Lattice HQET Calculation of the Isgur-Wise Function

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We calculate the Isgur-Wise function on the lattice, simulating the light quark with the Wilson action and the heavy quark with a direct lattice implementation of the heavy-quark effective theory. Improved smearing functions produced by a variational technique, MOST, are used to reduce the statistical errors and to minimize excited-state contamination of the ground-state signal. Calculating the required matching factors, we obtain $\xi'(1) = -0.64(13)$ for the slope of the Isgur-Wise function in continuum-HQET in the $\overline{\mathrm{MS}}$ scheme at a scale of 4.0 GeV.

1. The Tadpole-Improved Simulation

The Isgur-Wise function is the form factor of a heavy-light meson in which the heavy quark is taken to be much heavier than the energy scale, $m_Q \gg \Lambda_{\mathrm{QCD}}$. This calculation adds perturbative corrections to the simulation results of Draper & McNeile [1]. The Isgur-Wise function is calculated using the action first suggested by Mandula & Ogilvie [2]:

$$\begin{aligned} iS \;=\; \sum_x \Bigg\{ & v_0 \left[\psi^\dagger(x)\,\psi(x) - \psi^\dagger(x)\,\frac{U_t(x)}{u_0}\psi\big(x+\hat{t}\big) \right] \\ & + \sum_{j=1}^{3} \frac{-iv_j}{2} \Bigg[\psi^\dagger(x)\,\frac{U_j(x)}{u_0}\psi\Big(x+\hat{j}\Big) \\ & - \psi^\dagger(x)\,\frac{U_j^\dagger(x-\hat{j})}{u_0}\psi\Big(x-\hat{j}\Big) \Bigg] \Bigg\} \end{aligned}$$

This leads to the evolution equation:

$$\begin{aligned} G(x+\hat{t}) = \frac{U_t^\dagger(x)}{u_0} \Bigg\{ & G(x) \\ - \sum_{j=1}^{3} \frac{i\tilde{v}_j}{2} & \left[\frac{U_j(x)}{u_0} G(x+\hat{j}) - \frac{U_j^\dagger(x-\hat{j})}{u_0} G(x-\hat{j}) \right] \Bigg\} \end{aligned}$$

where $\tilde{v}_j = \frac{v_j}{v_0}$ and $G(\vec{x}, t=0) = \frac{1}{v_0} f(\vec{x})$. Better results can be obtained when a smeared source is used in place of a point source. The smearing function, $f(\vec{x})$, was calculated from a static simulation ($v_j = 0$) *via* the smearing technique MOST (Maximal Operator Smearing Technique [3]).

2. The Isgur-Wise Function

The Isgur-Wise function was extracted from the lattice simulation as a ratio of three-point functions which was suggested by Mandula & Ogilvie [2]. $\left|\xi^{\mathrm{lat}}_{\mathrm{unren}}(v \cdot v')\right|^2$ is the large Δt limit of

$$\frac{4v_0v_0'}{(v_0+v_0')^2} \frac{C_3^{vv'}(\Delta t) C_3^{v'v}(\Delta t)}{C_3^{vv}(\Delta t) C_3^{v'v'}(\Delta t)}$$

where Δt is the time separation between the current operator and each B-meson interpolating field.

Draper & McNeile have presented [1] the non-tadpole-improved unrenormalized slope of the lattice Isgur-Wise function to demonstrate the efficacy of the computational techniques.

3. Tadpole Improvement for HQET

Tadpole improvement grew from the observation that lattice links, U, have mean field value, $u_0 \neq 1$. Therefore, it is better to use an action written as a function of $({}^{U}\!/_{u_0})$. In the Wilson action, each link has a coefficient κ; u_0 can be paired with κ to easily tadpole improve *a posteriori* any previous non-tadpole-improved calculation.

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In the HQET, there is no common coefficient (analogous to κ) for both U_t and U_j. Correspondence between tadpole-improved and non-tadpole-improved HQET actions with $v^2 = 1$ cannot be made *via* a simple rescaling of parameters, as is done for the Wilson action with κ.

Fortunately, the evolution equation can be written (as noticed by Mandula & Ogilvie [4]) such that the u_0 is grouped with $\tilde{v}_j$. Thus, tadpole-improved Monte-Carlo data can be *constructed* from the non-tadpole-improved data by replacing $v^{\rm nt} \to v^{\rm tad}$ and by including two overall multiplicative factors:

$$G^{\rm tad}(t; \tilde{v}^{\rm tad}, v_0^{\rm tad}) \;=\; u_0^{-t} \frac{v_0^{\rm nt}}{v_0^{\rm tad}} G^{\rm nt}(t; \tilde{v}^{\rm nt}, v_0^{\rm nt}) \quad (1)$$

In addition to the multiplicative factors u_0^{-t} and $v_0^{\rm nt}/v_0^{\rm tad}$, the tadpole-improvement of the simulation requires adjusting the velocity according to $\tilde{v}^{\rm tad} = u_0 \tilde{v}^{\rm nt}$, subject to $(v^{\rm tad})^2 = 1$ and $(v^{\rm nt})^2 = 1$. Thus,

$$\begin{aligned} v_0^{\rm tad} &= v_0^{\rm nt}[1 + (1-u_0^2)(v_j^{\rm nt})^2]^{-1/2} \\ v_j^{\rm tad} &= u_0 v_j^{\rm nt}[1 + (1-u_0^2)(v_j^{\rm nt})^2]^{-1/2} \end{aligned}$$

4. Tadpole-Improved Renormalization

By comparing the unrenormalized propagator

$$\left[v_0^b \left(\frac{e^{ik_4}}{u_0} - 1 \right) + \frac{v_z^b}{u_0} \sin(k_z) + M_0^b - \Sigma(k,v) \right]^{-1}$$

to the renormalized propagator

$$iH(k,v) \;=\; Z_Q \left[v_0^r(ik_4) + v_z^r(k_z) + M^r \right]^{-1}$$

at $O(k^2)$ and using $(v^r)^2 = (v^b)^2 = 1$, the perturbative renormalizations can be obtained. Aglietti [5] has done this for a different non-tadpole-improved action, for the special case $\vec{v} = v_z \hat{z}$.

With momentum shift, $p \to p' = \langle p_4 + i\ln(u_0), \vec{p} \rangle$ [6], with $1/u_0 \exp(ik_4) = \exp(i(k_4 + i\ln(u_0)))$ and with $X_\mu \equiv \frac{\partial}{\partial p_\mu} \Sigma(p)|_{p=0}$, the tadpole-improved perturbative renormalizations are found to be

$$\begin{aligned} \delta M &= -\Sigma(0) - v_0 \ln(u_0) \\ \delta Z_Q &= Z_Q - 1 \;=\; -iv_0 X_4 - u_0 \sum_{j=1}^{3} v_j X_j \\ \delta \tfrac{v_i}{u_0} &= -iv_0 \frac{v_i}{u_0} X_4 - (1+v_i^2) X_i - v_i \sum_{j \neq i} v_j X_j \\ \delta v_0 &= -i \sum_{j=1}^{3} v_j^2 X_4 - u_0 v_0 \sum_{j=1}^{3} v_j X_j \end{aligned}$$

u_0 is the perturbative expansion and [4]

$$v_j^{r,\rm tad} = v_j^{b,\rm tad} Z_{v_j}^{\rm tad} \quad , \quad v_0^{r,\rm tad} = v_0^{b,\rm tad} Z_{v_0}^{\rm tad}$$

$$Z_{v_j}^{\rm tad} \equiv \frac{1}{u_0} \left(1 + \frac{\delta \frac{v_j}{u_0}}{\frac{v_j}{u_0}} \right) \quad , \quad Z_{v_0}^{\rm tad} \equiv 1 + \frac{\delta v_0}{v_0}$$

If one fits to $\exp\{-t\}$ rather than $\exp\{-(t+1)\}$, the tadpole-improved wave-function renormalization is reduced to $\delta Z_Q' = \delta Z_Q + (\delta M^{\rm tad} + v_0 \ln(u_0))/v_0$. Thus the $\ln(u_0)$ term cancels explicitly and, as in the static case [6], tadpole-improvement has no effect on $\delta Z_Q'$, to order α.

5. Perturbative Renormalizations

We will present our computations of the renormalization factors elsewhere, but include this comment: Although the tadpole-improved functions include factors of $u_0|_{\rm pt}$ [7], these effects are higher order in α and are dropped. Only the velocity renormalization is explicitly affected by the perturbative expansion of u_0:

$$Z_{v_j}^{\rm tad} \;=\; \left(1 + \frac{\delta \frac{v_j}{u_0}}{\frac{v_j}{u_0}} - \frac{g^2 C_F}{16\pi^2}(-\pi^2) \right)$$

The perturbative renormalizations favor a scale of $q^* a = 1.9(1)$ for α, which yields $\alpha \approx 0.19(1)$.

6. Velocity Renormalization

Mandula & Ogilvie [4] consider the perturbative velocity renormalization expanded in orders of $\tilde{v}$. Our numbers for the velocity renormalization agree with theirs.

Another option is to consider, as did both Mandula & Ogilvie and Hashimoto & Matsufuru [8], the non-perturbative renormalization of the velocity. From Hashimoto's & Matsufuru's graph, we estimate their $Z_{v,\rm np}^{\rm tad} \approx 1.05(5)$. From Mandula & Ogilvie's result, we notice that $Z_{\tilde{v},\rm np}^{\rm nt} =$

[4] Note: $(v^2 = 1) \Rightarrow \left(v_0 \delta v_0 = u_0^2 \sum_j \frac{v_j}{u_0} \delta \frac{v_j}{u_0} \right)$.

$u_0 \times 1.01(1)$. This is very close to the effect of tadpole-improving, and implies $Z_{\tilde{v}}^{\rm tad} = 1.01(1)$.

We therefore use $Z_v^{\rm tad} \approx 1$ as the non-perturbative velocity renormalization in our calculation to renormalize the slope of the Isgur-Wise function.

7. Renormalization of $\xi'(v \cdot v')$

We claim that we can convert our Monte-Carlo data into tadpole-improved results and can calculate a renormalized tadpole-improved slope for the Isgur-Wise function.

We use the notation $Z'_\xi = 1 + \delta Z'_\xi$ for the renormalization of the Isgur-Wise function, with $\delta Z'_\xi$:

$$\frac{g^2}{12\pi^2}\left[2\left(1 - v{\cdot}v' r(v{\cdot}v')\right)\ln(\mu a)^2 - f'(v,v')\right]$$

with $r(v{\cdot}v')$ defined in [9] and primes on Z and f to indicate the "reduced value."

For simplicity, we use the local current, which is not conserved on the lattice; $Z'_\xi(1) \neq 1$. However, the construction in §2 guarantees that the extracted renormalized Isgur-Wise function is properly normalized, $\xi_{\rm ren}(1) = 1$.

8. Conclusions

After renormalization of our tadpole-improved results, we obtain $\xi'_{\rm ren}(1) = -0.64(13)$ for the slope of the Isgur-Wise function in continuum HQET in the $\overline{\rm MS}$ scheme at a scale of 4.0 GeV. Without renormalization, the slope is $\xi'^{\,\rm tad}_{\rm unren}(1) = -0.56(13)$. Without tadpole-improvement, the slope is $\xi'^{\,\rm nt}_{\rm unren}(1) = -0.43(10)$.

We found that the tadpole-improved action (and therefore the tadpole-improved data) cannot be obtained from the non-tadpole-improved action (or data) by a simple rescaling of any parameter. However, the form of the evolution equation allows the *construction* of the tadpole-improved Monte-Carlo data from the non-tadpole-improved data as described in §3. After tadpole improvement, non-perturbative corrections to the velocity are negligible. Furthermore, tadpole improvement greatly reduces the perturbative corrections to the slope of the Isgur-Wise function.

Unrenormalized Isgur-Wise Slope

Not Tadpole Improved				
Δt	0.154	0.155	0.156	κ_c
2	0.38^{+1}_{-1}	0.38^{+1}_{-1}	0.38^{+1}_{-1}	0.39^{+1}_{-1}
3	0.42^{+2}_{-2}	0.41^{+2}_{-2}	0.41^{+2}_{-2}	0.41^{+2}_{-2}
4	0.50^{+8}_{-9}	0.48^{+8}_{-9}	0.45^{+9}_{-10}	0.43^{+10}_{-10}
Tadpole Improved				
Δt	0.154	0.155	0.156	κ_c
2	0.49^{+1}_{-1}	0.50^{+1}_{-1}	0.50^{+1}_{-1}	0.50^{+1}_{-1}
3	0.55^{+3}_{-2}	0.54^{+3}_{-2}	0.54^{+3}_{-2}	0.53^{+3}_{-3}
4	0.65^{+10}_{-12}	0.63^{+11}_{-12}	0.59^{+12}_{-13}	0.57^{+13}_{-13}

Renormalized Isgur-Wise Slope

Tadpole Improved				
Δt	0.154	0.155	0.156	κ_c
2	0.57^{+1}_{-1}	0.57^{+1}_{-1}	0.58^{+1}_{-1}	0.58^{+1}_{-1}
3	0.62^{+3}_{-2}	0.62^{+3}_{-2}	0.61^{+3}_{-2}	0.61^{+3}_{-3}
4	0.72^{+12}_{-11}	0.71^{+11}_{-11}	0.67^{+13}_{-11}	0.64^{+13}_{-13}

Table 1
The negative of the slope at the normalization point, $\xi'(1)$, from both the unrenormalized and the renormalized ratio of three-point functions. This ratio gives the (un)-renormalized Isgur-Wise function $\xi(v{\cdot}v')$ at asymptotically-large times Δt.

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