Optical materials are typically specified at selected spectral lines

Approximate λ (nm)	Designation	Spectral Line
656	C	red hydrogen
644	C'	red cadmium
588	d	yellow helium
546	e	green mercury
486	F	blue hydrogen
480	F'	blue Cadmium
436	g	blue mercury

Dispersion is the change in index of refraction with wavelength. The refractive index is typically designated with the subscript associated with the spectral lines

n_F is the index of refraction at $\lambda = 546\text{nm}$

The dispersion is characterized using the unitless quantity

$$V_d = \frac{n_d - 1}{n_F - n_c} \qquad \text{or} \qquad V_e = \frac{n_e - 1}{n_{F'} - n_{C'}}$$

Most common

It is called Abbe value, V number, or V value

A specific glass can be using a glass number

$XXX = n_d - 1$
$YYY = V_d$

Find BK7 on Figure 7.3

$n_d = 1.51680$
$V_d = 64.17$

glass $= 517\ 642$

Refractive index can be specified by Cauchy's formula

$$n(\lambda) = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} \qquad C \text{ is very small}$$

$$n(\lambda) = A + B\lambda^{-2}$$

$$B = \frac{(n_d - 1)\,\lambda_C^2\,\lambda_F^2}{V\,(\lambda_C^2 - \lambda_F^2)} \qquad A = n_d - \frac{B}{\lambda_d^2}$$

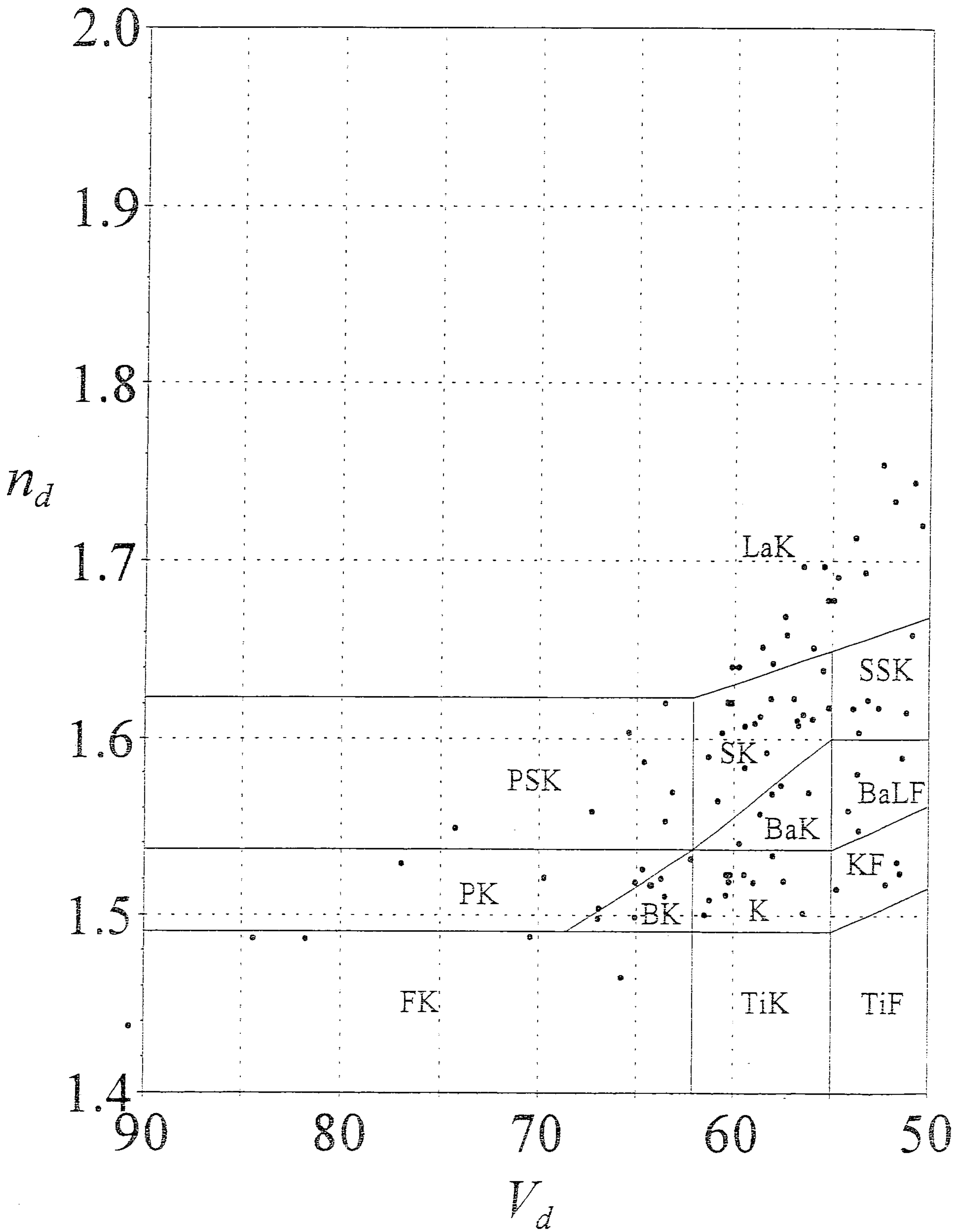


Figure A.1 The left side of the glass map showing mostly crown glasses.

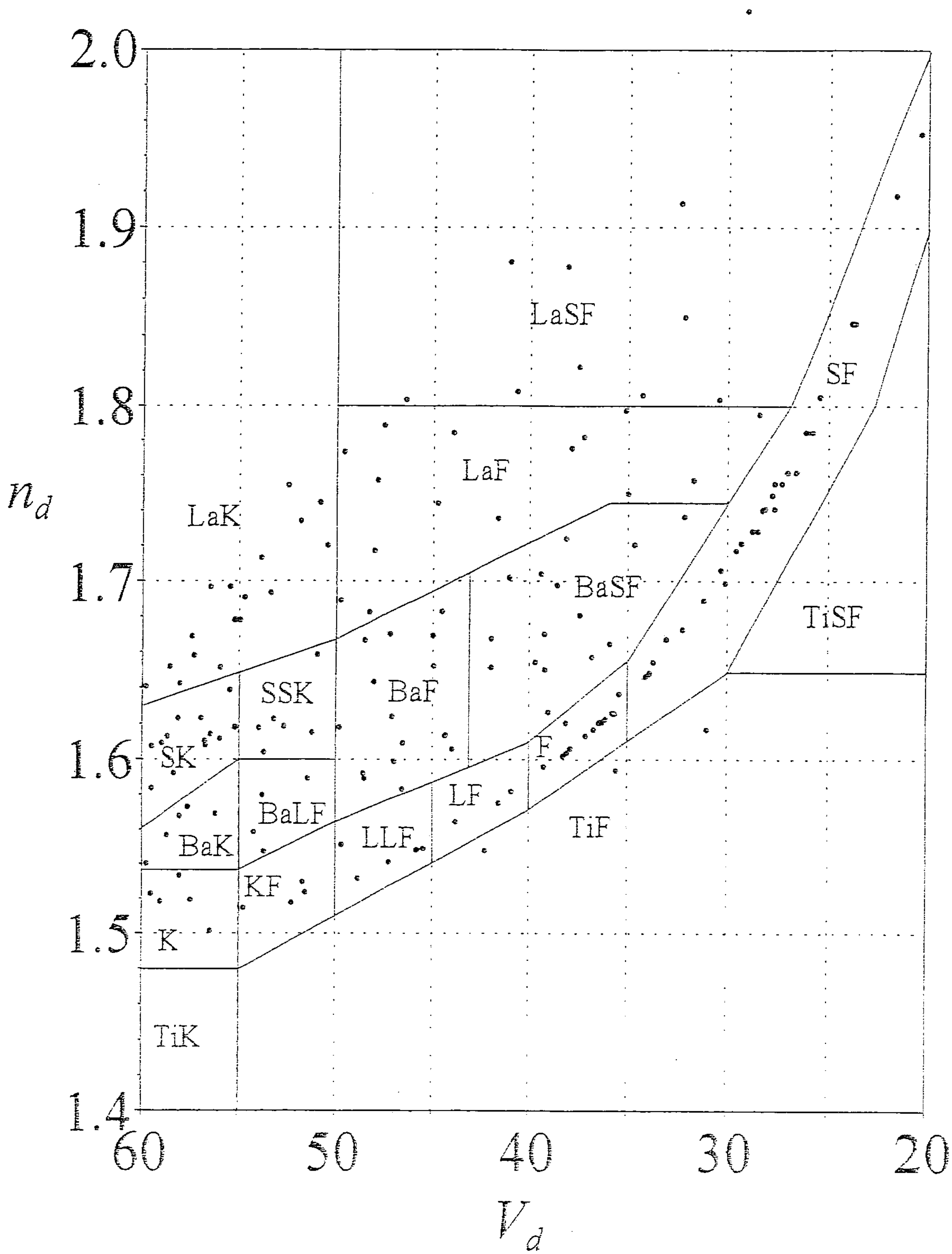


Figure A.2 The right side of the glass map showing all of the flint glasses a few crowns.

GLASS CATALOG AND MAP[1]

	Name	Abbe Value	Index
FK	FK 3	65.77	1.46450
	FK 5	70.41	1.48749
	FK 51	84.47	1.48656
	FK 52	81.80	1.48605
	FK 54	90.70	1.43700
PK	PK 1	66.92	1.50378
	PK 2	65.05	1.51821
	PK 3	64.66	1.52542
	PK 50	69.71	1.52054
	PK 51A	76.98	1.52855
PSK	PSK 2	63.08	1.56873
	PSK 3	63.46	1.55232
	PSK 50	67.28	1.55753
	PSK 52	65.41	1.60310
	PSK 53A	63.48	1.62014
	PSK 54	64.60	1.58599
BK	BK 1	63.46	1.51009
	BK 3	65.06	1.49831
	BK 6	62.15	1.53113
	BK 7	64.17	1.51680
	U BK 7	64.29	1.51680
	BK 8	63.69	1.52015
	BK 10	66.95	1.49782
BaLK	BaLK N3	60.26	1.51849

	Name	Abbe Value	Index
K	K 3	58.98	1.51823
	K 4	57.40	1.51895
	K 5	59.48	1.52249
	K 7	60.41	1.51112
	K 10	56.41	1.50137
	K 11	61.44	1.50013
	K 50	60.18	1.52257
	U K 50	60.38	1.52257
ZK	ZK 1	57.98	1.53315
	ZK N7	61.19	1.50847
BaK	BaK 1	57.55	1.57250
	BaK 2	59.71	1.53996
	BaK 4	56.13	1.56883
	BaK 5	58.65	1.55671
	BaK 50	57.99	1.56774

[1] Glass catalog and map are reproduced with the permission of Schott Glass Technologies, Inc.

	Name	Abbe Value	Index
SK	SK 1	56.71	1.61025
	SK 2	56.65	1.60738
	SK 3	58.92	1.60881
	SK 4	58.63	1.61272
	SK 5	61.27	1.58913
	SK 6	56.40	1.61375
	SK 7	59.46	1.60729
	SK 8	55.92	1.61117
	SK 10	56.90	1.62280
	SK 11	60.80	1.56384
	SK 12	59.45	1.58313
	SK 13	58.30	1.59181
	SK 14	60.60	1.60311
	SK 15	58.06	1.62299
	SK 16	60.32	1.62041
	SK 18A	55.42	1.63854
	SK 51	60.31	1.62090
	SK 55	60.12	1.62041
KF	KF 3	54.70	1.51454
	KF 6	52.20	1.51742
	KF 9	51.49	1.52341
BaLF	BaLF 4	53.71	1.57957
	BaLF 5	53.63	1.54739
	BaLF 50	51.37	1.58893
SSK	SSK 1	53.91	1.61720
	SSK 2	53.15	1.62230
	SSK 3	51.16	1.61484
	SSK 4A	55.14	1.61765
	SSK N5	50.88	1.65844
	SSK N8	49.77	1.61772
	SSK 50	52.61	1.61795
	SSK 51	53.63	1.60361

	Name	Abbe Value	Index
LaK	LaK N6	57.96	1.64250
	LaK N7	58.52	1.65160
	LaK 8	53.83	1.71300
	LaK 9	54.71	1.69100
	LaK 10	50.41	1.72000
	LaK 11	57.26	1.65830
	LaK N12	55.20	1.67790
	LaK L12	54.92	1.67790
	LaK N13	53.33	1.69350
	LaK N14	55.41	1.69680
	LaK 16A	51.78	1.73350
	LaK 21	60.10	1.64049
	LaK L21	59.75	1.64048
	LaK N22	55.89	1.65113
	LaK 23	57.38	1.66882
	LaK 28	50.77	1.74429
	LaK 31	56.42	1.69673
	LaK 33	52.43	1.75398
LLF	LLF 1	45.75	1.54814
	LLF 2	47.17	1.54072
	LLF 6	48.76	1.53172
	LLF 7	45.41	1.54883
BaF	BaF 3	46.47	1.58267
	BaF 4	43.93	1.60562
	BaF N6	48.45	1.58900
	BaF 8	47.00	1.62374
	BaF 9	47.96	1.64328
	BaF N10	47.11	1.67003
	BaF N11	48.42	1.66672
	BaF 13	44.96	1.66892
	BaF 50	44.50	1.68273
	BaF 51	44.93	1.65224
	BaF 52	46.44	1.60859
LF	LF 5	40.85	1.58144
	LF 7	41.49	1.57501
	LF 8	43.75	1.56444

	Name	Abbe Value	Index
F	F 1	35.70	1.62588
	F 2	36.37	1.62004
	F 3	37.04	1.61293
	F 4	36.63	1.61659
	F 5	38.03	1.60342
	F 6	35.34	1.63636
	F 7	35.56	1.62536
	F 8	39.18	1.59551
	F 9	38.08	1.62045
	FN 11	36.18	1.62096
	F 13	36.04	1.62237
	F 14	38.23	1.60140
	F 15	37.83	1.60565
BaSF	BaSF 1	38.96	1.62606
	BaSF 2	35.83	1.66446
	BaSF 6	41.93	1.66755
	BaSF 10	39.15	1.65016
	BaSF 12	39.20	1.66998
	BaSF 13	38.57	1.69761
	BaSF 51	38.11	1.72373
	BaSF 52	41.01	1.70181
	BaSF 54	32.15	1.73627
	BaSF 56	36.74	1.65715
	BaSF 57	41.90	1.65147
	BaSF 64A	39.38	1.70400
LaF	LaF 2	44.72	1.74400
	LaF 3	47.96	1.71700
	LaF N7	34.95	1.74950
	LaF N8	41.59	1.73520
	LaF 9	28.39	1.79504
	LaF N10	43.95	1.78443
	LaF 11A	31.70	1.75693
	LaF 13	37.84	1.77551
	LaF 20	48.20	1.68248
	LaF N21	47.47	1.78831
	LaF 22A	37.20	1.78179
	LaF N23	49.71	1.68900
	LaF N24	47.81	1.75719
	LaF N28	49.57	1.77314

	Name	Abbe Value	Index
LaSF	LaSF 3	40.61	1.80801
	LaSF N9	32.17	1.85025
	LaSF N15	38.07	1.87800
	LaSF 18A	32.36	1.91348
	LaSF N30	46.38	1.80318
	LaSF N31	41.01	1.88067
	LaSF 32	30.40	1.80349
	LaSF 33	34.24	1.80596
	LaSF 35	29.06	2.02204
	LaSF 36A	35.08	1.79712
SF	SF 1	29.51	1.71736
	SF 2	33.85	1.64769
	SF 3	28.20	1.74000
	SF 4	27.58	1.75520
	SF L4A	27.40	1.75520
	SF 5	32.21	1.67270
	SF 6	25.43	1.80518
	SF L6	25.39	1.80518
	SF 8	31.18	1.68893
	SF 9	33.65	1.65446
	SF 10	28.41	1.72825
	SF 11	25.76	1.78472
	SF 12	33.84	1.64831
	SF 13	27.60	1.74077
	SF 14	26.53	1.76182
	SF 15	30.07	1.69895
	SF 16	34.05	1.64611
	SF 18	29.25	1.72151
	SF 19	33.01	1.66680
	SF 53	28.69	1.72830
	SF 54	28.09	1.74080
	SF 55	26.95	1.76180
	SF 56A	26.08	1.78470
	SF L56	26.08	1.78470
	SF 57	23.83	1.84666
	SF L57	23.62	1.84666
	SF 58	21.51	1.91761
	SF 59	20.36	1.95250
	SF 63	27.71	1.74840
	SF 64A	30.30	1.70585
TiF	TiF 3	42.20	1.54765

	Name	Abbe Value	Index
KzF	KzF N1	49.64	1.55115
	KzF N2	51.63	1.52944
KzFS	KzFS 1	44.34	1.61310
	KzFS N2	54.16	1.55836
	KzFS N4	44.29	1.61340
	KzFS N5	39.63	1.65412
	KzFS 6	48.51	1.59196
	KzFS 7A	37.39	1.68064
	KzFS 8	34.61	1.72047
	KzFS N9	46.90	1.59856

Chromatic aberration is the dispersion variations

Axial color, longitudinal color, or longitudinal chromatic aberration is shift in focus with wavelength

$$K_F = (n_F - 1)(c_1 - c_2)$$
$$K_d = (n_d - 1)(c_1 - c_2)$$
$$K_C = (n_C - 1)(c_1 - c_2)$$

change in lens power $K_F - K_C = (n_F - 1)(c_1 - c_2) - (n_C - 1)(c_1 - c_2)$

$$= (n_F - n_C)(c_1 - c_2)$$
$$= \frac{n_F - n_C}{(n_d - 1)}(n_d - 1)(c_1 - c_2)$$

$$K_F - K_C = \frac{K}{V}$$

No subscript means d

Now look at shift is focus

$$\frac{1}{s_o} + \frac{1}{s_{i,F}} = K_F \qquad \frac{1}{s_o} + \frac{1}{s_{i,C}} = K_C$$

$$\frac{1}{s_{iF}} - \frac{1}{s_{iC}} = K_F - K_C = \frac{K}{V}$$

Axial color: $\boxed{\frac{1}{s_{iF}} - \frac{1}{s_{iC}} = \frac{K}{V}}$

Lateral or transverse chromatic aberration or color is change in image height or magnification

$$\boxed{H_{iF} - H_{iC} = -H_o\, s_i \frac{K}{V}}$$

Chromatic aberration is proportional to $\frac{K}{V}$.

Make this term as small as possible. Not possible to make it zero.

For 2 lenses in contact it is possible to make this equal to zero

$$K_1 + K_2 = K$$

$$\frac{K_1}{V_1} + \frac{K}{V_2} = 0$$

solve these 2 equations to get

$$K_1 = \frac{V_1 K}{V_1 - V_2} \quad \text{and} \quad K_2 = -\frac{V_2 K}{V_1 - V_2}$$

Good to pick glasses with distinct abbe numbers

Crown glasses : K designation on glass chart
Larger V

Flint glasses : F designation on glass chart

What determines the glass choice : Absorption
Cost
durability

Larger difference in abbe number means smaller powers

$V_1 = 40$ $V_2 = 30$ $K_1 = 4K$ $K_2 = -3K$

$V_1 = 60$ $V_2 = 20$ $K_1 = 1.5K$ $K_2 = -0.5K$

Newport PAC090 BK7 (crown) $n_d = 1.51680$ $V = 64.17$
SF5 (flint) $n_d = 1.67270$ $V = 32.21$

Monochromatic aberration.

Deviations from the paraxial approximation

$$n_1\theta_1 \neq n_2\theta_2 \quad \text{but} \quad n_1 \sin\theta_1 = n_2 \sin\theta_2$$

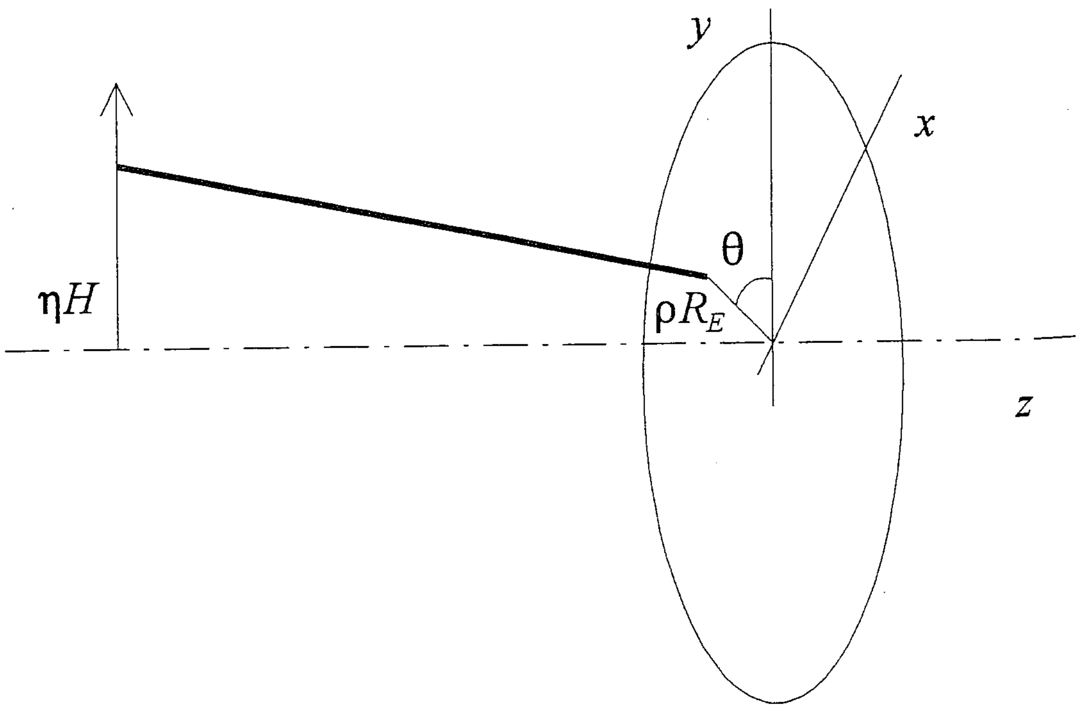
Aberration correction : Multiple surfaces with aberration are used to cancel out the aberrations.

Failure of rays to meet at the same point.

We assume the optical system is axially symmetric. We can always rotate the coordinate system so that the object lies on the x-axis. Thus, $x_0 = 0$ and $y_0 = h$.

η : fractional object height

ρ, θ polar coordinates of the entrance pupil



ϵ_x' : actual ray x-position in object space

ϵ_y' : actual ray y-position relative to the paraxial ray in object space

Aberration polynomial

$$\epsilon_x' = -\frac{1}{2 n_K u_K}\left[S\rho^3\sin\theta + C\eta\rho^2\sin(2\theta) + (A + P\Lambda^2)\eta^2\rho\sin\theta\right]$$

$$\epsilon_y' = -\frac{1}{2 n_K u_K}\left[S\rho^3\cos\theta + C\eta\rho^2(2+\cos(2\theta)) + (3A + P\Lambda)\eta^2\rho\cos\theta + D\eta^3\right]$$

(1) Spherical Aberration [S]

The only on-axis aberration

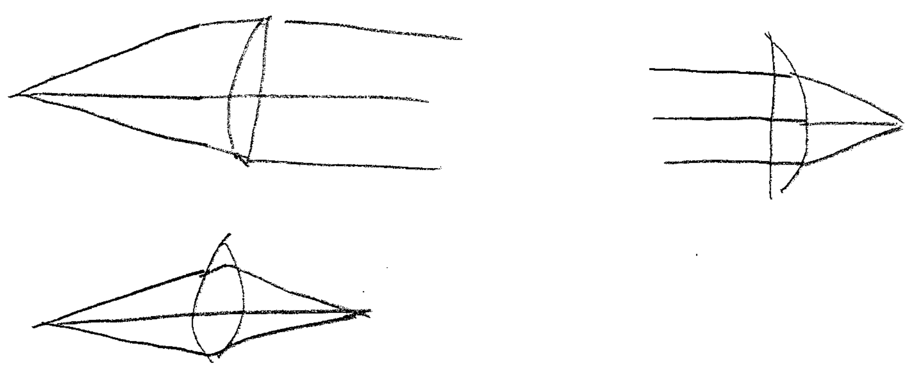
$\eta = 0$

$$\epsilon_x' = -\frac{1}{2 n_K u_K} S\rho^3\sin\theta \qquad \epsilon_y' = -\frac{1}{2 n_K u_K} S\rho^3\cos\theta$$

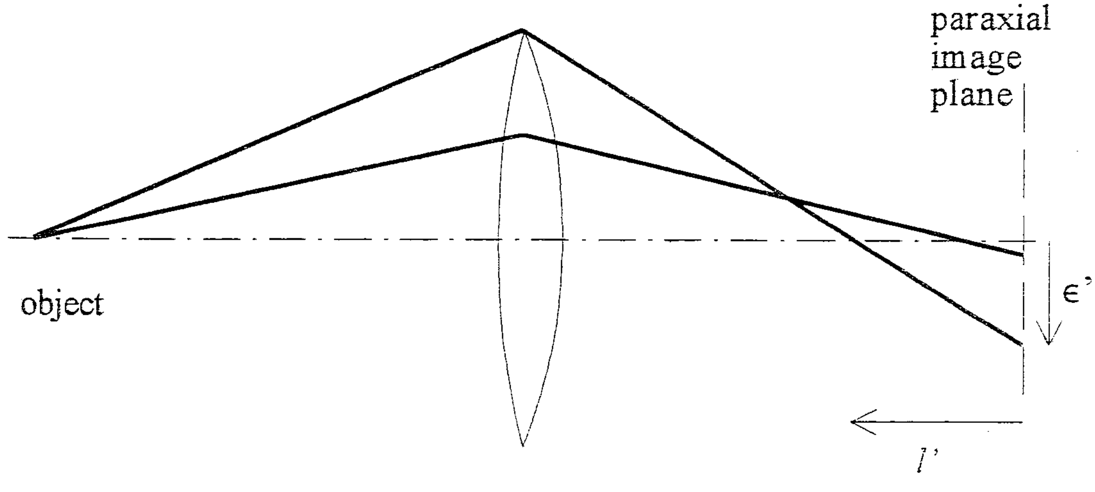
Spherical aberration varies as cube of ρ

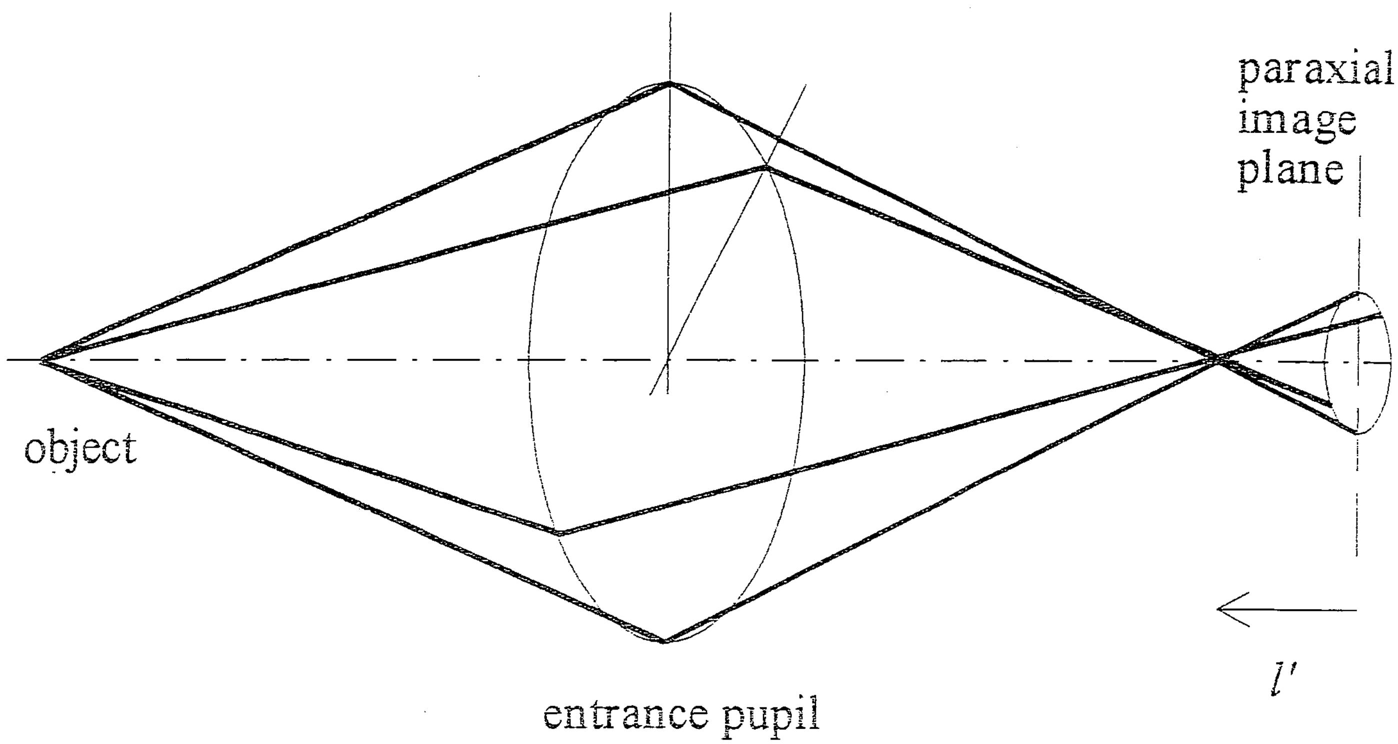
Different heights on the lens focus to different locations

Reduced by matching the lens curvature to the wavefront



Spherical aberration is worst with larger f#



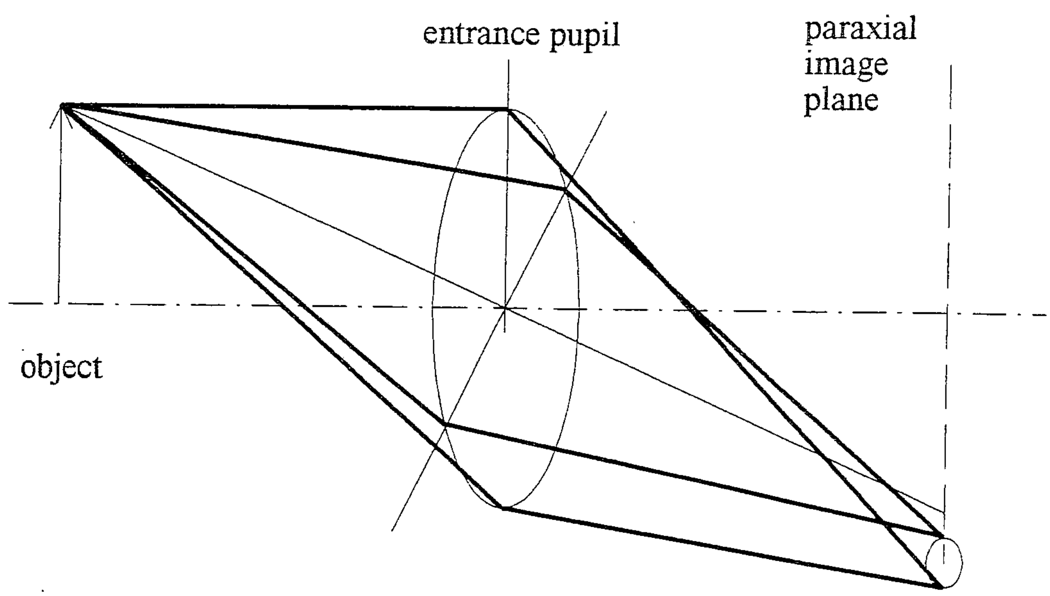

paraxial
image
plane
object
entrance pupil
l'

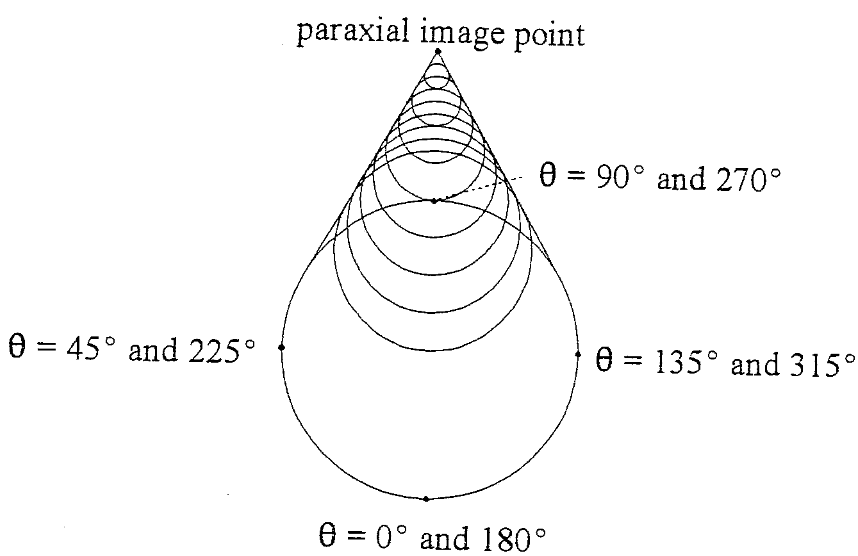
Coma

$$\epsilon_x' = \frac{-1}{2 n_k u_k} C \, \eta \rho^2 \sin 2\theta$$

$$\epsilon_y' = -\frac{1}{2 n_k u_k} C \, \eta \rho^2 (2 + \cos 2\theta)$$

Coma increases with field angle (η)





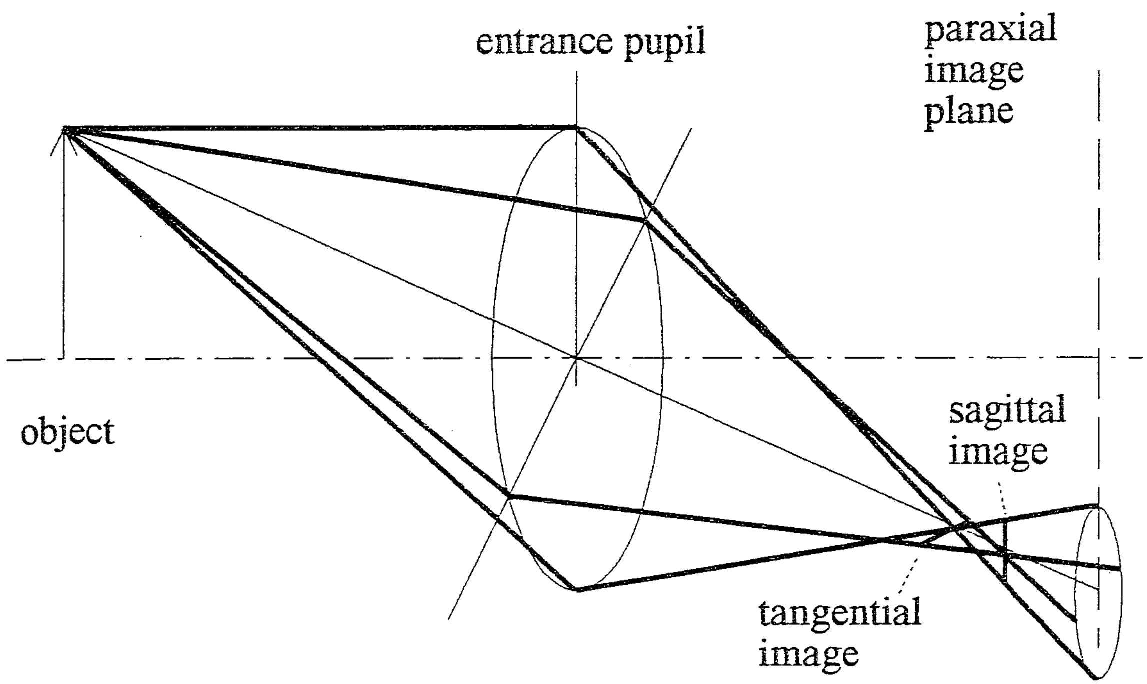
Astigmatism

Astigmatism is eye perscription is related to non-axial symmetry.

This astigmatism occurs even with perfect rotational symmetry

$$\epsilon_x = -\frac{1}{2 n_k u_k} A \, \eta^2 \rho \sin\theta$$

$$\epsilon_y = -\frac{3}{2 n_k u_k} A \, \eta^2 \rho \cos\theta$$



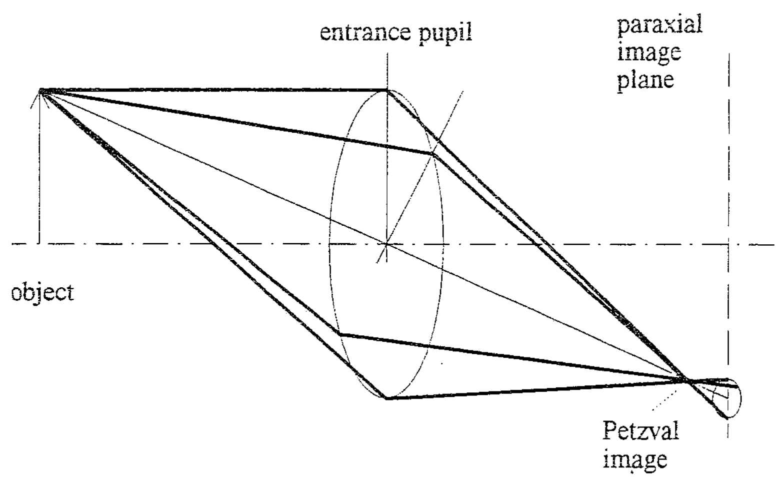
Petzval curvature
Field curvature

$$\epsilon_x' = -\frac{1}{2 n_k u_k} P \Lambda^2 \eta^2 \rho \sin\theta$$

$$\epsilon_y' = -\frac{1}{2 n_k u_k} P \Lambda^2 \eta^2 \rho \cos\theta$$

Produces a blur like spherical aberration but it depends on Field (η^2)

Leads to a curved image surface
Very similar to astigmatism but it is the same in ϵ_x and ϵ_y

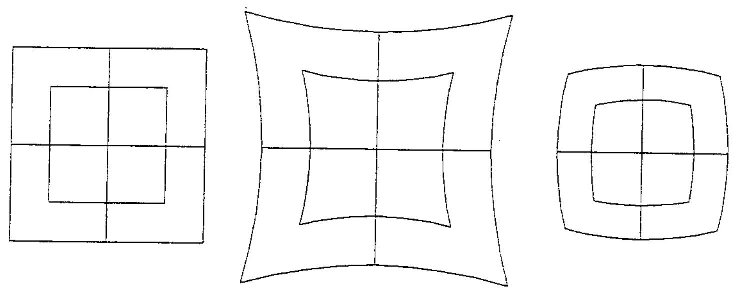


Distortion

$$\epsilon_x' = 0$$

$$\epsilon_y' = -\frac{1}{2 n_k u_k} D \eta^3$$

The real image is displaced from the paraxial image.
It depends on the object size (Fieldsize)



object

pincushion distortion

barrel distortion