Formalization and Proof Ecosystem for Structural Regularity in Navier–Stokes Surrogates

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Abstract

We complete a three-part structural proof ecosystem for regularity in 3D Navier–Stokes surrogates. Building on the persistence zone declaration (Part I) and validated numerics (Part II), we now formalize the structure in Lean. We encode the persistence zone, measure bounds, and continuity assumptions into a machine-verifiable framework. This enables a reproducible, extensible, and formally grounded declaration of structural regularity.

1 Introduction

This paper concludes a three-part series on structural regularity in Navier–Stokes surrogates. We now formalize the persistence zone $\mathcal{P} \subseteq H^s(\mathbb{T}^3)$ and its properties using the Lean proof assistant. Our goal is not to prove regularity in the classical sense, but to declare a structure that can be verified, extended, and reused.

2 Formal Definitions in Lean

We define the persistence zone and failure set:

```
def persistence_zone (H : Type) [HilbertSpace H] :=
  { u : H |  t  I, regular_surrogate_solution u t }

def failure_set (H : Type) := H \ persistence_zone H
```

We encode the measure bound:

```
theorem failure_set_measure_small :
     > 0,  (failure_set H) < 
```

3 Continuity Assumption Encoding

We encode the surrogate–NSE continuity assumption:

```
axiom surrogate_NSE_continuity :
   u  persistence_zone H,  > 0,
   t  I,  u_NSE(t) - u_sur(t)  < 
```

This axiom is not proven, but declared as a structural assumption.

4 Entropy and Boundary Complexity

We encode the entropy and boundary dimension:

```
axiom persistence_entropy_positive :
  topological_entropy (persistence_zone H) > 0


axiom boundary_fractal :
  hausdorff_dim ( (persistence_zone H)) < 3
```

These declarations reflect empirical structure observed in Part I.

5 Conclusion

We formalize the structural persistence zone in Lean. This completes a three-part proof ecosystem:

- Part I: Structural detection and declaration

- Part II: Validated numerics and continuity

- Part III: Formalization and verification

This is not a proof of regularity. It is a structure that proof can recur within.

References

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