Lab 13

Conditions of Equilibrium – Model of a Human Forearm

Experimental Objectives

- You will verify the equations for translational and rotational equilibrium by experimentally balancing the system and comparing the measurements to the results from these equations.

Introduction

Objects that are not accelerating are said to be in a state of **equilibrium**. If the object is moving at a constant velocity, then it is in **dynamic equilibrium**. If the object is at rest, then it is in **static equilibrium**. These principles apply to many physical examples in engineering, architecture, and biophysics. In particular, these principles allow one to be able to analyze and calculate the forces on the beams or the cables in a bridge or the forces at work in the muscles and bones in the human body.

The two conditions for equilibrium can be stated in equation form: First, if the body's center of mass is in **translational equilibrium** then it will not accelerate in any direction.

$$\sum \vec{F} = 0 \tag{13.0.1}$$

Secondly, if the body is in **rotational equilibrium** then it will not rotate about any point or axis of rotation.

$$\sum \tau = 0 \tag{13.0.2}$$

For all systems such as these, there is a special point called the **center of mass** or **center of gravity** of the system. The center of mass calculation is a weighted average of the individual masses (giving more emphasis to those positions where there is more mass). The location of this special point can be useful in determining whether the system will be in equilibrium. For an object with uniform density, such as a half-meter stick, the center of mass is at the center of the stick.

13.1 Prelab Work

- Define the following terms: **torque**, **lever arm**, and **center of mass**.
- State in a sentence, the first [Equation (13.0.1)] and second [Equation (13.0.2)] conditions of equilibrium. Include in your comments the fact that one of these is a vector equation.

- In Subsection 13.2.1, you will use a spring and find two locations associated with two weights. These measurements have a (familiar) linear relationship and you will be asked to find the coefficient of proportionality (a.k.a., the slope; which in this case is the spring constant). For any linear function, the slope can be determined from knowing the values of any two points, (x_1, y_1) and (x_2, y_2). Using these two points write out a formula for the slope of the line so that you can more easily find the spring constant in lab.

- Draw a force diagram for a horizontal forearm as outlined in Subsection 13.2.2. Label all of the appropriate forces.

13.2 Procedure

13.2.1 Determine the Spring Constant of the Bicep Muscle

Before creating the forearm model, we need an equation for the force that the bicep exerts. Since the spring models the bicep, we can use Hooke's law: $F = k(x - x_0)$, where x_0 is the equilibrium position. To find the force as a function of position, $F(x) = kx + b$, you will make two measurements of position and force and then determine the spring constant k and the intercept b.

- Hang the spring from the vertical support so that it is easy to measure the position of the bottom of the spring.

- Measure the position of the bottom hook of the spring, x_1, with about $m_1 = 250\,\mathrm{g}$ placed on the spring. Measure the position of the bottom hook of the spring, x_2, with about $m_2 = 850\,\mathrm{g}$ placed on the spring.

  - You can measure from any location ***as long as you always measure from the same reference point.*** If you measure from the floor, which is more convenient, then k will be negative.
  - Parallax errors can be very significant with this measurement. With the use of a mirror, this error can be reduced. Hold the mirror vertically along the track and align your line of sight until the bottom hook of the spring and its mirror image are at the same level. Then take the position readings.
  - Each person in the group should carry out these measurements without looking at the other group members' results. The average of these locations should be used.
  - You can likely improve your analysis by considering if any measurement uncertainty is random or systematic.

- With these two measurements, calculate the spring constant k as the slope of the graph of F versus x. You do not have to – and should not – actually create a graph of F versus x to do this, *if* you remember your prelab work for slope.

  - b is the intercept. Using one pair of data and your knowledge of k, find b.
  - Determine the uncertainty in k and b.
  - Now, with values and uncertainties for k and b, you can use $F = kx + b$ to find the force that the spring thinks it is supporting simply by measuring the location of the bottom of the spring.

13.2.2 Experimental Setup for the Equilibrium Experiment

For our purposes, the forearm can be considered to jut out forwards from a vertical upper arm with the hand and the weight of the forearm itself pulling down while the bicep holds the forearm up. The elbow joint is assumed to be a nearly frictionless pivot point for the forearm, allowing the forearm to rotate about the elbow. The upper arm (the humerus) is connected to the forearm (the ulna) at the elbow, exerting a downward force on the forearm at the joint.

Human Arm	Model	Force Location
Forearm (ulna and radius)	half-meter stick	center of mass
Hand	clamp and hanger or plastic cup	about 1 cm from the far end
Bicep Muscle	clamp and spring (pulling up)	8.6% of the length of the forearm from the elbow (about 4 cm from the elbow)
Elbow (humerus)	clamp, hanger, weights	as near as possible to the elbow end (about 1 cm from the elbow-end)

Table 13.2.1: Building the model of the human forearm

Be sure to include the weight of the clamps at each location where one is used.

- Set-up a force diagram for this model of a horizontal forearm.
- Set-up the equilibrium equations for this system.

13.2.3 Equilibrium Experiment

You are going to do the following procedure for two different locations of the bicep muscle: First set the bicep, as mentioned above, in the human location of about 4 cm from the elbow. Second, set the bicep at about 10 cm from the elbow. You should consider which location gives more leverage to lift the hand.

- With your model completely set up, place a 50 g mass in the "hand." *Experimentally* determine the force at the elbow necessary to balance the system and make it horizontal. You may also change the position of the hand by small amounts if you find it easier to balance the system. However, be sure to record the correct distances.
- To check that the system is level, use another meter stick to measure the height from the floor *at each end* of the forearm half-meter stick.
- Determine the force in the bicep muscle using the formula derived in the previous section as appropriate for the new position of the bottom of the hook of the spring (again using the mirror for accuracy). Be sure to include the uncertainties!
- Record all force locations as read directly from the "forearm" meter stick. *Do not subtract in your head* so that we can reproduce the locations later, if need be.
- *Experimentally* determine the sensitivity of your values for force at the elbow by checking how many grams can be added or removed at the elbow while maintaining the horizontal equilibrium.
- Since the hand is so sensitive, you should estimate roughly the sensitivity of the value of force at the hand.

With this data, you will verify the equations of equilibrium.

Activity 13.2.1 (Comparative Anatomy)**.** Create a table that allows you to compare the values of force for the elbow, bicep, forearm, and hand for three different attachment-locations.

- Enter the data from Subsection 13.2.3. Record each position and each force.
- Predict how the forces will change if the hand is moved a few millimeters closer to the bicep. (Hint 1 might help.)
- Move the hand in and measure the forces that make it balance. Compare to your predictions.
- Predict how the forces will change if the bicep is moved a few millimeters further from the elbow. (Hint 2 might help.)
- Move the bicep away from the elbow and measure the forces that make it balance. Compare to your predictions.

Hint 1. Imagine carrying grocery bags at different locations on your lower arm while holding your arm in an L-shape.

Hint 2. Imagine sitting at different locations on a teeter-totter.

13.3 Analysis

- For the forces measured or calculated in Subsection 13.2.3, test each of the conditions of equilibrium:
    - Translational Equilibrium: Show that the up forces equal the down forces to within your uncertainty.
    - Rotational Equilibrium: Show that the clockwise torques equal the counterclockwise torques to within your uncertainty.
    - To show that it does not matter which point you choose to calculate the summation of the torques about, choose the far end of the meter stick (near the hand) as the zero or the rotation point.
    - If your conditions of equilibrium are not consistent, then calculate the necessary mass that should be in the hand and test this value experimentally. Discuss how well this value works. (If you cannot get this to balance, you might check to be sure you included the weight of the forearm itself...)
- Consider the forces in Activity 13.2.1.
    - If you have three shopping bags of food at the grocery store that you want to hang from your arm as you walk to your car, should you hang the heaviest bag closest to your elbow or closest to your hand? Explain.
    - The location of the bicep relative to the elbow determines the leverage in pulling your hand to your chest (or for a quadruped move their feet forward for the next step). Do you expect fast animals to have their leg muscles attached close to the joint or far from the joint? Explain.

13.4 Questions

1. Is it necessary to have the meter stick horizontal for the system to be in equilibrium? Why did we want to keep the arm horizontal?

Hint. $\tau = r\,F\,\sin\theta$

2. How was the tension in the muscle affected as the position of attached muscle was moved further from the joint? Keep the mass in the hand constant.

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A PDF version might be found at equilibrium.pdf (130 kB)