Statistics II

Week 2: **Foundations**

The potential outcomes framework and experiments

Content for today

1. Review of core concepts from lecture
2. R refresher
3. Calculating NATE, ATT, ATU and ATE with in R using `dplyr`

Lecture Review

Causal Inference

The *reasoning* process of

- ruling out non-causal explanations of the observed association
- pointing out the assumptions necessary to rule out such sources

plus

- providing evidence to support or refute these assumptions

Remember these to make sense of every method we will see in the class.

Potential Outcomes Framework

Key concept: Every individual has a potential outcome (Y_i) both under treatment and under control (no treatment).

The fundamental problem of causal inference: we can only ever observe one of these states.

So, we <u>cannot</u> observe the individual treatment effect (ITE), nor directly observe the average treatment effect (ATE).

But we can understand them theoretically...

Getting familiar with POF notation

- Don't panic. Don't avoid the equations.
- Break down the equation into parts.
- Practice reading them *in English.*

NATE $= E[y_{1i}|d_i = 1] - E[y_{0i}|d_i = 0]$

"The expected outcome when treated, for those in the treatment group"

Key definitions (in POF notation)

ATE $= E[y_{1i} - y_{0i}] = E[y_{1i}] - E[y_{0i}]$

ATT $= E[y_{1i}|d_i = 1] - E[y_{0i}|d_i = 1]$

ATU $= E[y_{1i}|d_i = 0] - E[y_{0i}|d_i = 0]$

Unattainable: we cannot observe counterfactuals.

NATE $= E[y_{1i}|d_i = 1] - E[y_{0i}|d_i = 0]$

ATE, ATT, ATU

If we could observe counterfactuals...

Student (i)	Prejudice			Contact
	y_{0i}	y_{1i}	δ_i	
1	6	5	-1	0
2	4	2	-2	1
3	4	4	0	0
4	6	7	1	0
5	3	1	-2	1
6	2	2	0	1
7	8	7	-1	0
8	4	5	1	0

...we could know:

$$ATE = E[\delta_i] = \frac{-1 + (-2) + 0 + 1 + (-2) + 0 + (-1) + 1}{8} = -0.5 \quad (5)$$

$$ATT = \frac{-2 + (-2) + 0}{3} = -1.333$$

$$ATU = \frac{-1 + 0 + 1 + (-1) + 1}{5} = 0$$

NATE

We can only observe half of the potential outcomes we need to get to the ATE...

Student (i)	Prejudice y_{0i}	y_{1i}	δ_i	Contact
1	6			0
2		2		1
3	4			0
4	6			0
5		1		1
6		2		1
7	8			0
8	4			0

Information we *do* have

...so we can only calculate a naïve average treatment effect.

$$
\begin{aligned}
NATE &= E[y_{1i}|d_i = 1] - E[y_{0i}|d_i = 0] \\
&= \frac{2+1+2}{3} - \frac{6+4+6+8+4}{5} \\
&= 1.666 - 5.6 \\
&= -3.933
\end{aligned}
$$

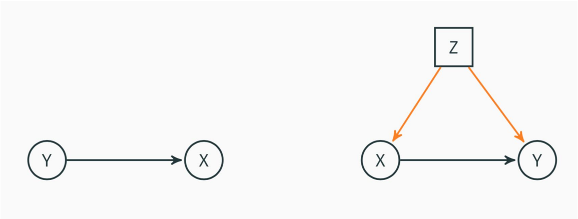
NATE and biases

Student (i)	Prejudice			Contact
	y_{0i}	y_{1i}	δ_i	
1	6			0
2		2		1
3	4			0
4	6			0
5		1		1
6		2		1
7	8			0
8	4			0

Information we *do* have

The treated and untreated groups may differ in more ways than just being treated or not and, therefore, have different potential outcomes.

$$NATE = ATE + \underbrace{E[Y_0|D=1] - E[Y_0|D=0]}_{\textit{selection bias}} + \underbrace{(1-p)(ATT - ATU)}_{\textit{HTE bias}}$$



Estimating Bias

Total bias of NATE = NATE − ATE

Selection bias: difference in average outcome without treatment for the treatment and control groups.

Heterogeneous treatment effect bias: the difference in the average treatment effect between the treatment and control groups, weighted by the proportion of the population in the control group.

Tackling biases

Randomization: randomly assigning subjects to D=0 or D=1.

- The probability of being assigned to treatment is the same for all subjects.
- Being assigned to treatment does not depend of any characteristic of the subjects.
- The treatment and control groups have the same potential outcomes (average)

Key point: *when using random assignment* (and the SUTVA holds), then ATE = NATE

$$ATE = E[Y_{1i}] - E[Y_{0i}] \longrightarrow ATE = E[Y_{1i}|D_i = 1] - E[Y_{0i}|D_i = 0]$$

Because, when randomly assigned, the expected outcomes of each group are the same as those of the entire population.

Questions?