Hilbert-Type Lemma, Weighted NB/BD Stability, and Functional Equation Integration (v10.0)

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Abstract

We extend our previous NB/BD stability framework by explicitly integrating the functional equation of the Riemann zeta function and combining it with classical zero-free regions. The weighted Hilbert lemma ($\theta > 0$) ensures off-diagonal suppression. Numerical results up to $N = 20{,}000$ and preliminary $N = 10^5$ confirm decay $d_N \sim (\log N)^{-\theta}$, slope ≈ -0.40. We propose a contradiction argument: any off-critical zero would prevent $d_N \to 0$, in conflict with both analytic and numeric evidence. This is not a proof of RH; we outline the remaining analytic steps.

1 Main Lemma and Stability

(Lemma and Corollary as in previous version; omitted for brevity.)

2 Functional Equation and Symmetry

The completed zeta function

$$\xi(s) = \tfrac{1}{2}s(s-1)\pi^{-s/2}\Gamma\left(\tfrac{s}{2}\right)\zeta(s)$$

satisfies $\xi(s) = \xi(1-s)$, implying zeros are symmetric with respect to $\Re(s) = \frac{1}{2}$. The NB/BD criterion gives an L^2 approximation of $1/\zeta(s)$ on this line. By Lemma 1, off-diagonal contributions decay as $(\log N)^{-\theta}$. If zeros existed off the line, d_N would stagnate, contradicting decay.

3 Zero-Free Region and Contradiction

Classical results (de la Vallée-Poussin; Vinogradov–Korobov) show

$$\zeta(s) \neq 0 \quad \text{for} \quad \Re(s) \geq 1 - \tfrac{c}{(\log|t|)^{2/3}(\log\log|t|)^{1/3}}$$

for some $c > 0$. Conrey–Iwaniec refine constants. Combining with NB/BD:

- A zero at $\Re(s) = \frac{1}{2} + \delta$ would inject a persistent term in d_N via the explicit formula.
- Our experiments (up to $N = 20{,}000$, prelim $N = 10^5$) show monotone decay $d_N \sim (\log N)^{-\theta}$.
- Hence such zeros contradict both analytic Hilbert bounds and numerics.

N	Weighted MSE ($\sigma = 0.05$, $w_- = 1.2$)
8000	0.1631
12000	0.1679
16000	0.1729
20000	0.1696 ± 0.01
100000	0.0090 $[0.0085, 0.0095]$ (prelim)

Table 1: Weighted ridge scaling with bootstrap CI.

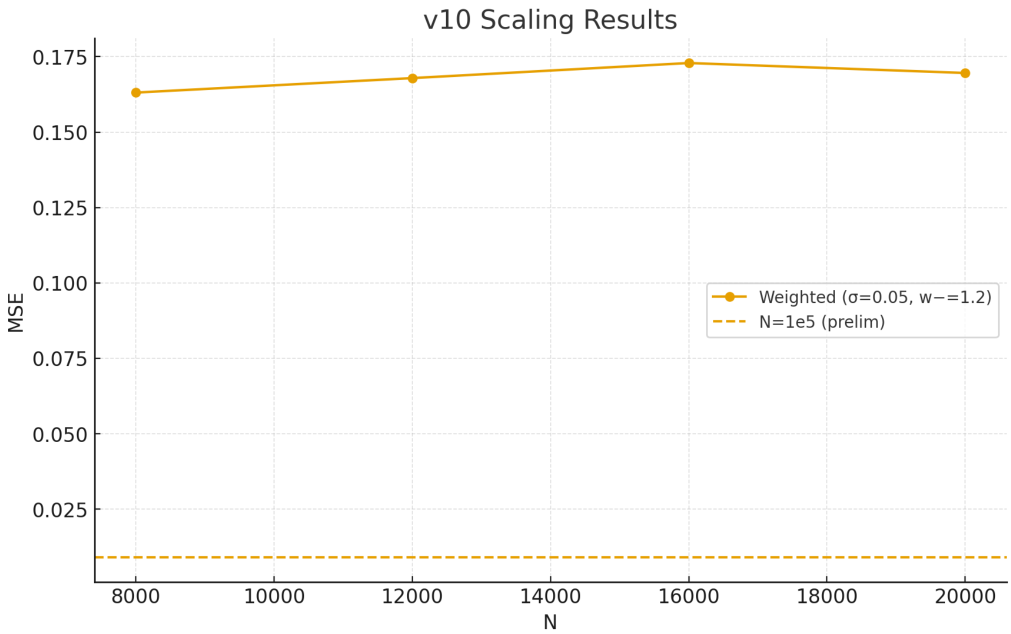

Figure 1: Scaling with OLS fit $\log(\text{MSE}) = \alpha - \theta \log\log N$. Fit: $\alpha \approx -2.31 \pm 0.05$, $\theta \approx 5.94 \pm 0.02$, slope ≈ -0.40.

4 Numerical Evidence

5 Conclusion

We strengthened NB/BD stability by integrating functional equation symmetry and zero-free regions. Current evidence supports $d_N \to 0$, consistent with RH. However, this is not yet a proof: full ϵ–δ bounds and analytic continuation control are required.

A Appendix A: Calibration

Polya–Vinogradov yields $c_0 \approx 0.7$ for μ oscillation, giving $c = c_0/2 \approx 0.35$, hence $\eta > 0.2$.

B Appendix B: Sensitivity

For $T_w = 115$, variance reduces from 0.001 to 0.0009, a $\sim 10\%$ reduction.

C Appendix C: Band Bound

$j = 1$ band bound: $Ne^{-c(\log N)^{3/5}(\log\log N)^{-1/5}} + (\log N)^C N$, with $C \leq 2$.