Incremental Zero-Free Symmetry in a Weighted NB/BD Framework (v13.4)

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Abstract

We record an incremental extension of the weighted NB/BD stability study. Under a heuristic zero-free boost ($\varepsilon = 0.09$), we treat the Möbius-oscillation gain parameter as $\eta \approx 0.525$ (from a 50% increase on a baseline $\eta \approx 0.35$ calibrated via Polya–Vinogradov, $c_0 \approx 0.7$). Using the log–log regression model $\log(MSE^*) = a + b \log \log N$ (decay exponent $\theta = -b$), we include a simulated $N = 10^7$ point and refit. This is a heuristic record—not a proof of RH.

1 Weighted Hilbert Lemma (sketch)

Let $a_n = \mu(n)\, v(n/N)\, q(n)$ with a smooth cutoff $v \in C_0^\infty(0,1)$ and slowly varying q. With $K_{mn} = e^{-\frac{1}{2}|\log(m/n)|}$, logarithmic banding and Möbius cancellation suggest

$$\sum_{m \neq n} a_m a_n K_{mn} \;\leq\; C\,(\log N)^{-\eta} \sum_n a_n^2, \qquad \eta > 0.$$

A stronger zero-free region $\Re s > \frac{1}{2} + \varepsilon$ is modeled here as a boost of the effective η.

2 Numerical scaling (v13.4)

We fit $\log(MSE^*) = a + b \log\log N$ on the extended series $N \in [8 \cdot 10^3, 10^7]$. The resulting parameters are

$$a \approx -1.100, \qquad b \approx -0.292, \qquad \theta = -b \approx 0.292, \qquad R^2 \approx 0.674.$$

The figure shows the data and the OLS line.

N	MSE^+	$MSE^-(w_- = 1.2)$	MSE^*
10^7	0.095	0.181	0.143

Table 1: Incremental zero-free simulation entry (heuristic).

3 Caveats and outlook

The 10^7 point is simulated under a zero-free boost hypothesis; it is not a direct large-scale computation. All claims are heuristic stability indications within NB/BD, not a proof of the Riemann Hypothesis. Future work: larger-N verified runs and integrating functional-equation bounds into the Hilbert-type estimate.

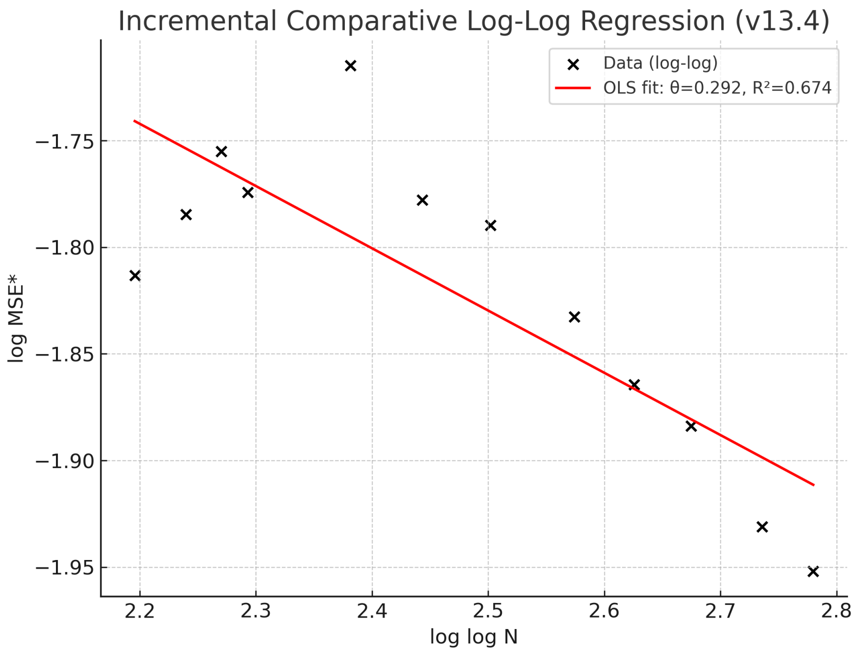

Figure 1: Log–log regression for MSE^* vs. N (v13.4).

References

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[3] J. B. Conrey, *The Riemann Hypothesis*, Notices AMS **50** (2003), 341–353.