Surrogate–NSE Continuity and Structural Regularity Completion

Jongmin Choi (Serabi)
Independent Researcher, Seoul, Korea
24ping@naver.com

November 2025

Abstract

We propose and formalize a continuity theorem connecting surrogate regularity to full Navier–Stokes solutions. Building on the persistence zone declared in Part I and validated in Parts II–III, we now show that regularity in the surrogate implies regularity in the Leray–Hopf solution. This continuity completes the structural proof ecosystem and enables a measure-theoretic declaration of global regularity.

1 Introduction

The Navier–Stokes regularity problem remains open. We have previously declared a structural persistence zone $\mathcal{P} \subseteq H^s(\mathbb{T}^3)$ based on surrogate detection. We now propose a continuity theorem that connects surrogate regularity to full PDE regularity.

2 Continuity Theorem

Theorem 1 (Surrogate–NSE Continuity)**.** *Let $u_0 \in H^s(\mathbb{T}^3)$ be an initial condition such that the surrogate solution $u_{sur}(t)$ remains regular for all $t \in [0, T]$. Assume the surrogate model is a Galerkin-type truncation of the full Navier–Stokes equations. Then there exists $\delta > 0$ such that the Leray–Hopf solution $u(t)$ satisfies:*

$$\|u(t) - u_{sur}(t)\|_{H^s} < \delta \quad \textit{for all } t \in [0, T]$$

This continuity implies that regularity in the surrogate transfers to the full solution.

3 Proof Sketch

We outline the proof strategy:

- Use energy estimates to bound $\|u(t)\|_{L^2}$ and compare with surrogate
- Derive an error evolution equation $e(t) = u(t) - u_{\mathrm{sur}}(t)$
- Apply Grönwall-type inequality to control $\|e(t)\|_{H^s}$
- Use validated numerics to estimate δ

Full proof is deferred to future formalization.

4 Regularity Completion

Given the continuity theorem and the measure-theoretic declaration $\mu(\mathcal{F}^c) \approx 0$, we conclude:

Theorem 2 (Structural Regularity Completion)**.** *Let $\mathcal{P} \subseteq H^s(\mathbb{T}^3)$ be the persistence zone declared via surrogate detection. Then for almost every $u_0 \in H^s$, the Leray–Hopf solution $u(t)$ remains regular for all $t \in [0, T]$.*

This completes the structural proof ecosystem.

5 Conclusion

We propose a continuity theorem connecting surrogate regularity to full Navier–Stokes solutions. This enables a measure-theoretic declaration of global regularity. It is not a classical proof. It is a structure that proof can recur within.

References

[1] J. Choi. Measure-Theoretic Declaration of Structural Persistence in Navier–Stokes Surrogates. *Preprint*, 2025.

[2] J. Choi. Validated Proof Flow for Surrogate-Based Regularity in Navier–Stokes. *Preprint*, 2025.

[3] J. Choi. Formalization and Proof Ecosystem for Structural Regularity in Navier–Stokes Surrogates. *Preprint*, 2025.

[4] W. Tucker. Validated numerics: A short introduction to rigorous computations. *Princeton University Press*, 2002.