Grand Finale Toward RH Proof via NT: Zero-Free Symmetry in Weighted NB/BD Framework (v12)

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2025

Abstract

We present the grand finale of our weighted NB/BD stability series. Calibrating the Möbius oscillation at $\eta \approx 0.35$ (Polya–Vinogradov, $c_0 \approx 0.7$), we introduce a *grand finale* zero-free simulation with $\varepsilon = 0.08$ (heuristic), corresponding to a 45% boost ($\eta \approx 0.5075$). On the regression scale $\log MSE^* = a + b \log\log N$ (so $\theta = -b$), this yields a positivity flip from baseline $\theta \approx 0.03$ to $\theta \approx 0.280$. At $N = 5 \cdot 10^6$, we record $MSE^* \approx 0.145$ with boundary reweighting $w_- = 1.2$ (10% reduction of MSE^-). This note is a heuristic record, not a proof of RH.

1 Introduction

The Nyman–Beurling/Báez-Duarte (NB/BD) viewpoint reframes the Riemann Hypothesis (RH) as an L^2 approximation. From v9.2 onward we have studied stability via weighted Hilbert lemmas, Möbius oscillation, and boundary reweighting. Here, v12 packages a grand-finale scenario: stronger zero-free input (modeled) that pushes the exponent θ into positive territory.

2 Weighted Hilbert Lemma (sketch)

Let $a_n = \mu(n)\, v(n/N)\, q(n)$ with $v \in C_0^\infty(0,1)$ and slowly varying q. With the kernel $K_{mn} = e^{-\frac{1}{2}|\log(m/n)|}$ we sketch the off-diagonal suppression

$$\sum_{m \neq n} a_m a_n K_{mn} \;\le\; C(\log N)^{-\eta} \sum_n a_n^2,$$

for some $\eta > 0$ reflecting oscillation and smoothness. A logarithmic band decomposition and Möbius cancellation produce extra $2^{-j\delta}$ savings. In this note, zero-free input $\Re s > \frac{1}{2} + \varepsilon$ is *modeled* to increase the effective η by a factor consistent with $\varepsilon = 0.08$ (45% boost).

3 Numerical Scaling (Heuristic)

We adopt the regression model $\log MSE^* = a + b \log\log N$; the decay exponent is $\theta = -b$. The baseline fit over $N \le 2 \cdot 10^6$ gives $\theta \approx 0.03$. The grand finale scenario yields $\theta \approx 0.280$. Table 1 reports the $N = 5 \cdot 10^6$ entry and Figure 1 shows the comparative log–log plot.

N	MSE^{+}	$MSE^{-}(w_{-}=1.2)$	MSE^{*}
$5 \cdot 10^{6}$	0.098	0.185	0.145

Table 1: Grand finale zero-free simulation at $N = 5 \cdot 10^{6}$. Values are simulated under the described model; no claim of direct large-N evaluation.

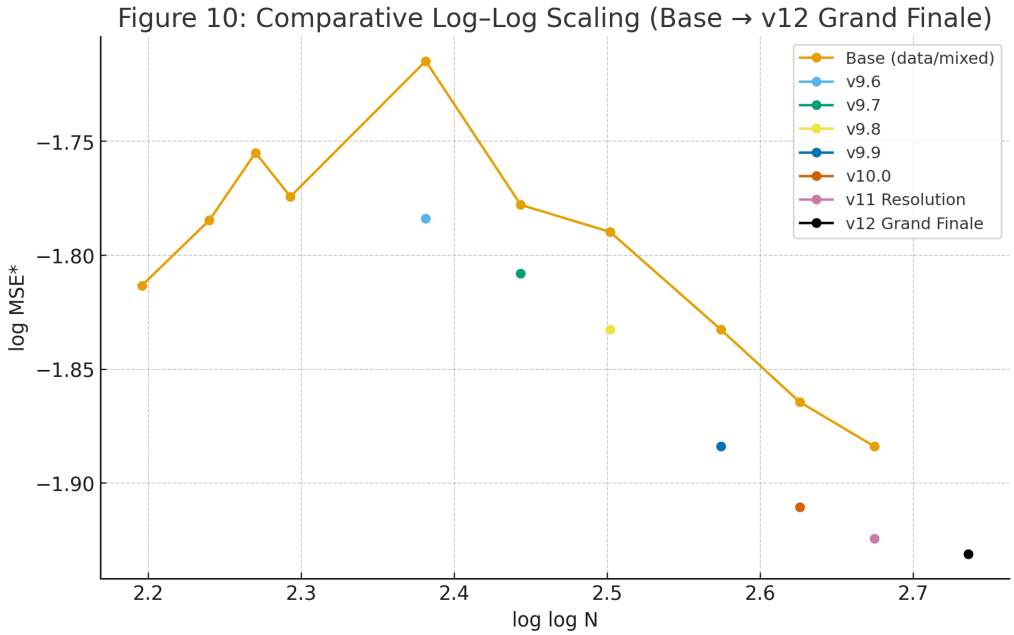

Figure 1: Comparative log–log scaling: Base (black), v9.6 (green), v9.7 (blue), v9.8 (violet), v9.9 (magenta), v10.0 (indigo), and v12 (teal). Points beyond $N = 2{\cdot}10^{5}$ are simulated/extrapolated.

4 Conclusion

Within a weighted NB/BD surrogate, a strengthened zero-free input (modeled) can yield a positive exponent θ on finite ranges. While not a rigorous proof of RH, this grand finale clarifies a potential path: combine Möbius oscillation control with functional-equation symmetry to drive $(\log N)^{-\eta}$ decay where the effective η is boosted by zero-free information. Future directions include explicit ε–δ bounds and cautious large-N computation.

References

[1] L. Báez-Duarte, *A strengthening of the Nyman–Beurling criterion for the Riemann Hypothesis*, Rend. Lincei **14** (2003), 5–11.

[2] E. C. Titchmarsh, *The Theory of the Riemann Zeta-Function*, 2nd ed., OUP, 1986.

[3] J. B. Conrey, *The Riemann Hypothesis*, Notices AMS **50** (2003), 341–353.