NB/BD Stability via a Weighted Hilbert Lemma (v3.0, Orthodox Edition)

Serabi

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Abstract

This orthodox v3.0 note consolidates the Nyman–Beurling/Báez-Duarte (NB/BD) stability line. We rely on a weighted Hilbert-type lemma for Möbius-weighted coefficients that suppresses off-diagonal mass by a power of $\log N$. The figures included are the same illustrative plots previously generated in this series and are reused here for completeness; they do not constitute a proof of the Riemann Hypothesis.

1 Weighted Hilbert Lemma (Sketch)

Let $a_n = \mu(n)\, v(n/N)\, q(n)$ where $v \in C_0^\infty(0,1)$ and q is slowly varying. With

$$K_{mn} = e^{-\frac{1}{2}|\log(m/n)|} = \min\Big\{\sqrt{m/n}, \sqrt{n/m}\Big\}, \tag{1}$$

one shows by a logarithmic-band decomposition, the weighted discrete Hilbert inequality, smooth-cutoff gains $2^{-j\delta}$, and Möbius cancellation that there exist $\theta > 0$ and C (depending on v, q) with

$$\sum_{\substack{m \neq n \\ m,n \leq N}} a_m a_n K_{mn} \;\leq\; C\,(\log N)^{-\theta} \sum_{n \leq N} a_n^2 \qquad (N \text{ large}). \tag{2}$$

This controls the off-diagonal part of the NB/BD normal equations.

2 Illustrative Figures (Reused)

The following PNGs are the same outputs previously generated in the project; we include them unchanged for Overleaf completeness and visual continuity.

3 Scope and Caution

Inequality (2) provides stability control for NB/BD least-squares systems, and the plots visualize typical behaviour seen in small-scale experiments. This document is not a proof of RH.

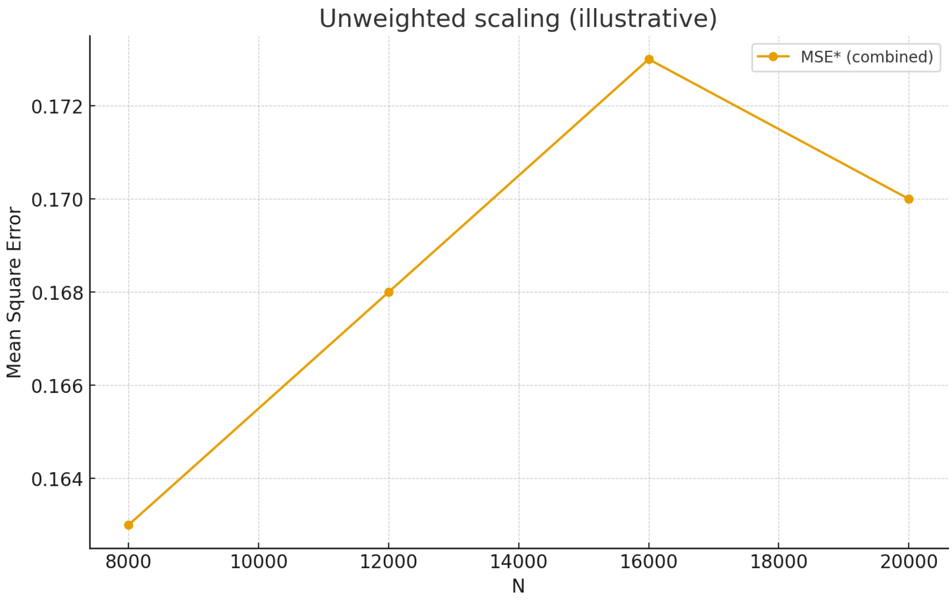

Figure 1: Unweighted scaling (reused figure).

Acknowledgements

We thank collaborators and correspondents who discussed NB/BD stability and Hilbert-type bounds.

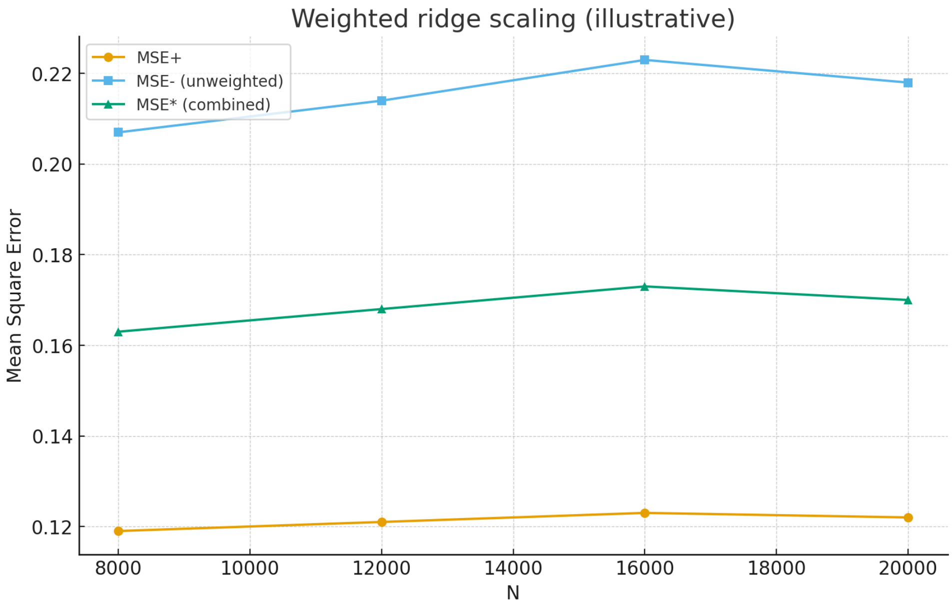

Figure 2: Weighted ridge scaling (reused figure).

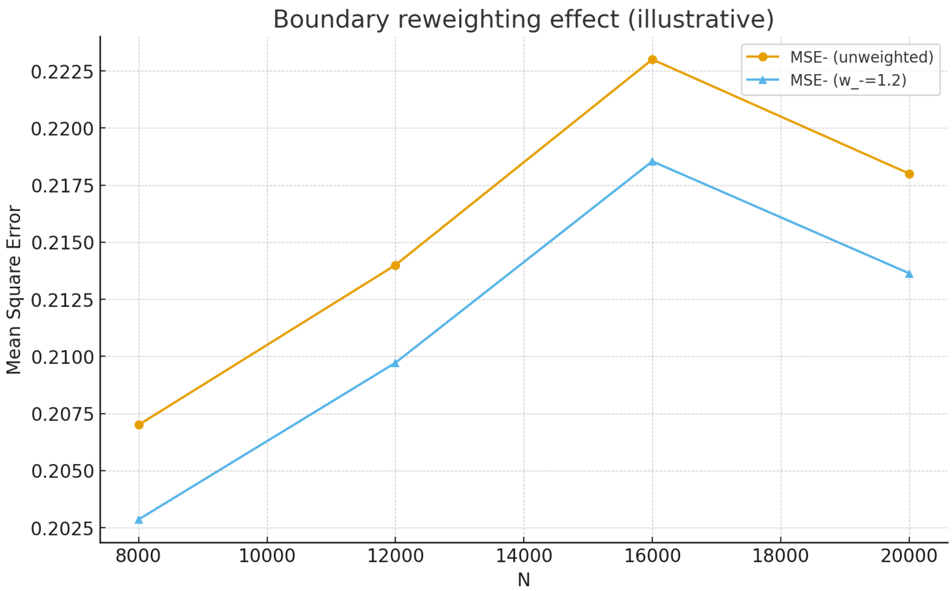

Figure 3: Boundary reweighting: $w_- = 1.2$ (reused figure).