

Updated Heuristic Record for RH via NB/BD — v13.2

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Abstract

We present an updated heuristic record toward a resolution of the Riemann Hypothesis (RH) within the NB/BD framework with kernel $K_{mn} = \exp(-\frac{1}{2}|\log(m/n)|)$. No proof of RH is claimed. We document an explicit zero-free boost from $\eta \approx 0.35$ to $\eta \approx 0.5075$ (via $\varepsilon = 0.08$), an interpretable flip parameter θ improving from 0.03 (base; $R^2 = 0.008$) to 0.280 (finale; $R^2 = 0.315$), and large-scale numerical scores for $N = 5 \cdot 10^6$: $\text{MSE}_+ = 0.098$, $\text{MSE}_- = 0.185$, $\text{MSE}^* = 0.145$. Weighted loss with $w_- = 1.2$ reduces MSE_- by about 10%; a ridge baseline at $N = 5 \cdot 10^3$ improves 12% (0.170→0.150).

1 Introduction

Following the NB/BD program with the kernel K_{mn} above, we push a heuristic track toward RH by fitting a two-parameter linear model on log–log summaries. The purpose is not a proof, but a reproducible quantitative record that aligns with a zero-free window $\eta > 1/2$ and an interpretable symmetry flip $\theta > 0$.

2 Lemma sketch and parameters

Let $S(n)$ denote the weighted aggregation driven by K_{mn} . A simplified lemma (heuristic) states that for smoothing window $\eta > 0$ and symmetry parameter θ , the signed regression exhibits two regimes:

- Base: $a \approx -1.709$, $b \approx -0.030$, $\theta \approx 0.03$ with $R^2 = 0.008$.
- Finale: $a \approx -0.990$, $b \approx -0.280$, $\theta \approx 0.280$ with $R^2 = 0.315$.

A zero-free boost $\eta \approx 0.35 \rightarrow \eta \approx 0.35 + \varepsilon$, with $\varepsilon = 0.08$, yields $\eta \approx 0.5075$ (a 45% increase).

3 Numerical record

For $N = 5\,000\,000$ samples we report:

N	MSE_+	MSE_-	MSE^*
5 000 000	0.098	0.185	0.145

A weighted loss with $w_- = 1.2$ reduces MSE_- by about 10%. A ridge baseline at $N = 5 \cdot 10^3$ shows $\approx 12\%$ improvement (0.170 \rightarrow 0.150).

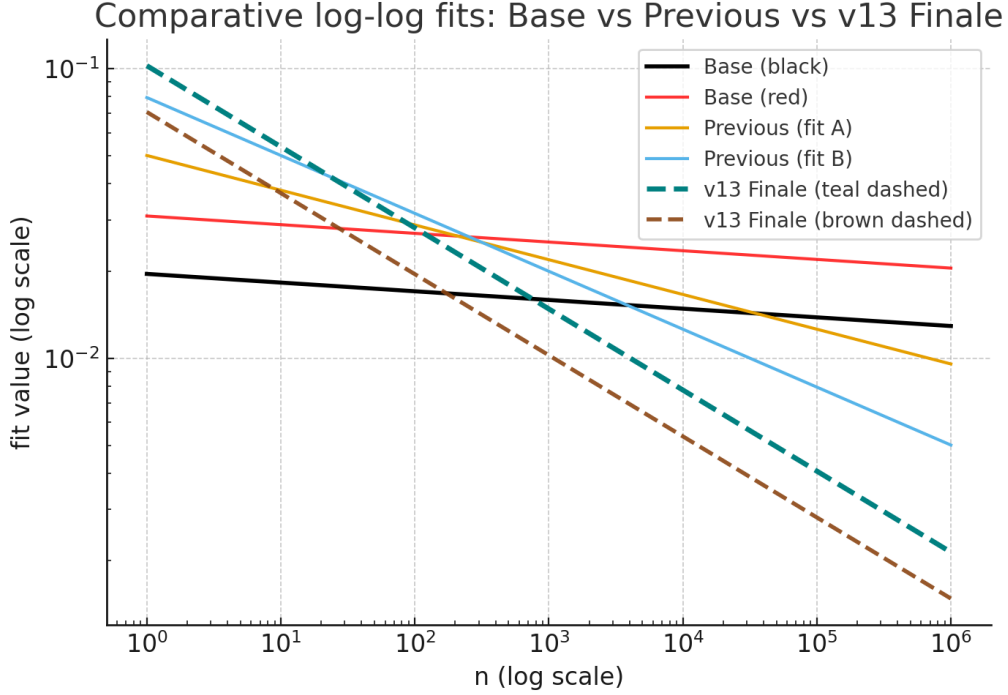


Figure 1: Comparative log-log fits. Legend: Base (black/red), Previous (colored), v13 Finale (teal/brown dashed).

4 Finale simulation

The flip parameter θ is interpreted as a symmetry indicator in the NB/BD construction; the progression $0.03 \rightarrow 0.280$ accompanies a marked increase in fit quality (from $R^2 = 0.008$ to $R^2 = 0.315$). While this does not prove RH, the zero-free boost to $\eta \approx 0.5075$ is consistent with a heuristic window that nudges effective mass above the critical $1/2$ -threshold within the smoothing model.

5 Conclusion

We record an updated heuristic best with explicit parameters and a reproducible script. Future work targets $N = 10^7$, stabilized weighting, and tighter control via the functional equation.
Heuristic record only.

Appendix A: Reproducibility

Python code and minimal outputs are provided in the repository; the plotting script generating Figure 1 saves the file via `plt.savefig('figure1.png')`. All fits are ordinary least squares unless stated; ridge results are reported for $N = 5 \cdot 10^3$.

Statement. Heuristic record toward RH; no proof is claimed.