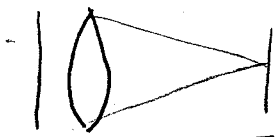
Optical Signal processing

The lens performs a Fourier transform of the input transmission



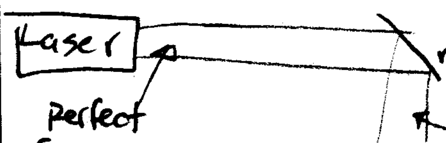
$t(x,y)$ $\qquad$ $T\big|_{f_x = \frac{x}{\lambda f},\ f_y = \frac{y}{\lambda f}}$

What are the constraints on this?
- optical coherence
- The aperture on the lens might change $t(x,y)$

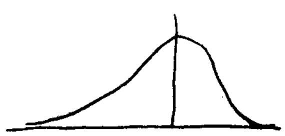
How can we use this Fourier transform property?
- Image modification
- Pattern recognition

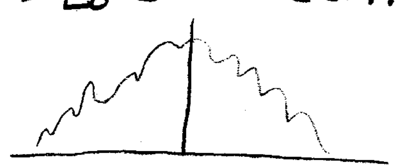
The simplest and most common image modification is spatial filtering.

Spatial Filtering



$$E(x,y) = E_0 e^{-(\rho/w)^2} \qquad E(x,y) = E_0 e^{-(\rho/w)^2} t(x,y)$$





Spatial filter gets rid of the non-pure Gaussian shape.

If we take a Fourier transform of a Gaussian we get a Gaussian

$$e(x,y) = e_0 e^{-(\rho/w)^2}$$

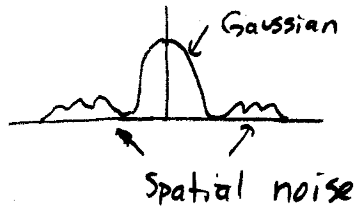
The transform is $e^{-\pi\rho^2} \Rightarrow e^{-\pi f_x^2}$ $\qquad a = \frac{1}{\pi w^2}$

$$E(x,y) = E_0 \exp\left[-\pi^2 w^2 \left(\frac{x}{\lambda f}\right)^2\right]$$

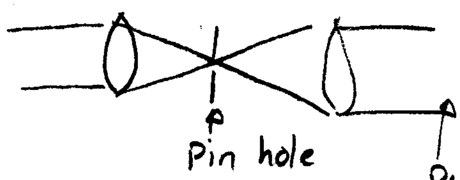
$$= E_0 \exp\left[-\left(\frac{\pi w \rho}{\lambda f}\right)^2\right]$$

$$I(x,y) = I_0 \exp\left[-2\left(\frac{\pi w \rho}{\lambda f}\right)^2\right]$$

The spatial frequency of the spatial noise is usually much larger than that of the Gaussian



If we eliminate the spatial noise components we will be left with just the Gaussian beam



How big should the pin hole be?

The total power through the pin hole is

$$P = 1 - \exp\left[-\frac{1}{2}\left(\frac{\pi a D}{\lambda f}\right)^2\right]$$

If we choose $D = \frac{f\lambda}{a}$

$$P = 1 - \exp\left[-\frac{1}{2}\left(\frac{\pi a f\lambda}{\lambda f a}\right)^2\right]$$

$$= 1 - \exp\left[-\frac{1}{2}\pi^2\right]$$

$$= 99.3\%$$

Example

Argon ion laser $\lambda = 364nm$
$a = 0.75mm$

Microscope objectives		D_{opt}
5x	$f = 25.4mm$	12.3 μm
10x	$f = 16.5mm$	8 μm
20x	$f = 9mm$	4.4 μm
40x	$f = 4.5mm$	2.2 μm
60x	$f = 2.9mm$	1.4 μm

which combination is best?

We use 10x and 5μm pin hole

If you want to do spatial filtering on an image you need to get the spatial image back from the Fourier transform image.

You need to perform an inverse Fourier transform. We can only perform a Fourier transform with a lens not an inverse Fourier transform.

What is the difference?

$$f(x,y) = \iint_{-\infty}^{\infty} F(f_x, f_y) \exp\left[+j\, 2\pi \left(f_x x + f_y y\right)\right] df_x \, df_y$$

$$F(f_x, f_y) = \iint_{-\infty}^{\infty} f(x,y) \exp\left[-j\, 2\pi \left(f_x x + f_y y\right)\right] dx \, dy$$

The difference is just a variable substitution

$$x = -f_x \qquad y = -f_y$$

If we use 2 lenses to perform 2 Fourier transforms we end up with an inverted image

we end up with a system called a 4F system.

If we use a mask at the Fourier transform plane we can perform Fourier filtering of the 2D image

We can also use the optical signal processing system to perform pattern recognition.

use a matched filter: A Fourier plane mask that matches the desired Fourier transform

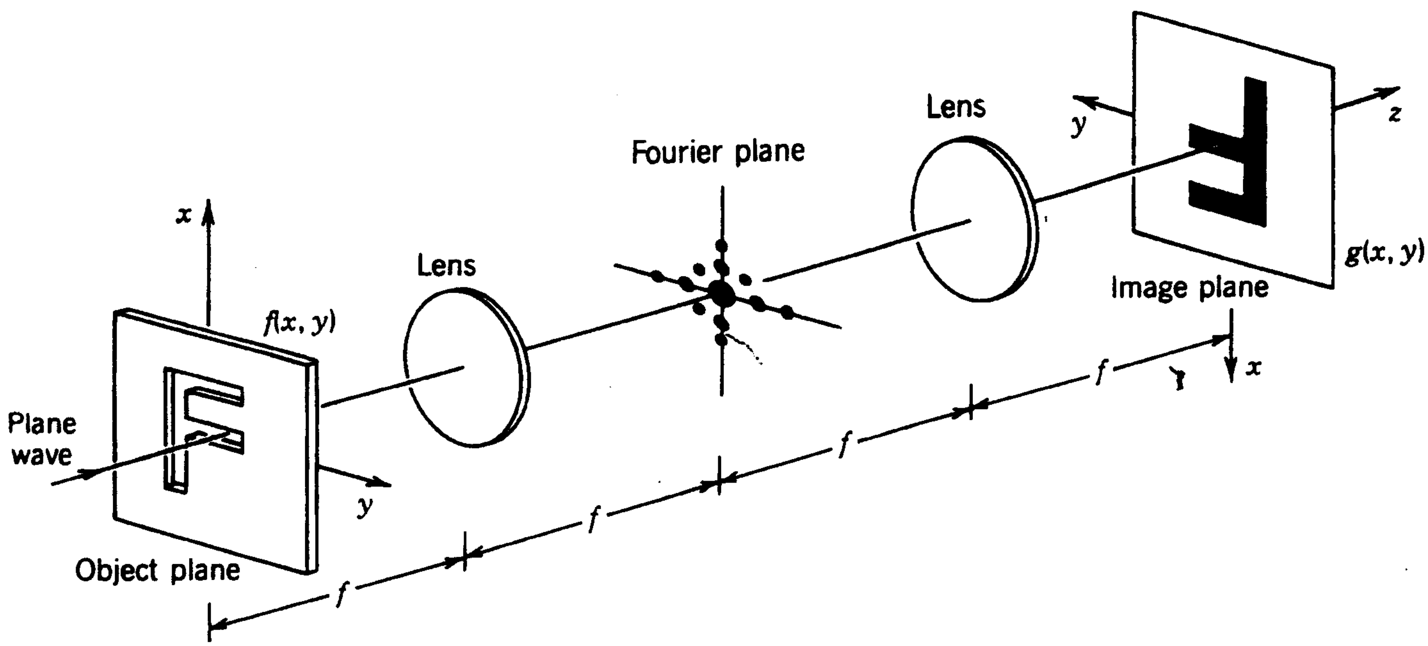


Figure 4.4-4 The 4–f system performs a Fourier transform followed by an inverse Fourier transform, so that the image is a perfect replica of the object.

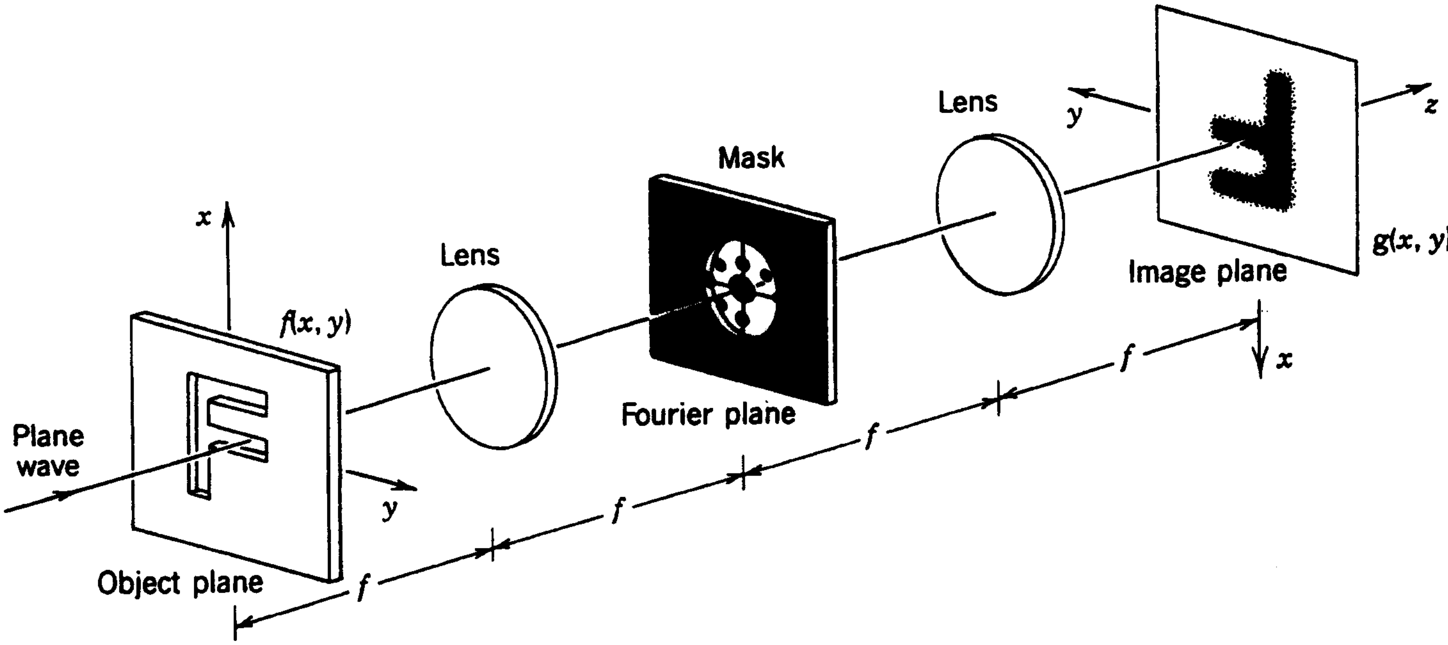


Figure 4.4-5 Spatial filtering. The transparencies in the object and Fourier planes have complex amplitude transmittances $f(x, y)$ and $p(x, y)$. A plane wave traveling in the z direction is modulated by the object transparency, Fourier transformed by the first lens, multiplied by the transmittance of the mask in the Fourier plane and inverse Fourier transformed by the second lens. As a result, the complex amplitude in the image plane $g(x, y)$ is a filtered version of $f(x, y)$. The system has a transfer function $\mathcal{H}(\nu_x, \nu_y) = p(\lambda f \nu_x, \lambda f \nu_y)$.

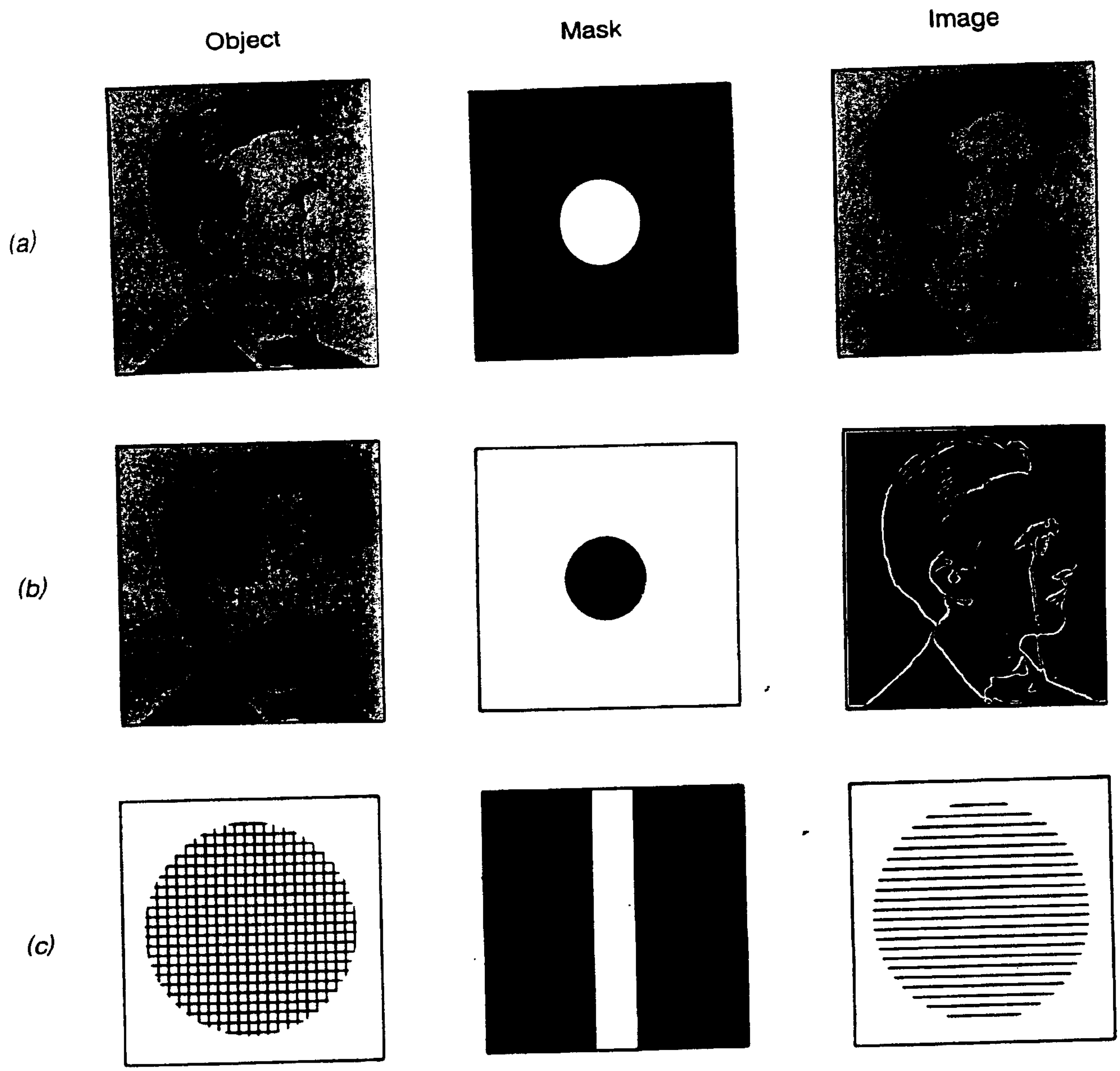
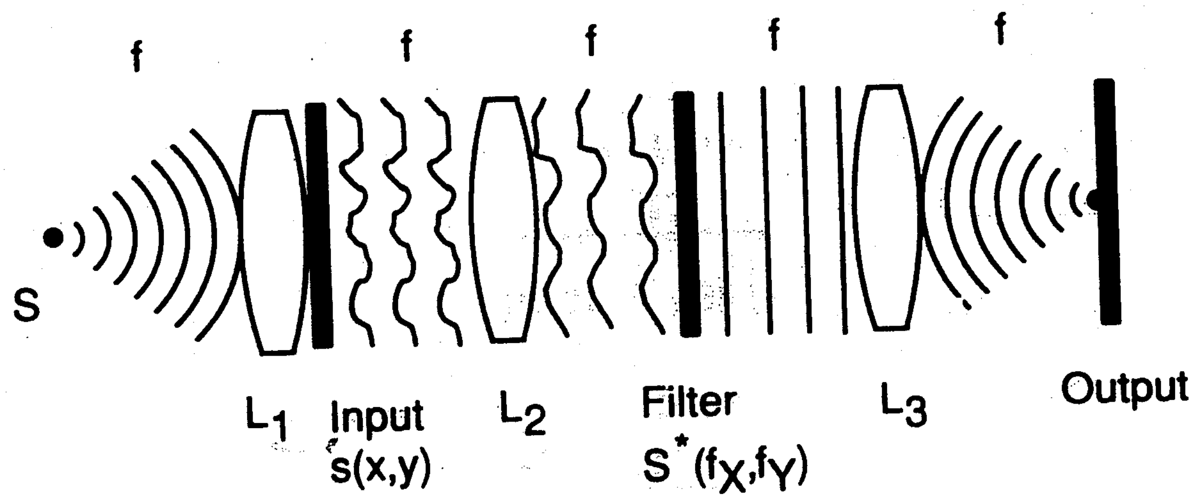
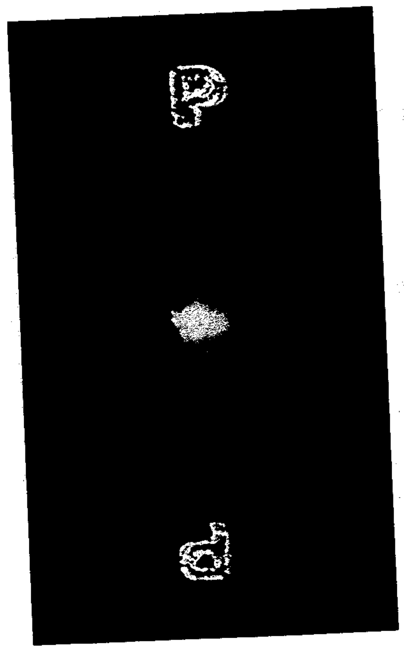
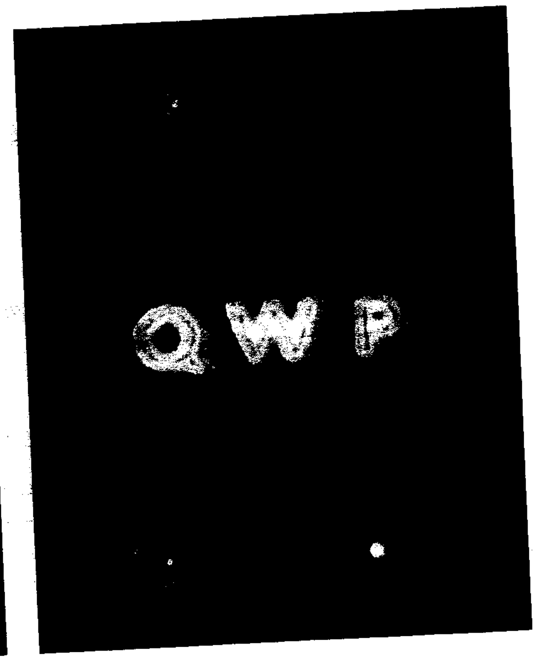


Figure 4.4-6 Examples of object, mask, and filtered image for three spatial filters: (*a*) low-pa filter; (*b*) high-pass filter; (*c*) vertical-pass filter. Black means the transmittance is zero and whi means the transmittance is unity.


f
f
f
f
f
S
L1
Input
s(x,y)
L2
Filter
S*(fX,fY)
L3
Output



(a)



(b)

FIGURE 8.19
Photographs of (a) the impulse response of a VanderLugt filter, and (b) the response of the matched filter portion of the output to the letters Q, W, and P.