# PHY 121 Lab Manual

Thomas More College, Algebra-based Introductory Physics

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## Thomas More College, Algebra-based Introductory Physics

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# Preface

This text is intended for a one or two-semester undergraduate course in introductory algebra-based physics.

# Download the PDF here

A full PDF version of this document can be found at fall-lab-manual.pdf (705 kB)

# Contents

# Lab 1

# Meaningful Measurements

### Experimental Objectives

- Determine the material of the objects by calculating their density and matching it to the accepted values for various common materials.

## Introduction

Physics is a science which is based on precise measurements of the seven fundamental physical quantities, three of which are: time (in seconds), length (in meters) and mass (in kilograms); and all of these measurements have an experimental uncertainty associated with them. It is very important for the experimenter to estimate these experimental uncertainties for every measurement taken. There are three factors that must be taken into account when estimating the uncertainty of a measurement:

1. statistical variations in the measurements,
2. using one-half of the smallest division on the measurement instrument,
3. any mechanical motions of the apparatus.

Physicists study the physical relationships between these defined fundamental quantities and usually give a name to the newly derived physical quantity. These derived physical quantities have units which are combinations of the units of the fundamental ones. For example, the product of the lengths (in meters, m) of the three sides of a cube is called volume and has units of $\mathrm{m}^3$. The ratio of mass to volume is called density and has units of $\frac{\mathrm{kg}}{\mathrm{m}^3}$. The concepts of volume and density are therefore derived from the fundamental physical quantities, rather than fundamental themselves.

## 1.1 Student Outcomes

Knowledge Developed: In this exercise, students should learn how to make precise and accurate length measurements with a meter stick and two types of calipers, how to read a vernier scale, and how to estimate uncertainty in a measurement. Students will make use of the relationship of the fundamental properties of mass and length to the derived concepts of volume and density.

Skills Developed:

- Proper use of a vernier caliper and scale
- Proper use of a micrometer caliper
- Evaluating and propagating uncertainties

## 1.2 Procedure

***Materials:*** Three measuring devices: a metric ruler, a vernier caliper, and a micrometer caliper.
Several objects convenient for measuring the mass and the physical dimensions.
***Procedure:***

- Check the measuring devices for any **zero-point errors** (verify that they are **calibrated**). The use of the caliper and micrometer is outlined below.
- There are also several solids available: a cylinder, a cube, and a sphere.
  - Measure the dimensions of two of the objects with each of the three instruments: the ruler, the vernier caliper, and the micrometer caliper. Take all measurements minimizing any parallax errors.
  - Estimate the experimental uncertainties of your measurements. Do Exercises 1.2.1, 1.2.2, and 1.2.3.
  - Repeat the measurements at several positions and orientations around the object, compute the average and the relative uncertainty.
- As outlined in the Analysis, compute the volume and the density.

**Exercise 1.2.1** (Zero-Point Errors)**.** You should verify the calibration of your instruments. Determine if there are the zero-point errors for each of the measuring instrument.

**Hint 1.** You might re-read the description of systematic error to remind yourself what a **zero-point error** is.

**Hint 2.** Is the meterstick rough on the end? Is the edge marked as zero actually at the zero value? Can you think of a way to ensure that the condition of the meterstick does not impact the measurement of length? How does the distance from zero to five compare to the distance from one to six?

**Hint 3.** Does the caliper read zero when it is measuring a zero length?
You should close the caliper to determine this.

**Hint 4.** Does the micrometer read zero when it is measuring a zero length?
You should close the micrometer to determine this.

**Answer.** You should make efforts to correct for any zero-point errors in your instrument. Especially in this week, your report should explain how you accommodated any zero-point errors.

**Solution.** You should figure out how to make efforts to correct for zero-point errors.

For each instrument, if the measurement you consider to have zero-length is not actually "zero", then you can handle that in the same way that you know the distance from 1 to 6 (in this case, subtract 1 from any measurement you make) or from 2 to 7 (in this case, subtract 2 from any measurement you make) or from 10 to 15 (in this case, subtract 10 from any measurement you make) .

You will also need to concern yourself with whether this uncertainty is **systematic** (always making the answer slightly too big or always slightly too small) or **random** (sometimes making the answer slightly too big and sometimes slightly too small). (Recall Subsection A.2.1.)

**Exercise 1.2.2** (Parallax)**.** Determine if the value you are reading depends on the location of your eye.

**Hint.** For the caliper and the micrometer, the instrument clamps around the item being measured. Decide if the location of your eye matters in the measurement.

For the meterstick, you have to align the edge with a tick-mark. Does the location of your eye impact the alignment of the tick-mark on the ruler with the edge of the object?

**Answer.** You should make efforts to correct for any zero-point errors in your instrument. Especially in this week, your report should explain how you accommodated any parallax errors.

**Solution.** You should figure out how to make efforts to correct any parallax errors. It may help to determine whether this uncertainty is **systematic** (always making the answer slightly too big or always slightly too small) or **random** (sometimes making the answer slightly too big and sometimes slightly too small). (Recall Subsection A.2.1.)

For each instrument, if the measurement does depend on the location of your eye, then you might try measuring it multiple times with your head in different locations each time to gauge the size of this uncertainty.

In the case of the meterstick, the object being measured should be as close as possible (touching?) the tick-marks of the meterstick in order to minimize parallax.

**Exercise 1.2.3** (Random or Systematic)**.** When you measure the diameter of a sphere, it might be difficult to get the caliper precisely at the full diameter. If you are off, then you will necessarily be measuring a smaller value. This is a systematic error that can be corrected for. Since any mistake necessarily gives a value that is too small, then measuring it multiple times and finding the largest value will minimize this uncertainty.

On the other hand, if you measure an egg, you would not expect the diameter to be the same. Clearly for an egg, there is no reasonable single value to use as The Diameter. In this case, the question of finding the diameter does not make sense, and by measuring in a systematic pattern, you can determine that the shape is not spherical.

Determine if the zero-point error and the parallax error are random or systematic.

**Hint 1.** If a meterstick is worn down at the zero-value, then is it more likely to be measuring too short or too long? Is that systematic or random?

**Hint 2.** For a measurement affected by parallax, is that systematic or random?

**Solution.** Since random errors might give a result too big or too small, measuring many times and averaging should minimize these errors.

Since systematic errors tend to be either too big or too small (in a predictable or explainable way), you should track the uncertainties and recognize if this measurement causes your result to be more likely too big or more likely too small.

### 1.2.1 The Vernier Caliper

The vernier scale was invented by Pierre Vernier in 1631. This scale has the advantage of enabling the user to determine one additional significant figure of precision over that of a straight ruler.

For example, this eliminates the need for estimating to the tenth of a millimeter on the metric ruler. The vernier caliper, shown in Figure 1.2.4, can measure distances using three different parts of the caliper: outside diameters (large jaws), inside diameters (small jaws), and depths (probe). You should locate these three places on your caliper. The vernier device consists of the main scale and a movable vernier scale. The fraction of a millimeter can be read off the vernier scale by choosing the mark on the vernier scale which best aligns with a mark on the main scale.

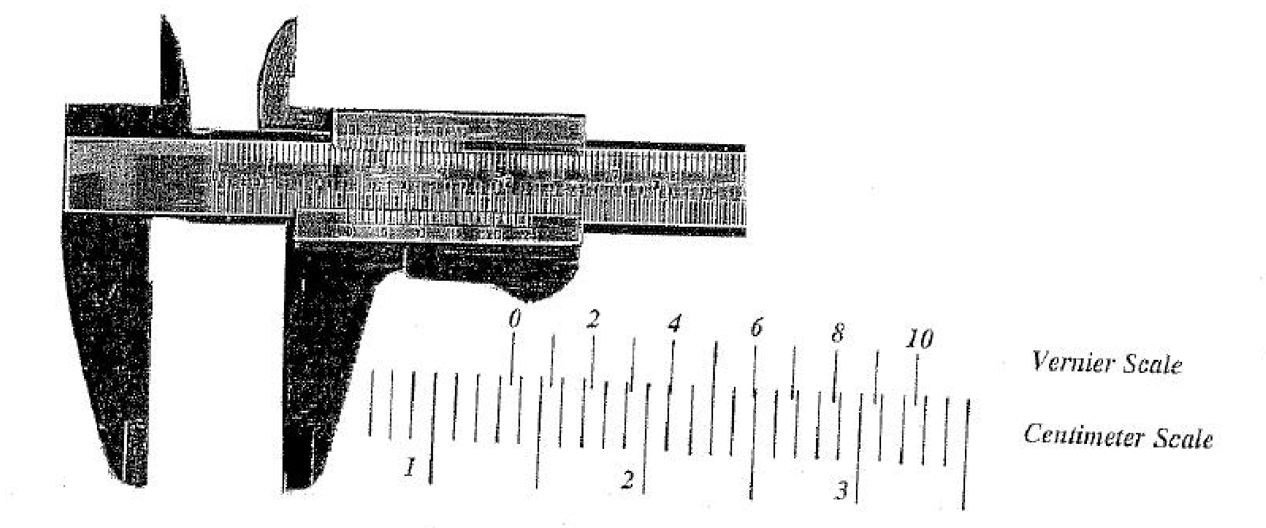


**Figure 1.2.4:** The location of the zero on the vernier scale tells you where to read the centimeter scale (1.3 cm). The vernier-scale line that lines up tells you the next digit (5). This picture measures $1.35 \pm 0.01\,\mathrm{cm}$ because we can distinguish 1.35 from 1.34 and 1.36, but we cannot gauge the result any more precisely.

To move the vernier scale relative to the main scale press down on the thumb-lock, this releases the lock and then move the vernier scale. Do not try to move the vernier scale without releasing the lock.

### 1.2.2 The Micrometer Caliper

A micrometer caliper is shown in Figure 1.2.5. This instrument is used for the precise length measurement of a small object. The object is placed with care between the anvil and the rod. It is very important to not tighten down on the object with a vise-like grip. Tightening with force will **decalibrate** the micrometer (causing a zero-point error). The rotating cylinder moves the rod, opening or closing the rod onto the object. There is a rachet, at the far end, for taking up the slack distance between the anvil, the object and the rod, so again do not over-tighten with the rotating cylinder. The linear dimension of the object can be read from the scale. Rotating the cylinder one revolution moves the rod 0.5 millimeters. The rotating cylinder has 50 marks on it. Read the mark on the rotating cylinder that aligns with the central line on the main scale.

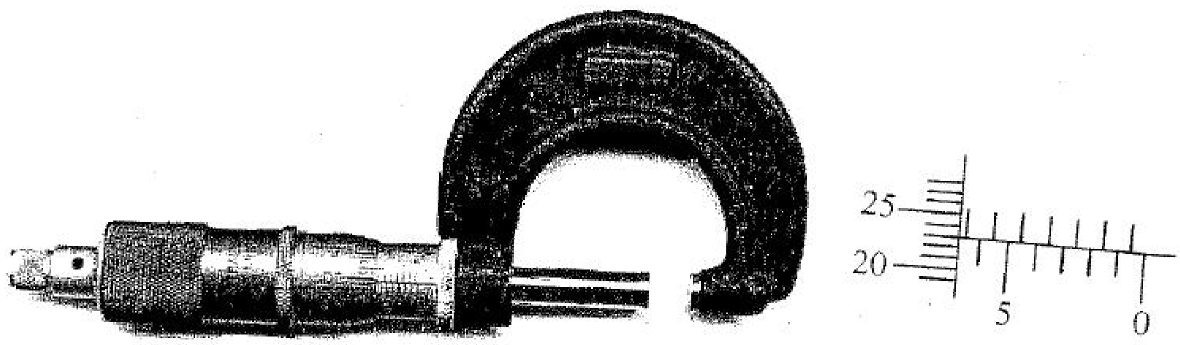


**Figure 1.2.5:** Notice on the coarse scale, that the lower lines read (1, 2, 3, ... 6 in this picture) and the higher lines read the half-marks (0.5, 1.5, 2.5, ... 6.5 in this picture). The location of the turning dial tells you where to read the coarse scale (6.5 mm). The center line of the coarse scale tells you where to read the fine scale. This is 23.0 (in units of $\times 10^{-2}$ mm), but not 23.5 and not 22.5 so the precision is 0.5 (in these units). This measurement in mm reads $6.5\,\text{mm} + 0.230\,\text{mm} = 6.730 \pm 0.005\,\text{mm}$. Since each mark corresponds to 0.01 mm and you can probably gauge a distance about half-way between the lines, the precision of this instrument is 0.005 mm.

The reading of the micrometer from Figure 1.2.5 is $6.730 \pm .005$ mm.

## 1.3 Analysis

- After finding the relevant dimensions of the object, calculate the volume of the object three times: once using the measurements from the ruler, once from the caliper, and once with the micrometer.
- Using the rules of propagation of uncertainty, compute the uncertainty in the volume for each.

**Exercise 1.3.1.** Which instrument is the most precise?

**Hint.** For which instrument can you measure to the most decimal places?

**Answer.** Your data should give you this answer. Your report should indicate how your data tells you the answer to this question.

- Measure the volume directly with the graduated cylinder.

**Exercise 1.3.2.** Are any of the volume measurements inconsistent (See Note A.1.1 about comparing values)? What can you infer about the accuracy of these instruments?

Using the most precise indirect measurement of volume (those calculated from other measurements), calculate a percent-difference with the direct measurement of volume.

**Hint 1.** When you measure the volume of (let's say the cylinder) with a meterstick, with a caliper, with a micrometer, and with a graduated cylinder, they are all measuring the volume of the same object, which does not change volume. You *expect* these to all give the same number. The question is whether or not *your data* do actually give the "same" values.

**Hint 2.** Since you are using measurement techniques that have different precision, you will have different ranges of uncertainty. Numbers are considered to be "the same" when their uncertainty ranges overlap.

**Hint 3.** Keep in mind that "imprecise" means "a large range in the uncertainty", whereas "inaccurate" means "inconsistent with the true value". (You might not know the true value.)

**Answer.** It is possible that your data do not give consistent results for the volume. You should notice if one result in particular is different than the others and then speculate on why *that* measurement is different. If all of your results are inconsistent with each other, then you might want to check your measurements. If they are again inconsistent, then you should check your results against a friend or the instructor.

- Measure the mass and then, using the overall most precise measurement of volume, compute the density with its uncertainty.
- Using your best density value, find the percent-error against the appropriate value given by the text, or the *Handbook of Physics & Chemistry*.
- Other considerations that might help with your analysis:

**Exercise 1.3.3.** What would be the best method to measure the volume of an irregularly shaped object? Why?

**Hint.** To answer this, it might help to think about *how* you would measure something that is irregularly shaped with each of the instruments you used today. Is one of them particularly good at conforming to the shape of an irregularly shaped object?

**Answer.** You should recognize which of the following objects (a meterstick, a caliper, a micrometer, and water) can touch all edges of an irregularly shaped object simultaneously. On the other hand, if you are measuring the volume of something that is water-soluble, then you might want to reconsider your reasoning process.

(Revised: Sep 13, 2017)
A PDF version might be found at measurement.pdf (308 kB)

# Lab 2

# Standard Deviation

### Experimental Objectives

- The goal of the experiment is to determine how
  - the mean,
  - the standard deviation, and
  - the standard deviation of the mean

  depend on the amount of data (number of samples) taken.
- You should also consider how well the data fit to a normal distribution.

## Introduction

Suppose you are standing in front of a dart-board. You have a large number of darts and you throw the darts one at a time, trying to hit a 1 cm thick vertical line drawn on the dart-board. Since you have a very good aim, let us say that 45% of the darts hit the line. This then means that you miss the line 55% of the time; with a significant number of these misses being between .5 to 1.5 cm either to the left or to the right from the center of the line, and with a smaller number of misses being between 1.5 to 2.5 cm from the center line.

Is there some mathematical way of characterizing how good you are at this game? What, for example, is the probability of missing the line by 1 cm? The statistical analysis of random fluctuations in data can help answer these questions. The word "statistical" implies that a relatively large set of similar measurements of a given physical quantity is available. The random fluctuations in the data can be measured with the use of a mathematical term called the "standard deviation."

Suppose you collect data on a large number of throws, separating the data into categories (bins): The number of darts on the line, the number within .5 to 1.5 cm from the center, the number within 1.5 to 2.5 cm, etc. A plot of this data with the dart positions on the x-axis and the number of darts hit within each bin plotted on the y-axis is called a histogram (see Figure 2.0.1). The envelope of this graphical data set is bell shaped, and is called a Gaussian or a Normal distribution curve.

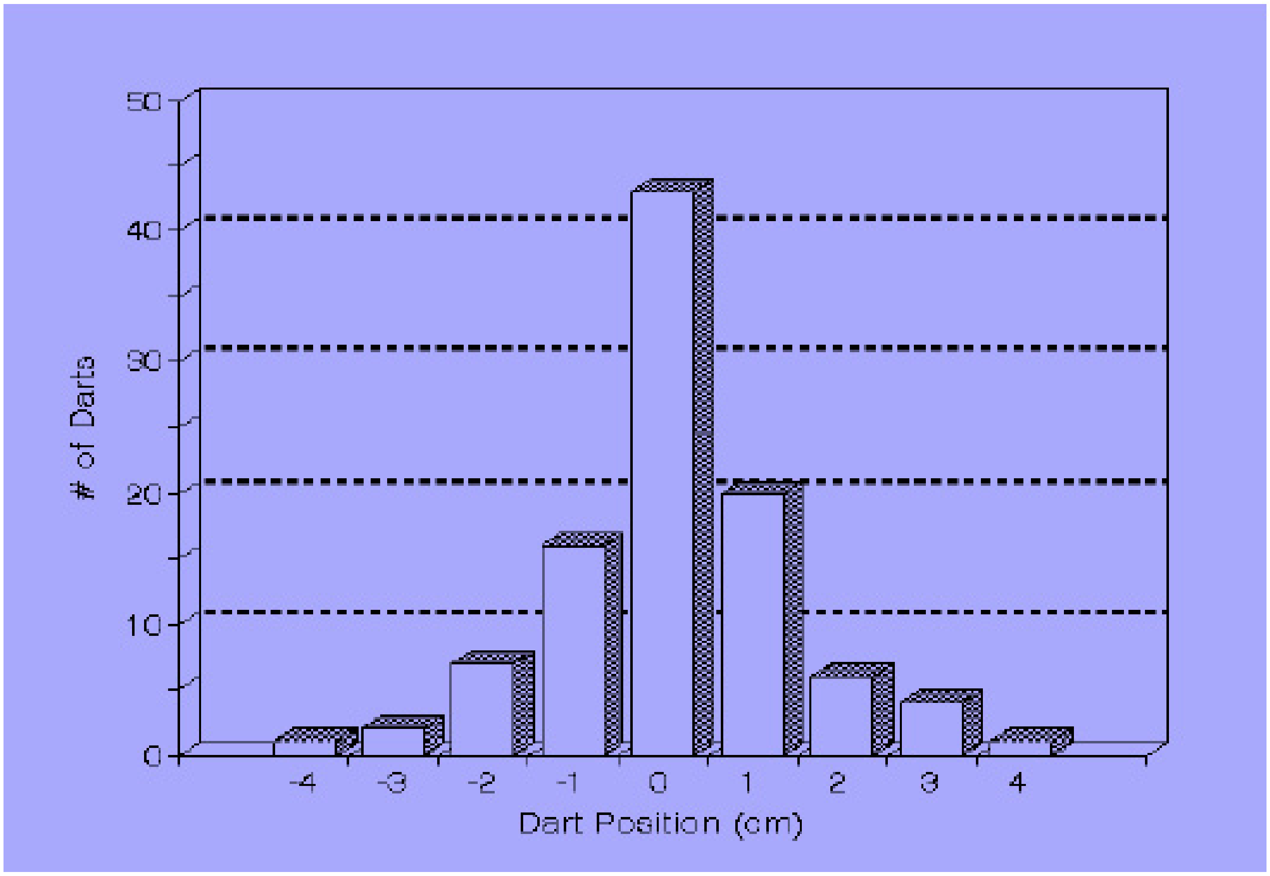


**Figure 2.0.1:** A sample histogram for the number of darts binned by distance from the centerline.

The "standard deviation" is a measure of the spread or width of the histogram data. A small standard deviation means that there is a small spread in the data about the central mean value and implies that the data cluster closely about one value. That is, there is a high degree of precision in the measurements.

The area beneath the curve, or below a part of the curve represents the probability of occurrence. For example, the area beneath the curve between plus and minus one standard deviation from the mean represents a 68% probability of your next throw falling within this range. The area beneath the curve between plus and minus two standard deviations from the mean represents a 95% probability of your next throw falling within this range.

In this experiment, you will study the use of the "standard deviation" in the statistical analysis and probability involved with flipping pennies.

## 2.1 Student Outcomes

In this experiment, you should learn

1. the formulas for and the roles of the mean, the standard deviation, and the standard deviation of the mean in the statistical analysis of data containing random errors,

2. how to create histograms and scatter plots,

3. how to include trendlines on scatter plots, and

4. why more data is always better.

## 2.2 Pre-Lab Considerations

- Define the following, both in a sentence and with a mathematical formula: mean, standard deviation (sometimes called the standard deviation of a single observation), and standard deviation of the mean. Be sure to describe the difference between these two terms.
- Define the following: histogram, probability, & probability distribution (Normal distribution).
- What does the total area under the Normal distribution curve represent?
- How should the axes of the histogram of the data you will take for this lab be labeled? Give a very specific scale for the x-axis. (Hint: Read the procedure below.)
- Make a sketch of an inverse function, like $y = 1/x$.

## 2.3 Procedure

**Materials** A supply of pennies.

**Procedure** Each individual will receive 20 pennies and will then simultaneously toss all of the pennies. Count the number of heads, and repeat 25 times. Each individual will then have collected 25 pieces of data.

After doing this, you will 25 "counts" of the number of heads among that 20 coins. Your lab partner will also have 25 "counts". The two of you together have access to 50 "counts". You, your partner, and the pair of people near you will together have 100 "counts" among you. You will therefore be able to consider a histogram for your $N = 25$, another histogram for your $N = 50$, etc. After you consider each of these histograms, you will consider how the shape a histogram changes as $N$ increases.

## 2.4 Analysis

- Calculate the mean, the standard deviation and the standard deviation of the mean for your first ten tosses and for all 25 tosses. Show your calculations.
- Make a histogram plot (by hand in your note book) for your 25 tosses. On this histogram superimpose a sketch of your best guess of the corresponding Normal distribution curve.
- How well does your data fit a normal distribution curve? Explain the reasons for any large discrepancies.
- Obtain histograms and calculations of the mean, the standard deviation and the standard deviation of the mean for the following data sets:
  1. your lab group,
  2. about 1/2 of the class, and
  3. the entire class.

  These histograms can be obtained with the use of a computer program, provided by the instructor.
- Draw graphs of: the mean, the standard deviation, the standard deviation of the mean, versus the number of data entries. Use the standard deviation of the mean as the error bars for both the mean and the standard deviation, draw these error bars on these two graphs.
- Describe how the values of the mean, the standard deviation, and the standard deviation of the mean, change as the number of data items in the set increases. What can you infer about the accuracy and about the precision of the data as more and more observations are made?
- Show that the standard deviation of the mean varies inversely as the square root of the number of data items in the sample. How well does the data in this lab agree with this prediction? (Make a graph of the standard deviation of the mean versus the square root of the number of data items in the sample.)

## 2.5 Questions

**1.** What is the probability of flipping the 20 pennies and getting 5 heads, or 8 heads, or 10 heads, or 15 heads? Answer this question by analyzing your data, the entire class' data and the normal distribution curve fit to your data. Explain any differences between these sets.

**Hint 1.** Consider the percentage of total throws that produce each number of heads.

**Hint 2.** The percentage of total throws that give 5 heads is the number of throws that gave 5 heads divided by the total number of throws.

**Hint 3.** You might consider this percentage (of throws that produces 5 heads) for $N = 25$ versus $N = 50$ versus other values of $N$ to explore the stability of this percentage. (Remember that more data is better.)

**2.** What percentage of your individual readings fall within plus or minus one standard deviation, two standard deviations? Compare your answer to the theoretical answer from a normal distribution curve. What are the percentages for the class data?

**Hint 1.** Hint 2.5.1.2 explains finding a percent of throws.

**Hint 2.** If the mean is 10.12 and the standard deviation is 2.74, then "within plus or minus one standard deviation" implies that you should consider the percentage of times you get between 7.38 and 12.86. This is larger than 7 and smaller than 13. So, consider how many throws gave 8, 9, 10, 11, or 12 heads.

**Hint 3.** To compare to the theoretical answer, you can figure out the actual mathematical formula, but you can also use the "Gaussian Prediction" column in the Excel spreadsheet for the corresponding range.

**3.** Does the height of the histograms change as a function of the number of trials? If so, how?

**4.** How does the width at 1/2 the maximum height for the histograms change as a function of the number of trials? Label this width on your histograms. Is this width a reasonable estimate of the standard deviation?

**Hint.** To get the "full-width at half-maximum" on your histogram, find the mean on the horizontal axis and note the vertical value of that peak (read the vertical axis). Half this peak value and then find the smallest number of heads thrown that is at least this half-value and the largest number of heads thrown that is at least this half-value.
For example, if the mean of your data is 10.12, then perhaps you had 6 throws that resulted in 10 heads (the number of heads closest to the mean). Half of the 6 throws is 3 throws. So, you will likely have find that you have fewer than 3 throws for 6-or-fewer heads but more than 3 throws for 7-or-more heads. On the other end, you will likely find that you have fewer than 3 throws for 14-or-more heads but more than 3 throws for 13-or-fewer heads. This says that the "full-width at half-maximum" is the range from 7 heads to 13 heads, which is a width of 6-heads thrown. You should consider how this "full-width at half-maximum" compares to your standard deviation.
(Please remember that you not actually throwing heads, you are throwing coins. Do not decapitate your lab partner.)

**5.** Does the standard deviation of a single observation and/or does the standard deviation of the mean change significantly as the number of tosses increase? What can you infer about the accuracy and about the precision of the data as more and more observations are made?

## 2.6 References and Suggested Readings

**[1]** Meyer, *Data Analysis for Scientists and Engineers.* John Wiley, 1975; p.19-48, 223-253

Revised: Aug 24, 2017
A PDF version might be found at StDev.pdf (242 kB)

# Lab 3

# Constant Acceleration

### Experimental Objectives

- Using position versus time and velocity versus time graphs, verify
  - that the equations of constant acceleration accurately describe the behavior of objects under constant acceleration and
  - that it is possible to distinguish acceleration due to gravity from acceleration due to friction.

## 3.1 Student Outcomes

Knowledge developed: In this exercise, the student should develop an understanding of the relationships between the position and the instantaneous velocity of an object, as well as how each of these can vary as functions of time. We will only consider the special case where the object experiences constant acceleration.

Skills developed:

- Evaluate the data for sources of uncertainty. Can you see an effect, such as a level track or the presence of friction, in the result?
- Using Pasco Capstone software
- Interpreting the slope and intercept of graphs

## 3.2 Procedure

***Materials***: An aluminum track, a low-friction cart, computer interface with PASCO Capstone$^{tm}$ software, a sonic motion sensor, a small steel ball.

You should notice that the subsections in this section parallel the subsections in Analysis.

### 3.2.1 Cart and Flat Track

**Note** The Pasco Capstone interface is also used in Lab 4, Lab 5, and Labs 7–8. You should start becoming familiar with the hardware and software.

Log into the computer (so you can save your data to your network drive) and then open Pasco Capstone. (Appendix C will provide some instructions for setting up the software and connecting the equipment.) Connect the motion sensor to the computer interface. Set the data rate of the motion sensor at 50 Hz. Place a steel ball on the track and adjust the leveling screw at one end of the track to see if the ball rolls one way or the other. This will roughly level your track. Place the sensor about 20 cm from the end of the track, because this is the minimum distance detected by the sensor. (You might need to use the "sail" for the sensor to see the cart.)

Place the cart on the track. Capstone, via the sonic ranger, can measure the position and velocity of the cart as a function of time. (This is explained in Appendix C.)

**Exercise 3.2.1.** Assume the track is frictionless and predict how the cart will move if the track is not perfectly level; include a comment about how the velocity versus time graph will look when it goes uphill versus when it goes downhill. Should these be the same?

**Hint 1.** If the track is not level, sending the cart in one direction, it will be going uphill; but in the other direction it will be going downhill. If you measure the motion in both directions, you should be able to see the difference.

**Hint 2.** You may still have an effect due to friction. (See Exercise 3.2.3.)

**Hint 3.** It is probably useful to describe this using terms such as "speeding up" or "slowing down". You may also want to practice describing this by comparing how the direction of the acceleration compares to the direction of the velocity.

**Exercise 3.2.2.** What do you expect the graph to look like if the track *is* perfectly level? Will it be the same going left versus going right?

**Hint 1.** If the track is level and then you compare the motion of the cart in one direction versus another, you should be able to see if there is a difference. You should also be able to predict how the motion in each direction for this case is similar or different from the case when the track is not level.

**Hint 2.** You may still have an effect due to friction. (See Exercise 3.2.3.)

**Hint 3.** It is probably useful to describe this using terms such as "speeding up" or "slowing down". You may also want to practice describing this by comparing how the direction of the acceleration compares to the direction of the velocity.

**Exercise 3.2.3.** Now, assuming it is perfectly level, what will friction do to the motion? How do you expect this to affect the graphs?

**Hint 1.** If the track is level and there is friction, then when you compare the motion of the cart in one direction versus another, you should be able to see if there is a difference. You should also be able to predict how the motion in each direction for this case is similar or different from the case when the track is not level.

**Hint 2.** If the track is tilted with no friction then describe the motion in each direction using the phrases "speeding up" or "slowing down".

If the track has friction with no tilt then describe the motion in each direction using the phrases "speeding up" or "slowing down". You should be able to indicate how you would see each effect in the graphs of the motion.

We will take four sets of data: a slow, constant velocity towards the ranger; a slow, constant velocity away from the ranger; a faster, constant velocity towards the ranger; and a faster, constant velocity away from the ranger. The two slow speeds should be about the same and the two faster speeds should be about the same. For each case, start the sonic ranger and then bump the cart firmly, but not violently(!).

On Capstone, you should have four curves of velocity versus time. Fit each with a trendline and display the equation of the trendline on the screen. Interpret the coefficients (slope and intercept) by noting their units, values, and uncertainties. You should also print out (in landscape mode) the position versus time graph, the velocity versus time graph, and the acceleration versus time graph. (You should notice that the acceleration versus time graph is *very* noisy.)

### 3.2.2 Cart and Sloped Track

Place a small block under one end of the track, so that the track is now tilted at a small angle with the sensor at the top of the incline. Measure the angle using a protractor or calculate it by measuring the two legs of the triangle and using the inverse sine. (Be careful about measuring the height.)

We will consider *three cases* for the sloped track: *First*, allow the cart to roll (without an initial push) down the ramp. *Second*, gently push the cart down the ramp. DON'T let it fly off or crash into anything.

**Exercise 3.2.4.** Should these two cases have the same acceleration while rolling down the ramp? How will that affect the shape of the velocity versus time graphs?

**Hint.** Do the graphs have the same slope?
Do the graphs have the same intercept?

**Exercise 3.2.5.** Should these have the same initial velocity? How will that affect the graphs?

**Hint.** Do the graphs have the same slope?
Do the graphs have the same intercept?

In the *third* case, start the cart at the bottom of the incline and roll it up the ramp, allowing it to roll back down on its own. Push it hard enough to get mostly up the ramp, but not so hard that it hits the sonic ranger at the top of the incline, because we want to watch it return to the bottom of the ramp. *This case is similar to throwing a ball into the air and allowing it to fall back down.*

**Exercise 3.2.6.** Should this case have the same acceleration while it goes up the ramp as while it goes down the ramp? How can we see that on the velocity versus time graphs?

**Hint.** If the track is tilted with no friction then describe the motion in each direction using the phrases "speeding up" or "slowing down".

If the track has friction with no tilt then describe the motion in each direction using the phrases "speeding up" or "slowing down".

In this case, there may be tilt *and* friction. You should be able to indicate how you would see each effect in the graphs of the motion.

**Exercise 3.2.7.** Should this case have the same acceleration (either while it goes up the ramp or while it goes down the ramp) as the previous two cases of rolling down the ramp?

In Capstone, you should be able to display all three graphs (position v time, velocity v time, and acceleration v time). You should also be able to display all three cases of data on each of these graphs. On the velocity versus time graph, fit each of the three graphs with a linear trendline. The next section will ask you to analyze how well the data match up to these lines. (It might be interesting to also fit the position vs time curves to parabolas. Be sure to print out copies of your three graphs.

Your lab should note the following results and explain their meaning: slope and y-intercept, the uncertainties (precision) in both the slope and intercept, and the $r$ value (correlation coefficient).

## 3.3 Analysis

You should notice that the subsections in this section parallel the subsections in Procedure.

### 3.3.1 Cart and Flat Track

Based on the results of Subsection 3.2.1, write a short analysis of the relationship between these two graphs (x and v versus time). From the velocity versus time graph (specifically from the trendline) determine the value of the acceleration of the cart down the track; be sure to include the uncertainty of the acceleration and the units.

**Exercise 3.3.1.** Do you see any evidence that the track was not perfectly level?

**Exercise 3.3.2.** Do you see any evidence that there is any friction as the cart moves along the track?

**Exercise 3.3.3.** What does the intercept of the velocity versus time graph tell you?

**Exercise 3.3.4.** If the slopes are different, then discuss any pattern that you see. If the slopes are (essentially) the same, then find an average and a standard deviation of the four values.

**Exercise 3.3.5.** Does the speed of the cart affect the slope of the velocity vs time graph?

Discuss any evidence observed in your data when answering these questions. Also consider the magnitude of the uncertainties when writing your conclusions.

### 3.3.2 Cart and Sloped Track

Based on the results of Subsection 3.2.2, write an analysis of the relationship between the two graphs (x and v versus time). From the velocity versus time graph determine the value of the acceleration of the cart down the track.

**Exercise 3.3.6.** For the two downhill cases, use your uncertainty analysis to determine if the acceleration of the cart changed when it was given a small push.

**Exercise 3.3.7.** Is there an accuracy that can be computed for this part of the experiment?

**Hint.** If the track were frictionless, then the acceleration should be $a = (9.81\ {}^{\mathrm{m}}\!/\!_{\mathrm{s}^2})(\sin\theta)$, where $\theta$ is the angle that the incline makes.

Inspect the line/curve that is defined by the data on the Distance traveled vs. time graph.

**Exercise 3.3.8.** What is its shape? Is the shape of the graph what you would expect for constant acceleration (straight line, parabola, etc.)? Explain your reasoning.

**Exercise 3.3.9.** Consider the trendline that you added. Does/should the trendline line go through the origin? What is the value of y-intercept of the X vs T graph? What physical quantity does the intercept represent? Explain why it has that value.

**Hint.** Think about where the sensor was located.

**Exercise 3.3.10.** What does the slope (whether it's constant or not) of the line on this graph signify?

Now consider the Instantaneous Velocity vs. Time graph.

**Exercise 3.3.11.** Does the curve/line on this graph have the shape you would expect for an object undergoing constant acceleration? Explain.

**Exercise 3.3.12.** What was the value of the y intercept on this graph (include units and uncertainty!)? Explain its significance. To what does it refer?

**Hint.** Think carefully about what you plotted on the X-axis!

(Revised: September 13, 2017)
A PDF version might be found at acceleration.pdf (115 kB)

# Lab 4

# Newton's $2^{\text{nd}}$ Law on a Linear Track with the Sonic Ranger

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**Experimental Objectives**

In this experiment, we will assume that Newton's first law is true and focus on Newton's second law.

- By measuring
  - the velocity versus time for a cart being pulled down a track and
  - the applied force that is pulling it,

  we can plot the acceleration versus the force and verify the validity of Newton's second law of motion: $\vec{F}_{\text{net}} = m\vec{a}$.

---

## Introduction to Forces

Forces are related to the natural motion of bodies, where one object can affect the motion of another object. That is, forces are interactions between objects affecting their motion. Although the famous Greek philosopher Aristotle claimed that a force was necessary to *maintain* any motion, careful analysis by Italian physicist Galileo Galilei in the mid-$17^{\text{th}}$ century and by Sir Isaac Newton, a British mathematician and physicist (1642-1727), eventually distinguished the effects of friction and allowed Newton to create a mathematically consistent theory of motion. These concepts were published in Newton's book "Mathematical Principles of Natural Philosophy" in 1687, for which (among other accomplishments) Newton is regarded as one of the greatest scientists of all time.

All forces can be placed in one of two main categories. First, there are natural (or fundamental) forces like the gravitational force, the electromagnetic force, or the nuclear forces. The gravitational force is a force on a body by another body (like the Earth), this force is an interaction between their two masses. The electromagnetic force is an interaction between the charges of two bodies. These forces may act on an object without any direct physical contact between the two bodies. This type of force is sometimes called an "action at a distance" force. All other forces are in a second category called "contact forces."

***Newton's First Law:***

> If there are no forces acting, then objects will remain at rest or, if not at rest, will maintain their velocity.

If this is true, then we can study the forces acting on a body based on the motion of the body, specifically through the change in the velocity of an object.

***Newton's Second Law:*** Not only is a force necessary to change the motion (to cause an acceleration), the amount of acceleration that a force causes is predictable and is inversely proportional to the mass. The same sized force causes a small mass to accelerate a lot and a large mass to accelerate a little. This is expressed by the equation:

$$\vec{F}_{\text{net}} = m\vec{a}.$$

The net force, $\vec{F}_{\text{net}}$, is the vector sum of all forces acting on an object. If we have an extended object (such as a weight hanging off of a table, but connected to a cart that is on the table), then we need only consider forces that are "external" to the system: So long as both objects accelerate at the same rate, we do not need to consider the "internal" tension that the string exerts between the connected bodies.

***Newton's Third Law:*** Inherent in the description of a force is that it is an interaction between objects: there must always be two objects that interact. These objects exert equal and opposite forces on each other. That is,

> If there is a force exerted on object 1 by object 2, then there is necessarily and simultaneously a force exerted on object 2 by object 1 that is equal (in magnitude) and opposite (in direction) to the original force.

Remember that these two forces are on different objects and that the two bodies in direct contact exert forces on each other. Remember then that if there is contact between the object (any part of the system) and anything else then there is an outside force on the object (system) and that if there is no contact (the two bodies break contact) then there is no force.

## 4.1 Pre-Lab Considerations

- Based on your understanding of Subsection 4.3.1, draw a free-body force diagram for the cart and for the hanging mass.
- You should be prepared to derive an equation for the acceleration of the system, in terms of, the mass of the cart and the hanging mass, while assuming that the cart has no friction with the track. (Hint: There is only one force accelerating the system.)

## 4.2 Student Outcomes

Knowledge Developed: In this exercise, students should learn how forces are related to the motion of a cart, how to use a free-body diagram, and gain a visceral understanding of Newton's second law.

Skills Developed:

- Evaluate the data for sources of uncertainty. Can you see an effect, such as a level track or the presence of friction, in the result?
- Using Pasco Capstone software
- Interpreting slope and intercept of graphs

## 4.3 Procedure

### 4.3.1 The Experimental Setup

**Materials** A low-friction linear track with a wheeled cart and a pulley at one end of the track. Weights that can ride in the cart without jostling. A string connecting the cart to a light-weight support for small masses, which sits over the pulley allowing the masses to fall vertically while pulling the cart horizontally. A "sonic ranger" that uses sonar to measure the position, velocity, and acceleration of the cart.

- A low-friction linear cart and track will be used, this reduces the friction between the cart and the track.

- A string will be connected to the cart and a known mass will be hanging from the end of the string (and over a pulley). The hanging mass will exert a constant horizontal force on the cart as the mass falls all the way to the floor. This gives a constant acceleration to the cart.

- The sonic motion sensor will be used to measure the position of the cart as a function of time.

- The carts and tracks need to be handled with care. Scratches can add friction to the system.

### 4.3.2 Procedure

- If the cart is given an initial push (without the hanging mass and string attached) then the cart should travel with a constant velocity down the horizontal track, if there are no other forces acting on the cart. Carry out a couple of constant velocity runs on the track, to check for the effects of friction and to see how level the track is. The track may need a level adjustment. Do runs in both directions. Maybe the track can be tilted so that the friction is countered by the tilt of the track.

  It might help to review Exercises 3.2.1, 3.2.2, and 3.2.3 from the Constant Acceleration lab.

- Connect a string to the cart and run it over a pulley. Measure the height of the string at both ends of the track, to ensure that the string is as level as the track.

**Exercise 4.3.1.** If the pulley has a very large wheel or is set so that the string is low at the cart, but high at the pulley, then the force pulling the cart is not horizontal. Draw the free-body diagram for the cart in this case. Comment on if this increases, decreases, or does not affect each of the other forces ($F_{\text{normal}}$, $F_{\text{gravity}}$, $F_{\text{friction}}$).

**Hint 1.** If the tension pulls up, then the normal force does not need to support all of the weight.

**Hint 2.** If the tension pulls slightly up, is there a convenient way to find then angle at which it pulls? Will that angle depend on how close the cart is to the pulley?

**Answer.** Based on the hints, decide how important it is for you to ensure that the string pulls horizontally.

- The hanging mass should be much less than the mass of the cart. Use a small plastic cup to hold the hanging masses. Measure the mass of this cup. The total mass of the system must be kept constant for all parts of the experiment. The hanging mass and the mass of the cart should vary, but their total must be kept constant, by moving small mass amounts from the cart to the hanging cup. Record the mass of the cart, the hanging cup mass, and the extra masses which are to be transferred from the cart to the cup.

**Exercise 4.3.2.** If the hanging mass is not "much less" than the mass of the cart, then the acceleration will be very large and the cart will move too quickly.

Your total mass should include the mass of the string because it is also being accelerated. As an interesting thought experiment, you might notice that the amount of string that hangs off the pulley is contributing to the mass of the basket (the amount pulling the cart). But this changes as the cart moves! Without using calculus it is impossible to include this consideration, so we *hope* this is a small effect. Do you have a way of ensuring that this is a small effect?

**Hint 1.** How many significant digits do you have in the mass of the cart? Is the mass of the basket large enough to be a *significant* effect in the overall mass?

**Hint 2.** How many significant digits do you have in the mass of the cart? Is the mass of the string large enough to be a *significant* effect in the overall mass?

**Hint 3.** What percentage of the total mass of the system is the mass of the string? What percentage of the mass of the cart is the mass of the string?

**Answer.** The effect of the mass of the string in the measured acceleration will be *insignificant*.

**Note** The Pasco Capstone interface is also used in Lab 3, Lab 5, and Labs 7–8. You should start becoming familiar with the hardware and software.

- Take data with Capstone and the motion sensor as the cart travels with constant acceleration down the track. Determine the acceleration of the cart from a linear regression using the velocity vs time data (a linear fit line in Capstone). Record the acceleration value and its uncertainty.
- Collect 7 data runs, where about 2-5 grams[1] is transferred each time from the cart to the hanging mass. Determine the acceleration of the cart (and the uncertainty for the acceleration) for each of these 7 runs.

## 4.4 Analysis

You should note that velocity-versus-time graphs are only useful for computing the acceleration in each case. Once you have the values for the acceleration, your attention should be on the graph of acceleration-versus-weight.

- In Excel, make a graph of the acceleration of the system (y-axis) versus the weight ($mg$) of the *hanging* body (x-axis). You should include at least 7 data points. Carry out a linear regression for this data set. Quote the slope and intercept values, their uncertainties, their p-values, and the $R^2$ value. Show a sample error bar (on the graph) for at least one of the points of this graph.
- Derive (show it completely) an equation for the acceleration of the system versus the weight of the hanging body. Plot this theory equation on your graph (as a second series, a line but no points).
- Compare your graph to the predicted theoretical equation, that is compare the values of the slopes and intercepts.

**Exercise 4.4.1. Note:** In almost every lab you will be comparing a theoretical equation to the equation of a line and interpreting what the slope and intercept mean.

What is the physical significance of the slope and of the intercept from the graph? That is, what physical quantity does the slope of this graph equal?

**Hint.** It should help to recall that Newton's second law looks very similar to the generic equation of a line: $y = mx + b$.

**Answer.** When you figure out which physical quantity the slope *should* compare to, compute the %-difference to that value.

- In many mechanics experiments, there may be deviations from the expected or theoretical results because of the effects of friction. (If you are lucky, you will get to investigate this effect in detail in Lab 5!) Frictional forces are sometimes difficult to take into consideration. If there are deviations between your results and the predicted theory then try to distinguish whether they are caused by a tilt of the track, friction between the cart and the track or the friction between the string and the pulley.

**Exercise 4.4.2.** What might be expected in the results from these different systematic effects? That is, would the slope be expected to increase or decrease slightly because of the effects of friction? Would the slope be expected to increase or decrease slightly because of the effects of an unlevel track?

- When designing experiments, it is important to keep control parameters; in this case a parameter which is kept constant.

**Exercise 4.4.3.** What parameter is held constant in this experiment? Is there an obvious reason for keeping this constant?

[1] Pennies are a reasonable mass to be moving. If you are provided with pennies as the mass being transferred, then (after finding the individual mass of each) you might consider using the date of minting to distinguish which penny was transferred in order to determine the specific mass each time.

## 4.5 Questions

**1.** Why is it important to keep the total mass of the system constant? If one simply added mass to the hanger without keeping the system's mass constant, how would the data appear on the graph of the acceleration vs $mg$?

**Hint 1.** In the equation $F_{\text{net}} = ma$, $m$ is the mass of the objects being accelerated. Is this the mass that is important to keep constant?

**Hint 2.** In the equation $F_{\text{net}} = ma$, $F_{\text{net}}$ is the weight of the object doing the pulling. Is this the mass that is important to keep constant?

**Hint 3.** When you draw a graph on 2-dimensional graph paper, you have two axes, one for each variable. If you change the weight that is pulling and the overall mass being accelerated and you have a new acceleration, then think about which variables would you graph and what that would look like.

**Hint 4.** The equation $F_{\text{net}} = ma$ looks like (has the same form as) $y = mx + b$ if $\cancelto{0}{b}$. Which variable goes in the place of $y$? of $x$? of the slope? What happens to $y = mx + b$ if the slope is not a constant?

**2.** How would the motion (and therefore your results) be different if the track was not level? Consider both cases: if the pulley end were higher and if the pulley end were lower.

**Hint 1.** One situation describes the cart going downhill during the experiment; the other, uphill.

**Hint 2.** In each case, would your measured acceleration be equal to, larger than, or smaller than expected?

**Hint 3.** Is your result different from the expected result in a way consistent with either of these situations (uphill or downhill)?

**3.** If your group has a discrepancy between the results and the theory, could the presence of friction explain why your results differ from what is expected? Explain how.

**Hint 1.** Would friction tend to make the measured acceleration larger than or smaller than the expected value?

**Hint 2.** Is your result larger than or smaller than the expected result?
***Some sources of uncertainty***: When answering this next subset of Questions, consider your responses to Questions 2 and 3. Recall also that physicists often use the terms "source of error" and "source of uncertainty" interchangeably and do *not* mean error-as-in-mistake.

**4.** When considering possible sources of uncertainty, is it possible for tilt and friction to combine to make your observed result different from the expected result in the same way (i.e., both making your result too large or both making your result too small)?

**5.** When considering possible sources of uncertainty, is it possible for tilt and friction to act against each other to make your observed result closer to the expected result (i.e., friction causing an error one way and the tilt causing an error the other way)?

**6.** Regardless of how well your observed result agrees with the expected result, indicate how the possible tilt and friction might have impacted your results. If you are able to determine that one source of uncertainty clearly did not affect your result then comment on how the pattern of your data reveal this to you.

**7.** How would the cart's acceleration change, if at all, if the cart was given an initial push? Decide if this is a source of uncertainty.

**Hint.** Recall Exercise 3.3.5.

**8.** What are the two greatest sources of uncertainty in this experiment? Are they random or systematic errors? Be specific and quantify your answer.

(Revised: Oct 11, 2017)
A PDF version might be found at Newton.pdf (162 kB)

# Lab 5

# Dry Sliding Friction

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### Experimental Objectives

- In this experiment you will devise methods
  - to investigate the nature of both the frictional force and the coefficient of friction and
  - to test the validity of da Vinci's empirical rule.

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## Introduction

Friction is a force which retards the relative motion of any body while sliding over another body. The frictional force acting on a body is parallel to the surface that the object is sliding upon and it is directed opposite to the direction of motion. The phenomenon of friction is rather complicated, especially at the microscopic level, because it is dependent on the nature of the materials of both contacting surfaces. The frictional force depends on the roughness or the irregularities of both surfaces. At the macroscopic level, the nature of this force can be described by a simple empirical law, first given by Leonardo da Vinci:

> The magnitude of the force of friction between unlubricated, dry surfaces sliding one over the other is proportional to the normal force pressing the surfaces together and is independent of the (macroscopic) area of contact and of the relative speed.

At the microscopic level, the frictional force ($F_f$) does depend on the actual area of contact of the irregularities of the surfaces. This actual area of contact then increases as the force pressing the two surfaces together increases, this force is called the load. Using Newton's 2$^{\text{nd}}$ Law in this perpendicular direction we can conclude that the magnitude of the load is equal to the Normal force ($F_N$) of the surface pushing on the object. Therefore we may write that

$$F_f \propto F_N \qquad \Rightarrow \qquad F_f = \mu F_N$$

where the Greek letter $\mu$ ("mew") is a dimensionless constant of proportionality called the coefficient of friction.

When a body is pushed or pulled parallel to the surface of contact and no motion occurs, we can conclude that the force of the push or pull is equal to the frictional force, using Newton's 2$^{\text{nd}}$ Law of motion. As the applied force is increased, the frictional force remains equal to the applied force until motion results. At this maximum value of the applied force, the frictional force is also a maximum and is given by

$$F_f = \mu_s F_N$$

where the subscript $s$ stands for static (non-moving) friction, and $\mu_s$ is **the coefficient of static friction**. This equation can only be used at this maximum static point also called the point of impending motion. At the instant that the applied force becomes greater than the maximum $f_s$, the body is set into motion and this motion is opposed by a frictional force called the kinetic (sliding) frictional force and is given by

$$F_f = \mu_k F_N$$

where the subscript $k$ stands for the kinetic (moving) friction, and $\mu_k$ is **the coefficient of kinetic friction**. In general, $\mu_k < \mu_s$; that is, it takes more force to overcome the static friction than to overcome the kinetic friction. The coefficients of friction are generally less than one, but may be greater than one, and they depend on the nature of both surfaces.

Consider a system comprised of a block on a horizontal surface being pulled horizontally by a string connected to a hanging weight. We can use $M$ is the mass of the block on the horizontal surface and $m$ is the hanging mass. The force that accelerates the system forward is $mg$. The frictional force depends on the normal force of the block $\mu_k(Mg)$. Then, the whole system is accelerating with a constant acceleration so that Newton's second law gives:

$$(mg) + [-(\mu_k Mg)] = (M + m)a. \tag{5.0.1}$$

From this, $\mu_k$ can be solved for, giving:

$$\mu_k = \frac{mg - (M + m)a}{Mg}. \tag{5.0.2}$$

## 5.1 Pre-Lab Considerations

- Draw force diagrams for the following case: a block on a horizontal surface pulled by a hanging mass and a string (include the friction force).
- Write out the corresponding Newton's 2$^{\text{nd}}$ Law equations for forces both parallel and perpendicular to the contact surface.
- Derive the relevant equations for each of the above two cases for which the coefficients of friction can be determined:
    - Case one is static, but at the point of motion.
    - Case two is the kinetic case.

## 5.2 Student Outcomes

Knowledge Developed: In this exercise, students should learn how forces are related to the motion of a cart, how to use a free-body diagram, and gain a visceral understanding of Newton's second law with the (more realistic) inclusion of the effects of friction.

Skills Developed:

- Evaluate the data for sources of uncertainty. Can you see an effect, such as a level track or the presence of friction, in the result?
- Using Pasco Capstone software
- Interpreting slope and intercept of graphs
- Evaluating and propagating uncertainties

## 5.3 Procedure

For the block on the horizontal plane:

1. Clean the block and the plane, so that they are free of dust and other contaminants.
2. Make sure the track is level, as in previous labs.

You will use the force transducer to measure the force directly in Subsection 1 and Subsection 2. However, Subsection 3 will require an indirect measurement (calculation) of the force by measuring the velocity and using the velocity-versus-time graph to get the acceleration.

### 5.3.1 Break Static Friction — pull until moves

> **Note** By this time, you should already be familiar with the Pasco Capstone interface, which is also used in Lab 3, Lab 4, and Labs 7–8. You may remind yourself of the format by reading Appendix C

1. Set up the Dynamics Track, cart, force transducer and friction block. The force transducer attaches to the dynamics cart, the friction block rests on the track (felt side down).
2. Attach a string to the force transducer. The force transducer needs to be zeroed before data collection starts. Collect data, and slowly start pulling on the string (*be sure to pull the string horizontally*) and slowly increase the pull force until the cart is moving down the track. Using just the maximum force (at the point of impending motion) the coefficient of static friction can be calculated.
3. Test the relationship between the force of friction and the normal force, by changing the load force (normal force) and measuring the force of friction at the point of motion impending. Carry this out for a total of five data points. Graph the frictional force versus the normal force. Calculate the coefficient of static friction from this graph.

### 5.3.2 Effect of Surface Area — distinguish pressure from force

Consider pushing a pencil into your arm. (Well, don't *actually* do it!) If you use the erasure end, then you can feel the force, but it doesn't hurt. If you use the sharpened tip with the *same* force then it will certainly hurt! So, you have the idea that the same force spread over a different surface area *can* have a different effect; but it doesn't *always* have a different effect. For this part of the lab, you will test the relationship between the coefficient of friction and the macroscopic area of contact between the block and the surface.

1. Place the friction block on its side (felt side down) and repeat Item 2 and Item 3 for three (rather than five) of the previous load forces.
2. Add the plot of this $F_f$ versus $F_N$ as a new series to the graph of Subsection 5.3.1.

### 5.3.3 Friction while Accelerating

1. Apply a force (hanging mass, pulley, and string) large enough to accelerate the block. Use the Sonic Ranger to collect data. Note: This should accelerate fast enough to measure the acceleration, but not so fast that it crashes at the end of the track. (Depending on the normal forces being used, you might try 300 g as the hanging mass.)
2. Graph the velocity vs time. Determine the acceleration of the block from the slope of the line.
3. Repeat this part four or five times with a different normal forces. (You may use any hanging mass.)
4. Since we are not measuring the frictional force, you will need to calculate it; See Exercise 5.3.1.
5. Add the plot of this $F_f$ versus $F_N$ as a new series to the graph of Subsection 5.3.1 and Subsection 5.3.2.

6. Calculate the coefficient of kinetic friction from the slope.

**Exercise 5.3.1.** Draw the free-body diagram for the cart being dragged by the hanging mass. Set-up Newton's second law for the forces involved. Solve this for the frictional force in terms of quantities you can measure.

**Hint 1.** Lab 4 might help you set-up the free-body diagram and equation. In that lab, we made a point of keeping the total mass constant. In this lab, that is not important because that lab allows us to trust Newton's second law and we are now testing a different relationship.

**Hint 2.** Equation (5.0.1) wrote out Newton's second law for you; but you want to solve it for $F_f$, not for $\mu$ and not in terms of $\mu$.

**Hint 3.** You can measure the total mass directly. You can measure the hanging mass directly. You can compute (an indirect measurement) the acceleration from the velocity-versus-time graph.

**Answer.** Do not compute the frictional force using the normal force, that is the relationship you are trying to investigate!

## 5.4 Analysis

The experimental precision should be estimated for all parts of this experiment and care should be taken for all of the measurements. , but it is more important to investigate the relationships than it is to repeat the experiment many times or to try to achieve high precision in the data. In Exercise 5.3.1 you found an equation for $F_f$ in terms of measured values. You should track the uncertainties from measurement, through the calculation, to the result (this is called the Propagation of Uncertainties) so that you can see how the uncertainty in the measurements impact the uncertainty of the final result. The following will step you through how it would work for Equation (5.0.2), which is much more complicated than your equation.

**Example 5.4.1.** In order to propagate the uncertainty for $\mu_k = \dfrac{mg - (M+m)a}{Mg}$ we should notice that it has both addition (See Rule 1) and multiplication (See Rule 2). I will assume some values with uncertainty; I will also list the relative uncertainty:

- $m = 0.30 \pm 0.01\,\mathrm{kg}$, $\dfrac{0.01\,\mathrm{kg}}{0.30\,\mathrm{kg}} = 0.033 = 3.3\%$
- $M = 2.50 \pm 0.02\,\mathrm{kg}$, $\dfrac{0.02\,\mathrm{kg}}{2.50\,\mathrm{kg}} = 0.008 = 0.8\%$
- $a = 0.45 \pm 0.04\ \mathrm{m/s^2}$, $\dfrac{0.04\ \mathrm{m/s^2}}{0.45\ \mathrm{m/s^2}} = 0.089 = 8.9\%$
- $g = 9.81 \pm 0.01\ \mathrm{m/s^2}$, $\dfrac{0.01\ \mathrm{m/s^2}}{9.81\ \mathrm{m/s^2}} = 0.001 = 0.1\%$

You should note that while the uncertainty of $m$ and $M$ are about the same, the relative uncertainty of $m$ is much larger than the relative uncertainty of $M$ (and similarly for $a$ and $g$). While the uncertainty for $m$ and $g$ have the same numeric value, they cannot be meaningfully compared because they have different units.

Recall the "order of operations" implies that we ***first*** add $(M + m)$, ***then*** multiply that value by $a$, ***then*** add that value to (the product of) $mg$, and ***finally*** divide by (the product of) $Mg$. We do the propagation of uncertainty in the same order.

Using Rule 1, add the masses $(M + m)$:

$$\begin{aligned}(0.30 \pm 0.01\,\mathrm{kg}) + (2.50 \pm 0.02\,\mathrm{kg}) &= [(0.30 + 2.50) \pm (0.01 + 0.02)]\,\mathrm{kg}\\ &= 2.80 \pm 0.03\,\mathrm{kg}\end{aligned}$$

$$\frac{0.03\,\mathrm{kg}}{2.80\,\mathrm{kg}} = 0.011 = 1.1\%$$

This overall uncertainty is affected a little bit more by $M$ (in this case) because we are adding uncertainty (not relative uncertainty).

Using Rule 2, multiply by the acceleration $(M+m)a$:

$$\begin{aligned}(2.80 \pm 0.03\,\text{kg})(0.45 \pm 0.04\,{}^{\text{m}}\!/\!_{\text{s}^2}) &= [(2.80\,\text{kg})(0.45\,{}^{\text{m}}\!/\!_{\text{s}^2}] \pm [(0.011 + 0.089)] \\ &= 1.26\,\text{N} \pm [0.10],\end{aligned}$$

since $(1.26\,\text{N})(0.10) = 0.126\,\text{N}$, this is $1.26 \pm 0.13\,\text{N}$.[1] In this case, $a$ contributed about 8 times as much uncertainty.[2]

We also need to use Rule 2 to find the uncertainty for $mg$ before combining the numerator:

$$\begin{aligned}(0.30 \pm 0.01\,\text{kg})(9.81 \pm 0.01\,{}^{\text{m}}\!/\!_{\text{s}^2}) &= [(0.30\,\text{kg})(9.81\,{}^{\text{m}}\!/\!_{\text{s}^2}] \pm [(0.033 + 0.001)] \\ &= 2.94\,\text{N} \pm [0.034],\end{aligned}$$

since $(2.94\,\text{N})(0.034) = 0.10\,\text{N}$, this is $2.94 \pm 0.10\,\text{N}$.[3] Notice that the mass had a larger impact on the uncertainty of this term.

Using Rule 1, subtract[4] the two terms in the numerator [$mg$ and $(M+m)a$]:

$$\begin{aligned}(2.94 \pm 0.10\,\text{N}) - (1.26 \pm 0.13\,\text{N}) &= [(2.94 - 1.26) \pm (0.10 + 0.13)]\,\text{N} \\ &= 1.68 \pm 0.23\,\text{N}\end{aligned}$$

$$\frac{0.23\,\text{N}}{1.68\,\text{N}} = 0.14 = 14\%$$

Notice that both terms had a similar contribution to the uncertainty.[5]

Finally, use Rule 2 to find the uncertainty for the final combination[6] of the numerator/denominator:

$$\begin{aligned}\frac{(1.68 \pm 0.23\,\text{N})}{(2.50 \pm 0.02\,\text{kg})(9.81 \pm 0.01\,{}^{\text{m}}\!/\!_{\text{s}^2})} &= [\frac{1.68\,\text{N}}{(2.50\,\text{kg})(9.81\,{}^{\text{m}}\!/\!_{\text{s}^2})}] \pm [(0.14 + 0.008 + 0.001)] \\ &= 0.0685 \pm [0.15],\end{aligned}$$

since $(0.0685\,\text{N})(0.15) = 0.010\,\text{N}$, this is $0.07 \pm 0.01 = (7 \pm 1) \times 10^{-2}$, rounded to the appropriate number of significant digits.

You should note that the uncertainty of the numerator swamps by far the uncertainty of the denominator. In order to make this measurement more precise, we should focus on improving the precision of $m$ and $a$, as indicated in Footnote 5.4.5.

The graphs and data should also be evaluated as usual. In this particular experiment, you should

- Explain why the normal force on the block by the surface rather than the weight of the object is related to the frictional force.
- Interpret the slope and intercept of the graphs.
- Compare the slopes from each of the three parts. Decide which should be the same and which should be different.
- Calculate the % decrease of the static to kinetic coefficient of friction.
- Comment on the validity of the empirical rules of friction.

[1] If this were the final answer, you should report it as $1.3 \pm 0.1\,\text{N}$; but since we are continuing to use it in calculations, for the purpose of managing appropriate rounding errors, you can keep an extra insignificant digit, which will maintain consistency. Please be sure to round your final answer to the appropriate number of significant digits.

[2] To improve this answer we should focus on a more precise measurement of $a$, rather than improving the precision of $m$ or $M$!

[3] If this were the final answer, you should report it as $2.9 \pm 0.1\,\text{N}$; but since we are continuing to use it in calculations, for the purpose of managing appropriate rounding errors, you can keep an extra insignificant digit, which will maintain consistency. Please be sure to round your final answer to the appropriate number of significant digits.

[4] Remember the rule is to ***add*** the uncertainty even if you are ***subtracting*** the values!

[5] Even if we improved the precision of $a$, as mentioned in Footnote 5.4.2, the uncertainty of $mg$ would keep this uncertainty near the 0.1 to 0.2 range. So we need to be more precise with $m$ for the $mg$ term *and* with $a$ for the $(M+m)a$ term.

[6] Remember the rule is to ***add*** the relative uncertainty whether you are multiplying ***or*** dividing the values!

## 5.5 Questions

For all questions, and when possible, use your experimental or theoretical results to demonstrate your answers to the questions.

**1.** Does the coefficient of friction depend on the area of contact?

**2.** Does the coefficient of friction depend on the mass of the object?

**3.** Does the coefficient of friction depend on the normal force of the object?

**4.** Does the frictional force depend on the normal force of the object?

**5.** Does the coefficient of kinetic friction depend on the speed of travel?

**6.** When the object was pulled by a string, how would the forces be affected if the cord was not horizontal?

**7.** What would happen to the coefficient of friction if the surfaces were lubricated with oil?

(Revised: Oct 11, 2017)
A PDF version might be found at friction.pdf (176 kB)

# Lab 6

# Centripetal Force

### Experimental Objectives

- It is our purpose to experimentally verify the equation for centripetal force by
  - measuring the applied force and
  - comparing it to the specific combination of variables expressed as either $\frac{mv^2}{r}$ or $m\omega^2 r$.

## Introduction

We will be investigating the force which is necessary to maintain the circular motion of an object. The apparatus used will allow you to spin an F-shaped arm which has a mass suspended from the top arm. This mass will be held in place by a spring which makes up the lower arm. The spring will provide the centripetal force. You will need to be familiar with the ideas of circular motion, centripetal versus centrifugal force, centripetal acceleration, and angular velocity. In addition to these concepts, try to understand how we will measure the angular speed in lab.

The centripetal acceleration $a_c$ is calculated from the following equation written either $a_c = \frac{v^2}{r}$ or $a_c = \omega^2 r$ where $v$ is the linear velocity of the particle, $\omega$ is the angular velocity, and $r$ is the radius of the circle. Note that angular velocity is measured in radians per second.

From Newton's second law, $\vec{F} = m\vec{a}$. Therefore, the force required to keep the particle of mass $m$ moving in a circle with constant speed is

$$F = ma_c = \frac{mv^2}{r} = m\omega^2 r \tag{6.0.1}$$

Recall that the centripetal force is not a force applied *in addition* to other existing forces. The centripetal force is *whatever combination* of existing force act to maintain circular motion.

## 6.1 Pre-Lab Considerations

1. Why do we say that an object moving with *constant speed* in a circular path is being accelerated?
2. In which direction is that acceleration? How do you know?
3. Is this $a_c$ "centripetal" or "centrifugal?"

## 6.2 Student Outcomes

Knowledge Developed: In this exercise, students should learn how the forces required to hold spinning objects in place changes with the speed.

Skills Developed:

- Evaluating and propagating uncertainties from the source of uncertainty to the result

## 6.3 Procedure

The centripetal force is supplied by a spring. Since we cannot directly measure "the force exerted by the spring while it is rotating" while it is rotating, determine how we can measure "the force exerted by the spring during the rotation" when the spring is not rotating.

- By means of the lab apparatus, a mass $m$ can be made to rotate with a constant (and measurable) angular speed $\omega$. With some practice, it is possible to adjust the speed so that the radius of the path remains constant. The value of the radius, $r$, is marked on the apparatus and so can be measured easily.
- Measuring the mass should be an obvious task.
- Measuring the angular speed $\omega$ is straightforward, but may not be obvious. To do so, consider the following:
  - Angular speed ($\omega$) is measured in $\frac{\text{rad}}{\text{s}}$.
  - $\omega$ is related to the rotational speed which is measured in $\frac{\text{rev}}{\text{s}}$.
  - There are $2\pi$ radians in 1 revolution.
  - We can count the number of revolutions.
  - The "period of rotation," $T$, is defined as the number of seconds per revolution.
  - We measure the period not by timing a single revolution, but by measuring the time for multiple (20) revolutions divided by the number of revolutions. (This averages out any uncertainty due to reaction-time.)

  As the mass rotates, its period of rotation can be measured. This allows you to calculate the angular speed using the hints above.
- Repeat the entire procedure for a second value of F.
  - To change the force used, you have to adjust the spring as follows.
  - There is a small dial at the end of the spring away from the mass. If you turn it one way, it tightens the spring (increasing the force). If you turn it the other way, it loosens the spring (decreasing the force).
  - When you change this dial, it is unlikely that you will be able to put it back to where you originally had it, so you should be certain that you have all the information you need from the first measurements before you turn this dial!

## 6.4 Analysis

From measurements of $m$, $\omega$, and $r$, calculate the theoretical centripetal force on the mass (which experimentally is supplied by the spring). You can, of course use Rule 2 to determine the uncertainty of this centripetal force (as expected from the theory).

The (unmeasurable) amount of force exerted by the spring to hold the spinning mass at a particular radius is equal to the (measurable) force required to stretch the spring to that radius. Experimentally determine the force necessary to stretch the spring to a specific radius. You should, of course, find the uncertainty in this value as well.

In order to verify the theory of centripetal force, and Equation (6.0.1) in particular, compare the force needed to stretch the spring with the amount of theoretical centripetal force calculated above.

(Revised: Oct 11, 2017)
A PDF version might be found at centripetal.pdf (83 kB)

# Lab 7

# Conservation of Energy on a Linear Track – (Single Week Version)

### Experimental Objectives

- The purpose of this experiment will be to verify the validity of the law of conservation of mechanical energy, which says that $\Delta E = 0$ as a cart runs along a track.

## Introduction

Conservation laws play a very important role in our understanding of our physical world. For example, the law of conservation of energy can be applied in all physical processes. This is a fundamental and independent statement about the nature of the physical world. It is not necessarily derivable from other laws like Newton's Laws of motion. Though for simple point mass systems, the law of conservation of energy can be derived from Newton's Laws. It can be shown that the net work done on a system is equal to the change in the kinetic energy ($W_{\text{net}} = \Delta K$) of the system; this is called the work-energy theorem and it can be written in a variety of forms. When a net positive work is done on a system, the kinetic energy of the system increases, and when a net negative work is done on the system (as from a friction force), the kinetic energy of the system decreases.

When the gravitational force acts on a system, the work it does on the system, $W_g$, is the gravitational force ($mg$) times the vertical displacement ($h = \Delta y$): $W_g = mg\Delta y$. For convenience, this is called the change in gravitational potential energy ($W_g = -\Delta P$). *If* the gravitational force is the only force acting on the system then $W_g = W_{\text{net}}$ and therefore, $-\Delta P = \Delta K$ for the system. When a force can be associated with a potential energy, it is called a "conservative force." Another kind of potential energy deals with an elastic potential energy, like in a spring. The energy stored in a spring is given by the formula $P_s = \frac{1}{2}k\Delta x^2$.

*If*, on the other hand, a force dissipates energy, then it is called a "nonconservative force" and it will have no associated potential energy. Frictional forces are an example of a nonconservative force and the work done by a frictional force is negative because (physically) the frictional force removes energy from the system and (mathematically) the frictional force and the displacement are in opposite directions. This work done by friction is converted into heat or sound. To distinguish the energy of heat or sound from the potential and kinetic energy, we define the total mechanical energy, $E = K + P$ at any point. Since frictional forces remove mechanical energy, we say $W_f = \Delta E = \Delta K + \Delta P$.

In general then, the law of conservation of energy states that energy can not be created or destroyed, but can only change from one form to another; or the total energy of the system at point A is equal to the total energy of the system at point B.

## 7.1 Procedure

We would like for you to verify the conservation of mechanical energy in two different situations; so, there are two parts to this experiment. We will first consider a flat track with accelerated motion, as in the Newton's Law lab and the Friction lab. We can then consider an inclined plane. You will not be given an explicit procedure, but rather you will be given a series of questions with answers that will imply the procedure. Part of the experiment is for you to figure out for yourself what the best course of action is. Please answer the questions as they are asked.

### 7.1.1 Flat Track

Set up the dynamics cart on a horizontal dynamics track. Set up the motion sensor at one end of the track and a pulley at the other end so that the pulley partly extends past the edge of the table. Hang the basket over the pulley so that it can accelerate the cart along the track – you might need extra weight in the cart to keep it from accelerating too fast. In order to use this motion to verify the validity of the conservation of mechanical energy, we need to measure some variables. Answering Exercise 7.1.1 and Exercise 7.1.2 will help you decide on the relevant variables. Exercise 7.1.3 should help you determine how to finish setting up the equipment.

**Exercise 7.1.1.** In order to verify $\Delta E = 0$, we will need to calculate $E$ as $E = K + P$. Therefore, we need to know the kinetic energy, $K = \frac{1}{2}mv^2$, the energy of *some mass*, $m$, moving at a speed $v$. Which mass do you need to measure? How can you measure the velocity?

**Exercise 7.1.2.** In order to verify $\Delta E = 0$, we will need to calculate $E$ as $E = K + P$. Therefore, we need to know the potential energy, $P = mgy$, the energy of *some mass*, $m$, located some height, $y$, above the ground. Which mass do you need to measure? How can you measure the position?

**Exercise 7.1.3.** In order to measure the position of the falling mass and the velocity of the system, do you need two motion sensors? Can you manage with one? Considering that it is a fairly expensive piece of equipment, where should you NOT put the sonic ranger? Where could you put it? Depending on where you put the ranger, decide if you need to "translate" the position or velocity data in order to find the specific values that you actually need.

Once you decide what variables to measure, run the experiment for one set of masses while measuring the appropriate variables. Put the data into Excel and decide what plot(s) will allow you to verify the validity of the conservation of mechanical energy. Exercise 7.1.4 may help with this. Decide if you need a trendline. Relate the information in Exercise 7.1.5 to the statement you are trying to verify.

**Exercise 7.1.4.** To verify $\Delta E = 0$, we will need to graph $E$, the total mechanical energy, as a function of time. What do you expect this graph to look like, if the law is valid? If not?

1. Does the kinetic energy change during this motion? Is $\Delta K = 0$? Considering the initial and final values of the kinetic energy, $K_i$ and $K_f$, what would a graph of $K$ versus time look like?

2. Does the potential energy change during this motion? Is $\Delta P = 0$? Considering the initial and final values of the potential energy, $P_i$ and $P_f$, what would a graph of $P$ versus time look like?

3. Assuming that the mechanical energy is conserved, what would a graph look like if it included $E$, $K$, and $P$? What if the mechanical energy is not conserved? How would $K$ and $P$ be affected in these two cases?

4. (Subsection 7.1.2 only) When the cart is at the bottom of the track during the motion, the values of position become negative (less than zero!). Why? Is there some other place where the energy might go?

**Exercise 7.1.5.** Please note the overall change in potential energy, $\Delta P$, and the overall change in the kinetic energy, $\Delta K$. Should either of these be related to the overall change in energy $\Delta E$ and, if so, how?

### 7.1.2 Sloped Track

Remove the pulley from the track. Your cart will have either a spring-loaded "battering ram" on the front or a pair of magnets. If you have the battering ram, then you will want the end of the track with the rubber nub at the bottom of the incline. If you have the magnets, then you need to replace the pulley with a "C" shaped "catch-bar." *Ask for help from the instructor!* The catch-bar has magnets in it that will repel the magnets in the cart. In this case, the cart must not be going so fast as to come into physical contact with the magnets on the catch-bar.

Raise one end of the dynamics track. Exercise 7.1.6 should help decide how tilted. Measure the tilt angle of the track with two methods: use a protractor, and measure the vertical rise and track length and calculate the tilt angle using the inverse-sine function. Answer Exercise 7.1.7. As you continue to set up the track for measurements, consider answering Exercise 7.1.1, Exercise 7.1.2, and Exercise 7.1.3 again for this situation to help you decide on the appropriate accessories (sensors); but note Exercise 7.1.8 as you think about the answers to the previous questions.

**Exercise 7.1.6.** We want the cart to accelerate down the track (not too slow), but not to fly off at the bottom (not too fast). How fast is *too fast*? Don't use that slope! How fast is *too slow*? Use a slope somewhere in between.

**Exercise 7.1.7.** After you measure the angle of incline in these two ways, consider the uncertainty in the measurements. Which of these measurement is more precise?

**Exercise 7.1.8.** The motion sensor will measure the motion of the cart *along* the ramp, but the potential energy needs the *vertical* position of the cart. Which trig function relates the distance along the ramp to the corresponding vertical distance?

Once you decide on the variables to be measured, but before you make the measurements, you will need to calibrate your position measurements. We would like zero to correspond to being at the bottom of the ramp, so place the cart stationary at the bottom and use the motion sensor to measure this position. In order to verify the validity of the conservation of mechanical energy, release the cart from rest near the top of the ramp and let it roll down the incline, bouncing three times before you stop the measurement. Do this for one value of mass.

Transfer these data to Excel again and decide on the best graph to verify the objective. Again, Exercise 7.1.4 may help with this; however, you will also need to consider Item 7.1.4.4. Decide if you need a trendline and where it would be fit. Relate the information in Exercise 7.1.5 to the statement you are trying to verify.

## 7.2 Analysis

We are now going to take a closer look at the irregularities of the data and investigate some variations to try to explain what those data say.

- Before drawing conclusions about the validity of the conservation of mechanical energy, consider Exercise 7.2.1.

**Exercise 7.2.1.** We need to look for the energy lost in each graph.

1. When you look at the graph from Subsection 7.1.1 for $E$, is the energy conserved or is there energy lost? If lost, calculate the energy lost or gained from the graph. (It might help to have a trendline.) If energy is lost, come up with at least two explanations for where this energy goes.

2. When you look at the graph from Subsection 7.1.2 for $E$, there are jumps in the energy. Why?

   (a) What is happening between the jumps? Does Subsection 7.1.1 help to explain these sections of the graph? Compared to the jumps, can we assume that the mechanical energy is conserved between the jumps?

   (b) What is happening at the time of those "jumps"? From the trend of the graph, calculate the amount of energy lost during each sudden change, call it the energy discrepancy, and the percent of this discrepancy relative to the total energy before the corresponding collision. Discuss where this "missing" energy goes. Is the ratio of "energy discrepancy" to total prior energy the same for each jump?

3. Comment in general, on the law of Conservation of Mechanical Energy. Can you predict any effects that might invalidate the conservation of mechanical energy? Can these effects be minimized? Is it possible to run the experiment again minimizing this effect?

- One explanation of a loss of energy (non-conservation) is friction. List all of the places where two pieces of material rub against each other. Since $F_f = \mu F_N$, do any of these locations have a normal force that can be varied? (Note Exercise 7.3.2.) As an independent measure of the amount of friction, we can also consider the actual acceleration versus the expected acceleration. Exercise 7.2.2 will help you determine the expected acceleration and the variable necessary to find it. Exercise 7.2.3 will help decide on the relationship between the friction and the acceleration.

**Exercise 7.2.2.** Given an ramp inclined at some angle $\theta$, what is the component of the gravitational force aimed down the ramp? Assuming that there is no friction, what is the net force? Since $F_{\text{net}} = ma$, the acceleration should be ... ?[1] From your expression, what do you need to measure in order to find the expected value of $a$? (Recall Exercise 7.1.7.)

**Exercise 7.2.3.** If there is friction, then how do you expect the actual acceleration to compare to the expected acceleration? If there is no friction? So, how would you interpret finding an acceleration that is exactly equal to the expected value? less than the expected value? Larger than the expected value?

- A second explanation for the loss of energy is that some component is gaining rotational kinetic energy. The formula for this is $K_R = \frac{1}{2}I\omega^2$, where $I$ is the moment of inertia[2], and $\omega$ is the angular speed $\omega = v/r$. Assuming that any discrepancy that you found in the conservation of energy is due to the rotational kinetic energy of the pulley, how much energy would the pulley need to have at the end of the run (while spinning full speed)? Based on the final velocity of the cart, what is the angular speed of the pulley? Based on these numbers, $K_R$ and $\omega$, what is the moment of inertia for the pulley? Can you tell if this is a reasonable estimate?

## 7.3 Questions

**1.** Does the mass of the cart matter? If you run it again at a different value of mass, would you expect the overall conclusion to be different? Would you expect the specific values to be different?

**2.** If the mechanical energy is conserved, then

$$\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f$$

What do you notice about the mass? Is your graph different if the mass of the cart changes? Does this support or conflict with the idea that the total mechanical energy is conserved? On the other hand, if the mechanical energy is not conserved, then

$$W_{\text{nc}} = \frac{1}{2}mv_f^2 + mgy_f - \frac{1}{2}mv_i^2 - mgy_i$$

What do you notice about the mass now? Does your graph support or conflict with the idea that the total mechanical energy is conserved?

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[1] $a = g\sin\theta$.

[2] In this case, the moment of inertia is probably a little less than $\frac{1}{2}mr^2$, where $m$ is the mass of the rotating object and $r$ is the radius of the rotating object. This is not a convenient way to calculate $I$ at this time.

# Lab 8

# Conservation of Energy on a Linear Track – (Two Week Version)

## Experimental Objectives

- The purpose of this experiment will be to verify the validity of the law of conservation of mechanical energy, which says that $\Delta E = 0$ as a cart runs along a track.

## Introduction

Conservation laws play a very important role in our understanding of our physical world. For example, the law of conservation of energy can be applied in all physical processes. This is a fundamental and independent statement about the nature of the physical world. It is not necessarily derivable from other laws like Newton's Laws of motion. Though for simple point mass systems, the law of conservation of energy can be derived from Newton's Laws. It can be shown that the net work done on a system is equal to the change in the kinetic energy ($W_{\text{net}} = \Delta K$) of the system; this is called the work-energy theorem and it can be written in a variety of forms. When a net positive work is done on a system, the kinetic energy of the system increases, and when a net negative work is done on the system (as from a friction force), the kinetic energy of the system decreases.

When the gravitational force acts on a system, the work it does on the system, $W_g$, is the gravitational force ($mg$) times the vertical displacement ($h = \Delta y$): $W_g = mg\Delta y$. For convenience, this is called the change in gravitational potential energy ($W_g = -\Delta P$). *If* the gravitational force is the only force acting on the system then $W_g = W_{\text{net}}$ and therefore, $-\Delta P = \Delta K$ for the system. When a force can be associated with a potential energy, it is called a "conservative force." Another kind of potential energy deals with an elastic potential energy, like in a spring. The energy stored in a spring is given by the formula $P_s = \frac{1}{2}k\Delta x^2$.

*If*, on the other hand, a force dissipates energy, then it is called a "nonconservative force" and it will have no associated potential energy. Frictional forces are an example of a nonconservative force and the work done by a frictional force is negative because (physically) the frictional force removes energy from the system and (mathematically) the frictional force and the displacement are in opposite directions. This work done by friction is converted into heat or sound. To distinguish the energy of heat or sound from the potential and kinetic energy, we define the total mechanical energy, $E = K + P$ at any point. Since frictional forces remove mechanical energy, we say $W_f = \Delta E = \Delta K + \Delta P$.

In general then, the law of conservation of energy states that energy can not be created or destroyed, but can only change from one form to another; or the total energy of the system at point A is equal to the total energy of the system at point B.

## 8.1 Procedure

We would like for you to verify the conservation of mechanical energy in two different situations; so, there are two parts to this experiment. We will first consider a flat track with accelerated motion, as in the Newton's Law lab and the Friction lab. We can then consider an inclined plane. You will not be given an explicit procedure, but rather you will be given a series of questions with answers that will imply the procedure. Part of the experiment is for you to figure out for yourself what the best course of action is. Please answer the questions as they are asked.

**NOTE:** There is enough analysis for this lab that you will have two weeks to complete the lab. During the first week, you will do the two parts of the experiment and begin to write up your report. During the second week, you will do some analysis and re-run the experiment to determine the cause of differences from expectations. A single lab report will be due after the second week of experimentation.

### 8.1.1 Flat Track

Set up the dynamics cart on a horizontal dynamics track. Set up the motion sensor at one end of the track and a pulley at the other end so that the pulley partly extends past the edge of the table. Hang the basket over the pulley so that it can accelerate the cart along the track – you might need extra weight in the cart to keep it from accelerating too fast. In order to use this motion to verify the validity of the conservation of mechanical energy, we need to measure some variables. Answering Exercise 8.1.1 and Exercise 8.1.2 will help you decide on the relevant variables. Exercise 8.1.3 should help you determine how to finish setting up the equipment.

**Exercise 8.1.1.** In order to verify $\Delta E = 0$, we will need to calculate $E$ as $E = K + P$. Therefore, we need to know the kinetic energy, $K = \frac{1}{2}mv^2$, the energy of *some mass*, $m$, moving at a speed $v$. You will have to decide which mass you need to measure. You will also have to decide how to measure the velocity.

**Hint 1 (mass).** The mass that should be used for the kinetic energy is the mass that is moving at this speed.

**Hint 2 (how to measure velocity).** You have measured the speed of these carts several times in previous labs. Do you recall how you did it then?

**Hint 3 (where to measure velocity).** How does the velocity of the basket compare to the velocity of the block? Is velocity the quantity you need in this equation?

**Exercise 8.1.2.** In order to verify $\Delta E = 0$, we will need to calculate $E$ as $E = K + P$. Therefore, we need to know the potential energy, $P = mgy$, the energy of *some mass*, $m$, located some height, $y$, above the ground. You will have to decide which mass you need to measure. You will also have to decide how to measure the ~~position~~ height.

**Hint 1 (mass).** The mass that should be used for the potential energy is all of the mass that is changing its vertical position.

**Hint 2 (how to measure position).** You have measured the position of these carts several times in previous labs. Do you recall how you did it then? In this case, which position do you need to know to compute the potential energy?

**Hint 3 (where to measure position).** The location that you need in order to compute the potential energy is the height of the thing that is moving vertically. See also Exercise 8.1.3.

**Exercise 8.1.3.** In order to measure the position of the falling mass and the velocity of the system, do you need two motion sensors? Can you manage with one?

**Hint 1.** The cart and the basket both move the same distance and move with the same speed.

**Hint 2.** If you the position/speed of the basket directly, then, considering that the motion sensor is a fairly expensive piece of equipment, where should you NOT put the sonic ranger? Where could you put it so that it will not get hit?

**Hint 3.** Depending on where you put the ranger, decide if you need to "translate" the position or velocity data in order to find the specific values that you actually need.

Once you decide what variables to measure, run the experiment for one set of masses while measuring the appropriate variables. Put the data into Excel and decide what plot(s) will allow you to verify the validity of the conservation of mechanical energy. Exercise 8.1.4 may help with this. Decide if you need a trendline. Relate the information in Exercise 8.1.5 to the statement you are trying to verify.

**Exercise 8.1.4.** To verify $\Delta E = 0$, we will need to graph $E$, the total mechanical energy, as a function of time. What do you expect this graph to look like, if the law is valid? If not?

***Note:*** Hint 6 is only relevant to Subsection 8.1.2.

**Hint 1 (KE-graph).** Does the kinetic energy change during this motion? Is $\Delta K = 0$? Considering the initial and final values of the kinetic energy, $K_i$ and $K_f$, what would a graph of $K$ versus time look like?

See Hint 4 to think about how a graph of velocity might help.

**Hint 2 (PE-graph).** Does the potential energy change during this motion? Is $\Delta P = 0$? Considering the initial and final values of the potential energy, $P_i$ and $P_f$, what would a graph of $P$ versus time look like?

See Hint 5 to think about how a graph of position might help.

**Hint 3 (E-graph).** Assuming that the mechanical energy is conserved, what would a graph look like if it included $E$, $K$, and $P$? What if the mechanical energy is not conserved? How would $K$ and $P$ be affected in these two cases?

**Hint 4 (velocity-graph).** Can you think of an equation of motion that relates the velocity to the time? Does this produce a linear or quadratic (parabolic) dependence on the time? Since the $K \sim v^2$, what dependence should $K$ have with the time?

**Hint 5 (position-graph).** Can you think of an equation of motion that relates the position to the time? Does this produce a linear or quadratic (parabolic) dependence on the time? Since the $P \sim y$, what dependence should $P$ have with the time?

**Hint 6 (Negative PE?).** (Subsection 8.1.2 only) When the cart is at the bottom of the track during the motion, the values of position become negative (less than zero!). Why? Is there some other place where the energy might go?

1. If you are using the force transducer, then it has a spring and a spring potential energy, $\Delta P_{\text{spring}}$. This can (and should!) also be included in the total mechanical energy. You can calculate the elastic potential energy stored in the spring of the force transducer with $P = \frac{1}{2}k\Delta x^2$, which, since we do not know $k$, can be written $P = \frac{1}{2}F\Delta x$, where F is the force in Newtons (measurable with the force transducer) and $\Delta x$ is the distance from the spring's equilibrium position, not the height (derivable from the position data). Be sure to match up the force values and the $x$ values at those same times.

**Exercise 8.1.5.** Please note the overall change in potential energy, $\Delta P$, and the overall change in the kinetic energy, $\Delta K$. Should either of these be related to the overall change in energy $\Delta E$ and, if so, how?

> **NOTE:** Save your data so that you can do further analysis next week.

### 8.1.2 Sloped Track

Remove the pulley from the track. Your cart will have either a spring-loaded "battering ram" on the front or a pair of magnets. If you have the battering ram, then you will want the end of the track with the rubber nub at the bottom of the incline. If you have the magnets, then you need to replace the pulley with a "C" shaped "catch-bar." *Ask for help from the instructor!* The catch-bar has magnets in it that will repel the magnets in the cart. In this case, the cart must not be going so fast as to come into physical contact with the magnets on the catch-bar.

Raise one end of the dynamics track. Exercise 8.1.6 should help decide how tilted. Measure the tilt angle of the track with two methods: use a protractor, and measure the vertical rise and track length and calculate the tilt angle using the inverse-sine function. Answer Exercise 8.1.7. As you continue to set up the track for measurements, consider answering Exercise 8.1.1, Exercise 8.1.2, and Exercise 8.1.3 again for this situation to help you decide on the appropriate accessories (sensors); but note Exercise 8.1.8 as you think about the answers to the previous questions.

**Exercise 8.1.6.** We want the cart to accelerate down the track (not too slow), but not to fly off at the bottom (not too fast). How fast is *too fast*? Don't use that slope! How fast is *too slow*? Use a slope somewhere in between.

**Exercise 8.1.7.** After you measure the angle of incline in these two ways, consider the uncertainty in the measurements. Which of these measurement is more precise?

**Exercise 8.1.8.** The motion sensor will measure the motion of the cart *along* the ramp, but the potential energy needs the *vertical* position of the cart. Which trig function relates the distance along the ramp to the corresponding vertical distance?

Once you decide on the variables to be measured, but before you make the measurements, you will need to calibrate your position measurements. We would like zero to correspond to being at the bottom of the ramp, so place the cart stationary at the bottom and use the motion sensor to measure this position. In order to verify the validity of the conservation of mechanical energy, release the cart from rest near the top of the ramp and let it roll down the incline, bouncing three times before you stop the measurement. Do this for one value of mass. Answer Exercise 8.1.9.

**Exercise 8.1.9.** Does the mass of the cart matter? If you run it again at a different value of mass, would you expect the overall conclusion to be different? Would you expect the specific values to be different?

**Exercise 8.1.10.** If the mechanical energy is conserved, then

$$\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f$$

What do you notice about the mass? Is your graph different if the mass of the cart changes? Does this support or conflict with the idea that the total mechanical energy is conserved? On the other hand, if the mechanical energy is not conserved, then

$$W_{\text{nc}} = \frac{1}{2}mv_f^2 + mgy_f - \frac{1}{2}mv_i^2 - mgy_i$$

What do you notice about the mass now? Does your graph support or conflict with the idea that the total mechanical energy is conserved?

Transfer these data to Excel again and decide on the best graph to verify the objective. Again, Exercise 8.1.4 may help with this; however, you will also need to consider Hint 8.1.4.6. Decide if you need a trendline and where it would be fit. Relate the information in Exercise 8.1.5 to the statement you are trying to verify.

**NOTE:** Save your data so that you can do further analysis next week.

## 8.2 Analysis

For the second week, you should already have your graphs from the experiment and you should have written a significant portion of the theory and the analysis. We are now going to take a closer look at the irregularities of the data and investigate some variations to try to explain what those data say.

- One of the factors you were asked to consider last week was Exercise 8.1.9. In order to verify this, re-run Subsection 8.1.1 with a noticeably different massed cart. Re-create the graph and use this only to note the effect of a different mass. Answer Exercise 8.1.10.
- Before drawing conclusions about the validity of the conservation of mechanical energy, consider Exercise 8.2.1.

**Exercise 8.2.1.** We need to look for the energy lost in each graph.

1. When you look at the graph from Subsection 8.1.1 for $E$, is the energy conserved or is there energy lost? If lost, calculate the energy lost or gained from the graph. (It might help to have a trendline.) If energy is lost, come up with at least two explanations for where this energy goes.

2. When you look at the graph from Subsection 8.1.2 for $E$, there are jumps in the energy. Why?
   (a) What is happening between the jumps? Does Subsection 8.1.1 help to explain these sections of the graph? Compared to the jumps, can we assume that the mechanical energy is conserved between the jumps?
   (b) What is happening at the time of those "jumps"? From the trend of the graph, calculate the amount of energy lost during each sudden change, call it the energy discrepancy, and the percent of this discrepancy relative to the total energy before the corresponding collision. Discuss where this "missing" energy goes. Is the ratio of "energy discrepancy" to total prior energy the same for each jump?
3. Comment in general, on the law of Conservation of Mechanical Energy. Can you predict any effects that might invalidate the conservation of mechanical energy? Can these effects be minimized? Is it possible to run the experiment again minimizing this effect?

- As you evaluate Subsection 8.1.2, you might be asked to re-run the experiment with a force transducer placed at the bottom of the track. (This should imply where the motion sensor will go.) Make sure that the cart will bounce from the force sensor. Make sure that the force sensor is zeroed before the start. There might be some information here based on work as a force-through-a-distance versus work as a change-in-energy.
- One explanation of a loss of energy (non-conservation) is friction. List all of the places where two pieces of material rub against each other. Since $F_f = \mu F_N$, do any of these locations have a normal force that can be varied? (Recall Exercise 8.1.9 and Exercise 8.1.10.) As an independent measure of the amount of friction, we can also consider the actual acceleration versus the expected acceleration. Exercise 8.2.2 will help you determine the expected acceleration and the variable necessary to find it. Exercise 8.2.3 will help decide on the relationship between the friction and the acceleration.

**Exercise 8.2.2.** Given an ramp inclined at some angle $\theta$, what is the component of the gravitational force aimed down the ramp? Assuming that there is no friction, what is the net force? Since $F_{\text{net}} = ma$, the acceleration should be ... ?[1] From your expression, what do you need to measure in order to find the expected value of $a$? (Recall Exercise 8.1.7.)

**Exercise 8.2.3.** If there is friction, then how do you expect the actual acceleration to compare to the expected acceleration? If there is no friction? So, how would you interpret finding an acceleration that is exactly equal to the expected value? less than the expected value? Larger than the expected value?

- A second explanation for the loss of energy is that some component is gaining rotational kinetic energy. The formula for this is $K_R = \frac{1}{2}I\omega^2$, where $I$ is the moment of inertia[2], and $\omega$ is the angular speed $\omega = v/r$. Assuming that any discrepancy that you found in the conservation of energy is due to the rotational kinetic energy of the pulley, how much energy would the pulley need to have at the end of the run (while spinning full speed)? Based on the final velocity of the cart, what is the angular speed of the pulley? Based on these numbers, $K_R$ and $\omega$, what is the moment of inertia for the pulley? Can you tell if this is a reasonable estimate?

(Revised: Oct 25, 2017)
A PDF version might be found at energy-2.pdf (166 kB)


[1] $a = g \sin\theta$.
[2] In this case, the moment of inertia is probably a little less than $\frac{1}{2}mr^2$, where $m$ is the mass of the rotating object and $r$ is the radius of the rotating object. This is not a convenient way to calculate $I$ at this time.

# Lab 9

# Peer Review

The most important part of doing science is the peer-review process. After one completes a research project, the report is submitted for publication. The publisher has a number of reviewer (usually made up of respected authors) and the submission is sent to two (sometimes three) reviewers who advise the publisher on the merits of the work. Once you make a submission, it might be two months before the reviewers finish reviewing the work. Generally the publisher will return the reviewers' comments to the author. If all reviewers agree that the paper is viable, then the publisher accepts it. If they agree not to accept a paper, then it is rejected. If the reviewers are split, then the decision is at the discretion of the publisher. In most cases, the reviewer makes suggestions for how to improve the paper or where to clarify the discussion. In some cases, the author must either significantly revise the entire project or make an argument why the reviewer is either mistaken or is merely pointing out the specific point-of-contention that the author was intending to spark in the readers. In most cases, the process of an accepted paper is

1. Author submits article.
2. Publishers submit to reviewers, who read and return comments to the publisher.
3. Publisher gives author a chance to respond; most do (!).
4. Publishers provide authors' response to reviewers, who then give final approval (or not).
5. Paper goes to Editor, who returns paper to author for grammar, spelling, and formatting corrections.
6. After the author fixes or refuses to fix the editor's "suggestions," the paper goes to publication.

This process can take anywhere from 1-2 months to a year and a half. This week, we will do Item 1. Next week, we will do Item 2.

Usually during the review process, the reviewer is not informed of the name of the submission author – to minimize influencing the reviewer. Similarly, the names of the reviewers are not revealed to the submission author. This is called "double-blind review." Some disciplines are specialized enough that all of the active researchers are familiar with each other's work. In those cases, it is possible to guess who an author is (based on the approach to the project) or to guess who the reviewer is (based on the style of comments). In principle, both sides are civil in their comments and reactions because they are members of the same community and see each other annually at the topic meetings. Researchers are competitors and collaborators who only progress by working off of each others' ideas.

## 9.1 The Assignment

In order to manage the double-blind review process, before you leave lab today you will all turn in your (personally selected) code name. Do not tell anybody what you selected and do not use a nickname that is easily recognizable by others – the point of a secret identity is to keep the secret!

In this week's lab, one lab section will do Hooke's Law and Simple Harmonic Motion and the other lab section will do The Simple Pendulum. The underlying ideas are similar to each other and will help you next week when you review an article submitted by a colleague who did the other experiment. When you submit your lab this week, you will submit your notebook and two (almost identical) copies of your report. One copy will have your name *and* your secret code name. The other copy will have *only* your secret identity.

(Revised: Sep 1, 2008)
A PDF version might be found at peer.pdf (47 kB)

# Lab 10

# Hooke's Law and Simple Harmonic Motion

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### Experimental Objectives

The stiffness of springs can be measured by stretching or by bouncing. Because we do not have an independent verification of the value of the stiffness of the spring, we will need to be clever about how to verify the relevant equations. It turns out that the measurement of stiffness through bouncing gives a value for the spring constant, whereas the measurement of stiffness by hanging involves both the spring constant and the acceleration due to gravity.

- By measuring and graphing
  - the relationship between mass and elongation when stretching a spring and
  - the relationship between mass and the period of a bouncing spring,

  we can compute the value of the acceleration due to gravity and thereby verify the relationships describing the stretch of a spring.

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## Introduction

Oscillatory motion is one of the most common types of motions and can occur in any physical system. Mechanical systems can experience a periodic motion, and then will vibrate at a natural frequency. This phenomenon is called resonance. Sound is a vibration in the air, which we hear with our ears; light is an oscillation of electric and magnetic fields, which we can see. The atoms and molecules in all objects are in a state of continual vibration, which we can detect as the temperature of the object, and the atomic vibrations of a quartz crystal can be used as a very accurate timer. The study of repetitive motion is not just an intellectual exercise, but actually enables us to model complicated systems with simple harmonic motion.

In this lab, we will consider spring as an example of oscillation. This oscillation is due to the elasticity of a spring. We will need to measure the stiffness of the spring and relate this to the rate of oscillation.

Most systems have elastic properties, such that when the system is deformed or vibrated, there is a force which tries to restore the system to its original state. If the restoring force is proportional to the displacement from its equilibrium position, then the object is said to be in simple harmonic motion (SHM). A linear restoring force can be expressed mathematically by the equation

$$\vec{F} = -k\vec{x} \quad \text{or as} \quad a = \frac{d^2x}{dt^2} = -\frac{kx}{m} \tag{10.0.1}$$

where $F$ is the **restoring force**, $x$ is the **elongation** (the displacement from the equilibrium position, which is also called the "zero position"), $k$ is a proportionality constant, and the minus sign indicates that the restoring force is always opposite the direction of the displacement. For a spring system, $k$ is called the **spring constant**, and represents the ratio of the applied force to the elongation. The spring constant is an inherent physical property of the spring itself (an elastic property). The value of $k$ gives a relative indication of the stiffness of the spring. If the spring system is in equilibrium ($\sum F_i = 0$) then the restoring force is equal to the force pulling on the spring, and this force is proportional to the extension of the spring from its equilibrium position. This relationship for elastic behavior is known as Hooke's law, after Robert Hooke (1635-1703).

We can investigate Hooke's law by hanging a mass on a spring, measuring the stretch, and plotting the mass versus the elongation. If we rewrite Equation (10.0.1) relating the mass to the elongation

$$m = \left[\frac{k}{g}\right] x \tag{10.0.2}$$

then we see an equation of the form $y = mx + \overset{0}{\cancel{b}}$, where the slope depends on both $k$ and $g$.

Simple Harmonic Motion (SHM) systems can be described by harmonic functions (cosines), where the displacement as a function of the time $x(t)$ can be written as

$$x(t) = A\cos(2\pi f t)$$

where $A$ is the amplitude of the motion, and $f$ is the frequency of the motion in units of cycles per second (sec$^{-1}$) commonly called a hertz (Hz) after Heinrich Hertz. The period ($T$, in units of seconds per cycle) equals the inverse of the frequency ($f$), $T = {}^{1}\!/_{f}$. For a mass on a spring, the period $T$ depends on the physical parameters of the system (the mass, and the spring constant), and can be given by

$$T = 2\pi\sqrt{\frac{m}{k}} \tag{10.0.3}$$

We can investigate this relationship by bouncing a mass on a spring, measuring the period, and plotting the mass versus the period. If we rewrite Equation (10.0.3) relating the mass to the period

$$m = [k]\left(\frac{T}{2\pi}\right)^2 \tag{10.0.4}$$

then we see an equation of the form $y = ax^2 + \overset{0}{\cancel{b}}x + \overset{0}{\cancel{c}}$, where the coefficient only depends on $k$. Although this is a nonlinear relationship, we can linearize the expression to find the parameter (slope with uncertainty) more easily. (See Section B.2 for more discussion.) You may also note that $T/2\pi$ is a more convenient variable than $T$ by itself because it produces a slope equal to $k$ rather than $k/(2\pi)^2$.

When you compare these relationships of the spring, you should be able to find a value for the acceleration due to gravity as a verification of these two equations.

## 10.1 Pre-Lab Considerations

- Make a sketch of your expectation for the displacement of a mass on a spring as a function of the time.
- On this graph, locate and label: the equilibrium positions ($x = 0$), and the places of maximum and minimum velocity.
- Based on the information in the introduction, make a sketch of the pull force as a function of the displacement from the equilibrium position (initial position).

## 10.2 Procedure

### 10.2.1 Hooke's law

We will first measure the elasticity of the spring, using Equation (10.0.1).

- With the available spring, attach it rigidly and hang it vertically against the Dynamics Track. Hang various masses and measure the elongation of the spring, to a maximum of 60 cm. Do not over stretch the spring. Record the bottom end of the mass hanger for the initial reference position. If a tapered spring is used, the small end should be at the top.
- Measure the elongation both when the masses are added and then when they are removed.

  *Perfectly elastic* objects (possibly your spring) will return to the exact same location when pulled with the same force whether they are being stretched out or being allowed to relax back after stretching.

  Objects that are elastic, but not perfectly elastic, will return to approximately the same location, but might retain some deformation.
- You will be graphing the relationship between the mass and the displacement, Equation (10.0.2).

### 10.2.2 Oscillating Spring

We will next consider the periodicity of an oscillating spring.

- With the same range of masses as in Subsection 10.2.1, measure the period of oscillation for each mass. You *can* but do not *have to* use the same values of mass, as long as the set of masses sampled are in the same range.
- You will be graphing the relationship between the mass and the period, Equation (10.0.4). I recommend using $T/(2\pi)$ as the variable representing the period (because it gives nice results for the graphical parameters – slope and intercept).
- ***Advice***: Keep the amplitude of vibration small, because there is a small but measurable effect with the period as a function of the amplitude.

## 10.3 Analysis

- Graph both data sets (Subsection 10.2.1 and Subsection 10.2.2) in such a way that the spring constant can be determined graphically (from a linear fit model).
  - When you graph the relationship between the mass and the displacement, recall that Equation (10.0.1) depends on two specific parameters.
  - When you graph the relationship between the mass and the period, recall that Equation (10.0.3) depends on one specific parameter.
  - With some effort, you should be able to recognize the units of the slope and intercept and find the relevant values of those parameters.
- Physically interpret the meaning and value for the slopes, and the x and y intercepts for both graphs.
- Calculate the spring constant for both data sets, using a linear regression method.
- So far in the analysis, the mass of the spring has been neglected. How would including the spring mass (or a partial %) affect the slopes or intercepts of the two graphs?

  For the period graph, one would expect to get a zero period with a zero mass. Why? What was your observation for the y-intercept? If the data was modified by adding a constant amount of mass to each mass value (say 1/3 the mass of the spring) and then re-compute the linear regression, then what happens to the slope and intercept values? And do you get a higher linear correlation coefficient?
- If you assume a value for $g$, then both graphs will give you $k$. Compare the precision for these two methods.
- If you do not assume a value for $g$, then you can use one graph to find $k$ and use this calculated value and the other graph to compute $g$. How does this value of $g$ compare to your expectations?

- Compare the elongations when the masses were added and then removed. Explain any differences. Is your spring perfectly elastic?
- Quantify the major sources of uncertainty in this experiment. Which of the experimental measurements has the largest relative uncertainty?

## 10.4 Questions

**1.** Why should the amplitude of vibration be kept as small as possible?

**2.** Is the spring totally elastic? (Does the elongation return to the same position when the masses are removed?)

**3.** Based on the data, which method do you think is more precise?

**4.** Does the force of gravity affect the value of $k$ (as derived from each method)? Why or why not?

**5.** If this experiment were conducted on the moon, would either method give a different result for the value of $k$? Explain.

(Revised: Oct 11, 2017)
A PDF version might be found at springs.pdf (122 kB)

# Lab 11

# The Simple Pendulum

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### Experimental Objectives

- Determine the relationship between the period of the pendulum and its amplitude.
- Determine the relationship between the period of the pendulum and its mass.
- Determine the relationship between the period of the pendulum and the length of the pendulum.
- Use a graphical analysis to investigate these relationships, and from the best linear graph determine an empirical equation for the period of a pendulum.
- Gravity also plays a part in this experiment, so include gravity into your empirical equation, and use unit analysis to help figure out this relationship.

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## Introduction

A simple pendulum consists of a small bob of mass ($m$) suspended by a light (assumed to be massless) string of length ($L$), and the string is firmly attached at its upper end. This pendulum is a mechanical system which we will assume exhibits simple harmonic motion. That is, the restoring force on the pendulum is proportional to the displacement from the equilibrium position.

Oscillatory motion is one of the most common types of motions and can occur in any physical system. Mechanical systems can experience a periodic motion, and then will vibrate at a natural frequency. This phenomenon is called resonance. Sound is a vibration in the air, which we hear with our ears; light is an oscillation of electric and magnetic fields, which we can see. The atoms and molecules in all objects are in a state of continual vibration, which we can detect as the temperature of the object, and the atomic vibrations of a quartz crystal can be used as a very accurate timer. The study of repetitive motion is not just an intellectual exercise, but actually enables us to model complicated systems with simple harmonic motion.

Galileo (1564-1642) investigated the natural motions of a simple pendulum. From his observations he concluded that "vibrations of very large and very small amplitude all occupy the same time." Galileo's time interval of measurement was his own pulse rate. With today's modern technology we have much more precise measuring instruments. This experiment will investigate the relationships between the physical characteristics of the pendulum and the period of the pendulum.

## 11.1 Procedure

You will have available for your use: pendulum bobs, string, timers, and a protractor. Be careful to fix the string to a point of support which will not move or vibrate as the pendulum swings. You will test each of the three relationships above (period vs amplitude, vs mass, and vs length). While measuring one relationship, you should ensure that – if they matter – then the other two variables are not varied. For example, when changing the pendulum mass do not vary the pendulum's length or its amplitude.

Some considerations while doing this lab:

- It turns out that the convenient quantity when graphing is not the period, $T$, but rather $T/(2\pi)$.
- The amplitude of oscillation is the maximum angle which the string makes from the vertical.
- In general when testing the mass or the length, it is best to keep the amplitude of oscillation small.
- When testing any of the relationships, you should measure a few widely-separated values. If these seem to vary significantly, then fill in the gaps between those measurements to make a reliable graph. See Question 11.2.3.
- If you can prove that the period is not affected by one of these variables, then you do not need to worry about keeping it constant while you measure the other variables.
- Your graphical analysis will be better if your graph is linear. Consider Question 11.2.7 for advice on making your graphs.

## 11.2 Questions

**1.** Was Galileo's statement precise?

**2.** Does this pendulum follow simple harmonic motion?

**3.** How many observations should you take in order to obtain good data?

**4.** Air resistance gradually decreases the amplitude of the pendulum. What effect does this have on the period of the pendulum?

**5.** What effect would stretching of the string have on your results?

**6.** How does gravity affect this experiment? What would happen to the results if this experiment were conducted on the moon?

**7.** If you have a parabolic graph, such as $y = ax^2$, then you might consider graphing $y$ versus $x^2$ to get a linear graph. (See also Section B.2.) What is the physical meaning of the slope and the intercept of each of your graphs?

**8.** Why is it a good idea to keep the amplitude of vibration small?

**9.** Where to and how should the pendulum length be measured?

(Revised: Oct 11, 2017)
A PDF version might be found at pendulum.pdf (67 kB)

# Lab 12

# Ballistic Pendulum

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### Experimental Objectives

- To verify that the equations of conservation of momentum and conservation of mechanical energy give the same results as the equations of two-dimensional motion by comparing the initial velocity (the "muzzle velocity") of the ballistic projectile from two different sets of experimental measurements:
  - the range and vertical height measurements of the projectile motion, and
  - through the use of the ballistic pendulum.

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## Introduction

Conservation laws play a significant part in this ballistic pendulum experiment. A ballistic pendulum is a pendulum that swings after catching a small ball that has been fired into it. The amount of swing depends on the energy and momentum transfer of the collision. During this collision between the ball and the pendulum, the momentum of the total system should be conserved from the instant just prior, to the instant just after the collision. Physicists recognize a general conservation law for momentum which applies in all interactions of two or more objects where there are no other outside forces acting on the system. For collisions on the Earth, the force of gravity is an outside force but momentum is still considered to be conserved if the time of the interaction is small.

There are three categories of collisions: **elastic collisions**, **inelastic collisions**, and **totally inelastic collisions**. The "elasticity" of the collision refers to whether or not the total kinetic energy of the objects involved is conserved (remains the same before and after). Totally inelastic collisions are those inelastic collisions in which either the objects start together and end apart (some form of spontaneous separation, like throwing) or the objects start apart and end together (some form of connecting, like catching). Since the pendulum bob catches the ball, they move off with the same velocity ($\vec{v}_{1f} = \vec{v}_{2f}$) and the kinetic energy of the system is cannot be conserved during the collision. Using the general conservation of momentum law for the collision described,

$$m_1\vec{v}_{1i} + M_2\vec{v}_{2i} = m_1\vec{v}_{1f} + M_2\vec{v}_{2f} = (m_1 + M_2)\vec{v}_f$$

an equation can be written for the initial velocity of the ball ($v_{1i}$) in terms of the velocity of the system at the instant after the collision ($v_f$) and the individual masses of the ball ($m_1$) and the pendulum ($M_2$). After the collision the pendulum and ball will swing and at the highest point in the swing they will be caught. The *KE of the system at the instant after the collision* is converted totally to the *PE of the system at the highest point in the swing*. The velocity of the system *at the instant after the collision* can therefore be determined using the law of conservation of energy. Then with these two conservation laws, the initial velocity of the ball can be determined.

The initial velocity of the ball can also be determined by firing the ball horizontally off the edge of the table and analyzing the 2-dimensional projectile motion of the ball moving under the influence of the gravitational force. This analysis involves separating the motion into its component directions, using the standard kinematic equations of motion and an appropriate set of measurements.

For these two very different techniques, calculate the same initial velocity of the projectile. An analysis and comparison of the two methods will help to illustrate the interconnections between these physics topics.

## 12.1 Pre-Lab Considerations

You can do the two parts of this lab in any order; but it will help lab go more smoothly if you answer the following questions before you come to lab.

**conservation principles** To prepare for the calculation of the muzzle velocity using conservation principles, consider the following questions:

**Exercise 12.1.1.** Draw before and after pictures for a totally inelastic collision between two masses, $m_1$ and $M_2$. Assume that $M_2$ is initially stationary, and that $m_1$ is initially moving horizontally with a muzzle velocity of $v$.

**Hint 1.** The "before" is intended to be the instant before they make contact. The "after" is intended to be the instant after they combine, but before they have had time to move off. You should bear in mind that they do not first hit, then combine, and then move off; rather the "combining" takes some small amount of time, during which the one object is slowing down and the other is speeding up until their velocities match each other.

**Hint 2.** It will be useful to bear in mind that the "final" state for the collision (conservation of momentum) is *the same* instant as the "initial" for the pendulum swing (conservation of energy).

**Exercise 12.1.2.** Draw before and after pictures for the pendulum swing of the masses, $m_1$ and $M_2$. Assume that they travel back until the speed is zero as they are just about to swing back down.

**Hint 1.** The "before" is intended to be the instant before they begin swinging after having just collided. You should bear in mind that this is a slight approximation because they do not first hit, then combine, and then move off; rather the "combining" takes some small amount of time, during which the one object is slowing down and the other is speeding up until their velocities match each other, so the swing will have already begun during the collision. However, this is a small effect. The "after" is intended to be the instant that they reach peak height and before they begin to swing back down.

**Hint 2.** You *should* consider the "final" state for the collision (conservation of momentum) to be *the same* instant as the "initial" for the pendulum swing (conservation of energy).

**Exercise 12.1.3.** For this collision, write out the conservation of momentum equation. Identify which velocity in this equation is the "muzzle velocity". Identify which velocity in this equation is related to the velocity in Exercise 4. Be prepared to solve this equation for the appropriate velocity.

**Hint.** It will be useful to bear in mind that the "final" state for the collision (conservation of momentum) is *the same* instant as the "initial" for the pendulum swing (conservation of energy).

**Exercise 12.1.4.** After the collision, the pendulum and ball will swing together. The KE of the pair at the instant after the collision will be converted to PE as it swings. Write out a conservation of energy equation for this process, in terms of the mass of the pendulum and ball, the change in height of the system and the velocity of the system at the instant after the collision. Identify which velocity in this equation is related to the velocity in Exercise 3.

**Hint 1.** It will be useful to bear in mind that the "final" state for the collision (conservation of momentum) is *the same* instant as the "initial" for the pendulum swing (conservation of energy).

**Hint 2.** You might have some trouble calling one state "initial"and another state "final". You might consider "before the collision" as "time 1", "after the collision" as "time 2", "before the swing" as "***time 2***", and "after the swing" as "time 3".

**Exercise 12.1.5.** Combine these two conservation laws (Exercise 3 and Exercise 4) to derive an expression for the initial velocity of the ball (before the collision) to the final height of the ball and pendulum system. ***List those quantities*** that you will need to measure during the lab in order to compute this number.

**ballistic motion** To prepare for the calculation of the muzzle velocity using 2-dimensional (ballistic) motion principles, consider the following questions:

**Exercise 12.1.6.** Draw a picture of the ball's path when fired horizontally off of a table. Draw the ball in its initial position (at the moment it begins its free fall) and in its final position (at the moment just before it hits the floor). Label the relevant quantities that you should measure during the lab in order to compute the muzzle velocity. (You might need to do Exercise 7 first.)

**Hint 1.** Make the ball larger than its scale size so that it size can be easily seen in your picture.

**Hint 2.** On the picture, label the height and the range of the projectile.

**Hint 3.** Think about whether the measurements should be taken from the top, bottom, or the middle of the ball. What part of the ball will hit the floor? Think about this for both the horizontal and the vertical measurements.

**Exercise 12.1.7.** For this projectile motion, use the kinematic equations of motion to derive an equation for the initial velocity of the ball in terms of the height and range measurements. ***List those quantities*** that you will need to measure during the lab in order to compute this number.

**Hint.** If the gun is not level, such as if it shoots slightly upwards, then decide if this changes your resulting equation and the quantities you would need to measure.

## 12.2 Procedure

The ballistic pendulum apparatus consists of three parts: (1) a ballistic spring-loaded gun for the firing of the projectile (used in both parts), (2) a hollow pendulum bob suspended by a light rod for catching the fired projectile, and (3) an angled platform for catching the pendulum bob at the highest position of the bob's swing. The pendulum bob can be lifted to rest in the angled platform so that it is out of the way for Subsection 2.

Notice that the initial velocity of the projectile can be changed by adjusting the spring tension. (Your instructor can explain how to do this.)

**Warning 12.2.1.** You should ***not change*** this tension for the trials that compare the projectile motion to the pendular motion because ***you want the muzzle velocity to be the same*** in those cases.

However, you might be asked to run this experiment for two different tensions. Alternatively, you might change the tension to make it easier to cock the gun. If you do loosen the spring to make it easier to cock the gun, be sure that the gun fires strongly enough to actually cause the pendulum to swing up to the available notches.

The two parts of this experiment can be done in either order. Before doing each part, be sure to clamp the apparatus to the table, using cardboard pads so that the table does not develop dents, so that the apparatus does not move due to the kick when firing the gun.

### 12.2.1 The Ballistic Pendulum

Some versions of the apparatus have a metal "lip" in the pendulum bob to hold the ball in. Other versions have a rubber O-ring. If you have the version with the lip, then when you are removing the ball from the pendulum, be sure to push up on the spring catch (the lip) in the pendulum bob so as not to damage the equipment.

- The pointy arm attached to the side of the pendulum indicates the position of the center of mass of the system.
- Do not try to take the apparatus apart, the instructor will give you the mass of the pendulum. (It might be written on a piece of tape attached to the apparatus.)
- Clamp the apparatus to the table, using cardboard pads so that the table does not develop dents, so that the apparatus does not move due to the kick when firing the gun.
- Fire the ball into the pendulum bob and mark the final notch position of the pendulum.

- Repeat the experiment with a sufficient number of trials (15) so that an average and a standard deviation of the notch positions can be obtained.
- Measure the change in height of the pendulum's pointer from its initial position to the average notch position. Calculate the uncertainty in this distance.
- Calculate (using conservation principles pre-lab work) the initial velocity (and uncertainty) of the projectile ball.

### 12.2.2 Projectile Motion

- Set-up the ballistic spring gun so that it will fire the projectile ball horizontally off the edge of the table.
  - Clamp the apparatus to the table, using cardboard pads so that the table does not develop dents, so that the apparatus does not move due to the kick when firing the gun.
  - Use a bubble level (or by seeing if the ball itself rolls) to make sure that the gun is level. (Recall Hint 1.)
  - Move the pendulum out of the way.
- Tape a piece of paper to the floor where the ball will land, then tape a sheet of carbon paper at this spot. Consider taping a second piece of paper above the carbon paper to keep the carbon paper from tearing.
- Be careful not to hit anything or anybody with the ballistic projectile! Use larger pads or boxes to protect the tables and the walls.
- Repeat the experiment for a sufficient number of trials (15-20), and calculate an average and a standard deviation of the range.
- Calculate (using ballistic motion pre-lab work) the initial velocity (and uncertainty) of the projectile after taking the appropriate measurements.

## 12.3 Analysis

Quantitatively compare the two methods by considering the following: Calculate a percent difference between the two methods. Calculate the uncertainties for the velocity in both methods (Propagation of Uncertainties) and, also, write these in a % form. Which method is more precise? Decide whether this experiment has random or systematic errors. Discuss and show your experimental evidence for this decision.

Determine if the kinetic energy is conserved during the collision. (Recall the introductory discussion.) If it is not conserved, then calculate the percentage of energy lost $\frac{(E_f - E_i)}{E_i} \times (100\%)$. Comment on the size of this number.

## 12.4 Questions

**1.** For each portion of the motion in this experiment (ballistic motion, during the collision, and after the collision during the upward swing), indicate if the momentum and/or energy should be conserved and why. Conclude whether the collision between the steel ball and the pendulum bob is elastic or inelastic.

**2.** During the collision, what percent of the kinetic energy of the ball was transferred to the combination of the pendulum and ball? If energy is lost, where does it go?

**3.** If this gun was aimed and fired vertically from the table top, would the ball hit the ceiling? Assume a vertical height of 1.5 meters. Show all of your work.

**4.** What effect does the force of gravity have on the horizontal velocity of the projectile?

**5.** Does the air resistance on the ball have a significant effect on the results of this experiment? If it does, which result would be different without air resistance? For each part of the experiment, determine if your estimate of the initial velocity is too large or too small due to the effects of air resistance.

(Revised: Nov 9, 2017)
A PDF version might be found at ballistic.pdf (138 kB)

# Lab 13

# Conditions of Equilibrium – Model of a Human Forearm

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### Experimental Objectives

- You will verify the equations for translational and rotational equilibrium by experimentally balancing the system and comparing the measurements to the results from these equations.

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## Introduction

Objects that are not accelerating are said to be in a state of **equilibrium**. If the object is moving at a constant velocity, then it is in **dynamic equilibrium**. If the object is at rest, then it is in **static equilibrium**. These principles apply to many physical examples in engineering, architecture, and biophysics. In particular, these principles allow one to be able to analyze and calculate the forces on the beams or the cables in a bridge or the forces at work in the muscles and bones in the human body.

The two conditions for equilibrium can be stated in equation form: First, if the body's center of mass is in **translational equilibrium** then it will not accelerate in any direction.

$$\sum \vec{F} = 0 \tag{13.0.1}$$

Secondly, if the body is in **rotational equilibrium** then it will not rotate about any point or axis of rotation.

$$\sum \tau = 0 \tag{13.0.2}$$

For all systems such as these, there is a special point called the **center of mass** or **center of gravity** of the system. The center of mass calculation is a weighted average of the individual masses (giving more emphasis to those positions where there is more mass). The location of this special point can be useful in determining whether the system will be in equilibrium. For an object with uniform density, such as a half-meter stick, the center of mass is at the center of the stick.

## 13.1 Prelab Work

- Define the following terms: **torque**, **lever arm**, and **center of mass**.
- State in a sentence, the first [Equation (13.0.1)] and second [Equation (13.0.2)] conditions of equilibrium. Include in your comments the fact that one of these is a vector equation.

- In Subsection 13.2.1, you will use a spring and find two locations associated with two weights. These measurements have a (familiar) linear relationship and you will be asked to find the coefficient of proportionality (a.k.a., the slope; which in this case is the spring constant). For any linear function, the slope can be determined from knowing the values of any two points, $(x_1, y_1)$ and $(x_2, y_2)$. Using these two points write out a formula for the slope of the line so that you can more easily find the spring constant in lab.

- Draw a force diagram for a horizontal forearm as outlined in Subsection 13.2.2. Label all of the appropriate forces.

## 13.2 Procedure

### 13.2.1 Determine the Spring Constant of the Bicep Muscle

Before creating the forearm model, we need an equation for the force that the bicep exerts. Since the spring models the bicep, we can use Hooke's law: $F = k(x - x_0)$, where $x_0$ is the equilibrium position. To find the force as a function of position, $F(x) = kx + b$, you will make two measurements of position and force and then determine the spring constant $k$ and the intercept $b$.

- Hang the spring from the vertical support so that it is easy to measure the position of the bottom of the spring.

- Measure the position of the bottom hook of the spring, $x_1$, with about $m_1 = 250\,\mathrm{g}$ placed on the spring. Measure the position of the bottom hook of the spring, $x_2$, with about $m_2 = 850\,\mathrm{g}$ placed on the spring.

  - You can measure from any location ***as long as you always measure from the same reference point.*** If you measure from the floor, which is more convenient, then $k$ will be negative.
  - Parallax errors can be very significant with this measurement. With the use of a mirror, this error can be reduced. Hold the mirror vertically along the track and align your line of sight until the bottom hook of the spring and its mirror image are at the same level. Then take the position readings.
  - Each person in the group should carry out these measurements without looking at the other group members' results. The average of these locations should be used.
  - You can likely improve your analysis by considering if any measurement uncertainty is random or systematic.

- With these two measurements, calculate the spring constant $k$ as the slope of the graph of $F$ versus $x$. You do not have to – and should not – actually create a graph of $F$ versus $x$ to do this, *if* you remember your prelab work for slope.

  - $b$ is the intercept. Using one pair of data and your knowledge of $k$, find $b$.
  - Determine the uncertainty in $k$ and $b$.
  - Now, with values and uncertainties for $k$ and $b$, you can use $F = kx + b$ to find the force that the spring thinks it is supporting simply by measuring the location of the bottom of the spring.

### 13.2.2 Experimental Setup for the Equilibrium Experiment

For our purposes, the forearm can be considered to jut out forwards from a vertical upper arm with the hand and the weight of the forearm itself pulling down while the bicep holds the forearm up. The elbow joint is assumed to be a nearly frictionless pivot point for the forearm, allowing the forearm to rotate about the elbow. The upper arm (the humerus) is connected to the forearm (the ulna) at the elbow, exerting a downward force on the forearm at the joint.

| Human Arm | Model | Force Location |
|---|---|---|
| Forearm (ulna and radius) | half-meter stick | center of mass |
| Hand | clamp and hanger or plastic cup | about 1 cm from the far end |
| Bicep Muscle | clamp and spring (pulling up) | 8.6% of the length of the forearm from the elbow (about 4 cm from the elbow) |
| Elbow (humerus) | clamp, hanger, weights | as near as possible to the elbow end (about 1 cm from the elbow-end) |

**Table 13.2.1:** Building the model of the human forearm

Be sure to include the weight of the clamps at each location where one is used.

- Set-up a force diagram for this model of a horizontal forearm.
- Set-up the equilibrium equations for this system.

### 13.2.3 Equilibrium Experiment

You are going to do the following procedure for two different locations of the bicep muscle: First set the bicep, as mentioned above, in the human location of about 4 cm from the elbow. Second, set the bicep at about 10 cm from the elbow. You should consider which location gives more leverage to lift the hand.

- With your model completely set up, place a 50 g mass in the "hand." *Experimentally* determine the force at the elbow necessary to balance the system and make it horizontal. You may also change the position of the hand by small amounts if you find it easier to balance the system. However, be sure to record the correct distances.
- To check that the system is level, use another meter stick to measure the height from the floor *at each end* of the forearm half-meter stick.
- Determine the force in the bicep muscle using the formula derived in the previous section as appropriate for the new position of the bottom of the hook of the spring (again using the mirror for accuracy). Be sure to include the uncertainties!
- Record all force locations as read directly from the "forearm" meter stick. *Do not subtract in your head* so that we can reproduce the locations later, if need be.
- *Experimentally* determine the sensitivity of your values for force at the elbow by checking how many grams can be added or removed at the elbow while maintaining the horizontal equilibrium.
- Since the hand is so sensitive, you should estimate roughly the sensitivity of the value of force at the hand.

With this data, you will verify the equations of equilibrium.

**Activity 13.2.1** (Comparative Anatomy)**.** Create a table that allows you to compare the values of force for the elbow, bicep, forearm, and hand for three different attachment-locations.

- Enter the data from Subsection 13.2.3. Record each position and each force.
- Predict how the forces will change if the hand is moved a few millimeters closer to the bicep. (Hint 1 might help.)
- Move the hand in and measure the forces that make it balance. Compare to your predictions.
- Predict how the forces will change if the bicep is moved a few millimeters further from the elbow. (Hint 2 might help.)
- Move the bicep away from the elbow and measure the forces that make it balance. Compare to your predictions.

**Hint 1.** Imagine carrying grocery bags at different locations on your lower arm while holding your arm in an L-shape.

**Hint 2.** Imagine sitting at different locations on a teeter-totter.

## 13.3 Analysis

- For the forces measured or calculated in Subsection 13.2.3, test each of the conditions of equilibrium:
  - Translational Equilibrium: Show that the up forces equal the down forces to within your uncertainty.
  - Rotational Equilibrium: Show that the clockwise torques equal the counterclockwise torques to within your uncertainty.
  - To show that it does not matter which point you choose to calculate the summation of the torques about, choose the far end of the meter stick (near the hand) as the zero or the rotation point.
  - If your conditions of equilibrium are not consistent, then calculate the necessary mass that should be in the hand and test this value experimentally. Discuss how well this value works. (If you cannot get this to balance, you might check to be sure you included the weight of the forearm itself. . . )
- Consider the forces in Activity 13.2.1.
  - If you have three shopping bags of food at the grocery store that you want to hang from your arm as you walk to your car, should you hang the heaviest bag closest to your elbow or closest to your hand? Explain.
  - The location of the bicep relative to the elbow determines the leverage in pulling your hand to your chest (or for a quadruped move their feet forward for the next step). Do you expect fast animals to have their leg muscles attached close to the joint or far from the joint? Explain.

## 13.4 Questions

**1.** Is it necessary to have the meter stick horizontal for the system to be in equilibrium? Why did we want to keep the arm horizontal?

**Hint.** $\tau = r\,F\,\sin\theta$

**2.** How was the tension in the muscle affected as the position of attached muscle was moved further from the joint? Keep the mass in the hand constant.

(Revised Nov 15, 2017)
A PDF version might be found at equilibrium.pdf (130 kB)

# Lab 14

# Scaling Laws Among Biological Specimens

**Procedure** You will be given many animal skulls. Measure some dimension that can be measured on all skulls. Verify graphically (make the plot of mass versus your measured dimension) if this measurement can be reliably correlated to the mass of the skull. We will assume that the mass of the skull is directly proportional to the overall size of the animal. This technique allows mammalogists to gauge the size of animals when they can only find partial skeletons.

**Analysis** When you fit this data with a trendline, the relationship might be the familiar linear relationship, or it might instead be something else. Some possibilities are:

- Linear $y = mx + b$. In this case, you should plot the skull mass versus your measurement, determine the slope and intercept and the relevant regression data. Discuss why the intercept might have the value that it does.
- Geometric $y = ax^b$, where the coefficient $a$ and the exponent $b$ are to be determined. In general, it is useful to fit your data this way first. If you find $b$ is close to an integer, then try a second fit with the appropriate polynomial. For example, if $b \approx 1$ then use a linear fit.
- Exponential $y = ae^{bx}$, where the coefficient $a$ and the base $b$ are to be determined.

Excel can fit to any of these and provide you with the equation for the trendline. However, Excel cannot provide the uncertainties unless the graph is linear. If one uses logarithms, then we can make a power-fit trendline or an exponential-fit trendline plot as a line so that Excel can provide the slope and intercepts with uncertainties. This will allow us to create a reliable mathematical relationship between the variables. In general, it is true that linear regression is significantly easier than a general functional regression.

If your power fit is better then we can apply the logarithm to the geometric relation:

$$
\begin{aligned}
\ln y &= \ln\left(ax^b\right) \\
\ln y &= \ln a + \ln x^b \\
\ln y &= \ln a + b \ln x \\
\ln y &= [b] \ln x + [\ln a]
\end{aligned}
$$

In this case, it is useful to make a "log-log plot" of $\ln y$ versus $\ln x$, which should be a straight line. The slope of the log-log relation is $b$, the exponent, and the intercept of the log-log relation is $\ln a$, so that $e^{(\ln a)} = a$ the coefficient.

If your exponential fit is better then we can apply the logarithm to the exponential relation:

$$
\begin{aligned}
\ln y &= \ln\left(ae^{bx}\right) \\
\ln y &= \ln a + \ln\left(e^{bx}\right) \\
\ln y &= \ln a + bx \ln(e) \\
\ln y &= [b \ln e]\, x + [\ln a]
\end{aligned}
$$

In this case, it is useful to make a "semi-log plot" of $\ln y$ versus $x$, which should be a straight line. The slope of the semi-log relation is $b$ (because $\ln e = 1$) and the intercept of the semi-log relation is $\ln a$, so that $e^{(\ln a)} = a$ the coefficient.

In biological relationships, it is common to find an exponential relationship with a base of $e = 2.717182\cdots$;

this is why we find it useful to use the "natural logarithm."

Once you determine if you have an exponential or a geometric relationship, decide if there are any "outliers," which animals those are, and justify why those data points might reasonably be excluded. Re-fit the data after excluding the outliers.

A PDF version might be found at scaling.pdf (59 kB)

# Lab 15

# Archimedes' Principle

### Experimental Objectives

- After verifying some properties of the buoyant force using Archimedes' principle, each group will predict the maximum amount of cargo (the number of pennies) that a ship (glass beaker) will hold without sinking.

Archimedes was a Greek scientist who lived from 287-212 BCE. As the story goes, the king thought that his crown was not pure gold and asked Archimedes to determine if this was true. Archimedes had previously observed that a totally submerged object displaces a volume of fluid which is equal to the volume of the object, and deduced what is now called Archimedes' principle: that the buoyant force on the submerged object is equal to the weight of the displaced fluid. Knowing the weight and density of gold, Archimedes measured the weight of the crown and the weight and volume of the water displaced by the crown, enabling him to determine that the crown was not pure gold.

As any swimmer who has had their head more than about 8 feet deep can tell you, the pressure exerted on the diver by the water increases as the diver swims deeper into the water. We can express this as: the pressure within a fluid is dependent on the depth at which the object sits ($h$), the gravitational acceleration of the earth ($g$), and the density of the fluid ($\rho_f$): $P = \rho_f gh$. Because of this, the bottom surface of any object in a fluid will have more pressure on it than the top surface. This $\Delta P$ gives rise to a net upward force acting on the submerged object: the buoyant force ($F_B$). The magnitude of the upward force depends on the density of the fluid and the size of the object: $F_B = \rho_f gV_o$, where $V_o$ is ***the submerged portion*** of the volume of the object. A partially submerged object has a smaller buoyant force than a completely submerged object. Interestingly, unless the fluid density depends on the depth of the fluid, the $F_B$ is independent of the depth of the object.

Knowing the buoyant force will help you determine if an object will float or sink. To determine whether an object will sink or float one should use Newton's second law to determine the direction of the net force. Assuming the simplest case of a submerged object feeling only its weight ($F_g = m_o g = \rho_o gV_o$) and the buoyant force ($F_B = \rho_f gV_o$), if the weight of the object is greater than $F_B$ then the net force on the object is down and it will sink. If $F_B$ is larger than the weight then the net force on the object is up and then it will accelerate upwards. However, when an object is floating, it is in equilibrium and the object's weight is equal to $F_B$, which depends on the submerged volume. While floating, not all of the object is submerged.

## 15.1 Pre-Lab Work

- Using the definitions of force as pressure times area, $F = PA$, and pressure, $P = \rho_f gh$, derive the equations for the buoyant force $F_B = \rho_f gV_o$ (if totally submerged) and for the object's weight $F_g = \rho_o gV_o$, where $\rho_o$ is the object's density.

- Show that although the pressure on an object does depend on depth within the fluid, the $\Delta P = P_{\text{bottom}} - P_{\text{top}}$ (and therefore $F_B$) is independent of the depth within the fluid.
- Draw a force diagram for a completely submerged object:

  **case 1** object's density is greater than that of the fluid's,

  **case 2** object's density is less than that of the fluid's.
- Using Newton's Second Law show that the object in case 1 will sink, and that the object in case 2 will accelerate up.
- Draw a force diagram for a partially submerged object which is floating.
- Using Newton's Second Law show that if an object has a density $\rho_o = .5\rho_f$ that only 0.5 of the object's volume is submerged.
- Look up the density of water.

## 15.2 Procedure

### 15.2.1 Develop Your Understanding

- Select an object for which you can easily measure the volume using a caliper.
- Measure the dimensions and calculate the volume.
- Measure the mass using the triple beam balance at your table.
- Calculate the density with uncertainty.
- Predict (with uncertainty) what buoyant force this object would have if it were completely submerged.
- Predict (with uncertainty) what a scale would read if this object were completely submerged.

### 15.2.2 Verify Your Understanding

1. Place a balance on top of a support rod so that the side with the string dangling can drop into a container of water.
2. Find the mass of your catch beaker – this should be a graduated cylinder because you will be using this beaker to measure both the volume and the mass of the fluid that overflows from the overflow container.
3. Place an overflow container under the scale so that the string can dip into the container if the lab jack is raised or lowered.
4. Place a catch beaker next to the overflow container and slowly fill the overflow container until it just starts dripping. Gently tap the overflow container once. Be very careful not to bump your table after this.
5. As thoroughly as possible, dry off your catch beaker and put it back in place. If you cannot dry it off completely, you will not be able to accurately complete all of the necessary steps.
6. Attach your object to the string, measure the mass of the object, $m_o$, in air while the object dangles.
7. Raise the lab jack until the object is completely submerged. *Be sure to catch all of the water that overflows from the container!*
8. The scale should now read a reduced mass, $m_s$, for the submerged object. Record this value.
9. You now have three ways of calculating the buoyant force: (Pay attention to which is the most precise and why.)

(a) The water (via the buoyant force) supports the difference in weights between not submerged and totally submerged. Calculate the buoyant force, with uncertainty, as the difference in these *weights*. Does it agree with your prediction?

(b) Measure the volume of the overflow water. This is equal to the volume of the object submerged, $V_f = V_o$. Calculate (with uncertainty) the buoyant force from this volume, $F_B = \rho_f g V_o$. Does this agree with your prediction?

(c) Note that $\rho_f V_f = m_f$, so instead of measuring $V_f$, we can measure the mass of the overflow water. The buoyant force can also be found from: $F_B = m_f g$, the weight of the fluid. Does this agree with your prediction?

You should ensure that you understand these results *with uncertainty* before continuing. If this doesn't make sense, then check your numbers, check your units, remeasure quantities that you thought you were sure of, and ask your instructor.

### 15.2.3 Consider How the Buoyant Force Changes as an Object is Submerged

For this part, you are going to repeat the previous procedure, with three modifications. First, you won't be able to measure the volume with a caliper. Second, based on the previous procedure, we now know which measurement provides the best estimate (smallest uncertainty) for the buoyant force, so you should only need to do one of Step 9.a, Step 9.b, or Step 9.c from the previous section. Finally, we will consider an object in air, partially submerged, and fully submerged.

- Select a fairly large object for which you cannot easily measure the volume using a caliper. This must fit within the overflow container without touching the sides.
- Refill the overflow container, as before, until is just starts to overflow.
- As thoroughly as possible, dry off your catch beaker. If you cannot dry it off, you will have to measure its mass with whatever water is in it and you will not be able to use the previous section technique Step 9.b because you cannot accurately measure the volume of new water.
- Attach it to the string hanging from the balance and find the mass in air, $m_o$.
- *You will need to do the rest of this* very *carefully. Go slowly and do not bump the table.*
- Raise the lab jack until the object is about half submerged. Catch the overflow water.
- Measure the apparent mass of the object and the mass or volume of the overflow water, as appropriate
    - Calculate the buoyant force with uncertainty.
    - Calculate the volume (with uncertainty) of the object that is submerged.
- Without removing any of the water from the catch beaker, continue to submerge the object and catch the overflow water until the object is completely submerged.
- Measure the new apparent mass of the object and the mass or volume of the overflow water, as appropriate.
    - Calculate the buoyant force with uncertainty.
    - Calculate the volume (with uncertainty) of the object that is submerged.
    - Calculate the density (with uncertainty) of the object.

### 15.2.4 Fill the Cargo-Hold

You have been provided with a beaker and some pennies. Use the information in the previous sections to predict the maximum number of pennies that can be gently placed into the beaker without allowing it to sink. You should measure the mass of the beaker and the dimensions of the beaker. The volume of a cylinder is $V = \pi r^2 h$. You should also measure about 30 pennies (all at once) to find an average mass for the pennies. After you have calculated your prediction with uncertainty, prepare the overflow canister with water and dry the catch beaker. Invite the instructor to your table and allow the instructor to test your prediction.

## 15.3 Questions

**1.** Why does an object weigh a different amount when in air and when submerged in water?

**2.** If the string was cut on your first object (while submerged), what would be the acceleration of the object?

**3.** Why is it important to make certain that no air bubbles adhere to the object during the submerged weighing procedures?

**4.** What would the buoyant force be for an object that was immersed in a fluid with the same density as the object?

**5.** A floating barge filled with coal is in a lock along the Ohio River. If the barge accidentally dumps its load into the water, will the water level in the lock rise or fall?

**6.** Does the mass of displaced water depend on the mass of the totally submerged object or on the volume of the submerged object?

**7.** There are two identical cargo ships. One has a cargo of 5 tons of steel and the other has a cargo of 5 tons of styrofoam. Which ship floats lower in the water?

Last revised: Nov. 23, 2008
A PDF version might be found at Archimedes.pdf (116 kB)

# Lab 16

# Standing Waves on a String

### Experimental Objectives

In this experiment, the conditions required for the production of standing waves in a string will be investigated in order to

1. empirically verify the relationship between the frequency ($f$), wavelength ($\lambda$), and speed of a standing wave ($v$), for a series of normal modes of oscillation, and
2. use the two relationships of the speed of the wave

$$v = \sqrt{\frac{F_T}{m/L}} \tag{16.0.1}$$

$$v = \lambda f \tag{16.0.2}$$

to empirically relate the string tension ($F_T$) and the wave velocity ($v$), for one mode of oscillation, specifically the fundamental mode.

A wave is the propagation of a disturbance in a medium. In a transverse wave, the disturbance of the medium is perpendicular to the direction of the propagation of the wave. In this experiment, transverse waves will be propagated along a taut flexible string. The frequency of the waves is determined by the source, the wave generator, which is connected to one end of the string. The speed of the waves, $v$, is determined by the medium itself; namely the linear mass density of the string, $(m/L)$, and the tension in the string which is determined by the static force of a mass suspended from the other end of the string, $F_T$. This is expressed by Equation (16.0.1) above.

Upon hitting at the fixed end of the string, the transverse waves will reflect back along the string. If the end of the string is rigidly held then the reflected waves will be inverted 180°. The initial propagated waves and the reflected waves will interfere constructively and destructively. At particular string tensions and/or wave frequencies, this wave superposition will give rise to waves called standing waves or stationary waves. There are many different standing waves patterns (normal modes) possible which have different wavelengths. The different modes are characterized by the number of nodes or antinodes in the wave. At one of the normal modes of oscillation, the amplitude of oscillation can become rather large, much larger than the amplitude of the original propagated wave. This phenomenon is called resonance. In this experiment, each end of the string will be close to a node position. The fundamental frequency (mode n=1) exhibits a wave pattern with one antinode. The second harmonic (mode n=2) exhibits a wave pattern with two antinodes, and so forth.

## 16.1 Pre-Lab Work

- Define the following terms: standing wave, node, and antinode.

- Draw a set of pictures of a standing wave on a string with two fixed ends showing the fundamental frequency (n=1), and the harmonics n=2, 3, 4, and 5. Label a wavelength, the nodes, and the antinodes on each picture.

- Given Equation (16.0.1), predict what will happen to the velocity of the wave as the static hanging mass (providing the tension) increases, decreases, or stays the same.

- Given Equation (16.0.2), how can measurements of $\lambda$ and $f$ be plotted to determine the wave velocity from the graph? (There is more than one way to do this; one way in particular produces a line.) How should the axes of the graph be labeled? Explain your reasoning.

## 16.2 Fixed Tension

In order to verify Equation (16.0.2), you can fix $F_T$ and compare multiple $f$ to the corresponding $\lambda$.

### 16.2.1 Procedure

- If you have not already done so, measure the mass and length of the string in order to calculate the linear mass density ($m/L$). You might note Question 16.4.3.

- A mechanical vibrator, driven with a (variable frequency) function generator, will be the wave source. Use a string slightly greater than the length of the table. Connect one end of the string to the vibrator and hang the other end over a pulley, keeping the string horizontal.

- Set up the system with some particular string tension (like 450 grams). This tension will be kept constant for this entire part.

- Produce 5 different normal modes by changing the frequency of the vibrator. Adjust the standing wave amplitude until it is at its maximum. Record the corresponding frequency for each normal mode.

- Measure the corresponding wavelengths for each normal mode, directly from the string with a meter stick. The vibrator end of the string is not a true node because the string is vibrating a small amount. Take this into account when measuring the wavelengths.

- Try touching the string at a node and at an antinode. What happens to the wave in each case?

- Optional: Try measuring the frequency of the vibrator with a strobe lamp.

### 16.2.2 Analysis

- Determine the mathematical relation between the wavelength of the standing wave and the frequency of the vibrator that will produce a straight line ($y = mx + b$) when graphed.

- Comment on the significance (and units) of the slope and intercept of this graph. Determine both the precision and the accuracy of the slope and of the intercept; are they consistent with what you expect? (Your expectation should be based on the "other" equation which we are not testing here.)

- Use one or both of these (slope and intercept) to determine the velocity (with uncertainty) of the standing wave. Relate this (%-error) to the expected value based on the tension you chose for your specific string.

## 16.3 Fixed Wavelength

In order to verify Equation (16.0.1), you can fix $\lambda$ and compare multiple $F_T$ to the corresponding $v$.

### 16.3.1 Procedure

- If you have not already done so, measure the mass and length of the string in order to calculate the linear mass density ($m/L$). You might note Question 16.4.3.
- A mechanical vibrator, driven with a (variable frequency) function generator, will be the wave source. Use a string slightly greater than the length of the table. Connect one end of the string to the vibrator and hang the other end over a pulley, keeping the string horizontal.
- Place 150 grams (including the hanger) on the end of the string. (If you start with too little mass, the string stretches different amounts – changing the string's effective density – during the experiment, causing unaccounted for curving in the graph.)
- Adjust the frequency of the generator until the n=2 mode is observed and is at its maximum amplitude. Record the frequency and corresponding wave velocity.
- Repeat the previous step for 6 hanging masses up to about 1200 grams.

### 16.3.2 Analysis

- Determine the mathematical relation between the hanging mass ($F_T$) and the wave velocity ($v$) that will produce a straight line ($y = mx + b$) when graphed.
- Determine which variables the slope and intercept of this graph should be related to. Determine both the precision and the accuracy of the slope and of the intercept; are they consistent with what you expect? (The equation we are testing should imply what to expect.)

## 16.4 Questions

**1.** What are the necessary and sufficient conditions for the production of standing waves?

**2.** Is it valid to consider the tightening of the string to be the same as a change of medium? (Does tightening the string change the way waves move or the way standing waves are produced on the string?) Why or why not?

**3.** When you measure the length of string, remember that the string will be under tension (pulled on) and may have some "give" to it. It may be possible for it to stretch by nearly 10%. If you don't account for this (if you measure the un-stretched length) will your value of density be incorrect too large or incorrect too small? Can you account for this either in the value of the length or in the uncertainty of the length. Will this affect comparison to either of your graphs?

**4.** Consider the Fixed Tension graph. We noted that the vibrator end of the string should not be considered to be a node. This means that the node might be a little in front of the post or a little behind the post and therefore your wavelength values might be a little wrong, but your error bars should already include the amount the values could be off. Based on the error bars on your Fixed Tension graph, would shifting the wavelength data a little larger or a little smaller have made a difference to the values of slope or intercept that you calculated? If so, would it have increased the value or decreased the value? If not, why not?

**5.** If the tension and the linear density of the string remain constant, but the pulley is shifted further from the post (so that the region which is vibrating lengthens), how is the resonant wavelength affected? How is the velocity of the wave affected?

**6.** What was the experimental velocity (and uncertainty) of the waves in part I of the experiment?

**7.** What will happen to the velocity of the wave if the tension is fixed but the frequency is changed.

Last revised: (Jan, 2012)
A PDF version might be found at standingwaves.pdf (98 kB)

# Appendix A

# Managing Uncertainties

One of the fundamental aspects of science is knowing the reliability of results. The mechanism for gaining this knowledge is first to gauge how well one knows any given measurement and then to propagate this to an indication of the reliability of the results that depend on those measurements. The primary goal in attending to the propagation of the uncertainty is that it allows scientists to determine which measurement is causing the most uncertainty in the result so that future experimenters know which measurement to improve to get an improved result.

In this section we will learn the terminology, determine how to gauge the measurement uncertainty, learn how to propogate this information through a calculation, and learn how to discuss this analysis in your lab reports.

## A.1 Experimental Uncertainties, Defining "Error"

Measurements are never exact. For example, if one apple is divided among three people, your calculator will tell you that each person has 0.3333333333 of an apple. A measurement of each slice will tell you two pieces of information: (1) how many 3s to keep and (2) how well you know the final 3. In this example, both $0.33 \pm 0.01$ and $0.33 \pm 0.04$ imply that the measurement is accurate to two decimal places, but the first implies that you trust the second 3 more than if you report it as the second number.

> **CAUTION:** Because physicists "know what we mean", they are often sloppy with their language and use the words "error" and "uncertainty" interchangeably.

Some technical terms and their use in physics (which may differ from common use):

**accuracy** How close a number is to the true (but usually unknowable) result. This is usually expressed by the (absolute or relative) error.

**precision** How well you trust the measurement. This is vaguely expressed by the number of decimals, or clearly expressed by the size of the (absolute or relative) uncertainty.

**uncertainty** The **uncertainty in a number** expresses the precision of a measurement or of a computed result. This can be expressed as the **absolute uncertainty** (explained in Finding the Precision of a Measurement), the **relative uncertainty**, or the **percent uncertainty**.

$$\text{relative uncertainty} = \left|\frac{\text{(absolute uncertainty)}}{\text{measured value}}\right|$$

$$\text{\%-uncertainty} = 100\% * \left|\frac{\text{(absolute uncetainty)}}{\text{measured value}}\right|$$

**error** The **error** is a number that expresses the accuracy by comparing the measurement to an accepted ("true") value. This can be expressed as the **absolute error**, the **relative error**, or the **percent error**.

$$\text{absolute error} = |\text{true value} - \text{measured value}|$$

$$\text{relative error} = \left|\frac{(\text{true value} - \text{measured value})}{\text{true value}}\right|$$
$$\text{\%-error} = 100\% * \left|\frac{(\text{true value} - \text{measured value})}{\text{true value}}\right|$$

**difference** The **difference** is a number that expresses the consistency of a multiple measurements by comparing one measurement to another. This can be expressed as the **absolute difference**, the **relative difference**, or the **percent difference**. You should notice that since we don't know *which* measurement to trust, we take the absolute difference relative to the *average* of the measurements (rather than choosing one measurement as "true").

$$\text{absolute difference} = (\text{measurement}_1 - \text{measurement}_2)$$
$$\text{relative difference} = \frac{(\text{measurement}_1 - \text{measurement}_2)}{\left[\frac{(\text{measurement}_1)+(\text{measurement}_2)}{2}\right]}$$
$$\text{\%-difference} = 100\% * \frac{(\text{measurement}_1 - \text{measurement}_2)}{\left[\frac{(\text{measurement}_1)+(\text{measurement}_2)}{2}\right]}$$

**Note A.1.1** (compare)**.** Whenever you are asked to "compare" values, it is expected that you will not only compute a %-error or %-difference (as appropriate, according to the above considerations); but will also comment on if the uncertainty of the values overlap. Recall that the uncertainty means that your measurement does not distinguish between values within that range, so if the uncertainties overlap, then the values are "the same to within your ability to measure them."

## A.2 Writing an Analysis of Error

The conclusion of your lab report should be based on an analysis of the error in the experiment. The analysis of error is one of the most certain gauges available to the instructor by which the student's scientific insight can be evaluated. To be done well, this analysis calls for comments about the factors that impacted the extent to which the experimental results agree with the theoretical value (what factors impact the percent error), the limitations and restrictions of the instruments used (what factors impact the uncertainty), and the legitimacy of the assumptions.

Physicists usually use the phrase "sources of error" (or "sources of uncertainty") to describe how the limits of measurement propagate through a calculation (see Propagation of Uncertainties) to impact the **uncertainty** in the final result. This type of "error analysis" gives insight into the **accuracy** of the result. Considerations for the Error Analysis provides questions that can help you describe which of several measurements can most effectively improve the **precision** of the result so that you can gain insight into the **accuracy** of the result. The accuracy allows one to gauge the veracity (truth) of an underlying relationship, but precision allows you to gauge accuracy. Said another way, a small **percent difference** usually is used to imply a small **percent error**. Said another way, imprecise measurements always *seem* accurate.

### A.2.1 Technically, Errors are not Mistakes

Your report should not list "human error" because most students misunderstand this term to mean "places I might have made a mistake" rather than "the limiting factor when using the equipment correctly." Finding the Precision of a Measurement discusses measurement uncertainties as defined above.

In the example of the apple above, the fact that one person has $0.33 \pm 0.04$ of an apple does *not* reflect a "mistake" in the cutting, but rather reflects that the cutter is limited in their precision. What is important is to use the uncertainty to express how well one can repeated cut the apple into thirds. The absolute uncertainty of 0.04 is generally interpreted to say that most instances (roughly 68%, as explained in Uncertainty of multiple, repeated measurements) of the cutting of an apple in this way will result in having between 0.29 to 0.37 of an apple for any given slice.

When describing the cause of an error (difference from the theoretical value) or of an uncertainty (the extent you trust a number), you can usually categorize this source of error as a random error (a cause that could skew the result too large *or* too small) or as a systematic error (a cause that tends to skew the result in one particular direction).

**Random Error** An environmental circumstances, generally uncontrollable, that sometimes makes the measured result too high and sometimes make it too low in an unpredictable fashion. Random errors may have a statistical origin – that is, they are due to chance. For example, if one hundred pennies are dumped on a table, on average we expect that fifty would land heads up. But we should not be surprised if fifty-three or forty-seven actually landed heads-up. This deviation is statistical in nature because the way in which a penny lands is due to chance. Random errors can sometimes be reduced by either collecting more data and averaging the readings, or by using instruments with greater precision.

**Systematic Error** A systematic error can be ascribed to a factor which would tend to push the result in a certain direction away from the theory value. The error would make all of the results either systematically too high or systematically to low. One key idea here is that systematic errors can be eliminated or reduced if the factor causing the error can be eliminated or controlled. This is sometimes a big "if", because not all factors can be controlled. Systematic errors can be caused by instruments which are not calibrated correctly, maybe a **zero-point error** (an error with the zero reading of the instrument). This type of error can usually be found and corrected. Systematic errors also often arise because the experimental setup is somehow different from that assumed in the theory. If the acceleration due to gravity was measured to be 9.52 $\mathrm{m/s^2}$ with an experimental uncertainty (precision) of 0.05 $\mathrm{m/s^2}$, rather than the textbook value of 9.81 $\mathrm{m/s^2}$, then we should be concerned with why the accuracy is not as good as the precision. This is most likely to mean that there is a significant systematic error in the experiment, where one of the initial assumptions may not be valid. The textbook value does not consider the effects of the air. The effects of the air may or may not be controllable, and the difference between the theory and the data may be (within appropriate limits or tests) considered a correction factor for the systematic error.

### A.2.2 Considerations for the Error Analysis

In order to help you get started on your discussion of error, the following list of questions is provided. It is not an exhaustive list. You need not answer all of these questions in a single report.

1. Is the error large or small? Is it random or systematic? ... statistical? ... cumulative?
   (a) What accuracy (precision) was expected? Why? What accuracy (precision) was attained? If different, why?
   (b) Was the experimental technique sensitive enough? Was the effect masked by noise?
2. Is it possible to determine which measurements are responsible for greater percent error by checking items measured and reasoning from the physical principles, the nature of the measuring instrument, and using the rules for propagation of error?
   (a) Is the error partly attributable to the fact that the experimental set-up did not approximate the ideal that was required by the physical theory closely enough? How did it fail?
   (b) If a systematic error skews high (low), then is your result too high (low)? Is this a reasonable explanation? Is the size of the skew enough to explain the result?
   (c) What can be done to improve the equipment and eliminate error? How can the influence of environmental factors be diminished? Why is this so?
3. Is the error (deviation) in the experiment reasonable?

**Note A.2.1** (compare)**.** Whenever you are asked to "compare" values, it is expected that you will not only compute a %-error or %-difference (as appropriate); but will also comment on if the uncertainty of the values overlap. Recall that the uncertainty means that your measurement does not distinguish between values within that range, so if the uncertainties overlap, then the values are "the same to within your ability to measure them."

## A.3 Finding the Precision of a Measurement

### A.3.1 Uncertainty of a single measurement

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### A.3.2 Uncertainty of multiple, repeated measurements

Calculate or estimate the precision of a measurement by one or more of the following methods:

1. by the precision of the measuring instrument, and take into account any uncertainties that are intrinsic to the object itself;

2. by the range of values obtained, the minimum and/or maximum deviation ($d$);

$$d_i = |X_i - X_{\text{ave}}|$$

3. by the standard deviation, which is the square root of the sum of the squares of the individual deviations ($d$) divided by the number of readings ($N$) minus one;

$$\sigma = \sqrt{\frac{1}{(N-1)} \sum d^2} = \sqrt{\frac{1}{(N-1)} \sum_i |X_i - X_{\text{ave}}|^2}$$

4. by the standard deviation of the mean, which is the standard deviation divided by the square root of the number of readings;

5. by the square root of the number of readings ($\sqrt{N}$), if $N$ is considered large;

If many data points were taken and plotted on a histogram, it would smooth out and approach the symmetrical graph typical of the binomial distribution (see the Figure 2.0.1). This distribution and many others in statistics may be approximated by the gaussian distribution.

The standard deviation, $\sigma$, can be estimated from the above graph. It is a measure of the "width" of the distribution. For the case shown, the standard deviation has the value of five. The greater the standard deviation, the wider the distribution and the less likely that an individual reading will be close to the average value. About 68% of the individual readings fall within one standard deviation (between 45 and 55 in this case). About 96% of the readings fall within two standard deviations (between 40 and 60 in this case).

As more and more readings are taken, the effect of the random error is gradually eliminated. In the absence of systematic error, the average value of the readings should gradually approach the true value. The smooth curve above was drawn assuming that there was no systematic error. If there were, the graph would merely be displaced sideways. The average value for the number would then be say 55.

The distribution of many average (mean) readings is also gaussian in shape. Comparing this to the distribution for individual readings, it is much narrower. We would expect this, since each reading on this graph is an average of individual readings and has much less random error. By taking an average of readings, a considerable portion of the random error has been canceled. The standard deviation for this distribution is called the standard deviation of the mean ($\sigma_m$). For this distribution, 68% of the averages of the readings are within one standard deviation of the mean, and 98% of the average readings fall within two standard deviations of the mean.

The standard deviation of the mean tells how close a particular *average* of several readings is likely to be to an overall average when many readings are taken. The standard deviation tells how close an *individual* reading is likely to be to the average.

There is one case for which the standard deviation can be estimated from one reading. In counting experiments (radioactivity, for example), the distribution is a Poisson distribution. For this distribution, the standard deviation is just the square root of the average reading. One reading can give an estimate of the average, and therefore, give an estimate of-the standard deviation.

## A.4 Propagation of Uncertainties

The previous sections discussed the uncertainties of directly measured quantities. Now we need to consider how these uncertainties affect the rest of the analysis. In most experiments, the analysis or final results are obtained by adding, subtracting, multiplying, or dividing the primary data. The uncertainty in the final result is therefore a combination of the errors in the primary data. The way in which the error propagates from the primary data through the calculations to the final result may be summarized as follows:

1. The error to be assigned to the sum or difference of two quantities is equal to the sum of their absolute errors.
2. Relative error is the ratio of the absolute error to the quantity itself. The relative error to be assigned to the product or quotient of two quantities is the sum of their relative errors.
3. The relative error to be assigned to the power of a quantity is the power times the relative error of the quantity itself.

These rules are not arbitrary, but rather they follow directly from the nature of the mathematical operations. These rules may be derived using calculus.

**Exercise A.4.1** (Try Propagating the Uncertainty When Adding Numbers)**.** Compute the perimeter of a table that is measured to be $176.7\,\mathrm{cm} \pm 0.2\,\mathrm{cm}$ along one side and $148.3\,\mathrm{cm} \pm 0.3\,\mathrm{cm}$ along the other side.

**Hint 1.** To find the perimeter, add the four sides of the rectangle. Use the values, but not the uncertainty.

**Hint 2.** To find the uncertainty, use Rule 1.

**Answer.** The perimeter is $P = 650\,\mathrm{cm} \pm 1\,\mathrm{cm}$.

**Solution.** The perimeter can be found as:

$$P = (176.7\,\mathrm{cm}) + (148.3\,\mathrm{cm}) + (176.7\,\mathrm{cm}) + (148.3\,\mathrm{cm})$$
$$P = 650.0\,\mathrm{cm}$$

but we do not know the precision (appropriate number of decimals) until we compute the uncertainty, which is

$$\Delta P = (0.2\,\mathrm{cm}) + (0.3\,\mathrm{cm}) + (0.2\,\mathrm{cm}) + (0.3\,\mathrm{cm})$$
$$\Delta P = 1.0\,\mathrm{cm}$$

The value of the uncertainty determines where you round the result. Because the first digit of the uncertainty is in the "one's place", we round *both* the value and the uncertainty to that place.

The perimeter is $P = 650\,\mathrm{cm} \pm 1\,\mathrm{cm}$.

**Exercise A.4.2** (Try Propagating the Uncertainty When Multiplying Numbers)**.** Compute the area of a table that is measured to be $176.7\,\mathrm{cm} \pm 0.2\,\mathrm{cm}$ along one side and $148.3\,\mathrm{cm} \pm 0.3\,\mathrm{cm}$ along the other side.

**Hint 1.** To find the area, multiple the length and width of the rectangle. Use the values, but not the uncertainty.

**Hint 2.** Because the area of the table is calculated using multiplication, use Rule 2to find the uncertainty.

**Answer.** The area is $A = (2.620 \times 10^4) \pm (0.008 \times 10^4)\mathrm{cm}^2$.

**Solution.** The area is found to be (significant digits are underlined)

$$A = (176.7\mathrm{cm}) \times (148.3\mathrm{cm})$$
$$A = \underline{2620}4.61\mathrm{cm}^2$$

The rules for **significant figures** gives a guide for the precision (appropriate number of decimals), that is only an approximation. To know with certainty, we need to compute the uncertainty, which is

$$\%\text{-uncertainty} = \left(\frac{.2\mathrm{cm}}{176.7\mathrm{cm}}100\%\right) + \left(\frac{0.3\mathrm{cm}}{148.3\mathrm{cm}}100\%\right)$$
$$\%\text{-uncertainty} = (.11\%) + (.20\%) = (.31\%)$$

**Insignificant** Please be aware that the reason some digits are called **insignificant** is that they are *insignificant*:

$$\begin{aligned}(.31548\%) \times (26204.61) &= 82.67\\(.31\%) \times (26204.61) &= 81.23\\(.31\%) \times (26200) &= 81.22\\(.3\%) \times (26204.61) &= 78.61\\(.3\%) \times (26200) &= 78.60\end{aligned}$$

All of these round to an uncertainty of $80\,\mathrm{cm}^2$.

To find the uncertainty, we calculate

$$(.31\%) \times (\underline{2620}4.61\mathrm{cm}^2) = 81.23\mathrm{cm}^2$$

This tells us that we need to round at the "ten's place". We can write the area in a variety of ways:

$$\begin{aligned}A &= (2.620 \times 10^4\,\mathrm{cm}^2) \pm 0.3\%\\&= (2.620 \times 10^4) \pm (0.008 \times 10^4)\,\mathrm{cm}^2\\&= 2.620(8) \times 10^4\,\mathrm{cm}^2\end{aligned}$$

## A.5 Significant Figures only *approximates* Uncertainty

The precision/accuracy of any measurement or number is approximated by writing the number with a convention called using **significant figures**. Every measuring instrument can be read with only so much precision and no more. For example, a meter stick can be used to measure the length of a small metal rod to one-tenth of a millimeter, whereas a micrometer can be used to measure the length to one-thousandth of a millimeter. When reporting these two measurements, the precision is indicated by the number of digits used to express the result. You should always record your data and results using the convention of significant figures.

To give a specific example, suppose that the rod mentioned above was 52.430 mm long. When making this measurement with the meter stick, you would count off the total number of millimeters in the length of the rod and then add your best guess that the rod was four-tenths of a millimeter longer than that. Using the micrometer, you would count off the hundredths of a millimeter and then add your best guess of the number of thousandths of a millimeter, to complete the measurement. How would you communicate the fact that one measurement is more precise than the other? If you wrote both quantities in the same way, you could not tell which was which.

The rules for **significant figures**:

1. Significant figures include all certain digits plus the first of the doubtful digits.

2. Zeros to the right of the number are significant; zeros on the left are not.

3. Round the number, increasing by one the last digit retained if the following digit is greater than five.

4. In addition and subtraction, carry the result only up to the first doubtful decimal place of any of the starting numbers.

5. In multiplication and division, retain as many significant figures in the answer as there are in the starting number with the smallest number of significant figures.

When determining or estimating the experimental uncertainty, the precision of the measuring instrument is important, as shown in the above examples. But you must also be aware of other experimental factors. For example, a good stopwatch may have a precision of 0.01 seconds. Is this the total uncertainty of the measurement? You must remember that our physical reaction time maybe another 0.3 seconds. This is more than 10 times larger than the precision of the timer. This is very significant. Another example is trying to measure the diameter of a fuzzy cotton ball with a micrometer. Why is this not a very productive procedure? There are major uncertainties here that are intrinsic to the object itself and are unrelated to the measuring instrument. One must use common sense when estimating these uncertainties.

The **actual uncertainty** written in the units of the measurement, may not convey a sense of how good the precision is. A better measure of the precision is given by the relative uncertainty. This is defined as the actual uncertainty divided by the measurement itself and multiplied by 100, the **relative uncertainty** does not carry any units, just a %.

# Appendix B

# Discovering Relationships – Graphical Analysis

The primary purpose of experimentation is to discover relationships between various physical quantities. This is usually best achieved with a graphical analysis. Graphs of data and/or graphs of other results can be very enlightening. We often try to choose to plot variables for a graph so that the resulting relationship is linear, whose slope and intercept may be of physical interest.

## B.1 Linear Relationships

When two quantities are related, there are many, many possible mathematical relationships. The simplest relationship is the direct proportion. This relationship is represented on a graph by a straight line which goes through the origin. The linear relationship is similar, it however, may have an intercept with a coordinate axes. The general equation for a straight line is:

$$y = mx + b,$$

where $x$ and $y$ are the plotted quantities, $m$ is the slope, $b$ is the $y$-intercept. The slope and intercepts of a linear relationship often have physical significance. It is therefore very important to calculate the slope and intercept and then for you to interpret their meaning, and always give the units of the slope and intercept. The slope can be computed by choosing two places, $(x_1, y_1)$ and $(x_2, y_2)$, on the straight line. The slope is then given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

## B.2 Linearizing Non-Linear graphs

Most relationships which are not linear, can be graphed so that the graph is a straight line. This process is called a linearization of the data. This does not change the fundamental relationship or what it represents, but it does change how the graph looks. The advantage of linearizing non-linear data is that the analysis of the parameters (slope and intercept) becomes significantly easier. Linear regression, which allows us to compute uncertainties in the slope and intercept as well as evaluating deviations from the equation (with residual plots), can be done by Pasco Capstone and Microsoft Excel; but, nonlinear regression requires specialized statistical software or lots of additional formulaic computations.

For example, the equation

$$X \times Y = \text{constant}$$

represents an *inverse proportion* between $X$ and $Y$. A graph of this equation is not a straight line with a negative slope. Think about this and sketch the curve for yourself. This relationship can be graphed in such a way so that the new graph is a straight line. This change is accomplished by choosing a new set of axes, and plotting new numbers which are related to the original set. In this case if we would plot $1/X$ on the x-axis instead of just $X$, this will yield a straight line graph. Try it.

As another example, the equation

$$y = ax^2$$

represents (a special case of) a *quadratic relationship* between $x$ and $y$. A graph of this equation is not a straight line; however, this relationship can be graphed in such a way so that the new graph is a straight line. This change is accomplished by choosing a new set of axes, and plotting new numbers which are related to the original set. In this case, if we would plot $(x^2)$ on the x-axis instead of just $x$, this will yield a straight line graph. Try it.

There are many other possible relationships which are easy to linearize. These include: exponential function, trigonometric functions, and power functions (squares, square roots, etc.) A change of either the x or y-axis may linearize a function for you.

To linearize the *power relationship*

$$Y = Bx^M,$$

take the natural logarithms of both sides to obtain

$$\ln(Y) = \ln(B) + M\ln(X).$$

If one plots $\ln(Y)$ on the vertical and $\ln(X)$ on the horizontal (a "log-log" graph), then the graph of this function yields a straight line with slope $M$ and intercept $\ln(B)$.

To linearize the *exponential relationship*,

$$Y = Be^{MX},$$

take the logarithm of both sides to obtain

$$\ln(Y) = \ln(B) + MX,$$

and again a graph of this function yields a straight line graph. If one plots $\ln(Y)$ on the vertical and $X$ on the horizontal (a "semi-log" graph), then the graph of this function yields a straight line with slope $M$ and intercept $\ln(B)$.

# Appendix C

# Using Capstone

**Figure C.0.1:** This is a PDF of a Power-Point description for how to use the important features of Capstone.

You may also download the PDF-version (2.5MB) or the power-point version (2.7MB) of this document.