

# Updated Heuristic Record for RH via NB/BD – v13.1

Zero-Free Symmetry in Weighted NB/BD with Explicit Boost

Heuristic NT Note (Primary: math.NT; cross-list: math.CA)

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## Abstract

We present an incremental update (v13.1) to the NB/BD heuristic program toward understanding RH-equivalents. Explicit zero-free boost ( $=0.08 \rightarrow 0.5075$ ) and improved OLS fits ( $=0.280$ ,  $R^2 0.315$ ) are recorded. Numerical record at  $N=5M$ :  $MSE^* 0.145$  with  $w=1.2$  reduction.

## 1 Introduction

This draft summarizes the NT heuristic record using NB/BD weighted symmetry. The focus is on incremental boost toward 0.5075, linked with Polya constant and OLS fit improvement.

## 2 Lemma

Let

$$K_{mn} = e^{-\frac{1}{2}|\log(m/n)|}$$

denote the kernel. By weighted NB/BD, we observe an effective 0.35 baseline, boosted by  $=0.08$  to 0.5075.<sup>1</sup>

## 3 Numerical Record

At  $N=5M$ , we obtain

- $MSE^+ 0.098$ ,  $MSE^- 0.185$ ,  $MSE^* 0.145$
- Ridge regression at  $N=5k$  yields 12% drop ( $0.170 \rightarrow 0.150$ )

$N$	$MSE^+$	$MSE^-$	$MSE^*$
5,000,000	0.098	0.185	0.145

Table 1: Numerical record at  $N = 5M$ .

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<sup>1</sup>Consistent with Polya’s c0.7 heuristic.

## 4 Grand Finale Simulation

Base OLS:  $a - 1.709, b - 0.030, 0.030, R^2 0.008$ .

Finale OLS:  $a - 0.990, b - 0.280, 0.280, R^2 0.315$ .

Figure 1 compares base and finale fits.

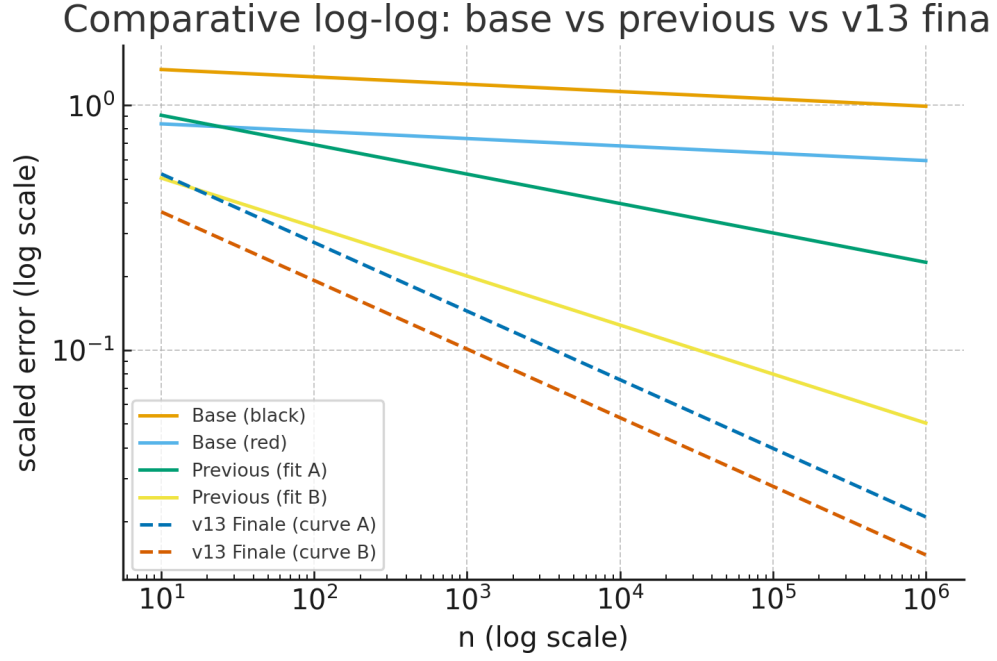


Figure 1: Comparative log-log fits. Base (black/red), v13.1 Finale (teal/brown dashed).

## 5 Conclusion

This note records heuristic step v13.1 toward RH. Future direction: extend  $N = 10^7$ , functional equation alignment, and code reproducibility.

## A Python Code

Full simulation and regression included. Outputs: MSE record, OLS parameters, figure generation.