

# A Structural Proof Ecosystem for Navier–Stokes Regularity

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November 2025

## Abstract

We construct a structural proof ecosystem for regularity in 3D Navier–Stokes equations. Rather than seeking a singular analytic proof, we declare a reproducible framework of surrogate detection, validated numerics, continuity formalization, and formal verification. This ecosystem consists of four interlinked papers, each contributing to a self-sustaining structure of proof recurrence.

## 1 Overview

The Navier–Stokes regularity problem remains unresolved. We propose a structural approach: detect regularity zones via surrogate models, validate them numerically, connect them analytically, and encode them formally.

This ecosystem consists of:

- **Part I: Measure-Theoretic Declaration of Structural Persistence in Navier–Stokes Surrogates** Declaration of persistence zone  $\mathcal{P} \subseteq H^s(\mathbb{T}^3)$  via surrogate detection and phase mapping
- **Part II: Validated Proof Flow for Surrogate-Based Regularity in Navier–Stokes** Interval-based numerics, continuity formalization, and measure-theoretic bounding
- **Part III: Formalization and Proof Ecosystem for Structural Regularity in Navier–Stokes Surrogates** Lean encoding of persistence zone, measure bounds, and structural assumptions
- **Part IV: Surrogate–NSE Continuity and Structural Regularity Completion** Continuity theorem connecting surrogate regularity to Leray–Hopf solutions, enabling measure-theoretic declaration of global regularity

## 2 Philosophy

This is not a proof in the classical sense. It is a structure that proof can recur within. Each part is self-contained, reproducible, and extensible. Together, they form a declaration of structural regularity.

### 3 Declaration

We declare:

- A persistence zone  $\mathcal{P} \subseteq H^s(\mathbb{T}^3)$  with negligible failure measure
- A critical boundary  $A^*(\nu) \sim C\nu^{-\alpha}$  separating regularity
- Structural entropy and fractal boundary complexity
- Validated numerics supporting surrogate regularity
- A continuity theorem connecting surrogate and full PDE solutions
- Formal encoding of all declarations in Lean

This ecosystem is not the end of the proof. It is the beginning of its recurrence.

### 4 References

See individual papers:

- Part I: \*Measure-Theoretic Declaration of Structural Persistence in Navier–Stokes Surrogates\*
- Part II: \*Validated Proof Flow for Surrogate-Based Regularity in Navier–Stokes\*
- Part III: \*Formalization and Proof Ecosystem for Structural Regularity in Navier–Stokes Surrogates\*
- Part IV: \*Surrogate–NSE Continuity and Structural Regularity Completion\*