Homework Chapter 04

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Question

use *repeated Simpson quadrature* and *repeated trapezoid quarature* Write a program to compute the integral

$$I(f) = \int_1^5 \sin(x)\mathrm{d}x,\ h = 0.1$$

Used function and algorithm.

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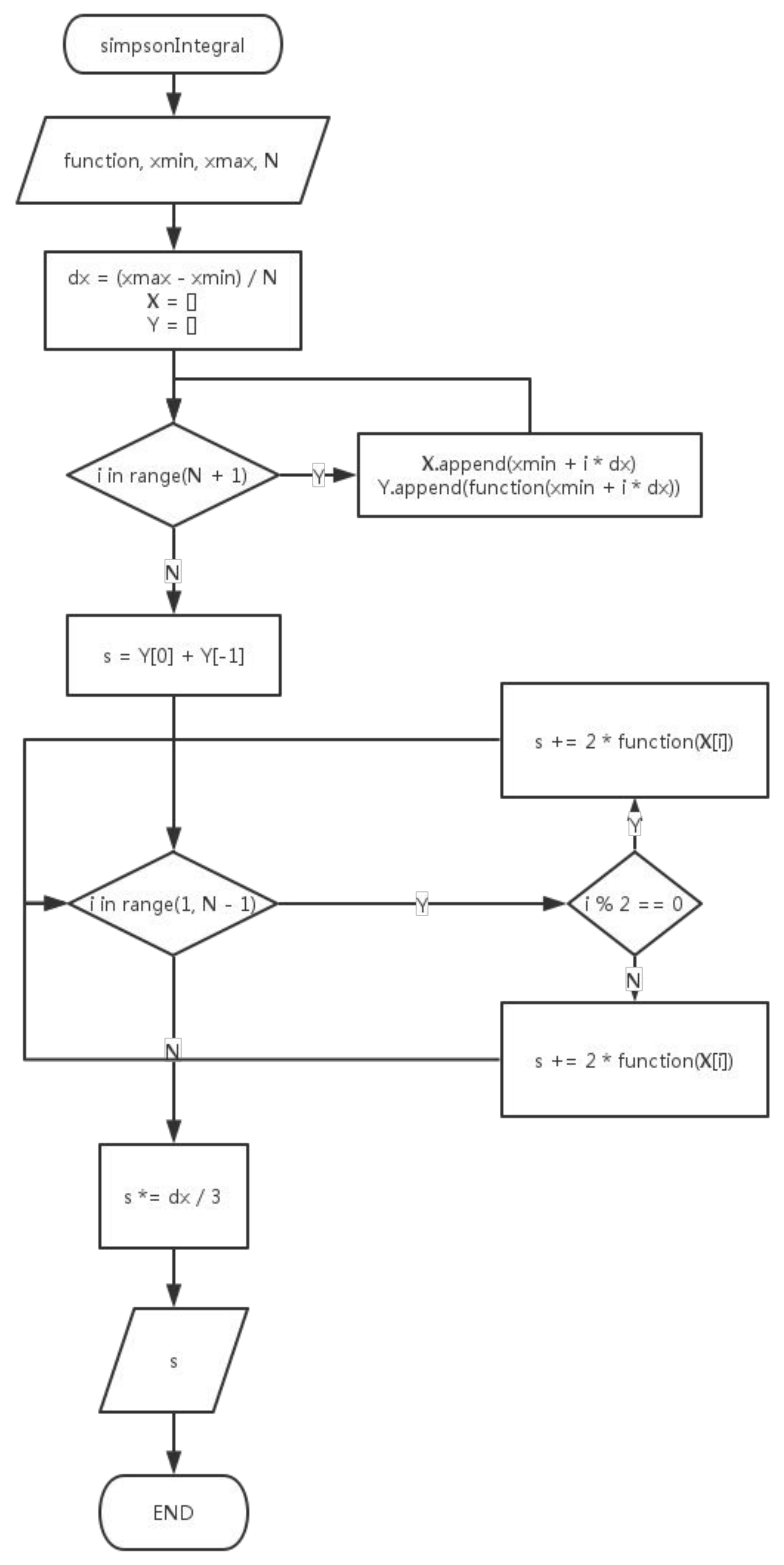
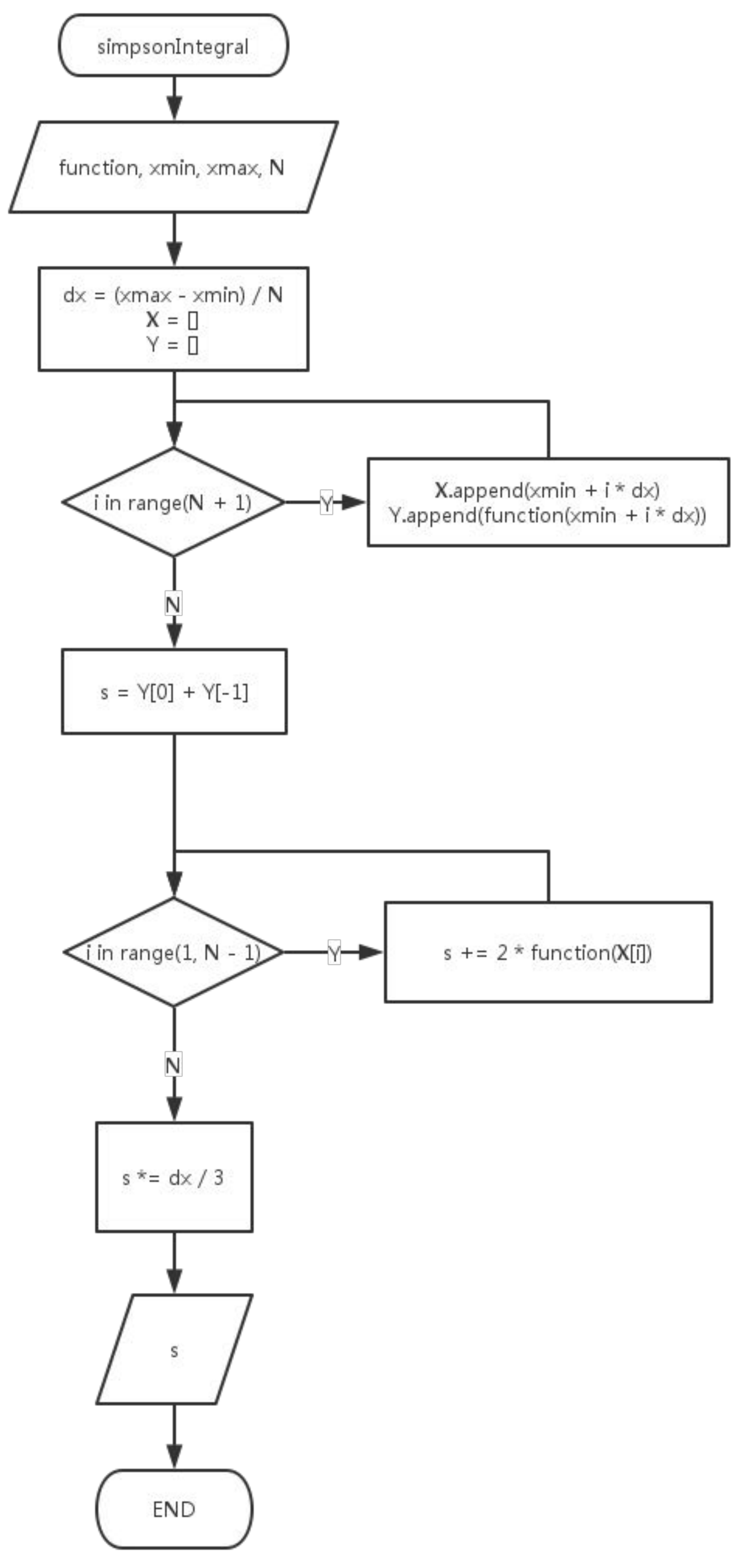
1. Simpson integrals

$$s = \frac{h}{2}\left(f(0) + 2\sum_{i=1}^{n-1} f(x_i) + f(n)\right.$$



2. Trapezoid integrals

$$s = \frac{h}{3}\left(f(0) + 4\sum_{i=1,i=2k}^{n-1} f(x_i) + 2\sum_{i=1,i=2k+1}^{n-1} f(x_i) + f(n)\right)$$

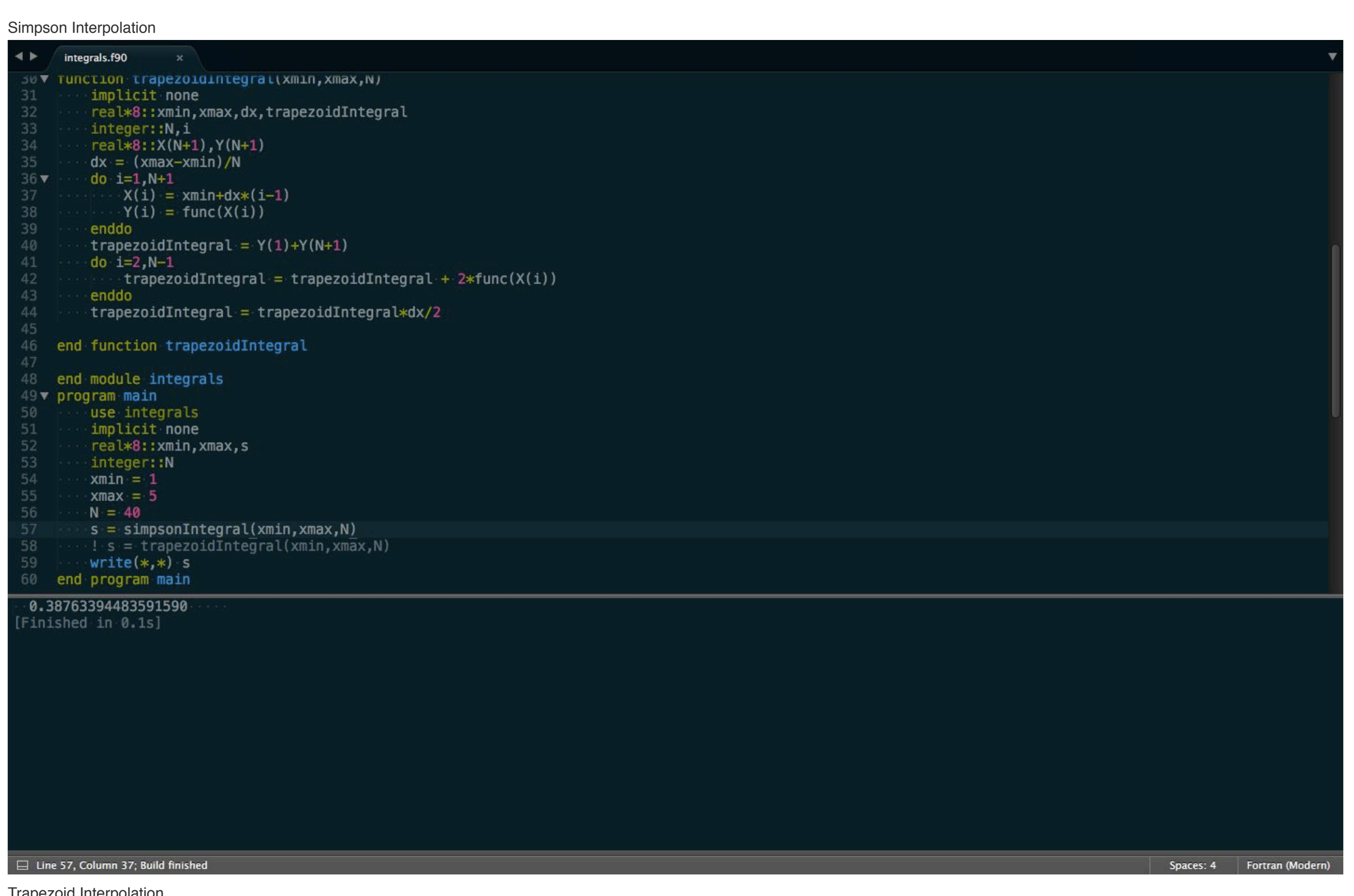


Source Code

```
module integrals
contains
function func(x)
    implicit none
    real*8::x,func
    func = sin(x)
end function func

function simpsonIntegral(xmin,xmax,N)
    implicit none
    real*8::xmin,xmax,dx,simpsonIntegral
    integer::N,i
    real*8::X(N+1),Y(N+1)
    dx = (xmax-xmin)/N
    do i=1,N+1
        X(i) = xmin+dx*(i-1)
        Y(i) = func(X(i))
    enddo
    simpsonIntegral = Y(1)+Y(N+1)
    do i=2,N-1
        if (mod(i,2) .eq. 0) then
            simpsonIntegral = simpsonIntegral+4 *func(X(i))
        else
            simpsonIntegral = simpsonIntegral+2*func(X(i))
        endif
    enddo
    simpsonIntegral = simpsonIntegral*dx/3
end function simpsonIntegral

function trapezoidIntegral(xmin,xmax,N)
    implicit none
    real*8::xmin,xmax,dx,trapezoidIntegral
    integer::N,i
    real*8::X(N+1),Y(N+1)
    dx = (xmax-xmin)/N
    do i=1,N+1
        X(i) = xmin+dx*(i-1)
        Y(i) = func(X(i))
    enddo
    trapezoidIntegral = Y(1)+Y(N+1)
    do i=2,N-1
        trapezoidIntegral = trapezoidIntegral + 2*func(X(i))
    enddo
    trapezoidIntegral = trapezoidIntegral*dx/2

end function trapezoidIntegral

end module integrals
program main
    use integrals
    implicit none
    real*8::xmin,xmax,s
    integer::N
    xmin = 1
    xmax = 5
    N = 40
    s = trapezoidIntegral(xmin,xmax,N)
    write(*,*) s
end program main
```

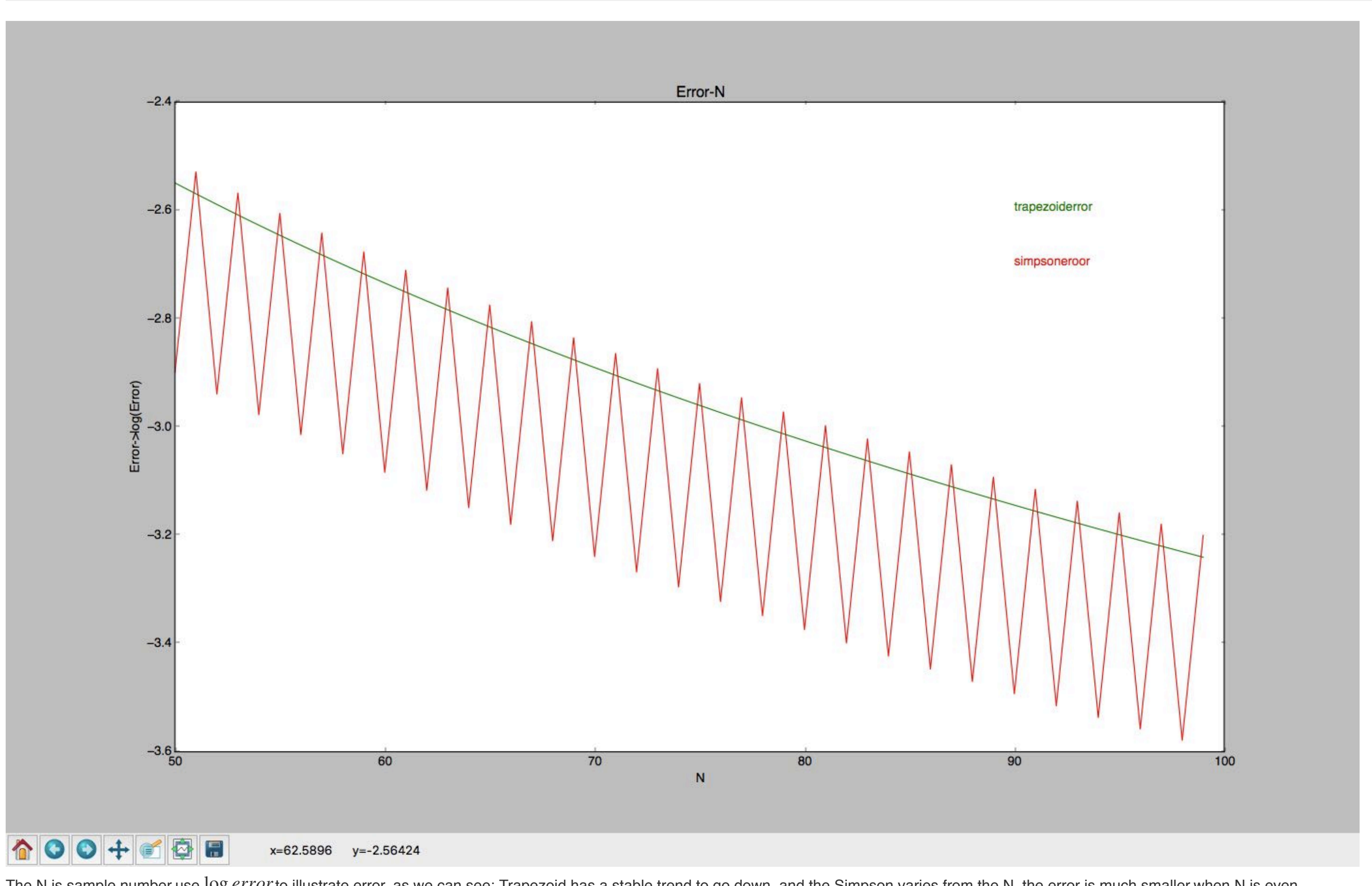
Running Screenshot

1. Simpson Interpolation



2. Trapezoid Interpolation

Error analytics



The N is sample number,use $\log error$ to illustrate error, as we can see: Trapezoid has a stable trend to go down, and the Simpson varies from the N, the error is much smaller,when N is even.

> The origin data has upload to Github