A Study of the Static-Light B_B Parameter

Joseph Christensen*, Terrence Draper and Craig McNeile^{† a ‡}

^aDepartment of Physics and Astronomy, University of Kentucky, Lexington, KY 40506

We calculate the B_B parameter, relevant for $\overline{B}{}^0-B^0$ mixing, from a lattice gauge theory simulation using the static approximation for the heavy quark and the Wilson action for the light quark and gauge fields. Improved sources, produced by an optimized variational technique, MOST, reduce statistical errors and minimize excited-state contamination of the ground-state signal. Renormalization of four-fermion operator coefficients, using the Lepage-Mackenzie procedure for estimating typical momentum scales, is linearized to reduce order α_s^2 uncertainties.

1. B_B Parameter

Since the lattice static effective theory has fewer symmetries than the full continuum theory, when calculating the static-light B_B parameter

$$B_{B} = \frac{\left\langle \overline{B}_{0} \left| \mathcal{O}_{L}^{\text{full}} \right| B_{0} \right\rangle}{\left\langle \overline{B}_{0} \left| \mathcal{O}_{L}^{\text{full}} \right| B_{0} \right\rangle_{\text{no}}} \tag{1}$$

operators besides $\mathcal{O}_L^{\mathrm{latt}}$ must be included. These correspond to the following full-theory fermion operators (see Flynn *et al.* [1]):

$$\mathcal{O}_{L} = \frac{1}{2} \left(\overline{b} \gamma^{\mu} P_{L} q \right) \left(\overline{b} \gamma_{\mu} P_{L} q \right)
\mathcal{O}_{R} = \frac{1}{2} \left(\overline{b} \gamma^{\mu} P_{R} q \right) \left(\overline{b} \gamma_{\mu} P_{R} q \right)
\mathcal{O}_{N} = \frac{1}{2} \left[\left(\overline{b} \gamma^{\mu} P_{L} q \right) \left(\overline{b} \gamma_{\mu} P_{R} q \right) + \left(\overline{b} \gamma^{\mu} P_{R} q \right) \left(\overline{b} \gamma_{\mu} P_{L} q \right) + \left(\overline{b} P_{L} q \right) \left(\overline{b} P_{L} q \right) \left(\overline{b} P_{R} q \right) + 2 \left(\overline{b} P_{R} q \right) \left(\overline{b} P_{L} q \right) \right]
\mathcal{O}_{S} = \frac{1}{2} \left(\overline{b} P_{L} q \right) \left(\overline{b} P_{L} q \right) \tag{2}$$

 \mathcal{O}_S is generated at order α_s in the continuum due to the mass of the heavy quark. \mathcal{O}_R and \mathcal{O}_N are generated at order α_s from the chiral symmetry

breaking Wilson mass term. The lattice calculation of the static-light B_B uses the ratio of two-and three-point hadronic correlation functions.

$$B(t_1, t_2) = \frac{C_3(t_1, t_2)}{8/3C_2(t_1)C_2(t_2)} \xrightarrow{|t_i| \gg 1} B_B$$
 (3)

where the required correlation functions are

$$\begin{split} C_3(t_1,t_2) &= \sum_{\vec{x}_1,\vec{x}_2} \left\langle 0 \left| \mathcal{T} \left(\chi(t_1,\vec{x}_1) \, \overline{b}(0,\vec{0}) \Gamma_{\!\!I} q(0,\vec{0}) \right. \right. \right. \\ &\left. \overline{b}(0,\vec{0}) \Gamma_{\!\!J} q(0,\vec{0}) \chi(t_2,\vec{x}_2) \right) \right| 0 \right\rangle \\ C_2(t_1) &= \sum_{\vec{x}_1} \left\langle 0 \left| \mathcal{T} \left(\chi(t_1,\vec{x}_1) \, \overline{b}(0,\vec{0}) \gamma_4 \gamma_5 q(0,\vec{0}) \right) \right| 0 \right\rangle \end{split}$$

The three-point function has a fermion operator inserted at the spacetime origin, between two external B-meson interpolating fields. The times are restricted to the range $t_2 > 0 > t_1$. The gamma matrices, Γ_I and Γ_J define the type of four fermion operator (see equation 2). A spatially extended B-meson operator

$$\chi(t,\vec{x}) = \sum_{\vec{r}} f(\vec{r}) \, \overline{q}(t,\vec{x} + \vec{r}) \, \gamma_5 b(t,\vec{x}) \tag{4}$$

is used, where f is a smearing function produced by MOST [2] for our static f_B study.

2. Scale Formulation

Using the integrand of the one-loop perturbative contribution from the coefficients as a weighting function, as per Lepage and Mackenzie [3],

^{*}Presented by J. Christensen at Lattice '96, St. Louis. †Currently at Department of Physics, University of Utah, Salt Lake City, UT 84112.

[‡]This work is supported in part by the U.S. Department of Energy under grant numbers DE-FG05-84ER40154 and DE-FC02-91ER75661, and by the University of Kentucky Center for Computational Sciences. The computations were carried out at NERSC.

| | $\langle \mathcal{O}_L angle$ | $\left\langle \mathcal{O}_L^{\mathrm{full}} ight angle$ | $\langle A_{\mu} \rangle$ | B_B |
|----------------|--------------------------------|--|---------------------------|-------|
| $q_i^{\star}a$ | 2.01 | 2.15 | 2.18 | 0.82 |

Table 1

"Typical" operator scales; using $\beta = 6.0$ and r = 1.

a "typical" momentum scale can be found (Table 1). Our value for the scale relevant for $\langle A_{\mu} \rangle$ agrees with that found by Hernández and Hill [4]. This is the scale which we claim is relevant for this calculation as well. We notice that the scale found for B_B is singularly different than the others and claim that each of the other matrix elements is describing physics at essentially the same scale. However, when a ratio is considered, the integrands should cancel, but the scale should not. Since the other values are similar, we choose 2.18, as it has been used for the f_B study.

3. Calculation of the Coefficients

The coefficients of the operators are calculated [1,5,6] by renormalization group techniques.

$$B_{B}^{\text{full}}(m_{b}) = \sum_{i=L,R,N,S} Z_{B_{i}} B_{i}^{l}$$

$$Z_{B_{L}} = \left(\frac{\alpha_{s}^{c}(m_{b})}{\alpha_{s}^{c}(\mu)}\right)^{\left(p_{0,L}^{c} - 2p_{0,f}^{c}\right)} \left[1 + \frac{\alpha_{s}^{c}(m_{b}) - \alpha_{s}^{c}(\mu)}{4\pi} \left(p_{1,L}^{c} - 2p_{1,f}^{c}\right) + \frac{\alpha_{s}^{c}(m_{b}) - \alpha_{s}^{c}(\mu)}{3\pi} \left(\frac{-13}{2}\right) + \frac{\alpha_{s}^{l}(\mu)}{3\pi} (11.56)\right]$$

$$Z_{B_{R}} = \left(\frac{\alpha_{s}^{c}(m_{b})}{\alpha_{s}^{c}(\mu)}\right)^{\left(p_{0,L}^{c} - 2p_{0,f}^{c}\right)} \frac{\alpha_{s}^{l}(\mu)}{3\pi} (-1.205)$$

$$Z_{B_{N}} = \left(\frac{\alpha_{s}^{c}(m_{b})}{\alpha_{s}^{c}(\mu)}\right)^{\left(p_{0,L}^{c} - 2p_{0,f}^{c}\right)} \frac{\alpha_{s}^{l}(\mu)}{3\pi} (-10.83)$$

$$Z_{B_{S}} = \left(\frac{\alpha_{s}^{c}(m_{b})}{\alpha_{s}^{c}(\mu)}\right)^{\left(p_{0,S}^{c} - 2p_{0,f}^{c}\right)} \frac{\alpha_{s}^{c}(m_{b})}{3\pi} (-6)$$

where $p_{0,i} = \frac{\gamma_{0,i}}{2b_0}$, and $p_{1,i} = p_{0,i} \left(\frac{\gamma_{1,i}}{\gamma_{0,i}} - \frac{b_1}{b_0} \right)$. For α_s^l , we use $\alpha_V(q^*) = 0.18$ [3].

The statistical uncertainties for the coefficients

The statistical uncertainties for the coefficients are listed in Table 2. There is a systematic error due to the linearization of the coefficients which is not listed. See reference [7] for complete details.

4. Results of Simulation

The raw lattice B parameters for the operators which appear in the lattice-continuum matching are determined from a Monte Carlo calculation of equation 3 and listed in Table 3. Table 4 lists the linear combination $B_{\mathcal{O}_L^{\text{full}}} = B_B$ as a function of κ and extrapolated to κ_c using fully-linearized tadpole-improved coefficients. For both tables, the first errors are statistical (bootstrap) and the second are systematic due to choice of fit range.

We find $B_B(m_b)=0.98^{+4}_{-4}(3)^{+1}_{-2}$ as our calculated value. The first two errors are as mentioned above. The final error is due to the statistical uncertainties in the coefficients. If we run to a scale of μ =2 GeV, with n_f =4, using

$$B_B(\mu) = \left(\frac{\alpha_s^c(\mu)}{\alpha_s^c(m_b)}\right)^{\left(p_{0,L}^c - 2p_{0,f}^c\right)} B_B(m_b) \tag{6}$$

we find $B_B(2 \text{ GeV})=1.05(4)$. When we convert $B_B(m_b)$ to a RG invariant quantity using

$$\hat{B}_B = (\alpha_s^c(m_b))^{-(p_{0,L}^c - 2p_{0,f}^c)} B_B(m_b)$$
 (7)

with 4 flavors, we find $\hat{B}_B=1.36(6)$. With 5 flavors, we find $\hat{B}_B=1.40(6)$.

5. Comparison to Others

The simulations using Wilson quarks calculate the B_B parameter for quark masses around charm and extrapolate up to the physical mass, using a fit model of the form $B = B^0 + \frac{B^1}{M}$. The value of B^0 ("extrapolated static" in Table 5) should be the same as the static theory. We scale the authors' numbers to 2.0 GeV and 4.33 GeV. The JLQCD collaboration cite their Λ as $n_f=0$ values. When Abada et al. quote a \hat{B}_B for the propagating Wilson quarks, they use $n_f=0$. When we scale these, we list values for both $n_f=0$ and $n_f=4$. Soni quotes numbers at 2.0 GeV, but no Λ is given. We use our value for $\Lambda^{(4)}$ as well as 200 MeV. We calculate $B_B(m_b)$, scale it to 2.0 GeV using $n_f=4$, and calculate a \hat{B}_B with both 4 and 5 flavors. Since all of the "raw" values are close to 1.0, differences between the estimates of the static B_B are due not to the choice of action, but to choices in the coefficients. See [7] for the justification of our choice.

| | | | | ,_, | | | | | | |
|-------------------|--------------|-------------------|---------------------|---------------------|---------|--|--|--|--|--|
| | $q^{\star}a$ | a^{-1} | m_b | $\Lambda_c^{(5)}$ | All | | | | | |
| | 2.18 | $2.1\mathrm{GeV}$ | $4.33\mathrm{GeV}$ | $0.175\mathrm{GeV}$ | | | | | | |
| $Z_{B_L} = 1.070$ | | | | | | | | | | |
| 10% | +0.002 | +0.003 | +0.003 | +0.0008 | +0.004 | | | | | |
| 1070 | -0.002 | -0.004 | -0.003 | -0.0005 | -0.005 | | | | | |
| 20% | +0.005 | +0.006 | +0.006 | +0.0019 | +0.008 | | | | | |
| 2070 | -0.003 | -0.009 | -0.005 | -0.0009 | -0.009 | | | | | |
| | | Z_{B_R} | =-0.0225 | | | | | | | |
| 10% | +0.0005 | | | | +0.0005 | | | | | |
| 1070 | -0.0006 | | | | -0.0006 | | | | | |
| 20% | +0.0009 | | | | +0.0009 | | | | | |
| 2070 | -0.0015 | | | | -0.0015 | | | | | |
| | | Z_{B_I} | $_{\rm v} = -0.202$ | | | | | | | |
| 10% | +0.005 | | | | +0.005 | | | | | |
| 1070 | -0.006 | | | | -0.006 | | | | | |
| 20% | +0.008 | | | | +0.008 | | | | | |
| 20% | -0.012 | | | | -0.012 | | | | | |
| | | Z_{B_S} | $_{\rm S} = -0.137$ | | | | | | | |
| 10% | | | +0.003 | +0.002 | +0.003 | | | | | |
| 10/0 | | | -0.003 | -0.003 | -0.004 | | | | | |
| 20% | | | +0.006 | +0.005 | +0.006 | | | | | |
| 20% | | | -0.005 | -0.007 | -0.008 | | | | | |

Table 2 The absolute changes from our preferred values of the coefficients Z_{BL} , Z_{BR} , Z_{BN} , and Z_{BS} as the parameters q^*a , a^{-1} , m_b , and $\Lambda_c^{(5)}$ are varied by 10%, and 20%, first individually, and then jointly ("All"), from our preferred input values. The coefficients are quite insensitive to the particular choice of input parameter.

| | $\kappa = 0.152$ | $\kappa = 0.154$ | $\kappa = 0.155$ | $\kappa = 0.156$ | $\kappa_c = 0.157$ |
|-------------------|-------------------------------|-------------------------------|---------------------|---------------------|---------------------|
| B_L | | $1.02^{+2}_{-2}(1)$ | | | $1.03^{+3}_{-3}(2)$ |
| B_R | | | $0.95^{+2}_{-2}(2)$ | $0.95^{+2}_{-3}(2)$ | $0.95^{+2}_{-3}(2)$ |
| B_N | $0.97^{+2}_{-2}(3)$ | $0.96^{+2}_{-2}(4)$ | $0.96^{+2}_{-2}(4)$ | $0.96^{+3}_{-2}(4)$ | $0.95^{+3}_{-2}(5)$ |
| $-\frac{8}{5}B_S$ | $1.00^{+\frac{1}{2}}_{-1}(2)$ | $1.00^{+\frac{7}{2}}_{-2}(2)$ | $1.00^{+2}_{-2}(3)$ | $1.01^{+3}_{-3}(3)$ | $1.01^{+3}_{-3}(3)$ |

Table 3

The raw lattice values for the various B_i parameters: \mathcal{O}_S has a vacuum-saturation value different from that of \mathcal{O}_L ; with a normalization in which the raw B_i have a common denominator equal to the vacuum-saturated value of \mathcal{O}_L , $\left(-\frac{8}{5}B_S\right)$ would identically equal 1.0 if vacuum-saturation were exact.

| | | | | $\kappa = 0.156$ | 0 |
|------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| $B_B(m_b)$ | $0.95^{+2}_{-2}(1)$ | $0.96^{+3}_{-2}(2)$ | $0.96^{+3}_{-3}(2)$ | $0.98^{+4}_{-4}(2)$ | $0.98^{+4}_{-4}(3)$ |

Table 4 $B_B(m_b)$ is calculated by combining the raw B_i parameters with the appropriate coefficients.

| | scale | | | one-loop | | | | | |
|----------------|----------------------|-----------|--------------|-----------|-------|-----------|----------|----------|-------------|
| Method | ref | β | (GeV) | B(scale) | n_f | Λ | B(2.0) | B(4.33) | \hat{B}_B |
| Static-Clover | [8] | 6.2 | $m_b = 5.0$ | 0.69(4) | 5 | 130 | - | - | 1.02(6) |
| | | 6.2 | $m_b = 5.0$ | 0.69(4) | 4 | 200 | 0.75(4) | 0.70(4) | 0.98(6) |
| Static-Wilson | this | 6.0 | $m_b = 4.33$ | 0.98(4) | 5 | 175 | - | - | 1.40(6) |
| | work | 6.0 | $m_b = 4.33$ | 0.98(4) | 4 | 226 | 1.05(4) | 0.98(4) | 1.36(6) |
| Extrap. Static | [9] | 5.7-6.3 | $\mu = 2.0$ | 1.04(5) | 4 | 200 | 1.04(5) | 0.97(5) | 1.36(7) |
| | | 5.7 - 6.3 | $\mu = 2.0$ | 1.04(5) | 4 | 226 | 1.04(5) | 0.97(5) | 1.34(6) |
| Extrap. Static | [10] | 6.4 | $\mu = 3.7$ | 0.90(5) | 0 | 200 | 0.94(5) | 0.89(5) | 1.21(7) |
| | | 6.4 | $\mu = 3.7$ | 0.90(5) | 4 | 200 | 0.95(5) | 0.89(5) | 1.25(7) |
| Wilson-Wilson | [9] | 5.7-6.3 | $\mu = 2.0$ | 0.96(6) | 4 | 200 | 0.96(6) | 0.90(6) | 1.25(8) |
| | | 5.7 - 6.3 | $\mu = 2.0$ | 0.96(6) | 4 | 226 | 0.96(6) | 0.89(6) | 1.24(8) |
| Wilson-Wilson | [9,11] | 6.1 | $\mu = 2.0$ | 1.01(15) | 4 | 200 | 1.01(15) | 0.94(13) | 1.32(20) |
| | | 6.1 | $\mu = 2.0$ | 1.01(15) | 4 | 226 | 1.01(15) | 0.94(14) | 1.30(19) |
| Wilson-Wilson | [12] | 6.1 | $m_b = 5.0$ | 0.895(47) | 0 | 239 | 0.96(5) | 0.90(5) | 1.21(6) |
| | | 6.1 | $m_b = 5.0$ | 0.895(47) | 4 | 239 | 0.98(5) | 0.91(5) | 1.25(7) |
| | | 6.1 | $m_b = 5.0$ | 0.895(47) | 5 | 183 | - | - | 1.29(7) |
| Wilson-Wilson | [12] | 6.3 | $m_b = 5.0$ | 0.840(60) | 0 | 246 | 0.90(6) | 0.85(6) | 1.14(8) |
| | | 6.3 | $m_b = 5.0$ | 0.840(60) | 4 | 246 | 0.92(6) | 0.85(6) | 1.17(8) |
| | | 6.3 | $m_b = 5.0$ | 0.840(60) | 5 | 189 | - | - | 1.20(9) |
| Wilson-Wilson | [10] | 6.4 | $\mu = 3.7$ | 0.86(5) | 0 | 200 | 0.90(5) | 0.85(5) | 1.16(7) |
| | | 6.4 | $\mu = 3.7$ | 0.86(5) | 4 | 200 | 0.91(5) | 0.85(5) | 1.19(7) |
| Sum Rule | [13] | | $m_b = 4.6$ | 1.00(15) | 5 | 175 | - | - | 1.43(22) |
| | | | $m_b = 4.6$ | 1.00(15) | 4 | 227 | 1.08(16) | 1.00(15) | 1.39(21) |

Table 5

The authors' numbers, quoted at the listed scale, have been scaled to μ =2.0 GeV and to m_b =4.33 GeV. If the authors quote a number which we used or reproduced, it is bold-faced in the table.

REFERENCES

- 1. J.M. Flynn, O.F. Hernández and B. Hill, Phys. Rev. D **43**, 3709 (1991).
- T. Draper and C. McNeile, in *Lattice '93*, Proceedings of the International Symposium, Dallas, Texas, 1993, edited by T. Draper *et al.* (*Nucl. Phys. B (Proc. Suppl.)* 34, 453, 1994).
- G.P. Lepage and P.B. Mackenzie, Phys. Rev. D 48, 2250 (1993).
- 4. O.F. Hernández and B.R. Hill, Phys. Rev. D **50**, 495 (1994).
- 5. V. Giménez, Nucl. Phys. **B401**, 116 (1993).
- 6. G. Buchalla, Renormalization of $\Delta B=2$ Transitions in the Static Limit Beyond Leading Logarithms, hep-ph/9608232, 1996.
- 7. J. Christensen, T. Draper and C. McNeile, to be published.

- (UKQCD Collaboration) A.K. Ewing et al., Heavy Quark Spectroscopy and Matrix Elements: A Lattice Study using the Static Approximation, hep-lat 9508030, 1995.
- 9. A. Soni, Weak Matrix Elements on the Lattice Circa 1995, hep-lat 9510036, 1995.
- A. Abada *et al.*, Nucl. Phys. **B376**, 172 (1992).
- C. Bernard, T. Draper, G. Hockney and A. Soni, Phys. Rev. D 38, 3540 (1988).
- 12. JLQCD Collaboration, Heavy-Light Matrix Elements with the Wilson Quark Action, heplat 9510033, 1995.
- S. Narison and A.A. Pivovarov, Phys. Lett. 327B, 341 (1994).