# Lab 5

# Dry Sliding Friction

### Experimental Objectives

- In this experiment you will devise methods
  - to investigate the nature of both the frictional force and the coefficient of friction and
  - to test the validity of da Vinci's empirical rule.

## Introduction

Friction is a force which retards the relative motion of any body while sliding over another body. The frictional force acting on a body is parallel to the surface that the object is sliding upon and it is directed opposite to the direction of motion. The phenomenon of friction is rather complicated, especially at the microscopic level, because it is dependent on the nature of the materials of both contacting surfaces. The frictional force depends on the roughness or the irregularities of both surfaces. At the macroscopic level, the nature of this force can be described by a simple empirical law, first given by Leonardo da Vinci:

> The magnitude of the force of friction between unlubricated, dry surfaces sliding one over the other is proportional to the normal force pressing the surfaces together and is independent of the (macroscopic) area of contact and of the relative speed.

At the microscopic level, the frictional force ($F_f$) does depend on the actual area of contact of the irregularities of the surfaces. This actual area of contact then increases as the force pressing the two surfaces together increases, this force is called the load. Using Newton's 2$^{\text{nd}}$ Law in this perpendicular direction we can conclude that the magnitude of the load is equal to the Normal force ($F_N$) of the surface pushing on the object. Therefore we may write that

$$F_f \propto F_N \qquad \Rightarrow \qquad F_f = \mu F_N$$

where the Greek letter $\mu$ ("mew") is a dimensionless constant of proportionality called the coefficient of friction.

When a body is pushed or pulled parallel to the surface of contact and no motion occurs, we can conclude that the force of the push or pull is equal to the frictional force, using Newton's 2$^{\text{nd}}$ Law of motion. As the applied force is increased, the frictional force remains equal to the applied force until motion results. At this maximum value of the applied force, the frictional force is also a maximum and is given by

$$F_f = \mu_s F_N$$

where the subscript $s$ stands for static (non-moving) friction, and $\mu_s$ is **the coefficient of static friction**. This equation can only be used at this maximum static point also called the point of impending motion. At the instant that the applied force becomes greater than the maximum $f_s$, the body is set into motion and this motion is opposed by a frictional force called the kinetic (sliding) frictional force and is given by

$$F_f = \mu_k F_N$$

where the subscript $k$ stands for the kinetic (moving) friction, and $\mu_k$ is **the coefficient of kinetic friction**. In general, $\mu_k < \mu_s$; that is, it takes more force to overcome the static friction than to overcome the kinetic friction. The coefficients of friction are generally less than one, but may be greater than one, and they depend on the nature of both surfaces.

Consider a system comprised of a block on a horizontal surface being pulled horizontally by a string connected to a hanging weight. We can use $M$ is the mass of the block on the horizontal surface and $m$ is the hanging mass. The force that accelerates the system forward is $mg$. The frictional force depends on the normal force of the block $\mu_k(Mg)$. Then, the whole system is accelerating with a constant acceleration so that Newton's second law gives:

$$(mg) + [-(\mu_k Mg)] = (M+m)a. \tag{5.1}$$

From this, $\mu_k$ can be solved for, giving:

$$\mu_k = \frac{mg - (M+m)a}{Mg}. \tag{5.2}$$

## 5.1 Pre-Lab Considerations

- Draw force diagrams for the following case: a block on a horizontal surface pulled by a hanging mass and a string (include the friction force).
- Write out the corresponding Newton's $2^{\text{nd}}$ Law equations for forces both parallel and perpendicular to the contact surface.
- Derive the relevant equations for each of the above two cases for which the coefficients of friction can be determined:
  - Case one is static, but at the point of motion.
  - Case two is the kinetic case.

## 5.2 Student Outcomes

Knowledge Developed: In this exercise, students should learn how forces are related to the motion of a cart, how to use a free-body diagram, and gain a visceral understanding of Newton's second law with the (more realistic) inclusion of the effects of friction.

Skills Developed:

- Evaluate the data for sources of uncertainty. Can you see an effect, such as a level track or the presence of friction, in the result?
- Using Pasco Capstone software
- Interpreting slope and intercept of graphs
- Evaluating and propagating uncertainties

## 5.3 Procedure

For the block on the horizontal plane:

1. Clean the block and the plane, so that they are free of dust and other contaminants.
2. Make sure the track is level, as in previous labs.

You will use the force transducer to measure the force directly in Subsection 1 and Subsection 2. However, Subsection 3 will require an indirect measurement (calculation) of the force by measuring the velocity and using the velocity-versus-time graph to get the acceleration.

### 5.3.1 Break Static Friction — pull until moves

**Note** By this time, you should already be familiar with the Pasco Capstone interface, which is also used in Lab 3, Lab 4, and Labs 6–7. You may remind yourself of the format by reading Appendix D

1. Set up the Dynamics Track, cart, force transducer and friction block. The force transducer attaches to the dynamics cart, the friction block rests on the track (felt side down).
2. Attach a string to the force transducer. The force transducer needs to be zeroed before data collection starts. Collect data, and slowly start pulling on the string (*be sure to pull the string horizontally*) and slowly increase the pull force until the cart is moving down the track. Using just the maximum force (at the point of impending motion) the coefficient of static friction can be calculated.
3. Test the relationship between the force of friction and the normal force, by changing the load force (normal force) and measuring the force of friction at the point of motion impending. Carry this out for a total of five data points. Graph the frictional force versus the normal force. Calculate the coefficient of static friction from this graph.

### 5.3.2 Effect of Surface Area — distinguish pressure from force

Consider pushing a pencil into your arm. (Well, don't *actually* do it!) If you use the erasure end, then you can feel the force, but it doesn't hurt. If you use the sharpened tip with the *same* force then it will certainly hurt! So, you have the idea that the same force spread over a different surface area *can* have a different effect; but it doesn't *always* have a different effect. For this part of the lab, you will test the relationship between the coefficient of friction and the macroscopic area of contact between the block and the surface.

1. Place the friction block on its side (felt side down) and repeat Item 2 and Item 3 for three (rather than five) of the previous load forces.
2. Add the plot of this $F_f$ versus $F_N$ as a new series to the graph of Subsection 5.3.1.

### 5.3.3 Friction while Accelerating

1. Apply a force (hanging mass, pulley, and string) large enough to accelerate the block. Use the Sonic Ranger to collect data. Note: This should accelerate fast enough to measure the acceleration, but not so fast that it crashes at the end of the track. (Depending on the normal forces being used, you might try 300 g as the hanging mass.)
2. Graph the velocity vs time. Determine the acceleration of the block from the slope of the line.
3. Repeat this part four or five times with a different normal forces. (You may use any hanging mass.)
4. Since we are not measuring the frictional force, you will need to calculate it; See Exercise 5.3.1.
5. Add the plot of this $F_f$ versus $F_N$ as a new series to the graph of Subsection 5.3.1 and Subsection 5.3.2.

6. Calculate the coefficient of kinetic friction from the slope.

**Exercise 5.3.1.** Draw the free-body diagram for the cart being dragged by the hanging mass. Set-up Newton's second law for the forces involved. Solve this for the frictional force in terms of quantities you can measure.

**Hint 1.** Lab 4 might help you set-up the free-body diagram and equation. In that lab, we made a point of keeping the total mass constant. In this lab, that is not important because that lab allows us to trust Newton's second law and we are now testing a different relationship.

**Hint 2.** Equation (5.1) wrote out Newton's second law for you; but you want to solve it for $F_f$, not for $\mu$ and not in terms of $\mu$.

**Hint 3.** You can measure the total mass directly. You can measure the hanging mass directly. You can compute (an indirect measurement) the acceleration from the velocity-versus-time graph.

**Answer.** Do not compute the frictional force using the normal force, that is the relationship you are trying to investigate!

## 5.4 Analysis

The experimental precision should be estimated for all parts of this experiment and care should be taken for all of the measurements. , but it is more important to investigate the relationships than it is to repeat the experiment many times or to try to achieve high precision in the data. In Exercise 5.3.1 you found an equation for $F_f$ in terms of measured values. You should track the uncertainties from measurement, through the calculation, to the result (this is called the Propagation of Uncertainties) so that you can see how the uncertainty in the measurements impact the uncertainty of the final result. The following will step you through how it would work for Equation (5.2), which is much more complicated than your equation.

**Example 5.4.1.** In order to propagate the uncertainty for $\mu_k = \dfrac{mg - (M+m)a}{Mg}$ we should notice that it has both addition (See Rule 1) and multiplication (See Rule 2). I will assume some values with uncertainty; I will also list the relative uncertainty:

- $m = 0.30 \pm 0.01\,\mathrm{kg}$, $\dfrac{0.01\,\mathrm{kg}}{0.30\,\mathrm{kg}} = 0.033 = 3.3\%$
- $M = 2.50 \pm 0.02\,\mathrm{kg}$, $\dfrac{0.02\,\mathrm{kg}}{2.50\,\mathrm{kg}} = 0.008 = 0.8\%$
- $a = 0.45 \pm 0.04\ \mathrm{m/s^2}$, $\dfrac{0.04\ \mathrm{m/s^2}}{0.45\ \mathrm{m/s^2}} = 0.089 = 8.9\%$
- $g = 9.81 \pm 0.01\ \mathrm{m/s^2}$, $\dfrac{0.01\ \mathrm{m/s^2}}{9.81\ \mathrm{m/s^2}} = 0.001 = 0.1\%$

You should note that while the uncertainty of $m$ and $M$ are about the same, the relative uncertainty of $m$ is much larger than the relative uncertainty of $M$ (and similarly for $a$ and $g$). While the uncertainty for $m$ and $g$ have the same numeric value, they cannot be meaningfully compared because they have different units.

Recall the "order of operations" implies that we ***first*** add $(M+m)$, ***then*** multiply that value by $a$, ***then*** add that value to (the product of) $mg$, and ***finally*** divide by (the product of) $Mg$. We do the propagation of uncertainty in the same order.

Using Rule 1, add the masses $(M+m)$:

$$\begin{aligned}(0.30 \pm 0.01\,\mathrm{kg}) + (2.50 \pm 0.02\,\mathrm{kg}) &= [(0.30 + 2.50) \pm (0.01 + 0.02)]\,\mathrm{kg} \\ &= 2.80 \pm 0.03\,\mathrm{kg}\end{aligned}$$

$$\frac{0.03\,\mathrm{kg}}{2.80\,\mathrm{kg}} = 0.011 = 1.1\%$$

This overall uncertainty is affected a little bit more by $M$ (in this case) because we are adding uncertainty (not relative uncertainty).

Using Rule 2, multiply by the acceleration $(M+m)a$:

$$\begin{aligned}(2.80 \pm 0.03\,\mathrm{kg})(0.45 \pm 0.04\,{}^{\mathrm{m}}\!/\!{}_{\mathrm{s}^2}) &= [(2.80\,\mathrm{kg})(0.45\,{}^{\mathrm{m}}\!/\!{}_{\mathrm{s}^2}] \pm [(0.011 + 0.089)] \\ &= 1.26\,\mathrm{N} \pm [0.10],\end{aligned}$$

since $(1.26\,\mathrm{N})(0.10) = 0.126\,\mathrm{N}$, this is $1.26 \pm 0.13\,\mathrm{N}$.[1] In this case, $a$ contributed about 8 times as much uncertainty.[2]

We also need to use Rule 2 to find the uncertainty for $mg$ before combining the numerator:

$$\begin{aligned}(0.30 \pm 0.01\,\mathrm{kg})(9.81 \pm 0.01\,{}^{\mathrm{m}}\!/\!{}_{\mathrm{s}^2}) &= [(0.30\,\mathrm{kg})(9.81\,{}^{\mathrm{m}}\!/\!{}_{\mathrm{s}^2}] \pm [(0.033 + 0.001)] \\ &= 2.94\,\mathrm{N} \pm [0.034],\end{aligned}$$

since $(2.94\,\mathrm{N})(0.034) = 0.10\,\mathrm{N}$, this is $2.94 \pm 0.10\,\mathrm{N}$.[3] Notice that the mass had a larger impact on the uncertainty of this term.

Using Rule 1, subtract[4] the two terms in the numerator [$mg$ and $(M+m)a$]:

$$\begin{aligned}(2.94 \pm 0.10\,\mathrm{N}) - (1.26 \pm 0.13\,\mathrm{N}) &= [(2.94 - 1.26) \pm (0.10 + 0.13)]\,\mathrm{N} \\ &= 1.68 \pm 0.23\,\mathrm{N}\end{aligned}$$

$$\frac{0.23\,\mathrm{N}}{1.68\,\mathrm{N}} = 0.14 = 14\%$$

Notice that both terms had a similar contribution to the uncertainty.[5]

Finally, use Rule 2 to find the uncertainty for the final combination[6] of the numerator/denominator:

$$\begin{aligned}\frac{(1.68 \pm 0.23\,\mathrm{N})}{(2.50 \pm 0.02\,\mathrm{kg})(9.81 \pm 0.01\,{}^{\mathrm{m}}\!/\!{}_{\mathrm{s}^2})} &= [\frac{1.68\,\mathrm{N}}{(2.50\,\mathrm{kg})(9.81\,{}^{\mathrm{m}}\!/\!{}_{\mathrm{s}^2})}] \pm [(0.14 + 0.008 + 0.001)] \\ &= 0.0685 \pm [0.15],\end{aligned}$$

since $(0.0685\,\mathrm{N})(0.15) = 0.010\,\mathrm{N}$, this is $0.07 \pm 0.01 = (7 \pm 1) \times 10^{-2}$, rounded to the appropriate number of significant digits.

You should note that the uncertainty of the numerator swamps by far the uncertainty of the denominator. In order to make this measurement more precise, we should focus on improving the precision of $m$ and $a$, as indicated in Footnote 5.4.5.

The graphs and data should also be evaluated as usual. In this particular experiment, you should

- Explain why the normal force on the block by the surface rather than the weight of the object is related to the frictional force.
- Interpret the slope and intercept of the graphs.
- Compare the slopes from each of the three parts. Decide which should be the same and which should be different.
- Calculate the % decrease of the static to kinetic coefficient of friction.
- Comment on the validity of the empirical rules of friction.

[1] If this were the final answer, you should report it as $1.3 \pm 0.1\,\mathrm{N}$; but since we are continuing to use it in calculations, for the purpose of managing appropriate rounding errors, you can keep an extra insignificant digit, which will maintain consistency. Please be sure to round your final answer to the appropriate number of significant digits.

[2] To improve this answer we should focus on a more precise measurement of $a$, rather than improving the precision of $m$ or $M$!

[3] If this were the final answer, you should report it as $2.9 \pm 0.1\,\mathrm{N}$; but since we are continuing to use it in calculations, for the purpose of managing appropriate rounding errors, you can keep an extra insignificant digit, which will maintain consistency. Please be sure to round your final answer to the appropriate number of significant digits.

[4] Remember the rule is to ***add*** the uncertainty even if you are ***subtracting*** the values!

[5] Even if we improved the precision of $a$, as mentioned in Footnote 5.4.2, the uncertainty of $mg$ would keep this uncertainty near the 0.1 to 0.2 range. So we need to be more precise with $m$ for the $mg$ term *and* with $a$ for the $(M+m)a$ term.

[6] Remember the rule is to ***add*** the relative uncertainty whether you are multiplying ***or*** dividing the values!

## 5.5 Questions

For all questions, and when possible, use your experimental or theoretical results to demonstrate your answers to the questions.

**1.** Does the coefficient of friction depend on the area of contact?

**2.** Does the coefficient of friction depend on the mass of the object?

**3.** Does the coefficient of friction depend on the normal force of the object?

**4.** Does the frictional force depend on the normal force of the object?

**5.** Does the coefficient of kinetic friction depend on the speed of travel?

**6.** When the object was pulled by a string, how would the forces be affected if the cord was not horizontal?

**7.** What would happen to the coefficient of friction if the surfaces were lubricated with oil?

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A PDF version might be found at friction.pdf (175 kB)
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