Lab 8

Conservation of Energy on a Linear Track – (Two Week Version)

Experimental Objectives

- The purpose of this experiment will be to verify the validity of the law of conservation of mechanical energy, which says that $\Delta E = 0$ as a cart runs along a track.

Introduction

Conservation laws play a very important role in our understanding of our physical world. For example, the law of conservation of energy can be applied in all physical processes. This is a fundamental and independent statement about the nature of the physical world. It is not necessarily derivable from other laws like Newton's Laws of motion. Though for simple point mass systems, the law of conservation of energy can be derived from Newton's Laws. It can be shown that the net work done on a system is equal to the change in the kinetic energy ($W_{\text{net}} = \Delta K$) of the system; this is called the work-energy theorem and it can be written in a variety of forms. When a net positive work is done on a system, the kinetic energy of the system increases, and when a net negative work is done on the system (as from a friction force), the kinetic energy of the system decreases.

When the gravitational force acts on a system, the work it does on the system, W_g, is the gravitational force (mg) times the vertical displacement ($h = \Delta y$): $W_g = mg\Delta y$. For convenience, this is called the change in gravitational potential energy ($W_g = -\Delta P$). *If* the gravitational force is the only force acting on the system then $W_g = W_{\text{net}}$ and therefore, $-\Delta P = \Delta K$ for the system. When a force can be associated with a potential energy, it is called a "conservative force." Another kind of potential energy deals with an elastic potential energy, like in a spring. The energy stored in a spring is given by the formula $P_s = \frac{1}{2}k\Delta x^2$.

If, on the other hand, a force dissipates energy, then it is called a "nonconservative force" and it will have no associated potential energy. Frictional forces are an example of a nonconservative force and the work done by a frictional force is negative because (physically) the frictional force removes energy from the system and (mathematically) the frictional force and the displacement are in opposite directions. This work done by friction is converted into heat or sound. To distinguish the energy of heat or sound from the potential and kinetic energy, we define the total mechanical energy, $E = K + P$ at any point. Since frictional forces remove mechanical energy, we say $W_f = \Delta E = \Delta K + \Delta P$.

In general then, the law of conservation of energy states that energy can not be created or destroyed, but can only change from one form to another; or the total energy of the system at point A is equal to the total energy of the system at point B.

8.1 Procedure

We would like for you to verify the conservation of mechanical energy in two different situations; so, there are two parts to this experiment. We will first consider a flat track with accelerated motion, as in the Newton's Law lab and the Friction lab. We can then consider an inclined plane. You will not be given an explicit procedure, but rather you will be given a series of questions with answers that will imply the procedure. Part of the experiment is for you to figure out for yourself what the best course of action is. Please answer the questions as they are asked.

NOTE: There is enough analysis for this lab that you will have two weeks to complete the lab. During the first week, you will do the two parts of the experiment and begin to write up your report. During the second week, you will do some analysis and re-run the experiment to determine the cause of differences from expectations. A single lab report will be due after the second week of experimentation.

8.1.1 Flat Track

Set up the dynamics cart on a horizontal dynamics track. Set up the motion sensor at one end of the track and a pulley at the other end so that the pulley partly extends past the edge of the table. Hang the basket over the pulley so that it can accelerate the cart along the track – you might need extra weight in the cart to keep it from accelerating too fast. In order to use this motion to verify the validity of the conservation of mechanical energy, we need to measure some variables. Answering Exercise 8.1.1 and Exercise 8.1.2 will help you decide on the relevant variables. Exercise 8.1.3 should help you determine how to finish setting up the equipment.

Exercise 8.1.1. In order to verify $\Delta E = 0$, we will need to calculate E as $E = K + P$. Therefore, we need to know the kinetic energy, $K = \frac{1}{2}mv^2$, the energy of *some mass*, m, moving at a speed v. Which mass do you need to measure? How can you measure the velocity?

Exercise 8.1.2. In order to verify $\Delta E = 0$, we will need to calculate E as $E = K + P$. Therefore, we need to know the potential energy, $P = mgy$, the energy of *some mass*, m, located some height, y, above the ground. Which mass do you need to measure? How can you measure the position?

Exercise 8.1.3. In order to measure the position of the falling mass and the velocity of the system, do you need two motion sensors? Can you manage with one? Considering that it is a fairly expensive piece of equipment, where should you NOT put the sonic ranger? Where could you put it? Depending on where you put the ranger, decide if you need to "translate" the position or velocity data in order to find the specific values that you actually need.

Once you decide what variables to measure, run the experiment for one set of masses while measuring the appropriate variables. Put the data into Excel and decide what plot(s) will allow you to verify the validity of the conservation of mechanical energy. Exercise 8.1.4 may help with this. Decide if you need a trendline. Relate the information in Exercise 8.1.5 to the statement you are trying to verify.

Exercise 8.1.4. To verify $\Delta E = 0$, we will need to graph E, the total mechanical energy, as a function of time. What do you expect this graph to look like, if the law is valid? If not?

1. Does the kinetic energy change during this motion? Is $\Delta K = 0$? Considering the initial and final values of the kinetic energy, K_i and K_f, what would a graph of K versus time look like?

2. Does the potential energy change during this motion? Is $\Delta P = 0$? Considering the initial and final values of the potential energy, P_i and P_f, what would a graph of P versus time look like?

3. Assuming that the mechanical energy is conserved, what would a graph look like if it included E, K, and P? What if the mechanical energy is not conserved? How would K and P be affected in these two cases?

4. (Subsection 8.1.2 only) When the cart is at the bottom of the track during the motion, the values of position become negative (less than zero!). Why? Is there some other place where the energy might go?

(a) If you are using the force transducer, then it has a spring and a spring potential energy, ΔP_{spring}. This can (and should!) also be included in the total mechanical energy. You can calculate the elastic potential energy stored in the spring of the force transducer with $P = \frac{1}{2}k\Delta x^2$, which, since we do not know k, can be written $P = \frac{1}{2}F\Delta x$, where F is the force in Newtons (measurable with the force transducer) and Δx is the distance from the spring's equilibrium position, not the height (derivable from the position data). Be sure to match up the force values and the x values at those same times.

Exercise 8.1.5. Please note the overall change in potential energy, ΔP, and the overall change in the kinetic energy, ΔK. Should either of these be related to the overall change in energy ΔE and, if so, how?

NOTE: Save your data so that you can do further analysis next week.

8.1.2 Sloped Track

Remove the pulley from the track. Your cart will have either a spring-loaded "battering ram" on the front or a pair of magnets. If you have the battering ram, then you will want the end of the track with the rubber nub at the bottom of the incline. If you have the magnets, then you need to replace the pulley with a "C" shaped "catch-bar." *Ask for help from the instructor!* The catch-bar has magnets in it that will repel the magnets in the cart. In this case, the cart must not be going so fast as to come into physical contact with the magnets on the catch-bar.

Raise one end of the dynamics track. Exercise 8.1.6 should help decide how tilted. Measure the tilt angle of the track with two methods: use a protractor, and measure the vertical rise and track length and calculate the tilt angle using the inverse-sine function. Answer Exercise 8.1.7. As you continue to set up the track for measurements, consider answering Exercise 8.1.1, Exercise 8.1.2, and Exercise 8.1.3 again for this situation to help you decide on the appropriate accessories (sensors); but note Exercise 8.1.8 as you think about the answers to the previous questions.

Exercise 8.1.6. We want the cart to accelerate down the track (not too slow), but not to fly off at the bottom (not too fast). How fast is *too fast*? Don't use that slope! How fast is *too slow*? Use a slope somewhere in between.

Exercise 8.1.7. After you measure the angle of incline in these two ways, consider the uncertainty in the measurements. Which of these measurement is more precise?

Exercise 8.1.8. The motion sensor will measure the motion of the cart *along* the ramp, but the potential energy needs the *vertical* position of the cart. Which trig function relates the distance along the ramp to the corresponding vertical distance?

Once you decide on the variables to be measured, but before you make the measurements, you will need to calibrate your position measurements. We would like zero to correspond to being at the bottom of the ramp, so place the cart stationary at the bottom and use the motion sensor to measure this position. In order to verify the validity of the conservation of mechanical energy, release the cart from rest near the top of the ramp and let it roll down the incline, bouncing three times before you stop the measurement. Do this for one value of mass. Answer Exercise 8.1.9.

Exercise 8.1.9. Does the mass of the cart matter? If you run it again at a different value of mass, would you expect the overall conclusion to be different? Would you expect the specific values to be different?

Exercise 8.1.10. If the mechanical energy is conserved, then

$$\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f$$

What do you notice about the mass? Is your graph different if the mass of the cart changes? Does this support or conflict with the idea that the total mechanical energy is conserved? On the other hand, if the mechanical energy is not conserved, then

$$W_{\text{nc}} = \frac{1}{2}mv_f^2 + mgy_f - \frac{1}{2}mv_i^2 - mgy_i$$

What do you notice about the mass now? Does your graph support or conflict with the idea that the total mechanical energy is conserved?

Transfer these data to Excel again and decide on the best graph to verify the objective. Again, Exercise 8.1.4 may help with this; however, you will also need to consider Item 8.1.4.4. Decide if you need a trendline and where it would be fit. Relate the information in Exercise 8.1.5 to the statement you are trying to verify.

NOTE: Save your data so that you can do further analysis next week.

8.2 Analysis

For the second week, you should already have your graphs from the experiment and you should have written a significant portion of the theory and the analysis. We are now going to take a closer look at the irregularities of the data and investigate some variations to try to explain what those data say.

- One of the factors you were asked to consider last week was Exercise 8.1.9. In order to verify this, re-run Subsection 8.1.1 with a noticeably different massed cart. Re-create the graph and use this only to note the effect of a different mass. Answer Exercise 8.1.10.
- Before drawing conclusions about the validity of the conservation of mechanical energy, consider Exercise 8.2.1.

Exercise 8.2.1. We need to look for the energy lost in each graph.

1. When you look at the graph from Subsection 8.1.1 for E, is the energy conserved or is there energy lost? If lost, calculate the energy lost or gained from the graph. (It might help to have a trendline.) If energy is lost, come up with at least two explanations for where this energy goes.
2. When you look at the graph from Subsection 8.1.2 for E, there are jumps in the energy. Why?
   (a) What is happening between the jumps? Does Subsection 8.1.1 help to explain these sections of the graph? Compared to the jumps, can we assume that the mechanical energy is conserved between the jumps?
   (b) What is happening at the time of those "jumps"? From the trend of the graph, calculate the amount of energy lost during each sudden change, call it the energy discrepancy, and the percent of this discrepancy relative to the total energy before the corresponding collision. Discuss where this "missing" energy goes. Is the ratio of "energy discrepancy" to total prior energy the same for each jump?
3. Comment in general, on the law of Conservation of Mechanical Energy. Can you predict any effects that might invalidate the conservation of mechanical energy? Can these effects be minimized? Is it possible to run the experiment again minimizing this effect?

- As you evaluate Subsection 8.1.2, you might be asked to re-run the experiment with a force transducer placed at the bottom of the track. (This should imply where the motion sensor will go.) Make sure that the cart will bounce from the force sensor. Make sure that the force sensor is zeroed before the start. There might be some information here based on work as a force-through-a-distance versus work as a change-in-energy.
- One explanation of a loss of energy (non-conservation) is friction. List all of the places where two pieces of material rub against each other. Since $F_f = \mu F_N$, do any of these locations have a normal force that can be varied? (Recall Exercise 8.1.9 and Exercise 8.1.10.) As an independent measure of the amount of friction, we can also consider the actual acceleration versus the expected acceleration. Exercise 8.2.2 will help you determine the expected acceleration and the variable necessary to find it. Exercise 8.2.3 will help decide on the relationship between the friction and the acceleration.

Exercise 8.2.2. Given an ramp inclined at some angle θ, what is the component of the gravitational force aimed down the ramp? Assuming that there is no friction, what is the net force? Since $F_{\text{net}} = ma$, the acceleration should be ... ?[1] From your expression, what do you need to measure in order to find the expected value of a? (Recall Exercise 8.1.7.)

Exercise 8.2.3. If there is friction, then how do you expect the actual acceleration to compare to the expected acceleration? If there is no friction? So, how would you interpret finding an acceleration that is exactly equal to the expected value? less than the expected value? Larger than the expected value?

- A second explanation for the loss of energy is that some component is gaining rotational kinetic energy. The formula for this is $K_R = \frac{1}{2}I\omega^2$, where I is the moment of inertia[2], and ω is the angular speed $\omega = v/r$. Assuming that any discrepancy that you found in the conservation of energy is due to the rotational kinetic energy of the pulley, how much energy would the pulley need to have at the end of the run (while spinning full speed)? Based on the final velocity of the cart, what is the angular speed of the pulley? Based on these numbers, K_R and ω, what is the moment of inertia for the pulley? Can you tell if this is a reasonable estimate?

[1] $a = g \sin\theta$.

[2] In this case, the moment of inertia is probably a little less than $\frac{1}{2}mr^2$, where m is the mass of the rotating object and r is the radius of the rotating object. This is not a convenient way to calculate I at this time.