hit the line. This then means that you miss the line 55% of the time; with a significant number of these misses being between .5 to 1.5 cm either to the left or to the right from the center of the line, and with a smaller number of misses being between 1.5 to 2.5 cm from the center line.

Is there some mathematical way of characterizing how good you are at this game? What, for example, is the probability of missing the line by 1 cm? The statistical analysis of random fluctuations in data can help answer these questions. The word "statistical" implies that a relatively large set of similar measurements of a given physical quantity is available. The random fluctuations in the data can be measured with the use of a mathematical term called the "standard deviation."

Suppose you collect data on a large number of throws, separating the data into categories (bins): The number of darts on the line, the number within .5 to 1.5 cm from the center, the number within 1.5 to 2.5 cm, etc. A plot of this data with the dart positions on the x-axis and the number of darts hit within each bin plotted on the y-axis is called a histogram (see Figure 2.3). The envelope of this graphical data set is bell shaped, and is called a Gaussian or a Normal distribution curve.

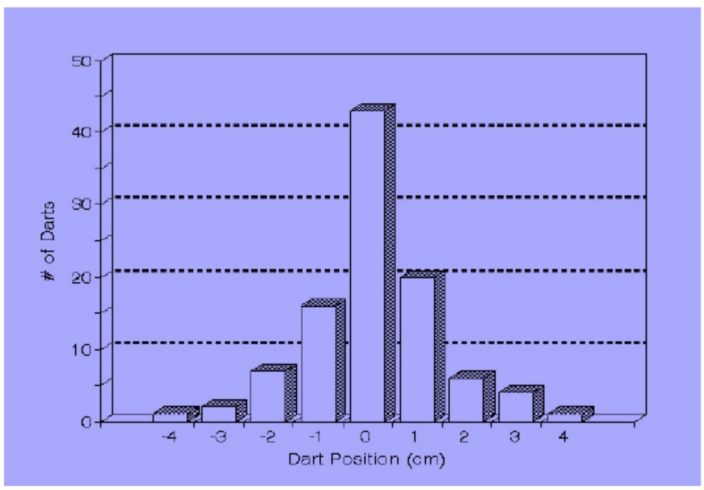

Figure 2.3: Histogram for the number of darts binned by distance from the centerline.

The "standard deviation" is a measure of the spread or width of the histogram data. A small standard deviation means that there is a small spread in the data about the central mean value and implies that the data cluster closely about one value. That is, there is a high degree of precision in the measurements.

The area beneath the curve, or below a part of the curve represents the probability of occurrence. For example, the area beneath the curve between plus and minus one standard deviation from the mean represents a 68% probability of your next throw falling within this range. The area beneath the curve between plus and minus two standard deviations from the mean represents a 95% probability of your next throw falling within this range.

In this experiment, you will study the use of the "standard deviation" in the statistical analysis and probability involved with flipping pennies.

2.2 Experimental Purpose

The goal of the experiment is to determine how the mean, standard deviation, and standard deviation of the mean depend on the amount of data (number of samples) taken. You should also consider how well the data fit to a normal distribution.

2.3 Student Outcome

In this experiment, you should learn

1. the formulas for and the roles of the mean, the standard deviation, and the standard deviation of the mean in the statistical analysis of data containing random errors,

2. how to create histograms and scatter plots,

3. how to include trendlines on scatter plots, and

4. why more data is always better.

2.4 Pre-Lab Work

⇒ Define the following, both in a sentence and with a mathematical formula: mean, standard deviation (sometimes called the standard deviation of a single observation), and standard deviation of the mean. Be sure to describe the difference between these two terms.

⇒ Define the following: histogram, probability, & probability distribution (Normal distribution).

⇒ What does the total area under the Normal distribution curve represent?

⇒ How should the axes of the histogram of the data you will take for this lab be labeled? Give a very specific scale for the x-axis. (Hint: Read the procedure below.)

⇒ Make a sketch of an inverse function, like $y = 1/x$.

2.5 Experimental Procedure

Each individual will receive 20 pennies and will then simultaneously toss all of the pennies. Count the number of heads, and repeat 25 times. Each individual will then have collected 25 pieces of data.

2.6 Analysis

⇒ Calculate the mean, the standard deviation and the standard deviation of the mean for your first ten tosses and for all 25 tosses. Show your calculations.

⇒ Make a histogram plot (by hand in your note book) for your 25 tosses. On this histogram superimpose a sketch of your best guess of the corresponding Normal distribution curve.

⇒ How well does your data fit a normal distribution curve? Explain the reasons for any large discrepancies.

⇒ Obtain histograms and calculations of the mean, the standard deviation and the standard deviation of the mean for the following data sets: i) your lab group, ii) about 1/2 of the class, and iii) the entire class. These histograms can be obtained with the use of a computer program, provided by the instructor.

⇒ Draw graphs of: the mean, the standard deviation, the standard deviation of the mean, versus the number of data entries. Use the standard deviation of the mean as the error bars for both the mean and the standard deviation, draw these error bars on these two graphs.

⇒ Describe how the values of the mean, the standard deviation, and the standard deviation of the mean, change as the number of data items in the set increases. What does this infer about the accuracy and about the precision of the data as more and more observations are made?

⇒ Show that the standard deviation of the mean varies inversely as the square root of the number of data items in the sample. How well does the data in this lab agree with this prediction? (Make a graph of the standard deviation of the mean versus the square root of the number of data items in the sample.)

2.7 Questions

1. What is the probability of flipping the 20 pennies and getting 5 heads, or 8 heads, or 10 heads, or 15 heads? Answer this question by analyzing your data, the entire class' data and the normal distribution curve fit to your data. Explain any differences between these sets.

2. What percentage of your individual readings fall within plus or minus one standard deviation, two standard deviations? Compare your answer to the theoretical answer from a normal distribution curve. What are the percentages for the class data?

3. Does the height of the histograms change as a function of the number of trials? If so, how?

4. How does the width at 1/2 the maximum height for the histograms change as a function of the number of trials? Label this width on your histograms. Is this width a reasonable estimate of the standard deviation?

5. Does the standard deviation of a single observation and/or does the standard deviation of the mean change significantly as the number of tosses increase? What does this infer about the accuracy and about the precision of the data as more and more observations are made?

Resource Materials

Meyer, Data Analysis for Scientists and Engineers John Wiley, (1975) p.19-48, 223-253

¡chapter¿¡title¿Constant Acceleration¡/title¿ ¡!– Revisions –¿

¡objectives¿¡title¿Experimental Purpose¡/title¿ ¡introduction¿¡p¿Using position versus time and velocity versus time graphs, verify that the equations of constant acceleration accurately describes the behavior of objects under constant acceleration and that it is possible to distinguish acceleration due to gravity from acceleration due to friction.¡/p¿ ¡/introduction¿ ¡/objectives¿

¡section¿¡title¿Student Outcomes¡/title¿ ¡p¿In this exercise, the student should develop an understanding of the relationships between the position and the instantaneous velocity of an object, as well as how each of these can vary as functions of time. We will only consider the special case where the object experiences constant acceleration.¡/p¿

¡/section¿

¡section¿¡title¿Materials¡/title¿ ¡p¿An aluminum track, a low-friction cart, computer interface with PASCO Capstone¡m¿$^{\rm tm}$ < */m* > *software, asonicmotionsensor, asmallsteelball.* < */p* >< */section* >

¡section¿¡title¿Procedure (to be paired with Section 2.7)¡/title¿

¡subsection¿¡title¿Cart and Flat Track¡/title¿ ¡p¿Log into the computer (so you can save your data to your network drive) and then open Pasco Capstone. (Section C will provide some instructions for setting up the software and connecting the equipment.) Connect the motion sensor to the computer interface. Set the data rate of the motion sensor at ¡m¿50 Hz¡/m¿. Place a steel ball on the track and adjust the leveling screw at one end of the track to see if the ball rolls one way or the other. This will roughly level your track. Place the sensor about 20 cm from the end of the track, because this is the minimum distance detected by the sensor. (You might need to use the ¡q¿sail¡/q¿ for the sensor to see the cart.)¡/p¿

¡p¿Place the cart on the track. Capstone, via the sonic ranger, can measure the position and velocity of the cart as a function of time. (This is explained in Section C.) ¡ol¿ ¡li¿¡p¿ Assume the track is frictionless and predict how the cart will move if the track is not perfectly level; include a comment about how the velocity versus time graph will look when it goes uphill versus when it goes downhill. Should these be the same? ¡/p¿¡/li¿ ¡li¿¡p¿ What do you expect the graph to look like if the track ¡em¿is¡/em¿ perfectly level? Will it be the same going left versus going right? ¡/p¿¡/li¿ ¡li¿¡p¿ Now, assuming it is perfectly level, what will friction do to the motion? How do you expect this to affect the graphs? ¡/p¿¡/li¿ ¡/ol¿ ¡/p¿

¡p¿We will take four sets of data: a slow, constant velocity towards the ranger; a slow, constant velocity away from the ranger; a faster, constant velocity towards the ranger; and a faster, constant velocity away from the ranger. The two slow speeds should be about the same and the two faster speeds should be about the same. For each case, start the sonic ranger and then bump the cart firmly, but not violently(!).¡/p¿

¡p¿On Capstone, you should have four curves of velocity versus time. Fit each with a trendline and display the equation of the trendline on the screen. Interpret the coefficients (slope and intercept) by noting their units, values, and uncertainties. You should also print out (in landscape mode) the position versus time graph, the velocity versus time graph, and the acceleration versus time graph. (You should notice that the acceleration versus time graph is ¡em¿very¡/em¿ noisy.)¡/p¿

¡/subsection¿ ¡subsection¿¡title¿Cart and Sloped Track¡/title¿ ¡p¿Place a small block under one end of the track, so that the track is now tilted at a small angle with the sensor at the top of the incline. Measure the angle using a protractor or calculate it by measuring the two legs of the triangle and using the inverse sine. (Be careful about measuring the height.)¡/p¿

¡p.¿We will consider ¡em¿three cases¡/em¿ for the sloped track: ¡em¿First¡/em¿, allow the cart to roll (without an initial push) down the ramp. ¡em¿Second¡/em¿, gently push the cart down the ramp. DON'T let it fly off or crash into anything. ¡ol¿ ¡li¿¡p¿ Should these two cases have the same acceleration while rolling down the ramp? How will that affect the shape of the velocity versus time graphs? ¡/p¿¡/li¿ ¡li¿¡p¿ Should these have the same initial velocity? How will that affect the graphs? ¡/p¿¡/li¿ ¡/ol¿ In the ¡em¿third¡/em¿ case, start the cart at the bottom of the incline and roll it up the ramp, allowing it to roll back down on its own. Push it hard enough to get mostly up the ramp, but not so hard that it hits the sonic ranger. ¡ol¿ ¡li¿¡p¿ Should this case have the same acceleration while it goes up the ramp as while it goes down the ramp? How can we see that on the velocity versus time graphs? ¡/p¿¡/li¿ ¡li¿¡p¿ Should this case have the same acceleration (either while it goes up the ramp or while it goes down the ramp) as the previous two cases of rolling down the ramp? ¡/p¿¡/li¿ ¡/ol¿ ¡/p¿

¡p¿In Capstone, you should be able to display all three graphs (position v time, velocity v time, and acceleration v time). You should also be able to display all three cases of data on each of these graphs. On the velocity versus time graph, fit each of the three graphs with a linear trendline. The next section will ask you to analyze how well the data match up to these lines. (It might be interesting to also fit the position vs time curves to parabolas. Be sure to print out copies of your three graphs.¡/p¿

¡p¿Your lab should note the following results and explain their meaning: slope and y-intercept, the uncertainties (precision) in both the slope and intercept, and the ¡m¿r¡/m¿ value (correlation coefficient). ¡/p¿

¡/subsection¿ ¡/section¿

¡section¿¡title¿Further Analysis and Discussion¡/title¿

¡subsection¿¡title¿Cart and Flat Track¡/title¿

¡p¿Based on the results of Sec. 2.7, write a short analysis of the relationship between these two graphs (x and v versus time). From the velocity versus time graph (specifically from the trendline) determine the value of the acceleration of the cart down the track; be sure to include the uncertainty of the acceleration and the units. ¡ol¿ ¡li¿¡p¿ Do you see any evidence that the track was not perfectly level? ¡/p¿¡/li¿ ¡li¿¡p¿ Do you see any evidence that there is any friction as the cart moves along the track? ¡/p¿¡/li¿ ¡li¿¡p¿ What does the intercept of the velocity versus time graph tell you? ¡/p¿¡/li¿ ¡li¿¡p¿ If the slopes are different, then discuss any pattern that you see. If the slopes are (essentially) the same, then find an average and a standard deviation of the four values. ¡/p¿¡/li¿ ¡li¿¡p¿ Does it matter how fast the cart travels? ¡/p¿¡/li¿ ¡/ol¿ Discuss any evidence observed in your data when answering these questions. Also consider the magnitude of the uncertainties when writing your conclusions.¡/p¿

¡/subsection¿ ¡subsection¿¡title¿Cart and Sloped Track¡/title¿

¡p¿Based on the results of Sec. 2.7, write an analysis of the relationship between the two graphs (x and v versus time). From the velocity versus time graph determine the value of the acceleration of the cart down the track. ¡ol¿ ¡li¿¡p¿ For the two downhill cases, use your uncertainty analysis to determine if the acceleration of the cart changed when it was given a small push. ¡/p¿¡/li¿ ¡li¿¡p¿ Is there an accuracy¡fn¿If the track were frictionless, then the acceleration should be
¡m¿a=(9.81 $^{\mathrm{m}}/_{\mathrm{s}^2}$)($\sin\theta$) *< /m >, where < m >* θ *< /m > istheanglethattheinclinemakes. < /fn >*
thatcanbecomputedforthispartoftheexperiment? < /p >< /li >< /ol > Inspecttheline/curvethatisdefinedbyth
ol >< li >< p > Whatisitsshape?Istheshapeofthegraphwhatyouwouldexpectforconstantacceleration(straightl
/p >< /li >< li >< p > Considerthetrendlinethatyouadded.Does/shouldthetrendlinelinegothroughtheorigin?W
interceptoftheXvsTgraph?Whatphysicalquantitydoestheinterceptrepresent?Explainwhyithasthatvalue.Hint :
(Thinkaboutwherethesensorwaslocated.) < /p >< /li >< li >< p > Whatdoestheslope(whetherit'sconstantorn
/p >< /li >< /ol > NowconsidertheInstantaneousVelocityvs.Timegraph. < ol >< li >< p >
Doesthecurve/lineonthisgraphhavetheshapeyouwouldexpectforanobjectundergoingconstantacceleration?Explo
/p >< /li >< li >< p > Whatwasthevalueoftheyinterceptonthisgraph(includeunitsanduncertainty!)?Explaini
axis! < /p >< /li >< /ol >< /p >

¡/subsection¿ ¡/section¿ ¡/chapter¿

¡chapter¿¡title¿Newton's ¡m¿2^{nd} < /m > *LawonaLinearTrackwiththeSonicRanger* < /title > <! − −*Revisions* − − >

¡introduction¿¡title¿¡/title¿Introduction to Forces

Forces are related to the natural motion of bodies, where one object can affect the motion of another object. That is, forces are interactions between objects affecting their motion. Although the famous Greek philosopher Aristotle claimed that a force was necessary to ¡em¿maintain¡/em¿ any motion, careful analysis by Italian physicist Galileo Galilei in the mid-¡m¿17^{th} < /m > *centuryandbySirIsaacNewton, a* 1727), *eventuallydistinguishedtheeffectsoffrictionandallowedNewtontocreateamathematicallyconsistenttheoryo* *q* > *MathematicalPrinciplesofNaturalPhilosophy* < /*q* > *in*1687, *forwhich*(*amongotheraccomplishments*)*Newt*

All forces can be placed in one of two main categories. First, there are natural (or fundamental) forces like the gravitational force, the electromagnetic force, or the nuclear forces. The gravitational force is a force on a body by another body (like the Earth), this force is an interaction between their two masses. The electromagnetic force is an interaction between the charges of two bodies. These forces may act on an object without any direct physical contact between the two bodies. This type of force is sometimes called an ¡q¿action at a distance¡/q¿ force. All other forces are in a second category called ¡q¿contact forces.¡/q¿

¡subsection¿¡title¿¡/title¿Newton's First law

¡quote¿ If there are no forces acting, then objects will remain at rest or, if not at rest, will maintain their velocity. ¡/quote¿ If this is true, then we can study the forces acting on a body based on the motion of the body, specifically through the change in the velocity of an object.

¡/subsection¿ ¡subsection¿¡title¿¡/title¿Newton's Second Law

Not only is a force necessary to change the motion (to cause an acceleration), the amount of acceleration that a force causes is predictable and is inversely proportional to the mass. The same sized force causes a small mass to accelerate a lot and a large mass to accelerate a little. this is expressed by the equation: ¡me¿ $\vec{F}_{\text{net}} = m\vec{a}$. < /*me* > *Thenetforce*, < *m* > $\vec{F}_{\text{net}}$ < /*m* > , *isthevectorsumofallforcesactingonanobject.Ifwehaveanextendedobject*(*suchasaweighthangingoffofatable*, *but* *q* > *external* < /*q* > *tothesystem* : *Solongasbothobjectsaccelerateatthesamerate*, *wedonotneedtoconsiderthe* < *q* > *internal* < /*q* > *tensionthatthestringexertsbetweentheconnectedbodies*.

¡/subsection¿ ¡subsection¿¡title¿¡/title¿Newton's Third Law

Inherent in the description of a force is that it is an interaction between objects: there must always be two objects that interact. These objects exert equal and opposite forces on each other. That is, ¡quote¿ If there is a force exerted on object 1 by object 2, then there is necessarily and simultaneously a force exerted on object 2 by object 1 that is equal (in magnitude) and opposite (in direction) to the original force. ¡/quote¿ Remember that these two forces are on different objects and that the two bodies in direct contact exert forces on each other. Remember then that if there is contact between the object (any part of the system) and anything else then there is an outside force on the object (system) and that if there is no contact (the two bodies break contact) then there is no force.

¡/subsection¿ ¡/introduction¿ ¡objectives¿¡title¿Experimental Objective¡/title¿

¡introduction¿¡p¿In this experiment, we will assume that the first law is true and focus on the second law. By measuring (a) the velocity versus time for a cart being pulled down a track and (b) the applied force that is pulling it, we can plot the acceleration versus the force and verify the validity of Newton's second law of motion: ¡m¿$\vec{F}_{\text{net}} = m\vec{a}$ < /*m* > . < /*p* >< /*introduction* >< /*objectives* >

¡/section¿¡section¿¡title¿¡/title¿The Experimental Setup

¡ul¿ ¡li¿¡p¿ A low-friction linear cart and track will be used, this reduces the friction between the cart and the track. ¡/p¿¡/li¿ ¡li¿¡p¿ A string will be connected to the cart and a known mass will be hanging from the end of the string (and over a pulley). The hanging mass will exert a constant horizontal force on the cart as the mass falls all the way to the floor. This gives a constant acceleration to the cart. ¡/p¿¡/li¿ ¡li¿¡p¿ The sonic motion sensor will be used to measure the position of the cart as a function of time. ¡/p¿¡/li¿ ¡li¿¡p¿ The carts and tracks need to be handled with care. Scratches can add friction to the system. ¡/p¿¡/li¿ ¡/ul¿

¡/section¿¡section¿¡title¿¡/title¿Pre-Lab Work

¡ul¿ ¡li¿¡p¿ Draw a free-body force diagram for the cart and for the hanging mass. ¡/p¿¡/li¿ ¡li¿¡p¿ Derive an equation for the acceleration of the system, in terms of, the mass of the cart and the hanging body. ¡/p¿¡/li¿ ¡/ul¿

¡/section¿¡section¿¡title¿¡/title¿Procedure

¡ul¿ ¡li¿¡p¿ If the cart is given an initial push (without the hanging mass and string attached) then the cart should travel with a constant velocity down the horizontal track, if there are no other forces acting on the cart. Carry out a couple of constant velocity runs on the track, to check for the effects of friction and to see how level the track is. The track may need a level adjustment. Do runs in both directions. Maybe the track can be tilted so that the friction is countered by the tilt of the track. ¡/p¿¡/li¿ ¡li¿¡p¿ Connect a string to the cart and run it through the hole at the end of the track, then over a pulley. Make sure that the string is horizontal. Measure the height of the string at both ends of the track, to see if the string is horizontal. ¡/p¿¡/li¿ ¡li¿¡p¿ The hanging mass should be much less than the mass of the cart. Use a small plastic cup to hold the hanging masses. Measure the mass of this cup. The total mass of the system must be kept constant for all parts of the experiment. The hanging mass and the mass of the cart should vary, but their total must be kept constant, by moving small mass amounts from the cart to the hanging cup. Record the mass of the cart, the hanging cup mass, and the extra masses which are to be transferred from the cart to the cup. ¡/p¿¡/li¿ ¡li¿¡p¿ Take data with Capstone and the motion sensor as the cart travels with constant acceleration down the track. Determine the acceleration of the cart from a linear regression using the velocity vs time data (a linear fit line in Capstone). Record the acceleration value and its uncertainty. ¡/p¿¡/li¿ ¡li¿¡p¿ Collect 7 data runs, where about 5 grams is transferred each time from the cart to the hanging mass. Determine the acceleration of the cart (and the uncertainty for the acceleration) for each of these 7 runs. ¡/p¿¡/li¿ ¡/ul¿

¡/section¿¡section¿¡title¿¡/title¿Analysis

¡ul¿ ¡li¿¡p¿ Make a graph of the acceleration of the system (y-axis) versus the weight (¡m¿mg¡/m¿) of the hanging body (x-axis), should include 7 data points. Do this in Excel. Carry out a linear regression for this data set. Quote the slope and intercept values, their uncertainties, their p-values, and the ¡m¿R^2 < /m > *value.Showasampleerrorbar(onthegraph)foratleastoneofthepointsofthisgraph.* <
/p >< /li >< li >< p > *Derive(showitcompletely)anequationfortheaccelerationofthesystemversustheweighto*
/p >< /li >< li >< p > *Compareyourgraphtothepredictedtheoreticalequation, thatiscomparethevaluesoftheslo*
/p >< /li >< li >< p > *Inmanymechanicsexperiments, theremaybedeviationsfromtheexpectedortheoreticalre*
/p >< /li >< li >< p > *Whendesigningexperiments, itisimportanttokeepcontrolparameters; inthiscaseaparam*
/p >< /li >< /ul >

¡/section¿¡section¿¡title¿¡/title¿Questions

¡ol¿ ¡li¿¡p¿ Why is it important to keep the total mass of the system constant? What would happen if the total mass of the system was not held constant? If one simply added mass to the hanger without keeping the system's mass constant, how would their data appear on the graph of the acceleration vs ¡m¿mg¡/m¿? ¡/p¿¡/li¿ ¡li¿¡p¿ What would happen if the track was not level? If the beginning end of the track is higher, how would the acceleration of the system be affected? ¡/p¿¡/li¿ ¡li¿¡p¿ If your group has a discrepancy between the results and the theory, could friction be used to explain your results? Explain how. In this case, how much of a tilt in the air track would be needed to explain the discrepancy? Show this calculation. ¡/p¿¡/li¿ ¡li¿¡p¿ What would happen to the cart's acceleration if the cart was given an initial push? ¡/p¿¡/li¿ ¡li¿¡p¿ What are the two greatest sources of uncertainty in this experiment? Are they random or systematic errors? Be specific and quantify your answer. ¡/p¿¡/li¿ ¡/ol¿

¡/chapter¿

¡chapter¿¡title¿Dry Sliding Friction¡/title¿ ¡!– Revisions –¿

¡introduction¿¡title¿¡/title¿Introduction

Friction is a force which retards the relative motion of any body while sliding over another body. The frictional force acting on a body is parallel to the surface that the object is sliding upon and it is directed opposite to the direction of motion. The phenomenon of friction is rather complicated, especially at the microscopic level, because it is dependent on the nature of the materials of both contacting surfaces. The frictional force depends on the roughness or the irregularities of both surfaces. At the macroscopic level, the nature of this force can be described by a simple empirical law, first given by Leonardo da Vinci: ¡quote¿ The magnitude of the force of friction between unlubricated, dry surfaces sliding one over the other is proportional to the normal force pressing the surfaces together and is independent of the (macroscopic) area of contact and of the relative speed. ¡/quote¿ At the microscopic level, the frictional force ¡m¿(F$_f$) $< /m >$
$doesdependontheactualareaofcontactoftheirregularitiesofthesurfaces.Thisactualareaofcontacttheninincreasesast$
$m > 2^{\text{nd}} < /m > LawinthisperpendiculardirectionwecanconcludethatthemagnitudeoftheloadisequaltotheNormal$
$m > (F_N) < /m > ofthesurfacepushingontheobject.Thereforewemaywritethat < me > F_f \propto$
F_N or $F_f = \mu F_N < /me > wheretheGreekletter < m > \mu < /m > (< q > mew < /q >$
$)isadimensionlessconstantofproportionalitycalledthecoefficientoffriction.$

When a body is pushed or pulled parallel to the surface of contact and no motion occurs, we can conclude that the force of the push or pull is equal to the frictional force, using Newton's ¡m¿2$^{\text{nd}}$ $<$
$/m > Lawofmotion.Astheappliedforceisincreased, thefrictionalforceremainsequaltotheappliedforceuntilmotion$
$me > F_f = \mu_s F_N < /me > wherethesubscript < m > s < /m > standsforstatic(non-$
$moving)friction.Thisequationcanonlybeusedatthismaximumstaticpointalsocalledthepointofimpendingmotion.At$
$me > F_f = \mu_k F_N < /me > wherethesubscript < m > k < /m > standsforthekinetic(moving)friction.Ingeneral,$
$m > \mu_k < \mu_s < /m >; thatis, ittakesmoreforcetoovercomethestaticfrictionthantoovercomethekineticfriction.The$

Consider a system comprised of a block on a horizontal surface being pulled horizontally by a string connected to a hanging weight. Assume that the system is accelerating with a constant acceleration. Then the ¡m¿$\mu_k < /m > canbesolvedforbythefollowingequation :< me > \mu_k = \frac{mg-(M+m)a}{Mg}, < /me > where < m > M < /m > isthemassoftheblockand < m > m < /m >$
$isthehangingmass.$

¡/introduction¿ ¡objectives¿¡title¿Experimental Objective¡/title¿

¡introduction¿¡p¿In this experiment you will devise methods to investigate the nature of both the frictional force and the coefficient of friction, and to test the validity of da Vinci's empirical rule.¡/p¿ ¡/introduction¿ ¡/objectives¿

¡section¿¡title¿¡/title¿Pre-Lab Analysis

¡ul¿ ¡li¿¡p¿ Draw force diagrams for the following case: a block on a horizontal surface pulled by a hanging mass and a string (include the friction force). ¡/p¿¡/li¿ ¡li¿¡p¿ Write out the corresponding Newton's ¡m¿2$^{\text{nd}}$ $< /m > Lawequationsforforcesbothparallelandperpendiculartothecontactsurface. <$
$/p >< /li >< li >< p > Derivetherelevantequationsforeachoftheabovetwocasesforwhichthecoefficientsoffrict$
$ul >< li >< p > Caseoneisstatic, butatthepointofmotion. < /p >< /li >< li >< p > Casetwoisthekineticcase. <$
$/p >< /li >< /ul >< /p >< /li >< /ul >$

¡/section¿¡section¿¡title¿¡/title¿The Experiment

For the block on the horizontal plane: ¡ol¿ ¡li¿¡p¿ Clean the block and the plane, so that they are free of dust and other contaminants.
Make sure the track is level, as in previous labs. ¡/p¿¡/li¿ ¡/ol¿

¡/subsection¿¡subsection¿¡title¿¡/title¿Break Static Friction - pull until moves ¡ol¿ ¡li¿¡p¿ Set up the Dynamics Track, cart, force transducer and friction block. The force transducer attaches to the dynamics cart, the friction block rests on the track (felt side down). ¡/p¿¡/li¿ ¡li¿¡p¿ Attach a string to the force transducer. The force transducer needs to be zeroed before data collection starts.

Collect data, and slowly start pulling on the string (¡em¿be sure to pull the string horizontally¡/em¿) and slowly increase the pull force until the cart is moving down the track. Using just the maximum force (at the point of impending motion) the coefficient of static friction can be calculated. ¡/p¿¡/li¿ ¡li¿¡p¿ Test the relationship between the force of friction and the normal force, by changing the load force (normal force) and measuring the force of friction at the point of motion impending. Carry this out for a total of five data points. Graph the frictional force versus the normal force. Calculate the coefficient of static friction from this graph. ¡/p¿¡/li¿ ¡/ol¿

¡/subsection¿¡subsection¿¡title¿¡/title¿Effect of Surface Area - distinguish pressure from force

Consider pushing a pencil into your arm. (Well, don't ¡em¿actually¡/em¿ do it!) If you use the erasure end, then you can feel the force, but it doesn't hurt. If you use the sharpened tip with the ¡em¿same¡/em¿ force then it will certainly hurt! So, you have the idea that the same force spread over a different surface area ¡em¿can¡/em¿ have a different effect; but it doesn't ¡em¿always¡/em¿ have a different effect. For this part of the lab, you will test the relationship between the coefficient of friction and the macroscopic area of contact between the block and the surface. ¡ol¿ ¡li¿¡p¿ Place the friction block on its side (felt side down) and repeat Steps **??** and **??** for three (rather than five) of the previous load forces. ¡/p¿¡/li¿ ¡li¿¡p¿ Add the plot of this ¡m¿$F_f < /m > versus < m > F_N < /m > asanewseriestothegraphofPart$ **??**$. < /p >< /li >< /ol >$

¡/subsection¿¡subsection¿¡title¿¡/title¿Friction while Accelerating

¡ol¿ ¡li¿¡p¿ Apply a force (hanging mass, pulley, and string) large enough to accelerate the block. Use the Sonic Ranger to collect data. ¡/p¿¡/li¿ ¡li¿¡p¿ Graph the velocity vs time. Determine the acceleration of the block from the slope of the line. ¡/p¿¡/li¿ ¡li¿¡p¿ Repeat this part four or five times with a different normal forces. (You may use any hanging mass.) ¡/p¿¡/li¿ ¡li¿¡p¿ Add the plot of this ¡m¿$F_f < /m > versus < m > F_N < /m > asanewseriestothegraphofParts$ **??**and **??**$. < /p >< /li >< li >< p > Calculatethecoefficientofkineticfrictionfromtheslope. < /p >< /li >< /ol >$

¡/section¿¡section¿¡title¿¡/title¿Analysis ¡ul¿ ¡li¿¡p¿ The experimental precision should be estimated for all parts of this experiment and care should be taken for all of the measurements, but it is more important to investigate the relationships than it is to repeat the experiment many times or to try to achieve high precision in the data. ¡/p¿¡/li¿ ¡li¿¡p¿ Explain why the normal force on the block by the surface rather than the weight of the object is related to the frictional force. ¡/p¿¡/li¿ ¡li¿¡p¿ Interpret the slope and intercept of the graphs. ¡/p¿¡/li¿ ¡li¿¡p¿ Compare the slopes from each of the three parts. Decide which should be the same and which should be different. ¡/p¿¡/li¿ ¡li¿¡p¿ Calculate the % decrease of the static to kinetic coefficient of friction. ¡/p¿¡/li¿ ¡li¿¡p¿ Comment on the validity of the empirical rules of friction. ¡/p¿¡/li¿ ¡/ul¿

¡/section¿¡section¿¡title¿¡/title¿Questions

For all questions, and when possible, use your experimental or theoretical results to demonstrate your answers to the questions. ¡ol¿ ¡li¿¡p¿ Does the coefficient of friction depend on the area of contact? ¡/p¿¡/li¿ ¡li¿¡p¿ Does the coefficient of friction depend on the mass of the object? ¡/p¿¡/li¿ ¡li¿¡p¿ Does the coefficient of friction depend on the normal force of the object? ¡/p¿¡/li¿ ¡li¿¡p¿ Does the frictional force depend on the normal force of the object? ¡/p¿¡/li¿ ¡li¿¡p¿ Does the coefficient of kinetic friction depend on the speed of travel? ¡/p¿¡/li¿ ¡li¿¡p¿ When the object was pulled by a string, how would the forces be affected if the cord was not horizontal? ¡/p¿¡/li¿ ¡li¿¡p¿ What would happen to the coefficient of friction if the surfaces were lubricated with oil? ¡/p¿¡/li¿ ¡/ol¿

¡/chapter¿

¡chapter¿¡title¿Centripetal Force¡/title¿ ¡!– Revisions –¿ ¡introduction¿¡title¿¡/title¿Introduction

We will be investigating the force which is necessary to maintain the circular motion of an object. The apparatus used will allow you to spin an F-shaped arm which has a mass suspended from the top arm. This mass will be held in place by a spring which makes up the lower arm. The spring will provide the centripetal force. You will need to be familiar with the ideas of circular motion, centripetal versus centrifugal force, centripetal acceleration, and angular velocity. In addition to these concepts, try to understand how we will measure the angular speed in lab.

The centripetal acceleration ¡m¿a_c *< /m > iscalculatedfromthefollowingequationwritteneither < m >* $a_c = \frac{v^2}{r}$ *< /m > or < m >* $a_c = \omega^2 r$ *< /m > where < m >* v *< /m > isthelinearvelocityoftheparticle, < m >* ω *< /m > istheangularvelocity, and < m >* r *< /m > istheradiusofthecircle.Notethatangularvelocityismea*

From Newton's second law, ¡m¿$\vec{F} = m\vec{a}$ *< /m > .Therefore, theforcerequiredtokeeptheparticleofmass < m >* m *< /m > movinginacirclewithconstantspeedis < me >* $F = ma_c = \frac{mv^2}{r} = m\omega^2 r$ *< /me > Recallthatthecentripetalforceisnotaforceapplied < em > inaddition < /em > tootherexistingforces.Thecentripe em > whatevercombination < /em > ofexistingforceacttomaintaincircularmotion.*

¡/introduction¿ ¡objectives¿¡title¿Purpose¡/title¿

¡p¿It is our purpose to verify the above equation experimentally by measuring the applied force and comparing it to the specific combination of variables expressed as either ¡m¿mv^2 *r</m>or<m>mω^{2}r</m>.</p></intro*

¡section¿¡title¿¡/title¿Pre-Lab

¡ol¿ ¡li¿¡p¿ Why do we say that an object moving with ¡em¿constant speed¡/em¿ in a circular path is being accelerated? ¡/p¿¡/li¿ ¡li¿¡p¿ In which direction is that acceleration? How do you know? ¡/p¿¡/li¿ ¡li¿¡p¿ Is this ¡m¿a_c *< /m >< q > centripetal < /q > or < q > centrifugal? < /q >< /p >< /li >< /ol >*

¡/section¿¡section¿¡title¿¡/title¿The Experiment

The centripetal force is supplied by a spring. Since we cannot directly measure ¡q¿the force exerted by the spring while it is rotating¡/q¿ while it is rotating, determine how we can measure ¡q¿the force exerted by the spring during the rotation¡/q¿ when the spring is not rotating.

¡ul¿ ¡li¿¡p¿ By means of the lab apparatus, a mass ¡m¿m¡/m¿ can be made to rotate with a constant (and measurable) angular speed ¡m¿ω *< /m > .Withsomepractice, itispossibletoadjustthespeedsothattheradiusofth m >* r *< /m >, ismarkedontheapparatusandsocanbemeasuredeasily. < /p >< /li >< li >< p > Measuringthemassshouldbeanobvioustask. < /p >< /li >< li >< p > Measuringtheangularspeed < m >* ω *< /m > isstraightforward, butmaynotbeobvious.Todoso, considerthefollowing :< ul >< li >< p > Angularspeed < m >* (ω) *< /m > ismeasuredin < m >* $^{\text{radians}}/_{\text{second}}$ *< /m > . < /p >< /li >< li >< p >< m >* ω *< /m > isrelatedtotherotationalspeedwhichismeasuredin < m >* $^{\text{revolutions}}/_{\text{second}}$ *< /m > . < /p >< /li >< li >< p > Thereare < m >* 2π *< /m > radiansin1revolution. < /p >< /li >< li >< p > Wecancountthenumberofrevolutions. < /p >< /li >< li >< p > The < q > periodofrotation, < /q >< m >* T *< /m >, isdefinedasthenumberofsecondsperrevol /p >< /li >< li >< p > Wemeasuretheperiodnotbytimingasinglerevolution, butbymeasuringthetimeformultiple(time.) < /p >< /li >< /ul >< /p >< /li >< li >< p > []Asthemassrotates, itsperiodofrotationcanbemeasured.T /p >< /li >< li >< p > Repeattheentireprocedureforasecondvalueofr. < /p >< /li >< /ul >*

¡/section¿¡section¿¡title¿¡/title¿Analysis

From measurements of ¡m¿m¡/m¿, ¡m¿ω *< /m >, and < m >* r *< /m >, calculatethetheoreticalcentripetalforce /chapter >*

¡chapter¿¡title¿Conservation of Energy on a Linear Track¡/title¿ ¡!– Revisions –¿

¡introduction¿¡title¿Introduction¡/title¿

Conservation laws play a very important role in our understanding of our physical world. For example, the law of conservation of energy can be applied in all physical processes. This is a fundamental and independent statement about the nature of the physical world. It is not necessarily derivable from other laws like Newton's Laws of motion. Though for simple point mass systems, the law of conservation of energy can be derived from Newton's Laws. It can be shown that the net work done on a system is equal to the change in the kinetic energy ¡m¿$(W_{\text{net}} = \Delta K) < /m > ofthesystem; thisiscalledthework-energytheoremanditcanbewritteninavarietyofforms.Whenanetpositivewor$

When the gravitational force acts on a system, the work it does on the system, ¡m¿$W_g < /m >, isthegravitationalforce < m > (mg) < /m > timestheverticaldisplacement < m > (h = \Delta y) < /m >:< m > W_g = mg\Delta y < /m > .Forconvenience, thisiscalledthechangeingravitationalpotentialenergy < m > (W_g = -\Delta P) < /m > . < em > If < /em > thegravitationalforceistheonlyforceactingonthesystemthen < m > W_g = W_{\text{net}} < /m > andtherefore, < m > -\Delta P = \Delta K < /m > forthesystem.Whenaforcecanbeassociated q > conservativeforce. < /q > Anotherkindofpotentialenergydealswithanelasticpotentialenergy, likeinaspring m > P_s = \frac{1}{2}k\Delta x^2 < /m > .$

¡em¿If¡/em¿, on the other hand, a force dissipates energy, then it is called a ¡q¿nonconservative force¡/q¿ and it will have no associated potential energy. Frictional forces are an example of a nonconservative force and the work done by a frictional force is negative because (physically) the frictional force removes energy from the system and (mathematically) the frictional force and the displacement are in opposite directions. This work done by friction is converted into heat or sound. To distinguish the energy of heat or sound from the potential and kinetic energy, we define the total mechanical energy, ¡m¿E=K+P¡/m¿ at any point. Since frictional forces remove mechanical energy, we say ¡m¿$W_f = \Delta E = \Delta K + \Delta P < /m >$.

In general then, the law of conservation of energy states that energy can not be created or destroyed, but can only change from one form to another; or the total energy of the system at point A is equal to the total energy of the system at point B.

¡/introduction¿ ¡objectives¿¡title¿Experimental Objective¡/title¿

¡introduction¿¡p¿The purpose of this experiment will be to verify the validity of the law of conservation of mechanical energy, that ¡m¿$\Delta E = 0 < /m > asacartrunsalongatrack. < /p >< /introduction >< /objectives >$

¡/section¿¡section¿¡title¿¡/title¿The Experiment

We would like for you to verify the conservation of mechanical energy in two different situations; so, there are two parts to this experiment. We will first consider a flat track with accelerated motion, as in the Newton's Law lab and the Friction lab. We can then consider an inclined plane. You will not be given an explicit procedure, but rather you will be given a series of questions with answers that will imply the procedure. Part of the experiment is for you to figure out for yourself what the best course of action is. Please answer the questions as they are asked.

¡/subsection¿¡subsection¿¡title¿¡/title¿Flat Track

Set up the dynamics cart on a horizontal dynamics track. Set up the motion sensor at one end of the track and a pulley at the other end so that the pulley partly extends past the edge of the table. Hang the basket over the pulley so that it can accelerate the cart along the track – you might need extra weight in the cart to keep it from accelerating too fast. In order to use this motion to verify the validity of the conservation of mechanical energy, we need to measure some variables. Answer Questions **??** and **??** to decide on the relevant variables. Question **??** should help you determine how to finish setting up the equipment.

Once you decide what variables to measure, run the experiment for one set of masses while measuring the appropriate variables. Put the data into Excel and decide what plot(s) will allow you to verify the validity of the conservation of mechanical energy. Question **??** may help with this.

Decide if you need a trendline. Relate the information in Question **??** to the statement you are trying to verify.

¡/subsection¿¡subsection¿¡title¿¡/title¿Sloped Track

Remove the pulley from the track. Your cart will have either a spring-loaded ¡q¿battering ram¡/q¿ on the front or a pair of magnets. If you have the battering ram, then you will want the end of the track with the rubber nub at the bottom of the incline. If you have the magnets, then you need to replace the pulley with a ¡q¿C¡/q¿ shaped ¡q¿catch-bar.¡/q¿ ¡em¿Ask for help from the instructor!¡/em¿ The catch-bar has magnets in it that will repel the magnets in the cart. In this case, the cart must not be going so fast as to come into physical contact with the magnets on the catch-bar.

Raise one end of the dynamics track. Question **??** should help decide how tilted. Measure the tilt angle of the track with two methods: use a protractor, and measure the vertical rise and track length and calculate the tilt angle using the inverse-sine function. Answer Question **??**. As you continue to set up the track for measurements, consider answering Questions **??**, **??**, and **??** again for this situation to help you decide on the appropriate accessories (sensors); but note Question **??** as you think about the answers to the previous questions.

Once you decide on the variables to be measured, but before you make the measurements, you will need to calibrate your position measurements. We would like zero to correspond to being at the bottom of the ramp, so place the cart stationary at the bottom and use the motion sensor to measure this position. In order to verify the validity of the conservation of mechanical energy, release the cart from rest near the top of the ramp and let it roll down the incline, bouncing three times before you stop the measurement. Do this for one value of mass.

Transfer these data to Excel again and decide on the best graph to verify the objective. Again, Question **??** may help with this; however, you will also need to consider Question **??**. Decide if you need a trendline and where it would be fit. Relate the information in Question **??** to the statement you are trying to verify.

¡/section¿¡section¿¡title¿¡/title¿Additional Analysis

We are now going to take a closer look at the irregularities of the data and investigate some variations to try to explain what those data say.

¡ul¿

¡li¿¡p¿ Before drawing conclusions about the validity of the conservation of mechanical energy, consider Question **??**.

¡/p¿¡/li¿ ¡li¿¡p¿ One explanation of a loss of energy (non-conservation) is friction. List all of the places where two pieces of material rub against each other. Since ¡m¿$F_f = \mu F_N$ < /m > *, doanyoftheselocationshaveanormalforcethatcanbevaried?(RecallQuestion* **??**.)*Asanindependentmeasureofthea*

¡/p¿¡/li¿ ¡li¿¡p¿ A second explanation for the loss of energy is that some component is gaining rotational kinetic energy. The formula for this is ¡m¿$K_R = \frac{1}{2}I\omega^2$ < /m >*, where* < m > I < /m > *isthemomentofinertia* < fn > *Inthiscase, themomentofinertiaisprobablyalittlelessthan* < m > $\frac{1}{2}mr^2$ < /m >*, where* < m > m < /m > *isthemassoftherotatingobjectand* < m > r < /m > *istheradiusoftherotatingobject.Thisisnotaconvenientwaytocalculate* < m > I < /m > *atthistime.* < /fn >*, and* < m > ω < /m > *istheangularspeed* < m > $\omega = v/r$ < /m > *.Assumingthatanydiscrepancythatyoufoundintheconservationofenergyisduetotherotationalkineticenergyofthep* m > K_R < /m > *and* < m > ω < /m >*, whatisthemomentofinertiaforthepulley?Canyoutellifthisisareasonablee* /p >< /li >< /ul >

¡/section¿¡section¿¡title¿¡/title¿Questions

¡ol¿ ¡li¿¡p¿ In order to verify ¡m¿$\Delta E = 0$ < /m >*, wewillneedtocalculate* < m > E < /m > *as* < m > $E = K + P$ < /m > *.Therefore, weneedtoknowthekineticenergy,* < m > $K = \frac{1}{2}mv^2$ < /m >*, theenergyof* < em > *somemass* < /em >*,* < m > m < /m >*, movingataspeed* < m > v < /m > *.Whichmassdoyouneedtomeasure?Howcanyoumeasurethevelocity?* < /p >< /li ><

li >< p > Inordertoverify < m > $\Delta E = 0$ < /m >, wewillneedtocalculate < m > E < /m >
as < m > $E = K + P$ < /m > .Therefore, weneedtoknowthepotentialenergy, < m > $P = mgy$ <
/m >, theenergyof < em > somemass < /em >, < m > m < /m >, locatedsomeheight, < m >
y < /m >, abovetheground.Whichmassdoyouneedtomeasure?Howcanyoumeasuretheposition? <
/p >< /li >< li >< p > Inordertomeasurethepositionofthefallingmassandthevelocityofthesystem, doyouneed
q > translate < /q > thepositionorvelocitydatainordertofindthespecificvaluesthatyouactuallyneed. <
/p >< /li >< li >< p > Toverify < m > $\Delta E = 0$ < /m >, wewillneedtograph < m > E < /m >
, thetotalmechanicalenergy, asafunctionoftime.Whatdoyouexpectthisgraphtolooklike, ifthelawisvalid?Ifnot? <
ol >< li >< p > Doesthekineticenergychangeduringthismotion?Is < m > $\Delta K = 0$ < /m >
?Consideringtheinitialandfinalvaluesofthekineticenergy, < m > K_i < /m > and < m > K_f <
/m >, whatwouldagraphof < m > K < /m > versustimelooklike? < /p >< /li >< li >< p >
Doesthepotentialenergychangeduringthismotion?Is < m > $\Delta P = 0$ < /m >?Consideringtheinitialandfinalval
m > P_i < /m > and < m > P_f < /m >, whatwouldagraphof < m > P < /m > versustimelooklike? <
/p >< /li >< li >< p > Assumingthatthemechanicalenergyisconserved, whatwouldagraphlooklikeifitincluded
m > E < /m >, < m > K < /m >, and < m > P < /m >?Whatifthemechanicalenergyisnotconserved?Howwo
m > K < /m > and < m > P < /m > beaffectedinthesetwocases? < /p >< /li >< li >< p >
(Part **??**only)Whenthecartisatthebottomofthetrackduringthemotion, thevaluesofpositionbecomenegative(less
/p >< /li >< /ol >< /p >< /li >< li >< p > Pleasenotetheoverallchangeinpotentialenergy, <
m > ΔP < /m >, andtheoverallchangeinthekineticenergy, < m > ΔK < /m > .Shouldeitherofthesebe relatedto
m > ΔE < /m > and, ifso, how? < /p >< /li >< li >< p > Wewantthecarttoacceleratedownthetrack(nottoosl
em > toofast < /em >?Don'tusethatslope!Howfastis < em > tooslow < /em >?Useaslopesomewhereinbetwee
/p >< /li >< li >< p > Afteryoumeasuretheangleofinclineinthesetwoways, considertheuncertaintyinthemea
/p >< /li >< li >< p > Themotionsensorwillmeasurethemotionofthecart < em > along <
/em > theramp, butthepotentialenergyneedsthe < em > vertical < /em > positionofthecart.Whichtrigfunctio
/p >< /li >< li >< p > Ifthemechanicalenergyisconserved, then < me > $\frac{1}{2}mv_i^2+mgy_i = \frac{1}{2}mv_f^2+$
mgy_f < /me > Whatdoyounoticeaboutthemass?Isyourgraphdifferentifthemassofthecartchanges?Doesthisissu
me > $W_{\text{nc}} = \frac{1}{2}mv_f^2+mgy_f-\frac{1}{2}mv_i^2-mgy_i$ < /me > Whatdoyounoticeaboutthemassnow?Doesyourgraphsupportor
/p >< /li >< li >< p > Weneedtolookfortheenergylostineachgraph. < ol >< li >< p >
WhenyoulookatthegraphfromPart **??**for < m > E < /m >, istheenergyconservedoristhereenergylost?Iflost, c
/p >< /li >< li >< p > WhenyoulookatthegraphfromPart **??**for < m > E < /m >, therearejumpsintheenerg
ol >< li >< p > Whatishappeningbetweenthejumps?DoesPart **??**helptoexplainthesesectionsofthegraph?Comp
/p >< /li >< li >< p > Whatishappeningatthetimeofthose < q > jumps? < /q > Fromthetrendofthegraph, co
q > missing < /q > energygoes.Istheratioof < q > energydiscrepancy < /q > tototalpriorenergythesameforea
/p >< /li >< /ol >< /p >< /li >< li >< p > Commentingeneral, onthelawofConservationofMechanicalEne
/p >< /li >< /ol >< /p >< /li >< li >< p > Givenanrampinclinedatsomeangle < m > θ <
/m >, whatisthecomponentofthegravitationalforceaimeddowntheramp?Assumingthatthereisnofriction, whati
m > $F_{\text{net}} = ma$ < /m >, theaccelerationshouldbe < ellipsis/ >? < fn >< m > $a = g\sin\theta$ < /m >
. < /fn > Fromyourexpression, whatdoyouneedtomeasureinordertofindtheexpectedvalueof <
m > a < /m >?(RecallQuestion **??**.) < /p >< /li >< li >< p > Ifthereisfriction, thenhowdoyouexpecttheactu
/p >< /li >< /ol >< /chapter >

¡chapter¿¡title¿Peer Review¡/title¿ ¡!– Revisions –¿

¡introduction¿ ¡p¿The most important part of doing science is the peer-review process. After one completes a research project, the report is submitted for publication. The publisher has a number of reviewer (usually made up of respected authors) and the submission is sent to two (sometimes three) reviewers who advise the publisher on the merits of the work. Once you make a submission, it might be two months before the reviewers finish reviewing the work. Generally the publisher will return the reviewers' comments to the author. If all reviewers agree that the paper is viable, then the publisher accepts it. If they agree not to accept a paper, then it is rejected. If the reviewers are split, then the decision is at the discretion of the publisher. In most cases, the reviewer makes suggestions for how to improve the paper or where to clarify the discussion. In some cases, the author must either significantly revise the entire project or make an argument why the reviewer is either mistaken or is merely pointing out the specific point-of-contention that the author was intending to spark in the readers. In most cases, the process of an accepted paper is ¡ol¿ ¡li¿¡p¿ Author submits article. ¡/p¿¡/li¿ ¡li¿¡p¿ Publishers submit to reviewers, who read and return comments to the publisher. ¡/p¿¡/li¿ ¡li¿¡p¿ Publisher gives author a chance to respond; most do (!). ¡/p¿¡/li¿ ¡li¿¡p¿ Publishers provide authors' response to reviewers, who then give final approval (or not). ¡/p¿¡/li¿ ¡li¿¡p¿ Paper goes to Editor, who returns paper to author for grammar, spelling, and formatting corrections. ¡/p¿¡/li¿ ¡li¿¡p¿ After the author fixes or refuses to fix the editor's ¡q¿suggestions,¡/q¿ the paper goes to publication. ¡/p¿¡/li¿ ¡/ol¿ This process can take anywhere from 1-2 months to a year and a half. This week, we will do Step **??**. Next week, we will do Step **??**.¡/p¿

¡p¿Usually during the review process, the reviewer is not informed of the name of the submission author – to minimize influencing the reviewer. Similarly, the names of the reviewers are not revealed to the submission author. This is called ¡q¿double-blind review.¡/q¿ Some disciplines are specialized enough that all of the active researchers are familiar with each other's work. In those cases, it is possible to guess who an author is (based on the approach to the project) or to guess who the reviewer is (based on the style of comments). In principle, both sides are civil in their comments and reactions because they are members of the same community and see each other annually at the topic meetings. Researchers are competitors and collaborators who only progress by working off of each others' ideas.¡/p¿

¡/introduction¿ ¡section¿¡title¿The Assignment¡/title¿

¡p¿In order to manage the double-blind review process, before you leave lab today you will all turn in your (personally selected) code name. Do not tell anybody what you selected and do not use a nickname that is easily recognizable by others – the point of a secret identity is to keep the secret!¡/p¿

¡p¿In this week's lab, one lab section will do Lab **??** and the other lab section will do Lab **??**. The underlying ideas are similar to each other and will help you next week when you review an article submitted by a colleague who did the other experiment. When you submit your lab this week, you will submit your notebook and two (almost identical) copies of your report. One copy will have your name ¡em¿and¡/em¿ your secret code name. The other copy will have ¡em¿only¡/em¿ your secret identity.¡/p¿

¡/section¿ ¡/chapter¿

¡chapter¿¡title¿Hooke's Law and Simple Harmonic Motion¡/title¿ ¡!– Revisions –¿

¡introduction¿¡title¿Introduction¡/title¿

Oscillatory motion is one of the most common types of motions and can occur in any physical system. Mechanical systems can experience a periodic motion, and then will vibrate at a natural frequency. This phenomenon is called resonance. Sound is a vibration in the air, which we hear with our ears; light is an oscillation of electric and magnetic fields, which we can see. The atoms and molecules in all objects are in a state of continual vibration, which we can detect as the temperature of the object, and the atomic vibrations of a quartz crystal can be used as a very accurate timer.

The study of repetitive motion is not just an intellectual exercise, but actually enables us to model complicated systems with simple harmonic motion.

In this lab, we will consider spring as an example of oscillation. This oscillation is due to the elasticity of a spring. We will need to measure the stiffness of the spring and relate this to the rate of oscillation.

Most systems have elastic properties, such that when the system is deformed or vibrated, there is a force which tries to restore the system to its original state. If the restoring force is proportional to the displacement from its equilibrium position, then the object is said to be in simple harmonic motion (SHM). A linear restoring force can be expressed mathematically by the equation ¡me¿ $\vec{F} = -k\vec{x}$ or as $a = \frac{d^2x}{dt^2} = -\frac{kx}{m}$ < /me > *where* < *m* > *F* < /*m* > *istherestoringforce*, <
m > *x* < /*m* > *isthedisplacementfromtheequilibriumposition*(*orzeroposition*), < *m* > *k* < /*m* >
isaproportionalityconstant, *andtheminussignindicatesthattherestoringforceisalwaysoppositethedirectionofthe*
m > *k* < /*m* > *iscalledthespringconstant*, *andrepresentstheratiooftheappliedforcetotheelongation*.*Thespringco*
m > *k* < /*m* > *givesarelativeindicationofthestiffnessofthespring*.*Ifthespringsystemisinequilibrium* <
m > ($\sum F_i = 0$) < /*m* > *thentherestoringforceisequaltotheforcepullingonthespring*, *andthisforceisproportiona*

¡ul¿ ¡li¿¡p¿ Make a sketch of your expectation for the displacement of a mass on a spring as a function of the time. ¡/p¿¡/li¿ ¡li¿¡p¿ On this graph, locate and label: the equilibrium positions ¡m¿(x=0)¡/m¿, and the places of maximum and minimum velocity. ¡/p¿¡/li¿ ¡li¿¡p¿ Based on the information in the introduction, make a sketch of the pull force as a function of the displacement from the equilibrium position (initial position). ¡/p¿¡/li¿ ¡/ul¿

¡/subsection¿¡subsection¿¡title¿¡/title¿Hooke's law We will first measure the elasticity of the spring, using Equation (**??**). ¡ul¿ ¡li¿¡p¿ With the available spring, attach it rigidly and hang it vertically against the Dynamics Track. Hang various masses and measure the elongation of the spring, to a maximum of 60 cm. Do not over stretch the spring. Record the bottom end of the mass hanger for the initial reference position. If a tapered spring is used, the small end should be at the top. ¡/p¿¡/li¿ ¡li¿¡p¿ Measure the elongation both when the masses are added and then when they are removed. Perfectly elastic objects (possibly your spring) will return to the exact same location when pulled with the same force whether they are being stretched out or being allowed to relax back after stretching. ¡/p¿¡/li¿ ¡li¿¡p¿ You will be graphing the relationship between the mass and the displacement. ¡/p¿¡/li¿ ¡/ul¿

¡/subsection¿¡subsection¿¡title¿¡/title¿Oscillating Spring We will next consider the periodicity of an oscillating spring. ¡ul¿ ¡li¿¡p¿ With the same range of masses as in Sec. **??**, measure the period of oscillation for each mass. You ¡em¿can¡/em¿ but do not ¡em¿have to¡/em¿ use the same values of mass, as long as the set of masses sampled are in the same range. ¡/p¿¡/li¿ ¡li¿¡p¿ You will be graphing the relationship between the mass and the period. I recommend using ¡m¿T/(2π) <
/*m* > *asthevariablerepresentingtheperiod*(*becauseitgivesniceresultsforthegraphicalparameters*−
−*slopeandintercept*). < /*p* >< /*li* >< *li* >< *p* > *Advice* : *Keeptheamplitudeofvibrationsmall*, *becausethereisas*
/*p* >< /*li* >< /*ul* >

¡/section¿¡section¿¡title¿¡/title¿Analysis

¡ul¿ ¡li¿¡p¿ Graph both data sets (Sections **??** and **??**) in such a way that the spring constant can be determined graphically (from a linear fit model). ¡ul¿ ¡li¿¡p¿ When you graph the relationship between the mass and the displacement, recall that Equation (**??**) depends on two specific variables. ¡/p¿¡/li¿ ¡li¿¡p¿ When you graph the relationship between the mass and the period, recall that Equation (**??**) depends on one specific variable. ¡/p¿¡/li¿ ¡li¿¡p¿ With some effort, you should be able to recognize the units of the slope and intercept and find the relevant values. ¡/p¿¡/li¿ ¡/ul¿ ¡/p¿¡/li¿ ¡li¿¡p¿ Physically interpret the meaning and value for the slopes, and the x and y intercepts for both graphs. ¡/p¿¡/li¿ ¡li¿¡p¿ Calculate the spring constant for both data sets, using a linear regression method. ¡/p¿¡/li¿ ¡li¿¡p¿ So far in the analysis, the mass of the spring has been neglected. How would including the spring mass (or a partial %) affect the slopes or intercepts of the two graphs? For the period graph one would expect to get a zero period with a zero mass. Why? What was your observation for the y-intercept? If the data was modified by adding a constant amount of mass to each mass value (say 1/3 the mass of the spring) and then re-compute the linear regression, then what happens to the slope and intercept values? And do you get a higher linear correlation coefficient? ¡/p¿¡/li¿ ¡li¿¡p¿ If you assume a value for ¡m¿g¡/m¿, then both graphs will give you ¡m¿k¡/m¿. Compare the precision for these two methods. ¡/p¿¡/li¿ ¡li¿¡p¿ If you do not assume a value for ¡m¿g¡/m¿, then you can use one graph to find ¡m¿k¡/m¿ and use this calculated value and the other graph to compute ¡m¿g¡/m¿. How does this value of ¡m¿g¡/m¿ compare to your expectations? ¡/p¿¡/li¿ ¡li¿¡p¿ Compare the elongations when the masses were added and then removed. Explain any differences. ¡/p¿¡/li¿ ¡li¿¡p¿ Quantify the major sources of uncertainty in this experiment. Which of the experimental measurements has the largest relative uncertainty? ¡/p¿¡/li¿ ¡/ul¿

¡/section¿¡section¿¡title¿¡/title¿Questions

¡ol¿ ¡li¿¡p¿ Why should the amplitude of vibration be kept as small as possible? ¡/p¿¡/li¿ ¡li¿¡p¿ Is the spring totally elastic? (Does the elongation return to the same position when the masses are removed?) ¡/p¿¡/li¿ ¡li¿¡p¿ Which method do you think is more precise? ¡/p¿¡/li¿ ¡li¿¡p¿ Does the force of gravity affect the value of ¡m¿k¡/m¿ (as derived from each method)? Why or why not? ¡/p¿¡/li¿ ¡li¿¡p¿ If this experiment were conducted on the moon, would either method give a different result for the value of ¡m¿k¡/m¿? Explain. ¡/p¿¡/li¿ ¡/ol¿ ¡/chapter¿

¡chapter¿¡title¿The Simple Pendulum¡/title¿ ¡!– Revisions –¿

¡introduction¿¡title¿Introduction¡/title¿

A simple pendulum consists of a small bob of mass ¡m¿(m)¡/m¿ suspended by a light (assumed to be massless) string of length ¡m¿(L)¡/m¿, and the string is firmly attached at its upper end. This pendulum is a mechanical system which we will assume exhibits simple harmonic motion. That is, the restoring force on the pendulum is proportional to the displacement from the equilibrium position.

Oscillatory motion is one of the most common types of motions and can occur in any physical system. Mechanical systems can experience a periodic motion, and then will vibrate at a natural frequency. This phenomenon is called resonance. Sound is a vibration in the air, which we hear with our ears; light is an oscillation of electric and magnetic fields, which we can see. The atoms and molecules in all objects are in a state of continual vibration, which we can detect as the temperature of the object, and the atomic vibrations of a quartz crystal can be used as a very accurate timer. The study of repetitive motion is not just an intellectual exercise, but actually enables us to model complicated systems with simple harmonic motion.

Galileo (1564-1642) investigated the natural motions of a simple pendulum. From his observations he concluded that ¡q¿vibrations of very large and very small amplitude all occupy the same time.¡/q¿ Galileo's time interval of measurement was his own pulse rate. With today's modern technology we have much more precise measuring instruments. This experiment will investigate the relationships between the physical characteristics of the pendulum and the period of the pendulum.

¡/introduction¿ ¡section¿¡title¿Experimental Objectives¡/title¿

¡introduction¿¡p¿ ¡ul¿ ¡li¿¡p¿ Determine the relationship between the period of the pendulum and its amplitude. ¡/p¿¡/li¿ ¡li¿¡p¿ Determine the relationship between the period of the pendulum and its mass. ¡/p¿¡/li¿ ¡li¿¡p¿ Determine the relationship between the period of the pendulum and the length of the pendulum. ¡/p¿¡/li¿ ¡li¿¡p¿ Use a graphical analysis to investigate these relationships, and from the best linear graph determine an empirical equation for the period of a pendulum. ¡/p¿¡/li¿ ¡li¿¡p¿ Gravity also plays a part in this experiment, so include gravity into your empirical equation, and use unit analysis to help figure out this relationship. ¡/p¿¡/li¿ ¡/ul¿ ¡/p¿ ¡/introduction¿ ¡/objectives¿

¡section¿¡title¿¡/title¿Procedure

You will have available for your use: pendulum bobs, string, timers, and a protractor. Be careful to fix the string to a point of support which will not move or vibrate as the pendulum swings. You will test each of the three relationships above (period vs amplitude, vs mass, and vs length). While measuring one relationship, you should ensure that – if they matter – then the other two variables are not varied. For example, when changing the pendulum mass do not vary the pendulum's length or its amplitude.

Some considerations while doing this lab: ¡ul¿ ¡li¿¡p¿ The amplitude of oscillation is the maximum angle which the string makes with the vertical. ¡/p¿¡/li¿ ¡li¿¡p¿ In general when testing the mass or the length, it is best to keep the amplitude of oscillation small. ¡/p¿¡/li¿ ¡li¿¡p¿ When testing any of the relationships, you should measure a few widely-separated values. If these seem to vary significantly, then fill in the gaps between those measurements to make a reliable graph. See question **??**. ¡/p¿¡/li¿ ¡li¿¡p¿ If you can prove that the period is not affected by one of these variables, then you do not need to worry about keeping it constant while you measure the other variables. ¡/p¿¡/li¿ ¡li¿¡p¿ Your graphical analysis will be better if your graph is linear. Consider question **??** for advice on making your graphs. ¡/p¿¡/li¿ ¡/ul¿

¡/section¿¡section¿¡title¿¡/title¿Questions

¡ol¿ ¡li¿¡p¿ Was Galileo's statement precise? ¡/p¿¡/li¿ ¡li¿¡p¿ Does this pendulum follow simple harmonic motion? ¡/p¿¡/li¿ ¡li¿¡p¿ How many observations should you take in order to obtain good data? ¡/p¿¡/li¿ ¡li¿¡p¿ Air resistance gradually decreases the amplitude of the pendulum. What effect does this have on the period of the pendulum? ¡/p¿¡/li¿ ¡li¿¡p¿ What effect would stretching of the string have on your results? ¡/p¿¡/li¿ ¡li¿¡p¿ How does gravity affect this experiment? What would happen to the results if this experiment were conducted on the moon? ¡/p¿¡/li¿ ¡li¿¡p¿ If you have a parabolic graph, such as ¡m¿y=ax^2 < /m >, *thenyoumightconsidergraphing* < m > *y* < /m > *versus* < m > x^2 < /m > *togetalineargraph.Whatisthephysicalmeaningoftheslopeandtheinterceptofeachofyourgraphs*? < /p >< /li >< li >< p > *Whyisitagoodideatokeeptheamplitudeofvibrationsmall*? < /p >< /li >< li >< p > *Wheretoandhowshouldthependulumlengthbemeasured*? < /p >< /li >< /ol >

¡/chapter¿

¡chapter¿¡title¿Ballistic Pendulum¡/title¿ ¡!– Revisions –¿

¡section¿¡title¿Introduction¡/title¿

Conservation laws will again play a significant part in this ballistic pendulum experiment. A ballistic pendulum is a devise which has a cavity in the pendulum bob, and a small ball will be fired into and captured in this cavity. When this happens, the initially stationary pendulum will swing about the pendulum's point of support. During this collision between the ball and the pendulum, the momentum of the total system should be conserved from the instant just prior, to the instant just after the collision. Physicist's hold true a general conservation law for momentum which applies in all interactions of two or more objects where there are no other outside forces acting on the system. For collisions on the earth, the force of gravity is an outside force but momentum is still considered to be conserved if the time of the interaction is small.

The collision in this experiment is called a totally inelastic collision because after the collision the two objects are held together, they move together with a single velocity, and the kinetic energy of the system is not conserved during the collision. Using the general conservation of momentum law for the collision described an equation can be written for the initial velocity of the ball in terms of the velocity of the system at the instant after the collision and the individual masses of the ball and the pendulum. After the collision the pendulum and ball will swing and at the highest point in the swing they will be caught. The KE of the system at the instant after the collision is converted totally to PE at the highest point in the swing. The velocity of the system at the instant after the collision can then be determined using the law of conservation of energy. Then with these two conservation laws, the initial velocity of the ball can be determined.

The initial velocity of the ball can also be determined by firing the ball horizontally off the edge of the table and analyzing the 2-dimensional projectile motion of the ball moving under the influence of the gravitational force. This analysis involves separating the motion into its component directions, using the standard kinematic equations of motion and an appropriate set of measurements.

For these two very different techniques calculate the same initial velocity of the projectile. An analysis and comparison of the two methods will help to illustrate the interconnections between these physics topics.

¡/introduction¿ ¡objectives¿¡title¿Experimental Objectives¡/title¿

¡introduction¿¡p¿To determine the initial velocity of the ballistic projectile from two different sets of experimental measurements, 1) the range and vertical height measurements of the projectile motion, and 2) through the use of the ballistic pendulum.¡/p¿ ¡/introduction¿ ¡/objectives¿

¡section¿¡title¿¡/title¿Pre-Lab Work

¡ul¿ ¡li¿¡p¿ Draw before and after pictures for a totally inelastic collision between two masses, ¡m¿m$_1 < /m > and < m > M_2 < /m > .Assumethat < m > M_2 < /m > isinitiallystationary, andthat <$
$m > m_1 < /m > isinitiallymovinghorizontallywithavelocityof < m > v < /m > . < /p ><$
$/li >< li >< p > Forthiscollision, writeouttheconservationofmomentumequation.Solvefortheshared velocityo$
$/p >< /li >< li >< p > Afterthecollision, thependulumandballwillswing.TheKEofthepairattheinstantaftert h$
$/p >< /li >< li >< p > Combinethesetwoconservationlawstoderiveanexpressionfortheinitialvelocityoftheball$
$/p >< /li >< li >< p > Drawapictureoftheball'spathwhenfiredhorizontallyoffofatable.Drawtheballinitsinit$
$/p >< /li >< li >< p > Forthisprojectilemotion, usethekinematicequationsofmotiontoderiveanequationforth$
$/p >< /li >< /ul >$

¡/section¿¡section¿¡title¿¡/title¿Procedure

¡/subsection¿¡subsection¿¡title¿¡/title¿Projectile Motion

¡ul¿ ¡li¿¡p¿ Set-up the ballistic spring gun so that it will fire the projectile ball horizontally off the edge of the table. Use a bubble level or the ball itself to make sure that the gun is level. Move the pendulum out of the way. Clamp the apparatus to the table, and use cardboard pads. The initial velocity of the projectile can be changed by adjusting the spring tension. ¡/p¿¡/li¿ ¡li¿¡p¿ Tape a piece of paper to the floor where the ball will land, then tape a sheet of carbon paper at this spot. ¡/p¿¡/li¿ ¡li¿¡p¿ Be careful not to hit anything or anybody with the ballistic projectile! Use larger pads or boxes to protect the tables and the walls. ¡/p¿¡/li¿ ¡li¿¡p¿ Repeat the experiment for a sufficient number of trials (15-20), and calculate a standard deviation of the range. ¡/p¿¡/li¿ ¡li¿¡p¿ Calculate the initial velocity (and uncertainty) of the projectile after taking the appropriate measurements. ¡/p¿¡/li¿ ¡/ul¿

¡/subsection¿¡subsection¿¡title¿¡/title¿The Ballistic Pendulum

The ballistic pendulum apparatus consists of three parts: 1) a ballistic spring-loaded gun for the firing of the projectile, 2) a hollow pendulum bob suspended by a light rod for catching the fired projectile, and 3) an angled platform for catching the pendulum bob at the highest position of the bob's swing.

¡ul¿ ¡li¿¡p¿ When removing the ball from the pendulum, be sure to push up on the spring catch in the pendulum so as to not to damage the pendulum. ¡/p¿¡/li¿ ¡li¿¡p¿ The pointer on the side of the pendulum indicates the position of the center of mass of the system. ¡/p¿¡/li¿ ¡li¿¡p¿ Do not try to take the apparatus apart, the instructor will give you the mass of the pendulum. ¡/p¿¡/li¿ ¡li¿¡p¿ Clamp the base to the table, so that there is no relative motion of the base. ¡/p¿¡/li¿ ¡li¿¡p¿ Fire the ball into the pendulum bob and mark the final notch position of the pendulum. ¡/p¿¡/li¿ ¡li¿¡p¿ Repeat the experiment with a sufficient number of trials (15) so that a standard deviation of the notch positions can be obtained. ¡/p¿¡/li¿ ¡li¿¡p¿ Measure the change in height of the pendulum's pointer from its initial position to the average notch position. Calculate the uncertainty in this distance. ¡/p¿¡/li¿ ¡li¿¡p¿ Calculate the initial velocity (and uncertainty) of the projectile ball. ¡/p¿¡/li¿ ¡/ul¿

¡/section¿¡section¿¡title¿¡/title¿Analysis

Quantitatively compare the two methods. Calculate a percent difference between the two methods. Calculate the uncertainties for the velocity in both methods (propagation of error), and also write these in a % form. Which method is more precise? Decide whether this experiment has random or systematic errors. Discuss and show your experimental evidence.

¡/section¿¡section¿¡title¿¡/title¿Questions

¡ol¿ ¡li¿¡p¿ Under what conditions are the laws of momentum and energy conserved in this experiment? State why. Why is the mechanical energy not conserved during the collision? Conclude whether the collision between the steel ball and the pendulum bob is elastic or inelastic. ¡/p¿¡/li¿ ¡li¿¡p¿ During the collision, what percent of the kinetic energy of the ball was transferred to the combination of the pendulum and ball? If energy is lost, where does it go? ¡/p¿¡/li¿ ¡li¿¡p¿ If this gun was aimed and fired vertically from the table top, would the ball hit the ceiling? Assume a vertical height of 1.5 meters. Show all of your work. ¡/p¿¡/li¿ ¡li¿¡p¿ What effect does the force of gravity have on the horizontal velocity of the projectile? ¡/p¿¡/li¿ ¡li¿¡p¿ Does the air resistance on the ball have a significant effect on the results of this experiment? ¡/p¿¡/li¿ ¡/ol¿

¡/chapter¿

¡chapter¿¡title¿Conditions of Equilibrium – Model of a Human Forearm¡/title¿ ¡!– Revisions –¿ ¡introduction¿¡title¿Introduction¡/title¿

Objects that are not accelerating are said to be in a state of equilibrium. If the object is moving at a constant velocity, then it is in equilibrium. If the object is at rest, then it is in ¡q¿static equilibrium.¡/q¿ These principles apply to many physical examples in engineering, architecture, and biophysics. In particular, these principles allow one to be able to analyze and calculate the forces on the beams or the cables in a bridge or the forces at work in the muscles and bones in the human body.

The two conditions for equilibrium can be stated in equation form: First, if the body's center of mass is in translational equilibrium then it will not accelerate in any direction. ¡me¿ $\sum \vec{F} = 0$ < /me > *Secondly, ifthebodyisinrotationalequilibriumthenitwillnotrotateaboutanypointoraxisofrotation.* < me > $\sum \tau = 0$ < /me >

For all systems such as these there is a special point called the center of mass or center of gravity of the system. The center of mass calculation is a weighted average of the individual masses (giving more emphasis to those positions where there is more mass). The location of this special point can be useful in determining whether the system will be in equilibrium. For an object with uniform density, such as a half-meter stick, the center of mass is at the center of the stick.

¡/introduction¿ ¡objectives¿¡title¿Experimental Objectives¡/title¿

¡introduction¿¡p¿You will verify the equations for translational and rotational equilibrium by experimentally balancing the system and comparing the measurements to the results from these equations.¡/p¿ ¡/introduction¿ ¡/objectives¿

¡section¿¡title¿¡/title¿Prelab Work

¡ul¿ ¡li¿¡p¿ Define the following terms: torque, lever arm, and center of mass. ¡/p¿¡/li¿ ¡li¿¡p¿ State in a sentence, the first and second conditions of equilibrium. ¡/p¿¡/li¿ ¡li¿¡p¿ For a linear function, the slope can be determined from knowing the values of any two points, ¡m¿(x_1, y_1) < /m > *and* < m > (x_2, y_2) < /m > *.Usingthesetwopointswriteoutaformulafortheslopeoftheline.* < /p >< /li >< li >< p > *DrawaforcediagramforahorizontalforearmasoutlinedinSection* **??**.*Labelalloftheappropriateforces* /p >< /li >< /ul >

¡/section¿¡section¿¡title¿¡/title¿Procedure

¡/subsection¿¡subsection¿¡title¿¡/title¿Determine the Spring Constant of the Bicep Muscle

Before creating the forearm model, we need an equation for the force that the bicep exerts. Since the spring models the bicep, we can use Hooke's law: ¡m¿F = k(x-x_0) < /m >, *where* < m > x_0 < /m >
istheequilibriumposition.Tofindtheforceasafunctionofposition, < m > $F(x) = kx + b$ < /m >
, youwillmaketwomeasurementsofpositionandforceandthendeterminethespringconstant < m >
k < /m > *andtheintercept* < m > *b* < /m > . < ul >< li >< p > *Hangthespringfromtheverticalsupportsothatitis*
/p >< /li >< li >< p > *Measurethepositionofthebottomhookofthespring,* < m > x_1 < /m >
, withabout < m > $m_1 = 250\,\mathrm{g}$ < /m > *placedonthespring.Measurethepositionofthebottomhookofthespring,* <
m > x_2 < /m >, *withabout* < m > $m_2 = 850\,\mathrm{g}$ < /m > *placedonthespring.(Ifyoumeasurefromthefloor, whichism*
m > *k* < /m > *willbenegative.)* < ul >< li >< p > *Parallaxerrorscanbeverysignificantwiththismeasurement.W*
/p >< /li >< li >< p > *Eachpersoninthegroupshouldcarryoutthesemeasurementswithoutlookingattheothergrou*
/p >< /li >< /ul >< /p >< /li >< li >< p > *Withthesetwomeasurements, calculatethespringconstant* <
m > *k* < /m > *astheslopeofthegraphof* < m > *F* < /m > *versus* < m > *x* < /m >
.Youdonothaveto − −andshouldnot − −actuallycreateagraphof < m > *F* < /m > *versus* < m >
x < /m > *todothis,* < em > *if* < /em > *youremember yourprelabworkforslope.* < ul >< li ><
p >< m > *b* < /m > *istheintercept.Usingonepairofdataandyourknowledgeof* < m > *k* < /m >
, find < m > *b* < /m > . < /p >< /li >< li >< p > *Determinetheuncertaintyin* < m > *k* < /m >
and < m > *b* < /m > . < /p >< /li >< li >< p > *Now, withvaluesanduncertaintiesfor* < m >
k < /m > *and* < m > *b* < /m >, *youcanuse* < m > $F = kx+b$ < /m > *tofindtheforcethatthespringthinksitissupp*
/p >< /li >< /ul >< /p >< /li >< /ul >

¡/subsection¿¡subsection¿¡title¿¡/title¿Experimental Setup for the Equilibrium Experiment

For our purposes, the forearm can be considered to jut out forwards from a vertical upper arm with the hand and the weight of the forearm itself pulling down while the bicep holds the forearm up. The elbow joint is assumed to be a nearly frictionless pivot point for the forearm, allowing the forearm to rotate about the elbow. The upper arm (the humerus) is connected to the forearm (the ulna) at the elbow, exerting a downward force on the forearm at the joint.

Human Arm	Model	Force Location
Forearm (ulna and radius)	half-meter stick	center of mass
Hand	clamp and hanger or plastic cup	about ¡m¿1 cm¡/m¿ from the far end
Bicep Muscle	clamp and spring (pulling up)	¡m¿8.6%¡/m¿ of the length of the forearm from the elbow (about ¡m¿4 cm¡/m¿ from the elbow)
Elbow (humerus)	clamp, hanger, weights	as near as possible to the elbow end (about ¡m¿1 cm¡/m¿ from the elbow-end)

Be sure to include the weight of the clamps at each location where one is used. ¡ul¿ ¡li¿¡p¿ Set-up a force diagram for this model of a horizontal forearm. ¡/p¿¡/li¿ ¡li¿¡p¿ Set-up the equilibrium equations for this system. ¡/p¿¡/li¿ ¡/ul¿

¡/subsection¿¡subsection¿¡title¿¡/title¿Equilibrium Experiment

You are going to do the following procedure for two different locations of the bicep muscle: First set the bicep, as mentioned above, in the human location of about ¡m¿4 cm¡/m¿ from the elbow. Second, set the bicep at about ¡m¿10 cm¡/m¿ from the elbow. You should consider which location gives more leverage to lift the hand.

¡ul¿ ¡li¿¡p¿ With your model completely set up, place a ¡m¿50 g¡/m¿ mass in the ¡q¿hand.¡/q¿ ¡em¿Experimentally¡/em¿ determine the force at the elbow necessary to balance the system and make it horizontal. You may also change the position of the hand by small amounts if you find it easier to balance the system. However, be sure to record the correct distances. ¡/p¿¡/li¿ ¡li¿¡p¿ To check that the system is level, use another meter stick to measure the height from the floor ¡em¿at each end¡/em¿ of the forearm half-meter stick. ¡/p¿¡/li¿ ¡li¿¡p¿ Determine the force in the bicep muscle using the formula derived in the previous section as appropriate for the new position of the bottom of the hook of the spring (again using the mirror for accuracy). Be sure to include the uncertainties! ¡/p¿¡/li¿ ¡li¿¡p¿ Record all force locations as read directly from the ¡q¿forearm¡/q¿ meter stick. ¡em¿Do not subtract in your head¡/em¿ so that we can reproduce the locations later, if need be. ¡/p¿¡/li¿ ¡li¿¡p¿ ¡em¿Experimentally¡/em¿ determine the sensitivity of your values for force at the elbow by checking how many grams can be added or removed at the elbow while maintaining the horizontal equilibrium. ¡/p¿¡/li¿ ¡li¿¡p¿ Since the hand is so sensitive, you should estimate roughly the sensitivity of the value of force at the hand. ¡/p¿¡/li¿ ¡/ul¿

With this data, you will verify the equations of equilibrium.

¡/subsection¿¡subsection¿¡title¿¡/title¿Comparative Anatomy

Create a table that allows you to compare the values of force for the elbow, bicep, and hand for three different attachment-locations. ¡ul¿ ¡li¿¡p¿ Enter the data from Sec. **??**. Record each position and each force. ¡/p¿¡/li¿ ¡li¿¡p¿ Predict how the forces will change if the hand is moved a few millimeters closer to the bicep. (Hint: Imagine carrying grocery bags at different locations on your lower arm while holding your arm in an L-shape.) ¡/p¿¡/li¿ ¡li¿¡p¿ Move the hand in and measure the forces that make it balance. Compare to your predictions. ¡/p¿¡/li¿ ¡li¿¡p¿ Predict how the forces will change if the bicep is moved a few millimeters further from the elbow. (Hint: Imagine sitting at different locations on a teeter-totter.) ¡/p¿¡/li¿ ¡li¿¡p¿ Move the bicep away from the elbow and measure the forces that make it balance. Compare to your predictions. ¡/p¿¡/li¿ ¡/ul¿

¡/section¿¡section¿¡title¿¡/title¿Analysis

¡ul¿ ¡li¿¡p¿ For the forces measured or calculated in Sec. **??**, test each of the conditions of equilibrium: ¡ul¿ ¡li¿¡p¿ Translational Equilibrium: Show that the up forces equal the down forces to within your uncertainty. ¡/p¿¡/li¿ ¡li¿¡p¿ Rotational Equilibrium: Show that the clockwise torques equal the counterclockwise torques to within your uncertainty. ¡/p¿¡/li¿ ¡li¿¡p¿ To show that it does not matter which point you choose to calculate the summation of the torques about, choose the far end of the meter stick (near the hand) as the zero or the rotation point. ¡/p¿¡/li¿ ¡li¿¡p¿ If your conditions of equilibrium are not consistent, then calculate the necessary mass that should be in the hand and test this value experimentally. Discuss how well this value works. (If you cannot get this to balance, you might check to be sure you included the weight of the forearm itself¡ellipsis /¿) ¡/p¿¡/li¿ ¡/ul¿ ¡/p¿¡/li¿ ¡li¿¡p¿ Consider the forces in Sec. **??**. ¡ul¿ ¡li¿¡p¿ If you have three shopping bags of food at the grocery store that you want to hang from your arm as you walk to your car, should you hang the heaviest bag closest to your elbow or closest to your hand? Explain. ¡/p¿¡/li¿ ¡li¿¡p¿ The location of the bicep relative to the elbow determines the leverage in pulling your hand to your chest (or for a quadruped move their feet forward for the next step). Do you expect fast animals to have their leg muscles attached close to the joint or far from the joint? Explain. ¡/p¿¡/li¿ ¡/ul¿ ¡/p¿¡/li¿ ¡/ul¿

¡/section¿¡section¿¡title¿¡/title¿Questions ¡ol¿ ¡li¿¡p¿ Is it necessary to have the meter stick horizontal for the system to be in equilibrium? Why did we want to keep the arm horizontal? ¡/p¿¡/li¿ ¡li¿¡p¿ How was the tension in the muscle affected as the position of attached muscle was moved further from the joint? Keep the mass in the hand constant. ¡/p¿¡/li¿ ¡/ol¿

¡/chapter¿

¡chater¿¡title¿Scaling Laws Among Biological Specimens¡/title¿ ¡!– Revisions –¿

¡introduction¿ ¡p¿You will be given many animal skulls. Measure some dimension that can be measured on all skulls. Verify graphically (make the plot of mass versus your measured dimension) if this measurement can be reliably correlated to the mass of the skull. We will assume that the mass of the skull is directly proportional to the overall size of the animal. This technique allows mammalogists to gauge the size of animals when they can only find partial skeletons.¡/p¿

When you fit this data with a trendline, the relationship might be the familiar linear relationship, or it might instead be something else. Some possibilities are: ¡ul¿ ¡li¿¡p¿ Linear ¡m¿$y = mx+b < /m > .Inthiscase, youshouldplottheskullmassversusyourmeasurement, determinetheslopeandinterceptandtherele$ $/p >< /li >< li >< p > Geometric < m > y = ax^b < /m >, wherethecoefficient < m > a < /m > andtheexponent < m > b < /m > aretobedetermined.Ingeneral, itisusefultofityourdatathiswayfirst.Ifyou$ $m > b < /m > isclosetoaninteger, thentryasecondfitwiththeappropriatepolynomial.Forexample, if < m > b \approx 1 < /m > thenusealinearfit. < /p >< /li >< li >< p > Exponential < m > y = ae^{bx} < /m >, wherethecoefficient < m > a < /m > andthebase < m > b < /m > aretobedetermined. < /p >< /li >< /ul >$

Excel can fit to any of these and provide you with the equation for the trendline. However, Excel cannot provide the uncertainties unless the graph is linear. If one uses logarithms, then we can make a power-fit trendline or an exponential-fit trendline plot as a line so that Excel can provide the slope and intercepts with uncertainties. This will allow us to create a reliable mathematical relationship between the variables. In general, it is true that linear regression is significantly easier than a general functional regression.

If your power fit is better then we can apply the logarithm to the geometric relation:

$$\begin{aligned}
\ln y &= \ln\left(ax^b\right) \\
\ln y &= \ln a + \ln x^b \\
\ln y &= \ln a + b\ln x \\
\ln y &= [b]\ln x + [\ln a]
\end{aligned}$$

In this case, it is useful to make a ¡q¿log-log plot¡/q¿ of ¡m¿$\ln y < /m > versus < m > \ln x < /m >, whichshouldbeastraightline.Theslopeofthelog-logrelationis < m > b < /m >, theexponent, andtheinterceptofthelog - logrelationis < m > \ln a < /m >, sothat < m > e^{(\ln a)} = a < /m > thecoefficient.$

If your exponential fit is better then we can apply the logarithm to the exponential relation:

$$\begin{aligned}
\ln y &= \ln\left(ae^{bx}\right) \\
\ln y &= \ln a + \ln\left(e^{bx}\right) \\
\ln y &= \ln a + bx\ln(e) \\
\ln y &= [b\ln e]\,x + [\ln a]
\end{aligned}$$

In this case, it is useful to make a ¡q¿semi-log plot¡/q¿ of ¡m¿$\ln y < /m > versus < m > x < /m >, whichshouldbeastraightline.Theslopeofthesemi-logrelationis < m > b < /m > (because < m > \ln e = 1) < /m > andtheinterceptofthesemi-logrelationis < m > \ln a < /m >, sothat < m > e^{(\ln a)} = a < /m > thecoefficient.$

In biological relationships, it is common to find an exponential relationship with a base of ¡m¿e=$2.717182\cdots < /m >; thisiswhywefindituseful touse the < q > naturallogarithm. < /q >$

Once you determine if you have an exponential or a geometric relationship, decide if there are any ¡q¿outliers,¡/q¿ which animals those are, and justify why those data points might reasonably be excluded. Re-fit the data after excluding the outliers.

¡/chapter¿