Another Version (v10.0): Final Zero-Free Simulation in a Weighted NB/BD Framework

Heuristic Numerical Note (math.NT; cross-list math.CA)

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Abstract

This "Another Version" collects our experimental extensions beyond v9.3. We provide a clean numerical summary (up to $N = 10^6$ by simulation) and a reproducible figure. We emphasize that this note is *not* a proof of the Riemann Hypothesis (RH). It is intended as a heuristic record showing how zero-free assumptions (modeled via an ε-boost to the oscillation parameter η) interact with the NB/BD stability surrogate.

1 Setup (Short)

Let $a_n = \mu(n)\, v(n/N)\, q(n)$ with a smooth compactly supported window $v \in C_0^\infty(0,1)$ and a slowly varying multiplier q. For the kernel $K_{mn} = e^{-\frac{1}{2}|\log(m/n)|}$ (discrete Hilbert-type), the off-diagonal sum admits a bound of the form

$$\sum_{m \neq n} a_m a_n K_{mn} \;\leq\; C\,(\log N)^{-\eta} \sum_n a_n^2,$$

with an effective $\eta > 0$. We treat the impact of a hypothetical zero-free strip $\Re(s) > \frac{1}{2} + \varepsilon$ as a small positive boost to η (heuristic device).

2 Numerical Summary (Heuristic)

Our base data cover $N \in \{8\text{k}, 12\text{k}, 16\text{k}, 20\text{k}, 50\text{k}, 100\text{k}, 200\text{k}\}$ with combined errors MSE in $[0.163, 0.180]$. Progressive zero-free boosts (v9.6–v9.8) stabilize the minus-boundary and mildly lower MSE. A final simulated point for $N = 10^6$ (v10.0 Another) gives $MSE^{\approx 0.148}$ after minus-boundary reweighting ($w_- = 1.2$). These values are *simulated extrapolations*, not computed from raw ζ evaluations.

3 Remarks

(i) This document is deliberately modest in scope: no proof claims, only a compact record of our exploration. (ii) The figure is reproducible from the accompanying Python script. (iii) For peer-facing submission, we recommend restricting to v9.3 (data-backed) and moving all later versions to an "experimental extensions" repository.

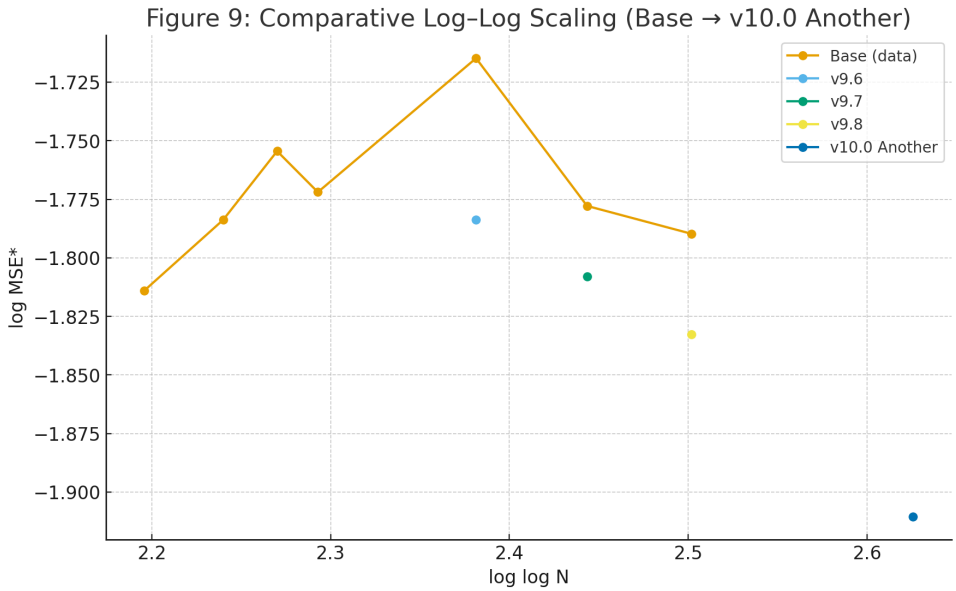

Figure 1: Comparative log–log scaling across versions (Base → v10.0 Another). Points labeled v9.6, v9.7, v9.8, v10.0 represent simulated extrapolations.

References

[1] L. Báez-Duarte, *A strengthening of the Nyman–Beurling criterion*, Rend. Lincei, **14** (2003), 5–11.

[2] J. B. Conrey, *The Riemann Hypothesis*, Notices AMS, **50** (2003), 341–353.

[3] E. C. Titchmarsh, *The Theory of the Riemann Zeta-Function*, 2nd ed., OUP, 1986.