Physics Connected

Learn like you think — an interconnected view of algebra-based physics

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Preface

This text is intended for a one or two-semester undergraduate course in introductory algebra-based physics.

The purpose of creating this book is to make better use of the technology that electronic texts allow for without losing the functionality of a print book. While this text should be comparable to any other print text, when this is provided in the online format it will provide links back and forth between early and later topics. Linking from later material to earlier material will allow students to refresh their memory of what was previously discussed. Linking from earlier material to later material will inspire students to look ahead to how that topic will be used in more interesting scenarios.

Having these links will allow for some other interesting features that can be placed in the back of the book and accessed through links. Examples of this might be:

1. "Dig Deeper" where some of the more tedious and some of the more interesting aspects can be investigated. For example in 5 on the equations of motion, one might see how these equations are direct applications of calculus for those students who happen to have taken that course (which is common for biology and pre-medical students).

2. "Every Equation Tells a Story" which discusses how the description-in-English and the description-with-math interrelate to build intuition in both directions.

3. "Examples", with the difference from a traditional textbook being that students can interact with the example as: "If you have this question, then go here. If you have that question, then go there."

4. "In the 'Real World"' where students see how the concept lives in the messy real world and why physicists simplify or ignore complicating aspects.

5. "Connections", which might take one of three forms:

   (a) *Looking Back* which links back to earlier material to address the question "Where have I seen this before?"

   (b) *Looking Ahead* which links ahead to later material to address the question "When will I ever use this?"

   (c) *In Detail* which links to popular or complex topics to address the question "Why is this interesting?"

The goal of the book is to encourage curiosity in the reader. Since there is an expectation that students will explore the material on their own, advanced topics will explicitly note where the reader can look for supporting material and basic topics will be motivated with links to more advanced topics. To help maintain the interest of the reader, recurring characters will be featured in the examples. These characters will live a storyline and interact with each other. It is possible to read the examples as a separate storyline for the N interacting characters.

I am choosing the approach described above based on the assumption that students will prefer to develop their knowledge by building a world-view that connects to their current understanding, their interests, and their world-view. Providing the cross-referencing links without distracting students with all of the information at once will enable them to explore the information. Writing the text in a narrative style that helps students see the explanations for the world they live in will encourage them to explore "what happens when I do this" in their real life. Fostering this spirit of exploration will enable the instructors to bring their own active-learning techniques into the classroom.

This textbook is in several Parts: *Part I* is for the preliminaries, including descriptions of science in general, physics in particular, and the use of math. *Part II* is intended to introduce three fundamental and powerful concepts. These concepts are motion, force, and energy. I have found that if a student can understand these ideas sufficiently well, then they can quickly pick up any other idea that we introduce, even if the idea seems initially unfamiliar. *Part III* develops the ideas in Part II by introducing momentum, circular motion, rotational motion, torque, and the Newtonian theory of gravitation. *Parts IV and V* are oscillations and thermodynamics. With the traditional organization of the two-semester introductory physics, these parts can be covered in either order and can be chosen to be put in either semester. *Part VI* covers electricity, magnetism, light, and optics. This is traditionally the meat of the second semester. *Part VII* touches on the topics that are usually referred to as "modern physics". The goal with including these chapters is to provide some inspiration for what some students see as the tedium of the standard material. These chapters will be linked to throughout the book as examples of how the traditional material supports the material that may be in the news and is more talked about in popular science. The last final part, *Part VIII*, holds the answers to the interactive examples mentioned above, the bulk of the adventures the reader can investigate in order to test their understanding of the material, and the story lines of each of the characters in the text.

> ***A note about viewing the PDF online:*** If you are viewing this as a PDF set to view "single page," then the links will take you to the top of the relevant page, rather than to the specific topic. If, on the other hand, you are viewing this in "continuous view" then you should go directly to the location of interest. If you are viewing this in "two-page" mode (whether continuous or not), it might not be immediately obvious to which page (left or right) you have jumped. Most of the PDF viewers I have encountered allow you to follow links and to return to your previous location. On most PCs, the way to return to your previous location is by holding the `ctrl` key and pressing the [←] (`Left-Arrow`) button. There are a few PDF viewers that do not allow you to "go back" to the location you linked from. Whether or not you have that capability, I have placed "return links" in the margins so that you can get back to the place from which you linked.

Contents

Part I

Prerequisites

The chapters in this Part are the preliminaries, including descriptions of science in general, physics in particular, and the use of math.

Chapter 1

The Story of Science

Once upon a time somebody saw the world around them and thought something equivalent to "well, that's an interesting pattern..." and predictions were born. Every human and many animals build their own world of expectations such as: objects will fall down, food will arrive at mealtime, or certain people will smile at me. Scientists study the patterns in the world around us and do so in a fairly specific way. Novelists, sociologists, historians, and cartoonists also look at the world around us in a very particular way. The story of humanity is a story about observing the world around us.

Scientists, in general, observe patterns through careful, detailed measurements ...

Physicists, in particular, consider the patterns in the physical world around us.

Some patterns that you might experience help us take very different experiences and group them together. For example, there are ways in which dropping your keys and throwing a dog toy are very similar. They both fall, even thought the fall along rather different paths. There are also patterns that you experience that might look very similar but can be treated very differently. For example, the path of baseball pitch (See Investigation 6.2.1.) is very different for a fast ball compared to a slider, a curve ball, or a knuckleball.

images/i-BoonePark.jpg

Figure 1.0.1: Life is full of examples of physics all around us.

1.1 Careful, Detailed Observation

[Discussion of "casual observer" as intuition versus "scientific observing" and mathematical modelling]

Paragraph referenced by Clarification of Newton's laws 7.2.1, discussion of simple piece of a complicated situation

[Discussion of common student comment: "in physics class it is this way, but in *real life* it is that way."]

1.2 Theory versus Law

Section referenced by Section 7.2

List of examples

Chapter 2

Seeing Physics

What you will find in this book is a series of chapters that, on the surface, feel like a list of isolated topics. Each chapter will have examples that focus your attention on examples of that specific concept. However, the really interesting aspect of physics is that these descriptions of the world around us come together in different ways to explain complex systems that might feel unrelated. For example, the thermodynamics of making your refrigerator work on Earth comes from the same theories of thermodynamics that help us understand the heat flow of the sun. Furthermore, in order to understand the sun, we also need to understand the gravitational interaction, which also describes how baseballs fly through the air.

This chapter will introduce a set of quick-overview explanations of phenomena to indicate how different ideas tie together in some complex systems. The point is specifically to over-simplify complex ideas in order to "get the idea". You will also be pointed to the various chapters that go into the details of the relevant physics where you can learn more. Then, at the end of the book in Chapter 33, we will revisit each of these ideas and go into the description in more depth assuming you have understood each of the relevant chapters, with reference back to the sections that provide the basis of our understanding.

Warning 2.0.1. Since this particular chapter is intended to be background introduction, rather than a place to study details, none of the links to other sections here will have return links in the rest of the text. So, if you intend to use this as a jumping off point, you might want to create a bookmark here so that you can return after you read the details in other sections.

2.1 The Flame Challenge and Other Brief Descriptions

The point of this section is to whet your whistle, to provide an appetizer before the main meal, to inspire your curiosity. This section will have very brief descriptions, inspired by The Flame Challenge. Since that is the inspiration, we will start by referencing it explicitly. After that you will find some brief explanations for complex ideas.

2.1.1 The Flame Challenge

- The Flame Challenge
- Useful? How Alan Alda Makes Science Understandable
  - 2012: What is a flame?
  - 2013: What is time?
  - 2014: What is color?
  - 2015: What is sleep?
  - 2016: What is sound?
  - 2017: What is energy?

2.1.2 The Forming of Matter in the Universe

In the early ages of the universe, which is an entirely different story that could be told, there were a ridiculously large number of particles created and drifting around. There were a variety of types (Subsection 29.3.4), some being positively charged (Section 22.1), some negatively charged, and some were neutral; but the larger ones tended to gradually decay (Subsection 29.3.5) into smaller ones. The smaller of the positively-charged baryons (Section 29.3), which we call protons, and the smallest of the negatively-charged leptons (Section 29.3), which we call electrons, also tended to stick together because of their electrical charges (Section 22.1), forming hydrogen atoms. You may note that as this happens, sometimes the more ambitious of the particles form larger clumps of two protons and two neutrons, making helium atoms that are held together by the strong nuclear force ([need ref])

2.1.3 Things in the Sky

Example 2.1.1 (*The Sun*)

The bright, shiny sun, which keeps us all alive, is a nice example of a rather complex system that allows us to verify our various theories of the world around us. As an over-simplification of the process, we can consider the existence of a star in three phases: the ignition (some have said "birth") of a star, the shining (some would say "life") of the star, and the snuffing ("death"?) of the star.

2.1.4 Things on the ground

Example 2.1.2 (*Hot Tea and Iced Tea*)

Example referenced by *Section 16.3*

On 28 April, 2017, CBC Broadcasting published a Quirks and Quarks episode discussing why hot water sounds different from cold water when they are poured. Spoiler Alert: It is due to surface tension, size of droplets when heated, and auditory perception.

2.1.5 Kitchen Appliances

Example 2.1.3 (*Oven*)

...

Example 2.1.4 (*Refrigerator*)

...

Example 2.1.5 (*Microwave*)

...

Example 2.1.6 (*Television*)

...

2.1.6 Automobile

Example 2.1.7 (*Coolant and Antifreeze*)

...

Example 2.1.8 (*Tires*)

...

Example 2.1.9 (*Torque*)

...

2.1.7 Cool Ideas

Example 2.1.10 (*Black Holes*)

...

Example 2.1.11 (*Quantum Mechanics*)

...

Example 2.1.12 (*Relativity*)

...

Example 2.1.13 (*String Theory*)

...

Example 2.1.14 (*Fusion*)

On 28 April, 2017, CBC Broadcasting published a Quirks and Quarks *episode discussing a documentary compares the massive scale ITER approach to fusion with the much smaller approach by a Canadian company.* I don't think I want to use this, but it might be helpful to listen again to the nice summary of fusion. *Maybe get some resources on "state of the art".*

2.2 Effective Theory

All of our explanations are approximations. This section will describe some physics in the world around us in one or two paragraphs with links to the sections in the book that provide the detailed understanding of that piece which connects to the mathematics and the underlying foundation. Each topic will also link to a more detailed discussion at the end of the book with a longer conversation that gets into more nitty-gritty details which assume you have learned the details from the book. In short, this section looks forward to what is possible to understand and that chapter looks back at how you do understand. Each of these topics will also be accompanied by a five-minute podcast describing the topic.

The term **effective theory** is used in physics to describe a wide-reaching phenomenon which can be approximated by a simpler theory in a smaller circumstance. So, for example, Einstein's theory of general relativity as a complex description of the gravitational interaction. It would be unwieldy and impractical to use that to describe our day-to-day interactions with the gravitational interaction. On the other hand, Newton's theory of the gravitational interaction is a special case of Einstein's general theory of relativity that works perfectly well so long as you behave yourself and do not try to travel at a significant fraction of the speed of light. We can say that Newton's theory of gravity is an effective theory for Einstein's theory of gravity that accounts for acceleration at low speeds. Likewise, Einstein's special theory of relativity is an effective theory of the general theory of relativity. The special theory is relevant when you do not allow for acceleration, but do allow for faster speeds. Once you reach beyond the limitations of the effective theory, the description "breaks down".

For example, see the discussion around Convention 8.1.11 regarding the acceleration due to gravity being "locally constant".

List of examples

(Continued on next page)

Chapter 3

Why so much math?

Mathematics is its own language. It is the language of patterns. Humans are very adept at tracking patterns. Physics is the study of patterns in the physical world. It turns out that the language of physics provides a natural and concise mechanism for expressing patterns in a uniquely precise manner.

3.1 Every equation tells a story

Equations allow us to connect physical reality to very specific predictions. For example, the equation for thermal conductivity, Equation (21.3.1) in Subsection 21.3.1, allows Abdul to predict the time it takes for his oven to warm up to a specific temperature because $\frac{Q}{\Delta t} = \kappa A \frac{\Delta T}{\Delta x}$ says that the rate at which energy flows depends on how well air allows energy to flow, the size of the oven, and the amount the temperature needs to change across the height of the oven as follows:

$\frac{Q}{\Delta t}$	$=$	κ	A	$\frac{\Delta T}{\Delta x}$
the rate at which energy flows	depends on	how well air allows energy to flow,	the size of the oven,	and the amount the temperature needs to change ――― across the height of the oven

Table 3.1.1: An example of how the math describes the "story-of" a physical situation.

We will see this particular story in more detail with Example 21.3.1 when Abdul prepares to bake some bread for his friends. Some of the more important equations are listed below. By jumping between these narratives, you can get a better sense of how to think about physics in general.

Equation	Location
$\vec{F}_{\text{net}} = m\vec{a}$	Translation 7.2.9
$F_f = \mu F_N$	Translation 8.4.11
$\vec{v} = \vec{\omega} \times \vec{r}$	Translation 11.1.1

Table 3.1.2: Locations of the "Story of" various equations

3.2 The Metric System

Section referenced by Subsection 8.1.1

The International System of Units (SI) [3.5.1] was adopted in 1960 at the eleventh meeting of the International Bureau of Weights and Measures (BIPM). [1]

In 1901 at the third meeting of the BIPM, it was declared that

1. The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram;
2. The word "weight" denotes a quantity of the same nature as a "force": the weight of a body is the product of its mass and the acceleration due to gravity; in particular, the standard weight of a body is the product of its mass and the standard acceleration due to gravity;
3. The value adopted in the International Service of Weights and Measures for the standard acceleration due to gravity is 980.665 cm/s^2, value already stated in the laws of some countries.

Other related decisions were made at the following meetings:

1. The 11th meeting (1960) redefined the meter in terms of wavelengths of light.
2. The 13th meeting (1967) redefined the second in terms of the frequency of radiation from ^{133}Cs.
3. The 17th meeting (1983) redefined the meter in terms of the speed of light and seconds.
4. The 24th (2011) and 25th (2014) meeting discussed redefining the kilogram in terms of the Planck constant, with an expectation that it will be redefined at the 26th meeting (Nov, 2018). See the references in Subsection 3.2.3 for more information.

Note Handbook 44, page B-6 talks about SI.

Note Handbook 44 webpage still links to the 2016 pdf instead of the the 2017 pdf even though it says it was updated in 2017.

There is also a special publication from NIST that summarizes the use and conversation between units in the SI.

3.2.1 Units Quantify Dimensions

In order to understand how big something is, we need to know what scale we are using. For example, Your thumb is about an inch across, but your foot is about a foot long. Both have the "amount" 1. but the unit is different. In this sense, $1 + 1$ might not equal 2.

$$1\,\mathrm{ft} + 1\,\mathrm{ft} = 2\,\mathrm{ft}$$
$$1\,\mathrm{in} + 1\,\mathrm{ft} \neq 2\,??$$

For the second line, you need to convert the units:

$$1\,\mathrm{in} + 1\,\mathrm{ft} = 1\,\mathrm{in} + 1\,\mathrm{ft} \times \left(\frac{12\,\mathrm{in}}{1\,\mathrm{ft}}\right) = 1\,\mathrm{in} + 12\,\mathrm{in} = 13\,\mathrm{in}$$

because we need to be speaking about the same thing. You have all had the experience of a miscommunication when you thought you were speaking about the same thing, but later found out you each meant something different.

Warning 3.2.1 (Clarify your intent)**.** You should always include every unit on every number throughout your entire math solution. This will help you ensure that the units are self-consistent and will occasionally help you either recognize which algebraic step to take next or even which equation to use in the first place.

After introducing some of the essential metric units here, we will build some intuition by considering how to convert from your likely more familiar English units in Subsection 2 and then build these fundamental units into the broader derived units that help express the relationships between interesting quantities in Subsection 3.

[1] In French this organization is the Bureau International des poids et mesures, so the acronym is BIPM.

Investigation 3.2.2 (*Identify familiar dimensions*)

(a) *What can be measured?*

Length feet, inches, (BASE 12?); mm, cm, m, miles (What's that number again?), kilometers, AU

time seconds (metric prefixes), minutes[2], hours (base 60? REALLY?)[3]

mass kilogram, slug (see stone!)

charge Coulomb

temperature? too soon?

A cooking website that nicely distinguishes weight from volume (ounce!): http://bowlofplenty.blogspot.com/2009/01/how-much-does-water-weigh.html

3.2.2 Conversion from English Units

Subsection referenced by Section 4.2, Exercise 5.3.1

Note internet search comments in Subsection 8.1.1 regarding the "conversion" of kilograms-to-pounds, with special attention to (See significant digits 4.2.).

A cooking website that nicely distinguishes weight from volume (ounce!): http://bowlofplenty.blogspot.com/2009/01/how-much-does-water-weigh.html

Example 3.2.3 (*Fahrenheit and Centigrade*)

Example referenced by

The $\frac{9}{5}$ *is actually* $\frac{180\,^\circ\mathrm{F}}{100\,^\circ\mathrm{C}} = \frac{212\,^\circ\mathrm{F}-32\,^\circ\mathrm{F}}{100\,^\circ\mathrm{C}-0\,^\circ\mathrm{C}}$ *as in slope is rise over run.*

3.2.3 Fundamental Units versus Derived Units

Subsection referenced by Subsubsection 7.2.2.1, Subsection 8.1.1

Note conversation in Subsubsection 7.2.2.1 about the Newton.

See the 2012 article from SciTechDaily.com and the NIST explanation about redefining the kilogram in terms of the Planck constant at the 26th meeting (Nov, 2018) of BIPM.

Paragraph referenced by historical discussion on the definition of the kilogram

Fundamental units This paragraph should describe a fundamental unit (as opposed to a derived unit).

Derived Units This paragraph should describe a derived unit (as opposed to a fundamental unit).

Length to area to volume

velocity as length-per-time

force intuition, visceral ... as a combination of several other units. (maybe note momentum is as yet unnamed)

---

[2]minute = (mine-oot = small); second = the second-order mine-oot = a mine-oot mine-oot

[3]Egyptians! Hooray! $60 = 2 \times 30 = 3 \times 20 = 4 \times 15 = 5 \times 12 = 6 \times 10$ versatile and broadly applicable. Practical.

Paragraph referenced by Footnote 8.4.1.3, How we walk Section 16.2, Subsection 19.1.2

Having defined the Newton, we can also define the Pascal. Describe force as a straightforward intuitive, visceral quantity. Example of pressure (same force tip of pencil versus erasure of pencil) and define unit.

$$P = F/A \qquad F = PA \tag{3.2.1}$$

This will be used in friction, fluids, and thermodynamics.

Do a Story Of for pressure, force, and area. Maybe reference an example of calculating the weight of the car or using the weight of the car to find the area of the tires... Note a flat tire covers more area.

Also make note that when you are on crutches in soft dirt or mud, you sink in deeper than when you have your whole foot to support you. This is relevant to the discussion of how we walk.

3.3 A graph is worth a thousand pictures

3.3.1 Coordinate Systems

Subsection referenced by Gravity's choice of coordinate system, Example 8.3.14

Discussion of the choice of origin (possible reference to zero-value of the potential energy)

Discussion of the choice of orientation of the axes. Look ahead to (1) how gravity prejudices us to use vertical and horizontal and (2) how inclined planes, like in Example 8.3.14, encourage us to skew the orientation.

Discussion of the choice of the positive-direction (possible reference to falling objects and using positive-up versus positive-down)

Paragraph referenced by Subsection 5.2.4 Subsection 5.6.1 Clarification of Newton's laws 7.2.1

Definition of a reference frame

- (different locations) The view from the roof versus from the ground
- (different speeds) The view from the sidewalk versus from a moving car (See also Subsection 5.6.1.)
- (different types of motion) The view from a park bench versus from a merry-go-round. (See also Section 11.5.)

3.3.2 The Vocabulary of Graphs

[Quick review of parameters and variables of $y = mx + b$ and $y = ax^2 + bx + c$.]

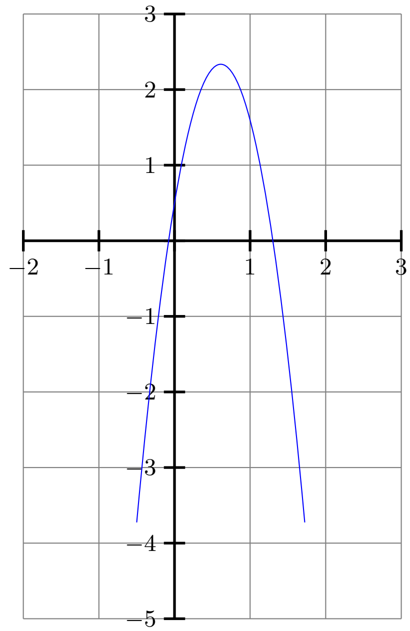


Figure 3.3.1: A random parabola

3.4 Trigonometry and Vectors

3.4.1 Trigonometry

.

3.4.2 Vectors

Subsection referenced by Subsubsection 7.2.2.2

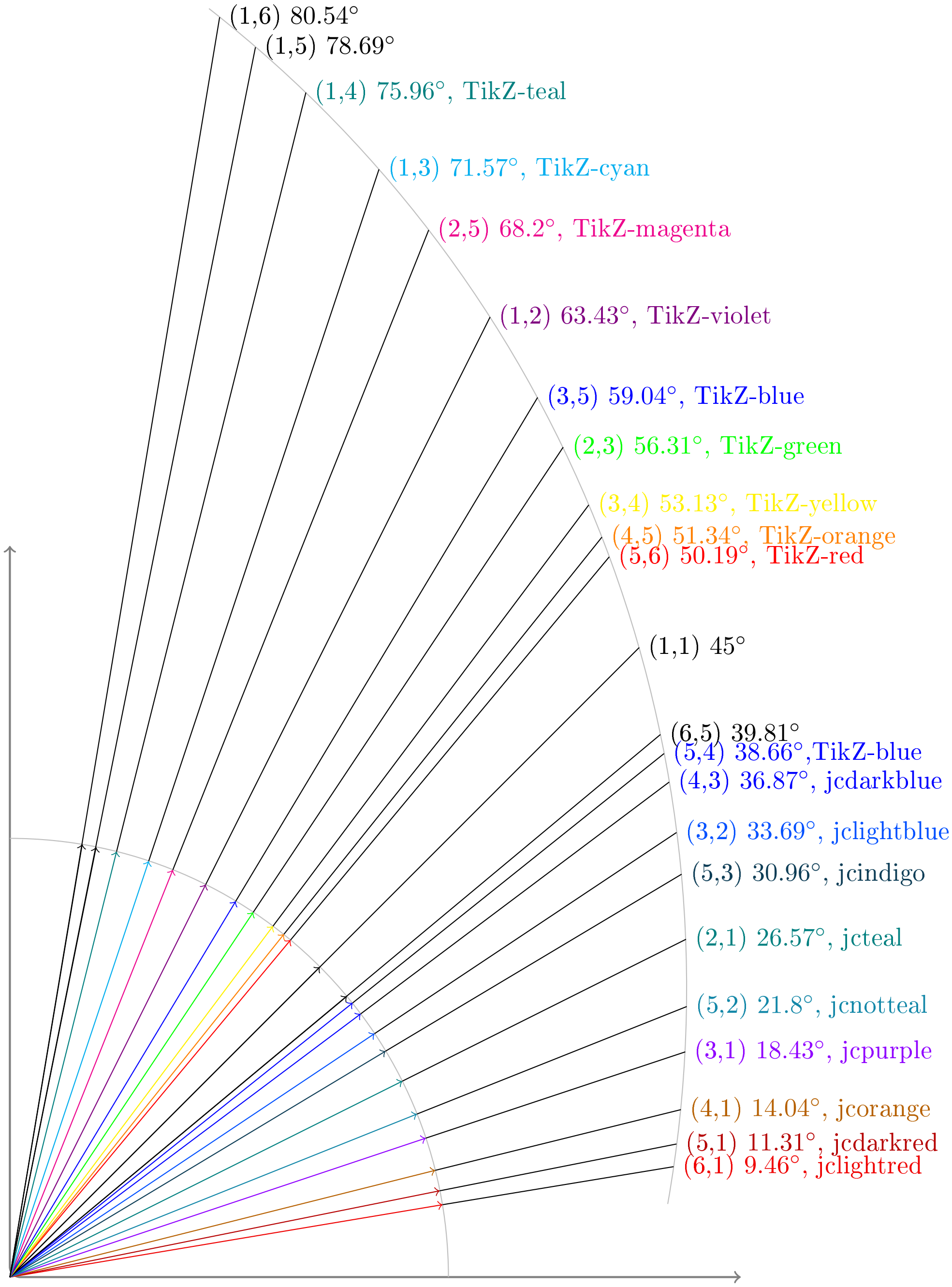


Figure 3.4.1: LaTeX lines and vectors. This will be deleted, but is here for reference.

3.4.2.1 Scalar Quantities versus Vector Quantities

definition of scalar

Paragraph referenced by Discussion of the direction of forces

definition of magnitude

Paragraph referenced by Discussion of the direction of forces

definition of direction (of a vector); how it gets expressed as an angle measured from a given reference point (usually the $+x$-axis).

Paragraph referenced by Discussion of the direction of forces

definition of vector

3.4.2.2 Vector Equations

Section referenced by the ballistic freefall $F = ma$

... If $\vec{A} = 3\vec{B}$, then this is true for each component.

$$A_x = 3B_x \quad (3.4.1)$$
$$A_y = 3B_y \quad (3.4.2)$$
$$A_z = 3B_z \quad (3.4.3)$$

This can also be written in two different ways: $A_x\hat{i} + A_y\hat{j} + A_z\hat{k} = 3\left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right) = (3B_x)\hat{i} + (3B_y)\hat{j} + (3B_z)\hat{k}$

Connection 3.4.2 (*Looking Ahead*)

> *This will be useful when we are discussing ballistics (2-dimensional motion), Newton's second law (combining multiple forces pushing on an object), 2-dimensional collisions, and the calculation of electrical fields.*

3.4.2.3 Multiplication, but Not Division

[define dot product]

[define cross-product]

Can do magnitude-equations like $F = ma$ or $m = F/a$. But for vector equations, while you can do $\vec{F} = m\vec{a}$, you cannot do something like $m = \dfrac{3\hat{i} + 4\hat{j}}{2\hat{i} - 5\hat{j}}$; but, in that case, you can use the magnitudes as follows $m = \frac{\sqrt{(3)^2+(4)^2}}{\sqrt{(2)^2+(-5)^2}} = \frac{\sqrt{25}}{\sqrt{29}} = \sqrt{\frac{25}{29}} = 0.928$

Paragraph referenced by Example 7.2.22

3.5 References and Further Reading

[1] *International System of Units (SI)*, Wikipedia (Visited June, 2017), `https://en.wikipedia.org/wiki/International_System_of_Units`.

[2] *General Conference on Weights and Measures*, Wikipedia (Visited June, 2017), `https://en.wikipedia.org/wiki/General_Conference_on_Weights_and_Measures`.

List of examples

Chapter 4

Estimating and Uncertainty

4.1 Precision and Accuracy

In this section, we will consider the benefits of being precise both in measurements and in our language. Sometimes people confuse the words precise and accurate, but they mean different things. It may help to remember that the opposite of precise is vague. Being precise makes it easier to determine if a statement is accurate. If we already know the answer, then we can know if a result is accurate. However, the exciting aspect of science is to study that which we do not already know. In this case, gauging accuracy can be tricky. If we are do not already know an answer, then we can try to be consistent within our accepted precision.

Since physics has its roots in the natural philosophy of the ancient Greeks and developed mathematically with Galileo and Newton, it has been around long enough for the technical language to both evolve (Newton used the word "action" for what we refer to as "force") and to be absorbed into everyday (colloquial) language. Words like force and energy have taken on broader meanings in English. In this text, we will try to be precise with the language. Hopefully we can avoid using the dismissive phrase, "Oh, you *know* what I *mean*."

One example of not being careful with the language comes when people use the term "massive" to mean "big." The word massive actually means "has a large amount of material" whereas big means "takes up a large amount of space" (which might be replaced by the word "voluminous" rather than "massive"). These are related by the density but it is possible to be massive and not voluminous (see, for example, the discussion of black holes). While it is *technically* inaccurate to use massive to mean big, "we" know what "we" mean.

Paragraph referenced by Subsection 8.1.1

4.2 Significant Figures

Section referenced by Subsection 3.2.2 Subsection 8.1.1

Note the comments in Subsection 8.1.1 regarding an internet search on the "conversion" of kilograms-to-pounds.

A short Google™ search by the author found that the conversation rate between pounds and kilograms was $1\,\text{kg} = 2.2046226218\,\text{lbs}$. Several sites go on to list about 10 decimal places for all of the conversions. First, you should recall our discussion about significant digits. Second you should note that the unit of pounds is a measure of force (how much the Earth pulls on you), whereas the unit of kilogram is a measure of mass (how much "stuff" there is). These are related in proportion to the strength of the gravitational field, which varies in the third digit (on the order of about 1%) around the globe. Some sites indicate that they are shortening their conversion factor to 3 digits for convenience, but this is not an issue of convenience, it is an issue of precision.

4.3 Scientific Notation

.

4.4 Effective Theories

Section referenced by non-inertial reference frames, air resistance Subsection 5.6.2, air resistance Subsection 6.2.1, Newton's first law, Newton's second law, Section 8.1, fundamental forces, Section 8.5.

Life is complicated. One mechanism that scientists in general and physicists in particular use to simplify their descriptions of the world around us is to build an effective theory. These are not intended to be true (accurate) to as many decimal places as can be calculated, but rather are intended to be good enough. In this context, good enough is most likely to mean something like: true to a reasonable number of decimal places.

A colloquial example of this is when you wear a smile to give the impression of happiness even if you are not in the mood. Most of your casual interactions will be the same as when you are in a good mood, but your friends who know you better will recognize the small discrepancies.

A technical example of this is that Newton's theory of gravity is very precise as long as none of the objects being described are travelling "close" to the speed of light. How close counts as close depends on the level of precision the measurement needs to be. If any of the objects are moving close to the speed of light, then we need Einstein's general theory of relativity. One way to describe this is that Newton's theory is a special case of Einstein's Theory. Another way is describe it is that Newton's theory is an effective theory for Einstein's theory, effective when the speed is low. It is possible for us to measure the difference between Newton's theory and Einstein's theory, but it is often not worth the effort of using the more complex theory in the cases where the simpler one will do; it is effectively true (rather than actually true).

Another technical example is that Einstein's special theory of relativity is a special case of the general theory of relativity. The aspect that makes it a special case is that the special theory only considers motion without acceleration.

One final case that should be mentioned up front is to notice that humans experience the Earth *as if* it were stationary.

Sometimes we imagine objects to be "massless" or "frictionless". In these instances we *usually* mean that the mass (or amount of friction) is small enough to not impact the significant digits of our calculation. In those cases, if we consider a more-precise measurement/computation, then the mass (or amount of friction) would impact the significant digits. (See comments in Example 8.5.16.)

I should define a term **mostly true** that is used in concert with **effective theory** to indicate the topic has some caveats. Then maybe list those in the **Touchstone** margin note? See, for example, the Touchstone on friction.

List of examples

Part II

Introducing Motion, Force, and Energy

The chapters in this Part are intended to introduce three fundamental and powerful concepts. These concepts are motion, force, and energy. I have found that if a student can understand these ideas sufficiently well, then they can quickly pick up any other idea that we introduce, even if the idea seems initially unfamiliar.

The trio of topics in this part of the book are fundamental and powerful concepts[1]. These are fundamental in that most other topics in physics are built upon them. They are powerful in that if they are well-understood, then one is empowered to use them to understand and develop an intuition for nearly any other topic that is experienced. It has been my experience that with these topics, students can jump into a surprisingly wide variety of other, significantly more esoteric, topics and develop a reasonable grasp of the key concepts. Furthermore, the development of understanding of these ideas introduce the language and thought processes of being a professional physicist such that it nicely bridges the language barrier that might otherwise exist due to the jargon of physics.

[1]The idea of Fundamental and Powerful Concepts (F&PC) is taken from Dr. Gerald Nosich, *Learning to Think Things Through*, Prentice Hall, 2012.

Chapter 5

1-D Motion, Relating position, Velocity, and Acceleration

5.1 How Physicists Use the Words (Notation)

- Position = where is it? Also discuss location as a vector and giving directions as defining a coordinate system (locate a common origin and unit-vector, then give a series of magnitudes and directions).
  - This chapter will distinguish location versus distance.
  - This chapter will distinguish distance traveled versus displacement.
- Velocity = which way did it go? Is its position changing?
  - This chapter will distinguish speed and velocity.
  - Introduce the language of "at rest".
- Acceleration = Is its velocity changing?
  - This chapter will clarify acceleration, deceleration, and changing direction.
  - This chapter will distinguish distance traveled versus displacement.

To help keep these concepts (velocity versus acceleration) separate, this text will use the following conventions:

Convention 5.1.1 (*Fast*)

This text will use ***fast*** *to indicate* moving with a large speed *(to be distinguished from quick).*

Convention 5.1.2 (*Quick*)

This text will use ***quick*** *to indicate* moving with a large acceleration *(to be distinguished from fast).*

With this notation, a drag-racer is "quick" but any car might eventually become "fast". Similarly, a cheetah is initially "quick" until it becomes "fast".

5.2 Connecting the Concepts- distance equals rate times time

5.2.1 Position

Identifying the position requires identifying a common known position (which we could call "the origin"), a distance from that known location (which we could call "a magnitude"), and a direction from the origin in which to travel such a distance. The common example that identifies the location as "I am in my room" references "your room" as the common, known origin. If the author of this text were to tell you that he was in his room, then your next obvious question is: "OK, but where is your room?"

Position can be seen to be a vector when you describe a meeting place or destination to a friend who has never been to that location: "Well, you know where the bookstore is, right?" (establishes a common origin). "OK, so, if you face the sports gear shop…" (sets the coordinate axis and defines the position direction) "… turn left and walk a block" (defines the magnitude and the direction).

5.2.2 Speed versus Velocity

When you are not moving, physicists will describe you as being "at rest". When you drive to the store, your car "starts from rest" and then travels some distance in some time. When you arrive at the store, your car "ends at rest" when you arrive at your destination.

Paragraph referenced by Discussion of Newton's First Law

When you are moving…

To be moving, you must be moving in a particular direction.

5.2.3 Relative Velocity

Subsection referenced by Example 8.4.7

Describe the motion of a car on a tree-lined road. From the typical observation location on the side of the road, the trees are stationary and the car is moving forwards (let's say east). On the other hand, from the car's perspective everything is still mathematically consistent if you consider the car to be stationary and imagine that the wheels are turning the Earth below it (like a treadmill) so that instead of the car moving forwards, it is the trees that are moving backwards (which would be west in this case).

Comment on inertial reference frames.

5.2.4 Adding Velocities

Comment on inertial reference frames.

5.3 Extending the Concepts: Changing How You Move

Section referenced by Clarification of Newton's laws 7.2.1 $F = ma$

The $F = ma$ reference to this section is about the statement that the direction of the acceleration does not determine the direction *of the motion*, but rather determines the direction *of the change* in motion. (adding sideways motion does not necessarily mean removing forwards motion.) Do we need to move that reference?

5.3.1 Moving versus Speeding Up

Paragraph referenced by Clarification of Newton's laws 7.2.2 Newton's First Law

Description of "moving" as *moving at constant velocity*.

Paragraph referenced by Section 7.1, Clarification of Newton's laws 7.2.1

The technical term **acceleration** means *changing the velocity*, which refers to *either* **speeding up** (colloquially "acceleration") *or* **slowing down** (colloquially "deceleration") *or* **changing the direction** (colloquially "turning").

Recall the use of fast versus quick.

Discussion of Exercise 5.3.1 and Exercise 5.3.2.

Exercise 5.3.1 (*How far will you go?*)

Exercise referenced by *Subsection 5.3.1*

You and your friend, Beth, are driving along at 55.0 mph *and run out of gas* 2.25 mi *from a gas station. You leave the car in gear and find that after* $t_1 = 1.00$ min, *you are travelling* $v_1 = 30$ mph. *Will you make it to the gas station?*

Hint. *The first thing we should do is notice what information is given to us and make sure that everything is in consistent units. I will convert everything to SI units.*

$$v_i = 55.0\ {}^{\mathrm{mi}}\!/\!_{\mathrm{hr}} \times \left(\frac{1609\,\mathrm{ft}}{1.0000\,\mathrm{mi}}\right)_{4\,\mathrm{sig}} \times \left(\frac{1\,\mathrm{hr}}{3600\,\mathrm{s}}\right)_{\mathrm{exact}} = \mathbf{24.58}\ {}^{\mathrm{m}}\!/\!_{\mathrm{s}}$$

$$\Delta x = 2.25\,\mathrm{mi} \times \left(\frac{1609\,\mathrm{ft}}{1.0000\,\mathrm{mi}}\right)_{4\,\mathrm{sig}} = \mathbf{362}0.3\,\mathrm{m} = \mathbf{3.62}0 \times 10^3\,\mathrm{m}$$

Answer. *You do not make it to the gas station.*

Solution. *[This example is not done, but the work will result in the following numbers: With* t_1 *and* v_1, *you can find* $a = -1500\ {}^{\mathrm{mi}}\!/\!_{\mathrm{hr}^2}$. *From that you can find, for* $v_f = 0\ {}^{\mathrm{m}}\!/\!_{\mathrm{s}}$, *that* $t = 2.2$ min *and* $\Delta x = \mathbf{1.00}8$ mi.*]*

You do not make it to the gas station.

Exercise 5.3.2 (*How fast should you start?*)

Exercise referenced by *Subsection 5.3.1*

You and your friend, Beth, are driving along at 55.0 mph *and run out of gas* 2.25 mi *from a gas station. You put the car in neutral because you know that the car will slow down with an acceleration of* $a = 500\ {}^{\mathrm{mi}}\!/\!_{\mathrm{hr}^2}$. *With what speed should you be going when you put your car into neutral in order to coast to a stop at the gas station?*

Hint. *The first thing we should do is notice what information is given to us and make sure that everything is in consistent units. I will convert everything to SI units.*

$$v_i = 55.0\ {}^{\mathrm{mi}}\!/\!_{\mathrm{hr}} \times \left(\frac{1609\,\mathrm{ft}}{1.0000\,\mathrm{mi}}\right)_{4\,\mathrm{sig}} \times \left(\frac{1\,\mathrm{hr}}{3600\,\mathrm{s}}\right)_{\mathrm{exact}} = \mathbf{24.58}\ {}^{\mathrm{m}}\!/\!_{\mathrm{s}}$$

$$\Delta x = 2.25\,\mathrm{mi} \times \left(\frac{1609\,\mathrm{ft}}{1.0000\,\mathrm{mi}}\right)_{4\,\mathrm{sig}} = \mathbf{362}0.3\,\mathrm{m} = \mathbf{3.62}0 \times 10^3\,\mathrm{m}$$

Answer. *So, if you start at* $55.0\ {}^{\mathrm{mi}}\!/\!_{\mathrm{hr}} - 27.8\ {}^{\mathrm{mi}}\!/\!_{\mathrm{hr}} = \mathbf{27.1}6\ {}^{\mathrm{mi}}\!/\!_{\mathrm{hr}}$ *you should make it exactly.*

Solution. *[This example is not done, but the work will result in the following numbers: With* $a = -500\ {}^{\mathrm{mi}}\!/\!_{\mathrm{hr}^2}$, *you can find, for* $v_f = 0\ {}^{\mathrm{m}}\!/\!_{\mathrm{s}}$, *that* $t = 6.6$ min *and* $\Delta x = 3.025$ mi. *You clearly make it to the gas station. You can also find that for* $\Delta x = 2.25$ mi, $t = \mathbf{3.25}9$ min *and* $v_f = \mathbf{27.8}388\ {}^{\mathrm{mi}}\!/\!_{\mathrm{hr}}$.*]*

So, if you start at $55.0\ {}^{\mathrm{mi}}\!/\!_{\mathrm{hr}} - 27.8\ {}^{\mathrm{mi}}\!/\!_{\mathrm{hr}} = \mathbf{27.1}6\ {}^{\mathrm{mi}}\!/\!_{\mathrm{hr}}$ *you should make it exactly.*

5.4 Connecting the English to the Math

Section referenced by Example 7.2.19

The equations of constant acceleration can be summarized as

$$x_f = x_i + v_i t + \frac{1}{2}at^2 \tag{5.4.1}$$

$$v_f = v_i + at \tag{5.4.2}$$

$$v_f^2 = v_i^2 + 2a\,\Delta x \tag{5.4.3}$$

Paragraph referenced by Exercise 5.6.1, discussion of herky-jerky motion

5.5 Examples

5.5.1 Freefall

Section referenced by Exercise 5.6.1

Since acceleration is the change in velocity (magnitude and/or direction), it is possible to select your own rate of change while driving your car. However, that acceleration is difficult to measure directly. Your speedometer measures the speed and you have to compute your acceleration based on how quickly your speed changes. It turns out that there is a convenient way to start from the acceleration and compute the expected velocity: Drop a ball or your keys. To convince yourself that objects do, in fact, accelerate when they fall, we can consider dropping items. One of the complications during such an experiment will be discussed in Subsection 5.6.2. If we drop a sheet of paper, air resistance causes an obvious effect (fluttering). For this section, I will assume that the mass-to-surface-area ratio is large enough that we can effectively ignore the air resistance.

Paragraph referenced by Discussion of the description of physics

The patterns that you see when you drop objects is that objects fall faster than humans are used to paying attention to. This Investigation shows you how you can pay close attention to the patterns that result from observing falling objects. You should go do those experiments before reading further. Go ahead. I'll wait.

Investigation 5.5.1 (*The motion of dropped objects*)

Investigation referenced by *Investigation 8.4.18*

Because Carl is a pitcher on the local baseball team, he decides to drop a ball and watch what happens. You and Diane decide to join him. Diane provides a few other objects that can also be dropped: a tennis ball, a hammer, a small Wonder Woman toy, and a broken cell phone. Some of these are dropped at the same time.

Diane notices that ***it is important to release the objects at exactly the same time.***

Carl notices that ***it is important to have the bottoms of the objects line up (rather than the tops of the objects) so that if they travel at the same speed, then they hit at the same time.***

(a) *Drop any two objects at the same time from the same height.*

(i) *Are there any objects that always hit first or last?*

Solution. *As long as you are careful about releasing at the same time, you should not see any object consistently land first or consistently land last. It is true that a piece of paper will consistently land last, but this is because of the air resistance that we previously agreed to avoid. If you crumple the paper into a tight ball (yes, it has to be a tight ball), then this will minimize the effect of air resistance and you might still be able to make the comparisons. It is possible that some of the objects you are dropping (such as those in Solution 5.5.1.1.a.ii.1) have a shape that makes air resistance relevant.*

(ii) *If so, what are the properties of those objects?*

Solution. *As long as you are careful about releasing at the same time, it is unlikely that you will find anything consistently falling faster or slower than the others. If you do notice a pattern, then the likely culprit is that air resistance is having an effect. If you have something flat, like a computer (!) or a book that is falling more slowly than something else, like a hammer, then* *try dropping the flat object in different orientations to see if that affects the air resistance. If you have something somewhat cylindrical, like a wine bottle (!) or a pencil that is falling more quickly than something else, like a hammer, then try dropping the cylindrical object in different orientations to see if that affects the air resistance. Remember that we are trying to eliminate differences due to air resistance so that we can study the effect of the gravitational force.* The effect you should notice is that so long as air resistance does not affect one object differently than the other, all objects fall at the same rate.

(b) *Drop one of these objects from about eye-level*

(i) *Observe the speed of the object as it falls.*

(ii) *Is the object moving at a constant speed?*

Solution. *When you drop something from eye-level, it takes less than a half-second for it to hit the ground. Due to the limited need to gauge speed, it is very difficult for most humans to distinguish constant speed from accelerated motion in this small of a time interval. Athletes can often tell is an object is moving fast or slow, but even then it is difficult to gauge acceleration. Practice measuring the time-of-flight by counting out loud: "one-one-thousand... two-one-thousand...". For this fall, you will likely only get to "one-one-thou".*

(c) *Climb a tall ladder, drop the ball from at least eight-feet high*[1]

(i) *Observe the time it takes the object to pass four rungs near the top of the ladder and compare it to the time it takes the object to pass four rungs near the bottom of the ladder*

(ii) *Is one set of four-rungs a shorter time or are they the same amount of time?*

Solution. *Measure the time-of-flight by counting out loud: "one-one-thousand... two-one-thousand...". For the four rungs near the top, you will likely only get to "one-one-thou". For the four rungs near the bottom, you will likely only get to "one-wa". Since those two distances are the same, it should be clear that the object is going faster at the bottom of the ladder.* Objects speed up (accelerate) while they fall.

You and your friends should get together to see if you can come up with a way to measure the acceleration due to the gravitational force.

Return to: freefall, the force of gravity

You did do them, right? You're not just reading ahead? Really? OK. Doing that experiment will help you see (1) that everything falls at the same rate and (2) that objects accelerate as they fall.It turns out that, ignoring the effect of air resistance, all objects fall with the same acceleration (due to the gravity). As discussed in Subsection 8.1.2, this varies slightly round the world, but only slightly. (See Table 8.1.13 for some representative samples.)

Definition 5.5.2 (freefall)**.** In this book, "being in freefall" will mean moving only under the influence of gravity and, when near the surface of the Earth, having an acceleration consistent with Convention 8.1.11.

We will start to discuss the reason for this in Section 8.1 and then get into more detail in Chapter 15. For now, Exercise 5.5.3 shows the type of experiment that can allow you to calculate the acceleration due to gravity.

Exercise 5.5.3 (*How quickly does it fall?*)

Exercise referenced by *freefall, discussion of falling objects.*

Your friend, Carl, is a baseball player and is curious to learn about the rate that baseballs fly through the air. You get on a 12 ft *ladder and he lays on the ground below you aiming his radar gun (which measures speed) upwards. Each rung is* 1.0 ft *apart and his gun is at the first rung. When you drop the ball three rungs above the gun, he measures the final velocity to be* $4.24\ \mathrm{m/s}$. *When you drop the ball six rungs above* *the gun, he measures the final velocity to be 6.00 $^{\mathrm{m}}\!/\!_{\mathrm{s}}$. When you drop the ball eleven rungs above the gun, he measures the final velocity to be 8.11 $^{\mathrm{m}}\!/\!_{\mathrm{s}}$. Find the acceleration of the ball in each case.*

Hint. *The first thing we should do is notice what information is given to us and make sure that everything is in consistent units. I will convert everything to metric.*

$$
\begin{aligned}
3\,\text{rungs} &= 3.00\,\text{ft} \times \left(\frac{0.3048\,\text{m}}{1.00000\,\text{ft}}\right) = 0.\mathbf{914}4\,\text{m} \\
6\,\text{rungs} &= 6.00\,\text{ft} \times \left(\frac{0.3048\,\text{m}}{1.00000\,\text{ft}}\right) = \mathbf{1.82}9\,\text{m} \\
11\,\text{rungs} &= 11.00\,\text{ft} \times \left(\frac{0.3048\,\text{m}}{1.00000\,\text{ft}}\right) = \mathbf{3.35}3\,\text{m}
\end{aligned}
$$

Solution. *To find the acceleration in each case, we can solve $v_f^2 = v_i^2 + 2a\,\Delta x$ for the acceleration:*

$$
\begin{aligned}
a_3 &= \frac{(4.23\ ^{\mathrm{m}}\!/\!_{\mathrm{s}})^2 - (0\ ^{\mathrm{m}}\!/\!_{\mathrm{s}})^2}{2(0.\mathbf{914}4\,\text{m})} &= \mathbf{9.83}1\ ^{\mathrm{m}}\!/\!_{\mathrm{s}^2} \\
a_6 &= \frac{(6.00\ ^{\mathrm{m}}\!/\!_{\mathrm{s}})^2 - (0\ ^{\mathrm{m}}\!/\!_{\mathrm{s}})^2}{2(\mathbf{1.82}9\,\text{m})} &= \mathbf{9.84}1\ ^{\mathrm{m}}\!/\!_{\mathrm{s}^2} \\
a_1 1 &= \frac{(8.11\ ^{\mathrm{m}}\!/\!_{\mathrm{s}})^2 - (0\ ^{\mathrm{m}}\!/\!_{\mathrm{s}})^2}{2(\mathbf{3.35}3\,\text{m})} &= \mathbf{9.80}8\ ^{\mathrm{m}}\!/\!_{\mathrm{s}^2}
\end{aligned}
$$

Notice that these have some variation due to the rounding. It turns out that the variation in the value of acceleration depends on the composition of the earth in your location as well as your altitude above sea-level. That will be discussed in detail in Chapter 15, for simplicity we will assume (by Convention 8.1.11) that all objects accelerate at the rate of 9.81 $^{\mathrm{m}}\!/\!_{\mathrm{s}^2}$ when they are solely under the influence of gravity.

It turns out that you can also see this acceleration when you throw an object straight up into the air.

5.6 Complications

5.6.1 Non-Inertial Accelerated Reference Frames

Subsection referenced by Reference Frames, Subsection 7.2.1.

[Discuss non-rotating linearly accelerating reference frames. See also Section 11.5 for a discussion on rotating reference frames.]

[Comment on the Earth as essentially stationary? See Section 4.4 on effective theories.]

5.6.2 Air Resistance

Subsection referenced by freefall, Answer 7.3.0.3.5, Section 8.1

Terminal velocity. . .

When do we include air resistance and when can we ignore it? . . .

[Comment on air resistance being a small effect in some cases? See Section 4.4 on effective theories.]

5.6.3 Multi-Step Solutions

Section referenced by discussion of herky-jerky motion

In physics we like to consider the simplest case that describes the relevant situation; however, these are often embedded in a more complicated situation and it is not always obvious to the casual observer. For example, if I throw a ball into the air, such as in Exercise 1, then we need to consider the motion of the throw separately from the motion of the ball moving through the air and separately from the motion of catching the ball. The general rule is that whenever the acceleration changes, we have to consider the motion separately. We will learn later (Subsection 7.2.2) that forces are related to acceleration, so for now, whenever you notice that different forces are acting on an object, then you should expect the acceleration to be different and, therefore, you should expect to break the problem into steps *during which* the forces (and acceleration) are constant and *between which* the forces (and acceleration) may change.

Exercise 5.6.1 (*Carl hits the ceiling!*)

Exercise referenced by *Example 8.3.8, Example 8.7.1, discussion of herky-jerky motion*

Carl gets bored one day in physics class[1] *and tossed a baseball*$(m_b = 0.145\,\mathrm{kg})$ *at the ceiling... a little too hard. The initial velocity is* $v_i = +5.00\ \mathrm{m/s}\,\hat{\jmath}$ *and it leaves his hand* $1.00\,\mathrm{m}$ *below the ceiling. The ball hits the ceiling and when it returns to his hand, it is travelling* $\vec{v}_f = -4.73\ \mathrm{m/s}\,\hat{\jmath}$*, slower than he expected.*

(a) *Assuming that the ball is in contact with the ceiling for* $\Delta t = 0.142\,\mathrm{s}$*, find the acceleration of the ball during the collision.* Solution 5.6.3.1.1

(b) *On the other hand, if the ceiling had not been there, then how high would the ball have gone and how fast would it have been going when it returned to Carl's hand?* Solution 5.6.3.1.2

Exercise Not Done *part b*

Hint 1 (First stage). *During* ***the first stage****, the ball is accelerating upwards and Carl is interacting with the ball. We are not going to consider this part of the motion at all because we are given the velocity that ends this stage (and begins the next stage).*

Hint 2 (Second stage). ***The second stage*** *of the motion is while the ball moves from Carl's hand up to the ceiling. During this stage only the gravitational force is acting on the ball. Since it has left Carl's hand, he is not interacting with it. Since it has not yet hit the ceiling, the ceiling is not interacting with it. We can therefore use the equations of constant acceleration to describe the motion. During this portion of the motion we know that the velocity at the bottom is* $\vec{v}_{\mathrm{bot}} = +5.00\ \mathrm{m/s}\,\hat{\jmath}$*, that it travels* $\Delta\vec{x} = +1.00\,\mathrm{m}\,\hat{\jmath}$*, and that (because it is in freefall) it is accelerating at* $\vec{a}_g = -9.81\ \mathrm{m/s^2}\,\hat{\jmath}$*.*

We can find the time of flight (not useful) and the velocity when the ball reaches the ceiling:

$$\begin{aligned} v_{\mathrm{top}} &= \sqrt{v_{\mathrm{bot}}^2 + 2a\,\Delta x} \\ v_{\mathrm{top}} &= \sqrt{(+5.00\ \mathrm{m/s})^2 + 2(-9.81\ \mathrm{m/s^2})(+1.00\,\mathrm{m})} \\ v_{\mathrm{top}} &= +\mathbf{2.31}9\ \mathrm{m/s} \end{aligned}$$

Note: When you take the square root, you have to decide if you should take the positive sign or the negative sign. In this case, the ball is still moving upwards, so we choose the positive sign.

Hint 3 (Third stage). ***The third stage*** *is while it is interacting with the ceiling. In order to find the acceleration during this motion, we need to know the velocity immediately before hitting the ceiling (which we just found) and the velocity just after it finishes hitting the ceiling (which we have not yet found). We will come back to this step.*

Hint 4 (Fourth stage). ***The fourth stage****, like the second, is while the ball moves from the ceiling down to Carl's hand. During this portion of the motion we know that the velocity at the bottom (final) is* $\vec{v}_{\mathrm{bot}} = -1.67\ \mathrm{m/s}\,\hat{\jmath}$*, that it travels* $\Delta\vec{x} = -1.00\,\mathrm{m}\,\hat{\jmath}$*, and that (because it is in freefall) it is accelerating at* $\vec{a}_g = -9.81\ \mathrm{m/s^2}\,\hat{\jmath}$*. We can find the time of flight (not useful) and the velocity when the ball leaves the ceiling (initial), solving* $v_{\mathrm{bot}}^2 = v_{\mathrm{top}}^2 + 2a\,\Delta x$ *for* v_{top}*:*

$$\begin{aligned} v_{\mathrm{top}} &= \sqrt{v_{\mathrm{bot}}^2 - 2a\,\Delta x} \\ v_{\mathrm{top}} &= \sqrt{(-1.67\ \mathrm{m/s})^2 - 2(-9.81\ \mathrm{m/s^2})(+1.00\,\mathrm{m})} \\ v_{\mathrm{top}} &= -\mathbf{4.73}4\ \mathrm{m/s} \end{aligned}$$

Note: When you take the square root, you have to decide if you should take the positive sign or the negative sign. In this case, the ball is now moving downwards, so we choose the negative sign.

Hint 5 (Fifth stage). *During **the fifth stage**, the ball is accelerating upwards while moving downwards and so Carl is stopping the ball. We are not going to consider this part of the motion at all.*

Hint 6 (Which stage?). *The acceleration is only constant during each stage. The acceleration is different from one stage to another. **This** is the observation that tells us that we must consider the stages separately.*

Solution 1. *In order to solve Question 5.6.3.1.a for the acceleration, we need to recognize that*

1. *there are five stages to the motion of the baseball: the throw (Hint 5.6.3.1.1), the ball moving from Carl's hand up to but not yet hitting the ceiling (Hint 5.6.3.1.2), the ball hitting the ceiling (Hint 5.6.3.1.3), the ball falling from the ceiling down to but not yet touching Carl's hand (Hint 5.6.3.1.4), and the catching of the ball (Hint 5.6.3.1.5), and*

2. *the equations of constant acceleration assume that the acceleration is constant (Hint 5.6.3.1.6).*

The acceleration is $a = \dfrac{v_f - v_i}{\Delta t}$, but the story of this equation says that since the acceleration is only during the interaction with the ceiling, then the velocities in this equation are just-before the ball hits and just-after the ball hits (not the very beginning velocity and not the very final velocity). Similarly, the Δt in this equation is only the time during which it was interacting with the ceiling, not the entire flight.

Now that we have the velocities immediately before (from Hint 5.6.3.1.2) and after (from Hint 5.6.3.1.4) the collision with the ceiling, we can find the acceleration: $a = \frac{v_f - v_i}{\Delta t} = \frac{(-\mathbf{4.734}\ \mathrm{m/s}) - (+\mathbf{2.319}\ \mathrm{m/s})}{(0.142\,\mathrm{s})} = -\mathbf{28.09}\ \mathrm{m/s^2}\ \hat{j}$ Notice that the acceleration is negative because the ball went from going up to going down.

Solution 2. *To solve Question 5.6.3.1.b, we can just consider from after-thrown to before-caught (both given in the question). During this motion, assuming there is no ceiling, the entire motion is in freefall, so we can use $v_f^2 = v_i^2 + 2a\,\Delta x$ and solve for Δx. However, we only want to consider from the lowest point to the highest point, not all the way back to Carl's hand.*

List of examples

Chapter 6

2-D Motion, The Vector Nature of Motion

6.1 Components of Motion

6.1.1 Cross-wind

Bikes and canoes

6.1.2 Ballistic Freefall

Subsection referenced by Subsubsection 3.4.2.2

Discussion about throwing a ball...

Paragraph referenced by Discussion of the description of physics

For 2-dimensional motion, we will use vector equations to describe the relationships. When we write $\vec{v}_f = \vec{v}_i + \vec{a}t$, we mean that this relationship holds for the x-components and separately for the y-components:

$$v_{fx} = v_{ix} + a_x t \qquad v_{fy} = v_{iy} + a_y t$$

Paragraph referenced by Discussion of $F = ma$

6.2 Complications

6.2.1 Air Resistance

Terminal velocity... non-parabolic paths ...

Words to indicate what is of interest in the IRL...

Investigation 6.2.1 (*Baseball pitches are not usually parabolic*)

Carl is a pitcher on the local baseball team. He throws a fast ball, a slider, a curve ball, and a knuckleball.

(a) *Go to a baseball game on a calm day. Sit near third base.*

(i) *Watch the path of fly balls to left field. Are they parabolic?*

Solution. *If you watch them carefully, you will notice that long fly balls are not parabolic. It turns out that the air resistance is fairly complicated, but in the case of baseballs, the part that is relevant is that air resistance is strong when the ball is moving faster and weak when the ball* *is moving slower. (This is different than the surface friction you will see in Section 8.4.) The effect of this is that the ball (usually) looks like it travels up into the air on a fairly straight path with a slight bend, which would produce a very wide parabola. As it slows, the horizontal motion decreases, which tightens the parabola. By the time the ball gets to its highest point, it is often travelling fairly slowly and has mostly all vertical motion by the time it drops into the outfielder's glove.*

(ii) *The path of pitch towards home plate. Are they parabolic?*

Solution. *The way a pitch travels is highly dependant on the way the pitcher releases the ball. As the ball rolls out of the pitcher's hand, a spin is (usually) given to the ball and this spin interacts with the air to modify the direction that the air presses on the ball during the flight. This will slightly affect the flight of the ball during the time it takes for the ball to get from the pitcher's mound to home plate. In addition, fast balls have less time for the gravitational force to pull the ball down, so they will curve downwards less than a slower pitch. This makes following the path of the ball somewhat difficult, but with some practice and careful attention, you should be able to see it. All balls will drop somewhat, but the effect of the air resistance is exactly the mechanism for making a pitch unpredictable, so it is unlikely that you see the ball drop in a clean parabolic path.*

(b) *Go to a baseball game. Sit up high behind home plate. Watch the path of the baseball for various pitches.*

(i) *Do they all fly straight over the plate?*

Solution. *The way a pitch travels is highly dependant on the way the pitcher releases the ball. As the ball rolls out of the pitcher's hand, a spin is (usually) given to the ball and this spin interacts with the air to modify the direction that the air presses on the ball during the flight. In many cases, this will affect the flight of the ball (especially to the right or to the left) during the time it takes for the ball to get from the pitcher's mound to home plate. If you watch from behind home plate, this sideways motion should be fairly clear.*

Air resistance has an effect on most objects that move through the air. The faster an object moves, the bigger the effect. Spinning objects also feel an effect. Objects that have a somewhat large cross sectional area and a somewhat small mass are also affected, but this depends on the actual area and the actual mass.

Return to: the description of physics, 6.2.1

[Comment on air resistance being a small effect in some cases? See Section 4.4 on effective theories.]

List of examples

Chapter 7

Force

Chapter referenced by Chapter 8

Having learned how to describe the way objects move, we now turn our attention to that which causes motion, although technically it causes motion to change. In this chapter we will consider the concept of force and frame your attention. In the next chapter we will consider the specific types of force and how to solve specific problems.

7.1 How Physicists Use the Words (Notation)

Section referenced by Discussion of heat as a verb

The technical term **force** refers to the general idea of pushing or pulling. In the same way that physicists use the technical word acceleration differently than the general population uses the colloquial word, force has a specific meaning. We will use *force as a noun* (not as a verb) referring to the act of pushing or pulling and having specific relationships that will be outlined in this chapter.

Etymology (force) We are using force in the sense of compulsion, although perhaps you might connect it more closely with a motive.

Connection 7.1.1 (*Looking Back*)

Forces are necessarily vectors, because pushing on something intrinsically involves both an amount and a direction.

You will use this property to show that multiple people pushing in the same direction increases the effect, whereas multiple people pushing in opposite directions reduces the effect. One might say that people who push an object in opposite directions work[1] against each other. Because the force is a vector, whenever you are answering a question about a force, you should always expect to give the strength of the force (the magnitude) and the direction of the force (relative to some specific axis, usually the positive x-axis).

Insight referenced by The answers in Example 8.4.7.

Insight 7.1.2

You can't have a push or pull without both *a thing that pushes or pulls* and *a thing that is pushed or pulled. Forces are* necessarily *an interaction between two objects.*

Clarification 7.1.3 (*Push or Pull*)

By now you may have noticed that it is tedious to keep saying "pushed or pulled," so we will only say "push" even when we are including the possibility of "pushing or pulling".

[1] After you study Section 9.2, this play on words will be hilarious!

Sometimes we care about the thing doing the pushing or pulling, sometimes we don't. We always care about the thing being pushed or pulled. We will *distinguish these objects* by referring to the object that is pushing as the object "causing the force" or "exerting the force" and by referring to the object that is being pushed as the object "feeling the force". We will *distinguish these forces* as follows:

Convention referenced by Example 7.3.4

Convention 7.1.4 (*The "on-by" notation*)

Let's imagine that Beth gives Abdul a good-natured shove in the arm. The following are useful descriptions and are different ways of describing the same action.

- *Beth exerted a force* **on** *Abdul.*
- *Abdul felt a force* **from** *Beth.*
- *There was a force* **on** *Abdul* **by** *Beth.*

The notation for this will be $F_{A,B}$ where the first subscript is the person who felt the force (who the force is "on") and the second subscript is who exerted the force (who the force is "by"). In those instances when we only care about who is feeling the force and not who is exerting the force, we might just use one subscript F_A.

In some cases, there may be two forces acting on one person (or object). In that case, it will be obvious who is feeling the force and we will use the subscript to distinguish which force it is in addition to *who feels the force, such as F_{A1} and F_{A2}, or rather than who feels the force, such as F_1 and F_2. This will be more relevant when we discuss in Chapter 8 the types of forces that might be applied. Convention 8.0.1 will expand this notation.*

Touchstone (law) Recall the distinction between Theory versus Law.

Connection 7.1.5 (*Looking Ahead*)

Looking ahead to Newton's Laws (Section 2), you should be ready to notice that the first law (Subsection 7.2.1) is about objects that are not feeling a force, the second law (Subsection 7.2.2) is about a specific object that is feeling a force, and the third law (Subsection 7.2.3) is about the interaction between the two objects. In all three of these, we care about the object feeling the force. It is only in the third law that we care about the object exerting the force.

7.2 Connecting the Concepts: Newton's Laws

Section referenced by Discussion of how to describe forces

Touchstone (law) Recall the distinction between Theory versus Law.

Paragraph referenced by Subsection 8.3.3

Newton's Laws describe our observations about three questions:

1. What happens to an object when I *don't* push on it?
2. What happens *to an object* when I do push on it?
3. What happens *to me* when I push on an object?

The answers to these questions have precise, concise, technical language and the point of the next three subsections is to translate that into (modern) English, into math, and into intuition. The statement of these laws has slightly different versions in different texts to emphasize different points. We will state them as follows:

Newton's Laws

1. When viewed from an inertial reference frame, an object with no forces acting on it will maintain its velocity, which may be zero.

   (Detailed in Subsection 1.)

2. When viewed from an inertial reference frame, the vector-sum of all forces acting on an object will cause that object to accelerate in proportion to its mass: $\vec{F}_{\text{net}} = m\vec{a}$.

   (Detailed in Subsection 2.)

3. For every force acting (the "action") on one object by an other object, there is an equal-in-magnitude reaction-force acting on the other object in the opposite direction.

   (Detailed in Subsection 3.)

There are a few terms that should be clarified in these laws.

Clarification 7.2.1 (*Inertial Reference Frame*)

Being in an inertial reference frame essentially means being in a place in which you do not have to hold on in order to maintain your position.

If you are accelerating then you are not in an inertial reference frame, but rather are in a non-inertial reference frame. In this case, you will misinterpret the forces acting. This will be discussed in more detail in Section 11.5 when we discuss the surface of the Earth as a non-inertial rotating reference frame. The fact that the Earth spins will be relevant for global-sized systems, such as the atmosphere and the ocean (discussed in Subsection 11.5.1 with the Coriolis effect).

For human-sized interactions, the effect of the Earth spinning is small enough that for most of what we casually observe, we can safely pretend that the Earth is stationary and that we are actually at rest while sitting on the curb watching the world go by. This is so true that our human brains already interpret everything around us as though it were an inertial reference frame. This psychological perspective is exactly the feature that both allows us to make fairly reliable predictions about the world around us and causes us to make incorrect judgements when we encounter human-sized non-inertial reference frames. That is to say, as long as we don't measure our world too closely, we are viewing it from an essentially inertial reference frame. This point is so implicit, that many books do not even include[1] the portion of the statement referring to the reference frame.

Touchstone (mostly true) Recall effective theories. Standing on the surface of the Earth provides an essentially-inertial reference frame.

Clarification 7.2.2 (*Objects in motion*)

Sometimes Newton's first law is written[2] to include the phrase "an object in motion", which I will be careful to link directly to velocity, as was done above. However, it technically should reference the momentum, which is discussed in Chapter 10.

Clarification 7.2.3 (*Action / Reaction*)

The way Newton's third law is often written[3] (and referred to) includes the words "action" and "reaction". Newton was referring to forces with these words and to keep it clear in our discussion, we will use the word force, with the occasional clarification of the action-force or the reaction-force.

7.2.1 Translating Newton's First Law: The Law of Inertia

Subsection referenced by Discussion of how to describe forces

Newton's First Law

When viewed from an inertial reference frame, an object with no forces acting on it will maintain its velocity, which may be zero.

Touchstone (inertial frames) You might also recall the discussion in Subsection 5.6.1.

Let's take this language apart and connect it to your daily experiences.

As indicated in Clarification 7.2.1, it is usually safe to assume you are in an inertial reference frame for your daily experience. So, at this point we won't worry about the words before the comma.

The rest of this statement is often written a little differently (and less concisely) as "an object at rest remains at rest unless acted on by an external force and an object in motion remains in motion unless acted on by an external force." Since being "at rest" is a statement about the velocity($\vec{v} = 0$) and being "in motion" is also a statement about the velocity, each of these statements can be understood as saying that

Insight 7.2.4

Forces are those things that cause a change in the velocity.

In other words, Newton's first law says that without a force, the velocity will not change.

Connection 7.2.5 (*Looking Ahead*)

In the discussion of equilibrium (Subsubsection 7.2.2.4), we will note that this is often extended to say that without a net force (as opposed to "without a force" as explained in Subsubsection 7.2.2.2) the velocity will not change, but that is a special case of Newton's second law (Subsection 7.2.2).

Paragraph referenced by Section 8.4

Building intuition Now that we have taken apart the language, let's connect it to our daily experience. Our intuition screams at us that if an object is at rest, then we do not need to do anything to keep it from moving: Objects just do not spontaneously move. This seems to be acceptable to most students of physics. The interesting piece of the first law is that it says that this is also true of objects that are moving. This seems to contradict many people's common sense. Most of the objects that we set in motion do not maintain that motion, but instead slow down. A clue to interesting physics is to think about when objects slow down and when they speed up without your help. In order to connect the first law with your intuition, we need to carefully observe what actually happens.

Investigation 7.2.6 (*Find a pattern*)

(a) *Make a list of situations in which a moving object slows down without your interference in its motion. Indicate if it is interacting with anything else.*

Task referenced by *Task 8.4.2.a*

Hint. *Investigation 7.3.2 might with this consideration.*

Answer. *There are many reasonable answers to this. It is likely that your answers will be relevant when we discuss friction in Section 8.4, and especially in Task 8.4.2.a.*

(b) *Make a list of situations in which a moving object speeds up without your interference in its motion. Indicate if it is interacting with anything else.*

Hint. *Investigation 7.3.1 might with this consideration.*

(c) *Make a list of situations in which a moving object maintains its motion without your interference in its motion. Indicate if it is interacting with anything else.*

(d) *Is there a pattern?*

If you slide a brick across an icy pond and compare that to sliding the same brick across a flat parking lot, to sliding that same brick across a gravel driveway, to sliding that brick across a grassy lawn, you will quickly see that the brick does not behave the same way. The brick is *interacting* with the surface is moves along. This is telling us (Insight 2) that there is a force acting on the brick, changing the way it moves. Newton's first law does not apply to the situation where there are forces acting on the object. When forces act, we need to consider Newton's second law, which we will do in Subsection 7.2.2.

7.2.1.1 Inertia

This law is often called the "law of inertia". The concept of inertia can be described as *the tendency of an object to maintain its velocity*. This is describing how the object behaves when you don't do anything to it. The inertia is not a quantity that physicists calculate, but physicists do refer to objects as having a lot of inertia, usually to indicate that it will take a large force to change the object's motion, or as having a small amount of inertia, usually to indicate that it should be relatively easy to change the object's motion. However, the inertia does not actually refer to the force needed. Instead, the inertia most often refers to the "inertial mass" of an object, which shows up in the second law.

Insight 7.2.7

Inertia is not a force.

Sometimes when physicists are not being careful with their language, they will appear to use the word inertia interchangeably with the term momentum, which we will discuss in more detail in Subsection 10.1.1.

7.2.1.2 How the Laws Work Together

You should notice that Newton's first law is about what happens when you are *not pushing* on the object, which is to say, the tendency of an object to maintain its own motion without a force acting on it; this is the inertia of the object. On the other hand, Newton's second law is about what happens *to the object* when you *do push* on an it. This is what we will consider next. After that, Newton's third law will describe what happens *to the thing pushing* rather than just to the thing being pushed. Subsubsection 5 of Subsection 7.2.2 will explore these ideas further.

7.2.2 Translating Newton's Second Law: The Equation Law

Subsection referenced by Subsubsection 3.4.2.2, how to describe forces, Newton's first law, discussion of falling objects, discussion of simple piece of a complicated situation

Newton's Second Law

When viewed from an inertial reference frame, the vector-sum of all forces acting on an object will cause that object to accelerate in proportion to its mass: $\vec{F}_{\text{net}} = m\vec{a}$.

Touchstone (inertial frames) You might also recall the discussion in Subsection 5.6.1.

Let's take this apart and connect it to your daily experiences. As with Newton's First Law, the non-inertial rotating reference frame of the surface of the Earth is a small enough effect that, as long as we don't measure our world too closely, we can pretend that we are viewing it essentially from an inertial reference frame.

Touchstone (mostly true) Recall effective theories. Standing on the surface of the Earth provides an essentially-inertial reference frame.

Touchstone (vector equation) Recall Subsubsection 3.4.2.2.

Insight 7.2.8 ($F = ma$)

For this law, it is often sufficient to write down the equation and know that the words are there for *back-up. While most people have no trouble remembering $F = ma$, it is important to pay attention to two aspects:*

- *This is a vector-equation, which means that*
  - *the equation is true for each component separately (recall Subsubsection 3.4.2.2), and*
  - *the direction of $\vec{F}_{\text{net}}$ is the same as the direction of $\vec{a}$ (which, of course, might be different than the direction of the velocity - recall Paragraph).*
- *The force in this equation is the* net force, *which means that we must consider* all *forces that are acting on this object and only those forces that are acting on* this *object.*

Translation 7.2.9 (*The Story of $\vec{F}_{\text{net}} = m\vec{a}$*)

Referenced by Exercise 7.2.17

This equation is all about what happens to a specific object, m. If the object, m, is accelerating in a particular direction, $\vec{a}$, then it is because the combination of forces, $\vec{F}_{\text{net}}$, do not entirely cancel each other out. This also can be expressed as: if the combination of forces, $\vec{F}_{\text{net}}$, do not entirely cancel each other out, then our friend m must be accelerating, $\vec{a}$, in a particular direction. Furthermore the resulting direction of the net force determines the direction of the acceleration. Connecting the English and the math:

$$\underbrace{\vec{F}_{\text{net}}}_{\substack{\text{the}\\ \text{combination}\\ \text{of all forces}\\ \text{acting on } m}} \underbrace{=}_{\text{causes}} \underbrace{m}_{\substack{\text{that}\\ \text{object}}} \underbrace{\vec{a}}_{\substack{\text{to}\\ \text{change}\\ \text{its}\\ \text{velocity}}}$$

Connection 7.2.10 (*Looking Back*)

You should recall, that the direction of the acceleration does not determine the direction of the motion, *but rather determines the direction* of the change *in motion.*

Connection 7.2.11 (*Looking Ahead*)

That idea will be important when we discuss how a tension acts as a *centripetal force, the relationship between velocity and acceleration in a spring* that *oscillates, and objects that are propelled through either a gravitational* or an *electrical* field.

7.2.2.1 Units of Force

Referenced by 3.2.3

Recall that the fundamental units of the SI-system are meters, kilograms, and seconds (MKS). With our relationship connecting force to mass (kg) and acceleration ($^{\text{m}}/_{\text{s}^2}$), we can see that the units of force are $^{\text{kg}\cdot\text{m}}/_{\text{s}^2}$. This quantity is so common that we would like to have a shorthand for it. Furthermore, Sir Isaac Newton did such ground-breaking work on the concept, that it was decided in 1948[4] to name the unit the Newton, such that

Definition 7.2.12 (Newton, unit)**.** The **unit of Newton** is defined to be the amount of force necessary to accelerate one kilogram by 1 meter-per-second-squared: $1\,\text{N} = 1\ ^{\text{kg}\cdot\text{m}}/_{\text{s}^2}$

[4]According to: International Bureau of Weights and Measures (1977), The international system of units (330-331) (3rd ed.), U.S. Dept. of Commerce, National Bureau of Standards, p. 17, which refers to the 7th resolution (Mar, 2017) of the 9th CGPM (Mar, 2017).

7.2.2.2 Calculating the Net Force

Subsubsection referenced by Discussion of Newton's first law

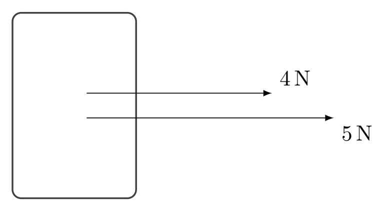
The word "net" that goes with force is here to indicate the total, which is useful to think of as "everything collected with the net." The intention here is that wherever there are multiple forces acting on a single object, we must combine them as vectors as follows:

Etymology (net) Although thinking of net as "everything collected with the net" is useful, according to etymonline.com (Mar, 2017), it is actually from the Old French *net* for "neat" or "clean", having the sense of trim and elegant.

Example 7.2.13 (*Net Force, Vector-Add Forces in the Same-Direction*)

Example referenced by *Example 7.2.19*, *Equilibrium*, *Example 7.2.30*, *Example 7.2.27*, *discussion about 7.2.30*

If there is a 5.0 N *force to the right and a* 4.0 N *force to the right, then the net force is* 9.0 N *to the right.*



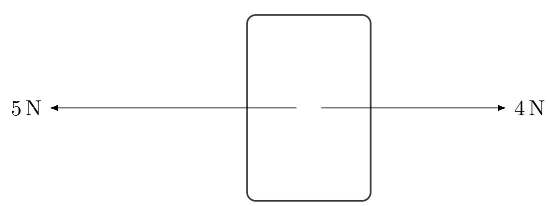
Solution.

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 = \left(5.0\,\text{N}\hat{\imath}\right) + \left(4.0\,\text{N}\hat{\imath}\right) = +9.0\,\text{N}\hat{\imath}$$

Example 7.2.14 (*Net Force, Vector-Add Forces in the Opposite-Direction*)

Example referenced by *Example 7.2.22*, *Equilibrium*

If there is a 5.0 N *force to the left and a* 4.0 N *force to the right, then the net force is* 1.0 N *to the left.*



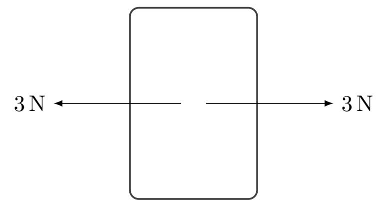
Solution.

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 = \left(-5.0\,\text{N}\hat{\imath}\right) + \left(4.0\,\text{N}\hat{\imath}\right) = -1.0\,\text{N}\hat{\imath}$$

Example 7.2.15 (*Net Force, Vector-Add Equal-Magnitude Opposite-Direction Forces*)

Example referenced by *Equilibrium*, *discussion about 7.2.15*

If there is a 3.0 N *force to the right and a* 3.0 N *force to the left, then the net force is* 0.0 N.



In this case, the object is said to be "in equilibrium."

Solution.

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 = \left(3.0\,\text{N}\hat{\imath}\right) + \left(-3.0\,\text{N}\hat{\imath}\right) = 0.0\,\text{N}\hat{\imath}$$

Connection 7.2.16 (*Looking Ahead*)

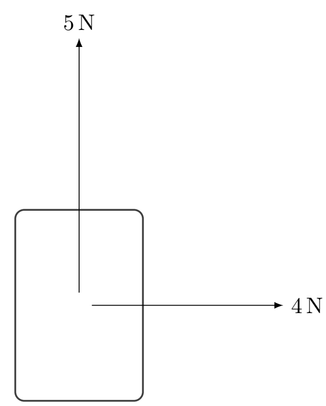
The images included in these examples will eventually be referred to as "free-body diagrams," but for now, you can just consider them images of the forces acting on the bodies.

Next, we should do a couple of examples that show the math for situations with forces in two dimensions. The first, Exercise 7.2.17, has one force in the x-direction and another in the y-direction. The second, Exercise 7.2.18, has one force in the x-direction and the other in the second quadrant.

Exercise 7.2.17 (*An object is pushed by perpendicular forces*)

Exercise referenced by *the discussion of the net force, Exercise 7.2.18*

A 2.0 kg mass is being pushed north with 5.0 N and east with 4.0 N. What is the net force?



Solution. *Since we have multiple forces acting on a mass to cause an acceleration, it should be clear (recall the story) that we need to use Newton's second law and find the net force in order to compute the acceleration. We will, as usual, start with a free-body diagram (at right).*

This example is made easier because the forces happen to be at right angles and so finding their x and y components is not difficult. By adding the x-components and separately adding the y-components, we have found the components of the net force.

	x-comp	y-comp
F_1	0 N	+5 N
F_2	+4 N	0 N
F_{net}	+4 N	+5 N

From there, we can easily find the magnitude and direction of the net force.

$$\text{Magnitude:}\quad F_{\text{net}} = \sqrt{(+4\,\text{N})^2 + (+5\,\text{N})^2} = \mathbf{6.40}\,\text{N}$$

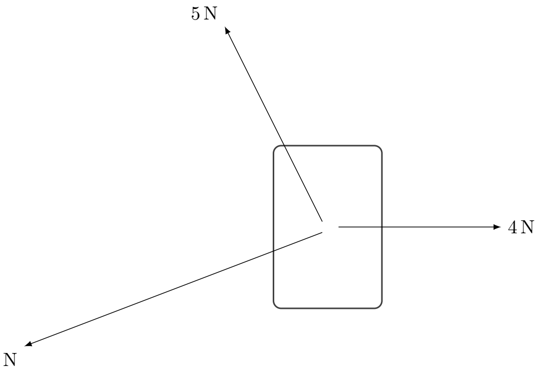
$$\text{Direction:}\quad \theta = \tan^{-1}\left(\frac{+5\,\text{N}}{+4\,\text{N}}\right) = \mathbf{51.3}\,^\circ\text{N of E}$$

(The direction can be stated as $\theta = 51^\circ$N of E or as $\phi = 39^\circ$E of N.) Exercise 7.2.20 will use this calculation to find the acceleration.

Exercise 7.2.18 (*Three forces act on an object*)

Exercise referenced by *the net force*

A 2.0 kg mass is being pushed northwest with 5.0 N at an angle 63.4°N of W, southwest with 6.0 N at an angle of 21.8°S of W, and east with 4.0 N. What is the net force?



Solution. *This follows the same logic as Exercise 7.2.17, which I will not restate here.*

This example is slightly harder because the forces have to be split into their x and y components. By adding the x-components and separately adding the y-components, we have found the components of the net force.

	x-comp	*y-comp*
F_1	$-(5.0\,\mathrm{N})\cos(63.4°) = -\mathbf{2.24}\,\mathrm{N}$	$+(5.0\,\mathrm{N})\sin(63.4°) = +\mathbf{4.47}\,\mathrm{N}$
F_2	$-(6.0\,\mathrm{N})\cos(21.8°) = -\mathbf{5.57}\,\mathrm{N}$	$-(6.0\,\mathrm{N})\sin(21.8°) = -\mathbf{2.23}\,\mathrm{N}$
F_3	$+4\,\mathrm{N}$	$0\,\mathrm{N}$
F_{net}	$-\mathbf{3.81}\,\mathrm{N}$	$+\mathbf{2.24}\,\mathrm{N}$

From there, we can easily find the magnitude and direction of the net force.

$$\begin{aligned} &\textit{Magnitude:} \quad F_{\text{net}} = && \sqrt{(-\mathbf{3.81}\,\mathrm{N})^2 + (+\mathbf{2.24}\,\mathrm{N})^2} && = \mathbf{4.42}\,\mathrm{N} \\ &\textit{Direction:} \quad \theta = && \tan^{-1}\left(\frac{+\mathbf{2.24}\,\mathrm{N}}{-\mathbf{3.81}\,\mathrm{N}}\right) && = \mathbf{30}.5\,°\textit{N of W} \end{aligned}$$

(The direction can be stated as $\theta = 31°$N of W or as $\phi = 60°$W of N.) Exercise 7.2.21 will use this calculation to find the acceleration.

7.2.2.3 Using the Net Force to Calculate Other Quantities

Generally, the point of finding the net force is that it causes an object to change its velocity. Let's also consider a few simple examples of this calculation.

Example 7.2.19 (*The acceleration of a box feeling a net force*)

Example referenced by *finding m from $F = ma$, Example 7.2.30, Example 7.2.27, discussion about 7.2.30, Example 8.1.6, Answer 8.1.2.6.1, Example 8.3.2, Example 8.4.8*

If the forces in Example 7.2.13 are applied to an object with mass 2.0 kg, then it will accelerate at the rate of

$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m} = \frac{+(9.0\,\mathrm{N})\hat{\imath}}{2.0\,\mathrm{kg}} = 4.5\ {}^{\mathrm{N}}\!/\!_{\mathrm{kg}}\hat{\imath} = 4.5\ {}^{\mathrm{kg\cdot m}}\!/\!_{\mathrm{s^2\cdot kg}}\hat{\imath} \ = \ 4.5\ {}^{\mathrm{m}}\!/\!_{\mathrm{s^2}}\hat{\imath}$$

which (recall Section 5.4), after acting for 1.6 s on an object originally at rest, would result in a final speed of

$$v_f = (0\ {}^{\mathrm{m}}\!/\!_{\mathrm{s}}) + (+4.5\ {}^{\mathrm{m}}\!/\!_{\mathrm{s^2}})(1.6\,\mathrm{s}) = 7.2\ {}^{\mathrm{m}}\!/\!_{\mathrm{s}}$$

We can do this same kind of procedure for the case when forces are in two dimensions.

Exercise 7.2.20 (*Accelerating a box from $\vec{F}_{\text{net}}$*)

A 2.0 kg mass is being pushed north with 5.0 N and east with 4.0 N. What is the acceleration?

Hint 1. *Exercise 7.2.17 already found the net force to be $\vec{F}_{\text{net}} = 4.0\,\text{N}\hat{\imath} + 5.0\,\text{N}\hat{\jmath}$ which is $F_{\text{net}} = 6.4\,\text{N}$ at $51°$ N of E. (What remains is to find the acceleration.)*

Hint 2. *It is possible to find the (vector) acceleration from the (vector) net force by using* either *the x- and y-components,* or *by using the magnitude and direction.*

Solution 1. ***components:*** *Given the mass and the components of the net force, we can calculate the acceleration.*

$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m} = \frac{4.0\,\text{N}\hat{\imath} + 5.0\,\text{N}\hat{\jmath}}{2.0\,\text{kg}} = 2.0\ {}^{\text{m}}\!/\!_{\text{s}^2}\,\hat{\imath} + 2.5\ {}^{\text{m}}\!/\!_{\text{s}^2}\,\hat{\jmath}$$

You can then use the components of the acceleration to find the magnitude and direction of the acceleration.

$$\begin{array}{lll} \textit{Magnitude:}\quad a = & \sqrt{(+2.0\ {}^{\text{m}}\!/\!_{\text{s}^2})^2 + (+2.5\ {}^{\text{m}}\!/\!_{\text{s}^2})^2} & = \mathbf{3.20}\,\text{N} \\ \textit{Direction:}\quad \theta = & \tan^{-1}\left(\dfrac{+2.5\ {}^{\text{m}}\!/\!_{\text{s}^2}}{+2.0\ {}^{\text{m}}\!/\!_{\text{s}^2}}\right) & = \mathbf{51}\,°\textit{N of E} \end{array}$$

Solution 2. ***magnitude and direction:*** *Given the mass and the magnitude and direction of the net force, we can calculate the acceleration. $a = \frac{6.4\,\text{N}}{2.0\,\text{kg}} = 3.2\ {}^{\text{m}}\!/\!_{\text{s}^2}$ and know that the direction of the acceleration is the same as the acceleration of the net force: $51°$ N of E.*

Exercise 7.2.21 (*Accelerating a box pushed by three forces*)

A $2.0\,\text{kg}$ mass is being pushed northwest with $5.0\,\text{N}$ at an angle $63.4°$N of W, southwest with $6.0\,\text{N}$ at an angle of $21.8°$S of W, and east with $4.0\,\text{N}$. What is the acceleration?

Hint 1. *Exercise 7.2.18 already found the net force to be $\vec{F}_{\text{net}} = 3.8\,\text{N}\hat{\imath} + 2.2\,\text{N}\hat{\jmath}$ which is $F_{\text{net}} = 4.4\,\text{N}$ at $31°$ N of W. (What remains is to find the acceleration.)*

Hint 2. *It is possible to find the (vector) acceleration from the (vector) net force by using* either *the x- and y-components,* or *by using the magnitude and direction.*

Solution 1. *Given the mass and the components of the net force, we can calculate the acceleration.*

$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m} = \frac{3.8\,\text{N}\hat{\imath} + 2.2\,\text{N}\hat{\jmath}}{2.0\,\text{kg}} = 1.9\ {}^{\text{m}}\!/\!_{\text{s}^2}\,\hat{\imath} + 1.1\ {}^{\text{m}}\!/\!_{\text{s}^2}\,\hat{\jmath}$$

You can then use the components of the acceleration to find the magnitude and direction of the acceleration.

$$\begin{array}{lll} \textit{Magnitude:}\quad a = & \sqrt{(+1.9\ {}^{\text{m}}\!/\!_{\text{s}^2})^2 + (+1.1\ {}^{\text{m}}\!/\!_{\text{s}^2})^2} & = \mathbf{2.2}\ {}^{\text{m}}\!/\!_{\text{s}^2} \\ \textit{Direction:}\quad \theta = & \tan^{-1}\left(\dfrac{+1.1\ {}^{\text{m}}\!/\!_{\text{s}^2}}{+1.9\ {}^{\text{m}}\!/\!_{\text{s}^2}}\right) & = \mathbf{31}\,°\textit{N of W} \end{array}$$

Solution 2. *Given the mass and the magnitude of the net force we can find the magnitude of the acceleration $a = \frac{4.4\,\text{N}}{2.0\,\text{kg}} = 2.2\ {}^{\text{m}}\!/\!_{\text{s}^2}$ and know that the direction of the acceleration is the same as the acceleration of the net force: $31°$ N of W.*

In Example 7.2.19, we used the forces to find the acceleration. It is also possible to use the forces to find the mass of an object, as follows:

Example 7.2.22 (*Finding the mass of a box from its force and acceleration*)

If the forces in Example 7.2.14 are applied to an object with unknown mass and produce an acceleration of 3.2 $\mathrm{m/s^2}$, then what is the mass of the object?

Solution. *Naively, one might consider $m = \frac{\vec{F}_{\text{net}}}{\vec{a}}$, but it does not make mathematical sense to divide vectors. In this case, you must consider the magnitudes of force and acceleration, knowing that their directions are the same. (We are not "cancelling" the directions.)*

$$m = \frac{F_{\text{net}}}{a} = \frac{9.0\,\mathrm{N}}{3.2\,\mathrm{m/s^2}} = \mathbf{2.8}1\ \mathrm{N{\cdot}s^2/m} = 2.8\ \mathrm{kg{\cdot}m{\cdot}s^2/s^2{\cdot}m} \;=\; 2.8\,\mathrm{kg}$$

Connection 7.2.23 (*Looking Ahead*)

Yet another example of using this equation can be seen in many bathrooms. The scale that people stand on uses a spring (introduced in Subsection 8.3.1 and discussed in detail in Section 8.6) to adjust the force provided until your acceleration is zero (placing you in equilibrium) and then tells you the force it needed to balance your weight.

It will be easier to visualize these ideas when we introduce the tool of a free-body diagram in Subsubsection 7.2.3.1.

7.2.2.4 Equilibrium

Subsubsection referenced by Example 7.2.15, Newton's first law, discussion of falling objects

This word can be traced back to Latin and Old English with the prefix *equi-* for *equal* and the root *libra* referring to a *pair of scales, as in a balance*, such as those depicted in images of the astronomical constellation Libra. When the scales are equal, they are in equilibrium. Since the second law asks us to calculate the sum of the forces acting on an object, one of the primary questions is to determine if those forces balance each other. In Example 7.2.13 and Example 7.2.14, the forces are not balanced, the object "is not in equilibrium", and it will be accelerated in a particular direction. In the Example 7.2.15, the forces are balanced, the object "is in equilibrium", and it will *not change* its velocity (in accord with the first law).

Definition 7.2.24 (equilibrium)**.** An object in **equilibrium** has $\vec{F}_{\text{net}} = 0\,\mathrm{N}$ and $\vec{a} = 0$.

7.2.2.5 How the Laws Work Together

Referenced by Subsubsection 7.2.1.2

When forces act on an object, Newton's second law applies, so we usually start with the second law. If those forces combine to give a net force of zero, such that the object is in equilibrium, then Newton's first law applies. If we also care about the person or thing pushing, then the third law also applies.

To better understand how the first and second laws work together, Investigation 7.3.2 provides some activities that you can do or consider in order to think about the patterns you can see when you are or aren't pushing on objects. Exploration 7.2.25 will help you think through some of the consequences of the first and second law. When you are ready to solve some problems, you can jump to Section 7.3, but some of those examples will also reference Newton's third law.

Exploration 7.2.25 (*Out of gas*)

Exploration referenced by *Subsubsection 7.2.2.5*

On a long road trip with your friend Beth, your car starts to sputter as it runs out of gas shortly before arriving in a new town. You see a sign for a gas station in the distance and have to decide what to do. You and Beth can think of three options.

Plan A *Pull over, park the car, walk to the gas station, buy a gas can, fill it up, carry it back to the car,* *and drive on! If you follow this plan, then read Answer 1.*

Plan B *Leave the car in drive, continue holding the gas-pedal down until there is absolutely no gas, and hope against all hope that you get the car to the gas station so that nobody needs to carry a heavy gas can. If you follow this plan, then read Answer 2.*

Plan C *Speed up to just over the speed limit, put the car in neutral, turn on your blinking hazard-lights, coast as far as you can possibly coast, and hope against all hope that you get the car to the gas station so that nobody needs to carry a heavy gas can. If you follow this plan, then read Answer 3.*

Answer 1. *Just as planned, you pull over and park the car. Beth suggests one of you stays with the car, probably because she has physics homework to do. If you decide to separate, read Answer 5. If you decide to journey together, read Answer 7.*

Answer 2. *As soon as you decide to do this, the gas runs out. Thinking you can make it to the gas station, you take your foot off of the gas pedal. You slow down fairly quickly and get nervous that you might get rear-ended. You turn on the hazard-lights. After about a minute you are travelling* 30 mph *and you pass the time by working out Exercise 1. People are honking at you as they try to pass. Beth turns to you and asks you why you are going so slow. If you start a discussion about Newton's First Law, then go to Answer 4. If you get embarrassed and decide to pull over, then read Answer 6.*

Answer 3. *You speed up to* 60 mph *before the gas runs out and then you quickly pop the car into neutral. You slow down gradually and, in an effort to not get rear-ended, you cleverly turn on the hazard-lights. After about a minute you are travelling* 52 mph *and you pass the time by working out Exercise 5.3.2). After* 2 min, *you are travelling* 43 mph *and people are getting impatient as they try to pass. After* 2.79 min, *you triumphantly coast into the gas station at a comfortable speed of* 36.7 mph. *Beth is so happy, she buys you a full tank of gas and the two of you start a discussion about Newton's First Law while pumping the gas. Please read Answer 8.*

Answer 4. *After some discussion, you and Beth realize that when the car is in drive, the transmission (the part of the car that converts how-fast-the-engine-spins to how-fast-the-axel-and-wheels-turn) is connected to the axel, which means that the rolling wheels are trying to turn the engine parts as well as the wheels themselves. The engine parts have grease and oil, but still take a lot of energy to turn. This causes friction, which dissipates energy and, more importantly, exerts a backwards force on the spinning wheels. Your car is not being described by Newton's First Law, which requires there to be no force applied. Instead your car is being described by Newton's Second Law and the force is changing the velocity to cause you to go slower. It only takes the car* 2.2 min *to stop and you still have to walk to the gas station. Beth laments "If only there were a way to reduce the force on the axel..." If it occurs to you to speculate about putting the car in neutral when the gas ran out, then imagine reading Answer 3. If you stop talking and walk to the gas station, then read Answer 10.*

Answer 5. *You leave Beth in the car and walk the 45 minutes to the gas station. You buy a gas can, fill it up, and start to carry it back to the car. It is very heavy and you notice vultures circling overhead. You hope you survive this. It might have been a better idea to bring Beth with you to share the burden. You stumble once, and then again. You swear to be more cautious about estimating your gas consumption. After walking for what seems like hours and stumbling back to the car, you find Beth very excited. She declares that she has invented a time machine so you can go back to the* adventure *and start over to learn something about Newton's First Law!*

Answer 6. *You pull over and park the car. Beth suggests one of you stays with the car, probably because she has physics homework to do. If you decide to separate, read Answer 9. If you decide to journey together, read Answer 7.*

Answer 7. *Everything goes as planned. You drive off into the sunset sadly ignorant of the physics you might have learned.* ***The end!***

Answer 8. *During the discussion, you and Beth realize that the rolling wheels and the spinning axel are still connected to the not-spinning frame of the car. While this causes less friction than if the car were in drive, there is still some friction, which dissipates energy and, more importantly, exerts a backwards force on the spinning wheels. Your car is not being described by Newton's First Law, which requires there to be no force applied. Instead your car is being described by Newton's Second Law and the force is changing* *the velocity to cause you to go slower. You finish getting gas and drive on to many happy adventures.* ***The end!***

Answer 9. *You leave Beth in the car and walk the 31 minutes to the gas station. You buy a gas can, fill it up, and start to carry it back to the car. It is very heavy and you notice vultures circling overhead. You hope you survive this. It might have been a better idea to bring Beth with you to share the burden. You stumble once, and then again. You swear to be more cautious about estimating your gas consumption. After walking for what seems like hours and stumbling back to the car, you find Beth very excited. She declares that she has invented a time machine so you can go back to the adventure and start over to learn something about Newton's First Law!*

Answer 10. *You and Beth happily walk the 25 minutes to the gas station, discussing and working out physics problems the whole way. You buy a gas can, fill it up, and share the burden of carrying a heavy gas can. You return to the car, add gas, and drive on to many happy adventures.* ***The end!***

7.2.3 Translating Newton's Third Law: Action & Reaction

Subsection referenced by how to describe forces, Example 7.3.4, Example 7.3.7, Task 8.4.13.c

Newton's Third Law

For every force acting *on* one object *by* an *other object*, there is an equal-in-magnitude reaction-force acting *on* the *other object* in the opposite direction.

Foreshadow This law is the force-version of the statement of the conservation of momentum.

This law is often shortened to "For every action, there is an equal and opposite reaction." The statement given above is meant to emphasize two points:

- These "actions" are specifically forces.
- Forces are an interaction in which the acting force is *on one object* by another and necessitates that there is a reaction force on the other object by the one. That is to say, an object cannot feel a force without also exerting a force back on the other object.

Another way to say this is that all forces come in action/reaction pairs that necessarily have equal magnitude and opposite direction and necessarily act on different objects.

Insight referenced by Subsection 8.3.3, the answers in Example 8.4.7

Insight 7.2.26 (*Third-law pairs*)

The action/reaction force pairs relevant to Newton's third law must *be of the same type because it is the same interaction.*

Let's take this apart and connect it to your daily experiences. Students of physics will often see the terms action and reaction and connect it to the way humans react to the actions of their friends. However, this implies a causal[5] response that is not true for Newton's forces. That is to say, this is not a "revenge law" whereby if you push on me, then I will choose to push you back. Instead, it is expressing that forces are intrinsically interactions between a pair of objects. When you push on me, I am – independent of my choosing to do so – necessarily pushing back on you. But, you might say, "if that were true, then why am I able to sneak up on you and push you over without falling over myself?" Well, you can think about how that works by reading Exploration 7.3.3. After we introduce the tool of a free-body diagram in the next subsection, you can also explore this idea by comparing Example 7.3.4 to Example 7.3.7.

7.2.3.1 The Free-Body Diagram (FBD)

[5] Please note that "causal" and "casual" are different words.

Referenced by Example 7.2.15, uses of $F = ma$

In the more interesting situations where there are several forces acting, it can be easy to lose track of what is pushing whom where. In order to better organize our information and direct our attention, we can make use of **free-body diagrams**. The basic idea is to make a diagram for each individual object that we care about in a given situation, and *free* from the overall picture. This allows us to identify the forces acting on a single object (relevant for Newton's second law) and more easily pair them with third-law pairs that act on different objects.

To see how this works, the next example will build on the previous examples to help us consider not only the (2nd law) forces on the object, but also the (3rd law) forces on the people doing the pushing and pulling.

Example 7.2.27 (*The reaction forces on people pushing a box*)

Let's reconsider Example 7.2.13 and imagine that Abdul stands to the left of the object and exerts a force to the right (pushes) with magnitude $\vec{F}_1 = +5.0\,\mathrm{N}\hat{\imath}$ and that Beth stands to the right of the object and exerts a force to the right (pulls) with magnitude $\vec{F}_2 = +4.0\,\mathrm{N}\hat{\imath}$, as drawn in Figure 7.2.28. Both of these forces are on the object and, by Newton's second law cause it to accelerate (as described in Example 7.2.19). Newton's third law tells us about the interaction between *objects and, from this, we can deduce the force on Abdul and Beth.*[6]

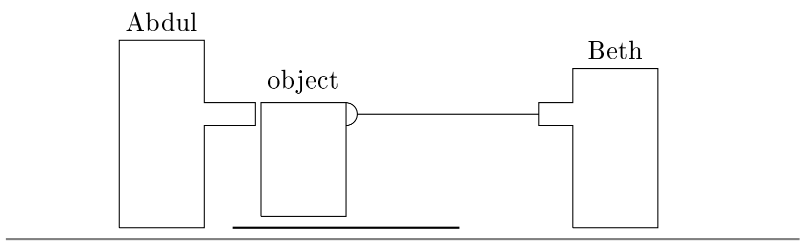
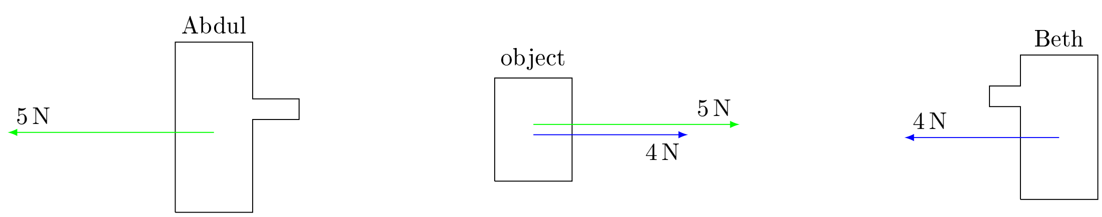


Figure 7.2.28: Abdul and Beth push on a box

Now we will draw a free-body diagram for each individual. Notice that each free-body *diagram is on its own, free from the rest of the picture.*



Because Abdul pushes the object to the right, Newton's third law tells us Abdul feels a force to the left *on* him *by* the object.

For the object (in the middle), Example 7.2.13 showed that $F_{\mathrm{net}} = 9.0\,\mathrm{N}$.

Because Beth pulls on the object to the right, Newton's third law tells us Beth feels a force to the left *on* her *by* the object.

Figure 7.2.29: These are the free fbody diagrams for Figure 26.

It turns out that this is a little oversimplified. When we get to Section 8.1 and *Section 8.3, we will see that we have to include a downwards force of gravity and an upwards support force. This will be explained in Example 8.3.2.*

Return to: 7.3.4, 7.3.7, rope-tension

As indicated above, the free-body diagram also helps us visualize the third-law (action/reaction) force pairs. By evaluating the colored forces in Figure 7.2.29, we can see that the green forces form an action-reaction pair and separately the blue forces form an action-reaction pair. Notice the following:

1. The third law action-reaction pairs are located of different objects and cannot be added. (You cannot add blue to blue nor green to green.)
2. The second law forces on the object should be added to find that specific object's acceleration. (You *should* add the blue force to the green force on the middle object.)

None of these three are in equilibrium, because they each have a net force. Example 7.2.19 calculates the acceleration of the object. Example 7.2.30 calculates the acceleration of the people.

Example 7.2.30 (*The acceleration of people pushing a box*)

***Referenced by** 7.2.27*

In the same way that Example 7.2.19 was able to use Newton's second law to compute the acceleration of an object after Example 7.2.13 found the net force, we can use the forces found in Example 7.2.27 to compute the acceleration of the people pushing the box. Recall that Figure 7.2.28 shows how Abdul and Beth exert forces on the box (and, from Newton's third law, how the box exerts force on the people).

1. *Since Abdul exerts a force of* 5.0 N *to the right* $(+\hat{\imath})$ *on the object, the* third law *reminds us that the object exerts a force of* 5.0 N *to the left* $(-\hat{\imath})$ *on Abdul. Since Abdul has a mass of* 85.0 kg*, the* second law *reminds us that he is accelerated at the rate of*

$$\vec{a}_1 = \frac{-5.0\,\mathrm{N}\hat{\imath}}{85.0\,\mathrm{kg}} = -0.0\mathbf{58}8\ {}^{\mathrm{m}}\!/_{\mathrm{s}^2}\hat{\imath} = -5.9\times10^{-2}\ {}^{\mathrm{m}}\!/_{\mathrm{s}^2}\hat{\imath}$$

   which is to the left with a small enough value that it is easy for him to brace against. Even if he doesn't brace, if he starts from rest, he will only be moving

$$v_{1f} = (0\ {}^{\mathrm{m}}\!/_{\mathrm{s}}) + (-0.0\mathbf{58}8\ {}^{\mathrm{m}}\!/_{\mathrm{s}^2})(1.6\,\mathrm{s}) = -0.0\mathbf{94}1\ {}^{\mathrm{m}}\!/_{\mathrm{s}},$$

   which is much slower than the object.

2. *Since Beth exerts a force of* 4.0 N *to the right on the object, the* third law *reminds us that the object exerts a force of* 4.0 N *to the left on Beth. Since Beth has a mass of* 75.0 kg*, the* second law *reminds us that she is accelerated at the rate of*

$$\vec{a}_2 = \frac{-4.0\,\mathrm{N}\hat{\imath}}{75.0\,\mathrm{kg}} = -0.0\mathbf{53}3\ {}^{\mathrm{m}}\!/_{\mathrm{s}^2}\hat{\imath} = -5.3\times10^{-2}\ {}^{\mathrm{m}}\!/_{\mathrm{s}^2}$$

   which is also to the left with a small enough value that it is easy for her to brace against. Even if she doesn't brace, if she starts from rest, she will only be moving

$$v_{2f} = (0\ {}^{\mathrm{m}}\!/_{\mathrm{s}}) + (-0.0\mathbf{53}3\ {}^{\mathrm{m}}\!/_{\mathrm{s}^2})(1.6\,\mathrm{s}) = -0.0\mathbf{85}3\ {}^{\mathrm{m}}\!/_{\mathrm{s}},$$

   which is also much slower than the object and different from Abdul.

It might be interesting to note that, since everybody and thing has a different mass, the accelerations are all different. This would also be true if the forces were all the same.

If we now do the same thing with the Example 7.2.15, but let Carl be the person on the left pushing to the right and Diane be the person on the right pushing to the left, then we see that while the green forces form an action-reaction pair showing the third-law interaction between Carl and the object and while the blue forces form an action-reaction pair showing the third-law interaction between Diane and the object, it is the combination of the blue and the green forces, which only act on the object itself, that coincidentally cancel to leave the object in (second law) equilibrium. In these images, Carl and Diane are *not* in equilibrium.

The point of Examples 7.2.27–7.2.30 in comparison to Examples 7.2.13 (which told us about the forces on the object) and 7.2.19 (which told us about how the object moved) is to highlight the different roles of Newton's second and third laws. The earlier examples were relevant to the second law and only affected the object itself. To say this more specifically, the forces within one free-body diagram are described by Newton's second law. They do get added together to form the net-force (which is to say that we can add the object's green force to the object's blue force). They are able to cancel each other if they *happen to* be equal in magnitude and opposite in direction. Finally, they will determine how that specific object accelerates. On the other hand, Newton's third law describes any specific pair of forces that interact between free-body diagrams (each colored pair); they *will necessarily be* equal in magnitude and opposite in direction, but they cannot be canceled because they cannot be added because they are on different objects.

7.3 Examples

Referenced by Subsubsection 7.2.2.5

Next, we can consider a simple interactive example that is intended to help you think about how you know a force is acting.

Investigation 7.3.1 (*Drop a Book*)

You hold a book a little above your desk. When you let go, it falls and then hits your desk.

1. *While you are holding it, it has no acceleration. Are there forces acting on it? Select one: "Yes" (Answer 1) or "No" (Answer 2).*
2. *While you are holding it, is it in equilibrium? Select one: "Yes" (Answer 3) or "No" (Answer 4).*
3. *After you let go and while the book falls, it accelerates downwards. Are there forces acting on it? Select one: "Yes" (Answer 5) or "No" (Answer 5).*
4. *While it is hitting the desk, is it accelerating? Select one: "Yes" (Answer 6) or "No" (Answer 7).*
5. *After it has landed and is sitting on the desk, is it in equilibrium? Select one: "Yes" (Answer 8) or "No" (Answer 9).*
6. *After it has landed and is sitting on the desk, how many forces are acting on it? Select one: "Zero" (Answer 11), "One" (Answer 12), or "Two" (Answer 13).*

Answer 1. *There are forces acting on it. You should be able to tell this because you are exerting one of the forces. While it is true that there are forces on it, it is also true that there is no* net *force. If you are exerting an upward force on the book, can you guess (Answer 10) what the downward force is?*

Answer 2. *It is true that while you hold the book, there is no* net *force, but that does not mean that there is no force acting. If there were no forces on the book, then your hand would not need to be there. In fact, if you remove the force your hand provides, then the book falls. This shows that there is an upward force (by your hand on the book) and a downward force (of gravity by the Earth on the book).*

Answer 3. *It is in equilibrium. When the acceleration is zero, then the net force must be zero and those properties are what define equilibrium.*

Answer 4. *The definition of equilibrium is that the forces balance. The result of this is that the net force must be zero and the acceleration is then zero. You can tell this is true because the velocity is* not changing. *It is not important that the velocity is zero, what is important is that the velocity* stays zero. *While you hold it, the book is in equilibrium.*

Answer 5. *Both "Yes" and "No" bring you to this answer. Yes, there is* a force *on the book while it falls (the force of gravity), but no, there are not force**s** (plural). There is only one force. "But, wait!" you say, "What about the force of air resistance?" Aha! You are correct; there is a force of air resistance, but in this case, it is negligible and we will not consider it. Please read Subsection 5.6.2 for more information about deciding when to use or ignore this phenomenon.*

Answer 6. *The book is accelerating. The velocity is changing from "moving downwards" to "stopped". The book is not in equilibrium.*

Answer 7. *While it is hitting the desk, the velocity is changing from "moving downwards" to "stopped". Since the velocity is changing,* the book is accelerating. *Since it is accelerating, the book is not in equilibrium.*

Answer 8. *It is in equilibrium. The book is at rest and* continues to be *at rest on the desk. There are forces acting, but they cancel each other, resulting in no net force.*

Answer 9. *After it has landed, the book stops moving. Once the book comes to rest on the desk, it* continues to stay at rest. *This says that the velocity is not changing, so the book is not accelerating. That means that the book is in equilibrium. There are forces acting, but they cancel each other, resulting in no net force.*

Answer 10. *It is the force of gravity.*

Answer 11. *Recall the situation when you were holding the book. Gravity is still pulling the book down and the desk is holding the book up. There are two forces acting on the book while it is at rest on the desk.*

Answer 12. *If there were only one force on the book, it could not be a balanced force, so the book could not be in equilibrium and the book would be accelerating. The book is not accelerating, so there are either two forces (Answer 13) or no forces (Answer 11).*

Answer 13. *There are two forces acting on the book while it is at rest on the desk. Similar to the situation when you were holding the book, gravity is pulling the book down and the desk is holding the book up.*

Next, we can consider pushing an object across the floor in Investigation 7.3.2 to get a different sense of observations we can make that help us recognize patterns that are due to forces we might not have thought to look for.

Investigation 7.3.2 (*Pushing an Object Across the Floor*)

(a) *Push a chair across a carpet floor. Notice that when you stop pushing, it stops moving. Based on this, do you think force causes motion?*

Solution. *If we refer to "motion" as describing the velocity, then no. Force causes* a change in *velocity. When you stop pushing, the chair stops because there is a force from the carpet acting to oppose the force you apply while you push the chair.*

(b) *Push a chair across a tile floor. Notice that when you stop pushing, it probably*[1] *stops moving. Based on this, do you think force causes motion?*

Solution. *This is essentially the same as Task a, but the carpet exerts more force than the tile. In either case, force causes a change in velocity. You are trying to speed the chair up and the floor is trying to slow the chair down. (Both are trying to change the velocity, but cancel to result in a constant velocity.) When you stop pushing, the chair stops moving because there is a force from the tile acting to oppose the force you apply while you push the chair; when you let go, this force slows the chair until the chair stops and then the force stops acting. (See Section 8.4* for more details.*)*

(c) *Push a chair* with wheels *across a tile floor, with some strength, then let it go.*

(i) *What happens when you stop pushing?*

Solution. *For a chair with wheels being pushed across a tile floor, when you stop pushing it probably continues to move across the floor for at least a short distance.*

(ii) *If force causes motion, why does the chair move after you stop touching it?*

Solution. *The chair continues to move for the same reason that the chair without wheels and the chair on carpet* all *continued to move when you let go. The reason is that this is* how all objects behave; they maintain their velocity when allowed to act without interference. *(This is why Newton's first law says what it does.) Because the chair with wheels has much less friction there is a smaller force trying to interfere with the motion and so it continues to move for a noticeable distance. The other chairs slowed to a stop almost immediately. The wheel-less chair* *on tile might have continued for a short distance if it was moving fast enough that it required a long enough time to change its velocity to zero.*

(d) *Push a chair* with wheels *across a tile floor, change your behavior after you let it go.*

(i) *Do your actions when you are not touching the chair have* any *impact on the chair?*

Solution 1. *No. But if it does not matter what you do after you let go of the chair, then why do coaches (in basketball free-throws, tennis serves and swings, baseball pitches, and all manner of arm and leg propulsion) tell you to pay attention to your "follow through"?*

Select one: "They have been fooled; follow-through doesn't matter" (Solution 2) or "they are right; follow-through does matter!" (Solution 3).

Solution 2. *They haven't been fooled, but follow-through matters in a different way. What does matter is not literally how you move* after *the release, but rather how you move* before *you release the ball. By paying attention to your follow-through, you are also changing the way you move before you release or impact the ball. You want a smooth flow throughout the motion and a sloppy follow-through often implies a sloppy initiation of the motion.*

Solution 3. *What does matter is not literally how you move* after *the release, but rather how you move* before *you release the ball. By paying attention to your follow-through, you are also changing the way you move before you release or impact the ball. You want a smooth flow throughout the motion and a sloppy follow-through often implies a sloppy initiation of the motion.*

(ii) *Is it possible that there is a "residual effect" that you have on the chair after letting it go?*

Solution. *No. When you throw a ball very high into the air, you can dance a jig or do any manner of things and it will obviously not affect the ball. The force you exerted on the chair goes away the instant you stop touching the chair. It is, however, true that your force gave the chair some velocity (actually momentum) and Newton's first law (inertia) says that the chair would prefer to keep that velocity. Unfortunately, the friction with the ground slows it down. The careful way to describe the situation is that your force gave the chair some velocity (actually momentum) and its characteristic inertia made it difficult for the frictional force to slow it down rapidly.*

(e) *Newton's First Law says that if you give the chair a velocity, it should keep that velocity. Repeat the first three suggestions and correlate the interaction-with-the-ground to the motion-after-you-push-and-release. Is there a force that the chair feels after you release it?*

Solution. *The force that the chair feels after you release it is friction. For the carpet, there is a lot of friction and the chair slows down very quickly (essentially instantaneously). For the wheel-less chair on the tile floor, the chair slows rapidly although it may leave your hand. The wheels provide the least amount of friction and that chair goes the furthest. You may note that the friction slowing the chair-with-wheels is primarily between the rolling wheel and its axel (where it connects to the non-rolling chair leg) rather than between the wheel and the floor (although the friction between the wheel and the floor also plays a role). This is discussed in more detail in* *Section 8.4**.*

(f) *Newton's Second Law says that a net force will change the velocity. Push a chair* gently *across the floor. A constant force (balanced by the force of friction) will move at a constant speed. What if there were no friction?*

Solution. *If there were no friction, then you could start the chair and it would move on its own at a constant speed; you wouldn't need to continue pushing to keep it moving. On the other hand, if you did continue to push, then the chair would continue to speed up and you would have to run faster and faster to keep up with it. On the other hand, if the chair were not experiencing friction, then you probably wouldn't either and you couldn't get enough traction to keep up with the chair, so it would sail away almost immediately, being then described by Newton's first law!*

(g) *Newton's Second Law says that a net force will change the velocity. Push a chair* forcefully *across the floor. A constant force (stronger than the force of friction) will accelerate the object away from your push. Can you list surfaces that are essentially frictionless?*

Hint 1. *You should include surfaces that are very easy to push objects across.*

Hint 2. *Ice is an obvious (?) choice. You might have experience with the table-top "air hockey" game. You can look-up "mag-lev trains".*

Hint 3. *You should also think about how smooth or rough the surfaces you list are.*

Notice in each case that you are not the only thing interacting with the chair. The floor is also interacting with the chair. The floor exerts a force of friction on the chair. So, when you interpret how your force causes the chair to move, you must also account for the interaction with the floor in your expectations. We can minimize the effect of friction, by modifying the floor surface. If you have ever driven on ice and felt out of control, you might have begun to develop your Newtonian intuition.

Return to: 7.2.2.5, section 7.3 reference to Theorem 7.3.2

Building on that, it is useful to also consider how human beings behave when they are pushing or getting pushed. Because people have *intention* in their actions, we subconsciously balance ourselves and we don't always recognize that we are doing it. Exploration 7.3.3 provides an interactive storyline that starts to show some of the patterns that can lead to a recognition of how we balance ourselves.

Exploration 7.3.3 (*The Town Bully*)

Exploration referenced by *the discussion of action-reaction forces, section 7.3 reference to Theorem 7.3.3, Example 7.3.4, Example 7.3.7*

Zambert is the town bully. One day, he spies a biology student, Carl, minding his own business studying an interesting ecological phenomenon. At the same time, you are standing across the street chatting with your friend Diane, who happens to be taking a psychology class. Diane has been quite fascinated lately with watching the way others interact and points out the way Zambert is menacingly approaching the unsuspecting Carl. You both predict that Zambert is going to push Carl over. Diane is mesmerized by the psychological effects and you, having just learned about Newton's laws, are excited to see if this action does indeed produce a reaction.

1. *If you watch the way Carl is standing before, during, and after Zambert pushes him, then read Answer 1. (See also Answer 8.)*

2. *If you watch the way Zambert is standing before, during, and after he pushes Carl, then read Answer 2. (See also Answer 8.)*

3. *If, on the other hand, you shout a warning to Carl and a criticism to Zambert, trying to keep the incident from becoming violent, then read Answer 3.*

Answer 1.

Foreshadow *The physics of why an object (or person) rotates when they fall over is discussed with torque.*

As Carl gets pushed, you notice that he was not aware of the pending doom. He is standing casually with his feet set to support his own weight, but not to brace him against the sideways force. When he gets pushed from the side, his feet stay in place and his torso topples, rotating him about his center of mass as he falls to the ground. Diane points to her phone and says, "I recorded the whole thing!" If you respond, "Awesome! Can I watch the part about how Zambert acts?", please read Answer 2. If you respond, "Awesome! Let's show the psychology and physics faculty our cool video!", please read Answer 6. If you respond, "Yeah, we probably should have intervened before this happened instead of just watching. Let's go talk to Campus Security.", please read Answer 7.

Answer 2. *As Zambert pushes, you notice that because he was being intentional, he put one foot behind him to brace his body during the push. He leans into the push and stays standing. You are intrigued.*

If you decide to do a follow-up experiment by pushing Diane over without bracing yourself, then read Answer 4. If you decide to exercise self-restraint, then read Answer 5.

Answer 3. *Being the thoughtful and considerate person you are, you rush over and startle Carl out of his reverie. Zambert is quite angry and now focuses his attention on you! He rushes towards you and shoves as hard as he can. You go flying backwards and land on your tailbone while he just stands there laughing. Carl and Diane both rush over to help you while Zambert wanders off. Surprisingly, Carl has an icepack, which helps. If you go speak to your faculty members about this, please read Answer 6. If you decide to talk to Campus Security, please read Answer 7.*

Answer 4. *You turn and push Diane over. Like Carl, she did not expect it and was not braced, so she falls over. Similarly, you decided not to brace yourself and in pushing Diane, you fall over backwards! Diane did not have to choose to push on you. The act of you deciding to push her necessarily and simultaneously produces a force on you, equal in magnitude and opposite in direction. Unfortunately, Diane doesn't think this was a useful exercise and shouts "I have the whole thing on video!" and storms off to Campus Security. You are arrested for assault, miss your physics class for a couple of weeks and ultimately fail all of your classes. I certainly hope this was all happening in your head and not in real life! You learned something about physics, but at what cost to your humanity?* ***The end!***

Answer 5. *Diane points to her phone and says, "I recorded the whole thing!" If you respond, "Awesome! Can I watch the part about how Carlacts?", please read Answer 1. If you respond, "Awesome! Let's show the psychology and physics faculty our cool video!", please read Answer 7.3.0.5.6. If you respond, "Yeah, we probably should have intervened before this happened instead of just watching. Let's go talk to Campus Security", please read Answer 7.*

Answer 6. *The psychology faculty member speaks to you both about how to be good citizens and about the psychological effects of bullies both on the bully and on the recipient. If you decide to learn more about this, please read Psychology Today. The physics faculty member points out that when one person pushes another, the person being pushed does not brace himself, whereas the person doing the pushing does. Furthermore one might imagine what would happen if you did not brace yourself when you pushed each other, such as in Answer 4. You are asked to review both Example 7.3.4 and Example 7.3.7 before the next exam. On your way out the door, you hear a voice suggest "... and you might want to talk to Campus Security about the incident..."*

Answer 7. *You speak with Campus Security about the incident and Zambert gets taken in for assault. The Dean thanks you for being brave enough to speak up.* ***The end!***

Answer 8. *You feel guilty for letting Zambert push Carl down despite your amazing score on the next physics test. It wasn't worth it.* ***The end!***

(To better understand Newton's third law, you should compare Example 7.3.4 to Example 7.3.7.)

Example 7.3.4 (*Zambert intentionally braces when pushing Carl*)

Example referenced by action-reaction, Example 7.3.4, Task 8.4.13.c

Zambert, the town bully (with $m_Z = 95.0\,\text{kg}$), decides to vent his frustration on Carl for all the times that Carl makes Zambert look bad in class. While Carl ($m_C = 90.0\,\text{kg}$) has his back turned, Zambert walks up, leans in, and shoves Carl with a force of $\vec{F}_{C,Z} = 215\,\text{N}\hat{\imath}$. How does Zambert accelerate during this exchange?

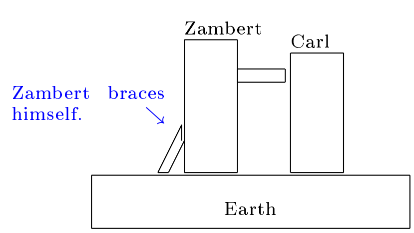


Figure 7.3.5: This image shows Zambert pushing Carl in order to see the reaction forces.

Solution. ***What do we know?*** *As usual, it is convenient to start with the picture to help decide on the appropriate coordinate system. We know m_Z, which is useful for relating $F_{Z,\text{net}}$ to a_Z. We know m_C, which is useful for relating $F_{C,\text{net}}$ to a_C. (This is not asked for, but is asked in homework problem Exercise 7.4.3.1.) We know $F_{C,Z}$, how hard Zambert pushes on Carl. We also know that Carl is not bracing himself (because he "has his back turned") so he only feels one force, and that Zambert is bracing himself (because the problem states that he "leans in") so he exerts multiple forces.*

What do we want to know? *We want to know about the forces acting on Zambert, in order to find $F_{Z,\text{net}}$ and therefore a_Z.*

How are what-we-know and what-we-want related? *First, since Zambert exerts a force on Carl, Newton's third law tells us that Zambert feels a force of $F_{Z,C} = -215\,\text{N}\hat{\imath}$.*

Second, because Zambert knew he was going to feel this reaction force, he compensates by bracing himself. This means he chooses to exert a force of 215 N on the Earth in the $-\hat{\imath}$ direction, probably by putting one leg behind himself and pushing the ground backwards with his foot. Newton's third law then tells us that Zambert feels a force of $F_{Z,C} = +215\,\text{N}\hat{\imath}$ from the ground.

Free-Body Diagrams *We are told of the force on Carl. We are told that Zambert braces himself, which tells us the force on the Earth. Newton's third law then helps us recognize the forces on Zambert. (Recall the "on-by" notation.)*

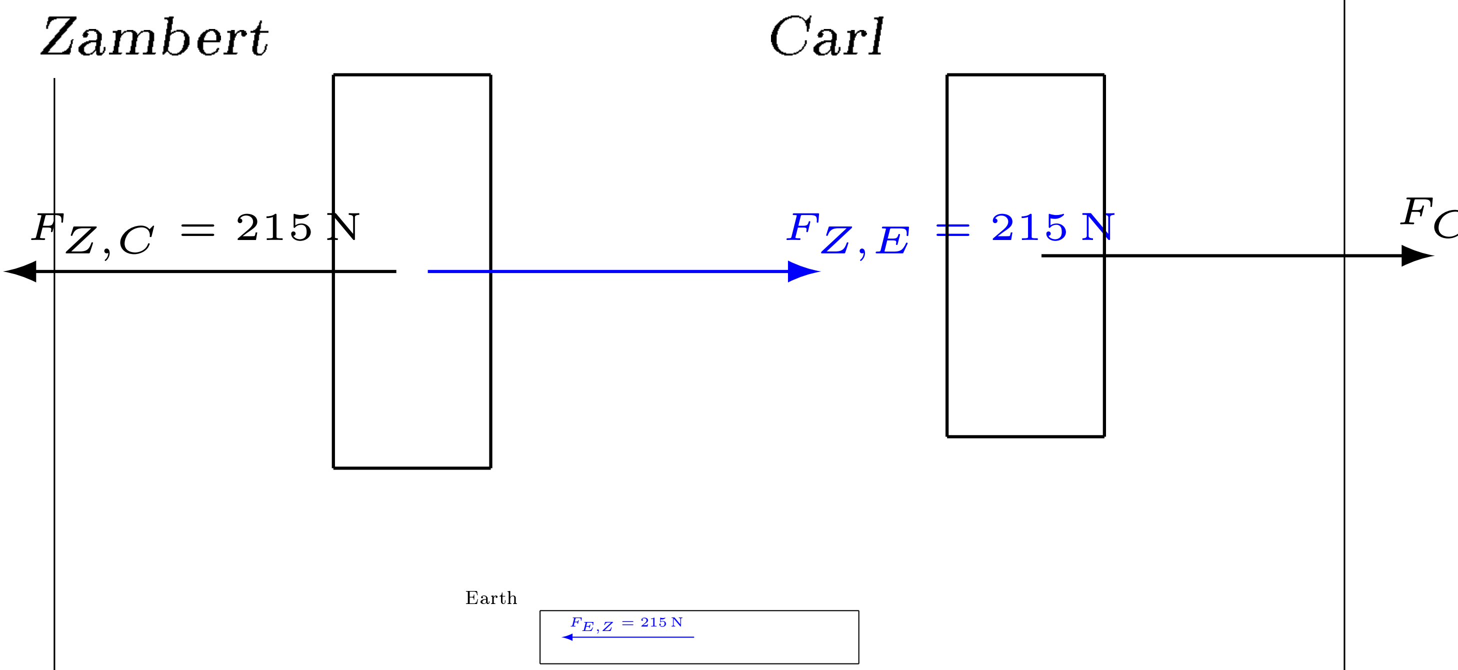


Figure 7.3.6: This image shows Zambert pushing Carl in order to see the reaction forces.

Concepts to Consider: *Newton's third law guarantees that the action-reaction force pairs, such as $F_{Z,C}$ and $F_{C,Z}$ or $F_{Z,E}$ and $F_{E,Z}$, are equal and opposite. There is no such guarantee on $F_{Z,C}$ and $F_{Z,E}$. These are equal because Zambert chose to make $F_{C,Z}$ and $F_{E,Z}$ equal. He pushed on the two others in equal amounts so that the reaction forces that act on him will balance for Newton's second law so that his acceleration would be zero.*

After using Newton's third law to find the forces on Zambert, we can use Newton's second law to find his acceleration:

$$a_Z = \frac{F_{Z,\text{net}}}{m_Z} = \frac{\left[\vec{F}_{Z,C} + \vec{F}_{Z,E}\right]}{95.0\,\text{kg}} = \frac{\left[\left(-215\,\text{N}\hat{\imath}\right) + \left(+215\,\text{N}\hat{\imath}\right)\right]}{95.0\,\text{kg}} = 0\ {}^{\text{m}}\!/_{\text{s}^2}$$

Aside about Example 34: This example only considers the left-right forces that act in order to make a point about our intuition regarding forces we intend to apply. Please consider how Example 8.3.2 updates Example 7.2.27 to make yourself aware of the other forces that are acting here, but are being ignored.

Example 7.3.7 (*Diane does not brace herself when pushing Carl.*)

Example referenced by *the discussion of action-reaction forces, Example 7.3.4, Task 8.4.13.c*

In the lab room one day, while waiting for the instructor, Diane (who has a mass of $m_D = 80.0\,\text{kg}$*) decides to try a physics experiment to test Newton's third law. She politely asks her lab partner, Carl (*$m_C = 90.0\,\text{kg}$*), to turn his back while she squares her feet underneath herself and pushes with a force of* $\vec{F}_{C,D} = 215\,\text{N}\hat{\imath}$*. Despite the experience of Example 7.3.4 (as told in Exploration 7.3.3), Carl reluctantly agrees. How does* Diane *accelerate during this exchange?*

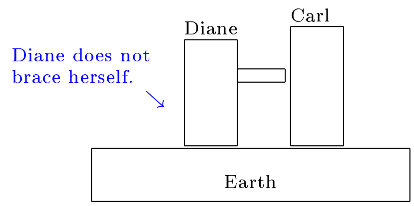


Figure 7.3.8: This image shows Diane pushing Carl in order to see the reaction forces.

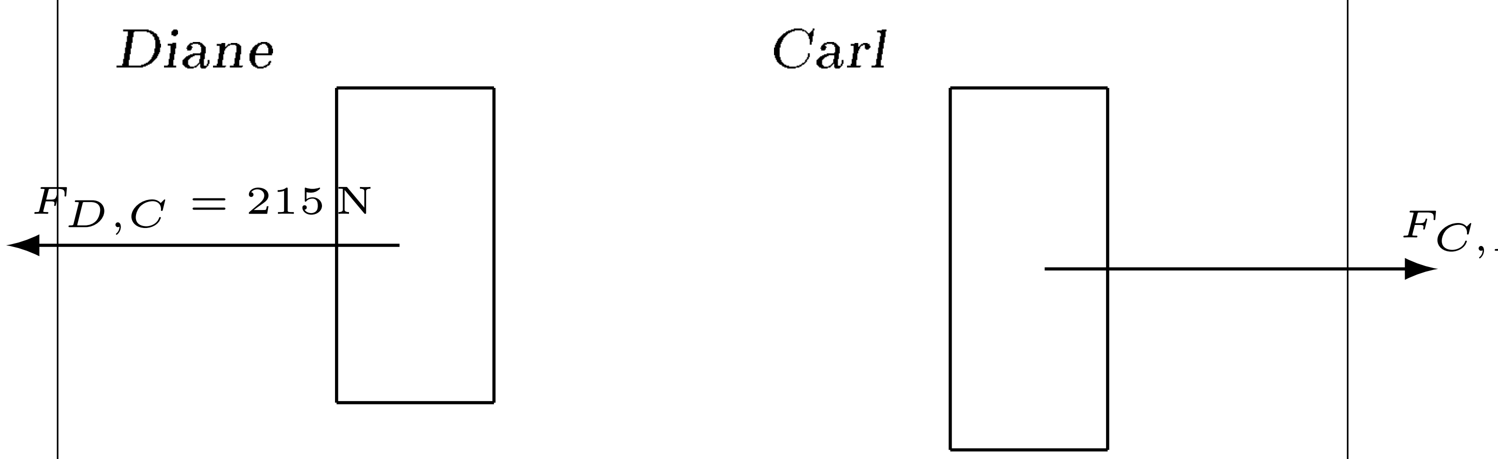
Solution. ***What do we know?*** *As usual, it is convenient to start with the picture to help decide on the appropriate coordinate system. We know* m_D*, which is useful for relating* $F_{D,\text{net}}$ *to* a_D*. We know* m_C*, which is useful for relating* $F_{C,\text{net}}$ *to* a_C*. (This is not asked for, but is asked in homework problem Exercise 7.4.3.1.) We know* $F_{C,D}$*, how hard Diane pushes on Carl. We also know that neither person is bracing for the push. So, both Carl and Diane each only feel one force.*

What do we want to know? *We want to know about the forces acting on Diane, in order to find* $F_{D,\text{net}}$ *and therefore* a_D*.*

How are what-we-know and what-we-want related? *First, since Diane exerts a force on Carl, Newton's third law tells us that Diane feels a force of* $F_{D,C} = -215\,\text{N}\hat{\imath}$*.*

Second, unlike Zambert in Example 7.3.4, Diane chooses not to exert a force on the Earth in the $-\hat{\imath}$ *direction.*

Free-Body Diagrams: *We again draw free-body diagrams:*



Concepts to Consider: *Newton's third law guarantees that the action-reaction force pairs,* $F_{D,C}$

and $F_{C,D}$, are equal and opposite. Because these forces are not on the same person, we cannot add these forces. Newton's second law will then indicate how each person accelerates.

After using Newton's third law to find the forces on Diane, we can use Newton's second law to find her acceleration:

$$a_D = \frac{F_{D,\text{net}}}{m_D} = \frac{\left[\vec{F}_{D,C}\right]}{80.0\,\text{kg}} = \frac{\left[\left(-215\,\text{N}\hat{\imath}\right)\right]}{80.0\,\text{kg}} = -\mathbf{2.6875}\ {}^{\text{m}}\!/\!_{\text{s}^2}\,\hat{\imath}$$

Aside about Example 37: This example only considers the left-right forces that act in order to make a point about our intuition regarding forces we intend to apply. Please consider how Example 8.3.2 updates Example 7.2.27 to make yourself aware of the other forces that are acting here, but are being ignored.

7.4 Summary and Homework

7.4.1 Summary of Concepts and Equations

This chapter introduced the way physicists describe forces. The concept of force encodes how objects interact. After reading this chapter, you should be comfortable responding to the following questions or comments. Unlike the other links in this book, if you follow the links in this summary section, there is no link to return to this page. (This is on purpose to encourage you to answer these points without following these links.)

- State Newton's Laws. (Answer)
- How is the unit of Newton related to the fundamental units of the SI system? (Answer)
- How do you know when a system is in equilibrium? (Answer)
- You should know how to draw a free-body diagram. (Example)

7.4.2 Conceptual Questions

1. In order to climb a tree, you reach up and grab a branch and pull. Most people refer to this as "pulling yourself up." In terms of Newton's third law, describe what is happening in more technical terms.

2. Some cars have a "cruise-control" feature that keeps your speed constant as you drive down the highway. (a) If you are driving due north with the cruise-control on, are you in equilibrium? (b) If, instead, you have the cruise-control set while you are following the road around a gradual curve of the road as it follows the shore of a lake, then are you in equilibrium? (c) In both cases, how can you tell if you are in equilibrium?

7.4.3 Problems

1. If Zambert, with $m_Z = 95.0\,\text{kg}$, braces himself (so that he does not accelerate) and pushes Carl ($m_C = 90.0\,\text{kg}$) with a force of $\vec{F}_{C,Z} = 215\,\text{N}\hat{\imath}$, find the following:

(a) What is the acceleration of Carl? (Solution 1)

(b) What net force does Zambert feel? (Solution 2)

(c) If Zambert braces himself against the Earth, then what must that bracing force be? (Solution 3)

(d) What are the individual forces that Zambert feels? (Solution 4)

(e) What is the acceleration of the Earth? (Solution 5)

(f) Which of Newton's laws allows you to answer each of these questions?

Solution 1. $\vec{a}_C = \dfrac{215\,\text{N}\hat{\imath}}{90.0\,\text{kg}} = \mathbf{2.389}\ {}^{\text{m}}\!/\!_{\text{s}^2}\,\hat{\imath}.$

Solution 2. $F_{Z,\text{net}} = 0\,\text{N}.$

Solution 3. $\vec{F}_{E,Z} = -215\,\text{N}\hat{\imath}.$

Solution 4. $F_{Z,C} = -215\,\mathrm{N}\hat{\imath}$ and $F_{Z,E} = 215\,\mathrm{N}\hat{\imath}$.

Solution 5. $\vec{a}_E = \dfrac{-215\,\mathrm{N}\hat{\imath}}{5.97 \times 10^{24}\,\mathrm{kg}} = -\mathbf{3.60}1 \times 10^{-23}\ \mathrm{m/s^2}\,\hat{\imath}$.

2. If you apply a force of 4.65 N to a mass of 2.18 kg, then how much will it accelerate?

3. How much force must you apply to cause a mass of 80.0 kg to accelerate at $a = 0.795\ \mathrm{m/s^2}$?

4. You arrive home to find a box that came in the mail. You find that you have to exert 54.3 N to cause it to accelerate $a = 1.25\ \mathrm{m/s^2}$. (a) What is its mass? (b) Is that a heavy box or a light box? (c) Is it likely that this box would fit in a mailbox?

5. Your 2538 kg car has run out of gas. So you ask your friend, Beth who has a mass of 75.0 kg, to put it in neutral, sit inside, and steer while you push. If you apply enough force to cause a net forward force of magnitude 37.5 N, how much time will it take for the car to move faster than you can walk? Assume your walking speed is $3.0\ \mathrm{mi/hr}$. How far will the car have travelled in that time?

6. Find the components of the net force on a large crate if three forces are applied: $\vec{F}_1 = -3.0\,\mathrm{N}\hat{\imath} + 2.5\,\mathrm{N}\hat{\jmath}$, $\vec{F}_2 = -6.25\,\mathrm{N}\hat{\jmath}$, and $\vec{F}_3 = 4.5\,\mathrm{N}\hat{\imath} + 1.63\,\mathrm{N}\hat{\jmath}$.

7. Find the components of the net force on a large crate if three forces are applied: $F_1 = 3.61\,\mathrm{N}$ at 71.6° north of east, $F_2 = 4.61\,\mathrm{N}$ due west, and $F_3 = 8.13\,\mathrm{N}$ at 21.8° south of east.

8. Find the magnitude and direction of the net force on a large crate if three forces are applied: $\vec{F}_1 = 4.25\,\mathrm{N}\hat{\imath} - 4.66\,\mathrm{N}\hat{\jmath}$, $\vec{F}_2 = -2.65\,\mathrm{N}\hat{\jmath}$, and $\vec{F}_3 = -5.4\,\mathrm{N}\hat{\imath} + 2.93\,\mathrm{N}\hat{\jmath}$.

9. Find the magnitude and direction of the net force on a large crate if three forces are applied: $F_1 = 2.65\,\mathrm{N}$ at 26.6° north of west, $F_2 = 2.22\,\mathrm{N}$ at 56.31° south of west, and $F_3 = 7.12\,\mathrm{N}$ at 28.4° north of east.

List of examples

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(Revised September 1, 2017)

Chapter 8

The Many Types of Force

Chapter referenced by Chapter 7, Discussion of subscript notation of forces

Having discussed the concept of forces and how they affect the motion, we can turn to the specific mechanisms for the introduction of forces on objects. Perhaps the most obvious mechanisms for imparting force is that objects fall towards the Earth. We call this force "gravity" and we will discuss it first. It turns out to be somewhat more complicated because it turns out that the force that helps us fall when we trip is the same force that holds the planets in their orbits. The complicated general theory will be discussed later, but for now Section 1 will describe the phenomenon at the surface of the Earth where we usually experience it.

After a brief introduction to the four fundamental forces in Section 2, we will consider the normal force, friction, tension (such as in ropes), springs, and end with the catch-all additional category of applied forces. Section 8 will wrap it up by using examples including multiple types of force to compute the net force, which connects back to Newton's Laws in the previous chapter.

Before we start, let's consider extending the subscript notation for forces introduced in Convention 7.1.4.

Convention referenced by Convention 7.1.4, Example 8.4.7

Convention 8.0.1 (*The "on-by-type" notation*)

Recall The "on-by" notation. Often we would like to indicate the type of force as well as which objects is it "on" or "by". In this chapter, we will add the type of force after the "on-by" notation. Considering the case where Abdul feels a gravitational force from the Earth, we say that there is a gravitational force on Abdul by the Earth and use the notation F_{AEg}, where the first subscript is the person who felt the force (who the force is "on"), the second subscript is who exerted the force (who the force is "by"), and the third subscript is the type of force.

Caution 1: *Sometimes it will be obvious who exerts the force and we might drop that subscript. For example, the Earth is the most common source for the gravitational force, so we might write F_{Ag}, distinguishing this from the normal force that acts on Abdul, F_{AN} (which will also be exerted by an obvious source, the surface on which Abdul stands).*

Caution 2: *Sometimes the interacting objects and type will have the same initial and the subscripts can be confusing, as in the case where Beth* applies *a good-natured force to Abdul. In that case, we would say that there is an applied force on Abdul by Beth and use the notation F_{ABa} where the first subscript (A) is the person who felt the force and the last subscript is the type of force (applied). The author of this book will endeavor to create examples which avoid this confusion, but if you create examples for yourself, just be aware of this possibility.*

8.1 Gravity at the Surface of the Earth

Section referenced by freefall, Example 7.2.27

Perhaps the force that is the most obvious to humanity is the one that helps us fall when we stumble: the gravitational force. This is one of the fundamental forces discussed in Section 8.2. In addition, the details about how the planets, moon, and the sun experience this force will be discussed in Chapter 15. For now, we can consider how this interaction manifests itself on our daily lives. In this section, we will start with how objects move when the gravitational force is the only force acting. Subsections 8.1.1 and 15.1.1 will clarify some subtleties and then we'll jump into the examples in Subsection 2.

We can investigate what happens when the gravitational force is the only force acting on an object by holding it in the air and dropping it. One of the complications during such an experiment was discussed in Subsection 5.6.2. If we drop a sheet of paper, there is air resistance in addition to the gravitational force. For this section, I will assume that the mass-to-surface-area ratio is large enough that we can effectively ignore the air resistance. A second consideration that allows this approximation is that (as discussed in Subsection 8.4.4) the air resistance depends on the speed of the object, so as long as the objects fall for a short distance or do not get a significant speed, then we can ignore air resistance. You should note that when you are deciding is a speed is significant, consider that baseballs hit (fly balls) by college athletes do not follow a parabolic path because of air resistance.

Touchstone (mostly true) Recall effective theories. For the vast majority of objects we drop (through the atmosphere) we don't have to worry about air resistance.

Paragraph referenced by Example 8.3.14

Before we get into too much detail, we should pause here and recognize that the force of gravity is likely the underlying reason for our intrinsically human perspective that vertical and horizontal are the "natural" orientation of any coordinate system. That is to say, it is the combination of the vector nature of Newton's Second Law combined with the fact that gravity pulls (and therefore accelerates) objects "downwards" (by which we mean "towards the center of the Earth"). When the object is in freefall, its acceleration is pointing along the vertical axis with no component along the horizontal axis. The awareness of this makes the horizontal component of Newton's second law convenient because $F_{\text{net},x} = m\cancelto{0}{a_x}$. On the other hand, if we drag something along the ground, then it has no vertical acceleration and only (possibly) a horizontal acceleration, which makes the vertical component convenient because $F_{\text{net},y} = m\cancelto{0}{a_y}$. It is (nearly) always convenient (meaning "easier to solve mathematically") if we can choose our coordinate system so that one of the components of the acceleration (either a_x or a_y) is equal to zero.

Since objects fall faster than humans are used to paying attention to, the patterns are difficult to see. Investigation 5.5.1 shows you how you can pay close attention to the patterns that result from observing falling objects. You should go do those experiments before reading further. Go ahead. I'll wait.

You did do them, right? You're not just reading ahead? Really? OK. Doing that experiment will help you see (1) that everything falls at the same rate and (2) that objects accelerate as they fall. This first point is a bit less intuitive and will be discussed further in Subsection 15.1.1. This second point should be exactly what you expect, when you consider Newton's second Law: If there is only one force (the gravitational force), then the object cannot be in equilibrium and it must be accelerating. (You should notice that this is the language of the story of Newton's second law.)

Touchstone (on-by) the on-by notation

In order to evaluate this further, let's consider a specific object, like a baseball. Our baseball has a mass of $m_b = 0.145\,\text{kg}$. If the only force acting *on* the ball is the gravitational force *by* the Earth, then the net force is the gravitational force: $\vec{F}_{\text{net}} = \vec{F}_{bEg}$. Here the subscripts are b (because the force is on the $b$all), E (because the force is exerted by the Earth), and g (because it is a $g$ravitational force). Since the acceleration is due to the gravitational force, I will use either a_g (usually when the object is in freefall and therefore accelerating at this rate) or g (usually when the object is not actually accelerating at that rate). With this notation, Newton's second law becomes:

$$\vec{F}_{bEg} = m_b \vec{a}_g$$

At this point, we know the mass, but we don't know the force or the acceleration. However, we have conveniently already done the experiment (recall Exercise 5.5.3) that will tell us the acceleration is $a_g = 9.81\ \mathrm{m/s^2}$ downwards. (Recall that "downwards" is the direction of the vector, which can be expressed as $-\hat{\jmath}$.) If we know the mass and the acceleration, then we can compute the force.

Example 8.1.1 (*Calculate the weight of a ball in freefall*)

> *If a baseball with mass $m_b = 0.145\,\mathrm{kg}$ is dropped (allowed to fall freely) so that it accelerates at $a_g = 9.81\ \mathrm{m/s^2}$ downwards, then while it falls it feels the gravitational force:*
>
> $$\vec{F}_g = m\vec{g} = (0.145\,\mathrm{kg})[-(9.81\ \mathrm{m/s^2})\,\hat{\jmath}] = -\mathbf{1.42}24\,\mathrm{N}\hat{\jmath} = -1.42\,\mathrm{N}\hat{\jmath}.$$

This is the force of the gravitational force on the baseball. Although we computed the force while the ball was falling, the gravitational force does not magically vanish when the ball is sitting on the floor. So, we can say that (as long as the ball is close to the surface of the Earth, as noted in Chapter 15) the force always has this value. Rather than continuing to say "the force of gravity" we call this force the weight.

Definition 8.1.2 (weight)**.** The **weight** of an object is literally the strength of the gravitational force acting on an object. Weight is computed by multiplying the mass times the gravitational field (which is functionally equal to the acceleration due to gravity, even when the object is not actually accelerating at that rate): $\mathbf{F_g} \equiv \mathbf{mg}$.

8.1.1 Weight versus Mass

Referenced by Subsection 3.2.2, Section 4.2

Since all objects have the same acceleration due to gravity at the surface of the Earth, the weight of an object and the mass of an object are very closely correlated, but they are not the same quantity. This tends to cause some confusion when the discussion is not explicitly technical. Recall the discussion about being precise in our language, Section 4.1. One complication for people in the United States is that there are two definitions of the pound; one is a unit of mass[1] and the other is a unit of force. Since the pound-force[2] is defined as the standard unit of mass times the standard unit for the acceleration due to gravity, as discussed in Section 3.2, the conversion directly from pound-force to Newtons will *not* match the longer, but more appropriate, conversion from pound-mass to kilogram that gets multiplied by the local acceleration due to gravity (as opposed to the standard g) into Newtons. It may also be useful to review the comments about unit-conversion in the section on significant digits.

In the discussion about being precise, we distinguished "massive" (the amount) from "voluminous" (the size). Now that we understand Newton's second law, we can distinguish "massive" (an amount of material causing a difficulty in making accelerate) from "weighty" (a strength needed to lift).

Touchstone (weight) Recall Definition 3

Clarification 8.1.3 (*mass versus weight*)

> *The mass describes the amount of material, whereas weight describes how strongly the gravitational force pulls on the object.*

Having mass affects both the inertia (ease of moving) and the weight (force of gravity). Having weight expresses the gravitational force due to whichever large object (moon, planet, sun, etc.) you happen to be on or near. Noticing that the SI-unit (Section 3.2) is different for different types of quantities, such as a kilogram (a fundamental unit) for mass and a Newton (a derived unit) for weight, may help you remember that these are different kinds of quantities.

The interesting aspect of this relationship is that while having more mass makes an object harder to move (the same force produces less acceleration for more massive objects), when objects fall under the influence of

[1] There are also multiple versions of the pound-mass. You can find these explained on the internet, but most of these are considered obsolete. The one I will use is the "avoirdupois-pound", which is defined in the NIST publication Handbook 44, page C-19, as exactly 453.59237 g.

[2] There is also a unit of force called the kilogram-force.

the gravitational force, they accelerate at the *same* rate. This reveals that the gravitational force must be stronger for more massive objects *by the exact amount* needed to compensate for that larger mass. This is called the equivalence principle and is discussed in Subsection 15.1.1.

8.1.2 Calculating the weight

When calculating the forces acting on a person or an object, we will often need to account for the force of gravity, while other forces may also be at work. As mentioned above, the weight is found by multiplying the mass times the local acceleration due to gravity, even if the object is not actually accelerating at that rate. Chapter 15 will clarify why it is true[3], but for now please note that the acceleration due to gravity is (1) different according to where we are and also (2) the same for all objects at that location.

Because of the peculiarities in the definition of pound it will be useful to build some intuition about masses in terms of kilograms and Newtons. Table 4 lists the mass of some common objects and, using the standard value for g, their corresponding weights. Exercise 8.9.2.1 asks you to estimate the mass of some other common objects. Exercise 8.9.2.2 asks you to think of common objects with a specified mass.

Object	pounds	mass (kg)	weight (N)
apple	0.33	0.15	1.5
lean, healthy cat	10	4.6	45
medium-sized dog	44	20	196
human	200	91	890
horse	1000	362	3.56×10^3
large pick-up truck	4000	1.81×10^3	1.78×10^4

Table 8.1.4: The list of objects is intended to give a sense of scale so that the reader can better estimate the value of the mass of an object. You might notice that (except for the apple) each of these is between 4 and 4.5 times heavier than the previous object. Note that these are rough estimates; for example, while the author weighs about 200 lbs this is not typical, nor average.

Now let's do some calculations...

Example 8.1.5 (*Calculate the mass from the weight*)

Abdul notices that he needs to exert $F = 1.5\,\mathrm{N}$ *to support the apple listed in Table 4. He then drops it and notices its acceleration of* $9.81\ \mathrm{m/s^2}$. *He computes the mass to be*

$$m = \frac{F_g}{a_a} = \frac{1.5\,\mathrm{N}}{9.81\ \mathrm{m/s^2}} = \frac{1.5\ \mathrm{kg{\cdot}m/s^2}}{9.81\ \mathrm{m/s^2}} = 0.\mathbf{153}\,\mathrm{kg}$$

(If you know the weight, you can compute the mass, even if the mass is not actually in freefall.)

[3] The short answer is that the altitude (distance from the surface of the Earth) and local geology affect the strength of the gravitational field. Since the Earth is slightly oblate (bulges at the equator), the altitude at different latitudes corresponds to a different distance from the center of the Earth. In addition, while the spin of the Earth does not affect the strength of the gravitational field, it does affect how objects accelerate. The GRACE project has measured the variations across the globe.

Example 8.1.6 (***Calculate weight from the mass***)

Referenced by Example 8.3.2, Example 8.5.6

Abdul, who knows his own mass (85.0 kg), then imagines dropping himself (!) from a (small) height. While he falls, he recognizes the gravitational force on him, which is computed to be

$$\vec{F}_g = m\vec{g} = (85.0\,\mathrm{kg})[-(9.81\,{}^{\mathrm{m}}\!/\!_{\mathrm{s}^2})\,\hat{j}] = -\mathbf{833}.85\,\mathrm{N}\hat{j} = -834\,\mathrm{N}\hat{j}$$

Since he is in freefall and there is only one force is acting on him, the net force is easy to compute: $\vec{F}_{\mathrm{net}} = -834\,\mathrm{N}$. However, if you know the mass something, you can compute the weight even if that object is not in freefall. You should repeat this calculation for the mass in Example 7.2.19.

Answer. *Since the object in Example 7.2.19 has a mass of 2.0 kg, we can find the weight by*

$$\vec{F}_g = m\vec{g} = (2.0\,\mathrm{kg})[-(9.81\,{}^{\mathrm{m}}\!/\!_{\mathrm{s}^2})\,\hat{j}] = -\mathbf{19}.62\,\mathrm{N}\hat{j} = -20\,\mathrm{N}\hat{j}.$$

Insight 8.1.7 (***ma versus mg***)

- *F_{net} $(= ma)$ is always related to the actual acceleration of the object.*
- *F_g $(= mg)$ is always related to the local acceleration due to gravity.*

You should also note that

Insight 8.1.8

the actual acceleration, (a), is only equal to the local acceleration due to gravity, (g), if the object is in freefall (by Definition 5.5.2).

Example 8.1.9 (***Deducing the existence of forces using Newton's second law***)

Referenced by Example 8.3.2, Example 8.5.6

If Beth is not falling, but rather standing safely on the floor, then the gravitational force is still acting. It can be computed as

$$\vec{F}_g = m\vec{g} = (75.0\,\mathrm{kg})[-(9.81\,{}^{\mathrm{m}}\!/\!_{\mathrm{s}^2})\,\hat{j}] = -\mathbf{735}.75\,\mathrm{N}\hat{j} = -736\,\mathrm{N}\hat{j}$$

However, since we can see that her acceleration is zero, the $\vec{F}_{\mathrm{net}}$ must be zero. The only way that can happen, though is if there is another force acting upwards on Beth. What could possibly be pushing up on her? Answer 1. Whatever it is pushing up on her, it is supplying a support force, which can be calculated since $\vec{F}_{\mathrm{net}} = \vec{F}_g + \vec{F}_{\mathrm{support}}$ and we can solve for

$$\vec{F}_{\mathrm{support}} = \vec{F}_{\mathrm{net}} - \vec{F}_g = m\,(0\,{}^{\mathrm{m}}\!/\!_{\mathrm{s}^2}) - \left[-(\mathbf{735}.8\,\mathrm{N})\hat{j}\right] = +736\,\mathrm{N}\hat{j}$$

Because it is in the direction opposite to $\vec{F}_g$, it is upwards $(+\hat{j})$.

Can you identify why the support force is equal in magnitude and opposite in direction to the gravitational force? Select one: Newton's second law or Newton's third law.

Answer 1. *I hope you guessed the floor. That is the only thing pushing up on Beth. One useful way to think about it is that the floor is the thing keeping her from falling. The direction of this force is normal (perpendicular) to the horizontal floor, so it is in the vertical direction. This will be discussed in more detail in Section 8.3.*

Answer 2. *You can tell that it is Newton's second law ($\vec{F}_{\mathrm{net}} = m\vec{a}$) because the forces we are considering are acting on the same object. In this case, the gravitational force is caused by the Earth and the normal force is caused by the floor by they are both felt by Beth. These forces happen to be equal and opposite* *because she happens to be in equilibrium. She does not* have to be *in equilibrium, such as when she jumps, in which case the forces would not be equal and might not be opposite.*

Answer 3. *If it were Newton's third law, then the two forces we were discussing would be acting on different objects and would be unrelated to the fact that the object (in this case, Beth) is in equilibrium. The gravitational force and the normal force in this case are both acting on Beth, so although they happen to be equal and opposite, this is not due to Newton's third law.*

You should, however, note that the force that is reaction-paired to the gravitational force on Beth by the Earth is a gravitational force on the Earth by Beth. Similarly, the reaction-paired force to the normal force on Beth by the floor is a normal force on the floor by Beth. (Please note the "on" and "by" in each case.)

As was mentioned earlier, the value of the acceleration due to gravity also varies across the surface, although this is less than about a percent or so (see Table 13). Nonetheless, this means that your weight can change even when your mass remains the same.

Example 8.1.10 (*Weight can vary even if mass does not*)

While talking to your friend Beth, you learn that her parents, Erik and Frances, grew up in Norway, visited Puerto Rico, and climbed Mount Everest before settling in the United States. Using Table 13, compute Erik's weight at each location, assuming his mass is 95.0 kg*: Norway, Puerto Rico, and Mt. Everest.*

Answer 1. *[Norway]* $F_g = mg = (95.0\,\mathrm{kg})(9.825\ ^{\mathrm{m}}\!/\!_{\mathrm{s}^2}) \;=\; \mathbf{933}.4\,\mathrm{N}$.

Answer 2. *[Puerto Rico]* $F_g = mg = (95.0\,\mathrm{kg})(9.782\ ^{\mathrm{m}}\!/\!_{\mathrm{s}^2}) \;=\; \mathbf{929}.3\,\mathrm{N}$.

Answer 3. *[Mount Everest]* $F_g = mg = (95.0\,\mathrm{kg})(9.763\ ^{\mathrm{m}}\!/\!_{\mathrm{s}^2}) \;=\; \mathbf{927}.5\,\mathrm{N}$.

Convention 8.1.11 (*The precision of g*)

Example 8.1.12 (*The acceleration due to gravity is only "locally constant"*)

Consider the values for the acceleration due to gravity at various locations around the Earth. Look for a pattern in the values as the latitude increases. You might notice the values for Mount Everest and Denver; Can you explain any peculiarity?

Answer 1. *Because the Earth was spinning as it cooled (forming the crust), it formed an oblate spheroid.*[4] *Since the strength of the gravitational interaction depends (among other things) on how far you are from the center (slightly weaker further away), the acceleration due to gravity is smaller when you are at smaller latitudes (closer to the equator).*

Answer 2. *In addition to being an oblate spheroid (Footnote 5), the Earth has mountains and valleys. Since the strength of the gravitational interaction depends (among other things) on how far you are from the center (slightly weaker further away), the acceleration due to gravity is smaller when you are at at high altitudes, such as Denver, CO and Mount Everest.*

Location	latitude	longitude	local g ($^{\mathrm{m}}\!/\!_{\mathrm{s}^2}$)
San Juan, Puerto Rico	18°26′24″ N	66°7′48″ W	9.782 $^{\mathrm{m}}\!/\!_{\mathrm{s}^2}$
Brownsville, TX	26°1′6″ N	97°27′14″ W	9.788 $^{\mathrm{m}}\!/\!_{\mathrm{s}^2}$
Mount Everest (Napal/China border)	27°59′17″ N	86°55′31″ E	9.763 $^{\mathrm{m}}\!/\!_{\mathrm{s}^2}$
Cincinnati, OH	39°8′24″ N	84°30′23″ W	9.801 $^{\mathrm{m}}\!/\!_{\mathrm{s}^2}$
Denver, CO	39°45′43″ N	104°52′50″ W	9.798 $^{\mathrm{m}}\!/\!_{\mathrm{s}^2}$
Paris, France	48°51′36″ N	2°20′24″ E	9.813 $^{\mathrm{m}}\!/\!_{\mathrm{s}^2}$
Oslo, Norway	59°54′36″ N	10°45′ E	9.825 $^{\mathrm{m}}\!/\!_{\mathrm{s}^2}$
Anchorage, AK	61°10′39″ N	149°16′28″ E	9.826 $^{\mathrm{m}}\!/\!_{\mathrm{s}^2}$

Table 8.1.13: Comparison of g at a few places on Earth. (See Footnote 8.1.2.4.) Example 12 considers some patterns in this table.

8.2 Fundamental Forces

The previous section describes our (macroscopic) experience of the gravitational interaction when standing on the surface of the Earth. This is essentially the same across the surface, but does change with altitude and the difference can be measured on mountain tops and in caves. In fact, one can use the differences from one location to another to predict where we might find a a pocket of oil.

In later sections, we will consider this and other interactions that depend on the physical properties, such as mass and charge. All particles with the property of mass (which we will start to call gravitational charge) will interact according to the gravitational force; however, this description is better described by the mathematics in Chapter 15. All particles with the property of electrical charge will interact according to the electrical force. The basic theory will be discussed in Chapter 22. A more complicated version that incorporates quantum mechanics is called quantum electrodynamics (QED) and this will be touched on in Subsection 29.3.2. Particles like protons and neutrons (hadrons) are actually made up of other particles (quarks) that are held together by an interaction that is sometimes called the strong nuclear force (Subsection 29.2.2) and is described by the theory of quantum chromodynamics (QCD); this will be touched on in Subsection 29.3.3. Finally, in Subsection 29.2.3 another fundamental force, called the weak nuclear force, will be discussed.

For the most part, these theories describe the interaction between microscopic particles, so we will not discuss them in detail here. However, the gravitational interaction is exception in a variety of ways. In particular, the gravitational interaction does affect macroscopic objects. These fundamental forces have a particular description that allows us to pretend (recall effective theories) that they are action-at-a-distance interactions. All other forces (introduced next) will require physical contact in order to exert the force.

8.3 Normal Force

Section referenced by 7.2.27, Answer 8.1.2.9.1, 8.5

The word "normal" originates with the idea of conformity to the pattern. While in everyday life this pattern is the typical state of being, the origins actually refer to a carpenter's square, which put corners into a right angle. In math and physics, the word is used to mean perpendicular.

Etymology (normal) See text.

Insight 8.3.1

In the context of forces, the normal force is the force that a surface exerts to keep objects from passing through them. The direction of this force is always in the outward direction, normal (perpendicular) to the surface.

Let's consider some specific situations... In Example 8.1.9, Beth felt the downwards gravitational force even while she was standing on the ground. We noticed that she was not falling (and so not accelerating). Colloquially, we say that the ground is supporting Beth. This support force is keeping Beth from passing through the floor; this is a normal force. The normal force from the floor is acting upwards, which is normal (perpendicular) to the surface of the floor. Example 8.3.2 updates the free-body diagrams of Example 7.2.27 to show how the gravitational and normal forces impact that calculation.

Example 8.3.2 (*People pushing a box also feel the gravitational and the normal forces*)

Example referenced by *Introduction of inclined planes*

Let's start from a familiar example so that we focus our attention on the new aspect. Starting from Example 7.2.27, we can observe that each of the three bodies also has a downwards gravitational force. This is analogous to the calculation in Example 8.1.6, which was only for Abdul; but you can calculate the weight for the mass in Example 7.2.19 and Beth's weight was computed in Example 8.1.9. In addition to the downward gravitational force (the weight), Newton's second law and the fact that nothing is accelerating up or down together tells us that there must also be a normal force on each body. This is analogous to the calculation in Example 8.1.9, which was only for Beth; but you can deduce it for the object and for Abdul.

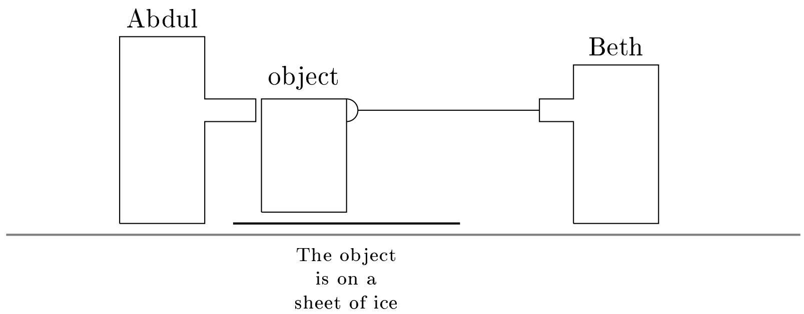
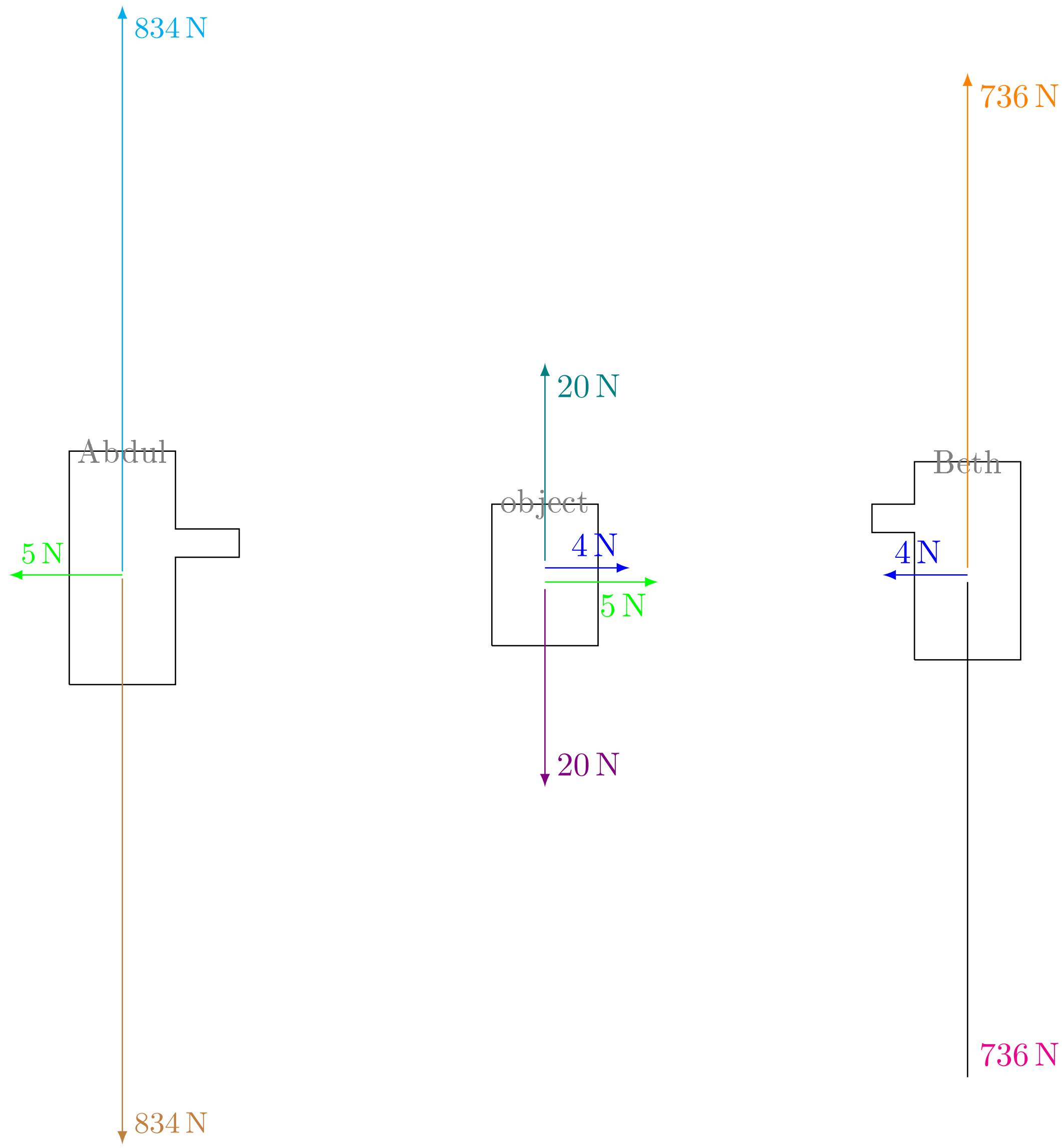


Figure 8.3.3: Abdul and Beth push on a box

Now, as in Figure 7.2.29, we will draw a free-body diagram for each individual separately. However, this time we will use the information calculated in Example 8.1.6 and Example 8.1.9 to include the gravitational force (the weight) and the normal force. I will use the "on-by" notation to distinguish the forces.



Even with the vertical forces, Abdul still has a $\vec{F}_{\text{net}} = -5.0\,\mathrm{N}\hat{\imath}$.

Even with the vertical forces, the object still has a $\vec{F}_{\text{net}} = +9.0\,\mathrm{N}\hat{\imath}$.

Even with the vertical forces, Beth still has a $\vec{F}_{\text{net}} = -4.0\,\mathrm{N}\hat{\imath}$.

Figure 8.3.4: This figure modifies Example 7.2.27 by allowing for the gravitational and the normal forces.

The normal forces and the gravitational forces are all caused by each object interacting with the Earth. So, while we can use Newton's second law to deduce the value of each normal force, the normal and gravitational forces are not third-law pairs. In order to see the third-law pairs, please review Example 8.3.16

Return to: 7.3.4, 7.3.7, 8.3, rope-tension, 8.5.6

Let's consider some other specific situations... If you decide to lean against a wall, the wall will provide a normal force that pushes horizontally, keeping you from moving through the wall.

Example 8.3.5 (*Ladders push on the wall and on the floor*)

Carl leans a $22.7\,\mathrm{kg}$ *ladder against a wall at an angle of* $75.5°$*, consistent with OSHA standard 1926.1053(a)(1)(ii), so that about* $^1/_8$ *of the weight is leaning into the wall.*

1. *Find the magnitude and direction of the normal force exerted* by the wall *on the ladder. (Answer 1)*

2. *Find the magnitude and direction of the normal force exerted* by the floor *on the ladder. (Answer 2)*

Note: *Example 13.2.1 goes into the full details of how one calculates the necessary values.*

Answer 1. *Since the weight is* $F_g = mg = (22.7\,\text{kg})(9.81\,{}^{\text{m}}\!/\!_{\text{s}^2}) = \mathbf{222}.69\,\text{N}$*, an eighth of this is* $\mathbf{27}.836\,\text{N}$*. This force is pressing into the wall (horizontally, which I will choose as the* $+\hat{\imath}$ *direction). By Newton's third law if the ladder presses into the wall with* $\mathbf{27}.836\,\text{N}$ *in the* $+\hat{\imath}$ *direction (this is also a normal force), then the wall pushes the ladder with a normal force of* $\mathbf{27}.836\,\text{N}$ *in the* $-\hat{\imath}$ *direction.* Notice that this is normal (perpendicular) to the surface of the wall.

Answer 2. *Since the full weight of the ladder,* $F_g = \mathbf{222}.69\,\text{N}$*, is still pressing downwards* $(-\hat{\jmath})$ *into the floor (as a normal force), Newton's third law says that the floor pushes the ladder upwards* $(+\hat{\jmath})$ *with a normal force of* $\mathbf{222}.69\,\text{N}$*.* Notice that this is normal (perpendicular) to the surface of the floor.

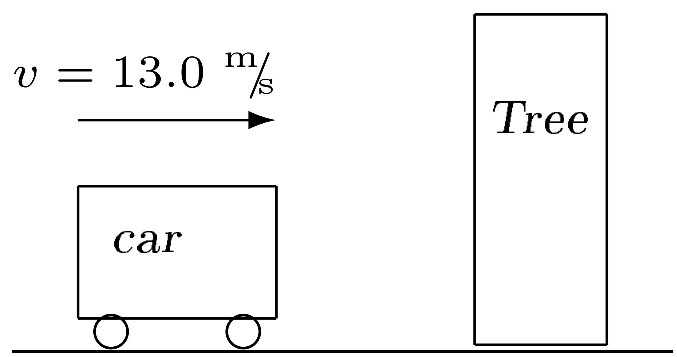
If you lose control of your car and run into a tree, the tree also provides a normal force pushing the car away from the tree; this normal force will stop the car.

Example 8.3.6 (*The normal force stops a crashing car*)

Example referenced by *Introduction of inclined planes*

Zambert is driving home after a late night of studying at the library. He is kind of tired and drifts off during the drive. While traveling $\vec{v}_i = 13.0\,{}^{\text{m}}\!/\!_{\text{s}}\hat{\imath}$*, Zambert runs into a tree, bringing his car* $(m = 2.1 \times 10^3\,\text{kg})$ *to a halt in* $\Delta t = 0.243\,\text{s}$*. (Zambert remains unharmed because he was awake enough to wear his seatbelt and had a car with a functioning airbag. Whew.) Find the normal force by the tree on the car.*

To be clear about what is happening, I will draw the picture.



Solution. *In order to find the force, we will first need to find the acceleration.*

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t} = \frac{(0\,{}^{\text{m}}\!/\!_{\text{s}}) - (13.0\,{}^{\text{m}}\!/\!_{\text{s}}\hat{\imath})}{0.243\,\text{s}} = -\mathbf{53}.49\,{}^{\text{m}}\!/\!_{\text{s}^2}\hat{\imath}$$

That the acceleration is in the direction opposite the velocity corresponds to the object slowing down. Now we can find the **net force** *from Newton's second law:*

$$\vec{F}_{\text{net}} = m\vec{a} = (2.1 \times 10^3\,\text{kg})(-\mathbf{53}.49\,{}^{\text{m}}\!/\!_{\text{s}^2}\hat{\imath}) = -\mathbf{1}.12 \times 10^5\,\text{N}\hat{\imath}$$

There are three forces acting on the car, as can be seen in analogy with the free-body diagrams of Example 8.3.2. So, we can draw a free-body diagram here as well.

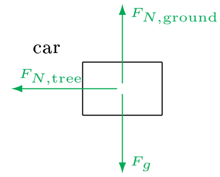


Figure 8.3.7: FBD for the car hitting the tree

The gravitational force on the car is

$$\vec{F}_g = m\vec{g} = (2.1\times 10^3\,\text{kg})(-9.81\,{}^{\text{m}}\!/_{\text{s}^2}\hat{\jmath}) = -\mathbf{2.06}\times 10^4\,\text{N}\hat{\jmath}$$

Since this in the vertical direction and the net force is in the horizontal direction, there must be an upwards normal force from the ground

$$F_{N,\text{ground}} = \mathbf{2.06}\times 10^4\,\text{N}\hat{\jmath}.$$

This is normal (perpendicular) to the surface of the ground. *The remaining horizontal force is the normal force from the tree,*

$$F_{N,\text{tree}} = -\mathbf{1.12}\times 10^5\,\text{N}\hat{\imath}.$$

This is normal (perpendicular) to the surface of the tree.

(Notice that Example 8.3.6 also shows why it is not always necessary to consider the vertical forces when we "know" that they cancel.) If you throw a ball at the ceiling, the ceiling will provide a normal force downwards, keeping the ball from moving through the surface.

Example 8.3.8 (*The normal force acts to reflect objects off a surface (ceiling)*)

Example referenced by *Introduction of inclined planes*

Carl recalls that one time he got bored one day in physics class (what?!?) and tossed a baseball ($m_b =$ 0.145 kg) at the ceiling... a little too hard ... as recounted in Exercise 5.6.1. The acceleration during that collision with the ceiling was $\vec{a} = -\mathbf{28.09}\,{}^{\text{m}}\!/_{\text{s}^2}\hat{\jmath}$. Find the normal force by the ceiling on the ball.

Solution. *There are five stages to the motion: (a) throwing, (b) falling up, (c) hitting the ceiling, (d) falling down, and (e) catching. We can show the forces involved in each (although we only care about the forces during (c) hitting the ceiling).*

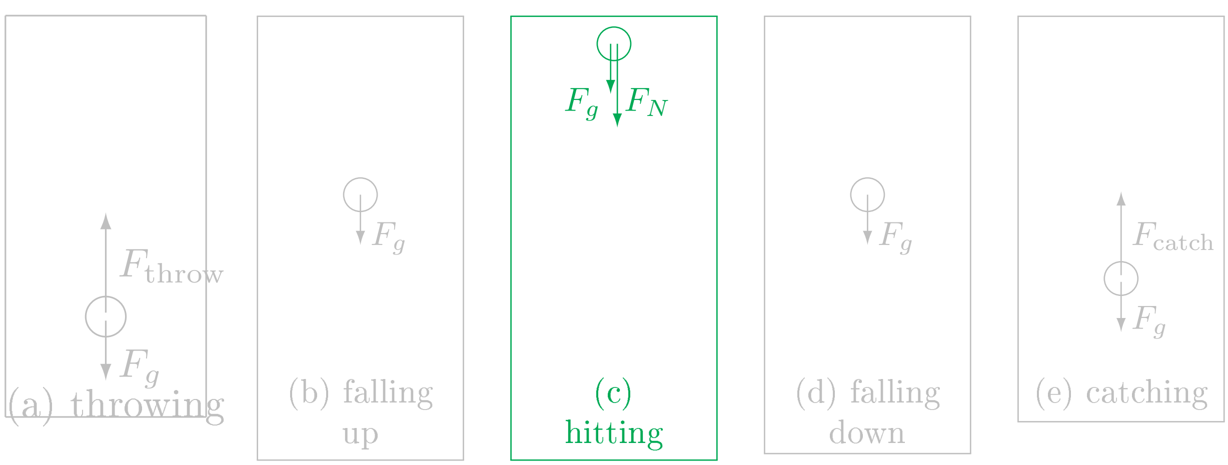


Figure 8.3.9: When a ball is thrown there are five distinct stages because the forces on the ball change and we can, at this point, only manage to describe a situation in which the forces do not change.

In this particular problem, we are only concerned with step (c) when the ball hits the ceiling, because that is the only part where the normal force acts. Example 8.7.1 will describe what happens during steps (a) and (e).

During step (c), we have the actual acceleration, which tells us about the net force. We will also need to know the weight of the baseball, because gravity is still acting during the collision.

$$\begin{aligned}
\vec{F}_N + \vec{F}_g = \vec{F}_{\text{net}} &= m\vec{a} \\
\vec{F}_N + m\vec{g} &= m\vec{a} \\
\vec{F}_N &= m\vec{a} - m\vec{g} \\
\vec{F}_N &= \left[(0.145\,\text{kg})(-\mathbf{28.0}9\ \tfrac{\text{m}}{\text{s}^2}\hat{\jmath})\right] - \left[(0.145\,\text{kg})(-9.81\ \tfrac{\text{m}}{\text{s}^2}\hat{\jmath})\right] \\
\vec{F}_N &= \left[-\mathbf{4.07}3\,\text{N}\hat{\jmath}\right] - \left[-\mathbf{1.42}2\,\text{N}\hat{\jmath}\right] = -\mathbf{2.65}1\,\text{N}\hat{\jmath}
\end{aligned}$$

You can see that the downward normal force ($\mathbf{2.65}1\,\text{N}$) *combined with the downward gravitational force* ($\mathbf{1.42}2\,\text{N}$) *together create the downward net force* ($\mathbf{4.07}3\,\text{N}$).

If you make a "bank shot" with either a basketball off the backboard or a pool ball[1] off the bumper, then the surface provides a normal force that is perpendicular to the surface, in this case redirecting the ball rather than stopping it. Unfortunately, the actual mechanism is somewhat more complicated than we are ready for; these are considered a little bit in the Investigation 1.

[1] Resources for specifications and a PDF version. These provide: weight ($5.5\,\text{oz} = 0.\mathbf{155}92\,\text{kg}$ and $6.0\,\text{oz} = 0.\mathbf{170}097\,\text{kg}$ cue), diameter ($2.250 \pm 0.005\,\text{in} = \mathbf{5.71}5\,\text{cm} \pm 0.0127\,\text{cm}$), rail height (63.5% of the ball height, $= \mathbf{3.62}9\,\text{cm}$), and dimension limits on the cue stick: $L_{\text{min}} = 40.00\,\text{in} = 1.016\,\text{m}$, $m_{\text{max}} = 25.0\,\text{oz} = 0.\mathbf{708}75\,\text{kg}$, and tip-width $w_{\text{max}} = 1.4\,\text{cm}$. You might also consider the information and calculations at Dr. Dave's site, which gives slow (1 mph), medium (3 mph), and fast (7 mph); coefficient of friction ball-to-ball $\mu = 0.06$; and ball-ball collision times as $250\,\mu$s-$300\,\mu$s.

Investigation 8.3.10 (*Pool balls and bumpers / cushions*)

Diane is relaxing with the local physics club, playing pool. She shoots a bank-shot and the ricochet reminds all of you about the normal force from the bumper on the ball.

(a) *Find a billiards table. Notice the felt, the bumpers (cushion), and the dimensions of the table. Does the ball roll as far on felt as it does on hardwood? (Answer 1) How soft is the bumper? (Answer 2)*

Answer 1. *First, you should not roll a pool ball across just any floor; there is felt on the pool table for a reason. However, if you have a clean, smooth surface and are able to reproduce your rolling speed, you will find that the pool ball rolls further on the stiff, nonyielding surface than it will on the felt. The reason for this is beyond the scope of this textbook, but you can read more from "Sliding and rolling: the physics of a rolling ball," J. Hierrezuelo and C. Carnero, Physics Education, Volume 30, Number 3* (unofficially at *this PDF*).

Answer 2. *The cushion (sometimes called a bumper) is pretty still to the touch, but it is made of a springy rubber that allows the balls to bounce reasonably well. The document indicates that you should be able to firmly strike a ball at some angle to the far wall and have it bounce around the table four to four-and-a-half times. If the bumpers were perfectly elastic, then the normal force would be normal to the resting surface; but since the bumper has some flexibility, when the ball hits the bumper with a glancing blow, then bumper bends inwards and the normal force is directed in a way that depends on the shape of the dent.*

(b) *Find a set of pool balls. Compare the solid-colored balls, the striped balls, and the cue ball. Are there differences in size of weight?*

Answer. *The specifications show that there is no difference between the solids and stripes, but the cue ball weighs* 9% *more that the other balls* (6.0 oz *versus* 5.5 oz*). The colored balls and the cue ball are otherwise identical.*

(c) *Hit the cue-ball off of a bumper in the manner intended for testing cushions. Compare the angle it leaves the bumper (reflected angle) match the angle at which it came in (incident angle). Does the spin of the ball matter?*

Answer. *Because the bumper is covered in felt, it has a small grip on the ball. Because the bumper has some give to it, it dents in when hit and provide more surface area, which increases the grip. Both of these mean that the spin of the ball gets transferred to the pool table somewhat and change the way a spinning ball exits from the bumper collision.*

(d) *Place a pool ball against the bumper and ricochet the cue ball off the pool ball instead of the bumper itself. Notice how the pool ball reacts. Why does the pool ball jump off the bumper?*

Does the pool ball move along the wall?

Where did you hit the pool ball?

Answer. *[Answer]*

Billiard tables have a lot of interesting physics, which can help us see a wide variety of physics, for example: friction, elastic versus inelastic collisions, rotational motion, and angular momentum.

Return to: the discussion of pool

8.3.1 Bathroom Scales Measure the Normal Force

Referenced by Discussion of uses of $F = ma$

To get a good sense of what how the normal force works, it helps to consider the way a bathroom scale works. Consider the concepts presented in the Investigation 8.3.11.

Investigation 8.3.11 (*Playing with a scale*)

While speaking to your friend, Beth about her recent accomplishment of losing 45 N, *you mention that your scale always gives a different number than the one in the doctor's office. You suggest she gets on your scale to verify the calibration. Beth currently has a mass of* 75.0 kg.

(a) *Imagine losing* 45 N. *Compare this to your weight. Is this a lot of weight to lose?*

Solution. *Since Beth weighs* $(75.0\,\mathrm{kg})(9.81\,\mathrm{m/s^2}) = 736\,\mathrm{N}$, $45\,\mathrm{N}$ *is about* 6% *of her weight. This is fairly substantial. You should compute how much* 6% *of your weight is and convert that to kilograms and Newtons.*

(b) *Place your toe on the scale while Beth weighs herself. This increases the value the scale reads. Does Beth weigh more?*

Solution. *When one person stands on the scale, the scale provides just enough of an upwards normal force to keep that person in equilibrium. In that case, the upwards force is balancing the weight of the person. This gives the impression that the scale is telling you your weight; however, when you press down or help support whomever is standing on the scale, the scale adjusts the amount it must provide. The scale is not trying to tell you your weight. Rather the scale is trying to create equilibrium by balancing whatever force(s) are pressing into it. Your weight is determined by the gravitational force and does not change when you press harder or lighter onto the scale.*

(c) *With your hands, press down on Beth's shoulders while she stands on the scale. Control the value read by the scale (HINT: Task b). Increase the reading by* 20 N, 30 N, *etc. Does Beth's weight change? (Solution 8.3.1.2.b.1) Are you adding weight to the scale? (Solution 8.3.1.2.c.1)*

Solution. *When you press down on Beth's shoulders, you are not adding weight. Weight has a specific definition: it is specifically the value that the gravitational force pulls on any object. Pushing the person does not change their weight; it does, however, change the amount that they press into the Earth. That is to say, it increases their downwards normal force on the scale, but not their weight.*

(d) *Have Beth lean on a nearby table or counter while she stands on the scale. Control the value read by the scale (HINT: Task c). Decrease the reading by* 20 N, 30 N, *etc. Does Beth's weight change? (Solution 8.3.1.2.b.1)*

Solution. *When Beth supports herself on something other than the scale, she is not removing weight. Weight has a specific definition: it is specifically the value that the gravitational force pulls on any object. Supporting oneself elsewhere does not change ones weight; it does, however, change* *the amount that they press into the Earth. That is to say, it decreases the downwards normal force on the scale, but not the weight.*

(e) *Find two bathroom scales and stand with one foot on each. Lean one way and then another. Explain what happens to the readings on the scale as you lean one way or the other. Do you notice a pattern in the values that the scales read?*

Solution. *The sum of the readings from the two scales should always add up to the total normal force needed to support your weight.*

(f) *Hold the scale against the wall and press into it. Control the value read by the scale (HINT: Task d). Increase the reading by* 20 N, 30 N, *etc. What is the scale measuring?*

Solution. *Since your weight is a force pulling downwards, having the scale on the wall shows that the scale cannot be balancing weight. Since you are pushing into the wall, you are exerting a normal force into the scale and the scale is exerting a normal force back at you. Both of these forces are horizontal (assuming the wall is plumb).*

Another way to think about this is: If you can control the value read by the scale (such as against the wall) while at the same time not changing your actual mass, the scale cannot literally be measuring the weight of the object on the scale.

(g) *Imagine placing a scale on a ramp that can be laid flat or raised to any angle up to a vertical (making it a wall). Imagine standing on the scale on the ramp while it is lifted from horizontal (like a floor) to vertical (like a wall). Does the scale always read the same value while it is raised to different angles?*

Solution. *When the scale is on the flat, horizontal floor, it balances your full weight. When the scale is on the vertical wall it does not carry any of your weight. At any angle in between those values, it carries some fraction of your weight while friction keeps you from sliding down the ramp. It will turn out that since the cosine function behaves in just the right way, we can use the cosine to find the component of the weight that the normal force from the scale has to support.*

Return to: Subsection 8.3.1, Clarification 8.3.15

Some digital scales are inconvenient for understanding how they work because they don't display the value until it has come to something close to equilibrium. If you have access to an analog scale, then you can watch the value change as it settles down and it might be easier to build your intuition.

As you consider the values that you read on the scale, you should consider what happens if you jump off of or land upon a scale. *Note that actually doing this can decalibrate your scale, if not break it entirely. Scales are not meant to be handled this way.* While you are jumping from your scale, it must provide not only the force necessary to support your weight, but also the upwards force require to accelerate you upwards. While you are landing on the scale, it musty provide not only the force necessary to support your weight, but also the upwards force necessary to decelerate you.

Bathroom scales use leverage (i.e., torque) and a spring-system to balance the force pressing into them. The mechanism can be seen at How Stuff Works.

8.3.2 The normal force and inclined planes

With our understanding of the normal force, we have seen examples of the normal force pointing upwards from the ground, downwards off the ceiling, and sideways off of a vertical surface. If you consider a the trunk of a tree or the edge of a wheel, the normal direction is radially outwards (in the direction of the spokes of the wheel).

Exercise 8.3.12 (*Practice drawing normal lines*)

When you draw a circle that represents the sun, you typically draw the light rays coming off the sun in the radially outwards direction. For this exercise, draw

1. *the rays coming from an egg-shaped (or pill-shaped) sun,*
2. *the rays coming from a hill-shaped sun,*
3. *the rays coming from a mountain-shaped sun.*

Hint 1 (egg-shape). *Eggs, as opposed to an ellipse, have one end larger than the other. Pills tend to be oval or rounded-rectangles. Any of these shapes will work for the effect we are going for. The point is to avoid sharp corners. When you add the outwards rays, notice how the rays connect with the surface of the shape.*

Hint 2 (hill-shape). *I am expecting a hill-shape to be similar to a "bell-curve" or the graph of a "normal distribution": flat on either side, curving upwards towards the middle and rounded at the top. The rays you draw should be "outwards".*

Hint 3 (mountain-shape). *I am expecting a mountain-shape to be similar to a triangle or a pyramid, although you might have a more detailed image. The rays you draw should be "outwards", but not like the lines drawn on the pyramid that appears on the back of a U.S. one-dollar bill. Those rays are radial like they come from a circular object. Your lines should come straight off the surface and emphasize the shape of the drawing rather than emitting from the central point.*

Answer. *Your lines should come straight off the surface and emphasize the shape of the drawing rather than emitting from the central point. If you do this, then your rays should indicate the normal direction, which is everywhere perpendicular to the surface.*

Another place where we see the normal force acting is on an inclined plane (a tilted flat surface).

Definition 8.3.13 (inclined plane)**. Definition referenced by** discussion of Inclined Planes

Any tilted, flat surface; such as a ramp, slide, ski jump, the side of a pyramid, etc.

You might experience this in a variety of places: driving up a steep hill, partially opening a hard-cover book or your laptop, and lifting one end of a table. Let's consider a tilted table and imagine putting a couple of books under one side of a table so that it tilts to the left. Most of the items you place on the table will not slide off because there is friction. This will be considered in detail in Subsubsection 8.4.1.3, especially under the topics of the angle of repose and sliding down inclined planes. For now, we will consider a very slick table top so that object slide very easily. (Subsection 8.4.2 will discuss rolling objects.)

Example 8.3.14 (*Sliding down an icy driveway*)

After an ice storm, Diane ($m_D = 80.0\,\mathrm{kg}$) decides to visit her neighbor across the street who has a broken leg to see if they need anything she can help with. When she arrives at the house, she notices that the driveway has a slight hill to it. The angle of incline is about $\theta = 3.00°$[2] and she is standing at the top of the hill.[3] The length of the driveway is about two-and-a-half car-lengths: $L = 11.25\,\mathrm{m}$.

As she slides down the hill, wondering how she will survive the crash at the bottom, you should compute

1. *her acceleration,*
2. *the time it takes to get to the bottom, and*
3. *her speed at the bottom (just before slamming into the garage door).*

Do you expect Diane to get hurt in the final collision?

Hint 1 (acceleration). *You should be able to use Newton's Second Law to find the acceleration. The tricky part is finding the components of the individual forces.*

Hint 2 (force components). *Subsection 3.3.1 discusses the choice of coordinate systems. In this case, since the acceleration is along the incline (and not normal to the incline), Newton's Second Law will be easier to manage if one axis (let's call it x) is along the incline (seeing all of the acceleration) so that the other component (which then is y) doesn't see any acceleration and has the convenience of $F_{\text{net},y} = m\cancelto{0}{a_y}$. The consequences of this are that the normal force only has a y component, but the gravitational force has* *both an x and a y component.*

For convenience, I am going to set my positive x-direction down-along the incline, since this is the direction of the motion. I am also going to set my positive y-direction up-normal off the incline.

Hint 3 (Free-body diagram). *Draw the incline. Point the normal force normal to the surface and the gravitational force directly towards the center of the Earth. These do not directly oppose each other (and do not cancel like they have in previous examples).*

Hint 2 implies we should use a coordinate system that has one axis along the incline and another normal to the incline.

Answer 1 (Acceleration). *Diane has an acceleration of $a_x = 0.\mathbf{5134}\ \mathrm{m/s^2}$. This is pretty low and likely will provide sufficient time to spin into a more advantageous position before the inevitable crash.*

Answer 2 (Time). *Diane takes* $\mathbf{6.62}0\,\mathrm{s}$ *to slide down the driveway. That's a long time to be sliding. This might even be enough time for you to get your phone out and film the crash!*

Answer 3 (Final speed). *Diane crashes into the garage door at $3.4\ \mathrm{m/s}$. This is faster than a brisk walk ($2.5\ \mathrm{m/s}$), but slower than most people run.*

Solution. *Diane's weight is $m_D g = \mathbf{784}.8\,\mathrm{N}$. We know the gravitational force (weight) and can compute the components of that. We do not (yet) know the normal force, but we do know that it only has a y-component. We do not know the net force, but we do know that it is necessarily (down) along the incline.*

	x-comp	y-comp
F_g	$mg\sin\theta = -\mathbf{783}.7\,\mathrm{N}$	$mg\cos\theta = +\mathbf{41.07}\,\mathrm{N}$
F_N	$0\,\mathrm{N}$	$F_{N,y}$
F_{net}	$m_D a_x$	$0\,\mathrm{N}$

We can use the y-component to find the value of the normal force $F_N = +\mathbf{783}.7\,\mathrm{N}$. We can use the x-component to find the acceleration of $a_x = F_{gx}/m_D = +0.\mathbf{5134}\ \mathrm{m/s^2}$. This allow us to find the time and final speed. It turns out to be convenient to find the speed first (to avoid the quadratic formula).

$$v_f = \sqrt{(\cancelto{0}{v_i})^2 + 2(0.5134\ \mathrm{m/s^2})(11.25\,\mathrm{m})} = \sqrt{\mathbf{11.55}\ \mathrm{m^2/s^2}} = \mathbf{3.399}\ \mathrm{m/s}$$

$$t = \frac{v_f - \cancelto{0}{v_i}}{a_x} = \frac{\mathbf{3.399}\ \mathrm{m/s}}{0.\mathbf{5134}\ \mathrm{m/s^2}} = \mathbf{6.62}0\,\mathrm{s}$$

That is a very slow motion slide. Diane likely has time to slow her fall and kind of stand up.

The study of inclined planes is important in introductory physics because it provides an example of a situation where it makes more sense (by which I am makes the math easier) to use a coordinate system that is not the usual horizontal-and-vertical. It is still useful (and will always be) to have the axes be perpendicular to each other. See the discussion in Hint 2 for the relevant reasoning.

8.3.3 The normal force and the third law

Recall that the third law tells us about what happens to the object that is doing the pushing. In Example 8.3.2, the gravitational and normal forces were included. In the same way that Example 7.2.27 explained the way the third law describes the sources of the horizontal forces on the object (by adding the people to Example 7.2.13), we can now consider the source of the vertical forces. The gravitational forces pulling down on the people and the object are caused by the gravitational interaction with the Earth. Since the Earth is pulling them down, the third law indicates that they must pull the Earth up *with a gravitational force.* These pairs of gravitational forces (Earth pulling down on the object and the object pulling up on the Earth, etc.) are "third-law pairs" (recall Insight 7.2.26).

Similarly, the normal forces that support the people and the object (against the gravitational force) are caused by the Earth's surface, which is keeping the people and objects from moving through that surface. Since the Earth's surface is pushing them up, the third law indicates that they must push the Earth down *with a normal force* (keeping the Earth from moving through their feet). These pairs of normal forces (Earth pushing up on the object and the object pushing down on the Earth, etc.) are "third-law pairs" (recall Insight 7.2.26).

Clarification 8.3.15 (*Third-law pairs*)

You should notice the language in the paragraph above. While the normal force does act against the gravitational force (in this case), the normal force is not the third-law pair to the gravitational force. If you keep Task 8.3.11.g in mind, it should be obvious that the normal force does not necessarily balance the gravitational force.

Since the source of both the gravitational and normal forces is the Earth, Example 14 adds the free-body diagram of the Earth to this set of examples.

Example 8.3.16 (*Using the second and third laws to find all forces acting*)

See Example 8.3.2 and add the FBD for the Earth.

8.4 Frictional Force

Section referenced by Task 7.3.2.b, Task 7.3.2.d.ii, Task 7.3.2.e, Solution 6.2.1.1.a.i.1

Recalling the discussion about Newton's first law, friction is the force that affects our intuition about how objects move. Friction is the force that is usually thought of as the reason that objects slow down, but it is also the reason we are able to move forward when we walk and to turn corners when we drive (for example). Whenever objects move, the motion is measured by what they move past. The four situations we (as humans living on Earth) are most familiar with are

- sliding across the ground,
- rolling along the ground,
- soaring through the air, and
- gliding through the water.

Each of these actions allow us to experience friction in a different way. If we start with sliding, then we can establish the concepts with some fairly simple mathematics and address the cases of trying to walk and turn a car. The concepts we will want to pay attention to in that case are the roughness of the surfaces that are in contact and the amount they are pressed together. Rolling friction will use these same ideas, but will apply them in a different way. The discussion of air resistance (and water resistance) will modify these ideas a bit, considering the thickness of the gas (or fluid) through which the object moves and the shape of the object. The shape of the object affects the aerodynamics (or hydrodynamics), which informs us about the amount of impact the gas (or fluid) has on the front of the object and is similar (conceptually) to the amount the surfaces are pressed together.

8.4.1 Dry Sliding Friction

Sliding friction is relevant to any problem in which one object is "trying to" move across another. You should note that the objects do not actually have to be moving across each other for this to be relevant. If they are moving across each other, then we will need to consider the kinetic friction (Subsubsection 1). If, on the other hand, you imagine pushing a heavy couch across a carpeted floor, then you can picture a case where the horizontal force is not sufficient to overcome the frictional force. In this case, we will consider the static friction (Subsubsection 2).

Etymology (kinetic) Kinetic is related to the Greek *kinetikos* and *kinesis*, which indicate motion.

Etymology (static) Static is related to stasis and the Greek *statikos* (which indicates standing) and *stasis* (which indicates standing still). It may also call to mind stationary. The connection to "radio static" seems to be that it is unchanging.

For both static and kinetic friction, we will consider the case of pushing on an object to get it to slide across the floor or a table or some other surface. For example, you can place your shoe on the floor and push it gently with your foot so that it doesn't move (static) or harder so that it does move (kinetic). If you are using a rubber-soled shoe, then you might notice that the shoe moves in a herky-jerky fashion. We will not be considering motion that comes in fits-and-starts because that indicates that the acceleration is changing, which (as discussed in Subsection 5.6.3, and especially Exercise 5.6.1) means that we cannot use the basic equations of motion for constant acceleration.

Clarification 8.4.1 (*Dry Sliding Friction*)

This section is describing "dry sliding friction". When lubrication is introduced, the relationship changes significantly. If you are interested in this, then you can find a nice summary at Explain that Stuff (Oct, 2017).

We will begin with the kinetic friction because static friction has a small extra complication.

8.4.1.1 Kinetic Friction

Here we will discuss the situation of two surfaces rubbing against each other as they slide past each other. This is not the case with walking (unless your feet slip out from under you!) or turning a car (even though the car *is* moving). Before introducing the relevant equation, please investigate how objects slide across different surfaces to build your intuition.

Investigation 8.4.2 (*Notice where friction is larger or smaller*)

(a) *Consider some instances of objects sliding across a surface and slowing down. List instances of objects that are heavier and lighter. List instances of objects sliding across smooth surfaces and rough surfaces.*

Task referenced by *Task 7.2.6.a*

Hint. *Do you recall your answers to Task 7.2.6.a?*

Answer. *While you may have different examples, for specificity, I will discuss the following cases:*

(a) *A mug full of cold root beer sliding across a counter. (Note that in the movies, you are probably seeing a counter that is slightly wet, contradicting Clarification 1.)*

(b) *An empty mug sliding across a counter.*

(c) *A cinder block sliding across a paved road.*

(d) *A cinder block sliding across a gravel road that is covered in fine-cut (small) gravel that is packed moderately tightly.*

(e) *Dragging a heavy chair across a hard-wood floor.*

(f) *Dragging a heavy chair across a carpeted floor.*

(b) *Considering each of the cases listed in Answer 1, rank them according to which has the most friction and which has the least.*

Hint 1. *You should be able to imagine which is the hardest (easiest) for you to keep moving (to overcome the friction).*

Hint 2. *You can also gauge this by how far they move because friction is the only force causing them to slow down. So, you can list them according to which one slows down the fastest (corresponding to the order of having the most friction).*

Hint 3. *The comparison asked for here is not really fair because multiple aspects are changing from example to example. For a fair comparison, you should also consider a cinder block and a chair* *being slid across a slightly wet counter (!), a cinder block being dragged across a hardwood floor (do not try that at home) and a carpet, and a heavy chair being slid across paved and gravel roads.*

You may also note that if the object you are dragging across a carpet is heavy enough, then the carpet might bunch-up in front of the object. This does not make for a good comparison. Similarly, the gravel is not really comparable because you will likely be digging ruts in the gravel, which takes more effort and is not equivalent to the friction seen in the other cases.

Answer 1. *The chair will experience more friction than the cinder block, which experiences more friction than the mug. To produce a fair comparison, you should consider dragging objects with various weights across the same surface. Furthermore, you should not use a surface (such as a gravel road) in which the object being dragged can pull the material of the surface along with it, creating ruts in the surface because that involves moving the material rather than simply sliding across it.*

Answer 2. *The slightly wet counter will provide less friction than the hardwood floor (or a dry counter), which provides less than the paved road. Assuming the carpet is a flat carpet (as opposed to a shaggy carpet), it is likely that this will be somewhere between the hardwood floor and the gravel road, but there is enough variation in these types of surface that this is not necessarily obvious without an experiment. To produce a fair comparison, you should drag the same object across each of the different surfaces.*

(c) *Try to determine (by creating an experiment) if it is the weight itself that matters.*

Hint 1. *You might try pushing a book across a floor and comparing the force necessary to overcome friction while simulating a heavier or lighter book without actually changing the book.*

Hint 2. *Recall Subsection 8.3.1 for ways to make something* seem *heavier or lighter without actually changing the weight.*

Answer. *The aspect that matters is the amount that the surfaces are pressed together. (See comments in the text for more information.*

There are several observations that you should notice from Investigation 8.4.2. The first observation is as follows:

Observation 8.4.3 (Friction is increased when two surfaces are pressed harder together)**.** If one object is above the other, then the weight contributes to the amount they are pressed together, but since objects can have friction when they rub their sides across each other (like a truck squeezing through a narrow alleyway), the weight cannot be the explicit quantity. It turns out that the normal force is the quantity that expresses how strongly objects are being pressed together because it reflects the force needed to keep one object from passing through the other.

Touchstone (normal force) Recall Bathroom Scales Measure the Normal Force.

This observation was made by Leonardo da Vinci in 1493 and, while still undiscovered in his notebooks, was rediscovered by Guillaume Amontons in 1699 ([8.10.1], [8.10.2], [8.10.3]) who wrote it as his first law of friction.

Touchstone (law) Recall the description of a "law".

Amontons' first law of friction

The force of friction is directly proportional to the applied load.

Example 8.4.4 (*Does full or empty stop sooner?*)

Considering the mugs in Answer 8.4.1.1.a.1, one full and one empty, let's assume that the mass of the full mug is double the mass of the empty mug. If both are slid at the same initial speed across the same *counter, select the answer that indicates which mug will come to rest sooner:*

1. *The full mug stops before the empty mug. (Answer 1)*
2. *The empty mug stops before the full mug. (Answer 2)*
3. *Neither; they stop after the same distance. (Answer 3)*

Hint 1. *The mug with twice the mass has twice the weight.*

Hint 2. *Because they are on a (presumably) flat surface with no other forces pushing up or down, the normal force must be balanced with the weight of each mug.*

Hint 3. *Recall Amontons' first law of friction.*

Answer 1. *Since the full mug has a larger normal force, it does have a larger frictional force than the empty mug. However, the relevant equation is Newton's second law in the horizontal direction with friction as the only horizontal force: $F_f = ma$. Since both the frictional force and the mass are double that of the empty mug, the acceleration is the same for both mugs. The full mug does not stop before the empty mug.*

Answer 2. *Since the empty mug has a smaller mass, you might expect it to have a larger acceleration (according to Newton's second law), but that conclusion assumes that the force is the same, which is not true. The empty mug has a smaller mass and a smaller frictional force (because it has a smaller normal force). The relevant equation is Newton's second law in the horizontal direction with friction as the only horizontal force: $F_f = ma$. Since both the frictional force and the mass are half of that for the full mug, the acceleration is the same for both mugs. The empty mug does not stop before the full mug.*

Answer 3. *The relevant equation is Newton's second law in the horizontal direction with friction as the only horizontal force: $F_f = ma$. The question requires an understanding of the acceleration,*

$$a = \frac{F_f}{m} = \frac{2F_f}{2m} = \frac{\frac{1}{2}F_f}{\frac{1}{2}m}$$

If the numerator and denominator are increased or decreased by the same amount, then the result is the same. The full and empty mugs should reach the same distance.

Since directly proportional indicates a linear relationship, you are probably thinking of the equation of a line, $y = mx + b$. Experiment shows that, consistent with your intuition, if the normal force is zero, then there is no friction, so the intercept is zero. The proportionality constant we use is called the **coefficient of friction** and the symbol is μ, which is the Greek letter equivalent to the English letter "m". In most of America, it is pronounced "mew". This relationship is expressed in the equation

$$F_f = \mu F_N \tag{8.4.1}$$

where F_f is the **frictional force**, F_N is the normal force.

One important aspect to Equation (8.4.1) is that:

Clarification 8.4.5 (*$F_f = \mu F_N$ is not a vector equation*)

The direction of the frictional force is necessarily *along the plane of the surface, whereas the normal force is* necessarily *normal (perpendicular) to the plane of the surface.*[1] *This is a "magnitude" relationship.*

Example 8.4.6 (*Hey! Let's get lunch (Direction of friction 1/2)*)

You decide to go to lunch with Beth. When you get to the table and pull your chair out away from the table, it makes an embarrassingly loud squeal as it drags across the ground. In your evaluation of the situation, draw a free-body diagram, clearly indicating the direction of all forces acting on the chair. Assume the chair starts at rest, accelerates as you pull, then decelerates as you gradually stop pulling, and stops as you release it. Let's assume that the table is in the $+\hat{\imath}$ direction and you are pulling the chair in the $-\hat{\imath}$ direction.

Hint 1. *There are four stages of motion: Before you pull, as you start to pull, while you reduce your pull, and after you release it.*

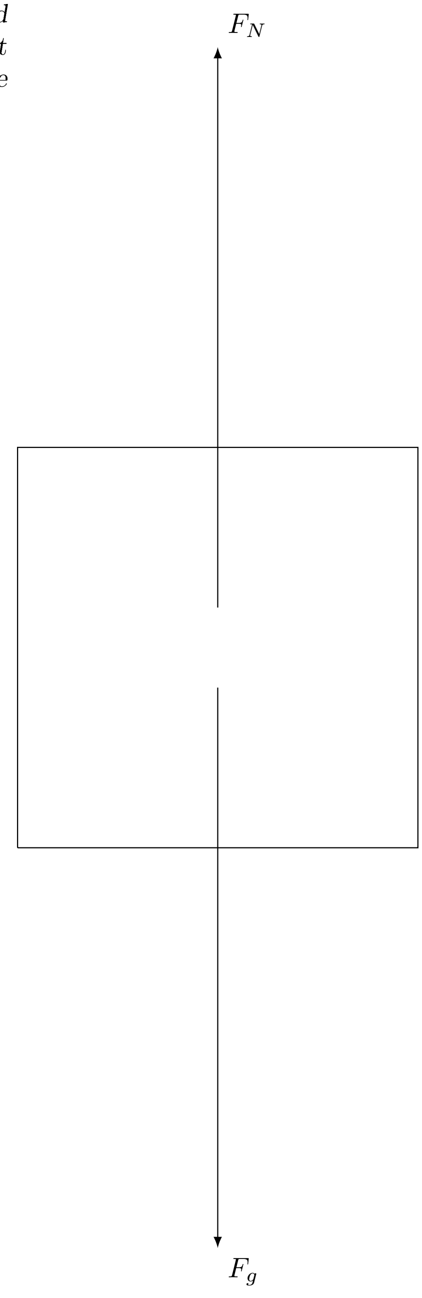
Hint 2. *Friction is a reactive force; its magnitude depends on what is happening to the chair.*

Hint 3. *Does the friction help you move the chair or interfere with your pulling? What does this imply about the direction of the force?*

Hint 4. *Is friction the same through out the problem?*

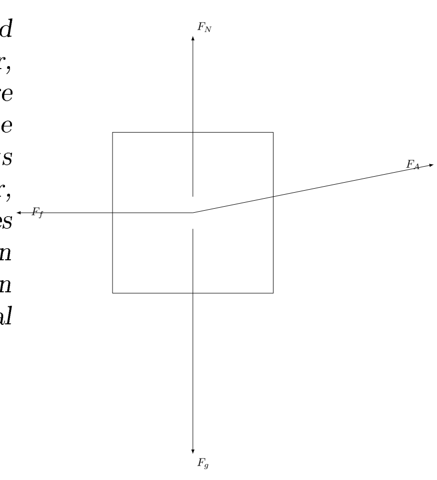
Answer 1.

Before you begin pulling, the chair has its weight pulling it down and the normal force keeping it from falling through the ground. Since it starts at rest, the acceleration is zero, so the net force is zero and the normal force must balance the weight.



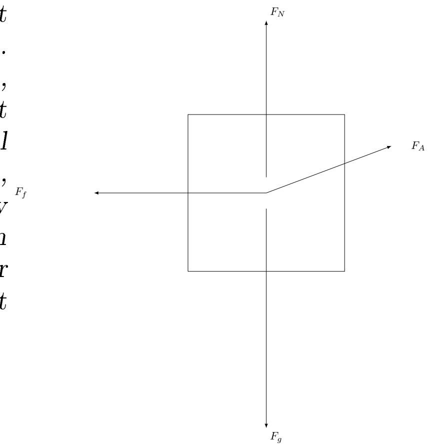
Answer 2.

As you start pulling, the chair still has its weight pulling it down and the normal force keeping it from falling through the ground. However, assuming that you are pulling at a slight upwards angle, you are helping to support the weight and the normal force does not need to be as large. (If we had numbers, you could use the vertical components to find the magnitude of the normal force.) The friction, however, does not help lift the chair, nor weigh it down; friction only interferes with the backward pulling. It is parallel to the ground in the direction opposite the motion. If you are accelerating the chair backwards, then the applied force has a larger horizontal component than the frictional force.



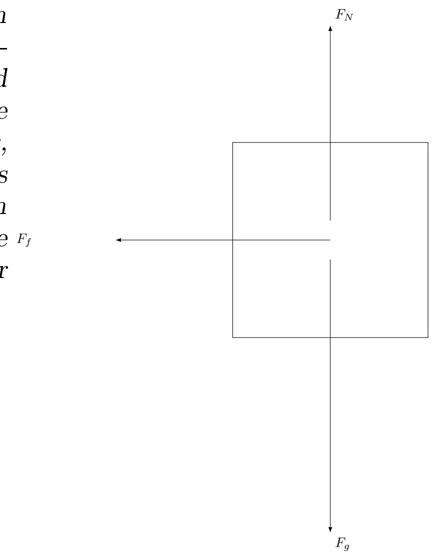
Answer 3.

As you start relaxing your pull, the chair still has its weight pulling it down and the normal force keeping it from falling through the ground. However, assuming that you are still pulling at a slight upwards angle, you are helping to support the weight and the normal force does not need to be as large. (If we had numbers, you could use the vertical components to find the magnitude of the normal force.) The friction, however, does not help lift the chair, nor weigh it down; friction only interferes with the backward pulling. It is parallel to the ground in the direction opposite the motion. If you are decelerating the chair backwards, then the applied force has a smaller horizontal component than the frictional force.



Answer 4.

As you release the chair, the chair still has its weight pulling it down and the normal force keeping it from falling through the ground. However, we might assume that you have stopped pulling the chair and are no longer helping to support the weight so that the normal force again needs to balance the gravitational force. The friction, however, does not help lift the chair, nor weigh it down; friction only interferes with the backward pulling. It is parallel to the ground in the direction opposite the motion. Since you have stopped pulling there is only the frictional force, in the direction opposite the motion, slowing the chair down.



Example 8.4.7 (*Click here! (Direction of friction 2/2)*)

You are moving the computer mouse to direct the arrow downwards along the screen by dragging the mouse towards you along the desk. What direction is each of the frictional forces acting between the mouse and the desk?

Hint. *If the mouse is moving towards you and friction hinders that motion, how will the acceleration be related to the velocity? Which of these is related to the force?*

Answer.

Touchstone (on-by) the on-by notation

Side note: there is a normal force *on the mouse* by *the desk* (F_{mdN}) upwards *that keeps the mouse from falling through the surface of the desk. This normal force has two relevant relationships. First, it has a "third-law pair" that implies there is a* downwards normal *force on the desk* by *the mouse,* F_{dmN}. *Second, having a normal force creates the potential for a frictional force, as seen by* Equation (8.4.1).

Touchstone (third-law pair) third-law pairs are the same type *of force acting* on different *objects*

As the mouse moves across the desk towards you, Hint 1 *indicates that the frictional force on the mouse* (F_{mdf}) *is away from you (and slows the mouse down). It may be less obvious that there is also a "third-law* *pair" frictional force on the desk by the mouse (F_{dmf}) which is towards you due to the relative motion between the desk and the mouse. You can see this if you imagine placing a tissue under the mouse and enacting the same motion. The tissue will be pulled towards you by the mouse due to that frictional force pair.*[2]

Example 8.4.7 is extended in Example 8.4.22 after we introduce Static Friction.

Example 8.4.8 (*How far the mug slides*)

Given a 0.75 kg mug pushed at 2.00 $\mathrm{m/s}$ across a counter with a coefficient of friction $\mu_k = 0.13$, how far will it go before stopping?

Hint 1. *There are two steps to this problem: Find the acceleration (see Hint 2), then the distance (see Hint 3).*

Hint 2 (Acceleration). *To find the acceleration, use Newton's second law for the horizontal direction. (See Hint 4.)*

Hint 3 (Distance). *There are two equations that involve the displacement. One of them requires the time of motion, which is difficult (but not impossible) to find. I suggest using the other equation.*

Hint 4 (net force).

$$\vec{F}_{\text{net}} = \vec{F}_g + \vec{F}_N + \vec{F}_f = m\vec{a}$$

In the horizontal direction, the net force is the sum of the x-component of the gravitational force (which has no x-component), of the normal force (which also has no x-component), and of the frictional force (which has no y-component);

$$\cancelto{0}{F_{Nx}} + \cancelto{0}{F_{gx}} + F_{fx} = ma_x$$

so you need the value of the frictional force. (See Hint 5.)

In the vertical direction, the net force is the sum of the y-component of the gravitational force (which has no x-component), of the normal force (which also has no x-component), and of the frictional force (which has no y-component);

$$F_{Ny} + F_{gy} + \cancelto{0}{F_{fy}} = m\cancelto{0}{a_y}$$

so you need to relate the vertical acceleration to the gravitational and normal forces. (See Hint 6.)

Hint 5 (Frictional force). *You can find the frictional force from the normal force with Equation (8.4.1), where μ is given and F_N is to be determined. (See Hint 6.)*

Hint 6 (Normal force). *You can find the normal force from Newton's second law in the vertical direction since the vertical-component of the acceleration is zero.*

$$\vec{F}_N + \vec{F}_g = m\cancelto{0}{a_y}$$

In this case $\vec{F}_N = -\vec{F}_g$.

Answer. *The mug travels 1.6 m across the counter.*

Solution. *In order to find the distance traveled by the mug, we should use the equation*

$$v_f^2 = v_i^2 + 2\,a\,\Delta x$$

and solve for the displacement (Δx). In order to do this, we need the initial and final velocities and the acceleration. We are given the initial velocity and, since the mug comes to rest, we can easily deduce the final velocity ($0\ \mathrm{m/s}$). As usual (recall Example 7.2.19), we can find the acceleration from Newton's second law (the hints give the justification for the steps)

$$a = \frac{F_f}{m} = \frac{\mu F_N}{m} = \frac{\mu(mg)}{m} = \mu g = (0.13)(9.81\ \mathrm{m/s^2}) = \mathbf{1.28}\ \mathrm{m/s^2}$$

where you might notice three relevant features:

1. *that the mass cancels (consistent with Example 3),*

2. *that the second step (connecting F_f to F_N) makes this a magnitude equation (not a vector equation) so you will need to add the sign of a "by hand", and*

3. *that you do not literally need to find the value of the normal or frictional forces.*

Given the acceleration, we can find the displacement:

$$\Delta x = \frac{(\cancelto{0}{v_f})^2 - v_i^2}{2\,a} = \frac{(0\ \mathrm{m^2/s^2}) - (2.00\ \mathrm{m/s})^2}{2(-\mathbf{1.28}\ \mathrm{m/s^2})} = \mathbf{1.57}\,\mathrm{m}$$

The second observation you might make from Investigation 8.4.2 might be as follows:

Observation 8.4.9 (Friction depends on the types of *both* surfaces rubbing together)**.** A chair on a hard-wood floor is different from a rubber-soled shoe on hard wood and both of these are different from that same chair (or shoe) on ceramic tile. It is not useful to talk about the friction caused by the floor independently of what is moving across the floor. Furthermore, it depends on how those surfaces interact. On the one hand, it is safe to conceptualize this as the rougher the surfaces, the more places they have to grab each other. On the other hand, there is also the effect of very flat, smooth surfaces: If two very flat, smooth surfaces are in contact, they have a "stickiness" that interferes with the easy of sliding past each other. For example, when you get an oil change in your car, they will often provide a small plastic reminder tag that sticks to your windshield with significant friction, but without stickiness. *The information about how these surfaces interact is encoded in the coefficient of friction.* (See also Footnote 5 in Amontons' second law of friction below, which indicates that some engineering texts indicate this as another law of friction.)

An interesting example of a recent development in friction is the development of "gecko-grip".

Equation (8.4.1) is also interesting for what it does not include.

Observation referenced by discussion of Pascals as a derived unit, Section 16.2, Subsection 19.1.2

Observation 8.4.10 (Friction does not depend on the area of contact)**.** One would not be wrong to imagine that the broader something is, the more it interacts with the surface it is dragged along, much like how a picnic blanket (or superhero cape!) collects many more twigs and leaves when it is held open and wide while being dragged through the park than when it is dragged by one corner. On the other hand, you might also notice that when you stand on the bed, you sink in further than when you lay on the bed. It turns out that while more surface does tend to increase the opportunity for friction, this is balanced by the reduced pressure due to having the weight spread out[3], thereby maintaining the same normal force regardless[4] of the amount of area in contact. Speaking of pressure, you might be interested to note that Amontons is also known for his work on barometers (see Barometrics).

As with Observation 2, this observation was also made by Leonardo da Vinci in 1493 and, while still undiscovered in his notebooks, was rediscovered by Guillaume Amontons in 1699 [8.10.1], who wrote it as his second law of friction.

Touchstone (law) Recall the description of a "law".

Amontons' second law of friction

The force of friction is independent of the apparent[a] area of contact.

[a]This indicates the "apparent" area because there is no relation to the macroscopic surface area. On the other hand, the microscopic contact does matter and this is accounted for through the coefficient of friction which describes the specifics of the surfaces that are interacting. Modern texts list an additional law of friction as: "Friction depends on the type of the surfaces in contact." In 1950, Frank Philip Bowden and David Tabor described how the microscopic surface area affects friction [8.10.2].

[3]Recall discussion of the units of pressure.

[4]You might be relieved to learn that this is only mostly true. The microscopic area of contact does matter, but that gets back to the roughness of surface discussion in Observation 8.

Equation (8.4.1) also does not include the relative velocity between the surfaces, as observed by Charles Coulomb in 1785, [8.10.3].

Touchstone (law) Recall the description of a "law".

Coulomb's law of friction

Kinetic friction is independent of the sliding velocity.

Touchstone (mostly true) Coulomb's law of friction is mostly true. If the object is moving extremely slowly then it gets into the fits-and-starts motion; the motion is not smooth and puts us back into that described above. If the object is moving very fast, then it can start to skip (imagine a speedboat skimming across a lake with a staccato thrumming and a choppy wake) and the nature of the surface-interaction becomes different enough that the description must also change. However, the range of speeds for which this *is* true is reasonably large.

Coulomb's law of friction turns out to be extremely convenient. Because of this law, we can do Example 7. If it were not true, then the magnitude of the frictional force would change as the object slowed down and the acceleration would not be constant! This requires mathematics called "differential equations", which most students take after completing three years of calculus. Unfortunately, this is what happens with Air Resistance.

Friction turns out to be quite interestingly complicated. Most of our understanding comes from experimental patterns, although the field of tribology is continuing to improve our theoretical descriptions of friction all the time. For our purposes, we can encapsulate all of these ideas into a single equation.

Translation 8.4.11 (*The Story of $F_f = \mu F_N$*)

This equation is all about those aspects that impact the strength of the frictional force between two surfaces. Connecting the English and the math:

$$\underbrace{F_f}_{\substack{\text{the strength}\\ \text{of the}\\ \text{frictional}\\ \text{force}}} \quad \underbrace{=}_{\substack{\text{depends}\\ \text{on}}} \quad \underbrace{\mu}_{\substack{\text{the}\\ \text{“grippy-ness”}\\ \text{of the}\\ \text{surface}}} \quad \text{and} \quad \underbrace{F_N}_{\substack{\text{the amount}\\ \text{the surfaces}\\ \text{are pressed}\\ \text{together}}}$$

Implicit in this story is that μ does not depend on the apparent surface area nor the relative speed between the surfaces.

Connection 8.4.12 (*Looking Ahead*)

The obvious question, if you have heard the phrase "energy is never created nor destroyed, only changed or transferred", is: Where does the lost energy go? Clearly this discussion will have to wait until we learn the language of energy (in Chapter 9), but we can start that conversation. Consider smoothing a piece of wood with sandpaper. There is a significant amount of friction. In this process, you are breaking off rough edges of the wood and detaching the sand from the sand paper. Those actions take effort. You are also moving the small particles that are broken off, which carries energy and momentum from the motion of the sandpaper. If you do this long enough, you will notice that the sandpaper and wood also get warmer. Friction is both doing work and heating. The friction when you vigorously rub your hands together involves much less "breaking off particles" but a similar amount of heating. Dragging an object across gravel or dirt involves rearranging small particles, but less heating.

8.4.1.2 Static Friction

Whereas kinetic friction describes the situation of two surfaces rubbing against each other as they slide past each other, static friction describes the case of two surfaces that *would slide* across each other *if* they weren't stuck. Unlike kinetic friction, this *is* what you intend to happen when you are walking. Your feet should not slip out from under you! Before developing this further, please investigate how objects can be "stuck" to build your intuition.

Investigation 8.4.13 (*Push without moving*)

Consider moving some heavy objects around your home. (A refrigerator, bed, or couch make good examples to try on your own. Although you might also try to push a small car or a heavy desk. If you push something tall, like a refrigerator or a bookshelf then you should careful not to topple the object or spill the contents.)

(a) *Push gently on the object. Notice that it is possible to be pushing on it while it continues to be in equilibrium. Identify all forces acting on the object and the size of each relative to the amount you are pushing.*

Hint. *Are you on the surface of the Earth? Is the object falling through the floor? Are you pushing on it? Why isn't it moving?*

Solution. *There is necessarily a gravitational force. You should be able to deduce the direction of that force.*

Since it is not falling through the floor, there is undoubtedly a normal force. The normal force will balance all downward forces minus any upward forces, which in this case happens to only be the gravitational force. You should be able to deduce the direction of the normal force.

Since you are pushing on it (however gently), there must be an applied force in the direction you are pushing. I will assume it is parallel to the floor.

Since it is not moving, there must be some force opposing the force you are applying. The only other thing in contact with the heavy object you are pushing is the floor, which exerts a frictional force on anything that tries to slide along it.

Since there is no vertical acceleration, the vertical components of the forces have to all balance. Since the applied and frictional forces do not have vertical components, the normal and gravitational forces must balance. Since I assume you are pushing lightly enough that the weight (and hence the normal force) are noticeably larger than your applied force.

Since the object is not accelerating in the horizontal direction, the horizontal components of the forces must also balance. Since the normal and gravitational forces do not have a horizontal component, the applied and frictional forces must balance. This says that the magnitude of the frictional force is the same as the magnitude of your applied force.

(b) *Continue to push gently on the object. Notice that it is possible to gradually increase the magnitude of your applied force while it continues to be in equilibrium. Recalling the last paragraph of the solution of Task a, comment on the value of the frictional force while you adjust the magnitude of your applied force (still keeping to gentle enough not to move the object).*

Solution. *Since the object is not accelerating in the horizontal direction, the horizontal components of the forces must also balance. Since the normal and gravitational forces do not have a horizontal component, the applied and frictional forces must balance regardless of the value of the applied force (so long as it remains static). This says that as you change the magnitude of the applied force, the magnitude of the frictional force* ***must*** *also be changing.*

(c) *Assuming (1) this is something that you can lean into when you push, that (2) it is on tile or linoleum, and that (3) you have slippery socks on, when you push without moving it, what happens to you? Why?*

Hint 1. *Newton's Third Law.*

Hint 2. Recall *Zambert intentionally braces when pushing Carl* and *Diane does not brace herself when pushing Carl.*.

(d) *By experimenting, determine the largest force you can apply without having the object move. Does it matter if you push near the top of the object or near the bottom of the object?*

Solution. *In this chapter, we will assume that it does not matter where you push on the object. However, when you do the experiment, you will likely see that it does matter; you might notice that if you push near the top, the object is likely to tip rather than slide whereas if you push near the bottom it is more likely to slide than to tip. This is likely more noticeable with a heavy object that is very difficult to move. When you do this, you are forming a force couple, which produces a torque.*

Based on Investigation 8.4.13, you should notice the following:

Insight referenced by Example 8.4.28

Insight 8.4.14 (*Can push with any value gentle force*)

Since you can gradually increase the applied force without moving the object, the static friction is able to vary its magnitude in such a way as to balance the applied force so long as the applied force is smaller than the maximum static force. (See also Observation 8.4.16.)

This observation means that Equation (8.4.1) cannot literally be true for the situations involving static friction. Instead, the frictional force can have any smaller value up to some maximum where the object moves and we can describe it with kinetic friction. Therefore we need to distinguish between the kinetic friction and the static friction. Equation (8.4.1) still holds for kinetic friction, but we have to use

$$F_{f,\text{static}} \leq F_{f,\max} = \mu_s F_N \tag{8.4.2}$$

to describe static friction. This describes the maximum value that the force of static friction can take before the object begins to move, while allowing the force of static friction to take any smaller value when the object is pushed more gently than this maximum value.

Example 8.4.15 (*Futility (pushing an immovable object)*)

You decide to rearrange your bedroom. Your bed weighs $F_g = 1111\,\text{N}$. Your floor is hardwood and the coefficient of friction between the bed and the floor is $\mu_s = 0.35$.

1. *Find the maximum static friction that can be created by pushing on the bed.*
2. *How hard do you have to push the bed to get it to move?*
3. *If you push the bed horizontally with an applied force of $F_A = 243\,\text{N}$ then how strong is the frictional force?*
4. *If you push the bed horizontally with an applied force of $F_A = 157\,\text{N}$ then how strong is the frictional force?*

Answer 1. *In order use Equation (8.4.2) to find the maximum frictional force, we first need to comment on the value of the normal force. Using Newton's second law in the vertical direction, we note that there is a downwards gravitational force of $F_g = 1111\,\text{N}$ and an unknown upwards force F_N. These have to add up to a net force that provides no acceleration because the bed is not accelerating in the vertical direction. Since $\vec{F}_g + \vec{F}_N = m\cancelto{0}{\vec{a}}$, the magnitude of the normal force is $F_N = 1111\,\text{N}$.*

We can now find the maximum static-frictional force:

$$\mu_s F_N = (0.35)(1111\,\text{N}) = \mathbf{38}8.85\,\text{N} = 3.9 \times 10^2\,\text{N}$$

(If you push with more force than this, then the bed will begin to move, which we will begin to discuss in Investigation 8.4.18.)

Answer 2. *Based on Answer 1, if you push with any value equal to or smaller than $3.9 \times 10^2\,\text{N}$, the bed will not move. If you push harder than that, then the bed will accelerate across the floor.*

Answer 3. *Since this number is smaller than Answer 1, the bed will not accelerate. Using Newton's second law in the horizontal direction (the horizontal components of F_g and F_N are zero and ignored), we note that there is a forwards applied force of $F_A = 243\,\text{N}$ and an unknown backwards frictional force F_f.*

These have to add up to a net force that provides no acceleration because the bed is not accelerating in the horizontal direction. Since $\vec{F}_A + \vec{F}_f = m\cancelto{0}{\vec{a}}$, *the magnitude of the frictional force is* $F_f = 243\,\mathrm{N}$.

Answer 4. *Since this number is smaller than Answer 1, the bed will not accelerate. Using Newton's second law in the horizontal direction (the horizontal components of* F_g *and* F_N *are zero and ignored), we note that there is a forwards applied force of* $F_A = 157\,\mathrm{N}$ *and an unknown backwards frictional force* F_f. *These have to add up to a net force that provides no acceleration because the bed is not accelerating in the horizontal direction. Since* $\vec{F}_A + \vec{F}_f = m\cancelto{0}{\vec{a}}$, *the magnitude of the frictional force is* $F_f = 157\,\mathrm{N}$.

Observation 8.4.16 ($F_{f,\max}$ is related to the value of μ_s)**.** The application of Insight 8.4.14 to the numbers in Example 8.4.15 indicates that neither $F_f = 243\,\mathrm{N}$ nor $F_f = 157\,\mathrm{N}$ come from using Equation (8.4.2), except insofar as to notice that these values are less than $F_{f,\max} = \mathbf{390}\,\mathrm{N}$. It is inappropriate to use the 243 N or 157 N to compute a value for the coefficient of static friction (or the normal force).

Example 8.4.17 (*Gosh, I'm thirsty (lifting objects uses static friction)*)

You are provided a pint of water[5] *with a weight of* $F_g = 8.78\,\mathrm{N}$. *Assuming the glass is cylindrical (not conical) and the coefficient of friction between your fingers and the glass is* $\mu_s = 0.35$, *compute the normal force needed to lift the glass.*

Hint 1. *Since lifting requires your fingers to not slide along the glass, this is a situation that requires static friction, not kinetic friction. It does not matter that the glass will be accelerating upwards (with your hand); rather, what matters is that your fingers are not moving across the glass.*

Hint 2. *Recall Observation 8.4.16. We can computer the normal force that corresponds to the maximum static friction. If this is larger than the weight, then we have produced enough frictional force to lift the glass. If the maximum frictional force is less than the weight, then this will not lift the glass. So we are actually computing the minimal normal force necessary to just balance the weight, and then any amount of normal force larger than this will lift the glass with acceleration and any normal force smaller than this will allow the glass to slip out from between our fingers.*

Hint 3. *Usually the normal force is in opposition to the gravitational force, but this is not the case here. The normal force is squeezing against the side of the glass. The frictional force is opposing the downwards gravitational force. So, the normal force is horizontal (into the surface, as normal forces always are) and the frictional force is vertical (along the surface, as frictional forces always are). You should be able to draw the free-body diagram.*

Answer. *Since the glass weighs* $F_g = 8.78\,\mathrm{N}$, *we need the frictional force to be at least this large. Assuming you are holding the glass with your thumb and finger, each of them will provide half of the support. So, there is* $F_f = 4.39\,\mathrm{N}$ *on either side. Using Equation (8.4.2),*

$$F_N = \frac{F_f}{\mu} = \frac{4.39\,\mathrm{N}}{0.35} = \mathbf{12.5}4\,\mathrm{N}$$

is necessary by each your finger and your thumb to just barely produce enough friction to support the weight of the glass. Fortunately, the human hand has evolved to have a very good gripping strength[6] *and this should not be a problem.*

Investigation 8.4.18 (*Get it moving*)

Consider moving some heavy objects around your home. A refrigerator, bed, or couch make good examples to try on your own. Unlike Investigation 2, we now want to investigate the boundary between not moving and moving. Where you were previously asked to push gently enough to not move the object, now you should investigate pushing just barely enough to make it move.

(a) *Push gently on the object and increase your pushing until the object moves. (Be careful about where you push as discussed in Task 8.4.13.d.)*

Try to pay attention to and to distinguish how much force it takes to get it moving compared to how *much force it takes to* keep it moving *at a constant velocity.*

This is difficult for two reasons: firstly, most people have spent their life paying attention to how the thing being pushed is moving rather than *how hard they are actually pushing and, second, when there is a large static friction, then the object being pushed tends to lurch forward as it starts moving.*

Hint. *If you are having trouble gauging the force you exert, you might find a small bathroom scale and place it between you and the object you are pushing so that you push against the scale while reading the value that it displays.*

Some electronic scales wait for you to settle into equilibrium while standing on them and then they read that value. This type of scale will not be useful for this experiment. If you can find one, use a mechanical scale ratehr than a digital one.

Answer. *Regardless of your ability to feel the difference, if you do the experiment carefully with a force-transducer (or a precision scale), it is possible to measure the difference between the larger force that it takes to break static friction (get it moving) and the smaller force needed to counter kinetic friction (keep it moving).*

(b) *Another situation where you can see the switch from static friction to kinetic friction as with a textbook*[7] *and something small like a coin*[8]*. If you don't have such a book, you can probably go to a local bookstore with your phone and try this – it should only take a flat surface and about 45 seconds of calm observation.*

Find a flat surface to rest the book on. Close the book. Place the small object near the edge away from the spine of the book. Slowly open the book cover until the object slides across the cover towards the spine.

Try to gauge the motion of the object. Is it moving at constant speed or is it accelerating? It might be difficult to tell, but try to compare the motion when it starts moving (from rest) to the motion at the spine.

Hint. *Note that it starts* from rest. *Recall what happens when you drop something in freefall from rest (Investigation 5.5.1). This object is not in freefall, but the observations you tried previously might help you distinguish if this is accelerating.*

Answer. *The object sliding across the cover is accelerating.*

Solution. *The object sliding across the cover is accelerating. The explanation for this is that there is some amount of force that breaks the static friction (gets it to move). Since amount of force is the component of the gravitational force that is directed along the incline; this will be explained below. This component of gravity is the same all the way down. This is just barely larger than friction as it escapes static friction and starts to move, but must be more than* barely larger *than kinetic friction in order to accelerate enough to notice.*

At this angle (called the Angle of repose), the component of the gravitational force along the cover is very similar to the force of static friction, but is larger than the kinetic friction. Since the gravitational force is not different, then we can conclude that the size of the kinetic friction is smaller than the magnitude of the static friction.

In fact, after you find the angle of repose, you should be able to get the object to slide and then lower the angle slightly and still have the object slide across the cover. This is another indication that you need less force to keep it moving than you do to get it moving.

Observation 8.4.19 (It is more difficult to get things moving than to keep things moving)**.** Although the amount of static friction will change depending on how hard you push so long as the object remains in static equilibrium, if you maintain the force used to initiate motion, then the object will move with acceleration rather than moving with a constant speed after it is moving. The maximum value of static friction is larger than the kinetic friction. Another way to say this is that $\mu_{\text{static}} > \mu_{\text{kinetic}}$. This difference is usually small and is often hard to notice.

Since this is also an empirically-determined statement, it becomes the fourth[9] of the laws of friction:

Another law of friction

The coefficient of static friction is slightly higher the value than the coefficient of kinetic friction.

Clarification 8.4.20 (*getting versus keeping something moving relative to Newton's first law*)

Newton's First Law indicates that objects "want to" maintain their current velocity (stay at rest or maintain their motion) and require a force to change this motion. You should not conflate Newton's first law about changing the state of motion with the need to overcome friction. Friction is a force that describes the way surfaces interact with each other. The inertia is a property that is independent of friction and is true regardless of contact with other surfaces. According to Newton's laws, without friction, it takes the same effort to change the velocity of an object from 0 $^{\mathrm{m}}/_{\mathrm{s}}$ to 2 $^{\mathrm{m}}/_{\mathrm{s}}$ as it does to change from 5 $^{\mathrm{m}}/_{\mathrm{s}}$ to 7 $^{\mathrm{m}}/_{\mathrm{s}}$. Furthermore, friction is only relevant where there are surfaces in contact and we can change the inertia with any force.

Example 8.4.21 (*Get it moving*)

Make an example of breaking through static to get to kinetic friction (note the difference with lifting, which continues as static even though the object is moving because there is no relative motion).

[9] Although if you count Footnote 5 as the fourth, this would be the fifth law of friction.

Example 8.4.22 (*More mouse friction*)

You are moving the computer mouse to direct the arrow on the screen by resting your forearm on the desk and curling your fingers to move the mouse towards you on the desktop. In how many places is friction acting and in what direction?

(This example extends Example 8.4.7 beyond the kinetic friction between the desk and the mouse to consider the static friction in moving the mouse. Connect moving the mouse to lifting an object.)

Hint 1. *There is friction where each surface is in contact.*

Hint 2. *If you had just finished running around the block and your arm were drenched in a coating of sweat, would anything be different about the interaction (ignoring the potentially disgusting nature of being drenched in sweat)?*

Hint 3. *If your arm were not touching the desk, would it be moving? If so, which way?*

Hint 4. *If somebody played a prank on you and glued the mouse to the desk, which way would your fingers be moving across the mouse?*

Answer 1. *The relevant points of contact are:*

1. *Your arm and the desktop (Hint 3) (Answer 2)*
2. *Your fingers and the mouse (Hint 4) (Answer 3)*
3. *The pads/feet attached to the underside of the mouse and the desk (or the mousepad) (See Example 8.4.7)*

Answer 2. *If you are truly using your fingers to move the mouse and not your arm, then there is no friction between your arm and the desk. It is only the normal force supporting the weight of your arm.*

> ***Touchstone (third-law pair)*** *third-law pairs* are the same type *of force acting* on different *objects*

On the other hand[10]*, if you were to drag your arm towards you as you move the mouse, then the friction will oppose the motion. Recall* Insight *7.1.2* and especially *Newton's Third Law, which indicate that there is a frictional force on your arm and another frictional force on the desk. These are "third-law pairs" (recall Insight 7.2.26). With that in mind, the frictional force on your arm by the desk is forwards (opposing the backward motion of your arm) and the frictional force on the desk by your arm is backwards (opposing the forward relative-motion of the desk moving along your arm). (It might help to recall the discussion of relative motion.)*

Answer 3. Given *Hint 4, while your fingers do not actually move across the mouse, they are gripping (which is a* normal force*) and dragging it (using static friction). That is, if the mouse were glued to the* *desk, then your fingers would move towards you (kinetic friction). Since the mouse stays with your fingers and there is no relative motion (static friction), that frictional force is the force moving the mouse.*

8.4.1.3 Applications of Dry Sliding Friction

Since this section is fairly long, you can use these links to jump to the topic of interest:

- Tabulated values of the coefficient of friction
- Complications in measuring the coefficient of friction
- How we walk
- Shifting loads in moving vehicles
- Angle of repose
- Inclined Planes

Tabulated values of the coefficient of friction If you are interested in investigating how widely the coefficient of friction can vary between different surfaces or between static and kinetic friction, then you can find some values of the coefficient of friction at the following websites:

- `http://www.engineershandbook.com/Tables/frictioncoefficients.htm` (referenced Oct, 2017)
- `http://www.physlink.com/Reference/FrictionCoefficients.cfm` (referenced Oct, 2017)
- `https://www.engineersedge.com/coeffients_of_friction.htm` (referenced Oct, 2017)
- `http://www.tribology-abc.com/abc/cof.htm` (referenced Oct, 2017)

Complications in measuring the coefficient of friction To indicate the complications of friction in general and friction due to gripping in particular, this paper [8.10.7] shows that the coefficient of friction between skin and glass vary with the actual pressure applied. A normal force of 5 N produces a coefficient near $\mu_s = 0.5$, but a normal force of 20 N has a coefficient near $\mu = 0.25$. The relationship is not linear.

How we walk There are many examples where friction is our friend. Walking is an example that is easy to overlook since we do that so often. However, the process is very similar to other situations where it is painfully obvious. Consider the online videos of animals (including people) trying to stand up (from an incline position) on ice. Imagine how you get out of a chair if you don't trust your legs to support you. Consider that many sports have cleats as part of their shows to get a better grip. This "grip" is quite explicitly the effect of friction allowing you to move your body in specific ways. During this discussion of walking, I would also like you to be thinking of the cutting-back-and-forth motion that is common in racquetball, tennis, basketball, volleyball, soccer, and football, not to mention the need for leverage (i.e., torque) in hitting a baseball or softball, wrestling, golf, and any event in track and field.

Rather than discussing the initiation of the walking motion, let's consider a person who is already walking. (Ask a friend to walk along a hallway without being self-conscious so you can study them.) We should consider both feet during the walk. Let's ***start with the landing of the front foot***, then consider the shifting of weight from the back foot to the front foot, and end with the lifting of the back foot. Generally people land their foot on their heel. You may have had the experience of having that heel slide forwards out from under you, which is not a pleasant experience. Since that forward-foot would slide forwards without friction, it should be clear (from Newton's Second Law) that the frictional force holding your foot in place must be backwards, against your forward-motion. Having this force on the bottom of your foot stops your foot while allowing your leg and body to continue to move (from Newton's First Law) forwards. Furthermore, while your foot is feeling this backwards force, it is exerting a forwards force on the Earth (as indicated by Newton's Third Law). This may not be obvious unless your forward-foot lands on anything with wheels, a sock (or other clothing) on a hardwood floor, or a stray piece of paper; you should notice that whatever you might step on slides forwards out from your foot indicating that it feels a forwards force from your foot.

While both feet are on the ground and you ***shift your weight from the back lag to the front***, your are doing two things. With your front foot, you are laying the rest of your foot on the ground to support your weight across a larger area. (see the discussion of pressure and recall Amontons' second law of friction.) This foot becomes the pivot point about which your body rotates, sort of like an inverted pendulum (see Subsubsection 8.5.1.2), and as such needs to be stationary. (Contrast this with ice skating in Investigation 4.) It is this recognition that your foot is stationary despite the motion of your body that tells you that static friction, rather than kinetic friction is relevant. What about the back foot? Although your toes are static, your moving leg is lifting your heel off the ground. This ideally prepares your foot for the third motion of walking in that it stretches the muscles that are on the bottom of your feet. Our muscles do their work by contracting and our limbs (hands and feet) have their strongest muscles on the inside of the gripping/curling motion[11].

In the final motion, while you ***lift your back foot***, your calf contracts and pulls your heel up and your toes down. Simultaneously, the muscles on the bottom of your foot contract pulling your toes into the ground and pushing the ground out from under you. That is, your toes are pushing the ground backwards. By Newton's Third Law, the ground is intrinsically also pushing your foot forwards; this is the process that propels you forward. As evidence of this, you might note that people who walk a lot or quickly tend to have larger calves. If you have ever ice skated or used roller-skates or roller-blades, you will notice that the usual walking motion is not very efficient at beginning to move from a standing position.

[11] When I describe the curling of your foot, you should think of the way you move your toes when you are asked to "curl your toes". That is you curl your toes down, not up. Similarly, your calf is stronger than your shin because walking involves curling your foot towards your heel and calf, not up towards your shin.

Investigation 8.4.23 (*The process of walking*)

The following items provide you with some useful perspectives on the aspects human locomotion with the intention of building your intuition about how we interact with friction.

(a) *How your feet land and behave: Ask a friend to walk casually down the hallway while you observe their feet. (It will be easy for them to feel awkward being watched, so you might have to do this long enough for them to relax a bit.) Have them walk quickly for a little while and slowly for a little while to see if anything becomes more obvious or less. Watch only one of their feet (likely the closer one) during the process so that you can follow the motion more easily. See if it matters which shoes they are wearing.*

- **(i)** *When your friend steps forward try to notice how the foot lands (heel) and rolls onto the flat part of the foot. If you are watching the foot (instead of the leg or hip) it will be quite obvious that the foot does not slide along the floor, but you should imagine how the foot would "slosh" forwards if your friend was wearing shoes that were a little too large for his or her feet, squashing into the toes during this first stage. Assuming the person is walking on tile, linoleum, or concrete, what would happen during this motion if a coin were squarely underneath where the person's heel touched the ground?*
- **(ii)** *When your friend shifts her or his weight from the back to the front, try to notice how the body moves past the foot while still looking directly at the foot. This will emphasize the stationary nature of the foot and, after viewing for several steps, you should start to notice when the heel is beginning to lift. Prepare to watch the push-off carefully.*
- **(iii)** *When your friend pushes off their back foot, try to notice how the angle between the foot and the shin increases and, at the same time, how the toes give a spring to the step. The lifting of the heel stretches the spring of the foot by bending the toes towards the shin and the push-off both extends the foot away from the shin and curls the toes back in line with the foot. Assuming the person is walking on tile, linoleum, or concrete, what would happen to a coin that had been on the ground just beneath the ball of the foot as the person stepped forward off of this foot?*

(b) *The difference with **crutches** is that they don't bend as your knee does. You will note that the toe of the crutch is usually rubber in order to increase the friction with the floor. If you have ever needed to walk with crutches, then you will be keenly aware of the stress involved in the being aware of what you are placing the crutch onto. This is especially true if you have inadvertently put the crutch onto a piece of paper while walking. If you have a pair of crutches and can adjust them to your proper height, hobbling on them for the better part of an afternoon will give you a new appreciation for the* *advantages of friction when you step. You will also notice that without crutches your foot is explicitly pushing the ground backwards (so that the ground pushes you forwards) because the crutches do not do that. When you walk with crutches, you have to plan for your momentum to carry you through until your weight is back on your other foot.*

(c) *Wearing a* ***walking boot*** *instead of using crutches provides you with a different experience. These boots touch the ground in wider area than a crutch and so it is much easier to "find purchase" and your attention about where your foot lands is more natural. However, the walking boots generally do not allow you to bend your ankle, which also removes your ability to push off when the boot is your back foot. If you are in this situation, then you should pay attention to your gait. Some people hop off their good foot more than when they walk. Some use their weaker leg in a different way. If you are wearing a walking boot, then try to attend to which muscles are being used in an unusual way (by noticing which ones get tired more quickly than usual).*

(d) *Contrast walking with* ***skating*** *either on ice skates, roller skates, or roller blades. I will consider ice skating, but the other forms have similar characteristics. When skating there is an additional aspect of the process described above. Once you are moving, you place one foot forwards and shift your weight over to that foot, but the shifting of your weight produces a sideways motion that allows you to twist your back foot (like a cow kick[12]) and push off the ice. You can see this in play if you watch speed skating (an olympic sport). The additional aspect of skating is that the skater takes much longer to move their back foot up to become the front foot, making this motion the part that takes the most time. You might also notice that ice skates have a "jagged toe" that allow the front tip to be used to grip the ice while the flat bladed bottom can slide smoothly along the ice.*

Two aspects to notice about ice skating which develop insight for walking (and friction). First, when you step forward on skates, you place your foot flat rather than heel-first because you want to keep gliding and you don't want to stop by driving your heel into the ground. Second, the jagged toe emphasizes that it is the toe that you push off with both in skating and in walking. The friction of pushing the ground backwards is what enables the ground to push you forwards.

(e) *In contrast to walking, if you are limber enough to be able to* ***do a cartwheel****, then you likely will recognize that you push off with you feet hard enough that your hands are merely placeholders as you roll over. The name cartwheel is reminiscent of having your limbs play the roll of spoke on the wheel of a cart, which is a nice segue to the next section on Rolling Friction. If you are comfortable doing a cartwheel, then you should pay attention to the action you use with your hands as you roll over. You might also try to do a roundoff and decide if the way your hands interact with the ground is different. You might note that some people do the roundoff with a push so that they bounce into the air and land on their feet. If you do this a couple of times and imagine that your feet and hands are hitting a patch of slippery ice, you should be able to build your intuition about how static friction helps you walk.*

Shifting loads in moving vehicles If you have ridden in a pick-up truck with heavy objects in the truck-bed, then you might notice when they shift. Driving up a steep incline or suddenly accelerating pulls objects to the back, driving downhill or stopping suddenly pulls objects to the front.

Exploration 8.4.24 (*Abdul and Carl use a pick-up truck to move a couch*)

One drives the pick-up with a couch, truck door down. No straps. The other watches from behind. Choose to measure the friction of the truck-bed. Choose to measure μ_s *and or* μ_k *with the couch. Choose to use straps? and they break? Choose to measure the mass of the couch? When you load it, maybe?*

Answer 1. *Accelerates slowly from the parking lot. friction and inertia (2nd law then 1st law) keeps the couch in place.*

Answer 2. *Accelerates quickly from the parking lot. Couch slides backwards. Choose honk or wait. choose traffic or no traffic. DO OVER: go back to the first choice.*

Answer 3. *Stops suddenly; couch slide back in.*

Answer 4. *Up hill, angle of repose*

Paragraph referenced by Investigation 8.4.18

Angle of repose One practical application of this idea is that if you have a stack of books and you want the one that is second or third from the top, you can lift the book you want at some angle until the books above it slide off and then you can take your book. The angle at which you have to tilt the book in order to get the other to slide off is called the angle of repose:

Definition 8.4.25 (angle of repose)**. Definition referenced by** discussion of Inclined Planes

When a surface is tilted at a gradually increasing angle, the **angle of repose** is the angle at which an object just breaks the static friction and begins to slide down the surface.

Because of Observation 8.4.19, after the object begins to slide, it will accelerate down the ramp, rather than sliding at a constant speed. On the other hand, since the surface (especially if it is unpolished wood) will likely have variable roughness along its length, it is possible that the coefficient of friction is different in different sections of the surface and the object might get stuck at some lower location.

Example 8.4.26 (*Finding the angle of repose*)

You meet a friend at the library to study for your physics class and she places her cell phone on top of your textbook! You remember from class that the coefficient of friction between this particular phone case and the book cover is $\mu_s = .62$. You did not get a chance to weigh her phone, so you will have to look it up on the internet. Compute the angle at which the phone would start to slide across the book cover as you slowly open the book.

Hint 1 (mass). *The mass should cancel from the equations, so you don't actually need to know the value of the mass.*

Hint 2 (normal force). *The normal force is normal to the surface even as the cover changes its orientation.*

Hint 3 (gravitational force). *The gravitational force is always straight towards the center of the Earth.*

Answer.

Answer referenced by *Example 8.4.28, Example 8.4.29*

The angle of repose turns out to be $\theta_r = \tan^{-1}(\mu_s) = \tan^{-1}(0.62) = \mathbf{31}.8\,°$.

Solution. *As the cover is lifted, the normal force is only able to balance the component of the gravitational force that is perpendicular to the book cover $F_N = (mg)\cos\theta$. While the angle is smaller than the angle of repose, the frictional force balances the component of the gravitational force that is parallel to the cover $F_f = (mg)\sin\theta$. It is only when we reach the angle of repose that the frictional force reaches its maximum and we can relate $\mu_s F_N = F_{f,max}$. In this case,*

$$\mu[(mg)\cos\theta_r] = (mg)\sin\theta_r$$

and the mg cancels leaving, $\mu = \frac{\sin\theta_r}{\cos\theta_r} = \tan\theta_r$. Since we are given μ_s, we can find the angle of repose from the inverse tangent (a.k.a. the arc-tangent, or atan):

$$\theta_r = \tan^{-1}(\mu_s) = \tan^{-1}(0.62) = \mathbf{31}.8\,°.$$

You should test this experimentally.

Example 8.4.27 (*Using the angle of repose*)

You meet a friend at the library to study for your physics class and she places her cell phone on top of your textbook! Wanting to read your book, you slowly lift the cover of your book and find that the phone slides off when you get to an angle of 38.1°. (You apparently are a very good judge of angle.) You did not get a chance to weigh her phone, so you will have to look it up on the internet. Compute the coefficient of static friction for the phone.

Hint 1 (mass). *The mass should cancel from the equations, so you don't actually need to know the value of the mass.*

Hint 2 (normal force). *The normal force is normal to the surface even as the cover changes its orientation.*

Hint 3 (gravitational force). *The gravitational force is always straight towards the center of the Earth.*

Answer. *The coefficient of static friction turns out to be* $\mu_s = \tan\theta = \tan(38.1°) = 0.\mathbf{784}1$

Solution. *As the cover is lifted, the normal force is only able to balance the component of the gravitational force that is perpendicular to the book cover* $F_N = (mg)\cos\theta$*. While the angle is smaller than the angle of repose, the frictional force balances the component of the gravitational force that is parallel to the cover* $F_f = (mg)\sin\theta$*. It is only when we reach the angle of repose that the frictional force reaches its maximum and we can relate* $\mu_s F_N = F_{f,\max}$*. In this case,*

$$\mu[(mg)\cos\theta_r] = (mg)\sin\theta_r$$

and the mg *cancels leaving,* $\mu = \frac{\sin\theta_r}{\cos\theta_r} = \tan\theta_r$*. Since we are given* θ_r*, we can find the coefficient of friction directly:*

$$\mu_s = \tan\theta = \tan(38.1°) = 0.\mathbf{784}1\,.$$

Paragraph referenced by Investigation 8.4.18

Inclined Planes In Subsection 8.3.2 we introduced the idea of an inclined plane and focused on the direction of the normal force. In the next couple of examples we will focus on the role of friction when objects are placed on tilted surfaces. It will be helpful to understand what it means to be above or below the angle of repose.

Example 8.4.28 (*Below the angle of repose*)

You are babysitting the neighbors preschool daughter and decide to go to the park. She decide to play on the slide, which is essentially an inclined plane at $25.7°$*. You notice that her pants have a coefficient of static friction of* $\mu_s = 0.60$ *and a coefficient of kinetic friction of* $\mu_k = 0.55$ *on the metal. When you lift her up, you estimate her mass to be* $m = 35.25\,\mathrm{kg}$*. Find the angle of repose and the force of friction while she tries to slide down the slide.*

Hint 1 (angle of repose). *See Answer 8.4.1.21.1.*

Hint 2 (friction). *If the angle is larger than the angle of repose, then we should use kinetic friction; recall Equation (8.4.1). If the angle is smaller than the angle of repose, then we should use static friction; recall* Insight *8.4.14* and *Equation (8.4.2).*

Answer 1 (angle of repose). *The angle of repose can be found from the coefficient of static friction:*

$$\theta = \tan^{-1}(\mu_s) = \tan^{-1}(0.60) = \mathbf{30}.96\,°$$

Since this is larger than the angle of the slide, the girl will not slide and we need to consider static friction*. (This is a pretty lame slide if it is so shallow.)*

Answer 2 (friction). *Based on Answer 1, we will find the static friction. Based on Insight 8.4.14, it will not help to use Equation (8.4.2) because the coefficient of static friction only tells us the maximum static friction and we are below the angle of repose. Since the component of the gravitational force along the slide is down the slide, the frictional force must be upwards along the slide. So, rather than trying to find the normal force, we only need to find the component of the gravitational force that needs to be balanced.*

the component of the gravitational force down the incline is

$$F_g = mg\sin\theta = (35.25\,\mathrm{kg})(9.81\,\mathrm{m/s^2})\sin(25.7°) = \mathbf{149}.96\,\mathrm{N}$$

Therefore, the static frictional force only needs to be $F_{fs} = \mathbf{150}\,\mathrm{N}$*.*

Solution (all forces). *Because the angle of the slide is below the angle of repose, the static friction will be able to hold the child in place. Since the child is not moving, she is not accelerating, and so is in equilibrium.*

	x-comp	*y-comp*
F_g	$mg\sin\theta = -\mathbf{150}.0\,\mathrm{N}$	$mg\cos\theta = +\mathbf{311}.6\,\mathrm{N}$
F_N	$0\,\mathrm{N}$	$F_{N,y}$
F_f	$F_{f,x}$	$0\,\mathrm{N}$
F_{net}	$0\,\mathrm{N}$	$0\,\mathrm{N}$

This implies that the normal force is $F_N = 312\,\mathrm{N}$ *and the frictional force is* $F_f = 150\,\mathrm{N}$. *It is also possible to compute the maximum value of the force of static friction:*

$$F_{f,max} = \mu_s F_N = (0.60)(\mathbf{311}.6\,\mathrm{N}) = \mathbf{187}\,\mathrm{N}$$

which is, in fact, larger than the actual frictional force.

Example 8.4.29 (*Above the angle of repose*)

You are babysitting the neighbors preschool daughter and decide to go to the park. She decide to play on the slide, which is essentially an inclined plane at 53.7°. You notice that her pants have a coefficient of static friction of $\mu_s = 0.60$ and a coefficient of kinetic friction of $\mu_k = 0.55$ on the metal. When you lift her up, you estimate her mass to be $m = 35.25\,\text{kg}$. Find the angle of repose and the force of friction while she tries to slide down the slide.

Hint 1 (angle of repose). *See Answer 8.4.1.21.1.*

Hint 2 (friction). *If the angle is larger than the angle of repose, then we should use kinetic friction; recall Equation (8.4.1). If the angle is smaller than the angle of repose, then we should use static friction; recall Insight 8.4.14 and Equation (8.4.2).*

Answer 1 (angle of repose). *The angle of repose can be found from the coefficient of static friction:*

$$\theta = \tan^{-1}(\mu_s) = \tan^{-1}(0.60) = \mathbf{30}.96\,^\circ$$

Since this is larger than the angle of the slide, the girl will certainly slide and we need to consider kinetic *friction. (This is a pretty awesome slide.)*

Answer 2 (friction). *Based on Answer 1, we will find the kinetic friction from Equation (8.4.1). To do this, we need the normal force, which we can do as is done in Example 8.3.14:*

$$F_N = F_{gy} = mg\cos\theta = (35.25\,\text{kg})(9.81\,{}^{\text{m}}\!/\!_{\text{s}^2})\cos(53.7°) = \mathbf{204}.7\,\text{N}$$

Then, the frictional force is

$$F_{fk} = \mu_k F_N = (0.55)(\mathbf{204}.7\,\text{N}) = \mathbf{11}2.6\,\text{N}.$$

Notice that since the coefficient is only known to two significant figures, we only know this value to two significant figures.

Solution (all forces). *Because the angle of the slide is above the angle of repose, the kinetic friction will not be able to hold the child in place. This means that the child is accelerating, and so is not in equilibrium.*

	x-comp	*y*-comp
F_g	$mg\sin\theta = -\mathbf{278}.7\,\text{N}$	$mg\cos\theta = +\mathbf{204}.7\,\text{N}$
F_N	$0\,\text{N}$	$F_{N,y}$
F_f	$F_{f,x}$	$0\,\text{N}$
F_{net}	ma	$0\,\text{N}$

We can compute the normal force is $F_N = 205\,\text{N}$ and the frictional force is $F_f = \mu_k F_N = 1.1\times10^2\,\text{N}$. With this information, we can find the acceleration:

$$\begin{aligned} ma &= F_{gx} + \cancelto{0}{F_{Nx}} + F_{fx} \\ ma &= (\mathbf{278}.7\,\text{N}) + (0\,\text{N}) + (-\mathbf{11}2.6\,\text{N}) \\ a &= \frac{+\mathbf{166}\,\text{N}}{35.25\,\text{kg}} && = +\mathbf{4.7}1\,{}^{\text{m}}\!/\!_{\text{s}^2} \end{aligned}$$

You should notice that the component of the gravitational force down the incline is larger than the frictional force up the incline, so the child does accelerate down the incline.

8.4.2 Rolling Friction

Student Outcomes

1. To recognize examples of (and be able to give new examples of) situations in which rolling friction (in contrast to dry-sliding friction) needs to be included
2. To connect the existence of rolling friction to specific effects in the motion and on the energy

Comparing and contrasting kinetic friction, static friction, and rolling friction is often difficult because the motion of the object is so different. Static and kinetic are likely pretty intuitive in that (1) the object is either moving or it isn't and (2) the objects are (or are trying to) slide against each other. Rolling friction, on the other hand, looks rather different. The surfaces are in contact, but it is not always obvious if they are sliding against each other. If you do an internet search, you will find that some authors group rolling friction in with kinetic (or dynamic) friction because the objects (I will say the center-of-mass of the objects) have some relative motion and you will find other authors who indicate that it should be compared to static friction because there is no relative motion between the surfaces of the objects.

Clarification referenced by the discussion of $\vec{v} = \omega \times \vec{r}$

Clarification 8.4.30 (*Rolling without slipping*)

There are three categories of motion we will consider in the discussion around friction.

1. *Objects that are sliding (and not rolling) will experience kinetic friction (unless they are on a frictionless surface, of course).*
2. *Objects that are "rolling without slipping" will experience rolling friction and not kinetic friction. This case assumes that there is sufficient static friction to keep the object rolling rather than sliding, but the static friction is not* interfering *with the motion, rather it is the process that* allows *the rotation.*
3. *Objects that are both rolling and sliding (think of skidding out in your car). While this text will acknowledge the existence of such motion, the mathematical description of such is beyond the scope of this course.*

Some of this will be easier to describe after we have the language of rotations (see Section 11.1) but for now suffice it to say that a rolling object, such as a bicycle wheel, has an axis-of-rotation (the wheel's axle) which *is moving* relative to the ground and, if it is not slipping, has a surface in contact with the ground that *is not moving* relative to the ground. The wheel surface is placed on the ground (not dragged along the ground) in much the same way as your hands are placed on the ground when you do a cartwheel. You can build some intuition by considering Investigation 8.4.31.

Investigation 8.4.31 (*Seeing rolling friction in rolling tires*)

Consider the spare tire from an automobile. You can lay it down and try to slide it across the ground, or you can stand it up and roll it across a large flat parking lot (which hopefully does not have a lot of cars or other obstacles).

(a) *Lay the tire down on its side and consider pushing it along the ground.*

(i) *While it lays on its side, push it gently, enough to be pushing, but not so much that it moves.*

Answer. *You should be able to gauge the amount of static friction between rubber and pave-*

ment. The coefficient of static friction for rubber on pavement ranges from 0.7 to 1.0 depending on the type of pavement.

(ii) *While it lays on its side, push it hard enough to move the tire across the ground.*

Answer. *You should be able to gauge the amount of kinetic friction between rubber and pavement. The coefficient of kinetic friction for rubber on pavement is smaller than the coefficient of static friction, ranging from 0.5 to 0.8 depending on the type of pavement.*

(b) *Stand the tire up on end and consider rolling it along the ground.*

(i) *Ensure the tire is fully inflated and it roll across the ground. If you have a large enough space that is safe from traffic and remains flat, see how far it will go on its own. (This will be easier with a truck tire which has a flatter profile.)*

Answer. *The tire is likely to roll for a long distance. It has very little rolling friction, but it does eventually does slow down; therefore there is some friction. Since the tire is not sliding, there is no kinetic friction.*

(ii) *Now deflate the tire to the point where you can see that it is "low" but not totally flat and it roll across the ground at approximately the same speed as in Task b.i. Compare the distance it travels.*

Answer. *While this flattish tire will roll far enough that you can see it has less friction than in the dry sliding case, it likely will not travel as far as the tire that is fully inflated. The malleability of the softer tire increases the rolling resistance, giving it a higher coefficient of rolling friction.*

(c) *Now consider a car with all four wheels firmly attached but placed in neutral so that the wheels can spin without engaging the engine. When you do this, a person should be in the car keeping it in a straight line and ready to stop if something gets in the path of the car.*

(i) *Try to push the car (in neutral) so that the tires spin at the same rate as in Task b.i and see how quickly the car slows down.*

Answer. *Even if the tires are rolling at the same rate as in Task b.i, the tires are more deformed than in Task b.i due to the weight of the car, so they will not roll as far. This indicates an increase in the amount of rolling resistance.*

(ii) *Deflate the tire to where you can see that it is "low" but not totally flat and try to push the car at the same speed as in Task c.i.*

Answer. *Even if the tires are rolling at the same rate as in Task c.i, the tires are even more deformed so they will not roll as far. This indicates a further increase in the amount of rolling resistance.*

Practical Consequence: *It is useful to keep your car tires inflated to the appropriate level for the following reasons:*

- *Under-inflated tires are more easily deformed causing*
  - *increased rolling resistance, which requires more effort by the engine and reduces you gas milage,*
  - *more sideways-wiggle, which can cause reduced control, affect alignment, and create a "shimmy",*
  - *wear on portions of the tire that were not designed to be in contact with the road (increasing the risk of creating a crack and a slow leak), and*
  - *uneven heating from the repeated deforming (increasing the risk of a blow-out).*
- *Over-inflated tires have less surface area in contact with the pavement and,*
  - *while kinetic friction is independent of the surface area in contact, the goal is to use static friction with rolling tires, which is larger than kinetic friction with skidding tires (see also ABS brakes),*

- *having less surface area makes it easier to skip along the road and lose contact altogether (frictionless and no control), even if only for brief instants, and*
- *depending on the tread of your tires, reduced surface area increases the opportunity for hydroplaning on wet surfaces.*

To be clear, the risks are small if you are close to the standard pressure and don't drive much. The further you are from the standard pressure and the more you drive, the bigger the risk. While you can find good information about tire inflation with a quick internet search, in October of 2017, `https://www.discounttiredirect.com/learn/tire-pressure` *had some good images of tires and the inappropriate wear patterns.*

We should also describe some additional effects that can accompany what you just observed in the Investigation.

- A tire without a car feels static friction to allow it to roll, but not kinetic friction (assuming it is not sliding along the ground).
- A tire that is connected to a car necessarily has a connection (at the axle) to the car, which is not rotating. This means that there is necessarily some kinetic friction at that connection, although this is lubricated friction, not dry-sliding friction. This connection is typically done with ball bearings, which means that there is additional friction in the case of the tire-and-car that is not present for the tire-alone. Ball-bearings should also have rolling friction rather than kinetic friction and should be designed to minimize the frictional effects.
- When a car is moving, there is also the effect of air resistance, which increases with speed. Your results from this Investigation will change if you do them at walking speeds versus at highway speeds.
- If you also compare the tasks of this Investigation to something that is not firmly connected (compare for example a hard-boiled egg to a raw egg), then you will have the added complication of the way the spinning shell interacts with in the liquid inside which also has its own fluid friction.

Observation 8.4.32 (Properties that affect rolling friction)**.** Rolling friction is a very small effect (as compared to dry-sliding friction), but is impacted by

- the softness of the rolling object (due to its deformation),
- the softness of the surface on which the object rolls (due to its deformation), and
- the motion beneath the surface due to these deformations

Although we can still write $F_f = \mu_r F_g$, this coefficient of rolling friction (a.k.a. rolling resistance) is significantly smaller (on the order of 0.001) and depends on[13] the size (radius) of the rolling object.

At this point, you should be able to imagine a variety of applications where you expect to see rolling friction rather than kinetic friction. Please answer Exercise 27 to test yourself.

Exercise 8.4.33

make an example

Note that desk chairs that tilt are essentially rocking chairs where the rocking mechanism is hidden inside the piece that connects the seat of the chair to the support post.

Investigation 8.4.34 (*Seeing rolling friction in rocking chairs*)

If you don't have access to a rocking chair, then you might be able to find an office chair that allows you to lean back. The problem with the office chair is that the mechanism that is like a rocking chair is hidden inside the shaft of the chair support. This is very similar to the mechanism in your knee, but you probably don't want to open that up either. If you are on a college campus, then you might be able to find an anatomy professor in the biology department. If you school has a program in physical therapy, then they might have a really nice model that they can show you.

[13] An extra complication is that the weight of the object impacts the deformation of the object and the surface, so μ also depends on the weight, meaning that the equation depends on F_g in multiple ways.

(a) *Look at the design of the rocking chair. Is the rocking circular? elliptical? something else?*

Answer. *(looks more like a hyperbola to me, but I doubt it is a specific shape; maybe I should get some photos and measure? Nah.)*

(b) *Prop your feet on the legs of the rocking chair. These are built for your center-of-mass to be similar to a natural sitting position, so don't bend yourself too much. Lean back and start the chair rocking. How many times does it go back-and-forth before it stops? What if you try to rock faster? slower?*

Answer. *(do the experiment. There is friction here. Static friction keeps it from scooching. The rolling friction dissipates the energy and slows the rocking motion.)*

(c) *If you drive it faster or give it a large amplitude, then they tend to scoot. Which direction does it scoot? Which part of the period are you in when it scoots (going back or going forth)? Why does it scoot the direction that it does?*

Answer. *(the scooching indicates the [static] friction which would otherwise hold it in place!)*

(d) *Does the surface on which you rock make a difference to the previous questions? why?*

Answer. *(this changes the friction in both cases)*

(e) *Why does a rocking chair stop going each way and turn around and go back the other way?*

Answer. *(not an issue of rolling friction. rather this is the conservation of energy [rolling friction is non-conservation of energy])*

Investigation 8.4.35 (*Rolling pool balls and friction*)

Diane is relaxing with the local physics club, playing pool. She hits the cue ball and counts the number of walls she can hit in one shot.

(a) *Hit the cue-ball off of a bumper in the manner intended for testing cushions. Compare the strength of the hit to the distance travelled.*

How much is the total distance affected by the number of bumpers hit? (Answer 1) Does it matter if you shoot along the length of the table versus the width of the table? (Answer 2) Why does friction slow the ball down instead of just make it turn $v = \omega r$ (no slip) (Answer 3)

Answer 1. *[Answer]*

Answer 2. *[Answer]*

Answer 3. *[Answer]*

Billiard tables have a lot of interesting physics, which can help us see a wide variety of physics, for example: normal force, elastic versus inelastic collisions, rotational motion, and angular momentum.

Return to: pool

8.4.3 Additional Detail: The field of Tribology

Etymology (tribology) See tribology.

Tribology, the study of friction from rubbing, is a field of engineering. There was a recent article

- Original
- Cambridge, likely best to use
    - Cambridge PDF

  - PDF general article on relevance of modern tribology

- Useful reference
- phys.org

that discusses the recent (Aug 2016) discovery of the notebook of Leonardo da Vinci which show a detailed study of friction. While it was known that he studied friction, this shows sketches of experiments that we might do today.

This article introduces historical figures, including Guillaume Amontons and Charles-Augustin de Coulomb (the same Coulomb from Coulomb's Law), as well as the three laws of friction:

1. The force of friction is directly proportional to the applied load. (Amontons 1st Law)
2. The force of friction is independent of the apparent area of contact. (Amontons 2nd Law)
3. Kinetic friction is independent of the sliding velocity. (Coulomb's Law)

Although, other sources indicate that there are five laws.

1. When a body is moving, the friction is directly proportional to normal force and frictional force direction is perpendicular to the normal force.
2. Friction doesn't depend on the area of contact so long as there is an area of contact.
3. The coefficient of static friction is slightly higher the value than the coefficient of kinetic friction.
4. Kinetic friction is independent of the velocity of the body.
5. Friction depends on the type of the surfaces in contact.

[8.10.3] indicates that tribology was born in 1933; but it the history of friction is much longer than this. [8.10.6] starts with Leonardo da Vinci[14] between 1452-1519, although XREF-ABOVE indicates those notes were temporarily lost to history. Amontons (ADD XREFS-ABOVE) in 1699 and contemporaries, Robert Hooke (of Hooke's Law fame) and Liebnitz who contributed to the studies of rolling friction. Hooke with surface yielding and adhesion; Liebnitz with distinguishing static and rolling friction. Leonard Euler distinguished static and kinetic friction in the mid 1700's with the angle of repose. Coulomb, among others, contributed in the late 1700's (Coulomb specifically started in 1779). [8.10.4] indicates that Coulomb studied what is now called the adhesion contribution and recognized that static friction increases with the length of time an object sits at rest. Although [8.10.3] indicates that Arthur Morin distinguished static friction from rolling friction in 1833, [8.10.4] indicates that Morin and Dupuit[15] had a public argument in 1841-1842 about whether Morin's expectation that rolling friction depended on the inverse of the wheel-radius (consistent with earlier work by Coulomb) or Dupuit's expectation that rolling friction depended on the inverse of the square root of the wheel-radius. [8.10.4] also indicates that both are mostly correct, but under different conditions. [8.10.5] indicates that this dispute was described by Tabor in 1962[16].

Cool current friction research List some here.

- `https://www.sciencedaily.com/terms/friction.htm` lists some current research articles.
- `https://phys.org/news/2010-10-friction-fundamental-physics-law.html`
- `http://physicsworld.com/cws/article/news/2012/oct/18/negative-friction-surprises-researchers`
- `https://www.engr.wisc.edu/in-static-friction-chemistry-is-key-to-stronger-bonds/` is a press release version of `https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.109.186102`

I should also find cool videos, maybe?

[14] add xref to above

[15] I can't find information about who Dupuit is.

[16] Google-documents did not provide that reference page, but I did find `https://www.smf.phy.cam.ac.uk/files/Taborpublications.pdf` which lists: Tabor, D. "Introductory remarks", in "Rolling Contact Phenomena", pp. 1-5, publ. Amsterdam, 1962, Elsevier. as the only 1962 publication of Tabor.

Gecko-like grip News from May 2015: https://news.stanford.edu/2015/05/27/grabber-robot-gecko-052715/ about a development at Stanford University.

8.4.4 Air Resistance

NOTE: [8.10.3] indicates that Reynolds introduced the equation of viscous fluid flow in 1866 (but referenced Reynolds 1886).

8.5 Tension

Section referenced by Discussion of $F = ma$

Insight 8.5.1

*Where the normal force is appropriate for pushing against surfaces, **tension** is the pulling force that is transmitted through materials such as cable, chain, or rope.*

Tension is closely related to the compression force experienced by support beams. One can simplistically think of tension as pulling and compression as pushing on the intermediate object that transmits force between the objects at either end.[1] When engineers design the skeleton of bridges and buildings, one of the primary considerations is the tension and compression of the steel beams. You can build your intuition by considering the Investigation 8.5.2.

[1] It doesn't usually make sense to talk about the compression of a rope or chain.

Investigation 8.5.2 (*Pull my finger.*)

We talk about tension and stress in our daily lives. This is an analogy to the physical version of tension, stress, and strain. While stress and strain come from the concept of tightening, tension comes from the concept of stretching.

Etymology (tension, stress, strain) *(See text.)*

(a) *Sit on a swing. Notice the tightness of the support ropes/chains. How tight are the supports when the swing is empty? When a small child is in the swing? When a full-sized adult is in the swing?*

Answer. *To make this comparison, let's consider a swing that is supported by chains. If you are sitting in the swing and take hold of the chains at about shoulder height, you should be able to shake them in (towards your chest) and out (away from you, towards your neighbor swings). You can do this same motion while standing next to the swing. If you do this when the swing is empty, it is very easy to do this. If you ask a series of successively larger people to sit in the swing, you will notice that it gets progressively more difficult to extend them very far. The chains are increasing in tension; they are pulled more taut. Your ability to move the chain in this way is exactly analogous to the way a bow draws across a violin or the way your fingers pluck a guitar, as described in Subsection 18.1.1.*

(b) *Install a fan or light fixture that hangs from the ceiling. You don't want the fan to be supported by the electrical wires, but rather by the metal shaft. How is the fan supported?*

Answer 1. *You might also consider Answer 2, which discusses the case of hanging a light fixture from the ceiling. If you have ever installed a fan in your house, then you will notice that you have to support the fan while the wires are connected. Usually the fan has a shaft that connects to the ceiling at one end and the fan at the other and provides a mechanism for supporting the fan while you manage the wires, which pass through the shaft. Since the fan houses the motor, it is usually reasonably heavy. The nice property of use a metal shaft to support the fan is that it doesn't stretch or wiggle like a chain might. The difficulty in this example is that it is more difficulty to notice the tension in the shaft. If you are the person hanging the fan, then one thing you might be able to notice is that if you flick the metal with a finger when it is not supporting anything, it will have a slightly different "ting" than when it is supporting the fan.*

Answer 2. *If you have ever installed a chandelier in your house, then you will notice that the light has to be supported between the joists of the ceiling. There will be an electrical box with a screw to* *which you will attach the support for the chain that holds the chandelier. The wires will run through the support chain. The heavier the chandelier, the tauter the chain, much as described in Answer 1. This tension is much easier to see than the tension in the shaft of the fan.*

(c) *Pull on a doorknob. Imagine replacing the knobs (inside and outside) with large knots on a rope that runs through the hole the doorknob used to occupy. What if the doorknob were replaced with a rope, knotted on either side of the door?*

Answer. *[Answer]*

(d) *Take a dog for a walk on a leash. Try to pay attention to Newton's second and third law when the dog changes its level of enthusiasm for pulling on the leash. If the dog pulls very hard on the leash and you balance that force without allowing the dog to move away from you, then describe the way the force connects you to the dog.*

Answer. *[Answer]*

Return to: Section 8.5

When considering the tension in the rope, the context is generally that the rope is connecting two objects that are trying to pull on each other. It is convenient to recognize that each object only "sees" the rope, not the object at the far side. This can be seen in a couple of contexts.

We will start with the simplistic approximation of ropes that only transmit the force. As your understanding improves, we will add some examples where the tension in the rope also affects the rope itself. In that more complicated situation, the tension will change across the rope and the rope may stretch. Since ropes and cables are twisted strands while chains are links, ropes and cables can also introduce a torsion that tends not to occur in chains.

Foreshadow ...

8.5.1 Tension as a Support Force

Ropes and chains (and beams) can use tension to support (from above) dangling objects.

Example 8.5.3 (*Tension supports hanging objects*)

Diane hangs her purse ($m = 1.36\,\text{kg}$) on a hook. How much tension is in the shoulder strap to keep it from falling?

Hint 1. *Since the strap supports the purse, what does the strap support against?*

Hint 2. *If we assume the purse is hanging peacefully, then it is not accelerating. Is there an equation that relates forces to accelerations?*

Solution. *The strap connects the hook to the purse. We can consider the interaction between the hook and the strap or between the purse and the strap. We will consider the latter since we don't know anything about the hook.*

Considering the forces on the purse, we know that there is a downwards gravitational force of $F_g = (1.36\,\text{kg})(9.81\,\text{m/s}^2) = \mathbf{13.3}4\,\text{N}$ *and that the net force must zero (because the purse is not accelerating). So, the strap must provide an upwards (tension) force.*

$$\begin{aligned} \vec{F}_T + \vec{F}_g &= m\cancelto{0}{\vec{a}} \\ \vec{F}_T + (-\mathbf{13.3}4\,\text{N}\hat{\jmath}) &= 0\,\text{N} \\ \vec{F}_T &= +\mathbf{13.3}4\,\text{N}\hat{\jmath} \end{aligned}$$

This is the upwards force that the strap applies to the purse.

Note: *The tension strap is doing two jobs: It is pulling up on the purse (as indicated above)* and *it is pulling down on the hook.*

The important thing to take away from Example 8.5.3 is not that we can compute the value (although that is, of course, a useful skill), but rather that

Insight 8.5.4

the tension is conveying the force between the two objects.

In the same way that the normal force on a scale does not measure your weight, but rather the amount you press into the scale, the tension passes force on to the attached object. The hook doesn't feel the weight of the purse, but does feel the tension required to support the purse.

In an upcoming section, we will consider what happens when multiple masses are hung from the rope.

8.5.1.1 How Physicists Use the Words (Vocabulary)

You can probably think of several examples of objects dangling: a purse on a hook, a flag on a pole, a shop sign attached to a post, a pendulum,a swing set, etc. Since these are all similar in some ways (although different in other ways), *we can treat all of them as a mass at the end of a rope*. Typically, because we do not want to deal with the complications that come from sagging supports, we will use the approximation of an "*immovable support*." This will be indicated by hashing the surface.

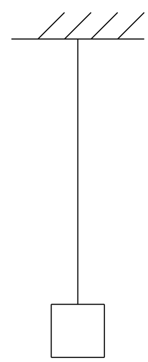


Figure 8.5.5: [Add caption]

8.5.1.2 Application to a Pendulum

The interplay of gravity and tension at various angles.

Paragraph referenced by How we walk

The discussion of walking refers to an inverted pendulum and uses the term "pivot point of an inverted pendulum".

8.5.2 Tension as Dragging Force

We can also consider the tension in a rope used to drag an object across the floor. You may recall that in Example 7.2.27 (and the updated version, Example 8.3.2) Beth pulled a box across a sheet of ice. It is possible that Beth was grabbing the object itself, but it is more likely that she was pulling on a rope that was attached to the object. In that case, the tension in the rope was 4.0 N. This tension is what pulled Beth leftwards *and* what pulled the object rightwards.

We can further update this by considering the case where Beth pulls the rope up at an angle. In that case, some of the tension is used to drag the box and some is used to reduce the normal force. In Example 8.5.6, we will have Abdul continue to push with 5.0 N horizontally and have Beth pull with 4.0 N at a 14° angle above the horizontal. You should note that since the tension on the object is pulling up, helping the normal force, this allows the normal force on the object (what a scale would read) to be a little smaller. You should also note that since the tension on Beth is pulling down, counter-acting the normal force, this requires the normal force on Beth (what a scale would read) to be a little larger.

Example 8.5.6 (*People pushing a box at an angle*)

Again, we can start by drawing a picture of the situation. The description is the same as it was for People pushing a box also feel the gravitational and the normal forces except that Beth pulls at a slight angle upwards. We will again need the gravitational force for Abdul (Example 8.1.6) and Beth (Example 8.1.9). As before, since nothing is accelerating up or down together, there must also be a normal force on each body.

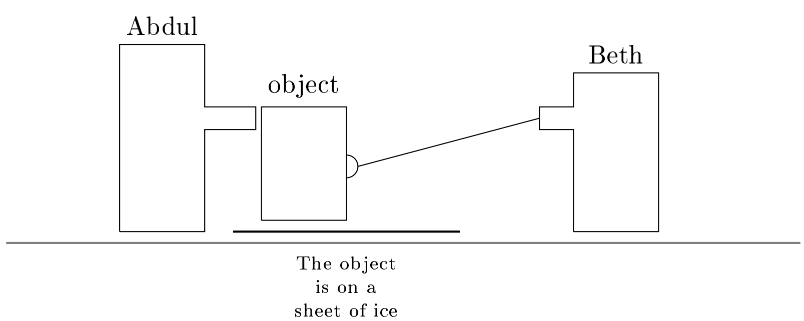


Figure 8.5.7: Abdul and Beth push on a box. Beth pulls at an angle.

Now, as in Example 8.3.2, we will draw a free-body diagram for each individual separately. However, this time we will put the tension of the rope at the appropriate angle. We will need to do a small calculation to find the value of the normal forces.

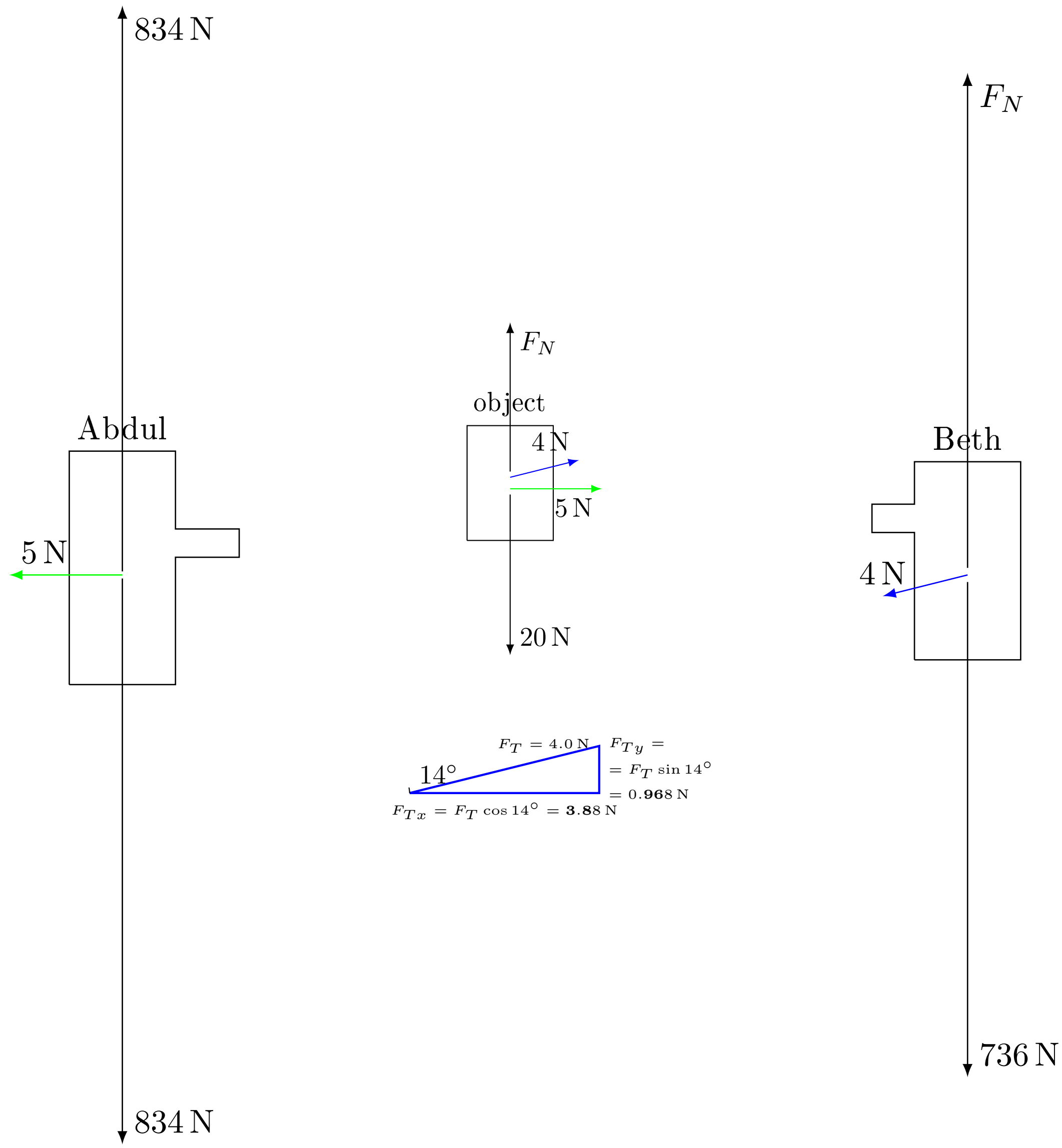


Figure 8.5.8: The forces on Abdul have not changed. ... The forces on the object *have* changed. ... The forces on Beth *have* changed.

For the object*: Since the y-component of the net force is zero, we can find the normal force to be* $F_N = -[(-20\,\mathrm{N}) + (+0.\mathbf{96}8\,\mathrm{N})] = 19\,\mathrm{N}$. *The x-component of the net force is* $F_{\mathrm{net},x} = (5.0\,\mathrm{N}) + (\mathbf{3}.\mathbf{8}8\,\mathrm{N}) = \mathbf{8}.\mathbf{8}8\,\mathrm{N}$.

For Beth*: Since the y-component of the net force is zero, we can find the normal force to be* $F_N = -[(-736\,\mathrm{N}) + (-0.\mathbf{96}8\,\mathrm{N}) = 737\,\mathrm{N}$. *The x-component of the net force is* $F_{\mathrm{net},x} = (-\mathbf{3}.\mathbf{8}8\,\mathrm{N})$.

Return to: rope-tension

8.5.3 Pulleys

While the flexibility of ropes makes them inconvenient for pushing, their flexibility makes them *very useful* for changing the direction of the pull. The mechanism for changing the direction is the pulley. Furthermore, by allowing us to change the direction of the pull, we are also able to double, triple, or further improve the strength of the pull. The term for this is **the mechanical advantage** of a pulley-system.

First we will consider three simple cases of redirecting the force. In each of these cases, I will assume that the pulley and rope have no mass and that there is no friction in the turning of the pulley (assume it is trivially easy to spin). If we do not make this assumption, then the problem gets significantly more complicated.

Example 8.5.9 (*The tension in a rope over a pulley*)

Abdul decides to hold a box that weighs 20 N using a pulley-system. What is the tension in the rope?

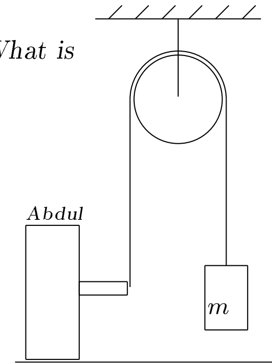


Figure 8.5.10: Tension in a rope hanging over a pulley

Solution. *Since the mass is in equilibrium, the net force is zero and the tension must balance the weight. This tells us that the tension in the rope is* 20 N*.*

If the pulley were difficult to turn (had friction) that stickiness could help support the mass and the tension on Abdul's side might be less than 20 N*;*[2] *but since we assumed the pulley to be frictionless, Abdul must provide the full* 20 N *of tension to the rope.*

The interesting aspect is that Abdul must pull *down* in order to produce the *upward* tension on the box. This means that both Abdul and the mass are pulling down. Since the rope is draped over the pulley, the pulley feels 40 N downwards, 20 N from the tension supporting the mass and 20 N from Abdul who is creating the tension that supports the mass. This means that the second rope that is connecting the pulley to the ceiling must be supporting the full 40 N in order to keep the pulley in equilibrium.

8.5.4 Interesting Complications

8.5.4.1 What is the net force on the rope itself?

The answer to this depends on how complicated you want the answer to be (recall the discussion about effective theories in Section 4.4). Some reasonable answers are:

- If the rope is static (whether massive or massless), then the net force on the rope must be zero even while it maintains the tension.
- If the rope is accelerating (and massive), the net force on the rope while it transfers the forces between the objects at each end is whatever is necessary to produce the acceleration $\vec{F}_{\text{net}} = m_{\text{rope}}\vec{a}_{\text{rope}}$. In English, you have to drag the rope as well as the sled to which it is attached.
- If we assume that the mass of the rope is small enough (**insignificant**) then whether it is in equilibrium or accelerating, it does not require a net force and it merely passes its tension through to the object at the other end.

On the other hand, if the rope has mass, then the answer is a different interesting complication, which you can read about in Subsubsection 8.5.4.3. In this case, the tension through the rope changes from one end to the other unless it is not supporting its own weight against gravity (which, unlike other forces, pulls on each portion of the rope) and is static (has no net force).

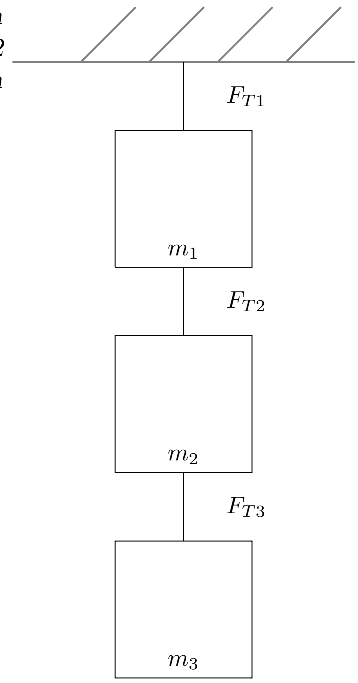
8.5.4.2 Multiple Masses

Referenced by Subsection 8.5.1

Now that we have a few examples of tension under our belts, we can consider some more interesting examples. Example 10 considers the case of hanging multiple masses, which extends the ideas of Subsection 1.

Example 8.5.11 (*Tension between masses hung in a chain*)

While preparing to hang some ornament on a tree, you chain them from a hook on the wall. You hang ornament 3 ($m_3 = 30\,\mathrm{g}$) below ornament 2 ($m_2 = 20\,\mathrm{g}$), which is below ornament 1 ($m_1 = 10\,\mathrm{g}$). What is the tension in each subsequent string?



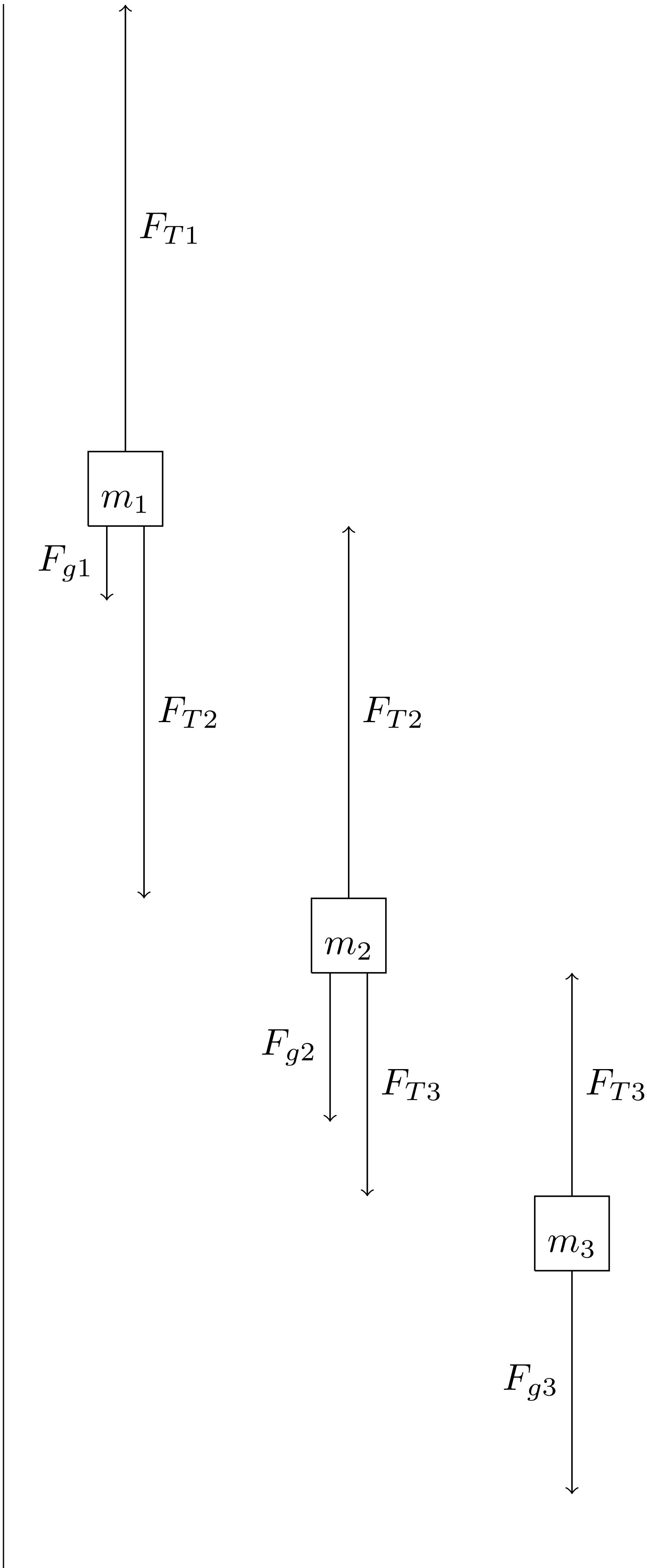
Hint 1. *The tension in the bottom rope only supports the mass beneath it.*

Hint 2. *The tension in the upper ropes supports the mass beneath it* and *whatever it has to carry.*

Solution. *The first thing we should do is notice what information is given to us and make sure that everything is in consistent units. I will convert everything to SI units and find the weight of each mass:*

$$
\begin{aligned}
F_{g1} = m_1 g &= (0.010\,\mathrm{kg})(9.81\,{}^{\mathrm{m}}\!/\!_{\mathrm{s}^2}) = 0.0\mathbf{981}\,\mathrm{N} \\
F_{g2} = m_2 g &= (0.020\,\mathrm{kg})(9.81\,{}^{\mathrm{m}}\!/\!_{\mathrm{s}^2}) = 0.\mathbf{196}\,\mathrm{N} \\
F_{g3} = m_3 g &= (0.030\,\mathrm{kg})(9.81\,{}^{\mathrm{m}}\!/\!_{\mathrm{s}^2}) = 0.\mathbf{294}\,\mathrm{N}
\end{aligned}
$$

The free-body diagrams show that the bottom mass is the easiest to manage. We should begin there.



Starting at the bottom of the picture, we can use Newton's second law to find the tension F_{T3}.

$$\begin{aligned}
\vec{F}_{T3} + \vec{F}_{g3} &= m_3 \cancelto{0}{\vec{a}_3} \\
\vec{F}_{T3} &= -\vec{F}_{g3} \\
\vec{F}_{T3} &= -(-0.\mathbf{29}4\,\mathrm{N}\,\hat{j}) \\
\vec{F}_{T3} &= 0.\mathbf{29}4\,\mathrm{N}(+\hat{j})
\end{aligned}$$

Since $\vec{F}_{g3}$ is down, we find that F_{T3} is upwards (as expected). *When we then jump to m_2, we note that the tension F_{T3} pulls m_2 downwards (even though it pulls m_3 upwards), so we have to change the sign. Once we do this, though, we can find F_{T2}.*

$$\begin{aligned}
\vec{F}_{T2} + \vec{F}_{T3} + \vec{F}_{g2} &= m_2 \cancelto{0}{\vec{a}_2} \\
\vec{F}_{T2} &= -\vec{F}_{T3} - \vec{F}_{g2} \\
\vec{F}_{T2} &= -(-0.\mathbf{29}4\,\mathrm{N}\,\hat{j}) - (-0.\mathbf{19}6\,\mathrm{N}\,\hat{j}) \\
\vec{F}_{T2} &= 0.\mathbf{49}0\,\mathrm{N}(+\hat{j})
\end{aligned}$$

Since both $\vec{F}_{g2}$ and $\vec{F}_{T3}$ are down, we find that F_{T2} is upwards (as expected). You should also note that m_2 does not experience the weight *of m_3, but rather the* tension *supporting that weight. This is more obvious when we consider m_1.* *Now we jump to the* top *of the picture with m_1 and note that the tension F_{T2} pulls m_1 downwards (even though it pulls m_2 upwards), so we have to change the sign. Once we do this, though, we can find F_{T1}.*

$$\begin{aligned}
\vec{F}_{T1} + \vec{F}_{T2} + \vec{F}_{g1} &= m_1 \cancelto{0}{\vec{a}_1} \\
\vec{F}_{T1} &= -\vec{F}_{T2} - \vec{F}_{g1} \\
\vec{F}_{T1} &= -(-0.\mathbf{49}0\,\mathrm{N}\,\hat{j}) - (-0.0\mathbf{98}1\,\mathrm{N}\,\hat{j}) \\
\vec{F}_{T1} &= 0.\mathbf{58}8\,\mathrm{N}(+\hat{j})
\end{aligned}$$

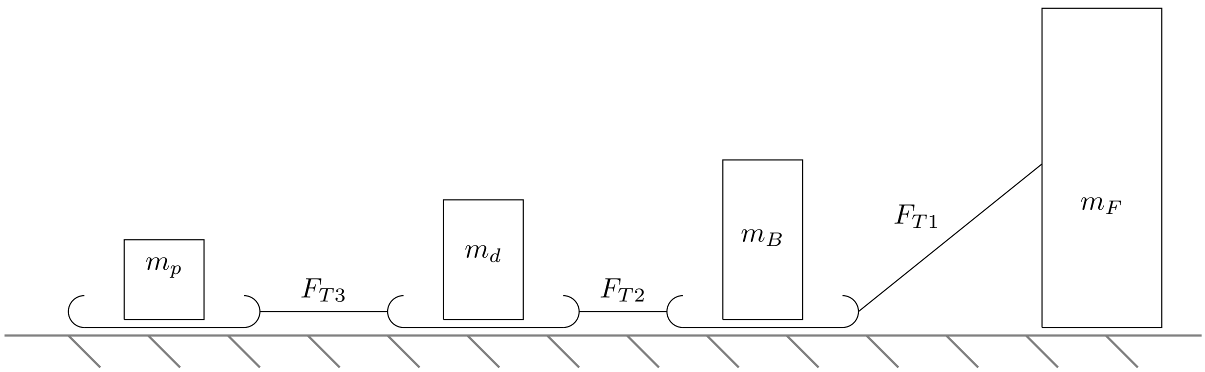
Since both $\vec{F}_{g1}$ and $\vec{F}_{T2}$ are down, we find that F_{T1} is upwards (as expected). You should also note that m_1 does not experience the weight *of the other masses, but rather experiences the* tension *supporting that weight.* *You might also note that F_{T1} pulls the ceiling downwards with a force of* $0.\mathbf{58}8\,\mathrm{N}(-\hat{j})$.

Example 8.5.12 considers the case of dragging multiple masses, which extends the ideas of Subsection 2. Since we have not yet introduced friction, we will assume this is a frictionless surface. We will update this example in Section 8.4 with Example 8.5.13.

Example 8.5.12 (*Tension in a caravan*)

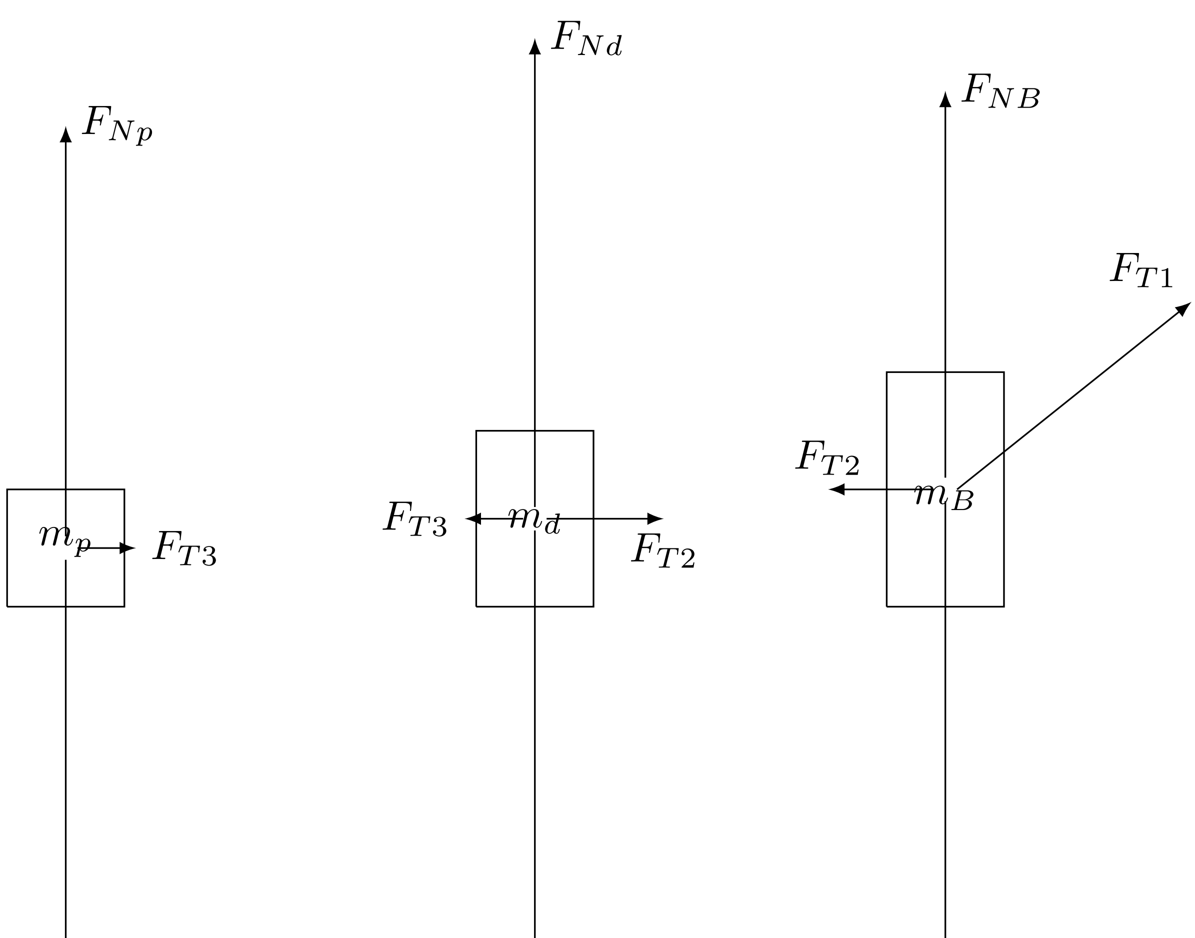
When Beth was a child in Norway, she was pulled through the woods in a sled by her parent, Frances. Beth's sled ($m_B = 50.0\,\mathrm{kg}$) was also tied to a sled on which her dog sat. The dog's sled ($m_d = 30.0\,\mathrm{kg}$) was then connected to a sled with provisions for the day ($m_p = 10.0\,\mathrm{kg}$). As they start their journey, the entire system is accelerated at $a = 0.215\ \mathrm{m/s^2}$.

Draw the free-body diagram (check against Answer 1) and find the tension in each string?



Hint. *If friction were pulling against the motion, then the vertical normal and gravitational forces would be necessary.*

Answer. *The free-body diagrams for this example are:*



Solution. *The first thing we should do is draw the free-body diagrams. These are given in Answer 1. If there were friction, then we would*

You should note that these examples are essentially expressing the same idea in two different contexts.

We can now update Example 8.5.12.

Example 8.5.13 (*Tension in a caravan with friction*)

While pulling a sled on which your son sits, your son's sled is tied to a sled on which your dog sits. Your dog's sled is then connected to a sled with provisions for the day. What is the tension in each subsequent string?

Solution. *The first thing we should do is notice what information is given to us and make sure that everything is in consistent units. I will convert everything to SI units.*

8.5.4.3 Ropes with Mass

If a rope has mass, then it can be thought of as a series of tiny pieces of mass hung by tiny massless strings.

8.5.4.4 Atwood's Machine

Note https://en.wikipedia.org/wiki/Atwood_machine. "invented in 1784 by the English mathematician George Atwood as a laboratory experiment to verify the mechanical laws of motion with constant acceleration. Atwood's machine is a common classroom demonstration used to illustrate principles of classical mechanics."

Example 8.5.14 (*The acceleration of masses on Atwood's Machine*)

The two crates in the figure (p. 114) hang over a pulley (in what is called an "Atwood's machine"). I will select $m_1 = 35\,\text{kg}$ (because it looks smaller) and $m_2 = 85\,\text{kg}$ (because it looks bigger). We will assume that the pulley is massless and frictionless (so that the tension is the same throughout the rope). Find the acceleration and the time it takes m_2 to accelerate down for the $12\,\text{m}$ to the floor.

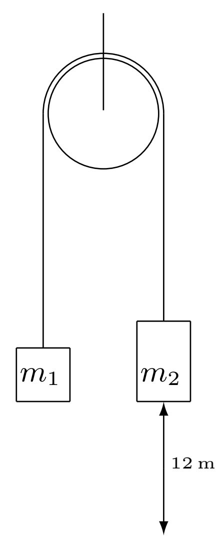


Figure 8.5.15: Masses hung from a rope draped over Atwood's Machine

Solution. *The easy way to do this is to say that m_1 pulls down on the left with $F_{g1} = (35\,\text{kg})(9.81\,{}^{\text{m}}\!/\!{}_{\text{s}^2}) = \mathbf{343}.4\,\text{N}$ and m_2 pulls down on the right with $F_{g2} = (85\,\text{kg})(9.81\,{}^{\text{m}}\!/\!{}_{\text{s}^2}) = \mathbf{833}.5\,\text{N}$ for a difference of $F_{net} = \mathbf{490}\,\text{N}$ down to the right. Since this has to move both m_1 and m_2, the acceleration is*

$$a = \frac{F_{\text{net}}}{m_1 + m_2} = \frac{\mathbf{490}\,\text{N}}{(35\,\text{kg}) + (85\,\text{kg})} = \frac{\mathbf{490}\,\text{N}}{\mathbf{120}\,\text{kg}} = \mathbf{4.0}87\,{}^{\text{m}}\!/\!{}_{\text{s}^2}$$

This acceleration then causes m_2 to drop and the time it takes is found from the equation that include *distance and time,*

$$y_f = y_i + v_i\,t + \frac{1}{2}a\,t^2$$
$$(0\,\mathrm{m}) = (12\,\mathrm{m}) + (0\ {}^{\mathrm{m}}\!/\!_{\mathrm{s}})\,t + \frac{1}{2}(-\mathbf{4.09}\ {}^{\mathrm{m}}\!/\!_{\mathrm{s}^2})\,t^2$$

which we can solve for time:

$$t \;=\; \sqrt{\frac{-(12\,\mathrm{m})}{\frac{1}{2}(-\mathbf{4.09}\ {}^{\mathrm{m}}\!/\!_{\mathrm{s}^2})}} \;=\; \sqrt{\mathbf{5.87}\,\mathrm{s}^2} \;=\; \mathbf{2.42}\,\mathrm{s}$$

Example 13 computes the acceleration, but does so in a way that avoids the question of the tension in the rope. Based on that example, it is possible to then deduce the tension by considering the net force on either of the masses.

Example 8.5.16 (*Atwood's tension from the acceleration*)

Given the acceleration from Example 13, find the tension in the rope.

Solution. *Since that example used the two masses as a single system, it did not reference the "internal force" of tension. By considering the two masses as separate objects, the tension is no longer "internal" to the system. This allows us to compute the tension with Newton's second law using one or the other mass. Here we do both to verifyt hat the result is the same.*

For the lighter mass (on the left),

$$\begin{aligned} F_{\text{net}} &= m_1 a_1 \\ &= (35\,\mathrm{kg})(\mathbf{4.09}\ {}^{\mathrm{m}}\!/\!_{\mathrm{s}^2}) \\ &= \mathbf{143}\,\mathrm{N} \end{aligned}$$

With the net force and the gravitational force (weight), we can find the tension:

$$\begin{aligned} \vec{F}_{\text{net}} &= \vec{F}_g + \vec{F}_T \\ \vec{F}_T &= \vec{F}_{\text{net}} - \vec{F}_g \\ \vec{F}_T &= (+\mathbf{143}\,\mathrm{N}\hat{\jmath}) - (-\mathbf{343}\,\mathrm{N}\hat{\jmath}) \\ \vec{F}_T &= +\mathbf{486}\,\mathrm{N}\hat{\jmath} \end{aligned}$$

For the heavier mass (on the right),

$$\begin{aligned} F_{\text{net}} &= m_1 a_1 \\ &= (85\,\mathrm{kg})(\mathbf{4.09}\ {}^{\mathrm{m}}\!/\!_{\mathrm{s}^2}) \\ &= \mathbf{348}\,\mathrm{N} \end{aligned}$$

With the net force and the gravitational force (weight), we can find the tension:

$$\begin{aligned} \vec{F}_{\text{net}} &= \vec{F}_g + \vec{F}_T \\ \vec{F}_T &= \vec{F}_{\text{net}} - \vec{F}_g \\ \vec{F}_T &= (-\mathbf{348}\,\mathrm{N}\hat{\jmath}) - (-\mathbf{833}\,\mathrm{N}\hat{\jmath}) \\ \vec{F}_T &= +\mathbf{485}\,\mathrm{N}\hat{\jmath} \end{aligned}$$

These are consistent to within the precision of our calculation. You should notice that

1. *the tension is larger than the weight of the lighter mass and therefore pulls it upwards,*
2. *the tension is smaller than the weight of the heavier mass and therefore keeps it from falling as fast as it would in freefall.*
3. *The tension is the same on both sides of the pulley specifically because we assumed that it did not take any effort to turn the pulley. This is usually expressed with three specific (and independent) assumptions:*
   (a) *The rope has an insignificant amount of mass – the rope is "massless". (This means that its mass is too small to impact the significant digits.)*
   (b) *The pulley has an insignificant amount of mass – the pulley is "massless". (This means that its inertial mass[3] is too small to impact the significant digits.)*
   (c) *The pulley has an insignificant amount of friction – the pulley is "frictionless". (This means that any force that might resist turning the pulley is too small to impact the significant digits.)*

The examples above are straightforward mathematically, but seem to cause some conceptual problems with students. Because of this, we can consider the same problem in a way that seems to be easier conceptually, although it is a little more involved mathematically.

Example 8.5.17 (*Computing the acceleration and tension for Atwood's machine*)

Given the description from Example 13, we can calculate the tension and acceleration.

Solution. *Figure 14 allows us to draw the free-body-diagrams from which we can write down Newton's second law for each individual object.*

For these equations, I am explicitly putting the sign in by hand to indicate the direction (positive is up and negative is down). Doing this, the value of F_T, F_g, and a will be positive because they are the magnitudes.
The equation for m_1 is

$$\begin{aligned} \vec{F}_{g1} + \vec{F}_{T1} &= m_1\vec{a}_1 \\ (-F_{g1}) + (+F_T) &= m_1(+a) \\ (-\mathbf{343}\,\mathrm{N}) + (+F_T) &= (35\,\mathrm{kg})(+a) \end{aligned}$$

The equation for m_2 is

$$\begin{aligned} \vec{F}_{g2} + \vec{F}_{T2} &= m_1\vec{a}_2 \\ (-F_{g2}) + (+F_T) &= m_1(-a) \\ (-\mathbf{833}\,\mathrm{N}) + (+F_T) &= (85\,\mathrm{kg})(+a) \end{aligned}$$

In these equations, I have noticed that the tensions F_{T1} and F_{T2} have to have the same magnitude, even though they are different forces. I am using F_T to indicate that magnitude. Similarly, the accelerations a_1 and a_2 have to have the same magnitude (because the masses are connected) even though they have opposite directions. I am using a to indicate that magnitude. In each case, the sign indicates the different directions.

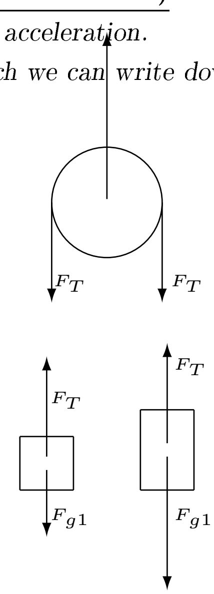


Figure 8.5.18: The free-body diagrams for the Atwood's machine system

These equations can be combined as two-equations-with-two-unknowns and solved in the familiar ways. It is useful to be aware that you can do these independently of each other. So, if you need the acceleration, you can solve Item 1 and if you need the tension, you can solve Item 2.

1. *If we solve one equation for F_T and plug it into the other, then we get the following equation. (Since the coefficient of F_T is the same in these equations, if we subtract the second equation from the first, then we also get the following equation.)*

   Computing the acceleration:

$$\begin{aligned} (-\mathbf{343}\,\mathrm{N}) - (-\mathbf{833}\,\mathrm{N}) &= [(35\,\mathrm{kg}) + (85\,\mathrm{kg})]\,(a) \\ a &= \frac{\mathbf{490}\,\mathrm{N}}{(35\,\mathrm{kg}) + (85\,\mathrm{kg})} = \mathbf{4.0}87\ \mathrm{m/s^2} \end{aligned}$$

2. *If we solve the first equation for a and plug it into the second, then we get the following equation.*

   Computing the tension:

$$\begin{aligned} (-\mathbf{833}\,\mathrm{N}) + (F_T) &= -(85\,\mathrm{kg})\left[\frac{(-\mathbf{343}\,\mathrm{N}) + (F_T)}{(35\,\mathrm{kg})}\right] \\ F_T &= \frac{-(35\,\mathrm{kg})(-\mathbf{833}\,\mathrm{N}) - (85\,\mathrm{kg})(-\mathbf{343}\,\mathrm{N})}{[(35\,\mathrm{kg}) + (85\,\mathrm{kg})]} = \mathbf{486}\,\mathrm{N} \end{aligned}$$

It is useful to verify the following: The tension is not enough to support m_2, so it falls:

$$\vec{a}_2 = \frac{\vec{F}_{\text{net}}}{m_2} = \frac{(+\mathbf{486}\,\mathrm{N}) + (-\mathbf{833}\,\mathrm{N})}{(85\,\mathrm{kg})} = -\mathbf{4.09}\ \mathrm{m/s^2}.$$

The tension is also more than enough to lift m_1, so it rises:

$$\vec{a}_2 = \frac{\vec{F}_{\text{net}}}{m_2} = \frac{(+\mathbf{486}\,\mathrm{N}) + (-\mathbf{343}\,\mathrm{N})}{(35\,\mathrm{kg})} = +\mathbf{4.09}\ \mathrm{m/s^2}.$$

8.5.4.5 Surface Tension

As a final note, surface tension is something else entirely. It is mentioned here only because it has the word "tension" in the name.

See Example 2.1.2 for a comment on the contribution to hot versus cold spoon noises.

8.6 Spring Force

Referenced by $F = ma$, uses of $F = ma$, Subsection 8.3.1

Hooke's Law

Hooke's law describes the dependence of the force on the elongation of an elastic material, typically a spring:

$$\vec{F} = -k\,\Delta\vec{x} \tag{8.6.1}$$

Initial definition of elastic (reference Paragraph

8.7 Applied Force

The term "an applied force" is used to describe any force applied by any object when there isn't really a formula to find it. So this is kind of a "any other force" category. I will use this type of force to describe forces exerted by people. We have seen some examples where a person throws an object. We can now revisit those examples and consider the force exerted (applied) by the person who threw the object.

Example 8.7.1 (*Applying a force to throw a ball*)

Carl recalls that one time he got bored one day in physics class (what?!?) and tossed a baseball ($m_b = 0.145\,\mathrm{kg}$) at the ceiling... a little too hard ... as recounted in Exercise 5.6.1. Recall that Example 8.3.8 found the normal force by the ceiling on the ball. Please now find the force Carl applied while throwing and catching the ball assuming that the throw took $0.200\,\mathrm{s}$ to gain the speed of $5.00\ \mathrm{m/s}$ and the catch took $0.250\,\mathrm{s}$ to slow the ball from $4.73\ \mathrm{m/s}$ to rest.

Solution.

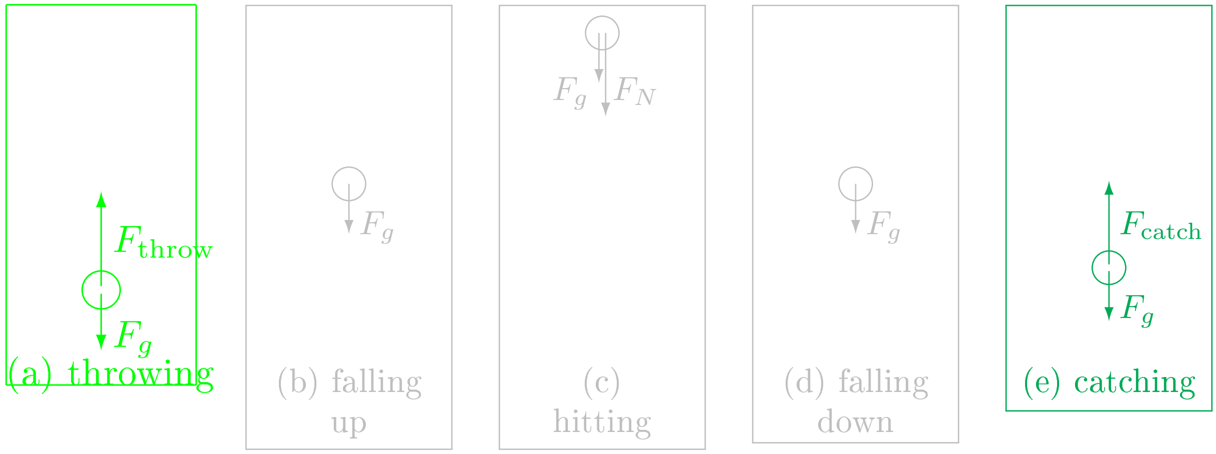


Figure 8.7.2: When a ball is thrown there are five distinct stages because the forces on the ball change and we can, at this point, only manage to describe a situation in which the forces do not change.

In this particular problem, we are only concerned with steps (a) and (e) because that's where Carl throws and catches the ball. In each case, we need the acceleration:

$$\vec{a}_{\text{throw}} = \frac{(+5.00\ {}^{\text{m}}\!/\!_{\text{s}}\hat{j}) - (0\ {}^{\text{m}}\!/\!_{\text{s}}\hat{j})}{0.200\,\text{s}} = +\mathbf{25.00}\ {}^{\text{m}}\!/\!_{\text{s}^2}\hat{j}$$

$$\vec{a}_{\text{catch}} = \frac{(0\ {}^{\text{m}}\!/\!_{\text{s}}\hat{j}) - (-4.73\ {}^{\text{m}}\!/\!_{\text{s}}\hat{j})}{0.250\,\text{s}} = +\mathbf{18.92}\ {}^{\text{m}}\!/\!_{\text{s}^2}\hat{j}$$

During each step, we have the actual acceleration, which tells us about the net force. We will also need to know the weight of the baseball $F_g = \mathbf{1.42}2\,\text{N}$*, because gravity is still acting during the collision. Let's consider the throwing part first.*

$$\begin{aligned}
\vec{F}_N + \vec{F}_g = \vec{F}_{\text{net}} &= m\vec{a} \\
\vec{F}_A &= m\vec{a} - \vec{F}_g \\
\vec{F}_A &= \left[(0.145\,\text{kg})(+\mathbf{25.00}\ {}^{\text{m}}\!/\!_{\text{s}^2}\hat{j})\right] - \left[-\mathbf{1.42}2\,\text{N}\hat{j}\right] \\
\vec{F}_A &= \left[+\mathbf{3.62}5\,\text{N}\hat{j}\right] - \left[-\mathbf{1.42}2\,\text{N}\hat{j}\right] = +\mathbf{5.04}7\,\text{N}\hat{j}
\end{aligned}$$

You can see that the upward applied force ($\mathbf{5.04}7\,\text{N}$) *has to be large enough so that when it is combined with the downward gravitational force* ($\mathbf{1.42}2\,\text{N}$) *they can together result in the necessary (but smaller) upward net force* ($\mathbf{3.62}5\,\text{N}$) *to get it going upwards.*

For the catching part, the ball is moving downwards and needs to be stopped, so the catching applied force must be upwards.

$$\begin{aligned}
\vec{F}_A + \vec{F}_g = \vec{F}_{\text{net}} &= m\vec{a} \\
\vec{F}_A &= m\vec{a} - \vec{F}_g \\
\vec{F}_A &= \left[(0.145\,\text{kg})(+\mathbf{18.92}\ {}^{\text{m}}\!/\!_{\text{s}^2}\hat{j})\right] - \left[-\mathbf{1.42}2\,\text{N}\hat{j}\right] \\
\vec{F}_A &= \left[+\mathbf{2.74}3\,\text{N}\hat{j}\right] - \left[-\mathbf{1.42}2\,\text{N}\hat{j}\right] = +\mathbf{4.16}5\,\text{N}\hat{j}
\end{aligned}$$

You can see that the upward applied force ($\mathbf{4.16}5\,\text{N}$) *has to be large enough so that when it is combined with the downward gravitational force* ($\mathbf{1.42}2\,\text{N}$) *they can together result in the necessary upward net force* ($\mathbf{2.74}3\,\text{N}$) *to stop it from continuing downwards.*

8.8 Putting it Together, F_{net}

8.8.1 Translational Equilibrium

blah blah blah Translational equilibrium: $F_{\text{net}} = m\not{a}$. blah blah blah

8.8.2 Static Equilibrium

.

8.8.3 Dynamic Equilibrium

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8.9 Summary and Homework

8.9.1 Summary of Concepts and Equations

...

8.9.2 Conceptual Questions

1. Estimate, preferably without using the internet, the mass of the following: (a) a four-door sedan, (b) dishwasher, (c) a pair of glasses, (d) a cell phone. You should be able to estimate to within one significant digit.

2. List at least one object, preferably without using the internet, that has the following mass: (a) 2500 kg (b) 41 kg, (c) 3 kg, (d) 50 g.

3. How long has it been since Amontons discovered his first and second laws of friction? Since Coulomb discovered his law of friction? Since da Vinci, his?

4. Do a web search to see what else Amontons is known for. Reference the sites and indicate which information can be corroborated with multiple unique references. (Note wikipedia is not a "primary" source.)

Answer. Wikipedia: barometer, hygrometer, thermometer, clepsydra, thermodynamics, friction; references the Galileo Project at Rice University.
Britannica: friction, thermometer
How Things Work: hygrometer, barometer, thermometer, [no references]
Purdue: Barometer
Oxford Reference: All of these and suggests it was an early indication of absolute zero (temperature)
Texas Women's University: cites the Deaf Scientists Corner, who lists all of these and suggests some work on earth quarks.

5. Do a web search to see what else Coulomb is known for. Reference the sites and indicate which information can be corroborated with multiple unique references. (Note wikipedia is not a "primary" source.)

8.9.3 Problems

1. They say "A pint's a pound, the world around", but really a pint of water is 1.04375 lbs. A pint glass, on the other hand, is 422 g. Compute the mass and weight of a full glass with a pint of water.

Answer. Recalling the discussion around Footnote 8.1.1.1, the water has a mass of

$$(1.04375\,\text{lbs}) \times \left(\frac{453.59237\,\text{g}}{1\,\text{kg}}\right) = \mathbf{473.437}04\,\text{g}$$

Together the glass and water have a mass of

$$(422\,\text{g}) + (\mathbf{473.437}04\,\text{g}) = \mathbf{895}.4\,\text{g}$$

The weight, then, is (after converting to kilograms)

$$(0.\mathbf{895}4\,\mathrm{kg})(9.81\ {}^{\mathrm{m}}\!/\!_{\mathrm{s}^2}) = \mathbf{8.78}4\,\mathrm{N}$$

8.10 References and Further Reading

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[2] van Beek, Anton. *History of Science Friction*. tribology-abc.com. Retrieved Oct, 2017. http://www.tribology-abc.com/abc/history.htm.

[3] Armstrong-Hélouvry, Brian (1991). Control of machines with friction. USA: Springer. [ISBN 0-7923-9133-0] pg 10.

[4] Popov, V.L. *Contact Mechanics and Friction: Physical Principles and Applications*, 2017, Springer Berlin Heidelberg; pg 4. (ISBN 9783662530818) https://books.google.com/books?id=qT5RDgAAQBAJ.

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[6] Blau, P.J. *Friction Science and Technology: From Concepts to Applications, 2e* (Mechanical Engineering Series), 2008, CRC Press; pg 12. (ISBN 9781420054101) https://books.google.com/books?id=5pIpV1LY5HMC.

[7] S. Derler & L-C. Gerhardt & A. Lenz & E. Bertaux & M. Hadad. (2009). Friction of Human Skin against Smooth and Rough Glass as a function of Contact Pressure. Tribology International. 42. 1565-1574. 10.1016/j.triboint.2008.11.009. pdf

List of examples

(Continued on next page)

Example 8.5.1.2	Tension supports hanging objects
Example 8.5.2.5	People pushing a box at an angle
Example 8.5.3.8	The tension in a rope over a pulley
Example 8.5.4.10	Tension between masses hung in a chain
Example 8.5.4.11	Tension in a caravan
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Example 8.5.4.13	The acceleration of masses on Atwood's Machine
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Example 8.5.4.16	Computing the acceleration and tension for Atwood's machine
Example 8.7.0.73	Applying a force to throw a ball

Chapter 9

Energy and the Transfer of Energy

Chapter referenced by Connection 8.4.12

Energy is a noun; objects can *have* energy.

Paragraph referenced by Discussions of force as a noun and heat as a verb, Connection 8.4.12

Work is a verb; doing work is the process of *exchanging* energy.

9.1 Objects Can Have Energy

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9.2 A Force Can Transfer Energy

Referenced by Discussion of the direction of forces

9.3 Dissipating Energy

pool balls on cushion/bumper

9.4 Conserving Energy

Investigation 9.4.1 (*1-D elastic collisions of pool balls and inelastic collisions off the bumper*)

Diane is relaxing with the local physics club, playing pool. She hits the cue ball and counts the number of walls she can hit in one shot.

(a) *collide some pool balls and notice where they hit each other. Can you determine the angle at which they roll away?*

Answer. 90°

Billiard tables have a lot of interesting physics, which can help us see a wide variety of physics, for example: normal force, elastic versus inelastic collisions, rotational motion, and angular momentum.

Return to: pool

9.4.1 Gravitational Potential Energy

Referenced by Section 15.2

See also Section 15.2.

9.4.2 Spring Potential Energy

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9.4.3 Conservative Forces in General

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Part III

Interesting Uses of Motion, Force, and Energy

The chapters in this Part develop the ideas in Part II by introducing momentum, circular motion, rotational motion, torque, and the Newtonian theory of gravitation.

Chapter 10

Momentum: A Better Way to Describe Force

Chapter referenced by Clarification of Newton's laws 7.2.2 Subsection 7.2.3, Task 7.3.2.d.ii

Paragraph referenced by Subsubsection 7.2.1.1

Define momentum
$$\vec{p} = m\vec{v}.$$

Useful to include? The Physics of Bullets Vs. Wonder Woman's Bracelets

10.1 Revising Newton's First and Second Laws

10.1.1 Inertia and Momentum

Referenced by Subsubsection 7.2.1.1

Recall Subsubsection 7.2.1.1.

Probably define elastic (Paragraph) and inelastic collisions

10.2 Revising Newton's Third Law: Conservation of Momentum

Referenced by Subsection 7.2.3

10.3 Two-Dimensional Collisions

Referenced by Subsubsection 3.4.2.2

pool balls? What about rolling?

Investigation 10.3.1 (*2-D collisions of pool balls*)

Diane is relaxing with the local physics club, playing pool. She hits the cue ball and counts the number of walls she can hit in one shot.

(a) *collide some pool balls and notice where they hit each other. Can you determine the angle at which they roll away?*

Answer. 90°

Billiard tables have a lot of interesting physics, which can help us see a wide variety of physics, for example: normal force, elastic versus inelastic collisions, rotational motion, and angular momentum.

Return to: pool

Chapter 11

Rotational Motion

11.1 How Physicists Use the Words

Description indicating that different parts of the object are moving at different speeds (depending on the radius) and, even at the same radius, different parts of the object are moving with different velocities (depending on the angle). This is a good place to introduce

$$v = \omega r \tag{11.1.1}$$

Translation 11.1.1 (*The Story of* $\vec{v} = \vec{\omega} \times \vec{r}$)

This equation describes the velocity (as a vector) to the spin rate and distance from the axis according to the right-hand rule. Connecting the English and the math:

$\vec{v}$	$=$	$\vec{\omega}$	$\times$	$\vec{r}$
the speed of a point on a spinning object	*depends on*	*the rate of spin*	*"crossed with"*	*the distance from the axis*
the velocity is directed in the plane of the spin	*such that it is perpendicular*	*to the axis of spin*	and	*the radial line to the point*

First, consider a merry-go-round for which $\vec{v}$ is the tangential velocity of the object, which is changing (and hence the object is accelerating). This motion is *relative to the axis of rotation*.

Second, consider the motion of a wheel along the ground. The edge of the wheel has this $\vec{v}$ as its tangential motion (*relative to the axle*). *Assuming* that the wheel rolls without slipping, it is *also true* that the edge of the wheel that is touching the ground is not moving *relative to the ground*, so the ground is moving (backwards) with this same velocity *relative to the axle*. Flipping the perspective, the axle of the wheel must be moving with $-\vec{v}$ *relative to the ground*.

11.2 The Equations of Rotational Motion

Use Section 11.1 to connect the translational motion to the rotational motion.

Investigation 11.2.1 (*Rolling pool balls*)

Diane is relaxing with the local physics club, playing pool. She hits the cue ball and counts the number of walls she can hit in one shot.

(a) *Roll a striped ball along the table. Use the stripe to notice the rate of rotation. How does the* *rotation compare to the translation?*

Answer. *[Answer]*

(b) *Roll a striped ball along the table. Notice the distance the ball travels. Why does friction slow the ball down instead of just make it turn $v = \omega r$ (no slip)*

Answer. *[Answer]*

Billiard tables have a lot of interesting physics, which can help us see a wide variety of physics, for example: normal force, elastic versus inelastic collisions, rotational motion, and angular momentum.

Return to: pool

11.3 Moment of Inertia

Paragraph referenced by Investigation 8.4.31

In this video of eggs in space from 14 Oct, 2011 created by Science Friday, we can watch how the momentum of inertia affects the way fresh eggs spin differently than hard-boiled eggs.

11.4 Angular Momentum

Investigation 11.4.1 (*Rolling pool balls*)

Diane is relaxing with the local physics club, playing pool. She hits the cue ball and counts the number of walls she can hit in one shot.

(a) *Roll a striped ball along the table. Use the stripe to notice the rate of rotation. How does the rotation compare to the translation?*

Answer. *[Answer]*

Billiard tables have a lot of interesting physics, which can help us see a wide variety of physics, for example: normal force, elastic versus inelastic collisions, rotational motion, and angular momentum.

Return to: pool

11.5 Non-inertial Rotational Reference Frames

Section referenced by Subsection 5.6.1, Clarification of Newton's laws 7.2.1

Paragraph referenced by Subsection 7.2.2

Because the Earth rotates, we are actually in a non-inertial reference frame. In fact, we can prove that the Earth rotates by observing the effects, such as the Coriolis effect, that in our non-inertial frame seem to require unexplainable forces but which, in a non-rotating frame, follow the expected laws of physics.

11.5.1 The Coriolis Effect

Subsection referenced by Clarification of Newton's laws 7.2.1

Paragraph referenced by Non-inertial Rotational Reference Frames

weather ... AND ... In her podcast, *Spacepod*[1] Dr. Carrie Nugent interviews Dr. Andy Thompson about "underwater flying objects" that investigate the ocean. He notes that ocean waters, because they are such a large-scale system, can see the effect of the rotation of the Earth.

11.5.2 The Foucault Pendulum

See youtube video by Sixty Symbols.

[1]Nugent, Carrie (Producer, Host). *Spacepod* [Audio podcast], episode 89 (19 May, 2017). Retrieved from http://spacepod.libsyn.com/ on 9 Apr. 2017.

Chapter 12

Circular Motion and Centripetal Force

12.1 Circular Motion

.

12.2 Centripetal Force

Referenced by Discussion of $F = ma$

Chapter 13

Torque and the $F = ma$ of Rotations

Chapter referenced by Answer 7.3.0.5.1

13.1 Leverage

Section referenced by Subsection 8.3.1, Task 8.4.13.d, the discussion of walking

discussion of **torque**: $\tau = Fl\sin\theta$.

Definition 13.1.1 (force couple)**.** A **force couple** is a pair of forces that act on an object at different locations in opposite directions, but with equal magnitudes. Force couples produce a torque on the object.

Definition referenced by Task 8.4.13.d

13.2 Putting it all together, τ_{net}

13.2.1 Rotational Equilibrium

blah blah blah. Rotational equilibrium: $\tau_{\text{net}} = I\cancelto{0}{\alpha}$. blah blah blah

13.2.2 Static (Rotational) Equilibrium

.

13.2.3 Dynamic (Rotational) Equilibrium

Example 13.2.1 (***Carluses a ladder***)

Example referenced by *Example 8.3.5, Paragraph*

Carl leans a 22.7 kg ladder against a wall at an angle of 75.5°, consistent with OSHA standard 1926.1053(a)(1)(ii). The coefficient of friction between the ladder and the floor is $\mu_f = 0.31$. The coefficient of friction between the ladder and the wall is $\mu_w = 0.19$. Use the rotational and translational equilibrium to determine if the ladder slides.

Answer 1. *Since the full weight of the ladder, $F_g = \mathbf{222}.69\,\mathrm{N}$, is still pressing downwards into the floor (as a normal force), it is tempting to say that Newton's third law implies that the floor pushes the ladder upwards with a normal force of $\mathbf{222}.69\,\mathrm{N}$ but this would not account for the frictional force on the wall, F_{fw}. If there were no friction between the ladder and the wall, then we could deduce F_{Nf}, but at this* *point, we cannot.*

Answer 2. *If we consider $\mu_w \to 0$, then $F_{fw} = 0\,\mathrm{N}$, $\vec{F}_{Nf} = -\vec{F}_g = \mathbf{222}.7\,\mathrm{N}\hat{\jmath}$, and $\vec{F}_{Nw} = -\vec{F}_{ff} = \mathbf{28}.\mathbf{7}9\,\mathrm{N}\hat{\imath}$. In this case, μ_f could be as small as $0.\mathbf{129}3$ and still hold the ladder in place, unless Carl climbs the ladder, in which case see Answer 13.2.3.1.3.*

Answer 3. *If we consider $\mu_w \to 0$ with Carl ($m = 90.0\,\mathrm{kg}$) at the third-rung-from-the-top of the ladder, (1.53 m up the ladder), then $F_{fw} = 0\,\mathrm{N}$, $\vec{F}_{Nf} = \mathbf{1105}.6\,\mathrm{N}\hat{\jmath}$, and $\vec{F}_{Nw} = -\vec{F}_{ff} = \mathbf{171}.97\,\mathrm{N}\hat{\imath}$. In this case, μ_f could be as small as $0.\mathbf{155}5$ and still hold the ladder in place.*

Solution. *Since we are asked to distinguish between two cases that cannot both be true, we should assume one (the easier one to calculate is that the ladder does not slip) and then verify that the result is consistent with that assumption.*

What do we know? *We know that the floor has a normal force (F_{Nf}) upwards and a frictional force (F_{ff}) to the left. We know that the wall has a normal force (F_{Nw}) to the right and a frictional force (F_{fw}) up (keeping the ladder from sliding down). We know the weight is $F_g = mg = (22.7\,\mathrm{kg})(9.81\,{}^{\mathrm{m}}\!/\!_{\mathrm{s}^2}) = \mathbf{222}.69\,\mathrm{N}$.*

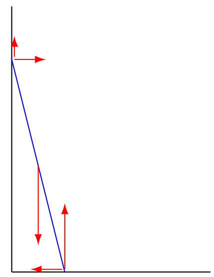

Figure 13.2.2: Diagram of the ladder against the wall

What do we want to know? *We want to know about the the magnitudes of both normal forces and both frictional forces. Can we easily deduce the magnitude of F_{Nf}? Answer 13.2.3.1.1.*

How are what-we-know and what-we-want related? *The forces acting on any body are related by static translational equilibrium*

$$\begin{aligned} x: &\quad 0 = \not{0}F_{gx} + \cancelto{0}{F_{Nfx}} + F_{ffx} + F_{Nwx} + \cancelto{0}{F_{fwx}} \\ y: &\quad 0 = F_{gy} + F_{Nfy} + \cancelto{0}{F_{ffy}} + \cancelto{0}{F_{Nwy}} + F_{fwy} \end{aligned}$$

and static rotational equilibrium, assuming the pivot point is at the ground, and using the relationship $F_f = \mu F_N$, we find

$$\begin{aligned} 0 &= \tau_g + \cancelto{0}{\tau_{Nf}} + \cancelto{0}{\tau_{ff}} + \tau_{Nw} + \tau_{fw} \\ 0 &= \left[F_g \frac{l}{2} \sin 14.5^\circ\right] + \left[-F_{Nw} l \sin(75.5^\circ)\right] + \left[-F_{fw} l \sin(14.5^\circ)\right] \\ F_{Nw} &= \left[F_g \frac{l}{2} \sin 14.5^\circ\right] / \left[l \sin(75.5^\circ) + \mu_w l \sin(14.5^\circ)\right] \end{aligned}$$

Concepts to consider: *First, the length of the ladder cancels from the expression; what matters is the angle at which it is propped.*

Second, every force value will be linearly dependent on the mass of the ladder. So once we solve this problem, we can easily scale the answers to any mass.

Third, the friction with the wall is, by far, the smallest effect and it might be interesting to approximate all of this with $\mu_w = 0$. You can check your calculation against Answer 13.2.3.1.2.

Solution to the example: *When we worry about significant figures,*

$$\begin{aligned} F_{Nw} &= \frac{[(\mathbf{222}.7\,\mathrm{N})(1/2)(0.\mathbf{250}4)]}{[(0.\mathbf{9682}) + (0.19)(0.\mathbf{250}4)]} = \frac{[(\mathbf{27}.88\,\mathrm{N})]}{[(0.\mathbf{9682}) + (0.0\mathbf{476})]} \\ F_{Nw} &= \frac{[(\mathbf{27}.88\,\mathrm{N})]}{[(\mathbf{1.0157})]} = \mathbf{27}.44\,\mathrm{N} \end{aligned}$$

$$F_{fw,\max} = (0.19)(\mathbf{27.44}\,\mathrm{N}) \;=\; \mathbf{5.2}15\,\mathrm{N}$$
$$F_{Nf} = F_g - F_{fw} = (\mathbf{222.7}\,\mathrm{N}) - (\mathbf{5.2}15\,\mathrm{N}) \;=\; \mathbf{217.5}\,\mathrm{N}$$
$$F_{ff,\max} = (0.31)(\mathbf{217.5}\,\mathrm{N}) \;=\; \mathbf{672.4}\,\mathrm{N}$$

Since $F_{ff} > F_{Nw}$, the friction is sufficient to hold the ladder in place, as assumed.

Note about Example 1: Since F_{ff} only needs to be $\mathbf{27.44}\,\mathrm{N}$ to hold the ladder in place, it is possible for the ladder to not slide on a floor that only has $\mu_{\min} = (\mathbf{27.44}\,\mathrm{N})/(\mathbf{217.5}\,\mathrm{N}) = 0.\mathbf{126}2$; but that would not allow a person to climb the ladder.

Homework related to Example 1: Homework problem Exercise 13.4.3.1 asks you to determine if the ladder slides when Carl climbs to different locations on the ladder.

13.3 Torsion

Referenced by Section 8.5

13.4 Summary and Homework

13.4.1 Summary of Concepts and Equations

...

13.4.2 Conceptual Questions

13.4.3 Problems

1. Carl leans a 22.7 kg ladder against a wall at an angle of 75.5°, consistent with OSHA standard 1926.1053(a)(1)(ii). The coefficient of friction between the ladder and the floor is $\mu_f = 0.31$. The coefficient of friction between the ladder and the wall is $\mu_w = 0.19$. Use the rotational and translational equilibrium to determine if the ladder slides when Carl (90.0 kg) climbs to ...

(a) the third-rung from the top of the ladder, so that he is 1.53 m from the bottom of the ladder. (Solution 1)

- (You might also consider Answer 13.2.3.1.3 for the case of $\mu_w = 0$.)

(b) the third-rung from the bottom of the ladder, so that he is 0.914 m from the bottom of the ladder. (Solution 2)

Solution 1. The normal force from the wall is the only force to the right. The frictional force from the floor is the only force to the left. Using the calculations in Example 13.2.1, we notice that

$$F_{Nw} = \mathbf{163.9}\,\mathrm{N}$$
$$F_{fw,\max} = (0.19)(\mathbf{163.9}\,\mathrm{N}) \;=\; \mathbf{31.1}4\,\mathrm{N}$$
$$F_{Nf} = \mathbf{1074.4}\,\mathrm{N}$$
$$F_{ff,\max} = \mathbf{333.0}\,\mathrm{N} > \mathbf{163.9}\,\mathrm{N}$$

The frictional force with the floor can support a normal force from the wall of 333 N, but the normal force is only 164 N so the ladder stays in place. In fact it is possible to compute the smallest coefficient of floor-friction that will keep the ladder in place: $\mu_{\min} = 0.\mathbf{152}56$.

Solution 2. The normal force from the wall is the only force to the right. The frictional force from the floor is the only force to the left. Using the calculations in Example 13.2.1, we notice that

$$F_{Nw} = \mathbf{108.9}7\,\mathrm{N}$$
$$F_{fw,\max} = (0.19)(\mathbf{108.9}7\,\mathrm{N}) \;=\; \mathbf{20.7}0\,\mathrm{N}$$
$$F_{Nf} = \mathbf{1084.9}\,\mathrm{N}$$

$$F_{ff,\max} = \mathbf{336}.3\,\mathrm{N} > \mathbf{108}.97\,\mathrm{N}$$

The frictional force with the floor can support a normal force from the wall of 336 N, but the normal force is only 109 N so the ladder stays in place. In fact it is possible to compute the smallest coefficient of floor-friction that will keep the ladder in place: $\mu_{\min} = 0.\mathbf{100}45$
If, hypothetically, $\mu_w = 0$, then

$$\begin{aligned} F_{Nw} &= \mathbf{114}.3\,\mathrm{N} \\ F_{fw,\max} &= 0\,\mathrm{N} \\ F_{Nf} &= \mathbf{1105}.6\,\mathrm{N} \\ F_{ff,\max} &= \mathbf{342}.7\,\mathrm{N} > \mathbf{114}.3\,\mathrm{N} \end{aligned}$$

The frictional force with the floor can support a normal force from the wall of 336 N, but the normal force is only 109 N so the ladder stays in place. In fact it is possible to compute the smallest coefficient of floor-friction that will keep the ladder in place: $\mu_{\min} = 0.\mathbf{103}4$

Chapter 14

Energy of Rotating Objects

14.1 Rotational Kinetic Energy

pool balls

Chapter 15

The Gravitational Force on a Large Scale

Chapter referenced by freefall, fundamental forces

This might be useful. http://www.physicsoftheuniverse.com/dates.html.

15.1 Gravitational Force and Field

Section referenced by Discussion of $F = ma$

The value of the acceleration due to gravity varies according to the mass and size of any celestial body. This means that, as was seen in Example 8.1.10, your weight can change even when your mass remains the same.

Example 15.1.1 (*Weight is not mass*)

In conversation with a visiting alien, Xerxes, you find that Xerxes has been to the moon and several planets both within and outside of our solar system. In addition to the Earth, Xerxes has visited our moon, Mars, Pluto, and Planet X. Using Table 2, compute Xerxes's weight are each location, assuming Xerxes's mass is 62.5 kg.

Solution.

1. [Earth] $F_g = (62.5\,\text{kg})\left[\frac{GM_E}{R_E^2}\right] = (62.5\,\text{kg})(9.825\,{}^{\text{m}}\!/\!_{\text{s}^2}) \;=\; \mathbf{933}.4\,\text{N}$
2. [moon] $F_g = (62.5\,\text{kg})\left[\frac{GM_m}{R_m^2}\right] = (62.5\,\text{kg})(9.782\,{}^{\text{m}}\!/\!_{\text{s}^2}) \;=\; \mathbf{929}.3\,\text{N}$
3. [Mars] $F_g = (62.5\,\text{kg})\left[\frac{GM_M}{R_M^2}\right] = (62.5\,\text{kg})(9.763\,{}^{\text{m}}\!/\!_{\text{s}^2}) \;=\; \mathbf{927}.5\,\text{N}$
4. [Pluto] $F_g = (62.5\,\text{kg})\left[\frac{GM_P}{R_P^2}\right] = (62.5\,\text{kg})(9.763\,{}^{\text{m}}\!/\!_{\text{s}^2}) \;=\; \mathbf{927}.5\,\text{N}$
5. [Planet X] $F_g = (62.5\,\text{kg})\left[\frac{GM_X}{R_X^2}\right] = (62.5\,\text{kg})(9.763\,{}^{\text{m}}\!/\!_{\text{s}^2}) \;=\; \mathbf{927}.5\,\text{N}$

Table referenced by Example 1

Planet	Mass (kg)	Mean Radius (m)	$g(\,{}^{\text{m}}\!/\!_{\text{s}^2})$

Table 15.1.2: Properties of various celestial bodies.

15.1.1 Inertial Mass versus Gravitational Mass

Referenced by Subsection 8.1.1

15.2 Gravitational Potential Energy

Referenced by Subsection 9.4.1

Recall Subsection 9.4.1

15.3 Making Connections

Referenced by Section 22.3

$$\begin{array}{ccc} & \vec{F} = m\vec{g} & \\ F = G\dfrac{m_1 m_2}{R^2} & \leftrightarrow g = G\dfrac{m}{R^2} & \\ \Delta\mathrm{PE} = -\vec{F}\cdot\Delta\vec{x} \updownarrow & \updownarrow & \text{[for later]} \\ \mathrm{PE} = G\dfrac{m_1 m_2}{R} & \leftrightarrow \text{[for later]} & \\ & \text{[for later]} & \end{array}$$

(Look ahead to the parallel with the electrical interaction in Section 22.3.)

15.4 Orbits

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Part IV

Making Waves

The chapters in this Part are oscillations and thermodynamics. With the traditional organization of the two-semester introductory physics, these parts can be covered in either order and can be chosen to be put in either semester.

Chapter 16

Fluids

16.1 Density

Section referenced by Subsection 8.1.1

16.2 Pressure

Section referenced by discussion of Pascals as a derived unit,

context of hydrostatic (and small reference to hydrodynamic) pressure, as opposed to pressure in the context of Amontons' second law of friction or pressure in gases (might connect nicely to kinetic theory of gases?).

16.3 Surface Tension

Referenced by Discussion of surface tension

Chapter 17

Oscillations

17.1 Oscillating Springs

Referenced by Discussion of $F = ma$

17.2 Oscillating Pendulums

.

17.3 Other Examples of Oscillations

On 13 April, 2017, CBC Broadcasting published a *Quirks and Quarks* episode discussing how we can find solutions to health issues caused by swaying office towers and vibrating floors.

Chapter 18

Sound

18.1 TBD

18.1.1 Musical Instruments

Subsection referenced by Answer 8.5.0.11.a.1

Part V

Is It Hot in Here?

The chapters in this Part are oscillations and thermodynamics. With the traditional organization of the two-semester introductory physics, these parts can be covered in either order and can be chosen to be put in either semester.

Chapter 19

Perspectives of Gases: Physics and Chemistry

19.1 Measuring the properties of a gas

19.1.1 Thermometrics

Thermometers, Fahrenheit, Celcius and centrigrade, Kelvin, Rankin

Reference unit conversion Example 3.2.3.

19.1.2 Barometrics

Section referenced by discussion of Pascals as a derived unit, discussion of friction independent of surface area 8.4.10 Section 16.2,

barometers

Reference derived units discussion of Pascals as a derived unit.

Connection to Amontons (of Amontons' second law of friction and Observation 8.4.10) who also built barometers?

19.2 Kinetic Theory of Gases

Section referenced by Discussions of thermal energy

19.2.1 Pressure as a statistical quantity

words

$$P = \frac{F}{A} = \frac{1}{A}\frac{dp}{dt} = \frac{l}{V}\frac{dp}{dt}$$

more words (might connect hydro-pressure to gaseous pressure?)

19.2.2 Temperature as a statistical quantity

Subsection referenced by Discussions of thermal energy

$K = \frac{3}{2}kT$

Chapter 20

Building the Ideal Gas Law

This should be: Boyle's, Charles', and Gay-Lussac's; each introducing the relevant VT, PT, and PV diagrams as subsections; then the last section is Ideal Gas Law.

Subsequent chapter is the Laws of Thermodynamics and flow of thermal energy, and P-V diagrams grow into their own there.

20.1 P-V Diagrams

Referenced by Discussion of heating versus warming

Chapter 21

The flow of thermal energy

Energy is a noun; objects can *have* energy. The Kinetic Theory of Gases shows us how the temperature of a gas can be an expression of the kinetic energy of the molecules of the gas.

Paragraph referenced by Discussions of force as a noun and heat as a verb, Connection 8.4.12

Heat is a verb; heating is a process of *exchanging* energy. Recall our discussions of force and work.

21.1 Specific Heat Capacity

Heating (positive Q) can warm (positive ΔT) a material.

$$Q = mc\,\Delta T \tag{21.1.1}$$

but (21.2.1) (as one example) shows that it is possible to heat (positive Q) a material without warming it (constant T). When we get to Section 20.1 we will see other examples of "isothermal processes" that have a non-zero Q (heat the system or heat the surroundings) without warming or cooling the system.

21.2 Latent Heat

Referenced by Discussion of Paragraph heating versus warming

Heating might also change the phase of a material.

$$Q = \pm mL \tag{21.2.1}$$

21.3 The Flow of Thermal Energy

21.3.1 Thermal Conductivity

Referenced by Section 3.1

$$\frac{Q}{\Delta t} = \kappa A\,\frac{\Delta T}{\Delta x} \tag{21.3.1}$$

Example 21.3.1 (*Abdulwarms his oven*)

Abdul decides to bake some bread for the dinner party at Beth's house, but he is on a tight schedule. In order to set his schedule, he needs to know how long it will take his oven to warm up.

Solution.

Referenced by Section 3.1

21.3.2 Convection

.

21.3.3 Radiation

Links to light below. Distinguish from nuclear radiation alpha, beta. Note thermal radiation versus gamma radiation?

Part VI

Let There Be Light!

The chapters in this Part cover electricity, magnetism, light, and optics. This is traditionally the meat of the second semester.

Chapter 22

The Electrical Interaction

Chapter referenced by Discussion of fundamental forces

22.1 Electrical Charge

.

22.2 The Big Picture

22.2.1 Electric Forces and Fields

Subsection referenced by Subsubsection 3.4.2.2, $F = ma$

pst-electricfield

Coulomb's Law

Coulomb's Law of the electrical interaction is

$$\vec{F} = k\frac{(q_1)(q_2)}{r_{12}^2}\hat{r}_{12} \tag{22.2.1}$$

22.2.1.1 Examples

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22.2.2 Electric Forces, Fields, and Potential Energy

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22.2.3 Electric Fields, Potential Energy, and Potential

.

22.3 Making Connections

Referenced by Section 15.3

$$\vec{F} = q\vec{E}$$

$$\begin{array}{ccc}
F = k\dfrac{q_1 q_2}{r^2} & \leftrightarrow E = k\dfrac{q}{r^2} & \\
\Delta\mathrm{PE} = -\vec{F}\cdot\Delta\vec{x}\ \updownarrow & \updownarrow & \Delta V = -\vec{E}\cdot\Delta\vec{x} \\
\mathrm{PE} = k\dfrac{q_1 q_2}{r} & \leftrightarrow V = k\dfrac{q}{r} &
\end{array}$$

$$\Delta\mathrm{PE} = q\Delta V$$

(Recall the parallel with the gravitational interaction in Section 15.3.)

Chapter 23

Electricity

See also https://www.facebook.com/UmairTOfficial/videos/1573430836018784/?hc_ref=ARTazrYkFpgDeSNC0xV45lwMO41lc9hWdnntKLNiVDm
pnref=story (referenced from Facebook™ Oct, 2017)

Chapter 24

The Magnetic Interaction

pst-magneticfield

Chapter 25

"Magnicity?"

.

Chapter 26

Light

.

Chapter 27

Optics

.

Part VII

What Have You Done for Me Lately?

The chapters in this Part touch on the topics that are usually referred to as “modern physics”. The goal with including these chapters is to provide some inspiration for what some students see as the tedium of the standard material. These chapters will be linked to throughout the book as examples of how the traditional material supports the material that may be in the news and is more talked about in popular science.

Chapter 28

Relativity

.

Chapter 29

Quantum Mechanics

This might be useful http://timeline.aps.org/APS/Timeline/.

29.1 Atomic Physics

29.1.1 The Periodic Table and Quantum Numbers

.

29.2 Nuclear Physics

29.2.1 Nuclear Decay

.

29.2.2 The Strong Nuclear Force

Referenced by Discussion of fundamental forces

29.2.3 The Weak Nuclear Force

Referenced by Discussion of fundamental forces

29.3 Particle Physics

29.3.1 Field Theory

.

29.3.2 Quantum Electrodynamics

Referenced by Discussion of fundamental forces

29.3.3 Quantum Chromodynamics

Referenced by Discussion of fundamental forces

29.3.4 The Standard Model

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29.3.5 Particle Decay

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Chapter 30

Condensed Matter

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Chapter 31

Astronomy

.

Chapter 32

Cosmology

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Part VIII

Supplements

This final part holds the answers to the interactive examples mentioned above, the bulk of the adventures the reader can investigate in order to test their understanding of the material, and the story lines of each of the characters in the text.

Chapter 33

Deeper Dive

33.1 [Need title]

This is where I will put the fuller explanations.

33.1.1 The Sun

The bright, shiny sun, which keeps us all alive, is a nice example of a rather complex system that allows us to verify our various theories of the world around us. We can consider the existence of a star in three phases: the birth of a star, the life of the star, and the death of the star.

33.1.1.1 The Birth of a Star

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33.1.1.2 The Life of a Star

.

33.1.1.3 The Death of a Star

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33.1.2 Kitchen Appliances

33.1.2.1 Oven

.

33.1.2.2 Refrigerator

.

33.1.2.3 Microwave

.

33.1.2.4 Television

.

33.1.3 Automobile

33.1.3.1 Coolant and Antifreeze

.

33.1.3.2 Tires

.

33.1.3.3 Torque

.

33.1.4 Cool Ideas

33.1.4.1 Black Holes

Referenced by Subsection 8.1.1

On 7 April, 2017, CBC Broadcasting published a *Quirks and Quarks* episode discussing how we can turn our planet into a giant telescope to get a photo of a black hole. The results should be available by the early 2018.

33.1.4.2 Quantum Mechanics

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33.1.4.3 Relativity

.

33.1.4.4 String Theory

.

Chapter 34

Internet Resources

34.1 Podcasts

Spacepod with Carrie Nugent
Science Friday with Ira Flatow

34.2 Videos

Physics Footnotes
Sixty Symbols

34.3 Websites

The Flame Challenge
NASA human performance capabilities

Chapter 35

Characters

This textbook has five characters who follow you throughout the book. They appear in the examples and some homework problems. They also remember previous experiences. I need to adjust the examples in Chapter 7 such that the people pushing boxes are helping the reader rearrange furniture.

The index lists the pages that the characters appear. The point of this chapter is to highlight some of the primary adventures of the characters according to their own perspectives. *None of the links in this chapter will be given a corresponding return link.* This chapter is for me to track relationships and will likely go away when the book is ready for publication

I can, at the header of the code, define the name, gender, mass, and dimensions of each individual.

35.1 The Story

This will be a dramatic story of college students in and out of relationships, stress about their major, and interacting with parents. Maybe some travel; certainly some growth. During their adventure, they casually interact with the reader in unimportant ways that produce the examples and explorations.

The next set of sections are there to help me storyboard the interactions.

35.2 Abdul

- In Section 7.1, Beth gives Abdul a good-natured shove in the arm in order to get the language clarified and begin the conversation about the on-by notation.
- In Example 7.2.30 Abdul helps Beth…
  - (in the current version) push an object to make it accelerate and feel a reaction force causing him to accelerate backwards.
  - (in the future version) will help the reader move into or out of their residence hall by pushing on heavier furniture.
  - [NOTE:] This is all drawn in Example 7.2.27, which is updated in Example 8.3.2.
- In Example 8.1.6, Abdul falls from a small height. (maybe he is jumping off a short ledge while taking a short-cut to class?)
- In Example 21.3.1, Abdul decides to bake some bread for a party at Beth's house, measuring the time it takes to warm his oven.

35.3 Beth

- Beth is a passenger in the reader's car in Exercise 5.3.1 when the reader runs out of gas and coasts to a stop.

- Beth is a passenger in the reader's car in Exercise 5.3.2 and speculates about how fast to go before putting the car in neutral to coast to a stop.
- Beth joins the reader on a road trip in Exploration 7.2.25 and runs out of gas. This results in multiple possible adventures:
  - Answer 7.2.2.2.1, which leads to either an end at Answer 7.2.2.2.5 or an end at Answer 7.2.2.2.7.
  - Answer 7.2.2.2.2, which leads to either Answer 7.2.2.2.4 (choose Answer 7.2.2.2.3 or end with Answer 7.2.2.2.10) or Answer 7.2.2.2.6 (choose Answer 7.2.2.2.7 or end at Answer 7.2.2.2.9)
  - Answer 7.2.2.2.3, which leads to an end at Answer 7.2.2.2.8.
- In Section 7.1, Beth gives Abdul a good-natured shove in the arm in order to get the language clarified and begin the conversation about the on-by notation.
- In Example 7.2.30, Beth helps Abdul...
  - (in the current version) pull an object to make it accelerate and feel a reaction force causing her to accelerate backwards.
  - (in the future version) will help the reader move into or out of their residence hall by pushing on heavier furniture.
  - [NOTE:] This is all drawn in Example 7.2.27, which is updated in Example 8.3.2.
- In Example 8.1.9, Beth has a normal force supporting her. (This touches Answer 8.1.2.9.1, Answer 8.1.2.9.2, and Answer 8.1.2.9.3.)
- At some point, Beth has a party, because in Example 21.3.1, Abdul decides to bake some bread for a party at Beth's house.
- According to Example 8.1.10, Beth's parents are Erik and Frances. They have lived in Norway, where Beth grew up, and visited both Puerto Rico and Mount Everest.

35.4 Carl

.

35.5 Diane

.

35.6 Erik

According to Example 8.1.10, Erik is the father of Beth. They have lived in Norway, where Beth grew up, and visited both Puerto Rico and Mount Everest.

35.7 Frances

According to Example 8.1.10, Frances is the mother of Beth. They have lived in Norway, where Beth grew up, and visited both Puerto Rico and Mount Everest.

35.8 Xerxes

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35.9 Zambert

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Part IX

Notation

The following table defines the notation used in this book. Page numbers or references refer to the first appearance of each symbol.

Symbol	Description	Page
acceleration	refers to the general idea of changing your motion (velocity), meaning *either* **speeding up** (colloquially "acceleration") *or* **slowing down** (colloquially "deceleration") *or* **changing the direction** (colloquially "turning")	27
fast	refers to moving with a large speed (to be distinguished from **quick**)	27
quick	refers to moving with a large acceleration (to be distinguished from **fast**)	27
acceleration	refers to the general idea of changing your motion (velocity), meaning *either* **speeding up** (colloquially "acceleration") *or* **slowing down** (colloquially "deceleration") *or* **changing the direction** (colloquially "turning")	28
freefall	"being in freefall" will mean moving only under the influence of gravity	31
force	refers to the general idea of pushing or pulling; every force is enacted **by** one person or object and is acted **on** another.	37
$1\,\mathrm{N} = 1\ {}^{\mathrm{kg}\cdot\mathrm{m}}\!/_{\mathrm{s}^2}$	Definition of the unit Newton	42
equilibrium	An object in equilibrium has $\vec{F}_{\text{net}} = 0\,\mathrm{N}$ and $\vec{a} = 0$.	48
$F_{\text{on,by,type}}$	Use of subscripts on force labels: If it is obvious which object feels the force, the subscript will only indicate *which force*. If it is not obvious which object feels the force, the subscript will also indicate which object the force is acted *on*. Sometimes we need to clarify who exerts the force as well as who feels the force. In this case, the subscripts indicate who the force is *on*, who the force is *by*, and which force we are referring to: $F_{\text{on,by,type}}$.	63
weight	$\mathbf{F_g} \equiv \mathbf{mg}$	65
inclined plane	Any tilted, flat surface; such as a ramp, slide, ski jump, the side of a pyramid, etc.	78
angleofrepose	The angle at which an object slides down a tilted ramp.	99
force couple	This is a pair of forces that act on an object at different locations in opposite directions, but with equal magnitudes. Force couples produce a torque on the object.	143

Part X

Solutions to Selected Exercises

7.4.3 Problems

Add more variety of problems.**1.** If Zambert, with $m_Z = 95.0\,\text{kg}$, braces himself (so that he does not accelerate) and pushes Carl ($m_C = 90.0\,\text{kg}$) with a force of $\vec{F}_{C,Z} = 215\,\text{N}\hat{\imath}$, find the following:

(a) What is the acceleration of Carl? (Solution 1)

(b) What net force does Zambert feel? (Solution 2)

(c) If Zambert braces himself against the Earth, then what must that bracing force be? (Solution 3)

(d) What are the individual forces that Zambert feels? (Solution 4)

(e) What is the acceleration of the Earth? (Solution 5)

(f) Which of Newton's laws allows you to answer each of these questions?

$$\vec{a}_C = \frac{215\,\text{N}\hat{\imath}}{90.0\,\text{kg}} = \mathbf{2.38}9\ {}^{\text{m}}\!/\!{}_{\text{s}^2}\hat{\imath}.$$

$$F_{Z,\text{net}} = 0\,\text{N}.$$

$$\vec{F}_{E,Z} = -215\,\text{N}\hat{\imath}.$$

$$F_{Z,C} = -215\,\text{N}\hat{\imath} \text{ and } F_{Z,E} = 215\,\text{N}\hat{\imath}.$$

$$\vec{a}_E = \frac{-215\,\text{N}\hat{\imath}}{5.97\times10^{24}\,\text{kg}} = -\mathbf{3.60}1\times10^{-23}\ {}^{\text{m}}\!/\!{}_{\text{s}^2}\hat{\imath}.$$

8.9.2 Conceptual Questions

4. Do a web search to see what else Amontons is known for. Reference the sites and indicate which information can be corroborated with multiple unique references. (Note wikipedia is not a "primary" source.)

Wikipedia: barometer, hygrometer, thermometer, clepsydra, thermodynamics, friction; references the Galileo Project at Rice University.
Britannica: friction, thermometer
How Things Work: hygrometer, barometer, thermometer, [no references]
Purdue: Barometer
Oxford Reference: All of these and suggests it was an early indication of absolute zero (temperature)
Texas Women's University: cites the Deaf Scientists Corner, who lists all of these and suggests some work on earth quarks.

8.9.3 Problems

1. They say "A pint's a pound, the world around", but really a pint of water is 1.04375 lbs. A pint glass, on the other hand, is 422 g. Compute the mass and weight of a full glass with a pint of water.

Recalling the discussion around Footnote 8.1.1.1, the water has a mass of

$$(1.04375\,\text{lbs})\times\left(\frac{453.59237\,\text{g}}{1\,\text{kg}}\right) = \mathbf{473.437}04\,\text{g}$$

Together the glass and water have a mass of

$$(422\,\text{g}) + (\mathbf{473.437}04\,\text{g}) = \mathbf{895}.4\,\text{g}$$

The weight, then, is (after converting to kilograms)

$$(0.\mathbf{895}4\,\text{kg})(9.81\ {}^{\text{m}}\!/\!{}_{\text{s}^2}) = \mathbf{8.78}4\,\text{N}$$

13.4.3 Problems

Add more problems.**1.** Carl leans a 22.7 kg ladder against a wall at an angle of 75.5°, consistent with OSHA standard 1926.1053(a)(1)(ii). The coefficient of friction between the ladder and the floor is $\mu_f = 0.31$. The coefficient of friction between the ladder and the wall is $\mu_w = 0.19$. Use the rotational and translational equilibrium to determine if the ladder slides when Carl (90.0 kg) climbs to ...

(a) the third-rung from the top of the ladder, so that he is 1.53 m from the bottom of the ladder. (Solution 1)

- (You might also consider Answer 13.2.3.1.3 for the case of $\mu_w = 0$.)

(b) the third-rung from the bottom of the ladder, so that he is 0.914 m from the bottom of the ladder. (Solution 2)

The normal force from the wall is the only force to the right. The frictional force from the floor is the only force to the left. Using the calculations in Example 13.2.1, we notice that

$$\begin{aligned} F_{Nw} &= \mathbf{163}.9\,\mathrm{N} \\ F_{fw,\max} &= (0.19)(\mathbf{163}.9\,\mathrm{N}) \ = \ \mathbf{31}.14\,\mathrm{N} \\ F_{Nf} &= \mathbf{1074}.4\,\mathrm{N} \\ F_{ff,\max} &= \mathbf{333}.0\,\mathrm{N} > \mathbf{163}.9\,\mathrm{N} \end{aligned}$$

The frictional force with the floor can support a normal force from the wall of 333 N, but the normal force is only 164 N so the ladder stays in place. In fact it is possible to compute the smallest coefficient of floor-friction that will keep the ladder in place: $\mu_{\min} = 0.\mathbf{152}56$.

The normal force from the wall is the only force to the right. The frictional force from the floor is the only force to the left. Using the calculations in Example 13.2.1, we notice that

$$\begin{aligned} F_{Nw} &= \mathbf{108}.97\,\mathrm{N} \\ F_{fw,\max} &= (0.19)(\mathbf{108}.97\,\mathrm{N}) \ = \ \mathbf{20}.70\,\mathrm{N} \\ F_{Nf} &= \mathbf{1084}.9\,\mathrm{N} \\ F_{ff,\max} &= \mathbf{336}.3\,\mathrm{N} > \mathbf{108}.97\,\mathrm{N} \end{aligned}$$

The frictional force with the floor can support a normal force from the wall of 336 N, but the normal force is only 109 N so the ladder stays in place. In fact it is possible to compute the smallest coefficient of floor-friction that will keep the ladder in place: $\mu_{\min} = 0.\mathbf{100}45$

If, hypothetically, $\mu_w = 0$, then

$$\begin{aligned} F_{Nw} &= \mathbf{114}.3\,\mathrm{N} \\ F_{fw,\max} &= 0\,\mathrm{N} \\ F_{Nf} &= \mathbf{1105}.6\,\mathrm{N} \\ F_{ff,\max} &= \mathbf{342}.7\,\mathrm{N} > \mathbf{114}.3\,\mathrm{N} \end{aligned}$$

The frictional force with the floor can support a normal force from the wall of 336 N, but the normal force is only 109 N so the ladder stays in place. In fact it is possible to compute the smallest coefficient of floor-friction that will keep the ladder in place: $\mu_{\min} = 0.\mathbf{103}4$

Part XI

List of Features

.1 List of Investigations

(Continued on next page)

.2 List of Tables

.3 List of Figures

(Continued on next page)

.4 List of Asides

(Continued on next page)

Subsection 2.1.4 Things on the ground

Aside	Example referenced by

Chapter 3 Why so much math?

Section 3.2 The Metric System

Aside	Section referenced by

Subsection 3.2.2 Conversion from English Units

Aside	Subsection referenced by
Aside	Example referenced by

Subsection 3.2.3 Fundamental Units versus Derived Units

Aside	Subsection referenced by
Aside	Paragraph referenced by
Aside	Paragraph referenced by

Section 3.3 A graph is worth a thousand pictures

Subsection 3.3.1 Coordinate Systems

Aside	Subsection referenced by
Aside	Paragraph referenced by

Section 3.4 Trigonometry and Vectors

Subsection 3.4.2 Vectors

Aside	Subsection referenced by
Aside	Paragraph referenced by
Aside	Paragraph referenced by
Aside	Paragraph referenced by
Aside	Section referenced by
Aside	Paragraph referenced by

Chapter 4 Estimating and Uncertainty

Section 4.1 Precision and Accuracy

Aside	Paragraph referenced by

Section 4.2 Significant Figures

Aside	Section referenced by

Section 4.4 Effective Theories

(Continued on next page)

(Continued on next page)

Subsection 5.6.3 Multi-Step Solutions

Aside	Section referenced by
Aside	Exercise referenced by
Aside	Exercise Not Done

Chapter 6 2-D Motion, The Vector Nature of Motion

Section 6.1 Components of Motion

Subsection 6.1.2 Ballistic Freefall

Aside	Subsection referenced by
Aside	Paragraph referenced by
Aside	Paragraph referenced by

Chapter 7 Force

Aside	Chapter referenced by

Section 7.1 How Physicists Use the Words (Notation)

Aside	Section referenced by
Aside	Etymology (force)
Aside	Insight referenced by
Aside	Convention referenced by
Touchstone	Touchstone (law)

Section 7.2 Connecting the Concepts: Newton's Laws

Aside	Section referenced by
Touchstone	Touchstone (law)
Aside	Paragraph referenced by
Touchstone	Touchstone (mostly true)

Subsection 7.2.1 Translating Newton's First Law: The Law of Inertia

Aside	Subsection referenced by
Touchstone	Touchstone (inertial frames)
Aside	Paragraph referenced by
Aside	Task referenced by

Subsection 7.2.2 Translating Newton's Second Law: The Equation Law

Aside	Subsection referenced by
Touchstone	Touchstone (inertial frames)
Touchstone	Touchstone (mostly true)
Touchstone	Touchstone (vector equation)
Aside	Referenced by
Aside	Referenced by
Aside	Subsubsection referenced by

(Continued on next page)

(Continued on next page)

Aside	Etymology (normal)
Aside	Example referenced by
Aside	Example referenced by
Aside	Example referenced by

Subsection 8.3.1 Bathroom Scales Measure the Normal Force

Aside	Referenced by

Subsection 8.3.2 The normal force and inclined planes

Aside	Definition referenced by

Section 8.4 Frictional Force

Aside	Section referenced by

Subsection 8.4.1 Dry Sliding Friction

Aside	Etymology (kinetic)
Aside	Etymology (static)
Aside	Task referenced by
Touchstone	Touchstone (normal force)
Touchstone	Touchstone (law)
Touchstone	Touchstone (on-by)
Touchstone	Touchstone (third-law pair)
Aside	Observation referenced by
Touchstone	Touchstone (law)
Touchstone	Touchstone (law)
Touchstone	Touchstone (mostly true)
Aside	Insight referenced by
Touchstone	Touchstone (third-law pair)
Aside	Paragraph referenced by
Aside	Definition referenced by
Aside	Answer referenced by
Aside	Paragraph referenced by

Subsection 8.4.2 Rolling Friction

Aside	Clarification referenced by

Subsection 8.4.3 Additional Detail: The field of Tribology

Aside	Etymology (tribology)

Section 8.5 Tension

Aside	Section referenced by
Aside	Etymology (tension, stress, strain)
Aside	Foreshadow

Subsection 8.5.1 Tension as a Support Force

(Continued on next page)

(Continued on next page)

Aside Section referenced by
Aside Paragraph referenced by

Subsection 11.5.1 The Coriolis Effect

Aside Subsection referenced by
Aside Paragraph referenced by

Chapter 12 Circular Motion and Centripetal Force

Section 12.2 Centripetal Force

Aside Referenced by

Chapter 13 Torque and the $F = ma$ of Rotations

Aside Chapter referenced by

Section 13.1 Leverage

Aside Section referenced by
Aside Definition referenced by

Section 13.2 Putting it all together, τ_{net}

Subsection 13.2.3 Dynamic (Rotational) Equilibrium

Aside Example referenced by

Section 13.3 Torsion

Aside Referenced by

Chapter 15 The Gravitational Force on a Large Scale

Aside Chapter referenced by

Section 15.1 Gravitational Force and Field

Aside Section referenced by
Aside

Subsection 15.1.1 Inertial Mass versus Gravitational Mass

Aside Referenced by

Section 15.2 Gravitational Potential Energy

Aside Referenced by

Section 15.3 Making Connections

(Continued on next page)

(Continued on next page)

Section 20.1 P-V Diagrams
Aside Referenced by

Chapter 21 The flow of thermal energy
Aside Paragraph referenced by

Section 21.2 Latent Heat
Aside Referenced by

Section 21.3 The Flow of Thermal Energy

Subsection 21.3.1 Thermal Conductivity
Aside Referenced by
Aside Referenced by

Chapter 22 The Electrical Interaction
Aside Chapter referenced by

Section 22.2 The Big Picture

Subsection 22.2.1 Electric Forces and Fields
Aside Subsection referenced by

Section 22.3 Making Connections
Aside Referenced by

Chapter 29 Quantum Mechanics

Section 29.2 Nuclear Physics

Subsection 29.2.2 The Strong Nuclear Force
Aside Referenced by

Subsection 29.2.3 The Weak Nuclear Force
Aside Referenced by

Section 29.3 Particle Physics

Subsection 29.3.2 Quantum Electrodynamics
Aside Referenced by

(Continued on next page)

Subsection 29.3.3 Quantum Chromodynamics

Aside Referenced by

Chapter 33 Deeper Dive

Section 33.1 [Need title]

Subsection 33.1.4 Cool Ideas

Aside Referenced by

.5 To-Do List

To Do storyboard the characters and how they develop.[NUM]
To Do Decide how many characters.[NUM]
To Do (em)Book Layout:(/em) Here we should add information about Adventures, Examples, Equation-Stories, and IRLs..[N

Chapter 1 The Story of Science

To Do Add description of science..[NUM]
To Do Add description of physics.[NUM]

Section 1.1 Careful, Detailed Observation

To Do Consider the ”casual to the obvious observer” joke.[NUM]

Chapter 2 Seeing Physics

To Do This chapter should mirror (xref ref="c-revisted" text="type-global" /).[NUM]

Section 2.1 The Flame Challenge and Other Brief Descriptions

Subsection 2.1.2 The Forming of Matter in the Universe

To Do Stopped mid-stream. This is a good place to jump back in when I am stuck someplace else..[NUM]

Section 2.2 Effective Theory

To Do Should this be here or in (xref ref="s-effective2" text="type-global" /)?.[NUM]

Chapter 3 Why so much math?

Section 3.1 Every equation tells a story

To Do I would love for this to be a mouse-over in the equation.[NUM]
To Do Consider "chunking" the "story" with colors to indicate the pieces..[NUM]
To Do Is there a way to include something in the LaTeXversion but not in the HTML version? (such as **??**).[NUM]

To Do Is there a way to include something in the LaTeXversion but not in the HTML version? (such as 42).[NUM]
To Do Decide if should use the "public version" or the "me version" (which uses listofstorys)..[NUM]

Section 3.2 The Metric System

Subsection 3.2.2 Conversion from English Units

To Do rephrase this. I moved that discussion to (xref ref="s-sigfig" text="type-global" /)..[NUM]

Chapter 4 Estimating and Uncertainty

Section 4.2 Significant Figures

To Do Remove this sentence and make the next paragraph sensible. It makes more sense here than in (xref ref="ss-weight
To Do Refocus this paragraph as an **example** about significant digits..[NUM]
To Do Add link: variation in g.[NUM]

Section 4.4 Effective Theories

To Do Should this be here or in (xref ref="s-effective1" text="type-global" /)?.[NUM]
To Do Decide if this should be filled out more or if it should reference the variety of places where the text fills out these ki

Chapter 5 1-D Motion, Relating position, Velocity, and Acceleration

Section 5.1 How Physicists Use the Words (Notation)

To Do Decide if the [notation] goes here (xref ref="intro-acc" text="type-global" /) or in (xref ref="d-acc" text="type-gl

Section 5.2 Connecting the Concepts- distance equals rate times time

Subsection 5.2.2 Speed versus Velocity

To Do Discussion of speed as ${}^{\Delta x}/_{\Delta t}$.[NUM]
To Do Discussion of velocity as a vector.[NUM]

Section 5.3 Extending the Concepts: Changing How You Move

Subsection 5.3.1 Moving versus Speeding Up

To Do Decide if the [notation] goes (xref ref="d-acc" text="type-global" /) or in (xref ref="intro-acc" text="type-global"
To Do Is there a way to include something in the LaTeXversion but not in the HTML version? (such as 29 and 29).[NUM]

Section 5.5 Examples

Subsection 5.5.1 Freefall

To Do Maybe add another exercise?.[NUM]

Section 5.6 Complications

Subsection 5.6.3 Multi-Step Solutions

To Do finish (xref ref="ex-ceiling" text="type-global" /). Maybe make it two examples, instead of one?.[NUM]

Chapter 7 Force

Section 7.2 Connecting the Concepts: Newton's Laws

To Do check the implications of this footnote..[NUM]

Subsection 7.2.1 Translating Newton's First Law: The Law of Inertia

To Do s-Ff.html references this section, poorly. Suggestion from forum was to make the preceding "p" a "prelude" and the s

Subsection 7.2.2 Translating Newton's Second Law: The Equation Law

To Do maybe note the MKS-to-SI transition. maybe leave that in (xref ref="ss-units" text="type-global" /)..[NUM]
To Do Maybe these should be sidebyside with the figure above the example?.[NUM]
To Do Make this object a desk so that we can have Abdul and Beth helping you rearrange your room in your residence hall..
To Do As before, make this object a desk so that we can have Abdul and Beth helping you rearrange your room in your resi
To Do Merge (xref ref="ex-2Dfa" text="type-global" /) and (xref ref="ex-2Dfa2" text="type-global" /). Also reference the
To Do I like "answer" better, but it doesn't fit. "hint" fits, but I don't like it. I can't figure out how to extend that line. side

Subsection 7.2.3 Translating Newton's Third Law: Action & Reaction

To Do As before, make this object a desk so that we can have Abdul and Beth helping you rearrange your room in your resi
To Do Figure out how to color text..[NUM]
To Do As before, make this object a desk so that we can have Abdul and Beth helping you rearrange your room in your resi
To Do Make an exercise that calculates the acceleration of Carl and Diane..[NUM]

Section 7.4 Summary and Homework

To Do Add more conceptual questions.[NUM]
To Do Add more variety of problems..[NUM]

Chapter 8 The Many Types of Force

To Do add reference to the general theory.[NUM]

Section 8.1 Gravity at the Surface of the Earth

Subsection 8.1.1 Weight versus Mass

To Do Update (xref ref="s-SI-MKS" text="type-hybrid" /) with this information..[NUM]

Subsection 8.1.2 Calculating the weight

To Do Gather values of g at various locations. Wiki has a list, but need to find the source. Wolfram has numbers, but they s

Section 8.3 Normal Force

To Do Do we need to repeat the example here? no?.[NUM]
To Do (xref ref="irl-poolcushion" text="type-hybrid" /) should be moved to a section that has more about friction and a
To Do This answer is getting too complex for the section it is in. I need to move the IRL before I finish considering how t

Subsection 8.3.1 Bathroom Scales Measure the Normal Force

To Do link equilibrium?.[NUM]
To Do link the gravitational force?.[NUM]
To Do link the gravitational force.[NUM]
To Do link the gravitational force.[NUM]
To Do link to the section on friction and ramps.[NUM]
To Do link to the trig section.[NUM]
To Do link "can use" to the section on ramps.[NUM]

Subsection 8.3.2 The normal force and inclined planes

To Do Should radially outwards reference the section on math?.[NUM]

Section 8.4 Frictional Force

Subsection 8.4.1 Dry Sliding Friction

To Do Use this link?.[NUM]
To Do Should maybe put an inclined plane in the normal force section to indicate that normal force does not always point
To Do Create this example.[NUM]
To Do Collect these in a references section? Esp. the Derler paper.[NUM]

Section 8.5 Tension

To Do add a link to (and the section itself) to a section on the modulus and stress/strain..[NUM]
To Do Maybe add an IRL about a house settling and the compression forces. Loading a pick-up truck and watching the b
To Do Still need to update the (xref ref="irl-tension" text="type-global"/)..[NUM]
To Do Is this sufficiently noticeable?.[NUM]
To Do maybe add links about the tension chaning in the rope.[NUM]
To Do maybe add links to the stretch of the rope.[NUM]

Subsection 8.5.1 Tension as a Support Force

To Do Add (insert)a description of the(/insert) (stale)an(/stale) image of an immovable surface to that section.[NUM]

Subsection 8.5.3 Pulleys

To Do add a reference to the section (problem?) where this is considered..[NUM]

Subsection 8.5.4 Interesting Complications

To Do YOU ARE HERE. Finish this example.[NUM]
To Do imported a homework problem from Giordano. Need to modify it to fit my purposes..[NUM]

Chapter 11 Rotational Motion

Section 11.1 How Physicists Use the Words

To Do Needs more detail and needs to be done better (magnitude and direction). Define the right-hand-rule..[NUM]
To Do Link this relative motion to the section on inertial reference frames..[NUM]
To Do Link this relative motion to the section on inertial reference frames..[NUM]

Chapter 13 Torque and the $F = ma$ of Rotations

Section 13.4 Summary and Homework

Subsection 13.4.2 Conceptual Questions

To Do Add conceptual problems..[NUM]
To Do Add more problems..[NUM]
To Do Make this a different homework problem..[NUM]

Chapter 15 The Gravitational Force on a Large Scale

Section 15.1 Gravitational Force and Field

To Do Reference a table of g on other planets and compute the weight of a space craft at each planet..[NUM]

Chapter 33 Deeper Dive

To Do This chapter should mirror (xref ref="c-physics" text="type-global" /)..[NUM]

Section 33.1 [Need title]

Subsection 33.1.4 Cool Ideas

To Do Follow-up in 2018 to find the results..[NUM]

Chapter 35 Characters

To Do The index will recognize the people in two different formats. One is by my name for them, which is Xerxes(where X i
To Do Check archivos.digital for storyboarding. $60/year.[NUM]

Index

This article was authored in PreTeXt.