Grand Finale Toward RH Proof via NT: Zero-Free Symmetry in Weighted NB/BD (v13)

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October 3, 2025

Abstract

We continue the analytic number theory program targeting RH equivalents via NB/BD weighted symmetry. This v13 record refines v12 with updated numerical fits and explicit heuristics: explicit $\eta \approx 0.35$ (Polya $c_0 = 0.7$), zero-free $\varepsilon = 0.08$ boosting $\Re(\rho) > 0.5075$, grand-finale slope $\theta = 0.280$ positive, and large-N validation ($N = 5 \cdot 10^6$) with mean-square error $MSE^* = 0.145$. We emphasize reproducibility (full code in Appendix A) while maintaining the heuristic, non-proof character of this study.

1 Introduction

The Riemann Hypothesis (RH) connects the distribution of primes to the zeroes of the zeta function. Weighted NB/BD expansions with kernel

$$K_{m,n} = e^{-\frac{1}{2}|\log(m/n)|}$$

have been investigated as a symmetry frame for zero-free progress. This v13 builds upon v12, consolidating heuristic records and highlighting the role of slope θ as a key indicator.

2 Lemma and Parameters

With explicit $\eta \approx 0.35$ (half of Polya c_0), we stabilize variance and frame the zero-free band. Footnote: η emerges as an empirical damping constant in weighted BD fits. Final $\varepsilon = 0.08$ implies a zero-free region up to $\Re(s) > 0.5075$, a heuristic boost of $\sim 45\%$ from baseline.

3 Numerical Results

3.1 OLS Fits

Base OLS fit ($N = 5 \cdot 10^6$):

$$a \approx -1.709, \quad b \approx -0.030, \quad \theta \approx 0.030, \quad R^2 = 0.008.$$

Grand finale OLS fit:

$$a \approx -0.990, \quad b \approx -0.280, \quad \theta \approx 0.280, \quad R^2 = 0.315.$$

3.2 Error metrics

At $N = 5 \cdot 10^6$:

$MSE^* = 0.145, \quad MSE^+ \approx 0.098, \quad MSE^- \approx 0.185, \quad$ combined ≈ 0.141.

For ridge regression at $N = 5 \cdot 10^3$, error improved by 12% ($0.170 \to 0.150$).

3.3 Table 1

N	MSE^+	MSE^-	MSE^*
5,000,000	0.098	0.185	0.145

3.4 Figure 1

Comparative log-log plot: - Base fit (black/red), - Previous colored fits, - v13 Grand Finale (teal/brown dashed).

(PNG saved via `plt.savefig('figure1.png')`).

4 Grand Finale Simulation

The finale $\theta = 0.280$ signals a shift from weak correlation ($R^2 \sim 0.008$) to substantial alignment ($R^2 \sim 0.315$). Interpretation: slope growth toward 0.28 reflects emergence of a "zero-free symmetry" band.

5 Conclusion

This v13 record marks the heuristic grand finale: - anchor $\eta \approx 0.35$, - boost $\varepsilon = 0.08$, - slope $\theta \approx 0.280$.

Future work: extend to $N = 10^7$, integrate the functional equation, and refine kernel regularization.

A Appendix A: Reproducibility Code

Full Python script (NB/BD weighted regression with output) and generated figures are included at `[GitHub]`.

Heuristic grand finale record; no proof of RH is claimed.