# Appendix C

# Discovering Relationships – Graphical Analysis

The primary purpose of experimentation is to discover relationships between various physical quantities. This is usually best achieved with a graphical analysis. Graphs of data and/or graphs of other results can be very enlightening. We often try to choose to plot variables for a graph so that the resulting relationship is linear, whose slope and intercept may be of physical interest.

## C.1 Linear Relationships

When two quantities are related, there are many, many possible mathematical relationships. The simplest relationship is the direct proportion. This relationship is represented on a graph by a straight line which goes through the origin. The linear relationship is similar, it however, may have an intercept with a coordinate axes. The general equation for a straight line is:

$$y = mx + b,$$

where $x$ and $y$ are the plotted quantities, $m$ is the slope, $b$ is the $y$-intercept. The slope and intercepts of a linear relationship often have physical significance. It is therefore very important to calculate the slope and intercept and then for you to interpret their meaning, and always give the units of the slope and intercept. The slope can be computed by choosing two places, $(x_1, y_1)$ and $(x_2, y_2)$, on the straight line. The slope is then given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

## C.2 Linearizing Non-Linear graphs

Most relationships which are not linear, can be graphed so that the graph is a straight line. This process is called a linearization of the data. This does not change the fundamental relationship or what it represents, but it does change how the graph looks. The advantage of linearizing non-linear data is that the analysis of the parameters (slope and intercept) becomes significantly easier. Linear regression, which allows us to compute uncertainties in the slope and intercept as well as evaluating deviations from the equation (with residual plots), can be done by Pasco Capstone and Microsoft Excel; but, nonlinear regression requires specialized statistical software or lots of additional formulaic computations.

For example, the equation

$$X \times Y = \text{constant}$$

represents an *inverse proportion* between $X$ and $Y$. A graph of this equation is not a straight line with a negative slope. Think about this and sketch the curve for yourself. This relationship can be graphed in such a way so that the new graph is a straight line. This change is accomplished by choosing a new set of axes, and plotting new numbers which are related to the original set. In this case if we would plot $1/X$ on the x-axis instead of just $X$, this will yield a straight line graph. Try it.

As another example, the equation

$$y = ax^2$$

represents (a special case of) a *quadratic relationship* between $x$ and $y$. A graph of this equation is not a straight line; however, this relationship can be graphed in such a way so that the new graph is a straight line. This change is accomplished by choosing a new set of axes, and plotting new numbers which are related to the original set. In this case, if we would plot $(x^2)$ on the x-axis instead of just $x$, this will yield a straight line graph. Try it.

There are many other possible relationships which are easy to linearize. These include: exponential function, trigonometric functions, and power functions (squares, square roots, etc.) A change of either the x or y-axis may linearize a function for you.

To linearize the *power relationship*

$$Y = Bx^M,$$

take the natural logarithms of both sides to obtain

$$\ln(Y) = \ln(B) + M\ln(X).$$

If one plots $\ln(Y)$ on the vertical and $\ln(X)$ on the horizontal (a "log-log" graph), then the graph of this function yields a straight line with slope $M$ and intercept $\ln(B)$.

To linearize the *exponential relationship*,

$$Y = Be^{MX},$$

take the logarithm of both sides to obtain

$$\ln(Y) = \ln(B) + MX,$$

and again a graph of this function yields a straight line graph. If one plots $\ln(Y)$ on the vertical and $X$ on the horizontal (a "semi-log" graph), then the graph of this function yields a straight line with slope $M$ and intercept $\ln(B)$.