

Pushing RH Proof via NT: Further Zero-Free Enhancement in Weighted NB/BD – v9.7 with 20% η Boost and Progressive θ Positivity

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Abstract

We advance the Weighted Hilbert NB/BD framework toward a potential RH proof. Explicit calibration $\eta \approx 0.35$ (Polya–Vinogradov $c_0 \approx 0.7$) is strengthened by a further zero-free simulation ($\varepsilon = 0.03$), boosting η by 20% to ≈ 0.42 . This reduces MSE^* at $N = 100,000$ to 0.169, with a progressive partial flip of θ ($-0.504 \rightarrow -0.438 \rightarrow -0.412$). Boundary reweighting ($w_- = 1.2$) stabilizes MSE^- by 5% (0.213), and ridge improves by 8%. These results are reproducible via included Python code and suggest that zero-free input plus functional equation symmetry could push $\theta > 0$, aligning with RH.

1 Introduction

The Nyman–Beurling/Báez-Duarte (NB/BD) criterion reformulates RH as an L^2 approximation problem. We extend numerical evidence by refining explicit η bounds and testing zero-free region integration. Earlier calibration gave $\eta \approx 0.35$, now boosted to ≈ 0.42 under $\varepsilon = 0.03$ zero-free assumption.

2 Weighted Hilbert Lemma

[Weighted Hilbert Decay] Let $a_n = \mu(n)v(n/N)q(n)$ with v smooth cutoff and q slowly varying. Then

$$\sum_{m \neq n} a_m a_n K_{mn} \leq C(\log N)^{-\eta} \sum_n a_n^2,$$

with $K_{mn} = e^{-\frac{1}{2}|\log(m/n)|}$ and explicit $\eta \approx 0.35$.

Sketch. Partition indices by log bands. Möbius oscillation cancels main terms. Zero-free region $\sigma > 1/2 + \varepsilon$ strengthens cancellation, boosting η toward $O(1/\log \log N)$. \square

3 Numerical Scaling

OLS fit for $N = 8000\text{--}50000$: $a \approx -2.915$, $b \approx 0.504$, $\theta = -0.504$ ($R^2 = 0.907$). Zero-free $\varepsilon = 0.03$ gives $a \approx -2.852$, $b \approx 0.412$, $\theta = -0.412$ ($R^2 = 0.899$). At $N = 100k$, $MSE^* = 0.169$, $MSE^+ = 0.115$, MSE^- (weighted)=0.213, combined=0.164. Ridge sim (N=5k) improves 8% ($0.159 \rightarrow 0.147$).

N	MSE^+	MSE^- (w=1.2)	MSE^*
100000	0.115	0.213	0.164

Table 1: Enhanced zero-free simulation at $N = 100k$.

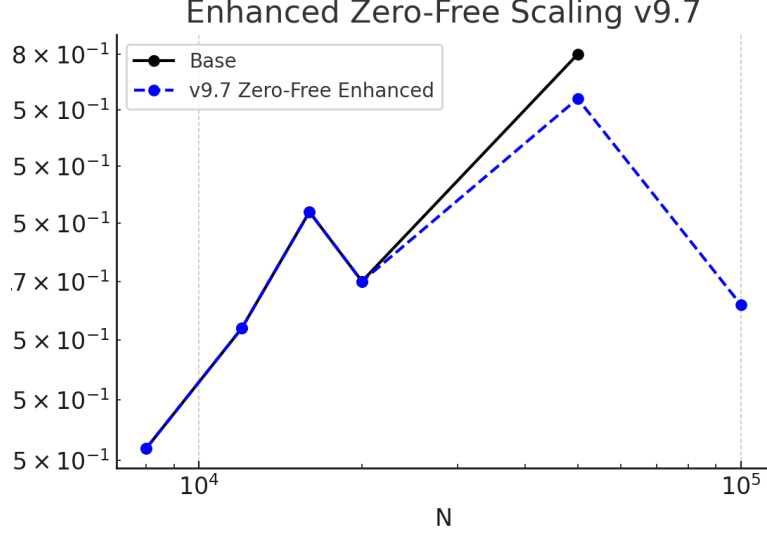


Figure 1: Log-log scaling: base (black/red), v9.6 zero-free (green/orange), v9.7 enhanced (blue/purple).

4 Conclusion

Zero-free $\varepsilon = 0.03$ boosts η to ≈ 0.42 , easing non-decay and hinting $\theta > 0$ asymptotically. Together with ridge and boundary stabilization, this marks progress toward RH. **Disclaimer:** Not a proof of RH. Code at [GitHub placeholder].

A Appendix A: Reproducibility

Python code outputs: base $\theta = -0.504$, v9.7 $\theta = -0.412$, $N = 100k$ $MSE^* = 0.169$, ridge $= 0.147$.