NSC 220 Lab Manual

Thomas More College, Anything Physics

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Latest update: January 14, 2018

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Edition: v 1.0

Website: TMC Physics

Acknowledgements

We would like to acknowledge the following reviewers and users for their helpful comments and suggestions.

- Tom Neal, Physics Adjunct
- Dr. Jeremy Huber, Physics Sabbatical Replacement

I would also like to thank Robert A. Beezer and David Farmer for their hard work and guidance with the MathBookXML / PreTeXt format.

Preface

This text is intended for a one-semester undergraduate course in conceptual physics, with a minimum of algebraic skills.

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A full PDF version of this document can be found at anything-lab.pdf (659 kB)

Contents

Lab 1

Meaningful Measurements

Experimental Objectives

- Determine the material of the objects by calculating their density and matching it to the accepted values for various common materials.

Introduction

Physics is a science which is based on precise measurements of the seven fundamental physical quantities, three of which are: time (in seconds), length (in meters) and mass (in kilograms); and all of these measurements have an experimental uncertainty associated with them. It is very important for the experimenter to estimate these experimental uncertainties for every measurement taken. There are three factors that must be taken into account when estimating the uncertainty of a measurement:

1. statistical variations in the measurements,
2. using one-half of the smallest division on the measurement instrument,
3. any mechanical motions of the apparatus.

Physicists study the physical relationships between these defined fundamental quantities and usually give a name to the newly derived physical quantity. These derived physical quantities have units which are combinations of the units of the fundamental ones. For example, the product of the lengths (in meters, m) of the three sides of a cube is called volume and has units of m^3. The ratio of mass to volume is called density and has units of $\frac{\mathrm{kg}}{\mathrm{m}^3}$. The concepts of volume and density are therefore derived from the fundamental physical quantities, rather than fundamental themselves.

1.1 Student Outcomes

Knowledge Developed: In this exercise, students should learn how to make precise and accurate length measurements with a meter stick and two types of calipers, how to read a vernier scale, and how to estimate uncertainty in a measurement. Students will make use of the relationship of the fundamental properties of mass and length to the derived concepts of volume and density.

Skills Developed:

- Proper use of a vernier caliper and scale
- Proper use of a micrometer caliper
- Evaluating and propagating uncertainties

1.2 Equipment

1.2.1 The Vernier Caliper

The vernier scale was invented by Pierre Vernier in 1631. This scale has the advantage of enabling the user to determine one additional significant figure of precision over that of a straight ruler.

For example, this eliminates the need for estimating to the tenth of a millimeter on the metric ruler. The vernier caliper, shown in Figure 1.2.1, can measure distances using three different parts of the caliper: outside diameters (large jaws), inside diameters (small jaws), and depths (probe). You should locate these three places on your caliper. The vernier device consists of the main scale and a movable vernier scale. The fraction of a millimeter can be read off the vernier scale by choosing the mark on the vernier scale which best aligns with a mark on the main scale.

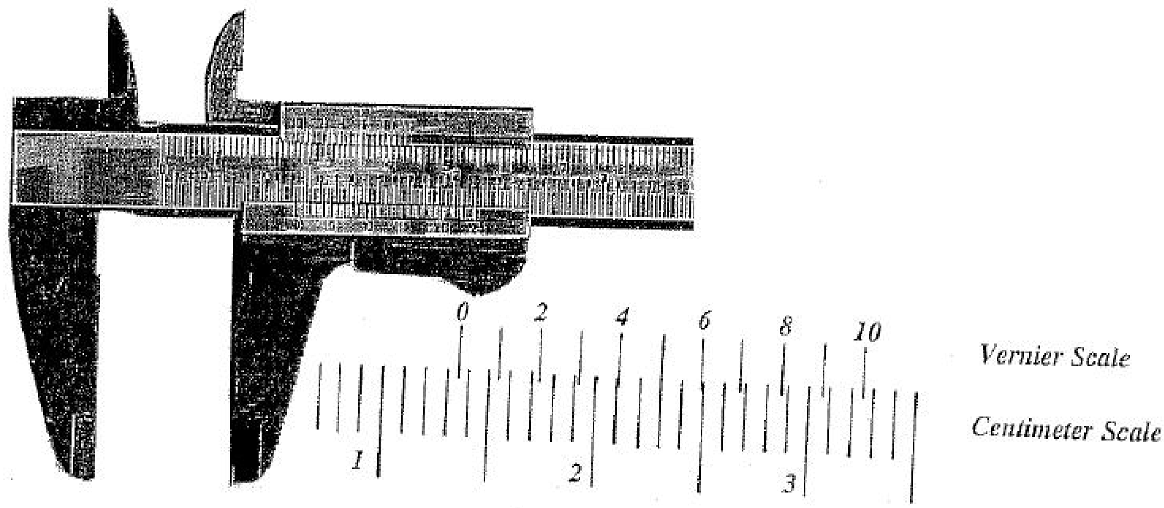

Figure 1.2.1: The location of the zero on the vernier scale tells you where to read the centimeter scale (1.3 cm). The vernier-scale line that lines up tells you the next digit (5). This picture measures $1.35 \pm 0.01\,\text{cm}$ because we can distinguish 1.35 from 1.34 and 1.36, but we cannot gauge the result any more precisely.

To move the vernier scale relative to the main scale press down on the thumb-lock, this releases the lock and then move the vernier scale. Do not try to move the vernier scale without releasing the lock.

1.2.2 The Micrometer Caliper

A micrometer caliper is shown in Figure 1.2.2. This instrument is used for the precise length measurement of a small object. The object is placed with care between the anvil and the rod. It is very important to not tighten down on the object with a vise-like grip. Tightening with force will **decalibrate** the micrometer (causing a zero-point error). The rotating cylinder moves the rod, opening or closing the rod onto the object. There is a rachet, at the far end, for taking up the slack distance between the anvil, the object and the rod, so again do not over-tighten with the rotating cylinder. The linear dimension of the object can be read from the scale. Rotating the cylinder one revolution moves the rod 0.5 millimeters. The rotating cylinder has 50 marks on it. Read the mark on the rotating cylinder that aligns with the central line on the main scale.

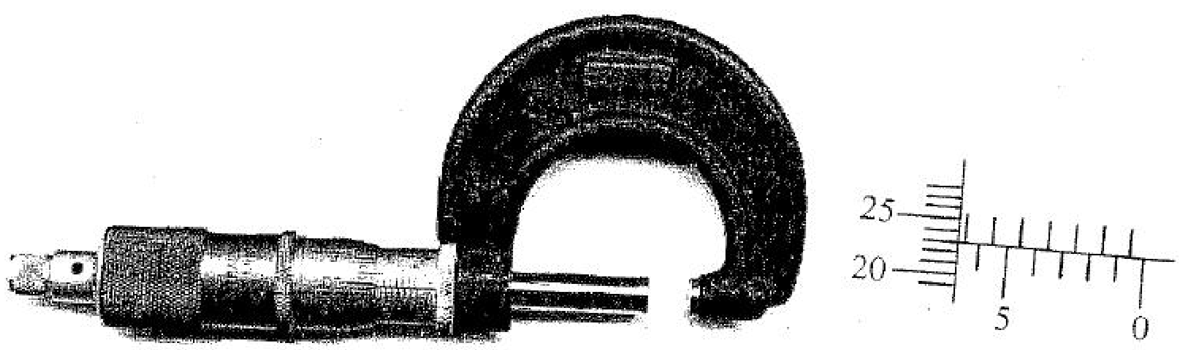

Figure 1.2.2: Notice on the coarse scale, that the lower lines read (1, 2, 3, ... 6 in this picture) and the higher lines read the half-marks (0.5, 1.5, 2.5, ... 6.5 in this picture). The location of the turning dial tells you where to read the coarse scale (6.5 mm). The center line of the coarse scale tells you where to read the fine scale. This is 23.0 (in units of $\times 10^{-2}$ mm), but not 23.5 and not 22.5 so the precision is 0.5 (in these units). This measurement in mm reads $6.5\,\text{mm} + 0.230\,\text{mm} = 6.730 \pm 0.005\,\text{mm}$. Since each mark corresponds to 0.01 mm and you can probably gauge a distance about half-way between the lines, the precision of this instrument is 0.005 mm.

The reading of the micrometer from Figure 1.2.2 is $6.730 \pm .005\,\text{mm}$.

1.3 Procedure

Materials: Three measuring devices: a metric ruler, a vernier caliper, and a micrometer caliper.
Several objects convenient for measuring the mass and the physical dimensions.
Procedure:

- Check the measuring devices for any **zero-point errors** (verify that they are **calibrated**).[1] The use of the caliper and micrometer are outlined below.

Exercise 1.3.1 (Zero-Point Errors)**.** You should verify the calibration of your instruments. Determine if there are the zero-point errors for each of the measuring instrument.

Hint 1. You might re-read the description of systematic error to remind yourself what a **zero-point error** is.

Hint 2. Is the meterstick rough on the end? Is the edge marked as zero actually at the zero value? Can you think of a way to ensure that the condition of the meterstick does not impact the measurement of length? How does the distance from zero to five compare to the distance from one to six?

Hint 3. Does the caliper read zero when it is measuring a zero length?
You should close the caliper to determine this.

Hint 4. Does the micrometer read zero when it is measuring a zero length?
You should close the micrometer to determine this.

Answer. You should make efforts to correct for any zero-point errors in your instrument. Especially in this week, your report should explain how you accommodated any zero-point errors.

[1] The instruments should read *exactly zero* when they are measuring zero — that is to say, when they are closed. If, for example, the point at which you should be measuring zero is actually labelled 2, then you need to subtract that 2 from every measurement.

Solution. You should figure out how to make efforts to correct for zero-point errors.

For each instrument, if the measurement you consider to have zero-length is not actually "zero", then you can handle that in the same way that you know the distance from 1 to 6 (in this case, subtract 1 from any measurement you make) or from 2 to 7 (in this case, subtract 2 from any measurement you make) or from 10 to 15 (in this case, subtract 10 from any measurement you make) .

You will also need to concern yourself with whether this uncertainty is **systematic** (always making the answer slightly too big or always slightly too small) or **random** (sometimes making the answer slightly too big and sometimes slightly too small). (Recall Subsection B.2.1.)

- There are also several solids available: a cylinder, a cube, and a sphere.
  - Measure the dimensions of two of the objects with each of the three instruments: the ruler, the vernier caliper, and the micrometer caliper. Take all measurements minimizing any parallax errors.[2] Do Exercise 1.3.2.
  - Estimate the experimental uncertainties of your measurements. Do Exercise 1.3.3.
  - Repeat the measurements at several positions and orientations around the object, compute the average and the relative uncertainty.

Exercise 1.3.2 (Parallax)**.** Determine if the value you are reading depends on the location of your eye.

Hint. For the caliper and the micrometer, the instrument clamps around the item being measured. Decide if the location of your eye matters in the measurement.

For the meterstick, you have to align the edge with a tick-mark. Does the location of your eye impact the alignment of the tick-mark on the ruler with the edge of the object?

Answer. You should make efforts to correct for any zero-point errors in your instrument. Especially in this week, your report should explain how you accommodated any parallax errors.

Solution. You should figure out how to make efforts to correct any parallax errors. It may help to determine whether this uncertainty is **systematic** (always making the answer slightly too big or always slightly too small) or **random** (sometimes making the answer slightly too big and sometimes slightly too small). (Recall Subsection B.2.1.)

For each instrument, if the measurement does depend on the location of your eye, then you might try measuring it multiple times with your head in different locations each time to gauge the size of this uncertainty.

In the case of the meterstick, the object being measured should be as close as possible (touching?) the tick-marks of the meterstick in order to minimize parallax.

Exercise 1.3.3 (Random or Systematic)**.** When you measure the diameter of a sphere, it might be difficult to get the caliper precisely at the full diameter. If you are off, then you will necessarily be measuring a smaller value. This is a systematic error that can be corrected for. Since any mistake necessarily gives a value that is too small, then measuring it multiple times and finding the largest value will minimize this uncertainty.

On the other hand, if you measure an egg, you would not expect the diameter to be the same. Clearly for an egg, there is no reasonable single value to use as The Diameter. In this case, the question of finding the diameter does not make sense, and by measuring in a systematic pattern, you can determine that the shape is not spherical.

Determine if the zero-point error and the parallax error are random or systematic.

Hint 1. If a meterstick is worn down at the zero-value, then is it more likely to be measuring too short or too long? Is that systematic or random?

Hint 2. For a measurement affected by parallax, is that systematic or random?

Solution. Since random errors might give a result too big or too small, measuring many times and averaging should minimize these errors.

Since systematic errors tend to be either too big or too small (in a predictable or explainable way), you should track the uncertainties and recognize if this measurement causes your result to be more likely too big or more likely too small.

[2] Parallax errors occur when you observed something from an angle rather than exactly straight-on. For example, when you pour water into a measuring cup that is sitting on the counter while you are standing next to the counter looking down at the measuring cup. There are ways to avoid this, but each instrument has its own solution. Your instructor should help you determine the best way to make a measurement.

- As outlined in the Analysis, compute the volume and the density.

1.4 Analysis

- After finding the relevant dimensions of the object, calculate the volume of the object three times: once using the measurements from the ruler, once from the caliper, and once with the micrometer.
  - The volume of a rectangular block is $V = lwh$ (length times width times height). You need to arbitrarily choose which dimension is which.
  - The volume of a cylinder is $V = \pi r^2 h = \frac{\pi D^2 h}{4}$ (the cross-sectional area times the height). Technically the formula is in terms of the radius, but it is more accurate to measure the diameter.
  - The volume of a sphere is $V = \frac{4}{3}\pi R^3$.
- Using the rules of propagation of uncertainty, compute the uncertainty in the volume for each.

Exercise 1.4.1. Which instrument is the most precise?

Hint. For which instrument can you measure to the most decimal places?

Answer. Your data should give you this answer. Your report should indicate how your data tells you the answer to this question.

- Measure the volume directly with the graduated cylinder.

Exercise 1.4.2. Are any of the volume measurements inconsistent (See Note B.1.1 about comparing values)?

What can you infer about the accuracy of these instruments?

Using the most precise indirect measurement of volume (those calculated from other measurements), calculate a percent-difference with the direct measurement of volume.

Hint 1. When you measure the volume of (let's say the cylinder) with a meterstick, with a caliper, with a micrometer, and with a graduated cylinder, they are all measuring the volume of the same object, which does not change volume. You *expect* these to all give the same number. The question is whether or not *your data* do actually give the "same" values.

Hint 2. Since you are using measurement techniques that have different precision, you will have different ranges of uncertainty. Numbers are considered to be "the same" when their uncertainty ranges overlap.

Hint 3. Keep in mind that "imprecise" means "a large range in the uncertainty", whereas "inaccurate" means "inconsistent with the true value". (You might not know the true value.)

Answer. It is possible that your data do not give consistent results for the volume. You should notice if one result in particular is different than the others and then speculate on why *that* measurement is different. If all of your results are inconsistent with each other, then you might want to check your measurements. If they are again inconsistent, then you should check your results against a friend or the instructor.

- Measure the mass and then, using the overall most precise measurement of volume, compute the density with its uncertainty.
- Using your best density value, find the percent-error against the appropriate value given by the text, or the *Handbook of Physics & Chemistry*.
- Other considerations that might help with your analysis:

Exercise 1.4.3. What would be the best method to measure the volume of an irregularly shaped object? Why?

Hint. To answer this, it might help to think about *how* you would measure something that is irregularly shaped with each of the instruments you used today. Is one of them particularly good at conforming to the shape of an irregularly shaped object?

Answer. You should recognize which of the following objects (a meterstick, a caliper, a micrometer, and water) can touch all edges of an irregularly shaped object simultaneously. On the other hand, if you are measuring the volume of something that is water-soluble, then you might want to reconsider your reasoning process.

1.5 Your Report

Since this is the first lab, we are not going to require you to do a full lab report as outlined in Writing a Lab Report.

For this week, please include

- your identifying information (listed above the abstract)
- Abstract: (write this after you've written everything else, but place it at the beginning of the report)
- Apparatus: please comment briefly on the use of the caliper and the micrometer and compare the precision of the ruler, caliper, and micrometer.
- (we are skipping the Theory this week)
- (we are skipping the Procedure this week)
- Data: Please organize your data into a clear table. (We can show you how to do this in Excel during class.)
- Analysis: There are no graphs for this week. Your discussion should include the following concepts:
  - Please point out which numbers (from the data) indicate the precision of each instrument.
  - For one object for which you computed a volume multiple times using different devices, indicate
    - if the results are consistent with each other
    - which result is the most precise
    - and if you think any one measurement is more accurate (or more trustworthy) than the others and why
  - For the object that you computed the density of, describe if the value your computed is consistent with the value associated with the material from which you think it is made. (You might need to compare your result with several materials, but you should not necessarily draw a conclusion about which it is here.)
- Conclusion: By referencing (rather than repeating) your Analysis, make a statement about each of the following:
  - which device is the most precise?
  - for the volume considered in your Analysis, what do you think the true value of the volume is and with what precision do you trust this result?
  - based on the comparison made in the Analysis, draw a conclusion about what the material is.

(Revised: Jan 9, 2018)
A PDF version might be found at measurement.pdf (331 kB)

Lab 2

Measuring Motion

Experimental Objectives

- By measuring various quantities with special attention to the precision of the instruments,
  - Predict the length of the hallway,
  - Predict the height of a lab table, and
  - Find a hidden treasure.

Last week, you learned about the uncertainty in some measuring equipment. This week you will build a more visceral understanding of these ideas and test the concepts of consistency and reproducibility. Throughout this experiment, you should bear in mind that "more data is better".

2.1 Procedure

There are three sections to the experiment; in each you will standardize a technique, use the technique to make a measurement (paying close attention to the variation within your group), and then verify that measurement against other groups.

Activity 2.1.1.

(a) ***Calibrate your paces — Do this first***

(i) Each person in the group should do this 3-5 times to check your consistency.

- Place your heel at an identifiable origin. See Hint 1.
- Take five paces in as natural and calm a manner as you are able. See Hint 2.
- Measure the distance from your first heel to your fifth heel. See Hint 3.
- Note the 5-step distance and variation. Use the average value of these measurements as your personal "standard step", include an uncertainty such that when added to or subtracted from the average, it encapsulates all of your measurements. Divide the average and the uncertainty by five: This is your "conversion to metric" per step. You can now measure distances in paces.
- Note Analysis Item 1.

Hint 1 (Set your origin). Place your heel against a wall or at the crack of a floor tile or on the edge of a piece of masking tape.

Hint 2 (Set your scale). Since you will be self-conscious, it will be difficult to be consistent. You might need to practice "being casual".

Hint 3 (Counting to five). Remember that you are measuring five paces. This means that your first foot (the one at the origin) is "zero" so that your first *step* is "one".

(b) ***Do these three in any order***

(i) Pace the Hallway:

- Measure the length of the hallway from end to end by pacing it off. Each person in the group should do this three times and should gauge the final fraction of a pace.
- As with the calibration, each person should average their number of paces and indicate their uncertainty. Convert paces to meters, with uncertainty.
- Using the distance in meters from each person in the group, average these values and provide two measures of uncertainty:
    - *your conservative estimate*: an uncertainty that encompasses the maximum and minimum of all group members (the largest variation in the measurements), and
    - *your "best" estimate*: an uncertainty that actually reflects the range that you believe the result to be within.
- Submit these answers to the instructor. Every individual who's group-measurement is consistent with the right answer (has the correct value within their uncertainty) gets 2 pts added to their lab report grades. The individuals in the group that is not only consistent with the right answer but also has the *smallest* relative uncertainty gets 5 extra points on their reports.
- Note Analysis Item 2.

(ii) ***Do this one group at a time.*** Create a treasure map: You are going to place a collection of pennies some place in an open area in the grass and create a "treasure map" to the location based on the number of paces. Your written instructions should be intended to help somebody actually find the pennies.

- Take 5 pennies (take one more penny than there are groups in the class), a pad of paper, and a pen and go *outside*, up the stairs, to the large flat grassy area (the "quad").
- Identify an obvious starting location (the origin) and get your bearings. Assume that while you are in the quad, the science building is "north", the cafeteria is "south", the back of the theatre/library is "west", and the large open space (the Five Seasons) is "east".
- Secretly choose which lab-partner will be pacing off the steps and do not tell any other group *which* person it was.
- On your paper, write down your origin, choose a cardinal direction (north, south, east, or west), and give specific clear instructions for how far to walk and in what direction. Your instructions must have at least one right-angle turn, but not more than three right-angle turns. All turns must be right-angles.
- When you return to lab, give your instructions to the professor, who will make copies and distribute them.
- Note Analysis Item 3.

Note that at the end of lab, you will need to be able to find your pennies. Each person in your group will lose one point on their lab report for each penny lost.

(iii) Using vector components, gauge the "area of uncertainty" when stepping 5 ± 1 tiles over and 15 ± 3 tiles up.

- Find a large open space on the floor of the lab room. It should be at least 10-tiles by 20-tiles. You will also need a "two-meter-stick" and a protractor.
- Select one tile as the origin. Place a piece of tape at its "bottom-left" corner. Write "origin" on the tape.
    - From this location, count five tiles to the right and fifteen tiles up.
    - Mark the bottom-left corner of that tile with a piece of tape. Write "expected" on the tape.
    - Measure the distance from the origin to the expected location using the meter-stick. (In units of tiles, this should be $\sqrt{(5\,\text{tiles})^2 + (15\,\text{tiles})^2} = 15.8\,\text{tiles}$.)
    - While the meter-stick is aiming from the origin towards the expected location, use the protractor to measure the angle from the horizontal (what you called "to the right" when you counted five tiles over).

- To account for the uncertainty, consider the smallest value you might be off by repeating those steps by counting over four $(5-1)$ and up twelve $(15-3)$.
  - Mark the bottom-left corner of that tile with a piece of tape. Write "short" on the tape.
  - Measure the distance from the origin to the short location using the meter-stick. (In units of tiles, this should be $\sqrt{(4\,\text{tiles})^2 + (12\,\text{tiles})^2} = 12.7\,\text{tiles}$.)
- To account for the uncertainty, consider the largest value you might be off by repeating those steps by counting over six $(5+1)$ and up eighteen $(15+3)$.
  - Mark the bottom-left corner of that tile with a piece of tape. Write "long" on the tape.
  - Measure the distance from the origin to the long location using the meter-stick. (In units of tiles, this should be $\sqrt{(6\,\text{tiles})^2 + (18\,\text{tiles})^2} = 18.9\,\text{tiles}$.)
- We might also make mixed errors where one is too large but the other is too small.
  - Repeat these steps but this time go over six $(5+1)$ and up twelve $(15-3)$.
    - Mark the bottom-left corner of that tile with a piece of tape. Write "theta 1 (θ_1)" on the tape.
    - While the meter-stick is aiming from the origin towards the theta-1 location, use the protractor to measure the angle from the horizontal (what you called "to the right" when you counted five tiles over).
  - Repeat these steps but this time go over four $(5-1)$ and up eighteen $(15+3)$.
    - Mark the bottom-left corner of that tile with a piece of tape. Write "theta 2 (θ_1)" on the tape.
    - While the meter-stick is aiming from the origin towards the theta-2 location, use the protractor to measure the angle from the horizontal (what you called "to the right" when you counted five tiles over).
- Remove all of your tape from the floor.
- Note Analysis Item 4.

If you finish these three exercises before the rest of the class, then you can move on to the next portion; but you cannot do that final experiment until everybody has created their treasure map.

(c) ***If you have time between experiments***, turn on the computer and the PASCO interface on the lab table, log in, and open the Capstone program on the desktop. Ask your instructor about the motion sensor and create a velocity versus time graph for: walking slowly away, quickly away, slowly towards, and quickly towards the motion sensor.

(d) Once all groups have created a treasure map, your instructor will distribute two maps to each group.

- One group at a time will go outside, following each map as best they can, and collect one penny from the treasure as evidence of success.
- After all groups have discovered some treasure, all groups will go out and follow their map to collect the remains of their treasure.
- Note Analysis Item 4.

For each map followed, the group should indicate if their pace-measurement differed from the map and by how much.

2.2 Analysis

Please consider the following for each of the tasks in Activity 2.1.1.

1. In Task 2.1.1.a, if you were to take 35 paces, you would multiply your distance-per-pace times the number of paces, but you would also multiple the uncertainty times the number of paces. For each person in the group, find the distance and uncertainty (in meters) if that person were to take 35 paces. You should notice that the uncertainty increases with the number of paces. This is also the premise of Task 2.1.1.b.iii.

2. In Task 2.1.1.b.i, your report should show how you chose your conservative estimate of uncertainty and how you chose your best estimate. These might be the same. If they are different, you should indicate why you feel your best estimate is better than the conservative estimate.

3. In Task 2.1.1.b.ii, your report should indicate how you chose the person who set the paces for the treasure map. Did you choose somebody with a peculiarly large or small pace? Did you select at random? You should also indicate any sources of uncertainty in creating the map. Comment on your pace length if you went up or down a hill. Did you have multiple people pace the path to check the values? If there was snow, comment on the effect and if you implemented any strategies.

4. The evaluation of the area in Task 2.1.1.b.iii should help you gauge what to do if your treasure map does not lead to an actual treasure (Task 2.1.1.d). Your report should include a calculation of the area in the region found. It should also indicate if the members in your group are exceptionally long-legged or short-legged compared to the other groups. Describe how this affected the way that you went about searching for the treasure.

2.3 Your Report

Since this is the first lab, we are not going to require you to do a full lab report as outlined in Writing a Lab Report.

For this week, please include

- your identifying information (listed above the abstract)
- Abstract: (write this after you've written everything else, but place it at the beginning of the report) Use the treasure hunt as the primary objective of this experiment and consider the other portions of this lab as mechanisms for calibrating your paces.
- (You can skip the Apparatus section this week.)
- (You can skip the Theory section this week.)
- Procedure: Please describe what you did and how; this should not be too detailed, but should give a reasonable picture. That is to say, this is a general description of the process, not detailed instructions. Note how you calibrated your measurements and minimized zero-point errors (as appropriate).
- Data: Please organize your data into a clear table or set of tables.
  - For Task 2.1.1.a your table should clearly indicate your calibration, but should also indicate the others in your group for comparison.
  - For Task 2.1.1.b.i you should include enough information to indicate how you found the results you turned in as the length of the hallway.
  - For Task 2.1.1.b.iii draw a picture of the layout and include relevant distances and angles.
  - Please include your treasure map as well as those you followed. These should be clearly labelled.
- Analysis: There are no graphs for this week. Your discussion should include a discussion regarding the points mentioned in the analysis section. This should be organized into paragraphs, each of which address one aspect of the experiment. In each paragraph, you should comment on any sources of uncertainty that you needed to worry about.
- Conclusion: This should be a few statements about how some piece of data or some portion of the analysis allowed you to verify or not verify a particular item. Do not simply answer the following questions, but use the ideas expressed by the questions as a guide for what you should discuss in a more narrative format. Since there are three topics listed, you should expect to write three short paragraphs.
  - Was your group able to accurately predict the length of the hallway? How did the sources of uncertainty mentioned in the Analysis enable or interfere with this?

- Were you able to follow other the treasure maps of the other groups? How did the sources of uncertainty mentioned in the Analysis enable or interfere with this?
- Were other groups able to follow your treasure map? What were the relevant issues (good or bad)?

Your Procedure and Analysis are probably the longest sections this week. The Conclusion should always be shorter than the analysis.

(Revised: Jan 6, 2018)
A PDF version might be found at motion.pdf (128 kB)

Lab 3

Constant Acceleration

Experimental Objectives

- Using position versus time and velocity versus time graphs, verify
  - that the equations of constant acceleration accurately describe the behavior of objects under constant acceleration and
  - that it is possible to distinguish acceleration due to gravity from acceleration due to friction.

3.1 Student Outcomes

Knowledge developed: In this exercise, the student should develop an understanding of the relationships between the position and the instantaneous velocity of an object, as well as how each of these can vary as functions of time. We will only consider the special case where the object experiences constant acceleration.

Skills developed:

- Evaluate the data for sources of uncertainty. Can you see an effect, such as a level track or the presence of friction, in the result?
- Using Pasco Capstone software
- Interpreting the slope and intercept of graphs

3.2 Procedure

Materials: An aluminum track, a low-friction cart, computer interface with PASCO Capstone^{tm} software, a sonic motion sensor, a small steel ball.

You should notice that the subsections in this section parallel the subsections in Analysis.

3.2.1 Cart and Flat Track

> **Note** The Pasco Capstone interface is also used in Lab 4, Lab 5, and Labs 6–7. You should start becoming familiar with the hardware and software.

Log into the computer (so you can save your data to your network drive) and then open Pasco Capstone. (Appendix D will provide some instructions for setting up the software and connecting the equipment.) Connect the motion sensor to the computer interface. Set the data rate of the motion sensor at 50 Hz. Place a steel ball on the track and adjust the leveling screw at one end of the track to see if the ball rolls one way or the other. This will roughly level your track. Place the sensor about 20 cm from the end of the track, because this is the minimum distance detected by the sensor. (You might need to use the “sail” for the sensor to see the cart.)

Place the cart on the track. Capstone, via the sonic ranger, can measure the position and velocity of the cart as a function of time. (This is explained in Appendix D.)

Exercise 3.2.1. Assume the track is frictionless and predict how the cart will move if the track is not perfectly level; include a comment about how the velocity versus time graph will look when it goes uphill versus when it goes downhill. Should these be the same?

Hint 1. If the track is not level, sending the cart in one direction, it will be going uphill; but in the other direction it will be going downhill. If you measure the motion in both directions, you should be able to see the difference.

Hint 2. You may still have an effect due to friction. (See Exercise 3.2.3.)

Hint 3. It is probably useful to describe this using terms such as “speeding up” or “slowing down”. You may also want to practice describing this by comparing how the direction of the acceleration compares to the direction of the velocity.

Exercise 3.2.2. What do you expect the graph to look like if the track *is* perfectly level? Will it be the same going left versus going right?

Hint 1. If the track is level and then you compare the motion of the cart in one direction versus another, you should be able to see if there is a difference. You should also be able to predict how the motion in each direction for this case is similar or different from the case when the track is not level.

Hint 2. You may still have an effect due to friction. (See Exercise 3.2.3.)

Hint 3. It is probably useful to describe this using terms such as “speeding up” or “slowing down”. You may also want to practice describing this by comparing how the direction of the acceleration compares to the direction of the velocity.

Exercise 3.2.3. Now, assuming it is perfectly level, what will friction do to the motion? How do you expect this to affect the graphs?

Hint 1. If the track is level and there is friction, then when you compare the motion of the cart in one direction versus another, you should be able to see if there is a difference. You should also be able to predict how the motion in each direction for this case is similar or different from the case when the track is not level.

Hint 2. If the track is tilted with no friction then describe the motion in each direction using the phrases “speeding up” or “slowing down”.

If the track has friction with no tilt then describe the motion in each direction using the phrases “speeding up” or “slowing down”. You should be able to indicate how you would see each effect in the graphs of the motion.

We will take four sets of data: a slow, constant velocity towards the ranger; a slow, constant velocity away from the ranger; a faster, constant velocity towards the ranger; and a faster, constant velocity away from the ranger. The two slow speeds should be about the same and the two faster speeds should be about the same. For each case, start the sonic ranger and then bump the cart firmly, but not violently(!).

On Capstone, you should have four curves of velocity versus time. Fit each with a trendline and display the equation of the trendline on the screen. Interpret the coefficients (slope and intercept) by noting their units, values, and uncertainties. You should also print out (in landscape mode) the position versus time graph, the velocity versus time graph, and the acceleration versus time graph. (You should notice that the acceleration versus time graph is *very* noisy.)

3.2.2 Cart and Sloped Track

Place a small block under one end of the track, so that the track is now tilted at a small angle with the sensor at the top of the incline. Measure the angle using a protractor or calculate it by measuring the two legs of the triangle and using the inverse sine. (Be careful about measuring the height.)

We will consider *three cases* for the sloped track: *First*, allow the cart to roll (without an initial push) down the ramp. *Second*, gently push the cart down the ramp. DON’T let it fly off or crash into anything.

Exercise 3.2.4. Should these two cases have the same acceleration while rolling down the ramp? How will that affect the shape of the velocity versus time graphs?

Hint. Do the graphs have the same slope?
Do the graphs have the same intercept?

Exercise 3.2.5. Should these have the same initial velocity? How will that affect the graphs?

Hint. Do the graphs have the same slope?
Do the graphs have the same intercept?

In the *third* case, start the cart at the bottom of the incline and roll it up the ramp, allowing it to roll back down on its own. Push it hard enough to get mostly up the ramp, but not so hard that it hits the sonic ranger at the top of the incline, because we want to watch it return to the bottom of the ramp. *This case is similar to throwing a ball into the air and allowing it to fall back down.*

Exercise 3.2.6. Should this case have the same acceleration while it goes up the ramp as while it goes down the ramp? How can we see that on the velocity versus time graphs?

Hint. If the track is tilted with no friction then describe the motion in each direction using the phrases "speeding up" or "slowing down".

If the track has friction with no tilt then describe the motion in each direction using the phrases "speeding up" or "slowing down".

In this case, there may be tilt *and* friction. You should be able to indicate how you would see each effect in the graphs of the motion.

Exercise 3.2.7. Should this case have the same acceleration (either while it goes up the ramp or while it goes down the ramp) as the previous two cases of rolling down the ramp?

In Capstone, you should be able to display all three graphs (position v time, velocity v time, and acceleration v time). You should also be able to display all three cases of data on each of these graphs. On the velocity versus time graph, fit each of the three graphs with a linear trendline. The next section will ask you to analyze how well the data match up to these lines. (It might be interesting to also fit the position vs time curves to parabolas. Be sure to print out copies of your three graphs.

Your lab should note the following results and explain their meaning: slope and y-intercept, the uncertainties (precision) in both the slope and intercept, and the r value (correlation coefficient).

3.3 Analysis

You should notice that the subsections in this section parallel the subsections in Procedure.

3.3.1 Cart and Flat Track

Based on the results of Subsection 3.2.1, write a short analysis of the relationship between these two graphs (x and v versus time). From the velocity versus time graph (specifically from the trendline) determine the value of the acceleration of the cart down the track; be sure to include the uncertainty of the acceleration and the units.

Exercise 3.3.1. Do you see any evidence that the track was not perfectly level?

Exercise 3.3.2. Do you see any evidence that there is any friction as the cart moves along the track?

Exercise 3.3.3. What does the intercept of the velocity versus time graph tell you?

Exercise 3.3.4. If the slopes are different, then discuss any pattern that you see. If the slopes are (essentially) the same, then find an average and a standard deviation of the four values.

Exercise 3.3.5. Does the speed of the cart affect the slope of the velocity vs time graph?

Discuss any evidence observed in your data when answering these questions. Also consider the magnitude of the uncertainties when writing your conclusions.

3.3.2 Cart and Sloped Track

Based on the results of Subsection 3.2.2, write an analysis of the relationship between the two graphs (x and v versus time). From the velocity versus time graph determine the value of the acceleration of the cart down the track.

Exercise 3.3.6. For the two downhill cases, use your uncertainty analysis to determine if the acceleration of the cart changed when it was given a small push.

Exercise 3.3.7. Is there an accuracy that can be computed for this part of the experiment?

Hint. If the track were frictionless, then the acceleration should be $a = (9.81\ \mathrm{m/s^2})(\sin\theta)$, where θ is the angle that the incline makes.

Inspect the line/curve that is defined by the data on the Distance traveled vs. time graph.

Exercise 3.3.8. What is its shape? Is the shape of the graph what you would expect for constant acceleration (straight line, parabola, etc.)? Explain your reasoning.

Exercise 3.3.9. Consider the trendline that you added. Does/should the trendline line go through the origin? What is the value of y-intercept of the X vs T graph? What physical quantity does the intercept represent? Explain why it has that value.

Hint. Think about where the sensor was located.

Exercise 3.3.10. What does the slope (whether it's constant or not) of the line on this graph signify?

Now consider the Instantaneous Velocity vs. Time graph.

Exercise 3.3.11. Does the curve/line on this graph have the shape you would expect for an object undergoing constant acceleration? Explain.

Exercise 3.3.12. What was the value of the y intercept on this graph (include units and uncertainty!)? Explain its significance. To what does it refer?

Hint. Think carefully about what you plotted on the X-axis!

(Revised: September 13, 2017)
A PDF version might be found at acceleration.pdf (115 kB)

Lab 4

Newton's 2^{nd} Law on a Linear Track with the Sonic Ranger

Experimental Objectives

In this experiment, we will assume that Newton's first law is true and focus on Newton's second law.

- By measuring
  - the velocity versus time for a cart being pulled down a track and
  - the applied force that is pulling it,

  we can plot the acceleration versus the force and verify the validity of Newton's second law of motion: $\vec{F}_{\text{net}} = m\vec{a}$.

Introduction to Forces

Forces are related to the natural motion of bodies, where one object can affect the motion of another object. That is, forces are interactions between objects affecting their motion. Although the famous Greek philosopher Aristotle claimed that a force was necessary to *maintain* any motion, careful analysis by Italian physicist Galileo Galilei in the mid-17^{th} century and by Sir Isaac Newton, a British mathematician and physicist (1642-1727), eventually distinguished the effects of friction and allowed Newton to create a mathematically consistent theory of motion. These concepts were published in Newton's book "Mathematical Principles of Natural Philosophy" in 1687, for which (among other accomplishments) Newton is regarded as one of the greatest scientists of all time.

All forces can be placed in one of two main categories. First, there are natural (or fundamental) forces like the gravitational force, the electromagnetic force, or the nuclear forces. The gravitational force is a force on a body by another body (like the Earth), this force is an interaction between their two masses. The electromagnetic force is an interaction between the charges of two bodies. These forces may act on an object without any direct physical contact between the two bodies. This type of force is sometimes called an "action at a distance" force. All other forces are in a second category called "contact forces."

Newton's First Law:

> If there are no forces acting, then objects will remain at rest or, if not at rest, will maintain their velocity.

If this is true, then we can study the forces acting on a body based on the motion of the body, specifically through the change in the velocity of an object.

Newton's Second Law: Not only is a force necessary to change the motion (to cause an acceleration), the amount of acceleration that a force causes is predictable and is inversely proportional to the mass. The same sized force causes a small mass to accelerate a lot and a large mass to accelerate a little. This is expressed by the equation:

$$\vec{F}_{\text{net}} = m\vec{a}.$$

The net force, $\vec{F}_{\text{net}}$, is the vector sum of all forces acting on an object. If we have an extended object (such as a weight hanging off of a table, but connected to a cart that is on the table), then we need only consider forces that are "external" to the system: So long as both objects accelerate at the same rate, we do not need to consider the "internal" tension that the string exerts between the connected bodies.

Newton's Third Law: Inherent in the description of a force is that it is an interaction between objects: there must always be two objects that interact. These objects exert equal and opposite forces on each other. That is,

> If there is a force exerted on object 1 by object 2, then there is necessarily and simultaneously a force exerted on object 2 by object 1 that is equal (in magnitude) and opposite (in direction) to the original force.

Remember that these two forces are on different objects and that the two bodies in direct contact exert forces on each other. Remember then that if there is contact between the object (any part of the system) and anything else then there is an outside force on the object (system) and that if there is no contact (the two bodies break contact) then there is no force.

4.1 Pre-Lab Considerations

- Based on your understanding of Subsection 4.3.1, draw a free-body force diagram for the cart and for the hanging mass.
- You should be prepared to derive an equation for the acceleration of the system, in terms of, the mass of the cart and the hanging mass, while assuming that the cart has no friction with the track. (Hint: There is only one force accelerating the system.)

4.2 Student Outcomes

Knowledge Developed: In this exercise, students should learn how forces are related to the motion of a cart, how to use a free-body diagram, and gain a visceral understanding of Newton's second law.

Skills Developed:

- Evaluate the data for sources of uncertainty. Can you see an effect, such as a level track or the presence of friction, in the result?
- Using Pasco Capstone software
- Interpreting slope and intercept of graphs

4.3 Procedure

4.3.1 The Experimental Setup

Materials A low-friction linear track with a wheeled cart and a pulley at one end of the track. Weights that can ride in the cart without jostling. A string connecting the cart to a light-weight support for small masses, which sits over the pulley allowing the masses to fall vertically while pulling the cart horizontally. A "sonic ranger" that uses sonar to measure the position, velocity, and acceleration of the cart.

- A low-friction linear cart and track will be used, this reduces the friction between the cart and the track.

- A string will be connected to the cart and a known mass will be hanging from the end of the string (and over a pulley). The hanging mass will exert a constant horizontal force on the cart as the mass falls all the way to the floor. This gives a constant acceleration to the cart.
- The sonic motion sensor will be used to measure the position of the cart as a function of time.
- The carts and tracks need to be handled with care. Scratches can add friction to the system.

4.3.2 Procedure

- If the cart is given an initial push (without the hanging mass and string attached) then the cart should travel with a constant velocity down the horizontal track, if there are no other forces acting on the cart. Carry out a couple of constant velocity runs on the track, to check for the effects of friction and to see how level the track is. The track may need a level adjustment. Do runs in both directions. Maybe the track can be tilted so that the friction is countered by the tilt of the track.

  It might help to review Exercises 3.2.1, 3.2.2, and 3.2.3 from the Constant Acceleration lab.
- Connect a string to the cart and run it over a pulley. Measure the height of the string at both ends of the track, to ensure that the string is as level as the track.

Exercise 4.3.1. If the pulley has a very large wheel or is set so that the string is low at the cart, but high at the pulley, then the force pulling the cart is not horizontal. Draw the free-body diagram for the cart in this case. Comment on if this increases, decreases, or does not affect each of the other forces (F_{normal}, F_{gravity}, F_{friction}).

Hint 1. If the tension pulls up, then the normal force does not need to support all of the weight.

Hint 2. If the tension pulls slightly up, is there a convenient way to find then angle at which it pulls? Will that angle depend on how close the cart is to the pulley?

Answer. Based on the hints, decide how important it is for you to ensure that the string pulls horizontally.

- The hanging mass should be much less than the mass of the cart. Use a small plastic cup to hold the hanging masses. Measure the mass of this cup. The total mass of the system must be kept constant for all parts of the experiment. The hanging mass and the mass of the cart should vary, but their total must be kept constant, by moving small mass amounts from the cart to the hanging cup. Record the mass of the cart, the hanging cup mass, and the extra masses which are to be transferred from the cart to the cup.

Exercise 4.3.2. If the hanging mass is not "much less" than the mass of the cart, then the acceleration will be very large and the cart will move too quickly.

Your total mass should include the mass of the string because it is also being accelerated. As an interesting thought experiment, you might notice that the amount of string that hangs off the pulley is contributing to the mass of the basket (the amount pulling the cart). But this changes as the cart moves! Without using calculus it is impossible to include this consideration, so we *hope* this is a small effect. Do you have a way of ensuring that this is a small effect?

Hint 1. How many significant digits do you have in the mass of the cart? Is the mass of the basket large enough to be a *significant* effect in the overall mass?

Hint 2. How many significant digits do you have in the mass of the cart? Is the mass of the string large enough to be a *significant* effect in the overall mass?

Hint 3. What percentage of the total mass of the system is the mass of the string? What percentage of the mass of the cart is the mass of the string?

Answer. The effect of the mass of the string in the measured acceleration will be *insignificant*.

> **Note** The Pasco Capstone interface is also used in Lab 3, Lab 5, and Labs 6–7. You should start becoming familiar with the hardware and software.

- Take data with Capstone and the motion sensor as the cart travels with constant acceleration down the track. Determine the acceleration of the cart from a linear regression using the velocity vs time data (a linear fit line in Capstone). Record the acceleration value and its uncertainty.

- Collect 7 data runs, where about 2-5 grams[1] is transferred each time from the cart to the hanging mass. Determine the acceleration of the cart (and the uncertainty for the acceleration) for each of these 7 runs.

4.4 Analysis

You should note that velocity-versus-time graphs are only useful for computing the acceleration in each case. Once you have the values for the acceleration, your attention should be on the graph of acceleration-versus-weight.

- In Excel, make a graph of the acceleration of the system (y-axis) versus the weight (mg) of the *hanging* body (x-axis). You should include at least 7 data points. Carry out a linear regression for this data set. Quote the slope and intercept values, their uncertainties, their p-values, and the R^2 value. Show a sample error bar (on the graph) for at least one of the points of this graph.

- Derive (show it completely) an equation for the acceleration of the system versus the weight of the hanging body. Plot this theory equation on your graph (as a second series, a line but no points).

- Compare your graph to the predicted theoretical equation, that is compare the values of the slopes and intercepts.

Exercise 4.4.1.

> **Note:** In almost every lab you will be comparing a theoretical equation to the equation of a line and interpreting what the slope and intercept mean.

What is the physical significance of the slope and of the intercept from the graph? That is, what physical quantity does the slope of this graph equal?

Hint. It should help to recall that Newton's second law looks very similar to the generic equation of a line: $y = mx + b$.

Answer. When you figure out which physical quantity the slope *should* compare to, compute the %-difference to that value.

- In many mechanics experiments, there may be deviations from the expected or theoretical results because of the effects of friction. (If you are lucky, you will get to investigate this effect in detail in Lab 5!) Frictional forces are sometimes difficult to take into consideration. If there are deviations between your results and the predicted theory then try to distinguish whether they are caused by a tilt of the track, friction between the cart and the track or the friction between the string and the pulley.

Exercise 4.4.2. What might be expected in the results from these different systematic effects? That is, would the slope be expected to increase or decrease slightly because of the effects of friction? Would the slope be expected to increase or decrease slightly because of the effects of an unlevel track?

- When designing experiments, it is important to keep control parameters; in this case a parameter which is kept constant.

Exercise 4.4.3. What parameter is held constant in this experiment? Is there an obvious reason for keeping this constant?

[1] Pennies are a reasonable mass to be moving. If you are provided with pennies as the mass being transferred, then (after finding the individual mass of each) you might consider using the date of minting to distinguish which penny was transferred in order to determine the specific mass each time.

4.5 Questions

1. Why is it important to keep the total mass of the system constant? If one simply added mass to the hanger without keeping the system's mass constant, how would the data appear on the graph of the acceleration vs mg?

Hint 1. In the equation $F_{\text{net}} = ma$, m is the mass of the objects being accelerated. Is this the mass that is important to keep constant?

Hint 2. In the equation $F_{\text{net}} = ma$, F_{net} is the weight of the object doing the pulling. Is this the mass that is important to keep constant?

Hint 3. When you draw a graph on 2-dimensional graph paper, you have two axes, one for each variable. If you change the weight that is pulling and the overall mass being accelerated and you have a new acceleration, then think about which variables would you graph and what that would look like.

Hint 4. The equation $F_{\text{net}} = ma$ looks like (has the same form as) $y = mx + b$ if $\cancelto{0}{b}$. Which variable goes in the place of y? of x? of the slope? What happens to $y = mx + b$ if the slope is not a constant?

2. How would the motion (and therefore your results) be different if the track was not level? Consider both cases: if the pulley end were higher and if the pulley end were lower.

Hint 1. One situation describes the cart going downhill during the experiment; the other, uphill.

Hint 2. In each case, would your measured acceleration be equal to, larger than, or smaller than expected?

Hint 3. Is your result different from the expected result in a way consistent with either of these situations (uphill or downhill)?

3. If your group has a discrepancy between the results and the theory, could the presence of friction explain why your results differ from what is expected? Explain how.

Hint 1. Would friction tend to make the measured acceleration larger than or smaller than the expected value?

Hint 2. Is your result larger than or smaller than the expected result?

Some sources of uncertainty: When answering this next subset of Questions, consider your responses to Questions 2 and 3. Recall also that physicists often use the terms "source of error" and "source of uncertainty" interchangeably and do *not* mean error-as-in-mistake.

4. When considering possible sources of uncertainty, is it possible for tilt and friction to combine to make your observed result different from the expected result in the same way (i.e., both making your result too large or both making your result too small)?

5. When considering possible sources of uncertainty, is it possible for tilt and friction to act against each other to make your observed result closer to the expected result (i.e., friction causing an error one way and the tilt causing an error the other way)?

6. Regardless of how well your observed result agrees with the expected result, indicate how the possible tilt and friction might have impacted your results. If you are able to determine that one source of uncertainty clearly did not affect your result then comment on how the pattern of your data reveal this to you.

7. How would the cart's acceleration change, if at all, if the cart was given an initial push? Decide if this is a source of uncertainty.

Hint. Recall Exercise 3.3.5.

8. What are the two greatest sources of uncertainty in this experiment? Are they random or systematic errors? Be specific and quantify your answer.

(Revised: Oct 11, 2017)
A PDF version might be found at Newton.pdf (161 kB)

Lab 5

Dry Sliding Friction

Experimental Objectives

- In this experiment you will devise methods
  - to investigate the nature of both the frictional force and the coefficient of friction and
  - to test the validity of da Vinci's empirical rule.

Introduction

Friction is a force which retards the relative motion of any body while sliding over another body. The frictional force acting on a body is parallel to the surface that the object is sliding upon and it is directed opposite to the direction of motion. The phenomenon of friction is rather complicated, especially at the microscopic level, because it is dependent on the nature of the materials of both contacting surfaces. The frictional force depends on the roughness or the irregularities of both surfaces. At the macroscopic level, the nature of this force can be described by a simple empirical law, first given by Leonardo da Vinci:

> The magnitude of the force of friction between unlubricated, dry surfaces sliding one over the other is proportional to the normal force pressing the surfaces together and is independent of the (macroscopic) area of contact and of the relative speed.

At the microscopic level, the frictional force (F_f) does depend on the actual area of contact of the irregularities of the surfaces. This actual area of contact then increases as the force pressing the two surfaces together increases, this force is called the load. Using Newton's 2$^{\text{nd}}$ Law in this perpendicular direction we can conclude that the magnitude of the load is equal to the Normal force (F_N) of the surface pushing on the object. Therefore we may write that

$$F_f \propto F_N \qquad \Rightarrow \qquad F_f = \mu F_N$$

where the Greek letter μ ("mew") is a dimensionless constant of proportionality called the coefficient of friction.

When a body is pushed or pulled parallel to the surface of contact and no motion occurs, we can conclude that the force of the push or pull is equal to the frictional force, using Newton's 2$^{\text{nd}}$ Law of motion. As the applied force is increased, the frictional force remains equal to the applied force until motion results. At this maximum value of the applied force, the frictional force is also a maximum and is given by

$$F_f = \mu_s F_N$$

where the subscript s stands for static (non-moving) friction, and μ_s is **the coefficient of static friction**. This equation can only be used at this maximum static point also called the point of impending motion. At the instant that the applied force becomes greater than the maximum f_s, the body is set into motion and this motion is opposed by a frictional force called the kinetic (sliding) frictional force and is given by

$$F_f = \mu_k F_N$$

where the subscript k stands for the kinetic (moving) friction, and μ_k is **the coefficient of kinetic friction**. In general, $\mu_k < \mu_s$; that is, it takes more force to overcome the static friction than to overcome the kinetic friction. The coefficients of friction are generally less than one, but may be greater than one, and they depend on the nature of both surfaces.

Consider a system comprised of a block on a horizontal surface being pulled horizontally by a string connected to a hanging weight. We can use M is the mass of the block on the horizontal surface and m is the hanging mass. The force that accelerates the system forward is mg. The frictional force depends on the normal force of the block $\mu_k(Mg)$. Then, the whole system is accelerating with a constant acceleration so that Newton's second law gives:

$$(mg) + [-(\mu_k Mg)] = (M + m)a. \tag{5.1}$$

From this, μ_k can be solved for, giving:

$$\mu_k = \frac{mg - (M + m)a}{Mg}. \tag{5.2}$$

5.1 Pre-Lab Considerations

- Draw force diagrams for the following case: a block on a horizontal surface pulled by a hanging mass and a string (include the friction force).
- Write out the corresponding Newton's 2$^{\text{nd}}$ Law equations for forces both parallel and perpendicular to the contact surface.
- Derive the relevant equations for each of the above two cases for which the coefficients of friction can be determined:
  - Case one is static, but at the point of motion.
  - Case two is the kinetic case.

5.2 Student Outcomes

Knowledge Developed: In this exercise, students should learn how forces are related to the motion of a cart, how to use a free-body diagram, and gain a visceral understanding of Newton's second law with the (more realistic) inclusion of the effects of friction.

Skills Developed:

- Evaluate the data for sources of uncertainty. Can you see an effect, such as a level track or the presence of friction, in the result?
- Using Pasco Capstone software
- Interpreting slope and intercept of graphs
- Evaluating and propagating uncertainties

5.3 Procedure

For the block on the horizontal plane:

1. Clean the block and the plane, so that they are free of dust and other contaminants.
2. Make sure the track is level, as in previous labs.

You will use the force transducer to measure the force directly in Subsection 1 and Subsection 2. However, Subsection 3 will require an indirect measurement (calculation) of the force by measuring the velocity and using the velocity-versus-time graph to get the acceleration.

5.3.1 Break Static Friction — pull until moves

> **Note** By this time, you should already be familiar with the Pasco Capstone interface, which is also used in Lab 3, Lab 4, and Labs 6–7. You may remind yourself of the format by reading Appendix D

1. Set up the Dynamics Track, cart, force transducer and friction block. The force transducer attaches to the dynamics cart, the friction block rests on the track (felt side down).
2. Attach a string to the force transducer. The force transducer needs to be zeroed before data collection starts. Collect data, and slowly start pulling on the string (*be sure to pull the string horizontally*) and slowly increase the pull force until the cart is moving down the track. Using just the maximum force (at the point of impending motion) the coefficient of static friction can be calculated.
3. Test the relationship between the force of friction and the normal force, by changing the load force (normal force) and measuring the force of friction at the point of motion impending. Carry this out for a total of five data points. Graph the frictional force versus the normal force. Calculate the coefficient of static friction from this graph.

5.3.2 Effect of Surface Area — distinguish pressure from force

Consider pushing a pencil into your arm. (Well, don't *actually* do it!) If you use the erasure end, then you can feel the force, but it doesn't hurt. If you use the sharpened tip with the *same* force then it will certainly hurt! So, you have the idea that the same force spread over a different surface area *can* have a different effect; but it doesn't *always* have a different effect. For this part of the lab, you will test the relationship between the coefficient of friction and the macroscopic area of contact between the block and the surface.

1. Place the friction block on its side (felt side down) and repeat Item 2 and Item 3 for three (rather than five) of the previous load forces.
2. Add the plot of this F_f versus F_N as a new series to the graph of Subsection 5.3.1.

5.3.3 Friction while Accelerating

1. Apply a force (hanging mass, pulley, and string) large enough to accelerate the block. Use the Sonic Ranger to collect data. Note: This should accelerate fast enough to measure the acceleration, but not so fast that it crashes at the end of the track. (Depending on the normal forces being used, you might try 300 g as the hanging mass.)
2. Graph the velocity vs time. Determine the acceleration of the block from the slope of the line.
3. Repeat this part four or five times with a different normal forces. (You may use any hanging mass.)
4. Since we are not measuring the frictional force, you will need to calculate it; See Exercise 5.3.1.
5. Add the plot of this F_f versus F_N as a new series to the graph of Subsection 5.3.1 and Subsection 5.3.2.

6. Calculate the coefficient of kinetic friction from the slope.

Exercise 5.3.1. Draw the free-body diagram for the cart being dragged by the hanging mass. Set-up Newton's second law for the forces involved. Solve this for the frictional force in terms of quantities you can measure.

Hint 1. Lab 4 might help you set-up the free-body diagram and equation. In that lab, we made a point of keeping the total mass constant. In this lab, that is not important because that lab allows us to trust Newton's second law and we are now testing a different relationship.

Hint 2. Equation (5.1) wrote out Newton's second law for you; but you want to solve it for F_f, not for μ and not in terms of μ.

Hint 3. You can measure the total mass directly. You can measure the hanging mass directly. You can compute (an indirect measurement) the acceleration from the velocity-versus-time graph.

Answer. Do not compute the frictional force using the normal force, that is the relationship you are trying to investigate!

5.4 Analysis

The experimental precision should be estimated for all parts of this experiment and care should be taken for all of the measurements. , but it is more important to investigate the relationships than it is to repeat the experiment many times or to try to achieve high precision in the data. In Exercise 5.3.1 you found an equation for F_f in terms of measured values. You should track the uncertainties from measurement, through the calculation, to the result (this is called the Propagation of Uncertainties) so that you can see how the uncertainty in the measurements impact the uncertainty of the final result. The following will step you through how it would work for Equation (5.2), which is much more complicated than your equation.

Example 5.4.1. In order to propagate the uncertainty for $\mu_k = \dfrac{mg - (M+m)a}{Mg}$ we should notice that it has both addition (See Rule 1) and multiplication (See Rule 2). I will assume some values with uncertainty; I will also list the relative uncertainty:

- $m = 0.30 \pm 0.01\,\mathrm{kg}$, $\dfrac{0.01\,\mathrm{kg}}{0.30\,\mathrm{kg}} = 0.033 = 3.3\%$
- $M = 2.50 \pm 0.02\,\mathrm{kg}$, $\dfrac{0.02\,\mathrm{kg}}{2.50\,\mathrm{kg}} = 0.008 = 0.8\%$
- $a = 0.45 \pm 0.04\ \mathrm{m/s^2}$, $\dfrac{0.04\ \mathrm{m/s^2}}{0.45\ \mathrm{m/s^2}} = 0.089 = 8.9\%$
- $g = 9.81 \pm 0.01\ \mathrm{m/s^2}$, $\dfrac{0.01\ \mathrm{m/s^2}}{9.81\ \mathrm{m/s^2}} = 0.001 = 0.1\%$

You should note that while the uncertainty of m and M are about the same, the relative uncertainty of m is much larger than the relative uncertainty of M (and similarly for a and g). While the uncertainty for m and g have the same numeric value, they cannot be meaningfully compared because they have different units.

Recall the "order of operations" implies that we ***first*** add $(M+m)$, ***then*** multiply that value by a, ***then*** add that value to (the product of) mg, and ***finally*** divide by (the product of) Mg. We do the propagation of uncertainty in the same order.

Using Rule 1, add the masses $(M+m)$:

$$\begin{aligned}(0.30 \pm 0.01\,\mathrm{kg}) + (2.50 \pm 0.02\,\mathrm{kg}) &= [(0.30 + 2.50) \pm (0.01 + 0.02)]\,\mathrm{kg} \\ &= 2.80 \pm 0.03\,\mathrm{kg}\end{aligned}$$

$$\frac{0.03\,\mathrm{kg}}{2.80\,\mathrm{kg}} = 0.011 = 1.1\%$$

This overall uncertainty is affected a little bit more by M (in this case) because we are adding uncertainty (not relative uncertainty).

Using Rule 2, multiply by the acceleration $(M+m)a$:

$$\begin{aligned}(2.80 \pm 0.03\,\mathrm{kg})(0.45 \pm 0.04\ {}^{\mathrm{m}}\!/\!_{\mathrm{s}^2}) &= [(2.80\,\mathrm{kg})(0.45\ {}^{\mathrm{m}}\!/\!_{\mathrm{s}^2}] \pm [(0.011 + 0.089)] \\ &= 1.26\,\mathrm{N} \pm [0.10],\end{aligned}$$

since $(1.26\,\mathrm{N})(0.10) = 0.126\,\mathrm{N}$, this is $1.26 \pm 0.13\,\mathrm{N}$.[1] In this case, a contributed about 8 times as much uncertainty.[2]

We also need to use Rule 2 to find the uncertainty for mg before combining the numerator:

$$\begin{aligned}(0.30 \pm 0.01\,\mathrm{kg})(9.81 \pm 0.01\ {}^{\mathrm{m}}\!/\!_{\mathrm{s}^2}) &= [(0.30\,\mathrm{kg})(9.81\ {}^{\mathrm{m}}\!/\!_{\mathrm{s}^2}] \pm [(0.033 + 0.001)] \\ &= 2.94\,\mathrm{N} \pm [0.034],\end{aligned}$$

since $(2.94\,\mathrm{N})(0.034) = 0.10\,\mathrm{N}$, this is $2.94 \pm 0.10\,\mathrm{N}$.[3] Notice that the mass had a larger impact on the uncertainty of this term.

Using Rule 1, subtract[4] the two terms in the numerator [mg and $(M+m)a$]:

$$\begin{aligned}(2.94 \pm 0.10\,\mathrm{N}) - (1.26 \pm 0.13\,\mathrm{N}) &= [(2.94 - 1.26) \pm (0.10 + 0.13)]\,\mathrm{N} \\ &= 1.68 \pm 0.23\,\mathrm{N}\end{aligned}$$

$$\frac{0.23\,\mathrm{N}}{1.68\,\mathrm{N}} = 0.14 = 14\%$$

Notice that both terms had a similar contribution to the uncertainty.[5]

Finally, use Rule 2 to find the uncertainty for the final combination[6] of the numerator/denominator:

$$\begin{aligned}\frac{(1.68 \pm 0.23\,\mathrm{N})}{(2.50 \pm 0.02\,\mathrm{kg})(9.81 \pm 0.01\ {}^{\mathrm{m}}\!/\!_{\mathrm{s}^2})} &= [\frac{1.68\,\mathrm{N}}{(2.50\,\mathrm{kg})(9.81\ {}^{\mathrm{m}}\!/\!_{\mathrm{s}^2})}] \pm [(0.14 + 0.008 + 0.001)] \\ &= 0.0685 \pm [0.15],\end{aligned}$$

since $(0.0685\,\mathrm{N})(0.15) = 0.010\,\mathrm{N}$, this is $0.07 \pm 0.01 = (7 \pm 1) \times 10^{-2}$, rounded to the appropriate number of significant digits.

You should note that the uncertainty of the numerator swamps by far the uncertainty of the denominator. In order to make this measurement more precise, we should focus on improving the precision of m and a, as indicated in Footnote 5.4.5.

The graphs and data should also be evaluated as usual. In this particular experiment, you should

- Explain why the normal force on the block by the surface rather than the weight of the object is related to the frictional force.
- Interpret the slope and intercept of the graphs.
- Compare the slopes from each of the three parts. Decide which should be the same and which should be different.
- Calculate the % decrease of the static to kinetic coefficient of friction.
- Comment on the validity of the empirical rules of friction.

[1] If this were the final answer, you should report it as $1.3 \pm 0.1\,\mathrm{N}$; but since we are continuing to use it in calculations, for the purpose of managing appropriate rounding errors, you can keep an extra insignificant digit, which will maintain consistency. Please be sure to round your final answer to the appropriate number of significant digits.

[2] To improve this answer we should focus on a more precise measurement of a, rather than improving the precision of m or M!

[3] If this were the final answer, you should report it as $2.9 \pm 0.1\,\mathrm{N}$; but since we are continuing to use it in calculations, for the purpose of managing appropriate rounding errors, you can keep an extra insignificant digit, which will maintain consistency. Please be sure to round your final answer to the appropriate number of significant digits.

[4] Remember the rule is to ***add*** the uncertainty even if you are ***subtracting*** the values!

[5] Even if we improved the precision of a, as mentioned in Footnote 5.4.2, the uncertainty of mg would keep this uncertainty near the 0.1 to 0.2 range. So we need to be more precise with m for the mg term *and* with a for the $(M+m)a$ term.

[6] Remember the rule is to ***add*** the relative uncertainty whether you are multiplying ***or*** dividing the values!

5.5 Questions

For all questions, and when possible, use your experimental or theoretical results to demonstrate your answers to the questions.

1. Does the coefficient of friction depend on the area of contact?

2. Does the coefficient of friction depend on the mass of the object?

3. Does the coefficient of friction depend on the normal force of the object?

4. Does the frictional force depend on the normal force of the object?

5. Does the coefficient of kinetic friction depend on the speed of travel?

6. When the object was pulled by a string, how would the forces be affected if the cord was not horizontal?

7. What would happen to the coefficient of friction if the surfaces were lubricated with oil?

(Revised: Oct 11, 2017)
A PDF version might be found at friction.pdf (175 kB)

Lab 6

Conservation of Energy on a Linear Track – (Single Week Version)

Experimental Objectives

- The purpose of this experiment will be to verify the validity of the law of conservation of mechanical energy, which says that $\Delta E = 0$ as a cart runs along a track.

Introduction

Conservation laws play a very important role in our understanding of our physical world. For example, the law of conservation of energy can be applied in all physical processes. This is a fundamental and independent statement about the nature of the physical world. It is not necessarily derivable from other laws like Newton's Laws of motion. Though for simple point mass systems, the law of conservation of energy can be derived from Newton's Laws. It can be shown that the net work done on a system is equal to the change in the kinetic energy ($W_{\text{net}} = \Delta K$) of the system; this is called the work-energy theorem and it can be written in a variety of forms. When a net positive work is done on a system, the kinetic energy of the system increases, and when a net negative work is done on the system (as from a friction force), the kinetic energy of the system decreases.

When the gravitational force acts on a system, the work it does on the system, W_g, is the gravitational force (mg) times the vertical displacement ($h = \Delta y$): $W_g = mg\Delta y$. For convenience, this is called the change in gravitational potential energy ($W_g = -\Delta P$). *If* the gravitational force is the only force acting on the system then $W_g = W_{\text{net}}$ and therefore, $-\Delta P = \Delta K$ for the system. When a force can be associated with a potential energy, it is called a "conservative force." Another kind of potential energy deals with an elastic potential energy, like in a spring. The energy stored in a spring is given by the formula $P_s = \frac{1}{2}k\Delta x^2$.

If, on the other hand, a force dissipates energy, then it is called a "nonconservative force" and it will have no associated potential energy. Frictional forces are an example of a nonconservative force and the work done by a frictional force is negative because (physically) the frictional force removes energy from the system and (mathematically) the frictional force and the displacement are in opposite directions. This work done by friction is converted into heat or sound. To distinguish the energy of heat or sound from the potential and kinetic energy, we define the total mechanical energy, $E = K + P$ at any point. Since frictional forces remove mechanical energy, we say $W_f = \Delta E = \Delta K + \Delta P$.

In general then, the law of conservation of energy states that energy can not be created or destroyed, but can only change from one form to another; or the total energy of the system at point A is equal to the total energy of the system at point B.

6.1 Procedure

We would like for you to verify the conservation of mechanical energy in two different situations; so, there are two parts to this experiment. We will first consider a flat track with accelerated motion, as in the Newton's Law lab and the Friction lab. We can then consider an inclined plane. You will not be given an explicit procedure, but rather you will be given a series of questions with answers that will imply the procedure. Part of the experiment is for you to figure out for yourself what the best course of action is. Please answer the questions as they are asked.

6.1.1 Flat Track

Set up the dynamics cart on a horizontal dynamics track. Set up the motion sensor at one end of the track and a pulley at the other end so that the pulley partly extends past the edge of the table. Hang the basket over the pulley so that it can accelerate the cart along the track – you might need extra weight in the cart to keep it from accelerating too fast. In order to use this motion to verify the validity of the conservation of mechanical energy, we need to measure some variables. Answering Exercise 6.1.1 and Exercise 6.1.2 will help you decide on the relevant variables. Exercise 6.1.3 should help you determine how to finish setting up the equipment.

Exercise 6.1.1. In order to verify $\Delta E = 0$, we will need to calculate E as $E = K + P$. Therefore, we need to know the kinetic energy, $K = \frac{1}{2}mv^2$, the energy of *some mass*, m, moving at a speed v. Which mass do you need to measure? How can you measure the velocity?

Exercise 6.1.2. In order to verify $\Delta E = 0$, we will need to calculate E as $E = K + P$. Therefore, we need to know the potential energy, $P = mgy$, the energy of *some mass*, m, located some height, y, above the ground. Which mass do you need to measure? How can you measure the position?

Exercise 6.1.3. In order to measure the position of the falling mass and the velocity of the system, do you need two motion sensors? Can you manage with one? Considering that it is a fairly expensive piece of equipment, where should you NOT put the sonic ranger? Where could you put it? Depending on where you put the ranger, decide if you need to "translate" the position or velocity data in order to find the specific values that you actually need.

Once you decide what variables to measure, run the experiment for one set of masses while measuring the appropriate variables. Put the data into Excel and decide what plot(s) will allow you to verify the validity of the conservation of mechanical energy. Exercise 6.1.4 may help with this. Decide if you need a trendline. Relate the information in Exercise 6.1.5 to the statement you are trying to verify.

Exercise 6.1.4. To verify $\Delta E = 0$, we will need to graph E, the total mechanical energy, as a function of time. What do you expect this graph to look like, if the law is valid? If not?

1. Does the kinetic energy change during this motion? Is $\Delta K = 0$? Considering the initial and final values of the kinetic energy, K_i and K_f, what would a graph of K versus time look like?

2. Does the potential energy change during this motion? Is $\Delta P = 0$? Considering the initial and final values of the potential energy, P_i and P_f, what would a graph of P versus time look like?

3. Assuming that the mechanical energy is conserved, what would a graph look like if it included E, K, and P? What if the mechanical energy is not conserved? How would K and P be affected in these two cases?

4. (Subsection 6.1.2 only) When the cart is at the bottom of the track during the motion, the values of position become negative (less than zero!). Why? Is there some other place where the energy might go?

Exercise 6.1.5. Please note the overall change in potential energy, ΔP, and the overall change in the kinetic energy, ΔK. Should either of these be related to the overall change in energy ΔE and, if so, how?

6.1.2 Sloped Track

Remove the pulley from the track. Your cart will have either a spring-loaded "battering ram" on the front or a pair of magnets. If you have the battering ram, then you will want the end of the track with the rubber nub at the bottom of the incline. If you have the magnets, then you need to replace the pulley with a "C" shaped "catch-bar." *Ask for help from the instructor!* The catch-bar has magnets in it that will repel the magnets in the cart. In this case, the cart must not be going so fast as to come into physical contact with the magnets on the catch-bar.

Raise one end of the dynamics track. Exercise 6.1.6 should help decide how tilted. Measure the tilt angle of the track with two methods: use a protractor, and measure the vertical rise and track length and calculate the tilt angle using the inverse-sine function. Answer Exercise 6.1.7. As you continue to set up the track for measurements, consider answering Exercise 6.1.1, Exercise 6.1.2, and Exercise 6.1.3 again for this situation to help you decide on the appropriate accessories (sensors); but note Exercise 6.1.8 as you think about the answers to the previous questions.

Exercise 6.1.6. We want the cart to accelerate down the track (not too slow), but not to fly off at the bottom (not too fast). How fast is *too fast*? Don't use that slope! How fast is *too slow*? Use a slope somewhere in between.

Exercise 6.1.7. After you measure the angle of incline in these two ways, consider the uncertainty in the measurements. Which of these measurement is more precise?

Exercise 6.1.8. The motion sensor will measure the motion of the cart *along* the ramp, but the potential energy needs the *vertical* position of the cart. Which trig function relates the distance along the ramp to the corresponding vertical distance?

Once you decide on the variables to be measured, but before you make the measurements, you will need to calibrate your position measurements. We would like zero to correspond to being at the bottom of the ramp, so place the cart stationary at the bottom and use the motion sensor to measure this position. In order to verify the validity of the conservation of mechanical energy, release the cart from rest near the top of the ramp and let it roll down the incline, bouncing three times before you stop the measurement. Do this for one value of mass.

Transfer these data to Excel again and decide on the best graph to verify the objective. Again, Exercise 6.1.4 may help with this; however, you will also need to consider Item 6.1.4.4. Decide if you need a trendline and where it would be fit. Relate the information in Exercise 6.1.5 to the statement you are trying to verify.

6.2 Analysis

We are now going to take a closer look at the irregularities of the data and investigate some variations to try to explain what those data say.

- Before drawing conclusions about the validity of the conservation of mechanical energy, consider Exercise 6.2.1.

Exercise 6.2.1. We need to look for the energy lost in each graph.

1. When you look at the graph from Subsection 6.1.1 for E, is the energy conserved or is there energy lost? If lost, calculate the energy lost or gained from the graph. (It might help to have a trendline.) If energy is lost, come up with at least two explanations for where this energy goes.
2. When you look at the graph from Subsection 6.1.2 for E, there are jumps in the energy. Why?
   (a) What is happening between the jumps? Does Subsection 6.1.1 help to explain these sections of the graph? Compared to the jumps, can we assume that the mechanical energy is conserved between the jumps?
   (b) What is happening at the time of those "jumps"? From the trend of the graph, calculate the amount of energy lost during each sudden change, call it the energy discrepancy, and the percent of this discrepancy relative to the total energy before the corresponding collision. Discuss where this "missing" energy goes. Is the ratio of "energy discrepancy" to total prior energy the same for each jump?

3. Comment in general, on the law of Conservation of Mechanical Energy. Can you predict any effects that might invalidate the conservation of mechanical energy? Can these effects be minimized? Is it possible to run the experiment again minimizing this effect?

- One explanation of a loss of energy (non-conservation) is friction. List all of the places where two pieces of material rub against each other. Since $F_f = \mu F_N$, do any of these locations have a normal force that can be varied? (Note Exercise 6.3.2.) As an independent measure of the amount of friction, we can also consider the actual acceleration versus the expected acceleration. Exercise 6.2.2 will help you determine the expected acceleration and the variable necessary to find it. Exercise 6.2.3 will help decide on the relationship between the friction and the acceleration.

Exercise 6.2.2. Given an ramp inclined at some angle θ, what is the component of the gravitational force aimed down the ramp? Assuming that there is no friction, what is the net force? Since $F_{\text{net}} = ma$, the acceleration should be ... ?[1] From your expression, what do you need to measure in order to find the expected value of a? (Recall Exercise 6.1.7.)

Exercise 6.2.3. If there is friction, then how do you expect the actual acceleration to compare to the expected acceleration? If there is no friction? So, how would you interpret finding an acceleration that is exactly equal to the expected value? less than the expected value? Larger than the expected value?

- A second explanation for the loss of energy is that some component is gaining rotational kinetic energy. The formula for this is $K_R = \frac{1}{2}I\omega^2$, where I is the moment of inertia[2], and ω is the angular speed $\omega = v/r$. Assuming that any discrepancy that you found in the conservation of energy is due to the rotational kinetic energy of the pulley, how much energy would the pulley need to have at the end of the run (while spinning full speed)? Based on the final velocity of the cart, what is the angular speed of the pulley? Based on these numbers, K_R and ω, what is the moment of inertia for the pulley? Can you tell if this is a reasonable estimate?

6.3 Questions

1. Does the mass of the cart matter? If you run it again at a different value of mass, would you expect the overall conclusion to be different? Would you expect the specific values to be different?

2. If the mechanical energy is conserved, then

$$\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f$$

What do you notice about the mass? Is your graph different if the mass of the cart changes? Does this support or conflict with the idea that the total mechanical energy is conserved? On the other hand, if the mechanical energy is not conserved, then

$$W_{\text{nc}} = \frac{1}{2}mv_f^2 + mgy_f - \frac{1}{2}mv_i^2 - mgy_i$$

What do you notice about the mass now? Does your graph support or conflict with the idea that the total mechanical energy is conserved?

[1] $a = g\sin\theta$.

[2] In this case, the moment of inertia is probably a little less than $\frac{1}{2}mr^2$, where m is the mass of the rotating object and r is the radius of the rotating object. This is not a convenient way to calculate I at this time.

Lab 7

Conservation of Energy on a Linear Track – (Two Week Version)

Experimental Objectives

- The purpose of this experiment will be to verify the validity of the law of conservation of mechanical energy, which says that $\Delta E = 0$ as a cart runs along a track.

Introduction

Conservation laws play a very important role in our understanding of our physical world. For example, the law of conservation of energy can be applied in all physical processes. This is a fundamental and independent statement about the nature of the physical world. It is not necessarily derivable from other laws like Newton's Laws of motion. Though for simple point mass systems, the law of conservation of energy can be derived from Newton's Laws. It can be shown that the net work done on a system is equal to the change in the kinetic energy ($W_{\text{net}} = \Delta K$) of the system; this is called the work-energy theorem and it can be written in a variety of forms. When a net positive work is done on a system, the kinetic energy of the system increases, and when a net negative work is done on the system (as from a friction force), the kinetic energy of the system decreases.

When the gravitational force acts on a system, the work it does on the system, W_g, is the gravitational force (mg) times the vertical displacement ($h = \Delta y$): $W_g = mg\Delta y$. For convenience, this is called the change in gravitational potential energy ($W_g = -\Delta P$). *If* the gravitational force is the only force acting on the system then $W_g = W_{\text{net}}$ and therefore, $-\Delta P = \Delta K$ for the system. When a force can be associated with a potential energy, it is called a "conservative force." Another kind of potential energy deals with an elastic potential energy, like in a spring. The energy stored in a spring is given by the formula $P_s = \frac{1}{2}k\Delta x^2$.

If, on the other hand, a force dissipates energy, then it is called a "nonconservative force" and it will have no associated potential energy. Frictional forces are an example of a nonconservative force and the work done by a frictional force is negative because (physically) the frictional force removes energy from the system and (mathematically) the frictional force and the displacement are in opposite directions. This work done by friction is converted into heat or sound. To distinguish the energy of heat or sound from the potential and kinetic energy, we define the total mechanical energy, $E = K + P$ at any point. Since frictional forces remove mechanical energy, we say $W_f = \Delta E = \Delta K + \Delta P$.

In general then, the law of conservation of energy states that energy can not be created or destroyed, but can only change from one form to another; or the total energy of the system at point A is equal to the total energy of the system at point B.

7.1 Procedure

We would like for you to verify the conservation of mechanical energy in two different situations; so, there are two parts to this experiment. We will first consider a flat track with accelerated motion, as in the Newton's Law lab and the Friction lab. We can then consider an inclined plane. You will not be given an explicit procedure, but rather you will be given a series of questions with answers that will imply the procedure. Part of the experiment is for you to figure out for yourself what the best course of action is. Please answer the questions as they are asked.

> **NOTE:** There is enough analysis for this lab that you will have two weeks to complete the lab. During the first week, you will do the two parts of the experiment and begin to write up your report. During the second week, you will do some analysis and re-run the experiment to determine the cause of differences from expectations. A single lab report will be due after the second week of experimentation.

7.1.1 Flat Track

Set up the dynamics cart on a horizontal dynamics track. Set up the motion sensor at one end of the track and a pulley at the other end so that the pulley partly extends past the edge of the table. Hang the basket over the pulley so that it can accelerate the cart along the track – you might need extra weight in the cart to keep it from accelerating too fast. In order to use this motion to verify the validity of the conservation of mechanical energy, we need to measure some variables. Answering Exercise 7.1.1 and Exercise 7.1.2 will help you decide on the relevant variables. Exercise 7.1.3 should help you determine how to finish setting up the equipment.

Exercise 7.1.1. In order to verify $\Delta E = 0$, we will need to calculate E as $E = K + P$. Therefore, we need to know the kinetic energy, $K = \frac{1}{2}mv^2$, the energy of *some mass*, m, moving at a speed v. You will have to decide which mass you need to measure. You will also have to decide how to measure the velocity.

Hint 1 (mass). The mass that should be used for the kinetic energy is the mass that is moving at this speed.

Hint 2 (how to measure velocity). You have measured the speed of these carts several times in previous labs. Do you recall how you did it then?

Hint 3 (where to measure velocity). How does the velocity of the basket compare to the velocity of the block? Is velocity the quantity you need in this equation?

Exercise 7.1.2. In order to verify $\Delta E = 0$, we will need to calculate E as $E = K + P$. Therefore, we need to know the potential energy, $P = mgy$, the energy of *some mass*, m, located some height, y, above the ground. You will have to decide which mass you need to measure. You will also have to decide how to measure the ~~position~~ height.

Hint 1 (mass). The mass that should be used for the potential energy is all of the mass that is changing its vertical position.

Hint 2 (how to measure position). You have measured the position of these carts several times in previous labs. Do you recall how you did it then? In this case, which position do you need to know to compute the potential energy?

Hint 3 (where to measure position). The location that you need in order to compute the potential energy is the height of the thing that is moving vertically. See also Exercise 7.1.3.

Exercise 7.1.3. In order to measure the position of the falling mass and the velocity of the system, do you need two motion sensors? Can you manage with one?

Hint 1. The cart and the basket both move the same distance and move with the same speed.

Hint 2. If you the position/speed of the basket directly, then, considering that the motion sensor is a fairly expensive piece of equipment, where should you NOT put the sonic ranger? Where could you put it so that it will not get hit?

Hint 3. Depending on where you put the ranger, decide if you need to "translate" the position or velocity data in order to find the specific values that you actually need.

Once you decide what variables to measure, run the experiment for one set of masses while measuring the appropriate variables. Put the data into Excel and decide what plot(s) will allow you to verify the validity of the conservation of mechanical energy. Exercise 7.1.4 may help with this. Decide if you need a trendline. Relate the information in Exercise 7.1.5 to the statement you are trying to verify.

Exercise 7.1.4. To verify $\Delta E = 0$, we will need to graph E, the total mechanical energy, as a function of time. What do you expect this graph to look like, if the law is valid? If not?

Note: Hint 6 is only relevant to Subsection 7.1.2.

Hint 1 (KE-graph). Does the kinetic energy change during this motion? Is $\Delta K = 0$? Considering the initial and final values of the kinetic energy, K_i and K_f, what would a graph of K versus time look like?

See Hint 4 to think about how a graph of velocity might help.

Hint 2 (PE-graph). Does the potential energy change during this motion? Is $\Delta P = 0$? Considering the initial and final values of the potential energy, P_i and P_f, what would a graph of P versus time look like?

See Hint 5 to think about how a graph of position might help.

Hint 3 (E-graph). Assuming that the mechanical energy is conserved, what would a graph look like if it included E, K, and P? What if the mechanical energy is not conserved? How would K and P be affected in these two cases?

Hint 4 (velocity-graph). Can you think of an equation of motion that relates the velocity to the time? Does this produce a linear or quadratic (parabolic) dependence on the time? Since the $K \sim v^2$, what dependence should K have with the time?

Hint 5 (position-graph). Can you think of an equation of motion that relates the position to the time? Does this produce a linear or quadratic (parabolic) dependence on the time? Since the $P \sim y$, what dependence should P have with the time?

Hint 6 (Negative PE?). (Subsection 7.1.2 only) When the cart is at the bottom of the track during the motion, the values of position become negative (less than zero!). Why? Is there some other place where the energy might go?

1. If you are using the force transducer, then it has a spring and a spring potential energy, ΔP_{spring}. This can (and should!) also be included in the total mechanical energy. You can calculate the elastic potential energy stored in the spring of the force transducer with $P = \frac{1}{2}k\Delta x^2$, which, since we do not know k, can be written $P = \frac{1}{2}F\Delta x$, where F is the force in Newtons (measurable with the force transducer) and Δx is the distance from the spring's equilibrium position, not the height (derivable from the position data). Be sure to match up the force values and the x values at those same times.

Exercise 7.1.5. Please note the overall change in potential energy, ΔP, and the overall change in the kinetic energy, ΔK. Should either of these be related to the overall change in energy ΔE and, if so, how?

NOTE: Save your data so that you can do further analysis next week.

7.1.2 Sloped Track

Remove the pulley from the track. Your cart will have either a spring-loaded "battering ram" on the front or a pair of magnets. If you have the battering ram, then you will want the end of the track with the rubber nub at the bottom of the incline. If you have the magnets, then you need to replace the pulley with a "C" shaped "catch-bar." *Ask for help from the instructor!* The catch-bar has magnets in it that will repel the magnets in the cart. In this case, the cart must not be going so fast as to come into physical contact with the magnets on the catch-bar.

Raise one end of the dynamics track. Exercise 7.1.6 should help decide how tilted. Measure the tilt angle of the track with two methods: use a protractor, and measure the vertical rise and track length and calculate the tilt angle using the inverse-sine function. Answer Exercise 7.1.7. As you continue to set up the track for measurements, consider answering Exercise 7.1.1, Exercise 7.1.2, and Exercise 7.1.3 again for this situation to help you decide on the appropriate accessories (sensors); but note Exercise 7.1.8 as you think about the answers to the previous questions.

Exercise 7.1.6. We want the cart to accelerate down the track (not too slow), but not to fly off at the bottom (not too fast). How fast is *too fast*? Don't use that slope! How fast is *too slow*? Use a slope somewhere in between.

Exercise 7.1.7. After you measure the angle of incline in these two ways, consider the uncertainty in the measurements. Which of these measurement is more precise?

Exercise 7.1.8. The motion sensor will measure the motion of the cart *along* the ramp, but the potential energy needs the *vertical* position of the cart. Which trig function relates the distance along the ramp to the corresponding vertical distance?

Once you decide on the variables to be measured, but before you make the measurements, you will need to calibrate your position measurements. We would like zero to correspond to being at the bottom of the ramp, so place the cart stationary at the bottom and use the motion sensor to measure this position. In order to verify the validity of the conservation of mechanical energy, release the cart from rest near the top of the ramp and let it roll down the incline, bouncing three times before you stop the measurement. Do this for one value of mass. Answer Exercise 7.1.9.

Exercise 7.1.9. Does the mass of the cart matter? If you run it again at a different value of mass, would you expect the overall conclusion to be different? Would you expect the specific values to be different?

Exercise 7.1.10. If the mechanical energy is conserved, then

$$\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f$$

What do you notice about the mass? Is your graph different if the mass of the cart changes? Does this support or conflict with the idea that the total mechanical energy is conserved? On the other hand, if the mechanical energy is not conserved, then

$$W_{\text{nc}} = \frac{1}{2}mv_f^2 + mgy_f - \frac{1}{2}mv_i^2 - mgy_i$$

What do you notice about the mass now? Does your graph support or conflict with the idea that the total mechanical energy is conserved?

Transfer these data to Excel again and decide on the best graph to verify the objective. Again, Exercise 7.1.4 may help with this; however, you will also need to consider Hint 7.1.4.6. Decide if you need a trendline and where it would be fit. Relate the information in Exercise 7.1.5 to the statement you are trying to verify.

NOTE: Save your data so that you can do further analysis next week.

7.2 Analysis

For the second week, you should already have your graphs from the experiment and you should have written a significant portion of the theory and the analysis. We are now going to take a closer look at the irregularities of the data and investigate some variations to try to explain what those data say.

- One of the factors you were asked to consider last week was Exercise 7.1.9. In order to verify this, re-run Subsection 7.1.1 with a noticeably different massed cart. Re-create the graph and use this only to note the effect of a different mass. Answer Exercise 7.1.10.
- Before drawing conclusions about the validity of the conservation of mechanical energy, consider Exercise 7.2.1.

Exercise 7.2.1. We need to look for the energy lost in each graph.

1. When you look at the graph from Subsection 7.1.1 for E, is the energy conserved or is there energy lost? If lost, calculate the energy lost or gained from the graph. (It might help to have a trendline.) If energy is lost, come up with at least two explanations for where this energy goes.

2. When you look at the graph from Subsection 7.1.2 for E, there are jumps in the energy. Why?
   (a) What is happening between the jumps? Does Subsection 7.1.1 help to explain these sections of the graph? Compared to the jumps, can we assume that the mechanical energy is conserved between the jumps?
   (b) What is happening at the time of those "jumps"? From the trend of the graph, calculate the amount of energy lost during each sudden change, call it the energy discrepancy, and the percent of this discrepancy relative to the total energy before the corresponding collision. Discuss where this "missing" energy goes. Is the ratio of "energy discrepancy" to total prior energy the same for each jump?
3. Comment in general, on the law of Conservation of Mechanical Energy. Can you predict any effects that might invalidate the conservation of mechanical energy? Can these effects be minimized? Is it possible to run the experiment again minimizing this effect?

- As you evaluate Subsection 7.1.2, you might be asked to re-run the experiment with a force transducer placed at the bottom of the track. (This should imply where the motion sensor will go.) Make sure that the cart will bounce from the force sensor. Make sure that the force sensor is zeroed before the start. There might be some information here based on work as a force-through-a-distance versus work as a change-in-energy.
- One explanation of a loss of energy (non-conservation) is friction. List all of the places where two pieces of material rub against each other. Since $F_f = \mu F_N$, do any of these locations have a normal force that can be varied? (Recall Exercise 7.1.9 and Exercise 7.1.10.) As an independent measure of the amount of friction, we can also consider the actual acceleration versus the expected acceleration. Exercise 7.2.2 will help you determine the expected acceleration and the variable necessary to find it. Exercise 7.2.3 will help decide on the relationship between the friction and the acceleration.

Exercise 7.2.2. Given an ramp inclined at some angle θ, what is the component of the gravitational force aimed down the ramp? Assuming that there is no friction, what is the net force? Since $F_{\text{net}} = ma$, the acceleration should be ... ?[1] From your expression, what do you need to measure in order to find the expected value of a? (Recall Exercise 7.1.7.)

Exercise 7.2.3. If there is friction, then how do you expect the actual acceleration to compare to the expected acceleration? If there is no friction? So, how would you interpret finding an acceleration that is exactly equal to the expected value? less than the expected value? Larger than the expected value?

- A second explanation for the loss of energy is that some component is gaining rotational kinetic energy. The formula for this is $K_R = \frac{1}{2}I\omega^2$, where I is the moment of inertia[2], and ω is the angular speed $\omega = v/r$. Assuming that any discrepancy that you found in the conservation of energy is due to the rotational kinetic energy of the pulley, how much energy would the pulley need to have at the end of the run (while spinning full speed)? Based on the final velocity of the cart, what is the angular speed of the pulley? Based on these numbers, K_R and ω, what is the moment of inertia for the pulley? Can you tell if this is a reasonable estimate?

(Revised: Oct 25, 2017)
A PDF version might be found at energy-2.pdf (165 kB)

[1] $a = g \sin\theta$.
[2] In this case, the moment of inertia is probably a little less than $\frac{1}{2}mr^2$, where m is the mass of the rotating object and r is the radius of the rotating object. This is not a convenient way to calculate I at this time.

Lab 8

Hooke's Law and Simple Harmonic Motion

Experimental Objectives

The stiffness of springs can be measured by stretching or by bouncing. Because we do not have an independent verification of the value of the stiffness of the spring, we will need to be clever about how to verify the relevant equations. It turns out that the measurement of stiffness through bouncing gives a value for the spring constant, whereas the measurement of stiffness by hanging involves both the spring constant and the acceleration due to gravity.

- By measuring and graphing
  - the relationship between mass and elongation when stretching a spring and
  - the relationship between mass and the period of a bouncing spring,

  we can compute the value of the acceleration due to gravity and thereby verify the relationships describing the stretch of a spring.

Introduction

Oscillatory motion is one of the most common types of motions and can occur in any physical system. Mechanical systems can experience a periodic motion, and then will vibrate at a natural frequency. This phenomenon is called resonance. Sound is a vibration in the air, which we hear with our ears; light is an oscillation of electric and magnetic fields, which we can see. The atoms and molecules in all objects are in a state of continual vibration, which we can detect as the temperature of the object, and the atomic vibrations of a quartz crystal can be used as a very accurate timer. The study of repetitive motion is not just an intellectual exercise, but actually enables us to model complicated systems with simple harmonic motion.

In this lab, we will consider spring as an example of oscillation. This oscillation is due to the elasticity of a spring. We will need to measure the stiffness of the spring and relate this to the rate of oscillation.

Most systems have elastic properties, such that when the system is deformed or vibrated, there is a force which tries to restore the system to its original state. If the restoring force is proportional to the displacement from its equilibrium position, then the object is said to be in simple harmonic motion (SHM). A linear restoring force can be expressed mathematically by the equation

$$\vec{F} = -k\vec{x} \quad \text{or as} \quad a = \frac{d^2x}{dt^2} = -\frac{kx}{m} \tag{8.1}$$

where F is the **restoring force**, x is the **elongation** (the displacement from the equilibrium position, which is also called the "zero position"), k is a proportionality constant, and the minus sign indicates that the restoring force is always opposite the direction of the displacement. For a spring system, k is called the **spring constant**, and represents the ratio of the applied force to the elongation. The spring constant is an inherent physical property of the spring itself (an elastic property). The value of k gives a relative indication of the stiffness of the spring. If the spring system is in equilibrium ($\sum F_i = 0$) then the restoring force is equal to the force pulling on the spring, and this force is proportional to the extension of the spring from its equilibrium position. This relationship for elastic behavior is known as Hooke's law, after Robert Hooke (1635-1703).

We can investigate Hooke's law by hanging a mass on a spring, measuring the stretch, and plotting the mass versus the elongation. If we rewrite Equation (8.1) relating the mass to the elongation

$$m = \left[\frac{k}{g}\right] x \tag{8.2}$$

then we see an equation of the form $y = mx + \cancelto{0}{b}$, where the slope depends on both k and g.

Simple Harmonic Motion (SHM) systems can be described by harmonic functions (cosines), where the displacement as a function of the time $x(t)$ can be written as

$$x(t) = A\cos(2\pi f t)$$

where A is the amplitude of the motion, and f is the frequency of the motion in units of cycles per second (sec^{-1}) commonly called a hertz (Hz) after Heinrich Hertz. The period (T, in units of seconds per cycle) equals the inverse of the frequency (f), $T = {}^{1}\!/_{f}$. For a mass on a spring, the period T depends on the physical parameters of the system (the mass, and the spring constant), and can be given by

$$T = 2\pi\sqrt{\frac{m}{k}} \tag{8.3}$$

We can investigate this relationship by bouncing a mass on a spring, measuring the period, and plotting the mass versus the period. If we rewrite Equation (8.3) relating the mass to the period

$$m = [k]\left(\frac{T}{2\pi}\right)^2 \tag{8.4}$$

then we see an equation of the form $y = ax^2 + \cancelto{0}{b}x + \cancelto{0}{c}$, where the coefficient only depends on k. Although this is a nonlinear relationship, we can linearize the expression to find the parameter (slope with uncertainty) more easily. (See Section C.2 for more discussion.) You may also note that $T/2\pi$ is a more convenient variable than T by itself because it produces a slope equal to k rather than $k/(2\pi)^2$.

When you compare these relationships of the spring, you should be able to find a value for the acceleration due to gravity as a verification of these two equations.

8.1 Pre-Lab Considerations

- Make a sketch of your expectation for the displacement of a mass on a spring as a function of the time.
- On this graph, locate and label: the equilibrium positions ($x = 0$), and the places of maximum and minimum velocity.
- Based on the information in the introduction, make a sketch of the pull force as a function of the displacement from the equilibrium position (initial position).

8.2 Procedure

8.2.1 Hooke's law

We will first measure the elasticity of the spring, using Equation (8.1).

- With the available spring, attach it rigidly and hang it vertically against the Dynamics Track. Hang various masses and measure the elongation of the spring, to a maximum of 60 cm. Do not over stretch the spring. Record the bottom end of the mass hanger for the initial reference position. If a tapered spring is used, the small end should be at the top.

- Measure the elongation both when the masses are added and then when they are removed.

  Perfectly elastic objects (possibly your spring) will return to the exact same location when pulled with the same force whether they are being stretched out or being allowed to relax back after stretching.

  Objects that are elastic, but not perfectly elastic, will return to approximately the same location, but might retain some deformation.

- You will be graphing the relationship between the mass and the displacement, Equation (8.2).

8.2.2 Oscillating Spring

We will next consider the periodicity of an oscillating spring.

- With the same range of masses as in Subsection 8.2.1, measure the period of oscillation for each mass. You *can* but do not *have to* use the same values of mass, as long as the set of masses sampled are in the same range.

- You will be graphing the relationship between the mass and the period, Equation (8.4). I recommend using $T/(2\pi)$ as the variable representing the period (because it gives nice results for the graphical parameters – slope and intercept).

- ***Advice***: Keep the amplitude of vibration small, because there is a small but measurable effect with the period as a function of the amplitude.

8.3 Analysis

- Graph both data sets (Subsection 8.2.1 and Subsection 8.2.2) in such a way that the spring constant can be determined graphically (from a linear fit model).

  - When you graph the relationship between the mass and the displacement, recall that Equation (8.1) depends on two specific parameters.
  - When you graph the relationship between the mass and the period, recall that Equation (8.3) depends on one specific parameter.
  - With some effort, you should be able to recognize the units of the slope and intercept and find the relevant values of those parameters.

- Physically interpret the meaning and value for the slopes, and the x and y intercepts for both graphs.

- Calculate the spring constant for both data sets, using a linear regression method.

- So far in the analysis, the mass of the spring has been neglected. How would including the spring mass (or a partial %) affect the slopes or intercepts of the two graphs?

  For the period graph, one would expect to get a zero period with a zero mass. Why? What was your observation for the y-intercept? If the data was modified by adding a constant amount of mass to each mass value (say 1/3 the mass of the spring) and then re-compute the linear regression, then what happens to the slope and intercept values? And do you get a higher linear correlation coefficient?

- If you assume a value for g, then both graphs will give you k. Compare the precision for these two methods.

- If you do not assume a value for g, then you can use one graph to find k and use this calculated value and the other graph to compute g. How does this value of g compare to your expectations?

- Compare the elongations when the masses were added and then removed. Explain any differences. Is your spring perfectly elastic?
- Quantify the major sources of uncertainty in this experiment. Which of the experimental measurements has the largest relative uncertainty?

8.4 Questions

1. Why should the amplitude of vibration be kept as small as possible?

2. Is the spring totally elastic? (Does the elongation return to the same position when the masses are removed?)

3. Based on the data, which method do you think is more precise?

4. Does the force of gravity affect the value of k (as derived from each method)? Why or why not?

5. If this experiment were conducted on the moon, would either method give a different result for the value of k? Explain.

(Revised: Oct 11, 2017)
A PDF version might be found at springs.pdf (122 kB)

Lab 9

The Simple Pendulum

Experimental Objectives

- Determine the relationship between the period of the pendulum and its amplitude.
- Determine the relationship between the period of the pendulum and its mass.
- Determine the relationship between the period of the pendulum and the length of the pendulum.
- Use a graphical analysis to investigate these relationships, and from the best linear graph determine an empirical equation for the period of a pendulum.
- Gravity also plays a part in this experiment, so include gravity into your empirical equation, and use unit analysis to help figure out this relationship.

Introduction

A simple pendulum consists of a small bob of mass (m) suspended by a light (assumed to be massless) string of length (L), and the string is firmly attached at its upper end. This pendulum is a mechanical system which we will assume exhibits simple harmonic motion. That is, the restoring force on the pendulum is proportional to the displacement from the equilibrium position.

Oscillatory motion is one of the most common types of motions and can occur in any physical system. Mechanical systems can experience a periodic motion, and then will vibrate at a natural frequency. This phenomenon is called resonance. Sound is a vibration in the air, which we hear with our ears; light is an oscillation of electric and magnetic fields, which we can see. The atoms and molecules in all objects are in a state of continual vibration, which we can detect as the temperature of the object, and the atomic vibrations of a quartz crystal can be used as a very accurate timer. The study of repetitive motion is not just an intellectual exercise, but actually enables us to model complicated systems with simple harmonic motion.

Galileo (1564-1642) investigated the natural motions of a simple pendulum. From his observations he concluded that “vibrations of very large and very small amplitude all occupy the same time.” Galileo’s time interval of measurement was his own pulse rate. With today’s modern technology we have much more precise measuring instruments. This experiment will investigate the relationships between the physical characteristics of the pendulum and the period of the pendulum.

9.1 Procedure

You will have available for your use: pendulum bobs, string, timers, and a protractor. Be careful to fix the string to a point of support which will not move or vibrate as the pendulum swings. You will test each of the three relationships above (period vs amplitude, vs mass, and vs length). While measuring one relationship, you should ensure that – if they matter – then the other two variables are not varied. For example, when changing the pendulum mass do not vary the pendulum's length or its amplitude.

Some considerations while doing this lab:

- It turns out that the convenient quantity when graphing is not the period, T, but rather $T/(2\pi)$.
- The amplitude of oscillation is the maximum angle which the string makes from the vertical.
- In general when testing the mass or the length, it is best to keep the amplitude of oscillation small.
- When testing any of the relationships, you should measure a few widely-separated values. If these seem to vary significantly, then fill in the gaps between those measurements to make a reliable graph. See Question 9.2.3.
- If you can prove that the period is not affected by one of these variables, then you do not need to worry about keeping it constant while you measure the other variables.
- Your graphical analysis will be better if your graph is linear. Consider Question 9.2.7 for advice on making your graphs.

9.2 Questions

1. Was Galileo's statement precise?

2. Does this pendulum follow simple harmonic motion?

3. How many observations should you take in order to obtain good data?

4. Air resistance gradually decreases the amplitude of the pendulum. What effect does this have on the period of the pendulum?

5. What effect would stretching of the string have on your results?

6. How does gravity affect this experiment? What would happen to the results if this experiment were conducted on the moon?

7. If you have a parabolic graph, such as $y = ax^2$, then you might consider graphing y versus x^2 to get a linear graph. (See also Section C.2.) What is the physical meaning of the slope and the intercept of each of your graphs?

8. Why is it a good idea to keep the amplitude of vibration small?

9. Where to and how should the pendulum length be measured?

(Revised: Oct 11, 2017)
A PDF version might be found at pendulum.pdf (67 kB)

Appendix A

Writing a Lab Report

A formal report will be required of each student for most experiments. However, in some experiments, you will only be required to do a piece of the report so that you can develop a sense of what the sections should include. Reports should be written as if they will be read by a fellow student with similar class experience but who did not experience this particular experiment.

The report is due by the end of the working day on Monday. (You should plan to turn it in before 4:30pm.) If you report is late, then you lose five additional points on every Thursday and Monday after the due date. Reports must be typed and can be submitted on paper or electronically. If you submit the report by email, then I will reply with a "Thank you." If you do not receive the thank-you, then you should assume that I did not receive your report.

Lab reports will be graded on clarity (which includes your overall organization, your use of paragraphs, grammar, and spelling, as well as using the technical terms correctly) as well as on scientific content. The report should contain the following sections (unless otherwise indicated for the specific lab exercise):

— Names of author *and co-experimenters*, name of experiment, date experiment was performed.

Abstract An abstract is a brief summary of your report; it will be easiest to write this after you have written everything else, but it should be placed at the beginning of the report. Although it accompanies your report, it is not considered *part of* your report and should not be referred to by any other section of the report. The abstract should only be about three sentences long (a brief summary) and is usually organized as follows: a simple statement of the objectives of the experiment, followed by a simple statement what you measured in order to achieve that objective, and ending with an indication of how successful your experiment was by citing the most important numerical results (with uncertainty). The important results are those that give evidence for your conclusion. You may also include a relevant percent difference.

The point of the abstract is to tell someone who is *already familiar* with the concept of your experiment how your specific experiment went.

Apparatus A list of the equipment used in the experiment including a description of any new or unfamiliar pieces or of unusual uses of some familiar piece. Diagrams should be given whenever it clarifies either what the equipment looks like or how it fits together to do the experiment.

Theory Although the abstract captures the essence of the concept explored by this exercise, this section should explain the underlying concepts. It should be written to an audience with the background knowledge of one of your fellow students who has been in class with you, but was unable to attend this particular lab exercise. You can treat ideas from class as familiar when you reference them, but some of these may still need an explanation to connect them to this experiment. You should begin by stating the objectives of the experiment, the principle which you are trying to verify. This might not be as simply stated as in the abstract since the goal here is to explain, rather than merely state, how it is that the principle can be proven. Once you have stated the objectives, you will *briefly* give the experiment a conceptual context by discussing the physical laws and concepts involved in reaching your objectives, by defining new terms, and by characterizing physical laws and relevant models. Equations used should be derived in order to relate the form of the equation that expresses the principle (as presented in class) to the form of the equation that will be useful for your calculation. All variables should be identified and it is important to relate the equations used to the physical situations that they represent, while showing how measured variables can be related to the final calculated result.

The point of the theory section is to connect what we knew before the lab to what we have discovered through the lab, to introduce the reader to the important ideas that can be applied to the physical relationships, to connect how the relationships (possibly expressed via equations) might be verified by considering a particular graph, and to compare what we should see if the theory is correct to what we might see if the theory were actually different than we are proposing.

Procedure A brief outline of the experimental procedure. ***This must be in the student's own words***, not copied or even transposed from the instruction sheets or from other laboratory manuals. Apply the general ideas of the theory to justify or explain the specific steps. Note that scientific reports are generally written in the past tense, passive voice: "measurements of the half-life were made using the technique..." rather than the active voice: "we measure the half-life by..."

The points of this section are: to enable the reader to visualize which measurements were made and how they relate to the theory, to allow you to repeat the experiment at a later date, and to provide enough information that another student (who has not done the lab) to reconstruct your experiment.

Data Provide a table of measured and calculated data including uncertainties and units. Show how your calculations were performed. Whenever possible, a graphical record of the data should be given. Even if you did not measure the data in order smallest-to-largest, you should report it this way so that patterns in the data are obvious by glancing down the column.

Analysis Analyze your data based on the predictions of the Theory Section. Describe important features of the data and how they express various features of the theory, such as: Is the graph linear or quadratic? What are the slope and intercept? Is your result reasonable and consistent in the context of the theoretical expectations? Cite uncertainty or %-uncertainty as applicable. Cite %-error or %-difference as applicable. Give a *quantitative* statement of the sources of uncertainty and their effect on the results of the experiment. Explain the steps taken to minimize the uncertainties.

The point of this section is to provide the connection between the data and the theory in order to draw a conclusion in the next section.

Conclusions A brief discussion of what you can conclude *about* the initial assumptions and objectives *based on* the analysis. Reference the theory section as appropriate. Cite the relevant results from your analysis which support your conclusion. Why do these numbers support your conclusion?

Appendix B

Managing Uncertainties

One of the fundamental aspects of science is knowing the reliability of results. The mechanism for gaining this knowledge is first to gauge how well one knows any given measurement and then to propagate this to an indication of the reliability of the results that depend on those measurements. The primary goal in attending to the propagation of the uncertainty is that it allows scientists to determine which measurement is causing the most uncertainty in the result so that future experimenters know which measurement to improve to get an improved result.

In this section we will learn the terminology, determine how to gauge the measurement uncertainty, learn how to propogate this information through a calculation, and learn how to discuss this analysis in your lab reports.

B.1 Experimental Uncertainties, Defining "Error"

Measurements are never exact. For example, if one apple is divided among three people, your calculator will tell you that each person has 0.3333333333 of an apple. A measurement of each slice will tell you two pieces of information: (1) how many 3s to keep and (2) how well you know the final 3. In this example, both 0.33 ± 0.01 and 0.33 ± 0.04 imply that the measurement is accurate to two decimal places, but the first implies that you trust the second 3 more than if you report it as the second number.

> **CAUTION:** Because physicists "know what we mean", they are often sloppy with their language and use the words "error" and "uncertainty" interchangeably.

Some technical terms and their use in physics (which may differ from common use):

accuracy How close a number is to the true (but usually unknowable) result. This is usually expressed by the (absolute or relative) error.

precision How well you trust the measurement. This is vaguely expressed by the number of decimals, or clearly expressed by the size of the (absolute or relative) uncertainty.

uncertainty The **uncertainty in a number** expresses the precision of a measurement or of a computed result. This can be expressed as the **absolute uncertainty** (explained in Finding the Precision of a Measurement), the **relative uncertainty**, or the **percent uncertainty**.

$$\text{relative uncertainty} = \left|\frac{\text{(absolute uncertainty)}}{\text{measured value}}\right|$$

$$\text{\%-uncertainty} = 100\% * \left|\frac{\text{(absolute uncetainty)}}{\text{measured value}}\right|$$

error The **error** is a number that expresses the accuracy by comparing the measurement to an accepted ("true") value. This can be expressed as the **absolute error**, the **relative error**, or the **percent error**.

$$\text{absolute error} = |\text{true value} - \text{measured value}|$$

$$\text{relative error} = \left|\frac{(\text{true value} - \text{measured value})}{\text{true value}}\right|$$

$$\%\text{-error} = 100\% * \left|\frac{(\text{true value} - \text{measured value})}{\text{true value}}\right|$$

difference The **difference** is a number that expresses the consistency of a multiple measurements by comparing one measurement to another. This can be expressed as the **absolute difference**, the **relative difference**, or the **percent difference**. You should notice that since we don't know *which* measurement to trust, we take the absolute difference relative to the *average* of the measurements (rather than choosing one measurement as "true").

$$\text{absolute difference} = (\text{measurement}_1 - \text{measurement}_2)$$

$$\text{relative difference} = \frac{(\text{measurement}_1 - \text{measurement}_2)}{\left[\frac{(\text{measurement}_1)+(\text{measurement}_2)}{2}\right]}$$

$$\%\text{-difference} = 100\% * \frac{(\text{measurement}_1 - \text{measurement}_2)}{\left[\frac{(\text{measurement}_1)+(\text{measurement}_2)}{2}\right]}$$

Note B.1.1 (compare)**.** Whenever you are asked to "compare" values, it is expected that you will not only compute a %-error or %-difference (as appropriate, according to the above considerations); but will also comment on if the uncertainty of the values overlap. Recall that the uncertainty means that your measurement does not distinguish between values within that range, so if the uncertainties overlap, then the values are "the same to within your ability to measure them."

B.2 Writing an Analysis of Error

The conclusion of your lab report should be based on an analysis of the error in the experiment. The analysis of error is one of the most certain gauges available to the instructor by which the student's scientific insight can be evaluated. To be done well, this analysis calls for comments about the factors that impacted the extent to which the experimental results agree with the theoretical value (what factors impact the percent error), the limitations and restrictions of the instruments used (what factors impact the uncertainty), and the legitimacy of the assumptions.

Physicists usually use the phrase "sources of error" (or "sources of uncertainty") to describe how the limits of measurement propagate through a calculation (see Propagation of Uncertainties) to impact the **uncertainty** in the final result. This type of "error analysis" gives insight into the **accuracy** of the result. Considerations for the Error Analysis provides questions that can help you describe which of several measurements can most effectively improve the **precision** of the result so that you can gain insight into the **accuracy** of the result. The accuracy allows one to gauge the veracity (truth) of an underlying relationship, but precision allows you to gauge accuracy. Said another way, a small **percent difference** usually is used to imply a small **percent error**. Said another way, imprecise measurements always *seem* accurate.

B.2.1 Technically, Errors are not Mistakes

Your report should not list "human error" because most students misunderstand this term to mean "places I might have made a mistake" rather than "the limiting factor when using the equipment correctly." Finding the Precision of a Measurement discusses measurement uncertainties as defined above.

In the example of the apple above, the fact that one person has 0.33 ± 0.04 of an apple does *not* reflect a "mistake" in the cutting, but rather reflects that the cutter is limited in their precision. What is important is to use the uncertainty to express how well one can repeated cut the apple into thirds. The absolute uncertainty of 0.04 is generally interpreted to say that most instances (roughly 68%, as explained in Uncertainty of multiple, repeated measurements) of the cutting of an apple in this way will result in having between 0.29 to 0.37 of an apple for any given slice.

When describing the cause of an error (difference from the theoretical value) or of an uncertainty (the extent you trust a number), you can usually categorize this source of error as a random error (a cause that could skew the result too large *or* too small) or as a systematic error (a cause that tends to skew the result in one particular direction).

Random Error An environmental circumstances, generally uncontrollable, that sometimes makes the measured result too high and sometimes make it too low in an unpredictable fashion. Random errors may have a statistical origin – that is, they are due to chance. For example, if one hundred pennies are dumped on a table, on average we expect that fifty would land heads up. But we should not be surprised if fifty-three or forty-seven actually landed heads-up. This deviation is statistical in nature because the way in which a penny lands is due to chance. Random errors can sometimes be reduced by either collecting more data and averaging the readings, or by using instruments with greater precision.

Systematic Error A systematic error can be ascribed to a factor which would tend to push the result in a certain direction away from the theory value. The error would make all of the results either systematically too high or systematically to low. One key idea here is that systematic errors can be eliminated or reduced if the factor causing the error can be eliminated or controlled. This is sometimes a big "if", because not all factors can be controlled. Systematic errors can be caused by instruments which are not calibrated correctly, maybe a **zero-point error** (an error with the zero reading of the instrument). This type of error can usually be found and corrected. Systematic errors also often arise because the experimental setup is somehow different from that assumed in the theory. If the acceleration due to gravity was measured to be 9.52 $^{m}/_{s^2}$ with an experimental uncertainty (precision) of 0.05 $^{m}/_{s^2}$, rather than the textbook value of 9.81 $^{m}/_{s^2}$, then we should be concerned with why the accuracy is not as good as the precision. This is most likely to mean that there is a significant systematic error in the experiment, where one of the initial assumptions may not be valid. The textbook value does not consider the effects of the air. The effects of the air may or may not be controllable, and the difference between the theory and the data may be (within appropriate limits or tests) considered a correction factor for the systematic error.

B.2.2 Considerations for the Error Analysis

In order to help you get started on your discussion of error, the following list of questions is provided. It is not an exhaustive list. You need not answer all of these questions in a single report.

1. Is the error large or small? Is it random or systematic? ... statistical? ... cumulative?
   (a) What accuracy (precision) was expected? Why? What accuracy (precision) was attained? If different, why?
   (b) Was the experimental technique sensitive enough? Was the effect masked by noise?
2. Is it possible to determine which measurements are responsible for greater percent error by checking items measured and reasoning from the physical principles, the nature of the measuring instrument, and using the rules for propagation of error?
   (a) Is the error partly attributable to the fact that the experimental set-up did not approximate the ideal that was required by the physical theory closely enough? How did it fail?
   (b) If a systematic error skews high (low), then is your result too high (low)? Is this a reasonable explanation? Is the size of the skew enough to explain the result?
   (c) What can be done to improve the equipment and eliminate error? How can the influence of environmental factors be diminished? Why is this so?
3. Is the error (deviation) in the experiment reasonable?

Note B.2.1 (compare)**.** Whenever you are asked to "compare" values, it is expected that you will not only compute a %-error or %-difference (as appropriate); but will also comment on if the uncertainty of the values overlap. Recall that the uncertainty means that your measurement does not distinguish between values within that range, so if the uncertainties overlap, then the values are "the same to within your ability to measure them."

B.3 Finding the Precision of a Measurement

B.3.1 Uncertainty of a single measurement

All equipment has a finite precision. For example, if you are stepping off the length of a room by placing one foot in front of the other and counting steps, then you are measuring in “shoe-lengths”. You can likely estimate a half-show length or a third-of-a-shoe-length, but it might be difficult to accurately gauge smaller intervals of a shoe-length. Conveniently for me, my size 11 shoe is 12 in long. So, I can replace a shoe-length with a “foot”. Furthermore, I could replace my measurement technique with a yard-stick (or better yet a meterstick). Since a meter-stick has increments of millimeters on it, it is straightforward to measure the distance to the nearest millimeter. In fact, you can probably guess the nearest half-millimeter. The uncertainty in your shoe-measurement is about 4 in (or about 10 cm = 100 mm). The uncertainty in the meter-stick measurement is about 0.5 mm, significantly better than your shoe.

In addition, to how well the equipment can make a measurement, there is also how well you can gauge the measurement with that piece of equipment. If you measure the length of the room using 12 in rulers, then you will need quite a few of them. It is possible that with so many individual rulers, you will either not measure in exactly a straight line or you might not be able to keep the rulers exactly parallel to each other. Both of these (inappropriate but difficult to control) uses of the rulers will introduce error and, since you can’t necessarily judge if you are doing this (especially since you are trying to not do so), you are introducing uncertainty. You do not necessarily know what errors are actually happening when they are so small. Nonetheless, you need to account for the possibility of such errors in your uncertainty. (Recall the types of error in Technically, Errors are not Mistakes.) In each of the cases mentioned here, the error would necessarily increase the measurement value to a number larger than the actual value. There are other instances (perhaps using a string to measure distance and mis-gauging the tautness of the string) where the value might be systematically small or perhaps randomly distributed.

To get a sense of how large the possible variations are due to this measurement uncertainty, it is advisable to always take more data. Having more data is always better (statistically). If you measure a quantity many times and you do not see any variation, then the precision of the instrument dominates the uncertainty; that is, your instrument is less precise than your technique. If you measure a quantity many times and see some variation, then your measurement technique dominates the uncertainty; that is, your technique is less precise than your instrument. Usually the actual uncertainty will be a combination of both. For our purposes, we will consider 10-20 measurements to be “many”, and hope it is sufficient. If you are asked to do a lab exercise on “Standard Deviation”, then you will explore what a “sufficient number of measurements” means.

In the next subsection (Uncertainty of multiple, repeated measurements), we will discuss how to do the statistics to account for multiple measurements.

B.3.2 Uncertainty of multiple, repeated measurements

Calculate or estimate the precision of a measurement by one or more of the following methods:

1. by the precision of the measuring instrument, and take into account any uncertainties that are intrinsic to the object itself;

2. by the range of values obtained, the minimum and/or maximum deviation (d);

$$d_i = |X_i - X_{\text{ave}}|$$

3. by the standard deviation, which is the square root of the sum of the squares of the individual deviations (d) divided by the number of readings (N) minus one;

$$\sigma = \sqrt{\frac{1}{(N-1)}\sum d^2} = \sqrt{\frac{1}{(N-1)}\sum_i |X_i - X_{\text{ave}}|^2}$$

4. by the standard deviation of the mean, which is the standard deviation divided by the square root of the number of readings;

5. by the square root of the number of readings ($\sqrt{N}$), if N is considered large;

If many data points were taken and plotted on a histogram, it would smooth out and approach the symmetrical graph typical of the binomial distribution (see the Figure B.3.1). This distribution and many others in statistics may be approximated by the gaussian distribution.

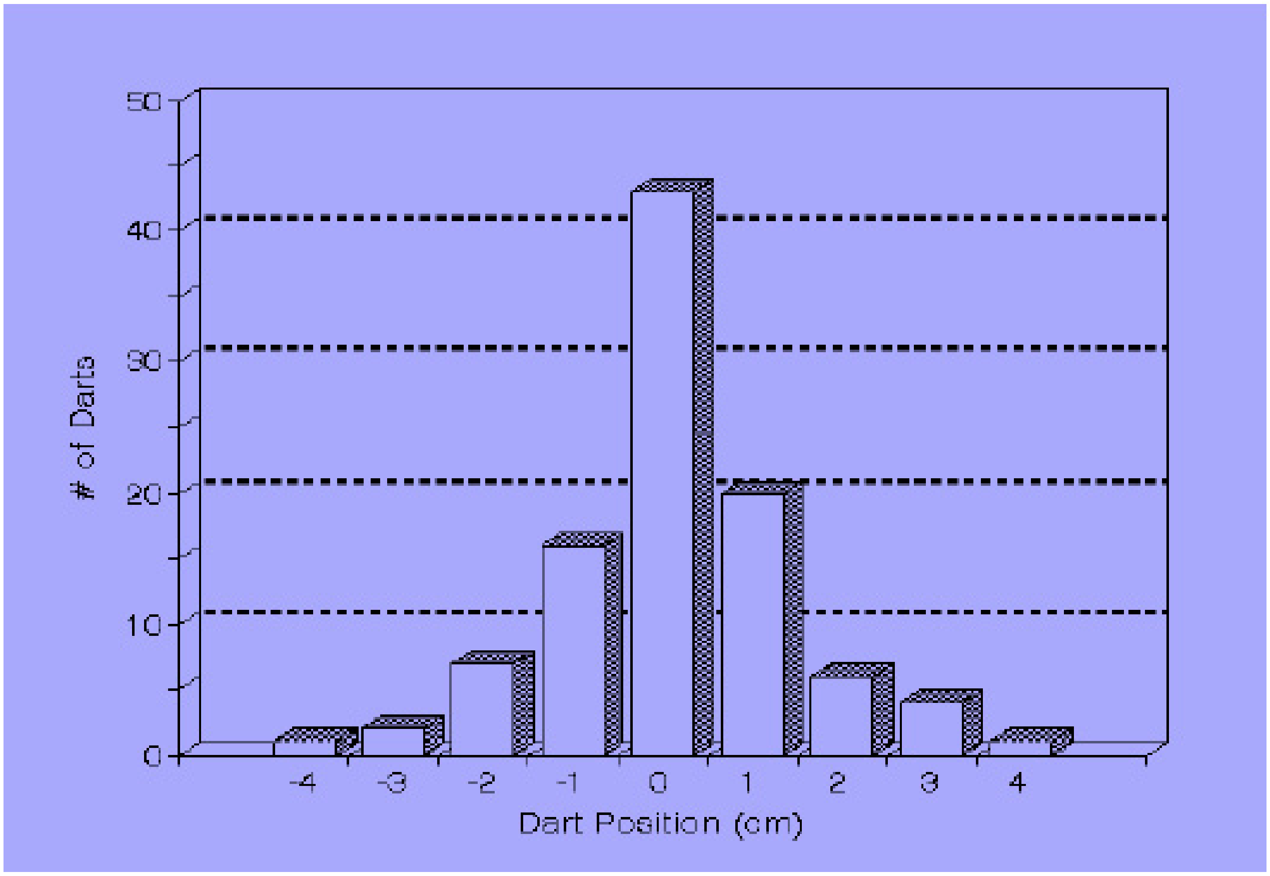

Figure B.3.1: A sample histogram for the number of darts binned by distance from the centerline.

The standard deviation, σ, can be estimated from the above graph. It is a measure of the "width" of the distribution. For the case shown, the standard deviation has the value of five. The greater the standard deviation, the wider the distribution and the less likely that an individual reading will be close to the average value. About 68% of the individual readings fall within one standard deviation (between 45 and 55 in this case). About 96% of the readings fall within two standard deviations (between 40 and 60 in this case).

As more and more readings are taken, the effect of the random error is gradually eliminated. In the absence of systematic error, the average value of the readings should gradually approach the true value. The smooth curve above was drawn assuming that there was no systematic error. If there were, the graph would merely be displaced sideways. The average value for the number would then be say 55.

The distribution of many average (mean) readings is also gaussian in shape. Comparing this to the distribution for individual readings, it is much narrower. We would expect this, since each reading on this graph is an average of individual readings and has much less random error. By taking an average of readings, a considerable portion of the random error has been canceled. The standard deviation for this distribution is called the standard deviation of the mean (σ_m). For this distribution, 68% of the averages of the readings are within one standard deviation of the mean, and 98% of the average readings fall within two standard deviations of the mean.

The standard deviation of the mean tells how close a particular *average* of several readings is likely to be to an overall average when many readings are taken. The standard deviation tells how close an *individual* reading is likely to be to the average.

There is one case for which the standard deviation can be estimated from one reading. In counting experiments (radioactivity, for example), the distribution is a Poisson distribution. For this distribution, the standard deviation is just the square root of the average reading. One reading can give an estimate of the average, and therefore, give an estimate of-the standard deviation.

B.4 Propagation of Uncertainties

The previous sections discussed the uncertainties of directly measured quantities. Now we need to consider how these uncertainties affect the rest of the analysis. In most experiments, the analysis or final results are obtained by adding, subtracting, multiplying, or dividing the primary data. The uncertainty in the final result is therefore a combination of the errors in the primary data. The way in which the error propagates from the primary data through the calculations to the final result may be summarized as follows:

1. The error to be assigned to the sum or difference of two quantities is equal to the sum of their absolute errors.

2. Relative error is the ratio of the absolute error to the quantity itself. The relative error to be assigned to the product or quotient of two quantities is the sum of their relative errors.

3. The relative error to be assigned to the power of a quantity is the power times the relative error of the quantity itself.

These rules are not arbitrary, but rather they follow directly from the nature of the mathematical operations. These rules may be derived using calculus.

Exercise B.4.1 (Try Propagating the Uncertainty When Adding Numbers)**.** Compute the perimeter of a table that is measured to be $176.7\,\text{cm} \pm 0.2\,\text{cm}$ along one side and $148.3\,\text{cm} \pm 0.3\,\text{cm}$ along the other side.

Hint 1. To find the perimeter, add the four sides of the rectangle. Use the values, but not the uncertainty.

Hint 2. To find the uncertainty, use Rule 1.

Answer. The perimeter is $P = 650\,\text{cm} \pm 1\,\text{cm}$.

Solution. The perimeter can be found as:

$$P = (176.7\,\text{cm}) + (148.3\,\text{cm}) + (176.7\,\text{cm}) + (148.3\,\text{cm})$$
$$P = 650.0\,\text{cm}$$

but we do not know the precision (appropriate number of decimals) until we compute the uncertainty, which is

$$\Delta P = (0.2\,\text{cm}) + (0.3\,\text{cm}) + (0.2\,\text{cm}) + (0.3\,\text{cm})$$
$$\Delta P = 1.0\,\text{cm}$$

The value of the uncertainty determines where you round the result. Because the first digit of the uncertainty is in the "one's place", we round *both* the value and the uncertainty to that place.

The perimeter is $P = 650\,\text{cm} \pm 1\,\text{cm}$.

Exercise B.4.2 (Try Propagating the Uncertainty When Multiplying Numbers)**.** Compute the area of a table that is measured to be $176.7\,\text{cm} \pm 0.2\,\text{cm}$ along one side and $148.3\,\text{cm} \pm 0.3\,\text{cm}$ along the other side.

Hint 1. To find the area, multiple the length and width of the rectangle. Use the values, but not the uncertainty.

Hint 2. Because the area of the table is calculated using multiplication, use Rule 2to find the uncertainty.

Answer. The area is $A = (2.620 \times 10^4) \pm (0.008 \times 10^4)\text{cm}^2$.

Solution. The area is found to be (significant digits are underlined)

$$A = (176.7\text{cm}) \times (148.3\text{cm})$$
$$A = \underline{2620}4.61\text{cm}^2$$

The rules for **significant figures** gives a guide for the precision (appropriate number of decimals), that is only an approximation. To know with certainty, we need to compute the uncertainty, which is

$$\begin{aligned}\%\text{-uncertainty} &= \left(\frac{.2\text{cm}}{176.7\text{cm}}100\%\right) + \left(\frac{0.3\text{cm}}{148.3\text{cm}}100\%\right)\\ \%\text{-uncertainty} &= (.11\%) + (.20\%) = (.31\%)\end{aligned}$$

Insignificant Please be aware that the reason some digits are called **insignificant** is that they are *insignificant*:

$$\begin{aligned}(.31548\%) \times (26204.61) &= 82.67\\ (.31\%) \times (26204.61) &= 81.23\\ (.31\%) \times (26200) &= 81.22\\ (.3\%) \times (26204.61) &= 78.61\\ (.3\%) \times (26200) &= 78.60\end{aligned}$$

All of these round to an uncertainty of $80\,\text{cm}^2$.

To find the uncertainty, we calculate

$$(.31\%) \times (\underline{2620}4.61\text{cm}^2) = 81.23\text{cm}^2$$

This tells us that we need to round at the "ten's place". We can write the area in a variety of ways:

$$\begin{aligned}A &= (2.620 \times 10^4\,\text{cm}^2) \pm 0.3\%\\ &= (2.620 \times 10^4) \pm (0.008 \times 10^4)\,\text{cm}^2\\ &= 2.620(8) \times 10^4\,\text{cm}^2\end{aligned}$$

B.5 Significant Figures only *approximates* Uncertainty

The precision/accuracy of any measurement or number is approximated by writing the number with a convention called using **significant figures**. Every measuring instrument can be read with only so much precision and no more. For example, a meter stick can be used to measure the length of a small metal rod to one-tenth of a millimeter, whereas a micrometer can be used to measure the length to one-thousandth of a millimeter. When reporting these two measurements, the precision is indicated by the number of digits used to express the result. You should always record your data and results using the convention of significant figures.

To give a specific example, suppose that the rod mentioned above was 52.430 mm long. When making this measurement with the meter stick, you would count off the total number of millimeters in the length of the rod and then add your best guess that the rod was four-tenths of a millimeter longer than that. Using the micrometer, you would count off the hundredths of a millimeter and then add your best guess of the number of thousandths of a millimeter, to complete the measurement. How would you communicate the fact that one measurement is more precise than the other? If you wrote both quantities in the same way, you could not tell which was which.

The rules for **significant figures**:

1. Significant figures include all certain digits plus the first of the doubtful digits. (Note Convention B.5.1.)
2. Zeros to the right of the number are significant; zeros on the left are not. (Note Convention B.5.2 and Example B.5.3.)
3. Round the number, increasing by one the last digit retained if the following digit is greater than five. (Note Convention B.5.4.)
4. In addition and subtraction, carry the result only up to the first doubtful decimal place of any of the starting numbers.

5. In multiplication and division, retain as many significant figures in the answer as there are in the starting number with the smallest number of significant figures.

Convention B.5.1 (One doubtful digit)**.** (Note Rule 1.) The reading obtained from the meter stick would be written as 52.4 mm; all digits up to and including the first doubtful digit. The reading from the micrometer would be written as 52.430 mm. The first doubtful digit in the case of the meter stick is .4 mm. The first doubtful digit in the case of the micrometer is zero-thousandths of a millimeter. Note that the number of significant figures is related to the precision of the measuring instrument – it is not an abstraction about the number.

Sometimes the character zero is confusing. See Rule 2.

Convention B.5.2 (Dealing with Zero)**.** (Note Rule 2 and Example B.5.3.) In the example above, the reading of the micrometer was given as 52.430 mm. The zero is a significant figure, it communicates the fact that the micrometer measurement is good to a thousandth of a millimeter. Zeros used to the left of significant digits to position the decimal point are not significant. They are not communicating the precision of the measurement. For example, if the measurement from the meter stick were written as 0.0524 meter, the zeros would not be significant digits. Because of the units (meters instead of millimeters), the decimal point had to be moved to the left. This measurement still has only three significant figures.

Example B.5.3 (Handling Zero)**.** Suppose that you wished to give the meter stick measurement in terms of microns (μ) (1 micron = 1 millionth of a meter). We determined that the meter stick reading has 3 sig. figs., one good way to write this is to use scientific notation. Write the number as 5.24×10^4 microns. The factor of 10^4 shows the order of magnitude, while the 5.24 retains the proper number of significant figures. Study the following examples:

$5.24\,\mathrm{cm}$	3 significant figures
$52.4\,\mathrm{mm}$	3 significant figures
$0.0524\,\mathrm{m}$	3 significant figures
$5.24 \times 10^4\,\mu\mathrm{m}$	3 significant figures
$52.430\,\mathrm{mm}$	5 significant figures
$0.052430\,\mathrm{m}$	5 significant figures
$5.2430 \times 10^4\,\mu\mathrm{m}$	5 significant figures
$52430\,\mu\mathrm{m}$	4 significant figures
$52430.\,\mu\mathrm{m}$	5 significant figures

Because all measurements are limited in their precision, then all results derived from these measurements are also limited in their precision. Many students get carried away with the number of digits produced by a calculator and mislead the reader by reporting their results with more significant figures than their data permits. Form the habit of rejecting all figures which will have no influence in the final result and report the result with only the number of significant digits allowed by the data. The following rules will help you do this successfully.

Convention B.5.4 (Rounding)**.** (Note Rule 3.) Because the use of significant digits is meant to be an approximation to the more precise use of measurement uncertainty, the convention for the rounding of the final digit is not critical and is treated a little differently by different scientists. Some round-up anything five-or-more. Some round-down anything five-or-less. Some round the five up or down based on whether the next digit is even or odd. This third choice is intended to express that we don't actually know which way to round it and letting the randomness of the next digit make the determination is like flipping a coin. ***You should ask your instructor, which approach they prefer.***

When determining or estimating the experimental uncertainty, the precision of the measuring instrument is important, as shown in the above examples. But you must also be aware of other experimental factors. For example, a good stopwatch may have a precision of 0.01 seconds. Is this the total uncertainty of the measurement? You must remember that our physical reaction time maybe another 0.3 seconds. This is more than 10 times larger than the precision of the timer. This is very significant. Another example is trying to measure the diameter of a fuzzy cotton ball with a micrometer. Why is this not a very productive procedure?

There are major uncertainties here that are intrinsic to the object itself and are unrelated to the measuring instrument. One must use common sense when estimating these uncertainties.

The **actual uncertainty** written in the units of the measurement, may not convey a sense of how good the precision is. A better measure of the precision is given by the relative uncertainty. This is defined as the actual uncertainty divided by the measurement itself and multiplied by 100, the **relative uncertainty** does not carry any units, just a %.

Appendix C

Discovering Relationships – Graphical Analysis

The primary purpose of experimentation is to discover relationships between various physical quantities. This is usually best achieved with a graphical analysis. Graphs of data and/or graphs of other results can be very enlightening. We often try to choose to plot variables for a graph so that the resulting relationship is linear, whose slope and intercept may be of physical interest.

C.1 Linear Relationships

When two quantities are related, there are many, many possible mathematical relationships. The simplest relationship is the direct proportion. This relationship is represented on a graph by a straight line which goes through the origin. The linear relationship is similar, it however, may have an intercept with a coordinate axes. The general equation for a straight line is:

$$y = mx + b,$$

where x and y are the plotted quantities, m is the slope, b is the y-intercept. The slope and intercepts of a linear relationship often have physical significance. It is therefore very important to calculate the slope and intercept and then for you to interpret their meaning, and always give the units of the slope and intercept. The slope can be computed by choosing two places, (x_1, y_1) and (x_2, y_2), on the straight line. The slope is then given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

C.2 Linearizing Non-Linear graphs

Most relationships which are not linear, can be graphed so that the graph is a straight line. This process is called a linearization of the data. This does not change the fundamental relationship or what it represents, but it does change how the graph looks. The advantage of linearizing non-linear data is that the analysis of the parameters (slope and intercept) becomes significantly easier. Linear regression, which allows us to compute uncertainties in the slope and intercept as well as evaluating deviations from the equation (with residual plots), can be done by Pasco Capstone and Microsoft Excel; but, nonlinear regression requires specialized statistical software or lots of additional formulaic computations.

For example, the equation

$$X \times Y = \text{constant}$$

represents an *inverse proportion* between X and Y. A graph of this equation is not a straight line with a negative slope. Think about this and sketch the curve for yourself. This relationship can be graphed in such a way so that the new graph is a straight line. This change is accomplished by choosing a new set of axes, and plotting new numbers which are related to the original set. In this case if we would plot $1/X$ on the x-axis instead of just X, this will yield a straight line graph. Try it.

As another example, the equation

$$y = ax^2$$

represents (a special case of) a *quadratic relationship* between x and y. A graph of this equation is not a straight line; however, this relationship can be graphed in such a way so that the new graph is a straight line. This change is accomplished by choosing a new set of axes, and plotting new numbers which are related to the original set. In this case, if we would plot (x^2) on the x-axis instead of just x, this will yield a straight line graph. Try it.

There are many other possible relationships which are easy to linearize. These include: exponential function, trigonometric functions, and power functions (squares, square roots, etc.) A change of either the x or y-axis may linearize a function for you.

To linearize the *power relationship*

$$Y = Bx^M,$$

take the natural logarithms of both sides to obtain

$$\ln(Y) = \ln(B) + M\ln(X).$$

If one plots $\ln(Y)$ on the vertical and $\ln(X)$ on the horizontal (a "log-log" graph), then the graph of this function yields a straight line with slope M and intercept $\ln(B)$.

To linearize the *exponential relationship*,

$$Y = Be^{MX},$$

take the logarithm of both sides to obtain

$$\ln(Y) = \ln(B) + MX,$$

and again a graph of this function yields a straight line graph. If one plots $\ln(Y)$ on the vertical and X on the horizontal (a "semi-log" graph), then the graph of this function yields a straight line with slope M and intercept $\ln(B)$.

Appendix D

Using Capstone

Figure D.0.1: This is a PDF of a Power-Point description for how to use the important features of Capstone.

You may also download the PDF-version (2.5MB) or the power-point version (2.7MB) of this document.