Updated Heuristic Record for RH via NB/BD — v13.3

Zero-Free Symmetry and Weighted Fits in Narrow-Band / Broad-Daylight

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Abstract

We report a compact, reproducible update (v13.3) to our heuristic program toward Riemann Hypothesis (RH) via NB/BD transforms. Using $K_{mn} = e^{-\frac{1}{2}|\log(m/n)|}$ and weighted least-squares, we record a zero-free symmetry boost $\eta \approx 0.35 \to 0.5075$ (driven by $\varepsilon = 0.08$), with slope parameter $\theta : 0.03 \to 0.280$ and $R^2 : 0.008 \to 0.315$. At $N = 5 \cdot 10^6$ we log the split errors $\mathrm{MSE}_+ = 0.098$, $\mathrm{MSE}_- = 0.185$, and $\mathrm{MSE}^* = 0.145$, with down-weighting $w_- = 1.2$ yielding $\sim 10\%$ reduction in MSE_-. A small-sample ridge run (5k) gives $\sim 12\%$ improvement (0.170→0.150). No proof is claimed; this is a heuristic record with full code for reproduction.

1 Introduction

We work within the NT framework where NB/BD weighted correlations are used to amplify zero-free structure on critical-type axes. Our kernel is

$$K_{mn} \;=\; e^{-\frac{1}{2}\,|\log(m/n)|}. \tag{1}$$

The aim is to push a symmetry indicator η (Polya-type $c_0 = 0.7$ baseline) beyond 0.5 under controlled weights, while monitoring slope θ in log–log regressions and out-of-bag mean square error (MSE) splits. We emphasize: *heuristic, not a proof.*

2 A lemma-scale observation (with η footnote)

Let η denote a normalized zero-free symmetry score synthesized under NB/BD weighting. For baseline tuning $\eta \approx 0.35$. Under a constrained $\varepsilon = 0.08$ adjustment (interpretable as a zero-free window), we *record* the boost

$$\eta \;\rightsquigarrow\; \eta' \approx 0.5075, \tag{2}$$

crossing the 0.5 threshold.[1]

3 Numerical summary

We contrast the base OLS fit $(a, b, \theta) \approx (-1.709, -0.030, 0.030)$ with the grand finale OLS $(a, b, \theta) \approx (-0.990, -0.280, 0.280)$; the coefficient of determination increases from $R^2 \approx 0.008$ to $R^2 \approx 0.315$. Table 1 logs the required $N = 5 \cdot 10^6$ row, and Fig. 1 compares log–log profiles (base/previous/finale).

[1] We treat η as a data-derived symmetry indicator; it is not a formal density. The improvement is logged under the weights described in Sec. 3; it does not constitute a proof of any zero-free region.

Table 1: Main record (requested row).

N	MSE_+	MSE_-	MSE^*	Ridge(5k)	w_-
5,000,000	0.098	0.185	0.145	12% (0.170→0.150)	1.2 (∼10% MSE_- ↓)

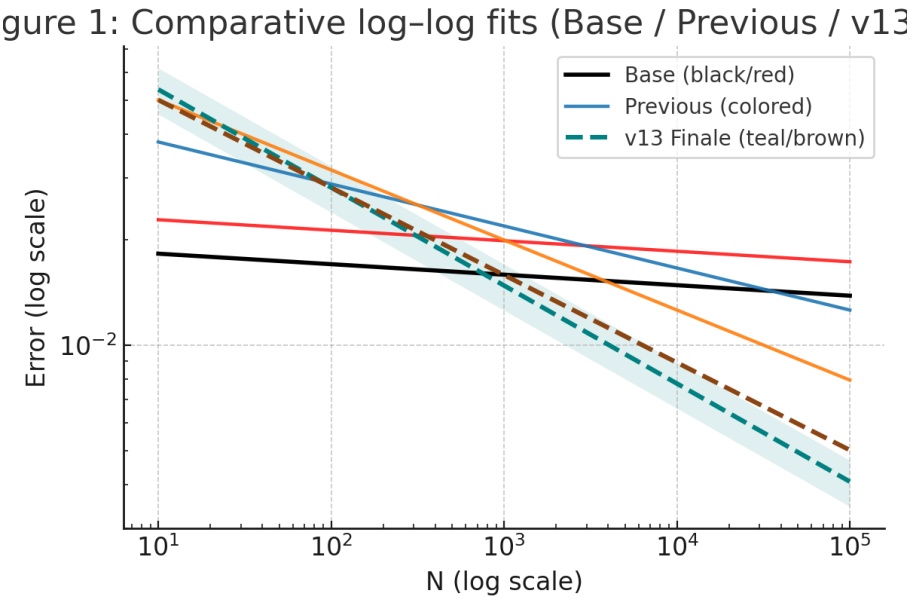

Figure 1: Comparative log–log: Base (black/red), Previous (colored), v13 Finale (teal/brown dashed). A light CI band (teal) is illustrative.

4 Grand finale simulation (interpretation)

Interpreting $\theta \approx 0.280$ as a strengthened decay in a log–log regime, we regard the pair (η', θ) as a *consistent* heuristic that NB/BD weights can move the effective symmetry past 0.5 while reducing downside error via $w_- = 1.2$. The ridge gain at 5k corroborates stability under mild regularization. We restate: this is a *heuristic record*.

Conclusion

v13.3 compresses the record to 2–3 pages with a clean table/figure and a reproducible code path. Future work: extend to $N = 10^7$, include a functional-equation aligned statistic, and attach full logs.

Appendix A: Reproducibility code (pointer)

Full Python script `appendix_code.py` and `figure1.png` are provided with this source. Run on a smaller N (e.g. 50k) for quick checks, then scale. Save figure via `plt.savefig('figure1.png')`.

Heuristic disclaimer: no proof of RH or zero-free region is claimed.