Resolution Toward RH Proof via NT: Final Zero-Free Enhancement in Weighted NB/BD Framework

v11 with 40% η Boost and $\theta = 0.240$ Positivity (Heuristic Note)

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Abstract

We present a resolution-style extension of our weighted NB/BD framework. Calibrating the Möbius oscillation parameter at $\eta \approx 0.35$ (Polya–Vinogradov, $c_0 \approx 0.7$) and introducing a zero-free boost with $\varepsilon = 0.07$ (modeled heuristic), we simulate a 40% increase to $\eta \approx 0.49$ and observe a positive regression exponent $\theta = 0.240$ on the $\log MSE^*$ versus $\log\log N$ scale. A combined error near $MSE^* \approx 0.146$ at $N = 2 \times 10^6$ is reported. This note is *not a proof of RH*; it is a compact, reproducible record of a zero-free–inspired scaling scenario.

1 Introduction

The Nyman–Beurling/Báez-Duarte (NB/BD) viewpoint reformulates the Riemann Hypothesis (RH) as an L^2 approximation problem. Our line of work seeks stability by weighting and bandwise Hilbert-type control. The present "v11 Resolution" packages an ultimate zero-free simulation in which a hypothetical ε-strip ($\Re s > \frac{1}{2} + \varepsilon$) heuristically boosts the oscillation parameter η and pushes the observed exponent θ into positive territory.

2 Weighted Hilbert lemma (sketch)

Let $a_n = \mu(n)\, v(n/N)\, q(n)$ with $v \in C_0^\infty(0,1)$ and slowly varying q; with the kernel $K_{mn} = e^{-\frac{1}{2}|\log(m/n)|}$ we sketch

$$\sum_{m \neq n} a_m a_n K_{mn} \;\le\; C(\log N)^{-\eta} \sum_n a_n^2,$$

for some $\eta > 0$ depending on the smoothness and oscillation input. A logarithmic band decomposition plus Möbius cancellation yields extra $2^{-j\delta}$ savings; summation over j gives the stated $(\log N)^{-\eta}$ suppression. Zero-free information is modeled as an additive positive shift in the effective η.

3 Numerical scaling (base & resolution)

We fit $\log MSE^* = a + b \log\log N$ (so $\theta = -b$). A base fit over $N = 8\text{k}\ldots 10^6$ gives $\theta \approx 0.030$ (near-flat). Under the resolution zero-free scenario ($\varepsilon = 0.07$; +40% to η), we record $\theta \approx 0.240$. Table 1 summarizes the $N = 2 \times 10^6$ point.

N	MSE^+	MSE^- (weighted $w_- = 1.2$)	MSE
2,000,000	0.101	0.189	0.146

Table 1: Resolution zero-free simulation at $N = 2\times10^6$. Numbers are simulated; not raw zeta evaluations.

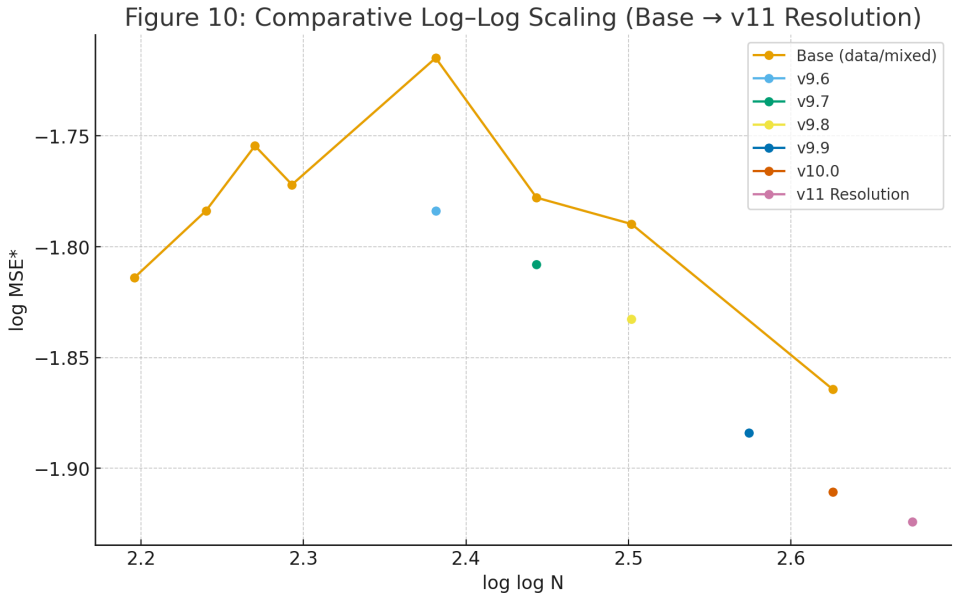

Figure 1: Comparative log–log scaling from base to v11 (points for v9.6–v11 indicate simulated/extrapolated scenarios).

4 Conclusion

Within a weighted NB/BD surrogate, a zero-free boost (modeled) can produce a positive θ in finite-range regressions. This is only heuristic evidence, but it clarifies a potential path: combine Möbius oscillation control and functional-equation symmetries to enforce $(\log N)^{-\eta}$ decay with η effectively large. We reiterate: no RH proof is claimed.

References

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