Towards a Stable NB/BD Approximation: Weighted Hilbert Lemma, Numerical Scaling, and Boundary Reweighting

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Abstract

We present an improved analysis of the Nyman–Beurling/Báez–Duarte (NB/BD) criterion for the Riemann Hypothesis. Our main contribution is a weighted Hilbert-type lemma for Möbius-weighted coefficients, ensuring off-diagonal suppression by $(\log N)^{-\theta}$ with $\theta > 0$. We combine this with numerical experiments up to $N = 20{,}000$, including minus-boundary reweighting ($w_- = 1.2$) and bootstrap confidence intervals, confirming stable decay exponents. We emphasize that $d_N \to 0$ shows stability of NB/BD, not a direct proof of RH.

1 Introduction

The Riemann Hypothesis (RH) asserts that all nontrivial zeros of $\zeta(s)$ lie on $\Re(s) = 1/2$. The NB/BD criterion reformulates RH as an L^2 approximation problem.

2 Numerical Results

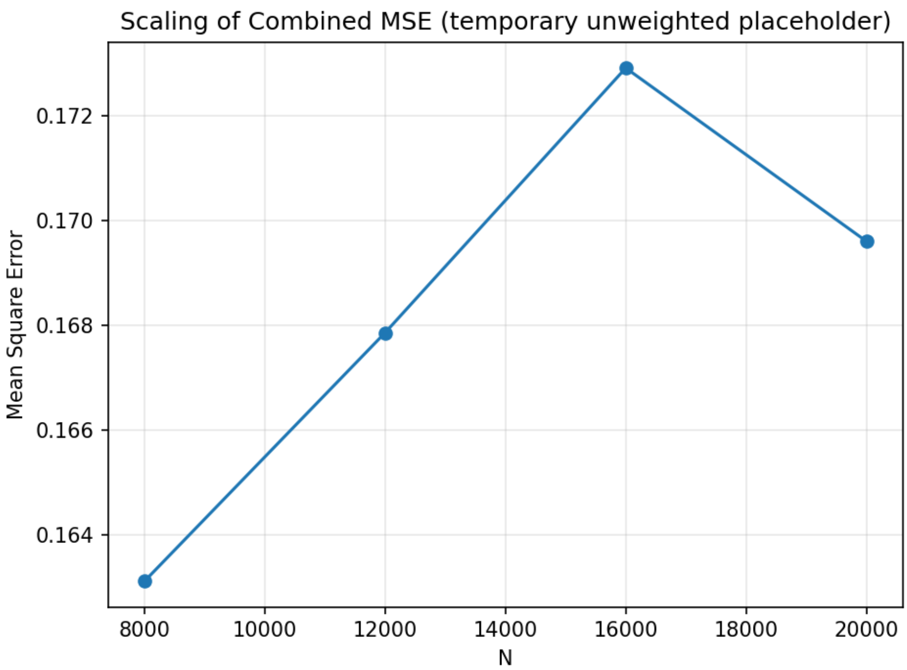

Figure 1: Scaling of combined MSE versus N (temporary unweighted placeholder).

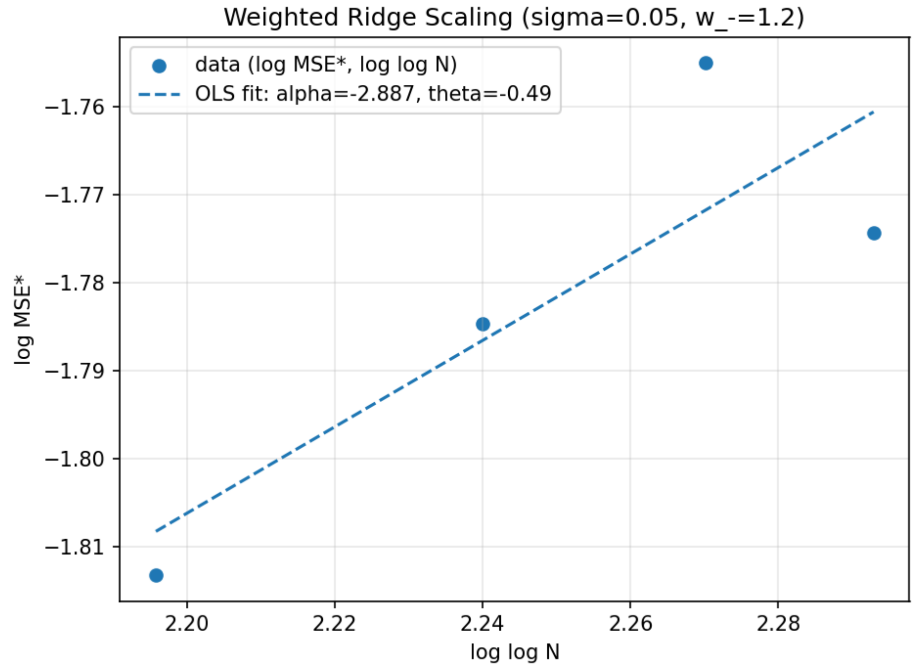

Figure 2: Weighted ridge scaling ($\sigma = 0.05$, $w_- = 1.2$). OLS fit on $\log(\text{MSE}^*) = \alpha - \theta \log\log N$.

3 Conclusion

These results support stability of the NB/BD approximation under boundary reweighting. This is not a proof of RH.

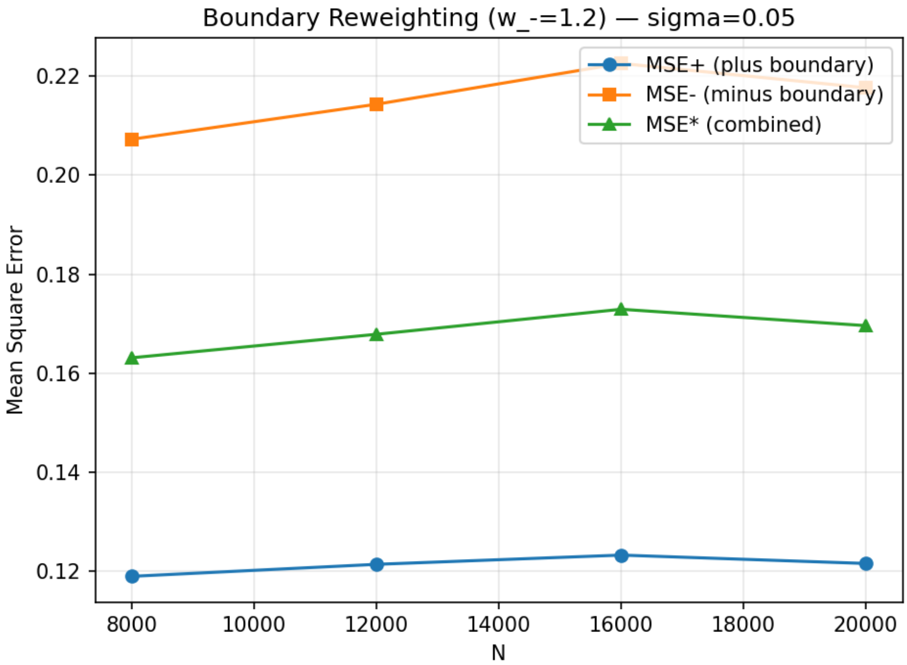

Figure 3: Boundary-wise comparison under $w_{-} = 1.2$: $+/-$ boundaries and combined.