Statistics II

Week 8:

Difference in differences

Content for today

1. Lecture review
2. Diff-in-diff in R
3. Overview of tasks for assignment 4

Lecture Review

Motivation

- Until now we had largely ignored the variable time. We have only compared treated vs. untreated units.
- But time is very important in causality: the cause always precedes the effect.
- By considering *both dimensions* -units and time-, we can:
  - Compare individuals to themselves, to account for units' characteristics that affect both outcome and treatment (~ permanent differences between groups).
  - Compare how outcomes for different units change across time, to account for characteristics of different periods (~ trends in Y that affect all units, regardless of treatment).

Assume we have a data set with **two outcome measurements:** before and after treatment. As usual, we have a problem of not knowing **counterfactuals**.

We could:

Compare the **treatment and control groups after treatment**

CSU vote shares

Unit	Y_{2014}	Y_{2020}	D
County A	42.1	38.5	0
County B	41.2	40.2	1
...	...	...	...

This assumes the PO of control group is the same as the counterfactual PO for those being treated.

Compare **before and after treatment for the treatment group**

CSU vote shares

Unit	Y_{2014}	Y_{2020}	D
County A	42.1	38.5	0
County B	41.2	40.2	1
...	...	...	...

This assumes no change in average PO over time.

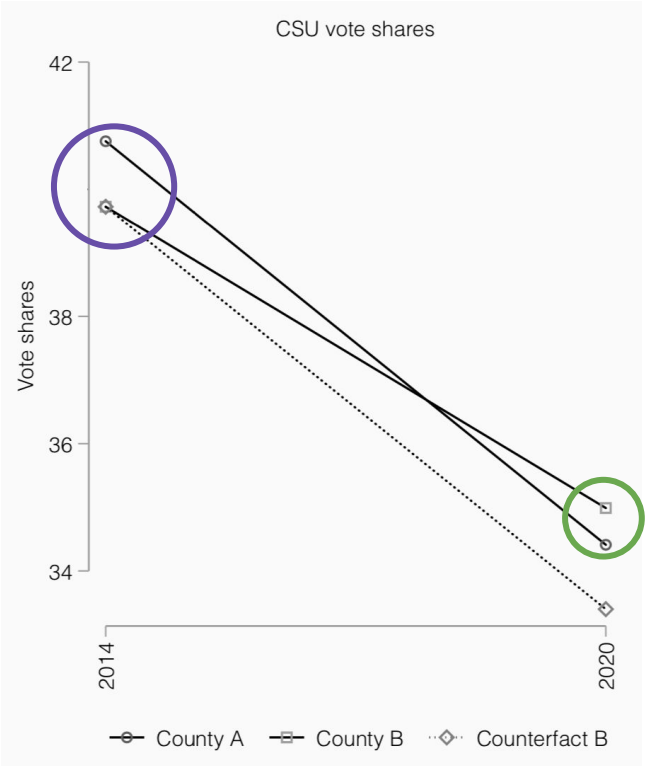
Or ...

Difference-in-Differences

Or consider both!

1. Get the difference between the treatment and control group **after** treatment
2. Get the difference between the treatment and control group **before** treatment
3. Subtract the second difference from the first

CSU vote shares				
Unit	Y_{2014}	Y_{2020}	D	$\Delta Y_{2020-2014}$
County A	42.1	38.5	0	-3.6
County B	41.2	40.2	1	-1
	-0.9	1.7		2.6

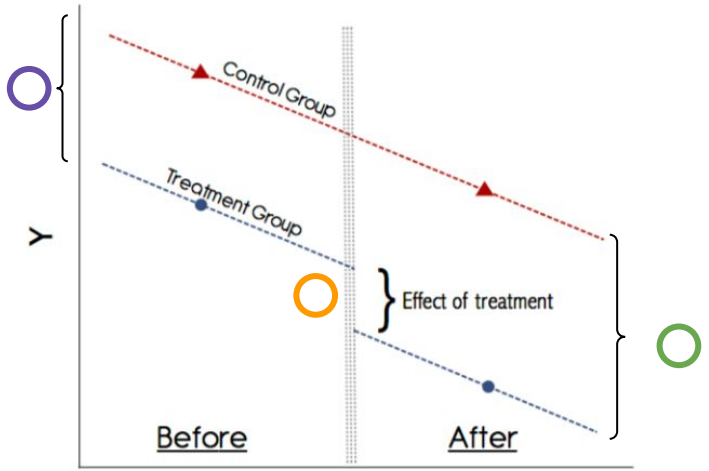


This approach uses the overtime difference in the control group as a counterfactual of *what would have happened* in the treatment group, had the treatment no taken place

Difference-in-Differences

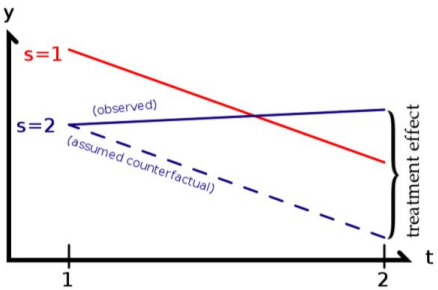
Main idea:

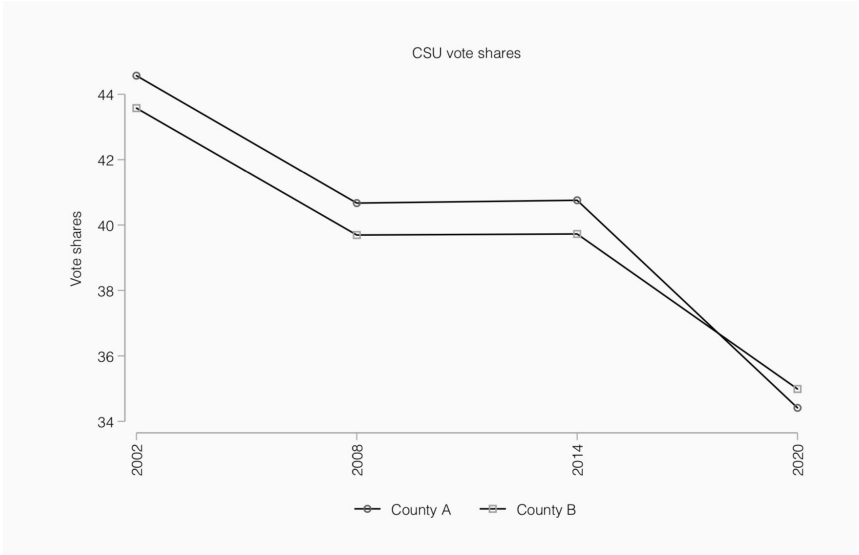
- Sometimes treatment and control units move in parallel in the absence of treatment.
- When they do, we can see how much do the treated units diverge from the post-treatment expected path, compared to the control units.
- We can estimate the treatment effect as the divergence from the expected outcome of the treatment group in the absence of treatment.



Parallel Trends Assumption

In order to validly use the overtime difference in the control group as a counterfactual, DiD estimation rests on the assumption that **observed overtime changes in the control group reflect, on average, unobserved changes in the treatment group** in the absence of treatment.





Three ways to estimate DiD

1. **Manually**, using average outcome values for subgroups defined by *D* and t.

$$DiD = \{E[Y_{1c}|D=1,t=1] - E[Y_{0c}|D=0,t=1]\} - \{E[Y_{1c}|D=1,t=0] - E[Y_{0c}|D=0,t=0]\}$$

2. Calculate **first differences** and regress on D. ⟶ wide format data.

$$\Delta Y_{ct_0-t_1} = \alpha + \delta D_c + \Delta_{v_c}$$

3. **Regression formulation** of the DiD model ⟶ long format data.

$$Y_{ct} = \alpha + \beta D_c + \gamma Post_t + \delta(D_c * Post_t) + v_{ct}$$

Generalized DiD: $Y_{ct} = \lambda_c + \gamma_t + \delta D_{ct} + u_{ct}$

Interpreting the results

Model: $Y_{ct} = \alpha + \beta D_c + \gamma Post_t + \delta(D_c * Post_t) + v_{ct}$ → In R: `lm(Y ~ D + Post_t + D*Post_t)`

Regression output:

	Share CSU
Treat	-1.03
	(1.56)
Post	-6.34***
	(0.72)
Treat × *Post*	1.61**
	(0.79)
Intercept	40.76***
	(1.39)
N	192
R^2	0.16

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

In POF notation:

	$t = 2014$ (pre)	$t = 2020$ (post)
$D_c = 0$	$E[Y_{0c2014} \mid D_c = 0]$	$E[Y_{0c2020} \mid D_c = 0]$
$D_c = 1$	$E[Y_{1c2014} \mid D_c = 1]$	$E[Y_{1c2020} \mid D_c = 1]$
$D_c = 0$	α	$\alpha + \gamma$
$D_c = 1$	$\alpha + \beta$	$\alpha + \beta + \gamma + \delta$

Wide vs long formatted data

Wide

Wide format table

Unit c	Y_{c2014}	Y_{c2020}	D_c
County A	42.1	38.5	0
County B	41.2	40.2	1
...	...	...	...

- Only one row per individual or unit.
- Outcome values included in different variables, by year.

Long

Long format table

Unit c	Year t	Y_c	D_c
County A	2014	42.1	0
County A	2020	38.5	0
County B	2014	41.2	1
County B	2020	40.2	1
...	...	...	...

- One column for every variable.
- One row for every unique observation

! To see how to change between these formats in R, heck out Grolemund & Wickham's R for Data Science, chapter 12.3 .

Questions?