Derivation of process covariance matrix Q for the EKF with CTRV motion model.

$$\text{noise} = \begin{pmatrix} a_a \\ a_{\ddot{\psi}} \end{pmatrix}$$

$$\nu = \begin{pmatrix} \frac{1}{2}(\Delta t)^2 \cos(\psi)\, a_a \\ \frac{1}{2}(\Delta t)^2 \sin(\psi)\, a_a \\ \Delta t\, a_a \\ \frac{1}{2}(\Delta t)^2 a_{\ddot{\psi}} \\ \Delta t\, a_{\ddot{\psi}} \end{pmatrix}$$

$$\nu = G a$$

$$G = \begin{pmatrix} \frac{1}{2}(\Delta t)^2 \cos(\psi) & 0 \\ \frac{1}{2}(\Delta t)^2 \sin(\psi) & 0 \\ \Delta t & 0 \\ 0 & \frac{1}{2}(\Delta t)^2 \\ 0 & \Delta t \end{pmatrix}$$

$$Q = E[G a a^T G^T] = G\, E[a a^T]\, G^T$$

$$E[a a^T] = \begin{pmatrix} \sigma_{a_a}^2 & 0 \\ 0 & \sigma_{a_{\ddot\psi}}^2 \end{pmatrix}$$

$$G^T \begin{pmatrix} \frac{1}{2}(\Delta t)^2 \cos\psi & \frac{1}{2}(\Delta t)^2 \sin\psi & \Delta t & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2}(\Delta t)^2 & \Delta t \end{pmatrix}$$

$$Q = \begin{pmatrix} \frac{1}{2}(\Delta t)^2 \cos\psi & 0 \\ \frac{1}{2}(\Delta t)^2 \sin\psi & 0 \\ \Delta t & 0 \\ 0 & \frac{1}{2}(\Delta t)^2 \\ 0 & \Delta t \end{pmatrix} \begin{pmatrix} \frac{1}{2}(\Delta t)^2 \cos\psi\, \sigma_{a_a}^2 & \frac{1}{2}(\Delta t)^2 \sin\psi\, \sigma_{a_a}^2 & \Delta t\, \sigma_{a_a}^2 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2}(\Delta t)^2 \sigma_{a_{\ddot\psi}}^2 & \Delta t\, \sigma_{a_{\ddot\psi}}^2 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{4}(\Delta t)^4 \cos^2\psi\, \sigma_{a_a}^2 & \frac{1}{4}(\Delta t)^4 \cos\psi \sin\psi\, \sigma_{a_a}^2 & \frac{1}{2}(\Delta t)^3 \cos\psi\, \sigma_{a_a}^2 & 0 & 0 \\ \frac{1}{4}(\Delta t)^4 \sin\psi \cos\psi\, \sigma_{a_a}^2 & \frac{1}{4}(\Delta t)^4 \sin^2\psi\, \sigma_{a_a}^2 & \frac{1}{2}(\Delta t)^3 \sin\psi\, \sigma_{a_a}^2 & 0 & 0 \\ \frac{1}{2}(\Delta t)^3 \cos\psi\, \sigma_{a_a}^2 & \frac{1}{2}(\Delta t)^3 \sin\psi\, \sigma_{a_a}^2 & \Delta t^2 \sigma_{a_a}^2 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{4}(\Delta t)^4 \sigma_{a_{\ddot\psi}}^2 & \frac{1}{2}(\Delta t)^3 \sigma_{a_{\ddot\psi}}^2 \\ 0 & 0 & 0 & \frac{1}{2}(\Delta t)^3 \sigma_{a_{\ddot\psi}}^2 & (\Delta t)^2 \sigma_{a_{\ddot\psi}}^2 \end{pmatrix}$$