NB/BD Program toward RH: Version 5 (Proof Roadmap)

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1 Introduction

Version 4 demonstrated numerically the simultaneous satisfaction of two core conditions: (i) $M < 0$ (negative main–diag) and (ii) off–diag suppression at the $1/\log N$ scale. Version 5 outlines the steps needed to convert this into a rigorous proof.

2 Correct Frame Alignment

We must work within the exact Nyman–Beurling/Báez-Duarte (NB/BD) frame:

- Admissible functions $f(t)$ with weight $w(t) = 1/(1/4 + t^2)$.
- Approximation of 1 in $L^2(w)$ by linear combinations of n^{it-1}.

Any modified kernel or basis (e.g. band-limited, phase-modulated Gaussians) must be shown equivalent to legal test functions under this frame.

3 Uniform Bounds

Prove that for sufficiently large N,

$$M(c)\le -\theta \sum |a_n|^2, \quad \frac{E_{\mathrm{off}}(c)}{\sum |a_n|^2} \le \frac{C}{\log N}, \quad (1)$$

with constants $\theta, C > 0$ independent of N.

4 Regularity of Basis

Check that chosen modes (Gaussian, phase-modulated, $\mu(n)/n$) belong to admissible closure. Demonstrate L^2 integrability and bounded growth.

5 Residual Terms

Control truncation and discretization errors explicitly. Bound remainder R_T for truncation $T = N^\gamma$.

6 Closing the Chain

Combine estimates to show

$$d_N^2 \leq -\theta \sum |a_n|^2 + \frac{C}{\log N} \sum |a_n|^2 + o(1). \tag{2}$$

For large N, $\frac{C}{\log N} < \theta$ holds, hence $d_N \to 0$, equivalent to RH.

7 Conclusion

Version 5 establishes a roadmap: aligning with NB/BD rigor, proving uniform bounds, and controlling residuals. This sets the stage for transforming numerical evidence (V4) into a proof strategy.