Radiometric (Photometric Terms)

Φ Radiant (Luminous) Power or Flux
units W (lm)

I Radiant (Luminous) Intensity

$$I = \frac{\text{Power}}{\text{Solid Angle}} = \frac{d\Phi}{d\Omega}$$

units $\frac{W}{sr}$ or $\frac{lm}{sr} = cd$

M Radiant (Luminous) Emittance or Exitance

$$M = \frac{\text{Power}}{\text{unit area}} \quad \text{of a source}$$

E Irradiance (Illuminance)

Same as M but for a receiving area

$$M = \frac{d\Phi}{dA} \qquad \frac{W}{m^2} \quad \text{or} \quad \frac{lm}{m^2} = lux$$

$$E = \frac{d\Phi}{dA} \qquad \frac{W}{m^2} \quad \text{or} \quad \frac{lm}{m^2} = lux$$

L Radiance (Luminance) of a source

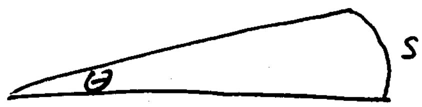
characterizes a source

$$L = \frac{\text{Power}}{(\text{Solid Angle})(\text{unit area})}$$

$$L = \frac{d^2\Phi}{d\Omega \, dA \, \cos\theta}$$

You are already use to measuring an angle using radians. Let's review the definition of radians.

From Wikipedia: "Radian is the ratio between the length of an arc and it's radius."



$$S = r\theta$$

$$\text{Full circle} = \frac{2\pi r}{r} = 2\pi$$

Now we extend the concept of a planar angle into 3-dimensions. It is called solid angle Ω

It is defined as the surface area divided by r^2. It has units of steradians.

A full sphere would have a solid angle of

$$\Omega = \frac{4\pi r^2}{r^2} = 4\pi \ Sr$$

What is the solid angle for a half angle of α?

$$d\Omega = \frac{dA}{r^2}$$

In spherical coordinates $d\Omega = \dfrac{r^2 \sin\theta \, d\phi \, d\theta}{r^2}$

$$d\Omega = \sin\theta \, d\theta \, d\phi$$

$$\Omega = \int_0^{2\pi} \int_0^{\alpha} \sin\theta \, d\theta \, d\phi$$

$$= 2\pi \left(-\cos\theta\right)\Big|_0^{\alpha}$$

$$= 2\pi \left(\cos 0 - \cos\alpha\right)$$

$$= 2\pi \left(1 - \cos\alpha\right)$$

If α is small we can use Taylor Series expansion.

$$\cos\alpha \approx 1 - \frac{\alpha^2}{2}$$

$$\Omega = 2\pi \left(1 - 1 + \frac{\alpha^2}{2}\right)$$

$$\Omega \approx \pi\alpha^2$$

SOLID Angle of the Moon:

distance from earth: 384,405 Km
diameter 1737.1 Km

$$\alpha = \frac{(1737.1/2)}{384\,405} = 0.0023 \text{ rad} \qquad (2.3 \text{ mrad}) \quad 0.13^\circ$$

$$\Omega = 2\pi\left(1 - \cos(0.0023)\right)$$
$$= 1.6 \times 10^{-5} \text{ Sr} = 16\,\mu\text{Sr}$$

SOLID Angle of Sun
distance from earth : 1.496×10^8 Km
diameter : 1.392×10^6 Km

$$\alpha = \frac{(1.392 \times 10^6/2)}{1.496 \times 10^8} = 0.0047 \qquad (4.7 \text{ mrad}) \quad (0.27^\circ)$$

$$\Omega = 2\pi\left(1 - \cos(0.0047)\right) = 6.8 \times 10^{-5} \text{ Sr}$$
$$68\,\mu\text{ Sr}$$

Let's look at some different light sources.
The main parameters of interest are

Φ : total light produced (lumens)

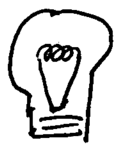
I : How directional the light is ($lm/sr = cd$)

L : How directional and concentrated the light is (cd/m^2)

η : Luminous efficiency ~~lm~~ $\frac{lm}{W}$

(1) Incandescent bulb

$$\Phi = 1050 \ lm$$



With a coated bulb the light from the filament is scattered from bulb.

$$I = \frac{1050}{4\pi} = 83. \ cd$$

L : from bulb $\quad L = \frac{83}{\pi (30 \times 10^{-3})^2} = 29 \ Kcd/m^2$

from filament $\quad L \approx 7 \times 10^6 \ cd/m^2$

$$\eta = \frac{1050}{100} = 10 \ lm/W$$

(2) Small LED

$$I = 38 \ cd$$
$$\theta = 15^\circ$$
$$\Omega = 2\pi(1 - \cos 7.5^\circ) = 0.0538$$
$$\Phi = I\Omega = 2 \ lm$$

L : from LED surface

$$L = \frac{38}{\pi (2.5 \times 10^{-3})^2} = 1.9 \times 10^6 \ cd/m^2$$

$$\eta = \frac{2}{(3.2)(20 \times 10^{-3})} = 31.25 \ lm/W$$

(3) LED light bulb replacement

$$\Phi = 1100 \text{ lm}$$
$$I = \frac{1100}{4\pi} = 87 \text{ cd}$$

L same as light bulb $L = \frac{87}{(\pi)(30\times10^{-3})^2} = 31 \text{ Kcd/m}^2$

$$\eta = \frac{1100}{13.5} = 81 \text{ lm/W}$$

(4) LED array $\Phi = 18000 \text{ lm}$

from spec sheet $\cancel{\Phi \approx \Phi_0}$ $I = I_0 \cos\theta$
find solid angle

$$d\Omega = \frac{dA}{r^2}$$

$$\Omega = \int_{dA} \cancel{I} \cos\theta \frac{dA}{r^2}$$

$$\int_0^{2\pi} \int_0^{\pi/2} \cos\theta \frac{r^2 \sin\theta}{r^2} d\theta\, d\phi$$

$$= 2\pi \int_0^{\pi/2} \cos\theta \sin\theta\, d\theta$$

$u = \cos\theta$
$du = -\sin\theta\, d\theta$

$$= 2\pi \int_0^{\pi/2} u \frac{\sin\theta\, du}{-\sin\theta}$$

$$= (2\pi)(-\tfrac{1}{2})\, u^2$$

$$= -\pi \cos^2\theta \Big|_0^{\pi/2}$$

$$= -\pi (\cos^2(\pi/2) - \cos^2 0)$$

$$= \pi$$ This is equivalent to $\pi = 2\pi(1 - \cos\theta)$
$\theta = 60°$

$$I = \frac{18000}{\pi} = 5.7 \text{ Kcd}$$

$$L = \frac{I}{A} = \frac{5.7\times10^3}{(\pi)(15\times10^{-3})^2} = 8\times10^6 \text{ cd/m}^2$$

$$\eta = \frac{18000}{(3.6)(36)} = 139 \text{ lm/W}$$

(5) Sun

from the book $L = 1.6 \times 10^{9}\ cd/m^2$

$\Phi = L A \Omega = (1.6\times10^{9})(2\pi)(\pi)\left(\frac{1.392\times10^{9}}{2}\right)^2$

$\Phi = 1.5\times10^{28}\ lm$

$I = \frac{\Phi}{2\pi} = 2.4\times10^{27}\ cd$

(6) Moon

from Wikipedia $L = 2.5\times10^{3}\ cd/m^2$

$\Phi = L A \Omega = (2.5\times10^{3})(2\pi)(\pi)\left(\frac{1.737\times10^{6}}{2}\right)^2$

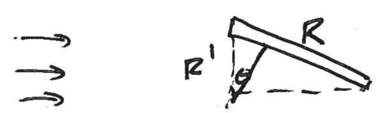
$\Phi = 3.7\times10^{16}\ lm$

$I = 5.9\times10^{15}\ cd$

Flux from one surface to another.
Characterized by luminance

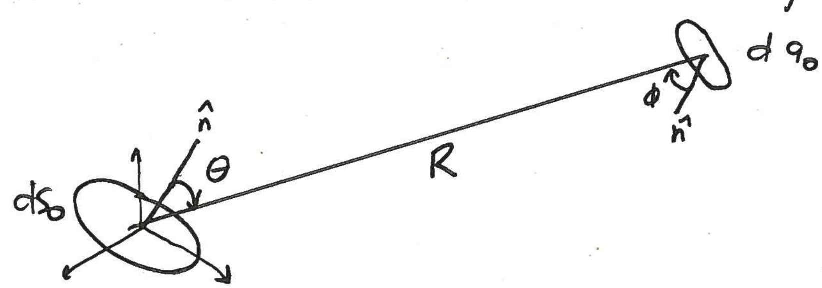
If source emits equally in all directions it is called ~~iso~~ Lambertian

Viewing a surface from an angle reduces flux by $\cos\theta$. Effective width is reduced



$$R' = R\cos\theta$$

Φ from an infinitesmal area emits into a direction
Add up all of the infinitesmal areas to get total flux



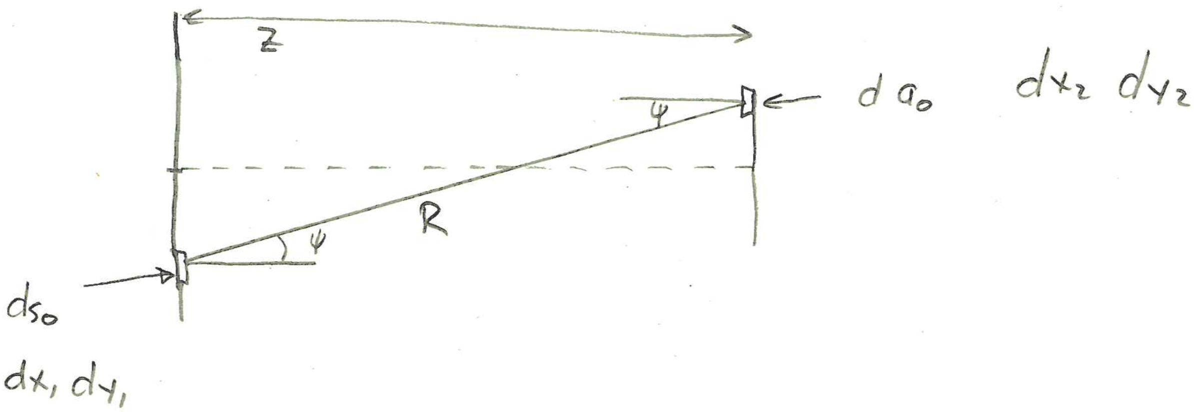
$$L = \frac{d^2\Phi}{d\omega\, dS_0 \cos\theta}$$

$d\omega$ is the infinitesmal solid angle

$$d\omega = \frac{da}{R^2} = \frac{da_0 \cos\phi}{R^2}$$

$$d^2\Phi = L\,\frac{dS_0 \cos\theta\, da_0 \cos\phi}{R^2}$$

Flux on receiver with
(1) Lambertian source
(2) Two sources facing each other

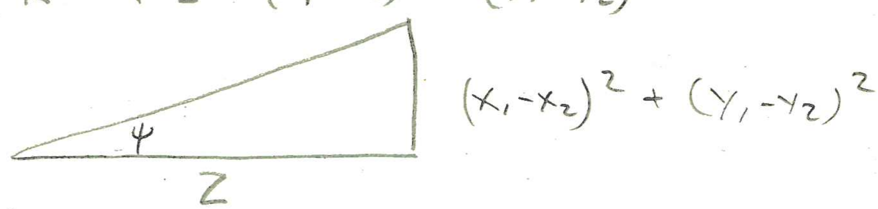


$$L = \frac{d^2\Phi}{d\omega\, ds_o \cos\theta}$$

$$d\omega = \frac{da}{R^2} = \frac{da_o \cos\psi}{R^2}$$

$$\Phi = L \int_{S_o} \int_{a_o} \frac{da_o \cos\psi}{R^2}\, ds_o \cos\psi$$

$$R^2 = Z^2 + (x_1 - x_2)^2 + (y_1 - y_2)^2$$



$$(\cos\psi)^2 = \frac{(x_1-x_2)^2 + (y_1-y_2)^2}{(x_1-x_2)^2 + (y_1-y_2)^2 + Z^2}$$

$$\Phi = \int_{S_o}\int_{a_o} L \frac{(x_1-x_2)^2 + (y_1-y_2)^2}{\left[(x_1-x_2)^2 + (y_1-y_2)^2 + Z^2\right]^2}\, dx_1\, dy_1\, dx_2\, dy_2$$

This is a big mess so we want to analyze simpler systems

CASE 1: $x_1, y_1 \ll R$ Source area is small

CASE 2: $x_1, y_1 \gg R$ Source approaches infinite

CASE 1

at receiver let's just look at illuminance

$$\Phi = \int_{S_0} \int_{A_0} L \frac{ds_0 \cos\theta \, da_0 \psi}{R}$$

$$E = \frac{\Phi}{A}$$

$$dE = \frac{d^2 \Phi}{da_0} = L \frac{\cos\theta \cos\psi \, ds_0}{R^2}$$

$$E = \int_{S_0} \frac{L \cos\theta \cos\psi \, ds_0}{R^2}$$

since $S_0 << R^2$

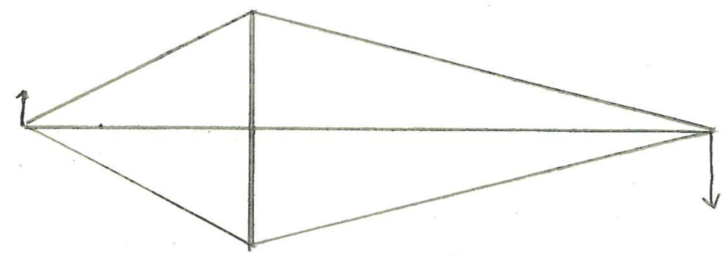
R, θ, ψ are approximately independent of the ds_0 integral

$$E = \frac{L \cos\theta \cos\psi}{R^2} \int_{S_0} ds_0$$

$$E = \frac{L \cos\theta \cos\psi \, S_0}{R^2}$$

$\max(X_1, Y_1) < 0.1 R$

Radiometry with an ideal lens



The total Φ collected by the lens is spread over the image.

How does L and E change between object and image?

$$\Phi_{ob} = (L_{obj}) \underbrace{(H_{ob})^2}_{A_{obj}} \underbrace{(\pi \Theta^2)}_{\Omega}$$

$\Theta = \frac{D}{S_o}$ paraxial approximation.

$$\Phi_{ob} = (L_{obj}) (H_{ob})^2 (\pi) \left(\frac{D}{S_o}\right)^2$$

Now look at image

$$\Phi_{im} = (L_{im}) (|M| H_{ob})^2 (\pi) \left(\frac{D}{S_i}\right)^2$$

$$|M| = \frac{S_i}{S_o}$$

$$\Phi_{im} = (L_{im}) \left(\frac{\cancel{S_i}}{S_o}\right)^2 (H_{ob}) (\pi) \left(\frac{D}{\cancel{S_i}}\right)^2$$

$$= L_{im} (H_{ob}) (\pi) \left(\frac{D}{S_o}\right)^2 = \Phi_{ob}$$

$L_{im} = L_{ob}$ RADIANCE is CONSTANT

$E_{ob} = \frac{\Phi}{H_{ob}^2}$ $E_{im} = \frac{\Phi}{(M H_{ob})^2} = E_{ob} \left(\frac{S_o}{S_i}\right)^2$

E changes with magnification