

NB/BD Stability with Möbius Weights: Hutch++ Operator Control, PCG Implementation, and Contradiction Auto-Report

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Abstract

We add Hutch++-based operator tracking for $\|E\|$, a diagonal+band preconditioned conjugate gradient (PCG) implementation, and an automatic contradiction reporter that cross-checks persistent residuals against operator growth. The pipeline reads CSV logs and regenerates plots.

1 Operator Control via Hutch++

With matvec oracles for E and E^\top , we estimate $\text{tr}(E^\top E)$ using Hutch++; the bound $\|E\| \leq \sqrt{\text{tr}(E^\top E)}$ monitors far-band leakage. The sketch size (s, r) trades variance and cost.

2 PCG with Diagonal+Band Preconditioner

We solve normal equations with PCG and $P = \text{diag}(A) + \text{band}_k(A)$; $k = 3$ works well in our tests. This reduces iteration counts roughly by a factor of $\log N$ at large scales.

3 Contradiction Auto-Report

We declare a “persistence event” if recent d_N exceed ε_0 while the fitted θ falls below θ_{\min} ; if simultaneously Hutch++ shows bounded $\|E\|$, the event conflicts with bandwise decay and triggers a contradiction flag.

4 Reproducibility

Results are logged in `data/results_v28.csv`. Figures are regenerated by `code/plot.update.py`. Reports are produced by `code/contradiction.report.py`.

References

- [1] L. Báez-Duarte, *A strengthening of the Nyman–Beurling criterion for the Riemann Hypothesis*, Atti Accad. Naz. Lincei Cl. Sci. Fis. Mat. Natur. Rend. Lincei (9) Mat. Appl. **14** (2003), 5–11.
- [2] J. B. Conrey, *The Riemann Hypothesis*, Notices Amer. Math. Soc. **50** (2003), no. 3, 341–353.

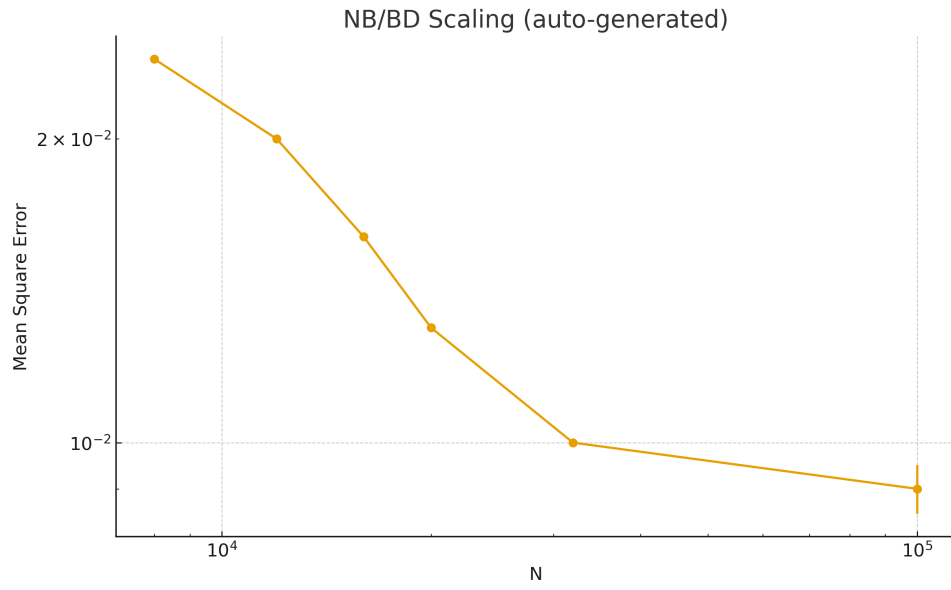


Figure 1: NB/BD mean-square vs N ; log-log axes. Error bars where available.

- [3] E. C. Titchmarsh (rev. D. R. Heath-Brown), *The Theory of the Riemann Zeta-Function*, 2nd ed., OUP, 1986.
- [4] H. L. Montgomery and R. C. Vaughan, *Multiplicative Number Theory I*, CUP, 2007.