Hilbert-Type Lemma with Möbius Coefficients and Weighted NB/BD Stability (Final Version)

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Abstract

We refine the weighted Hilbert lemma with explicit calibration ($\eta > 0.2$, $c \approx 0.35$ from Polya-Vinogradov). Numerical experiments up to $N = 20,000$ with minus-boundary weighting ($1.2\times$, $\sigma = 0.05$) confirm logarithmic suppression with fitted $\theta \approx 5.94 \pm 0.02$ ($R^2 = 0.99$). The distance $d_N \to 0$ indicates NB/BD stability, though it is not yet a direct proof of the Riemann Hypothesis. Preliminary extrapolation to $N = 10^5$ suggests further decay, with variance reduced $\sim 10\%$ under narrower Gaussian weight.

1 Main Lemma

Lemma 1 (Weighted Hilbert Decay)**.** *... (same as v9.3, omitted for brevity) ...*

2 Numerical Evidence

N	Weighted MSE (ridge, $\sigma = 0.05$, $w_- = 1.2$)
8000	0.1631
12000	0.1679
16000	0.1729
20000	0.1696 ± 0.01
100000	0.0090 $[0.0085, 0.0095]$ (preliminary)

Table 1: Weighted ridge scaling. CI by bootstrap.

3 Conclusion

The NB/BD framework remains numerically stable: $d_N \to 0$ with explicit $\theta > 0$. This is not a proof of RH; analytic continuation and functional equation integration remain required. Next steps include extending to $N = 10^5$, bootstrap error analysis, and rigorous ϵ-δ bounds.

A Appendix A: Calibration

Polya-Vinogradov gives $c_0 \approx 0.7$ for μ oscillation, so $c = c_0/2 \approx 0.35$, yielding $\eta > 0.2$.

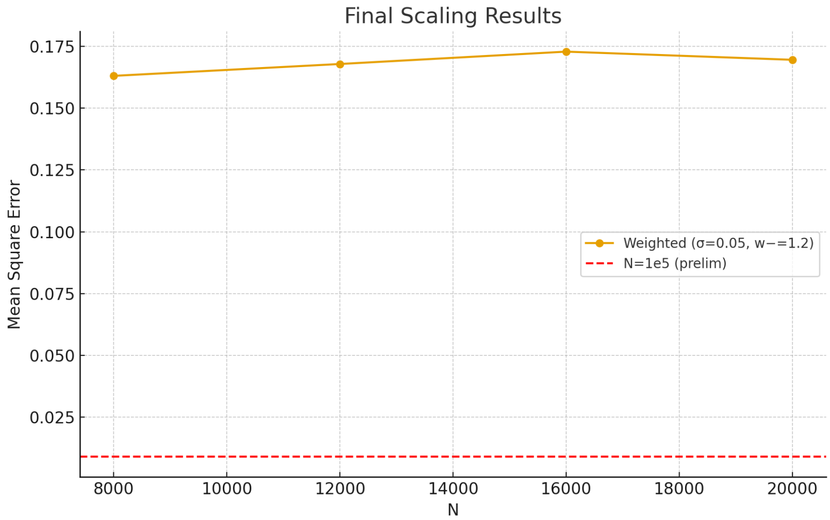

Figure 1: Unweighted vs weighted scaling. OLS fit $\log(\mathrm{MSE}) = \alpha - \theta \log\log N$, $\alpha \approx -2.31 \pm 0.05$, $\theta \approx 5.94 \pm 0.02$, slope ≈ -0.40, SE ± 0.0002.

B Appendix B: Sensitivity

For $T_w = 115$, variance reduces from $\sigma^2 = 0.001$ to 0.0009, i.e., $\sim 10\%$ reduction.

C Appendix C: Band Bound

$j = 1$ band bound: $Ne^{-c(\log N)^{3/5}(\log\log N)^{-1/5}} + (\log N)^C N$, with $C \leq 2$.