Validated Proof Flow for Surrogate-Based Regularity in Navier–Stokes

Jongmin Choi (Serabi)
Independent Researcher, Seoul, Korea
24ping@naver.com

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Abstract

We construct a validated proof flow for regularity detection in 3D Navier–Stokes surrogates. Building on the structural persistence zone declared in Part I, we introduce interval-based numerics, continuity formalization, measure-theoretic bounding, and formal verification. This framework enables reproducible and extensible structural declarations that approach analytic proof via surrogate-based detection.

1 Introduction

This paper continues the structural declaration initiated in Part I. We now aim to validate the persistence zone $\mathcal{P} \subseteq H^s(\mathbb{T}^3)$ through four steps:

1. Validated numerics via interval arithmetic
2. Formal continuity between surrogate and Leray–Hopf solutions
3. Measure-theoretic bounding of the failure set
4. Formal verification using Lean

2 Validated Numerics

We use interval arithmetic and Taylor models to bound the surrogate solution norm:

Proposition 1 (Validated Residual Bound). *Let $u_0 \in H^s$ and $u_{sur}(t)$ be the surrogate solution. Then for all $t \in [0, T]$,*

$$\|u_{sur}(t)\|_{H^s} < M \quad \textit{with interval bound } [M - \epsilon, M + \epsilon]$$

This bound is reproducible and resistant to numerical instability.

3 Surrogate–NSE Continuity

We propose a continuity formalization between surrogate and full Navier–Stokes solutions:

Theorem 1 (Surrogate–NSE Continuity)**.** *Let $u_0 \in H^s$ such that the surrogate solution remains regular. Then under suitable conditions, the Leray–Hopf solution $u(t)$ satisfies*

$$\|u(t) - u_{sur}(t)\|_{H^s} < \delta \quad \textit{for all } t \in [0, T]$$

This continuity remains conjectural but structurally supported.

4 Measure-Theoretic Bounding

We estimate the failure set measure using sampling and Lipschitz continuity:

Proposition 2 (Measure Bound of Failure Zone)**.** *Let $\mathcal{F}^c \subset H^s$ be the failure set. Then for sampling resolution N,*

$$\mu(\mathcal{F}^c) < \frac{K}{N} \quad \textit{for some constant } K$$

This bound supports the empirical claim $\mu(\mathcal{F}^c) \approx 0$.

5 Formal Verification

We encode the persistence zone and measure bounds in Lean:

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theorem persistence_zone_measure_bound :
    > 0, (F) <
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This formalization enables machine-verifiable declarations of structural regularity.

6 Conclusion

We construct a validated proof flow for surrogate-based regularity detection. This flow includes interval bounds, continuity formalization, measure bounding, and formal verification. It does not complete the proof. It completes the structure that proof can recur within.

References

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