

# **Introduction to Sequential Parameter Optimization**

# Isotropic and Anisotropic Kriging

## 1 Example: Isotropic Spot Surrogate and the 2-dim Sphere Function

```
import numpy as np
from math import inf
from spotPython.fun.objectivefunctions import analytical
from spotPython.spot import spot
from scipy.optimize import shgo
from scipy.optimize import direct
from scipy.optimize import differential_evolution
import matplotlib.pyplot as plt
```

### The Objective Function: 2-dim Sphere

- The spotPython package provides several classes of objective functions.
- We will use an analytical objective function, i.e., a function that can be described by a (closed) formula:

$$f(x, y) = x^2 + y^2$$

```
fun = analytical().fun_sphere
fun_control = {"sigma": 0,
               "seed": 123}
```

- The size of the lower bound vector determines the problem dimension.
- Here we will use `np.array([-1, -1])`, i.e., a two-dim function.

```
spot_2 = spot.Spot(fun=fun,
                   lower = np.array([-1, -1]),
                   upper = np.array([1, 1]))

spot_2.run()
```

<spotPython.spot.spot.Spot at 0x29b563970>

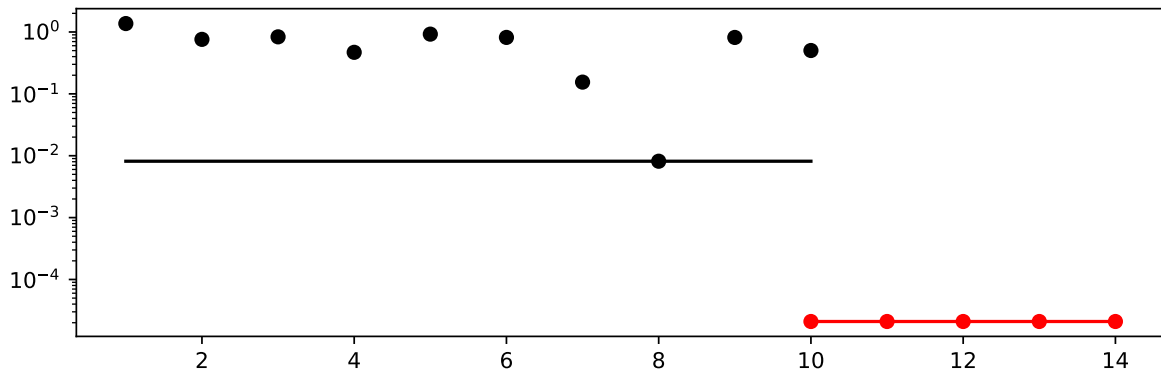
## Results

```
spot_2.print_results()
```

```
min y: 2.093282610941807e-05  
x0: 0.0016055267473267492  
x1: 0.00428428640184529
```

```
[['x0', 0.0016055267473267492], ['x1', 0.00428428640184529]]
```

```
spot_2.plot_progress(log_y=True)
```



## 2 Same, but with anisotropic Kriging

- The default parameter setting of `spotPython`'s Kriging surrogate uses the same `theta` value for every dimension.
- This is referred to as “using an isotropic kernel”.
- If different `theta` values are used for each dimension, then an anisotropic kernel is used.
- To enable anisotropic models in `spotPython`, the number of `theta` values should be larger than one.
- We can use `surrogate_control={"n_theta": 2}` to enable this behavior (2 is the problem dimension).

```
spot_2_anisotropic = spot.Spot(fun=fun,  
                                lower = np.array([-1, -1]),  
                                upper = np.array([1, 1]),
```

```
surrogate_control={"n_theta": 2})
spot_2_anisotropic.run()
```

```
<spotPython.spot.spot.Spot at 0x2c04aed10>
```

## Taking a Look at the theta Values

- We can check, whether one or several `theta` values were used.
- The `theta` values from the surrogate can be printed as follows:

```
spot_2_anisotropic.surrogate.theta
```

```
array([0.19447342, 0.30813872])
```

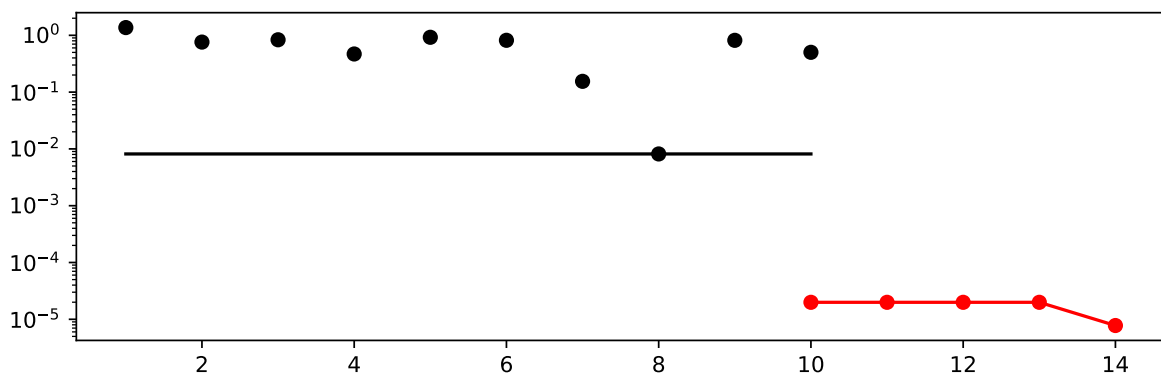
- Since the surrogate from the isotropic setting was stored as `spot_2`, we can also take a look at the `theta` value from this model:

```
spot_2.surrogate.theta
```

```
array([0.26287447])
```

- Next, the search progress of the optimization with the anisotropic model can be visualized:

```
spot_2_anisotropic.plot_progress(log_y=True)
```



```
spot_2_anisotropic.print_results()
```

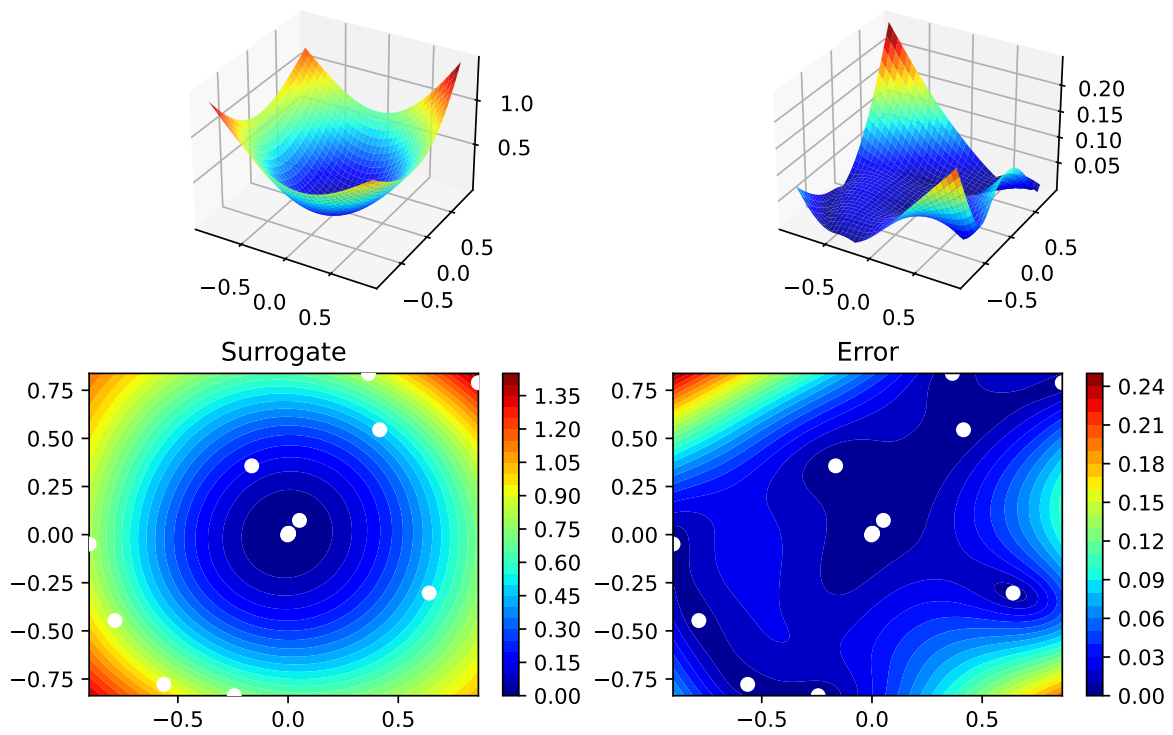
```
min y: 7.77061191821505e-06
```

```
x0: -0.0024488252797500764
```

```
x1: -0.0013318658594137815
```

```
[['x0', -0.0024488252797500764], ['x1', -0.0013318658594137815]]
```

```
spot_2_anisotropic.surrogate.plot()
```



# Exercises

- Important:
  - Results from these exercises should be added to this document, i.e., you should submit an updated version of this notebook.
  - Please combine your results using this notebook.
  - Only one notebook from each group!
  - Presentation is based on this notebook. No additional slides are required!
  - spotPython version 0.16.11 (or greater) is required (see <http://www.gm.fh-koeln.de/~bartz/site/download/>)

## Exercise 1

- Each team member should choose one of the following objective functions.
- Please add your name to the section title!

### 1. fun\_branin

- Describe the function.
  - The input dimension is 2. The search range is  $-5 \leq x_1 \leq 10$  and  $0 \leq x_2 \leq 15$ .
- Compare the results from spotPython run a) with isotropic and b) anisotropic surrogate models.
- Modify the termination criterion: instead of the number of evaluations (which is specified via fun\_evals), the time should be used as the termination criterion. This can be done as follows (max\_time=1 specifies a run time of one minute):

```
fun_evals=inf,  
max_time=1,
```

## 2. `fun_sin_cos`

- Describe the function.
  - The input dimension is 2. The search range is  $-2\pi \leq x_1 \leq 2\pi$  and  $-2\pi \leq x_2 \leq 2\pi$ .
- Compare the results from `spotPython` run a) with isotropic and b) anisotropic surrogate models.
- Modify the termination criterion (`max_time` instead of `fun_evals`) as described for `fun_branin`.

## 3. `fun_runge`

- Describe the function.
  - The input dimension is 2. The search range is  $-5 \leq x_1 \leq 5$  and  $-5 \leq x_2 \leq 5$ .
- Compare the results from `spotPython` run a) with isotropic and b) anisotropic surrogate models.
- Modify the termination criterion (`max_time` instead of `fun_evals`) as described for `fun_branin`.

## 4. `fun_wingwt`

- Describe the function.
  - The input dimension is 10. The search ranges are between 0 and 1 (values are mapped internally to their natural bounds).
- Compare the results from `spotPython` run a) with isotropic and b) anisotropic surrogate models.
- Modify the termination criterion (`max_time` instead of `fun_evals`) as described for `fun_branin`.

# Solutions

1. `fun_branin`
2. `fun_sin_cos`
3. `fun_runge`
4. `fun_wingwt`