CAMERA ILLUMINANCE

A camera is used to take a picture of an object. The object has a total reflectivity of 20% and scatters all of the light equally into a hemisphere (Lamberian).

If the camera has the following specs.

$f = 6.7\,mm$
$f/3.1$

Find the illuminance on the camera focal plane if the object is 1m away from the camera.

Let's assume the object is in full sun.

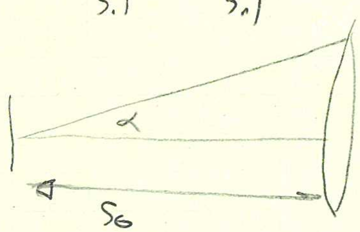
$$E_{sun} = 10^5\ lux = 10^5\ lm/m^2$$

20% of the light is reflected into a hemisphere

$$L_{obj} = \frac{(0.2)(10^5)}{2\pi} = 3.18 \times 10^3\ \frac{cd}{m^2}$$

Let's look at a disk or radius Δr.
Find the total flux that passes through the lens.

$$f/D = 3.1$$
$$D = \frac{f}{3.1} = \frac{6.7mm}{3.1} = 2.16\ mm$$



$$\alpha = \frac{D/2}{S_o} = \frac{D}{2S_o}$$
$$\Omega = 2\pi(1-\cos\alpha) = 4\pi \sin^2\left(\frac{\alpha}{2}\right)$$
$$= \pi\alpha^2$$
$$\Omega = \pi\left(\frac{D}{2S_o}\right)^2$$

$$\Phi = (L)(\Omega)(A)$$
$$= L\ \pi\left(\frac{D}{2S_o}\right)^2\ \pi\ \Delta r^2$$

This total flux is imaged down to a spot of radius $\Delta r'$

$$\frac{1}{S_i} + \frac{1}{S_o} = \frac{1}{f}$$
$$\Delta r' = \Delta r\ \frac{S_i}{S_o}$$

With $S_o \gg f \rightarrow S_i = f$

$$\Delta r' = \Delta r\ \frac{f}{S_o}$$

$$E_{CCD} = \frac{\Phi}{A_{CCD}} = L\ \pi^2\left(\frac{D}{2S_o}\right)^2 \frac{\Delta r^2}{\pi(\Delta r)^2(f/S_o)^2}$$

$$E_{CCD} = (L)(\pi)\left(\frac{1}{4}\right)\left(\frac{D}{S_o}\right)^2\left(\frac{S_o}{f}\right)^2$$

$$= L\left(\frac{\pi}{4}\right)\left(\frac{D}{f}\right)^2$$

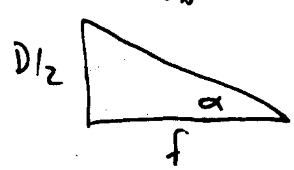
$$= L\left(\frac{\pi}{4}\right)\left(\frac{1}{f\#}\right)^2$$

$$E_{CCD} = (3.18\times10^3)\left(\frac{\pi}{4}\right)\left(\frac{1}{3.1}\right)^2$$

$$E_{CCD} = 260 \ \mathrm{lm/m^2} = 260 \ \mathrm{lux}$$

Now using the concept that $L_{obj} = L_{im}$

$$L_{ob} = \frac{0.2\times10^5}{2\pi} = L_{im} = \frac{E_{im}}{\Omega}$$



$$\alpha = \tan^{-1}\left(\frac{D}{2f}\right) = \tan^{-1}\left(\frac{1}{2f\#}\right)$$

$$\Omega = 2\pi\left(1-\cos\left(\tan^{-1}\left(\frac{1}{2f\#}\right)\right)\right)$$

$$\Omega = 0.0802$$

$$E_{im} = \left(\frac{0.2\times10^5}{2\pi}\right)(0.0802)$$

$$E_{im} = 255 \ \mathrm{lux}$$