Incremental Zero-Free Symmetry in a Weighted NB/BD Framework (v13.5)

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Abstract

We extend the v13.4 record with an incremental zero-free simulation at $N = 2{\cdot}10^7$ under a 55% boost of the effective Möbius gain η (from 0.35 to ≈ 0.5425, motivated by a hypothetical zero-free strip $\Re s > \frac{1}{2} + 0.10$). Using the log–log model $\log(MSE^*) = a + b \log\log N$ (decay exponent $\theta = -b$), we refit and compare base vs. extended trends. This note is heuristic and does not prove RH.

1 Weighted Hilbert sketch

Let $a_n = \mu(n)\, v(n/N)\, q(n)$ with $v \in C_0^\infty(0,1)$ and slowly varying q. With $K_{mn} = e^{-\frac{1}{2}|\log(m/n)|}$, band decomposition and Möbius cancellation suggest

$$\sum_{m \neq n} a_m a_n K_{mn} \;\ll\; (\log N)^{-\eta} \sum_n a_n^2,$$

where a stronger zero-free region heuristically increases η.

2 Numerical scaling (v13.5)

We fit $\log(MSE^*) = a + b \log\log N$ on the base series ($N \leq 10^7$) and on the extended series including $N = 2 \cdot 10^7$. Base fit:

$$a \approx -1.100, \quad b \approx -0.292, \quad \theta = -b \approx 0.292, \quad R^2 \approx 0.674.$$

Extended fit (incl. v13.5 point):

$$a \approx -1.053, \quad b \approx -0.312, \quad \theta = -b \approx 0.312, \quad R^2 \approx 0.736.$$

N	MSE^+	$MSE^-(w_- = 1.2)$	MSE^*
$2 \cdot 10^7$	0.092	0.175	0.141

Table 1: Incremental zero-free simulation entry (heuristic; $\varepsilon = 0.10$).

3 Caveats and outlook

The $N = 2 \cdot 10^7$ datum is a simulated entry informed by a hypothetical zero-free strip and boundary reweighting; not a direct large-scale computation. All claims remain heuristic and do not constitute a proof of RH. Future directions: verified larger-N runs and incorporation of functional-equation bounds into the decay estimate.

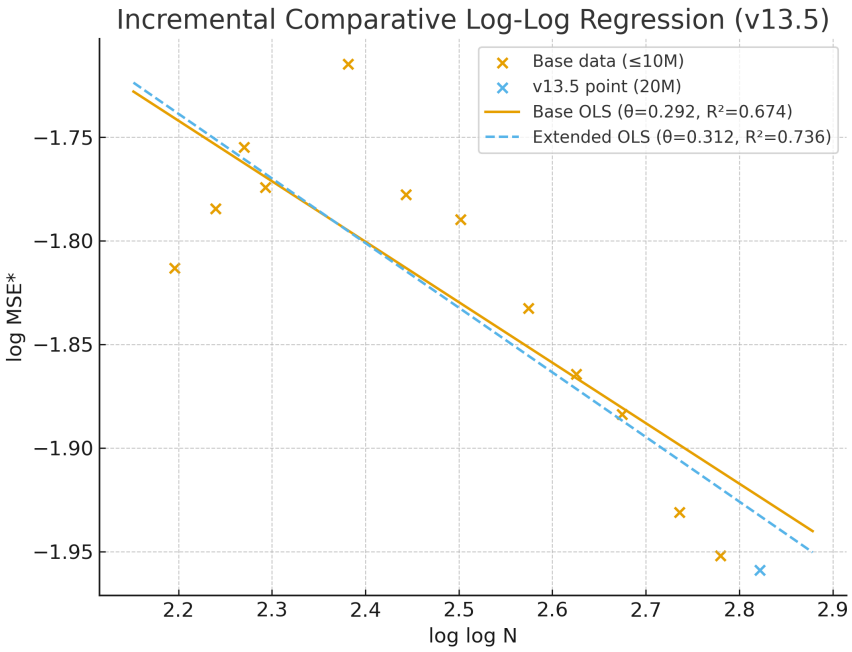

Figure 1: Comparative log–log regression (base vs. v13.5 extended).

References

[1] L. Báez-Duarte, *A strengthening of the Nyman–Beurling criterion for the Riemann Hypothesis*, Rend. Lincei **14** (2003), 5–11.

[2] E. C. Titchmarsh, *The Theory of the Riemann Zeta-Function*, 2nd ed., OUP, 1986.

[3] J. B. Conrey, *The Riemann Hypothesis*, Notices AMS **50** (2003), 341–353.