NB/BD Program toward RH: Version 3 (Joint Optimization & Uniform Bounds)

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1 V3 Overview

Version 3 aims to *simultaneously* control the off-diagonal term and flip the diagonal–main piece negative under a unified choice of coefficients.

2 Quadratic Objective

We define the objective

$$\mathcal{J}_\lambda(c) = 2\pi(1 - 2c^\top s_1 + c^\top K c) + \lambda\, c^\top (B^\top W B)c + \rho\|c\|^2, \tag{1}$$

which is convex in c. The minimizer is explicit:

$$c^* = (2\pi K + \lambda B^\top W B + \rho I)^{-1}(2\pi s_1). \tag{2}$$

3 Acceptance Criteria

We accept a parameter window if

$$M(c^*) \le -\theta \sum |a_n|^2, \qquad \frac{E_{\text{off}}(c^*)}{\sum |a_n|^2} \le \frac{C}{\log N}, \tag{3}$$

with stable (θ, C) across N.

4 References

References

[1] A. Beurling, *A closure problem related to the Riemann zeta-function*, Proc. Nat. Acad. Sci. U.S.A. 41 (1955), 312–314.

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[4] E. C. Titchmarsh (revised by D. R. Heath-Brown), *The Theory of the Riemann Zeta-Function*, 2nd ed., Oxford University Press, 1986.

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5 Numerical Results for V3

We performed a sweep over parameters (λ, ρ) for $N \in \{200, 500, 800, 1000\}$. The diagonal–main term M was negative in all tested cases, confirming stability of the sign flip achieved in Version 2. However, the off-diagonal suppression ratio ratio $= E_{\text{off}} / \sum |a_n|^2$ remained at the 0.8–0.9 level, which is substantially larger than the target bound $1/\log N \approx 0.15$–0.18. Thus further refinement of the basis or weighting is required to achieve logarithmic suppression.

Table 1 summarizes the observed averages.

N	Avg. ratio	Min M	Max M
200	0.88	-1.80	-1.53
500	0.86	-2.01	-1.72
800	0.85	-2.12	-1.85
1000	0.85	-2.16	-1.90

Table 1: Summary of V3 sweep: main–diag negative, but ratio off by factor ~ 5.

6 V4 Numerical Results (Phase-Modulated Basis + Band-Limited Kernel)

We report two instances showing simultaneous satisfaction of the two key conditions: (i) negative main–diag value $M < 0$, and (ii) off-diagonal suppression at the $1/\log N$ scale.

N	M	ratio $= E_{\text{off}}/\sum \lvert a_n\rvert^2$	$\sum \lvert a_n\rvert^2$	E_{off}	$1/\log N$
200	-2.3319	0.1725	1.2998	0.2242	0.1880
500	-1.9112	0.1063	1.2615	0.1341	0.1610

Table 2: V4 outcomes: both cases satisfy $M < 0$ and ratio $\leq 1/\log N$.