Example

Let's look at an LED illuminating the human eye. Rather than treat the LED as an extended source (like it really is), let's treat it as a perfect point souce.

LED specifications : 70 cd
No need for LED area or cone angle

Eye diameter outside $L = 10^2\ cd/m^2 \rightarrow D = 3mm$, $f = \frac{1}{60}$
Distance from LED to eye 3m

$$\Omega = \pi \left(\frac{1.5\times10^{-3}}{3}\right)^2 = 0.785\ \mu Sr$$

Total luminous flux $\Phi = (70)(0.785\times10^{-6})$
$= 55\ \mu lm$

At this distance the field should be very uniform

$$e(r) = E_0\ \mathrm{circ}\left(\frac{r}{1.5mm}\right)$$

$$E(f_r) = \mathcal{F}\{e(r)\}\Big|_{f_r = \frac{r}{\lambda f}}$$

$$E(f_r) = E_0\ \frac{J_1((2\pi)(1.5\times10^{-3})f_r)}{f_r}$$ ignore any amplitude factors

Substitute in $f_r = \frac{r}{\lambda f}$

$$E(r) = E_0\ \frac{J_1\left((2\pi)(1.5\times10^{-3})\frac{r}{(620\times10^{-9})(16.67\times10^{-3})}\right)}{r}$$

$$= E_0\ \frac{J_1(9.12\times10^5 r)}{r}$$

convert to illuminance

$$I = I_0\ \frac{J_1^2(9.12\times10^5 r)}{r^2}$$

Use conservation of power to find I_0

$$\Phi = 55\times10^{-6} = \int_0^{2\pi}\int_0^{\infty} I_0\ \frac{J_1^2(9.12\times10^5 r)}{r^2}\, r\, dr\, d\theta$$

$$= I_0\, 2\pi \int_0^{\infty} \frac{J_1^2(9.12\times10^5 r)}{r}\, dr$$

$$u = 9.12\times10^5 r \qquad du = 9.12\times10^5 dr$$

$$\Phi = 2\pi I_0 \int_0^{\infty} \frac{J_1^2(u)}{\left(\frac{u}{9.12\times10^5}\right)}\left(\frac{1}{9.12\times10^5}\right) du$$

$$= 2\pi I_0 \int_0^{\infty} \frac{J_1^2(u)}{u}\, du = (2\pi)(I_0)\left(\frac{1}{2}\right) = I_0 \pi$$

$$I_0 = \frac{55\times10^{-6}}{\pi} = 1.75\times10^{-5}\ \frac{lm}{m^2}$$

$$I = 1.75 \times 10^{-5} \frac{J_1^2(9.12 \times 10^5 r)}{r^2}$$

To find the maximum I: $I_{max} = 1.75 \times 10^{-5} \lim_{r \to 0} \frac{J_1^2(9.12 \times 10^5 r)}{r^2}$

look up small angle approximation for bessel function

$$J_n \sim \frac{x^n}{2^n n!} \qquad J_1 \sim \frac{x}{2}$$

$$I_{max} = 1.75 \times 10^{-5} \lim_{r \to 0} \frac{(9.12 \times 10^5 \cancel{r})^2}{4 \cancel{r^2}}$$

$$= (1.75 \times 10^{-5}) \frac{(9.12 \times 10^5)^2}{4}$$

$$I_{max} = 3.64 \times 10^6 \frac{lm}{m^2} = 3.64 \times 10^6 \text{ lux}$$

This is much higher than reality because it assumes the lens is perfect.

Let's go to oslo to look at a single lens

Main lobe of pattern is called the Airy Disk. It is when

$$J_1(9.12 \times 10^5 r) = 0$$

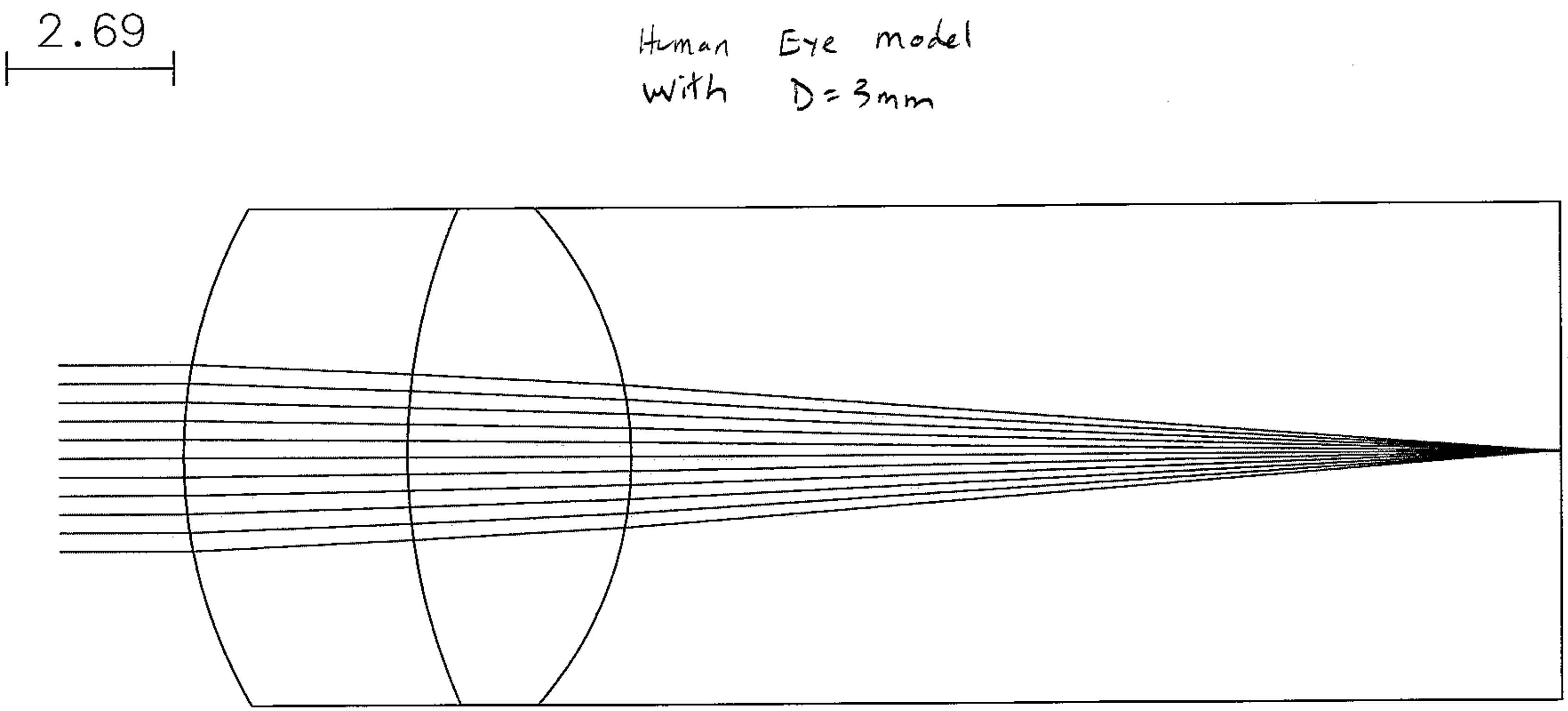
$$J_1(3.8317) = 0$$

$$r = \frac{3.8317}{9.12 \times 10^5} = 4.2 \mu m$$

$$I \propto \frac{55 \times 10^{-6} \text{ lm}}{(\pi)(0.0043 \times 10^{-3})^2} = 9.36 \times 10^5 \frac{lm}{m^2} = 0.94 \times 10^6 \text{ lux}$$

No name FOCAL LENGTH = 14.97 NA = 0.1002	UNITS: MM DES: OSLO

Human Eye model
with D=3mm

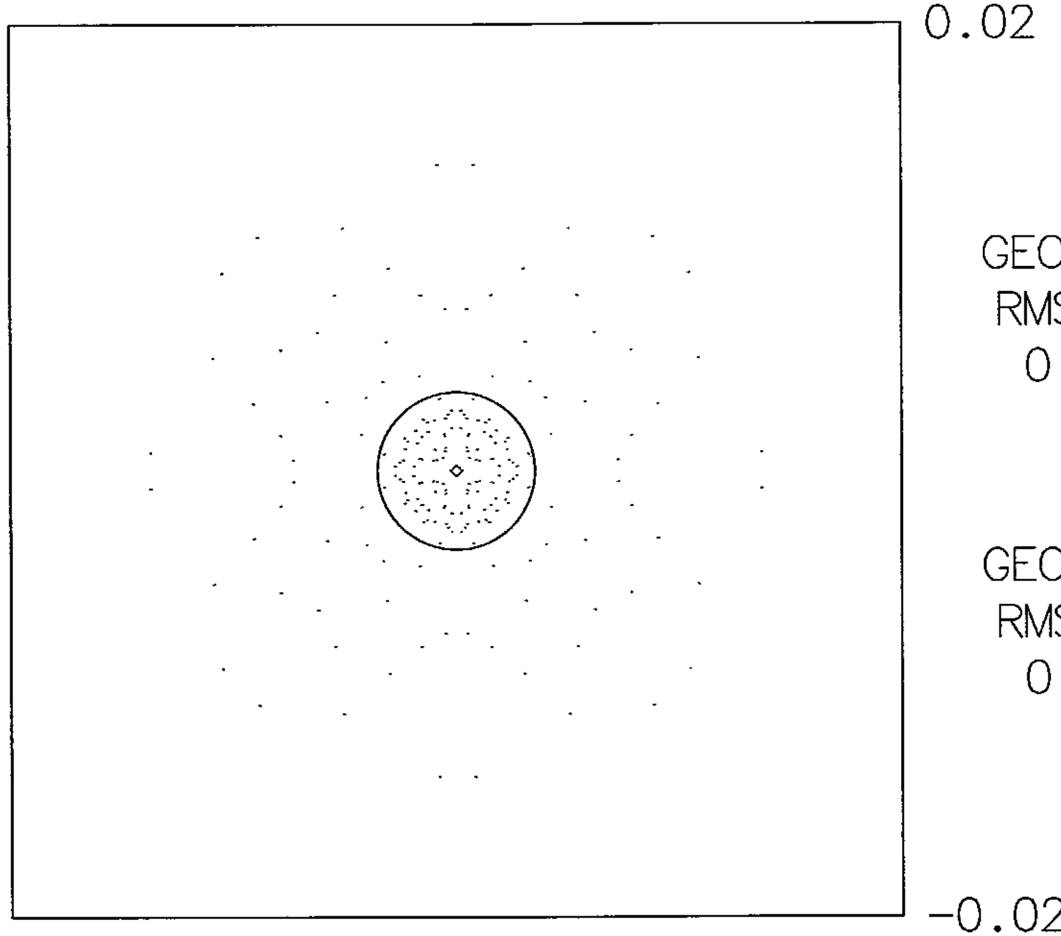


No name SPOT DIAGRAM	FBY 0 FBX 0 FOCUS 0	REFHT 0 UNITS: mm

GEOMETRICAL
RMS R SIZE
0.005965

Geometrical optics dominates

DIFFRACTION
LIMIT
0.003541



GEOMETRICAL
RMS Y SIZE
0.004218

GEOMETRICAL
RMS X SIZE
0.004218