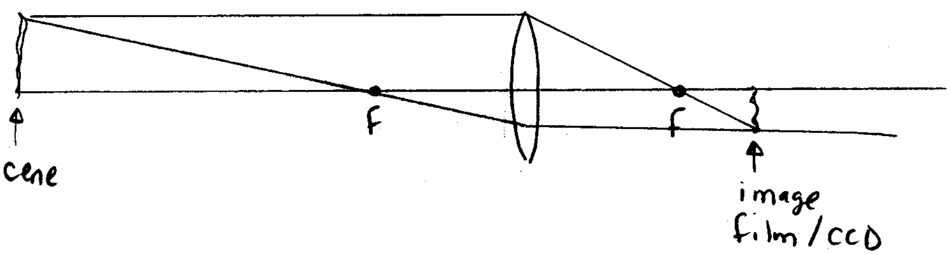
Camera

Place film/ CCD at the image focus



Questions: (1) Location of film/CCD.
(2) How do you change the picture size?
(3) How big is the picture?
(4) What is the field of view of the camera?

(1) use the thin lens equation

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

For a normal camera $f \sim 50mm$

Typical $s_o > f$

Let's say $0.5m < s_o < 30m$

$$\frac{1}{s_i} = \frac{1}{f} - \frac{1}{s_o}$$

$$s_i = \frac{s_o f}{s_o - f} \qquad 556mm < s_i < 50.08mm$$

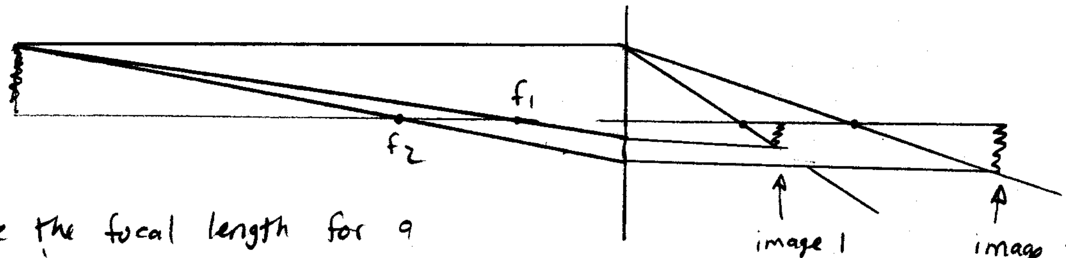
about a 6 mm shift in the film
This is the camera focus
usually camera auto-focuses
This is moving the lenses relative to the film/CCD

More motion is needed when s_o is close to f

What happens if $s_o < f$?

s_i is negative and you get a virtual image.
can't put the film at the correct location.

(2) How do you change the picture size (zoom)?



Increase the focal length for a bigger image

Wide angle	$24mm < f < 35mm$
Normal	$\sim 50mm$
Telephoto	$80mm < f < 300mm$ ← This is why telephoto lenses are really long

(3) and (4)

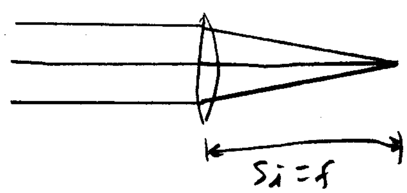
What is the magnification?

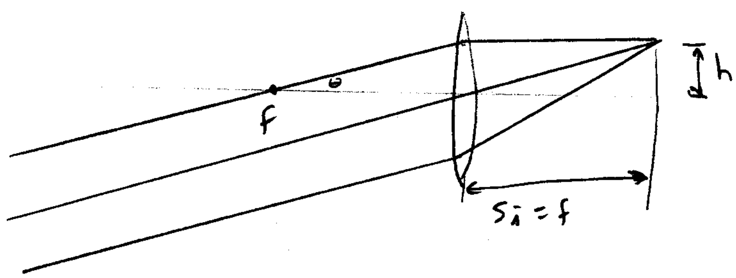
$S_o = 1m$

$f = 50mm$ $\quad M = \frac{h_i}{h_o} = \frac{S_i}{S_o} = \frac{f}{S_o - f} = 52.6 \times 10^{-3}$

This standard magnification equation is not actually the best way

Since $S_o \gg f$ we will treat the object points as plane wave with different incident angles





$\tan\theta = \frac{h}{f}$

$\theta = \frac{h}{f}$

$h = f\theta$

A particular point on the film/CCD corresponds to a specific incident angle.

There is a cone of angles for each pixel

A cone for the entire CCD array

The FOV of the camera is

$$H = f\theta$$
$$\theta = \frac{H}{f}$$ where H is the size of Film / CCD

The resolution of a CCD is essentially the FOV of a particular pixel.

$$\theta_{pixel} = \frac{h}{f}$$ so the larger f is the smaller the angular content of a single pixel

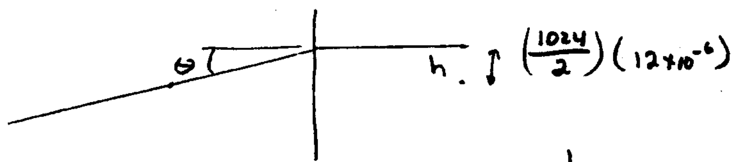
In class example

There are 2 optical imagers on the Cassini space craft.

Both use a 1024 x 1024 CCD with 12 μm pixels
This is approximately a 1/2 inch wide CCD

The 2 imagers are
Narrow angle camera (NAC) $f = 2m$
Wide angle camera (WAC) $f = 0.2m$

What is the total FOV for both imagers in degrees?



$$\tan\theta = \frac{h}{f}$$

$$\theta \simeq \frac{h}{f}$$

$$\theta \simeq \frac{(512)(12\times10^{-6})}{2}$$

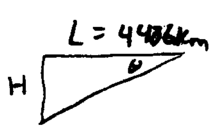
$$\theta \simeq 0.00307 \quad , \quad 0.0307 \text{ rad}$$

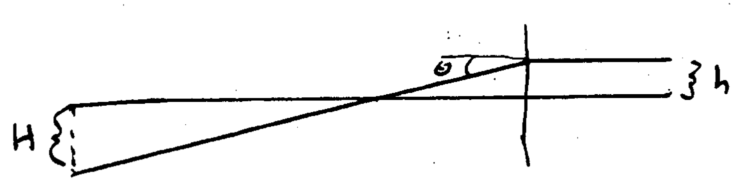
$$\pm\theta \simeq 0.176° \quad , \quad 0.176°$$

$$\text{FOV} = 0.35° \quad , \quad 3.5°$$

What is the resolution at a distance of 4486 Km using the NAC camera?

For a single pixel $h = 12 \mu m$





$$\theta = \frac{h}{f} = \frac{12\times10^{-6}}{2} = \frac{H}{4486}$$

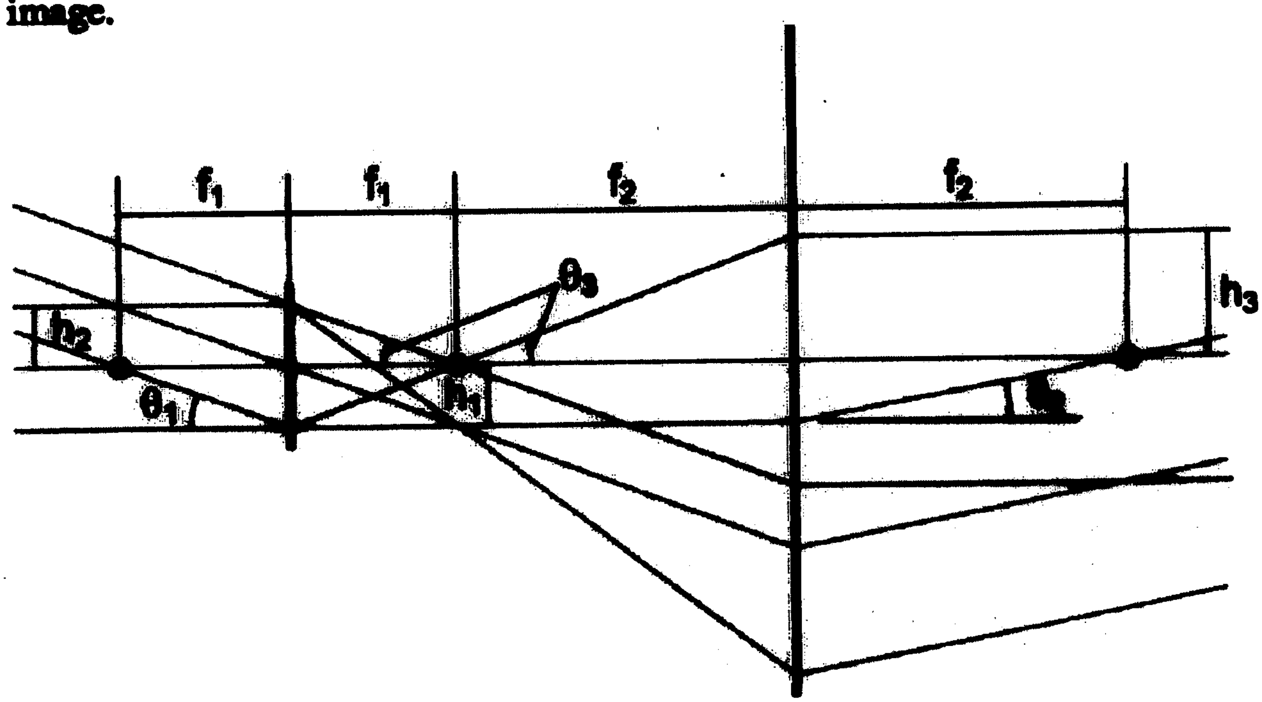
$$H = \left(\frac{12\times10^{-6}}{2}\right)(4486\times10^{3})$$

$$\boxed{H = 26.9 \text{ m}}$$

see cassini picture

Afocal (Beam Expander)

Called an afocal since neither the input nor output comes to an image.



Beam Expansion

$$\tan(\theta_3) = \frac{h_2}{f_1} \approx \theta_3 \qquad\qquad \tan(\theta_3) = \frac{h_3}{f_2} \approx \theta_3$$

$$\frac{h_3}{h_2} = \frac{f_2}{f_1}$$

Angle

$$\frac{h_1}{f_1} = \theta_1 \qquad\qquad \theta_2 = \frac{h_1}{f_2}$$

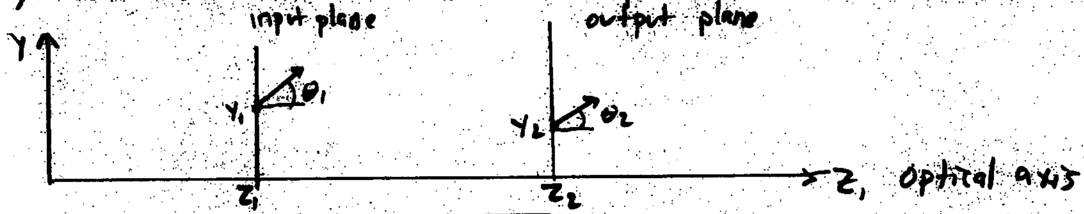
$$\frac{\theta_2}{\theta_1} = \frac{f_1}{f_2}$$

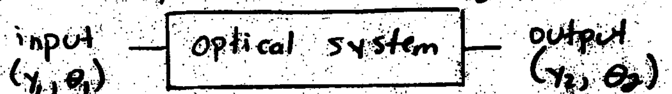
Cannon Powershot G7

3648 x 2736 pixels
7.18 mm x 5.32 mm

Wide angle lens f = 35mm
Telephoto lens f = 210mm
Normal focus range 50cm
Macro focus range 1cm
F2.8 - F4.8 aperture

In a homogeneous material rays travel in straight lines.
A ray is characterized by its position and direction.





An optical system is a set of optical components that change the position and direction of rays.

In the paraxial approximation, when all angles are sufficiently small so that $\sin\theta \approx \theta$, the relationship between (Y_2, θ_2) and (Y_1, θ_1) is linear

$$Y_2 = A\,Y_1 + B\,\theta_1$$
$$\theta_2 = C\,Y_1 + D\,\theta_1$$

$$\begin{bmatrix} Y_2 \\ \theta_2 \end{bmatrix} = \underbrace{\begin{bmatrix} A & B \\ C & D \end{bmatrix}}_{M} \begin{bmatrix} Y_1 \\ \theta_1 \end{bmatrix}$$

The matrix M characterizes the optical system.

This analysis technique is called "Matrix Optics" or "ABCD Matrices".

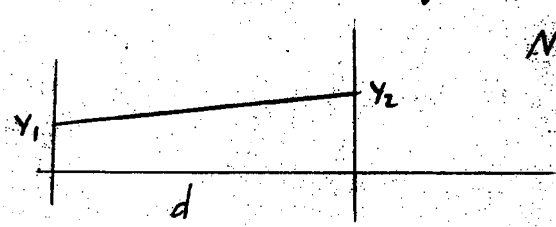
Let's calculate the ABCD matrices for some simple optical components.

<u>Free Space Propagation</u>

rays travel in straight lines

$$Y_2 = Y_1 + \theta_1 d$$
$$\theta_2 = \theta_1$$

$$M = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$$

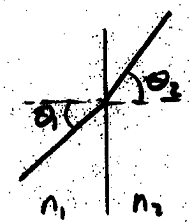


Refraction at a Planar Boundary

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$n_1 \theta_1 \approx n_2 \theta_2$

$y_2 = y_1$
$\theta_2 = \frac{n_1}{n_2}\theta_1 \qquad M = \begin{bmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{bmatrix}$



Refraction at a Spherical Boundary

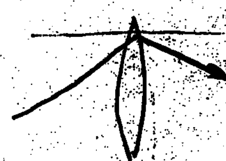
$y_2 = y_1$

$\theta_2 \approx \frac{n_1}{n_2}\theta_1 - \frac{n_2 - n_1}{n_2 R} y_1$

$$M = \begin{bmatrix} 1 & 0 \\ -\frac{n_2 - n_1}{n_2 R} & \frac{n_1}{n_2} \end{bmatrix}$$

Transmission through a Thin Lens

$y_2 = y_1$
$\theta_2 = \theta_1 - \frac{y}{f} \qquad M = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}$



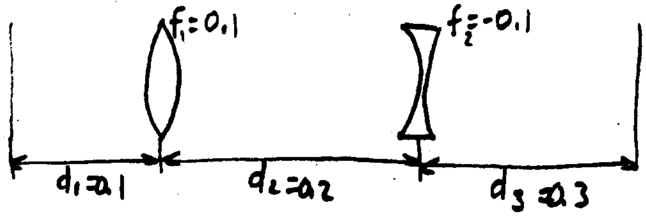
Reflection from a Planar Mirror

$y_2 = y_1$
$\theta_2 = \theta_1 \qquad M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Reflection from a Spherical Mirror

$$M = \begin{bmatrix} 1 & 0 \\ \frac{2}{R} & 1 \end{bmatrix}$$

Multiple elements are analyzed by multipling all of the matrices for the individual elements.



$$M = \begin{bmatrix} 1 & d_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_1} & 1 \end{bmatrix} \begin{bmatrix} 1 & d_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_2} & 1 \end{bmatrix} \begin{bmatrix} 1 & d_3 \\ 0 & 1 \end{bmatrix}$$

if the initial ray is $y_1 = 0$
$\theta_1 = 1^\circ = 0.0174$

$$\begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 1 & .1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -10 & 1 \end{bmatrix} \begin{bmatrix} 1 & .2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 10 & 1 \end{bmatrix} \begin{bmatrix} 1 & .3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ .0174 \end{bmatrix} = \begin{bmatrix} 0.007 \\ -.1218 \end{bmatrix}$$

Can use this technique to perform a simple ray trace through an arbitrary system.