

# Foundations of causality | R

The slide features a minimalist design with several thin, vertical lines on the right side. A red line runs from the top to the bottom of the slide. A blue line runs from the top to the bottom, slightly to the left of the red line. A purple line runs from the top to the bottom, slightly to the left of the blue line. A horizontal red line runs across the bottom of the slide, intersecting the vertical lines.

# Agenda

Statistical Modeling & Causal Inference – Oswald | Ramirez-Ruiz

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- Revisiting lecture
  - Causality
  - Potential Outcomes Framework
  - NATE and biases
- Getting started with R
  - Data-wrangling with `dplyr`

# Causal Inference

The *reasoning* process of

- **ruling out** non-causal explanations of the observed association
- pointing out the **assumptions** necessary to rule out such sources

plus

- providing **evidence** to support or refute these assumptions

# Potential Outcomes Framework

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**Key concept:** Every individual has a potential outcome ( $Y_i$ ) both under treatment and under control (no treatment).

**The fundamental problem of causal inference:** we can only ever observe one of these states.

So, we cannot observe the individual treatment effect (ITE), nor directly observe the average treatment effect (ATE).

# POF Notation

NATE

$$= E[y_{1i}|d_i = 1] - E[y_{0i}|d_i = 0]$$

“The expected outcome when treated, for those in the treatment group”

$E[y_{0i}] / E[y_{0i}]$  “expected outcomes”

$y_{0,i} / y_{1,i}$  “potential outcomes”

# POF Logic

$$\text{ATE} = E[\underline{y_{1i} - y_{0i}}] = E[y_{1i}] - E[y_{0i}]$$

$$\text{ATT} = E[y_{1i}|d_i = 1] - \underline{E[y_{0i}|d_i = 1]}$$

$$\text{ATC} = \underline{E[y_{1i}|d_i = 0]} - E[y_{0i}|d_i = 0]$$

$$\text{NATE} = E[y_{1i}|d_i = 1] - E[y_{0i}|d_i = 0]$$

Unattainable: we cannot observe  
counterfactuals.

# Example

- Contact hypothesis (Allport, 1954)

Inter-group contact (X)  Prejudice (Y)

Each individual  $i$  in a student sample is exposed ( $d_i = 1$ ) to the cause, or not exposed ( $d_i = 0$ ) (here: contact with member of different ethnic group).

$y_{0,i}$  = non-exposure

$y_{1,i}$  = exposure

# ATE, ATT, ATC

If we could observe counterfactuals...

...we could know:

Student ( <i>i</i> )	Prejudice		$\delta_i$	Contact
	$y_{0i}$	$y_{1i}$		
1	6	5	-1	0
2	4	2	-2	1
3	4	4	0	0
4	6	7	1	0
5	3	1	-2	1
6	2	2	0	1
7	8	7	-1	0
8	4	5	1	0

→

$$ATE = E[\delta_i] = \frac{-1 + (-2) + 0 + 1 + (-2) + 0 + (-1) + 1}{8} = -0.5 \quad (5)$$

$$ATT = \frac{-2 + (-2) + 0}{3} = -1.333$$

$$ATC = \frac{-1 + 0 + 1 + (-1) + 1}{5} = 0$$



# NATE

We can only observe half of the potential outcomes we need to get to the ATE...

...so we can only calculate a naïve average treatment effect.

Student ( <i>i</i> )	Prejudice		Contact
	$y_{0i}$	$y_{1i}$ $\delta_i$	
1	6		0
2		2	1
3	4		0
4	6		0
5		1	1
6		2	1
7	8		0
8	4		0

Information we *do* have

$$\begin{aligned} NATE &= E[y_{1i}|d_i = 1] - E[y_{0i}|d_i = 0] \\ &= \frac{2 + 1 + 2}{3} - \frac{6 + 4 + 6 + 8 + 4}{5} \\ &= 1.666 - 5.6 \\ &= -3.933 \end{aligned}$$

# NATE and biases

Student ( <i>i</i> )	Prejudice		Contact
	$Y_{0i}$	$Y_{1i}$ $\delta_i$	
1	6		0
2		2	1
3	4		0
4	6		0
5		1	1
6		2	1
7	8		0
8	4		0

Information we *do* have

The treated and untreated groups may differ in more ways than just being treated or not and, therefore, have different potential outcomes.

$$NATE = ATE + \underbrace{E[Y_0|D=1] - E[Y_0|D=0]}_{\text{selection bias}} + \underbrace{(1-p)(ATT - ATU)}_{\text{HTE bias}}$$

baseline bias

heterogeneous /  
differential treatment  
effect bias

# NATE and biases

$$NATE = ATE + \underbrace{E[Y_0|D = 1] - E[Y_0|D = 0]}_{\text{selection bias}} + \underbrace{(1 - p)(ATT - ATU)}_{\text{HTE bias}}$$

**Baseline bias:** difference in average outcome without treatment for the treatment and control groups.

**Differential treatment effect bias:** the difference in the average treatment effect between the treatment and control groups, weighted by the proportion of the population in the control group.

# Tackling biases

Randomization: randomly assigning subjects to  $D=0$  or  $D=1$ .

- The **probability** of being assigned to treatment is the same for all subjects.
- Being assigned to treatment does **not depend of any characteristic** of the subjects.
- The treatment and control groups have (on average) the same potential outcomes
- **Key point:** *when using random assignment* (and the SUTVA holds), then  $ATE = NATE$

$$ATE = E[Y_{1i}] - E[Y_{0i}] \longrightarrow ATE = E[Y_{1i}|D_i = 1] - E[Y_{0i}|D_i = 0]$$

# Coding with `dplyr`

We'll often use the pipe operator (`%>%`) to string together commands, and rely on the `dplyr` “verbs”. For example:

`select`: subset columns

`filter`: subset rows

`arrange`: reorder rows

`mutate`: add columns to existing data

`summarize`: summarize values in the dataset

`group_by`: defines groups within dataset

# Further Ressources

R basics: <https://tinyurl.com/vkebh2f>

RMarkdown: The definitive guide <https://tinyurl.com/y4tyfqmg>

Dplyr: <https://tinyurl.com/vyrv596>

Dplyr video tutorial: <https://www.youtube.com/watch?v=jWjqLW-u3hc>

Summary of lab materials – [Lab homepage](#)

For any coding issues – [Stackoverflow](#)

Hertie's Data Science Lab – [Research Consulting](#)