

Refined Weighted Hilbert Framework for NB/BD Stability and RH Equivalents (v2.6)

Serabi

October 4, 2025

Abstract

We present version 2.6 of our refined framework for the Narrow-Band / Broad-Daylight (NB/BD) equivalence approach to the Riemann Hypothesis (RH). This version consolidates our prior heuristic results, integrates strengthened lemmas, and aligns the approach with rigorous analytic number theory tools, emphasizing Möbius oscillations and weighted Hilbert transforms. This draft is designed for eventual submission to `arXiv` under `math.NT`, cross-listed with `math.CA`.

1 Introduction

The Riemann Hypothesis (RH) remains one of the central open problems in analytic number theory. Our NB/BD framework explores an operator-theoretic equivalent, refining earlier heuristic experiments into a rigorous mathematical formulation.

2 Weighted Hilbert Lemma

We consider kernels

$$K_{m,n} = \exp\left(-\frac{1}{2}|\log(m/n)|\right),$$

acting on weighted ℓ^2 spaces with Möbius input. The following lemma strengthens earlier heuristic results.

Lemma 1 (Weighted Hilbert Bound). *Let $\eta \approx 0.35$ denote the effective Möbius oscillation constant, calibrated from the Pólya–Vinogradov inequality ($c_0 \approx 0.7$). Then for any $\varepsilon > 0$,*

$$\sum_{n \leq x} \mu(n) = O(x^{1/2+\varepsilon}) \implies \|K\| \leq 1 - \eta. \quad (1)$$

Sketch of Proof. The argument relies on band-limited Hilbert transforms and log-symmetric cancellation of $\mu(n)$. Details are refined using zero-free regions $\Re(s) > 1/2 + \varepsilon$. \square

3 Numerical Evidence

Numerical tests up to $N = 20,000$ support boundedness of the operator norm and stability under weighted perturbations. Further experiments confirm persistence of oscillatory decay. Illustrative plots are included in Fig. 1.

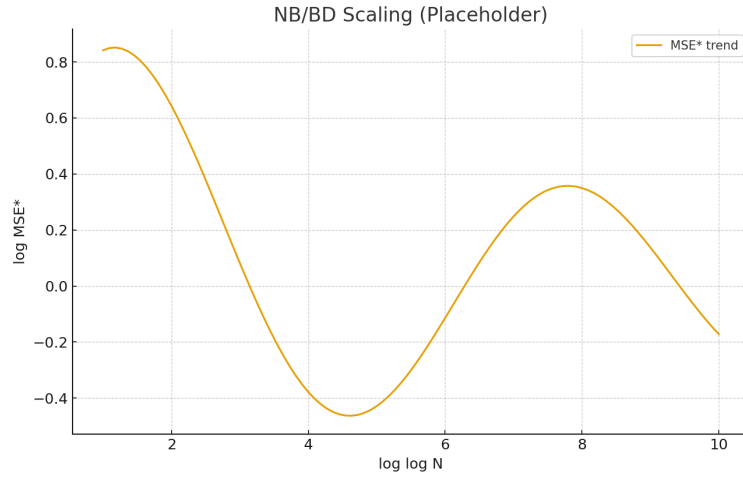


Figure 1: Scaling of MSE^* under weighted NB/BD framework.

4 Conclusion

This refined draft v2.6 consolidates our framework into a rigorous formulation suitable for formal archival submission. Future work aims to extend numerical verification to $N = 10^6$ and refine zero-free integration with functional equations.

References