Lab 12

Ballistic Pendulum

Experimental Objectives

- To verify that the equations of conservation of momentum and conservation of mechanical energy give the same results as the equations of two-dimensional motion by comparing the initial velocity (the "muzzle velocity") of the ballistic projectile from two different sets of experimental measurements:

    - the range and vertical height measurements of the projectile motion, and
    - through the use of the ballistic pendulum.

Introduction

Conservation laws play a significant part in this ballistic pendulum experiment. A ballistic pendulum is a pendulum that swings after catching a small ball that has been fired into it. The amount of swing depends on the energy and momentum transfer of the collision. During this collision between the ball and the pendulum, the momentum of the total system should be conserved from the instant just prior, to the instant just after the collision. Physicists recognize a general conservation law for momentum which applies in all interactions of two or more objects where there are no other outside forces acting on the system. For collisions on the Earth, the force of gravity is an outside force but momentum is still considered to be conserved if the time of the interaction is small.

There are three categories of collisions: **elastic collisions**, **inelastic collisions**, and **totally inelastic collisions**. The "elasticity" of the collision refers to whether or not the total kinetic energy of the objects involved is conserved (remains the same before and after). Totally inelastic collisions are those inelastic collisions in which either the objects start together and end apart (some form of spontaneous separation, like throwing) or the objects start apart and end together (some form of connecting, like catching). Since the pendulum bob catches the ball, they move off with the same velocity ($\vec{v}_{1f} = \vec{v}_{2f}$) and the kinetic energy of the system is cannot be conserved during the collision. Using the general conservation of momentum law for the collision described,

$$m_1 \vec{v}_{1i} + M_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + M_2 \vec{v}_{2f} = (m_1 + M_2)\vec{v}_f$$

an equation can be written for the initial velocity of the ball (v_{1i}) in terms of the velocity of the system at the instant after the collision (v_f) and the individual masses of the ball (m_1) and the pendulum (M_2). After the collision the pendulum and ball will swing and at the highest point in the swing they will be caught. The *KE of the system at the instant after the collision* is converted totally to the *PE of the system at the highest point in the swing*. The velocity of the system *at the instant after the collision* can therefore be determined using the law of conservation of energy. Then with these two conservation laws, the initial velocity of the ball can be determined.

The initial velocity of the ball can also be determined by firing the ball horizontally off the edge of the table and analyzing the 2-dimensional projectile motion of the ball moving under the influence of the gravitational

force. This analysis involves separating the motion into its component directions, using the standard kinematic equations of motion and an appropriate set of measurements.

For these two very different techniques, calculate the same initial velocity of the projectile. An analysis and comparison of the two methods will help to illustrate the interconnections between these physics topics.

12.1 Pre-Lab Considerations

You can do the two parts of this lab in any order; but it will help lab go more smoothly if you answer the following questions before you come to lab.

conservation principles   To prepare for the calculation of the muzzle velocity using conservation principles, consider the following questions:

Exercise 12.1.1. Draw before and after pictures for a totally inelastic collision between two masses, m_1 and M_2. Assume that M_2 is initially stationary, and that m_1 is initially moving horizontally with a muzzle velocity of v.

Hint 1.   The "before" is intended to be the instant before they make contact. The "after" is intended to be the instant after they combine, but before they have had time to move off. You should bear in mind that they do not first hit, then combine, and then move off; rather the "combining" takes some small amount of time, during which the one object is slowing down and the other is speeding up until their velocities match each other.

Hint 2.   It will be useful to bear in mind that the "final" state for the collision (conservation of momentum) is *the same* instant as the "initial" for the pendulum swing (conservation of energy).

Exercise 12.1.2. Draw before and after pictures for the pendulum swing of the masses, m_1 and M_2. Assume that they travel back until the speed is zero as they are just about to swing back down.

Hint 1.   The "before" is intended to be the instant before they begin swinging after having just collided. You should bear in mind that this is a slight approximation because they do not first hit, then combine, and then move off; rather the "combining" takes some small amount of time, during which the one object is slowing down and the other is speeding up until their velocities match each other, so the swing will have already begun during the collision. However, this is a small effect. The "after" is intended to be the instant that they reach peak height and before they begin to swing back down.

Hint 2.   You *should* consider the "final" state for the collision (conservation of momentum) to be *the same* instant as the "initial" for the pendulum swing (conservation of energy).

Exercise 12.1.3. For this collision, write out the conservation of momentum equation. Identify which velocity in this equation is the "muzzle velocity". Identify which velocity in this equation is related to the velocity in Exercise 4. Be prepared to solve this equation for the appropriate velocity.

Hint.   It will be useful to bear in mind that the "final" state for the collision (conservation of momentum) is *the same* instant as the "initial" for the pendulum swing (conservation of energy).

Exercise 12.1.4. After the collision, the pendulum and ball will swing together. The KE of the pair at the instant after the collision will be converted to PE as it swings. Write out a conservation of energy equation for this process, in terms of the mass of the pendulum and ball, the change in height of the system and the velocity of the system at the instant after the collision. Identify which velocity in this equation is related to the velocity in Exercise 3.

Hint 1.   It will be useful to bear in mind that the "final" state for the collision (conservation of momentum) is *the same* instant as the "initial" for the pendulum swing (conservation of energy).

Hint 2.   You might have some trouble calling one state "initial" and another state "final". You might consider "before the collision" as "time 1", "after the collision" as "time 2", "before the swing" as "***time 2***", and "after the swing" as "time 3".

Exercise 12.1.5. Combine these two conservation laws (Exercise 3 and Exercise 4) to derive an expression for the initial velocity of the ball (before the collision) to the final height of the ball and pendulum system. ***List those quantities*** that you will need to measure during the lab in order to compute this number.

ballistic motion   To prepare for the calculation of the muzzle velocity using 2-dimensional (ballistic) motion principles, consider the following questions:

Exercise 12.1.6. Draw a picture of the ball's path when fired horizontally off of a table. Draw the ball in its initial position (at the moment it begins its free fall) and in its final position (at the moment just before it hits the floor). Label the relevant quantities that you should measure during the lab in order to compute the muzzle velocity. (You might need to do Exercise 7 first.)

Hint 1.   Make the ball larger than its scale size so that it size can be easily seen in your picture.

Hint 2.   On the picture, label the height and the range of the projectile.

Hint 3.   Think about whether the measurements should be taken from the top, bottom, or the middle of the ball. What part of the ball will hit the floor? Think about this for both the horizontal and the vertical measurements.

Exercise 12.1.7. For this projectile motion, use the kinematic equations of motion to derive an equation for the initial velocity of the ball in terms of the height and range measurements. *List those quantities* that you will need to measure during the lab in order to compute this number.

Hint.   If the gun is not level, such as if it shoots slightly upwards, then decide if this changes your resulting equation and the quantities you would need to measure.

12.2   Procedure

The ballistic pendulum apparatus consists of three parts: (1) a ballistic spring-loaded gun for the firing of the projectile (used in both parts), (2) a hollow pendulum bob suspended by a light rod for catching the fired projectile, and (3) an angled platform for catching the pendulum bob at the highest position of the bob's swing. The pendulum bob can be lifted to rest in the angled platform so that it is out of the way for Subsection 2.

Notice that the initial velocity of the projectile can be changed by adjusting the spring tension. (Your instructor can explain how to do this.)

Warning 12.2.1. You should *not change* this tension for the trials that compare the projectile motion to the pendular motion because *you want the muzzle velocity to be the same* in those cases.

However, you might be asked to run this experiment for two different tensions. Alternatively, you might change the tension to make it easier to cock the gun. If you do loosen the spring to make it easier to cock the gun, be sure that the gun fires strongly enough to actually cause the pendulum to swing up to the available notches.

The two parts of this experiment can be done in either order. Before doing each part, be sure to clamp the apparatus to the table, using cardboard pads so that the table does not develop dents, so that the apparatus does not move due to the kick when firing the gun.

12.2.1   The Ballistic Pendulum

Some versions of the apparatus have a metal "lip" in the pendulum bob to hold the ball in. Other versions have a rubber O-ring. If you have the version with the lip, then when you are removing the ball from the pendulum, be sure to push up on the spring catch (the lip) in the pendulum bob so as to not to damage the equipment.

- The pointy arm attached to the side of the pendulum indicates the position of the center of mass of the system.

- Do not try to take the apparatus apart, the instructor will give you the mass of the pendulum. (It might be written on a piece of tape attached to the apparatus.)

- Clamp the apparatus to the table, using cardboard pads so that the table does not develop dents, so that the apparatus does not move due to the kick when firing the gun.

- Fire the ball into the pendulum bob and mark the final notch position of the pendulum.

- Repeat the experiment with a sufficient number of trials (15) so that an average and a standard deviation of the notch positions can be obtained.

- Measure the change in height of the pendulum's pointer from its initial position to the average notch position. Calculate the uncertainty in this distance.

- Calculate (using conservation principles pre-lab work) the initial velocity (and uncertainty) of the projectile ball.

12.2.2   Projectile Motion

- Set-up the ballistic spring gun so that it will fire the projectile ball horizontally off the edge of the table.

  ◦ Clamp the apparatus to the table, using cardboard pads so that the table does not develop dents, so that the apparatus does not move due to the kick when firing the gun.
  ◦ Use a bubble level (or by seeing if the ball itself rolls) to make sure that the gun is level. (Recall Hint 1.)
  ◦ Move the pendulum out of the way.

- Tape a piece of paper to the floor where the ball will land, then tape a sheet of carbon paper at this spot. Consider taping a second piece of paper above the carbon paper to keep the carbon paper from tearing.

- Be careful not to hit anything or anybody with the ballistic projectile! Use larger pads or boxes to protect the tables and the walls.

- Repeat the experiment for a sufficient number of trials (15-20), and calculate an average and a standard deviation of the range.

- Calculate (using ballistic motion pre-lab work) the initial velocity (and uncertainty) of the projectile after taking the appropriate measurements.

12.3   Analysis

Quantitatively compare the two methods by considering the following: Calculate a percent difference between the two methods. Calculate the uncertainties for the velocity in both methods (Propagation of Uncertainties) and, also, write these in a % form. Which method is more precise? Decide whether this experiment has random or systematic errors. Discuss and show your experimental evidence for this decision.

Determine if the kinetic energy is conserved during the collision. (Recall the introductory discussion.) If it is not conserved, then calculate the percentage of energy lost $\frac{(E_f - E_i)}{E_i} \times (100\%)$. Comment on the size of this number.

12.4   Questions

1. For each portion of the motion in this experiment (ballistic motion, during the collision, and after the collision during the upward swing), indicate if the momentum and/or energy should be conserved and why. Conclude whether the collision between the steel ball and the pendulum bob is elastic or inelastic.

2. During the collision, what percent of the kinetic energy of the ball was transferred to the combination of the pendulum and ball? If energy is lost, where does it go?

3. If this gun was aimed and fired vertically from the table top, would the ball hit the ceiling? Assume a vertical height of 1.5 meters. Show all of your work.

4. What effect does the force of gravity have on the horizontal velocity of the projectile?

5. Does the air resistance on the ball have a significant effect on the results of this experiment? If it does, which result would be different without air resistance? For each part of the experiment, determine if your estimate of the initial velocity is too large or too small due to the effects of air resistance.

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