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UNITED NATIONS  
UNIVERSITY

MCTE 4352  
ROBOTICS  
SEMESTER II || SESSION 2021/2022  
SECTION 1

## ROBOTICS PROJECT

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## 1.0 ROBOT DESIGN

In this project, we will design a 3 degree of freedom (DOF) robot program using MATLAB software. We will be discussing the mathematical modelling of the robot that involves the forward kinematics and the inverse kinematics. Next, we will simulate the working space of the robot. The sketch of the robot is shown in Figure 1.

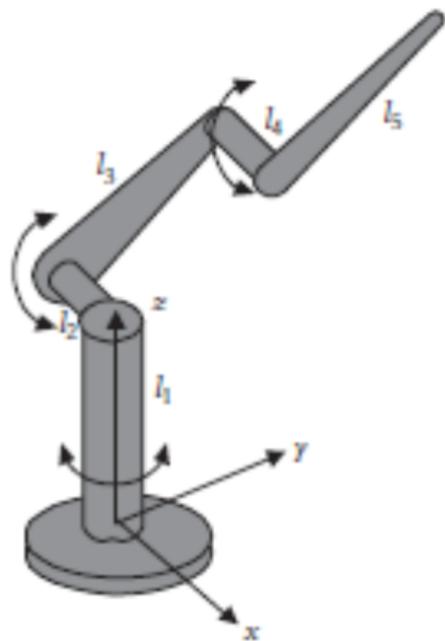


Figure 1: Robot Sketch

The robot has a specification that are as follows

Parameters	Length (mm)
L1	82
L2	0.182
L3	1.1582
L4	0.182
L5	0.7582

Table 1: Robot Specifications

## 2.0 MATHEMATICAL MODELLING

### 2.1 FORWARD KINEMATICS

Firstly, to model the system in mathematical form, we apply forward kinematics to find the end effector position of the robot. We are using Denavit-Hartenberg representation to find the related parameters. Figure 2 and Table 2 show the frame assignment and DH parameters of the robot.

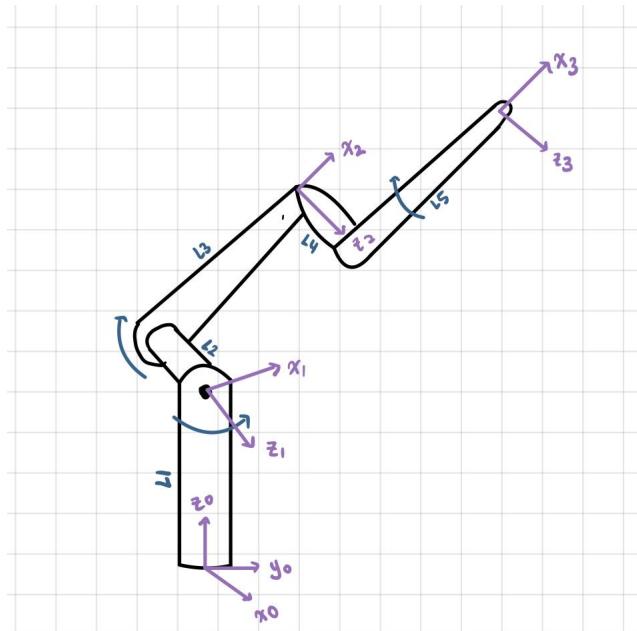
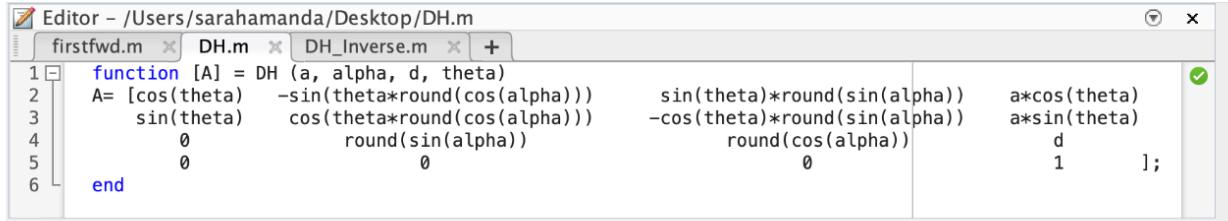


Figure 2: Frame Assignment

Joint	$a_i$ (mm)	$\alpha_i$	$d_i$ (mm)	$\theta_i$
0 - 1	0	90	L1	$\theta_1$
1 - 2	L3	0	-L2	$\theta_2$
2 - 3	L5	0	L4	$\theta_3$

Table 2: DH Parameters

From the parameters assigned in the DH table, the derivation of the position transformation matrix,  ${}^0T_3$ , could be obtained. The mathematical modelling and calculation was done using MATLAB software.

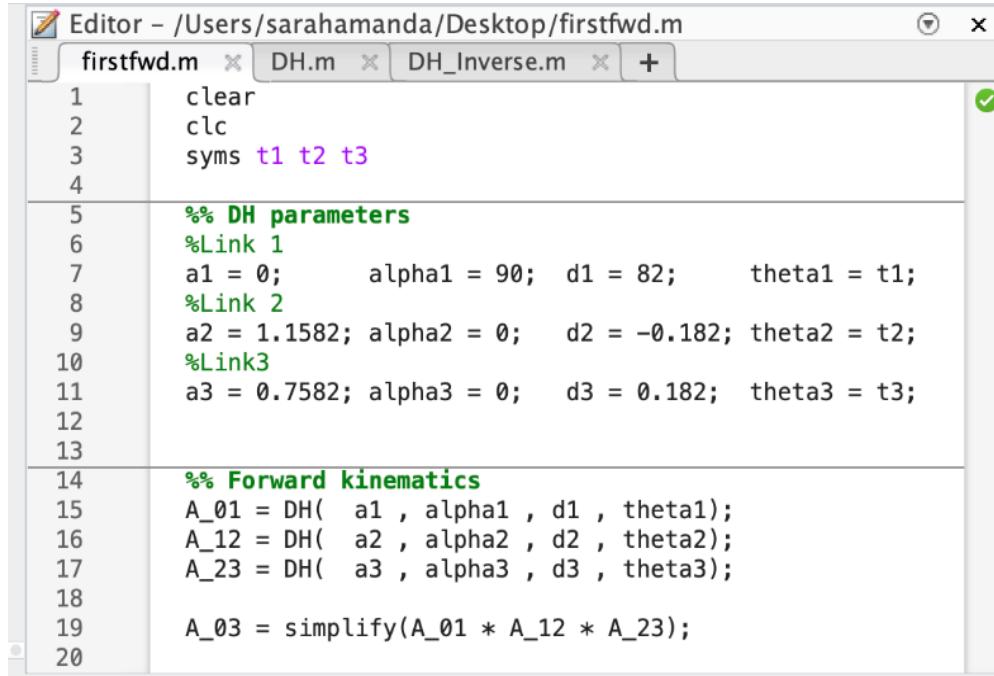


```

Editor - /Users/sarahamanda/Desktop/DH.m
firstfwd.m x DH.m x DH_Inverse.m x +
1 function [A] = DH (a, alpha, d, theta)
2 A= [cos(theta) -sin(theta)*round(cos(alpha)) sin(theta)*round(sin(alpha)) a*cos(theta)
3 sin(theta) cos(theta)*round(cos(alpha)) -cos(theta)*round(sin(alpha)) a*sin(theta)
4 0 round(sin(alpha)) round(cos(alpha)) 0 d
5 0 0 0 1 ];
6 end

```

Figure 3: DH Function in Matlab



```

Editor - /Users/sarahamanda/Desktop/firstfwd.m
firstfwd.m x DH.m x DH_Inverse.m x +
1 clear
2 clc
3 syms t1 t2 t3
4
5 %% DH parameters
6 %Link 1
7 a1 = 0; alpha1 = 90; d1 = 82; theta1 = t1;
8 %Link 2
9 a2 = 1.1582; alpha2 = 0; d2 = -0.182; theta2 = t2;
10 %Link3
11 a3 = 0.7582; alpha3 = 0; d3 = 0.182; theta3 = t3;
12
13 %% Forward kinematics
14 A_01 = DH( a1 , alpha1 , d1 , theta1);
15 A_12 = DH( a2 , alpha2 , d2 , theta2);
16 A_23 = DH( a3 , alpha3 , d3 , theta3);
17
18 A_03 = simplify(A_01 * A_12 * A_23);
19
20

```

Figure 4: Forward Kinematics Coding in Matlab

A DH function is firstly written in a separate script as shown in Figure 3. The DH function contains the formula for Denavit-Hartenberg. In another script, we inserted the DH parameters of the robot following Table 2 and finally the forward kinematics calculation is executed by calling the DH Function. The position transformation matrix,  ${}^0T_3$ , which is denoted as,  $A_{03}$ , in line 19, was attained.

```

Command Window
>> A_03
A_03 =
[cos(t2 + t3)*cos(t1), -sin(t2 + t3)*cos(t1), sin(t1), (cos(t1)*(3791*cos(t2 + t3) + 5791*cos(t2)))/5000]
[cos(t2 + t3)*sin(t1), -sin(t2 + t3)*sin(t1), -cos(t1), (sin(t1)*(3791*cos(t2 + t3) + 5791*cos(t2)))/5000]
[ sin(t2 + t3), cos(t2 + t3), 0, (3791*sin(t2 + t3))/5000 + (5791*sin(t2))/5000 + 82]
[ 0, 0, 0, 1]
f↓ >

```

Figure 5: Transformation Matrix  ${}_0T_3$

Figure 5 shows the transformation matrix output from the matlab software. Below we will simplify the transformation matrix to make it easier to read.

$${}^3{}_0A = \begin{bmatrix} n_x & o_x & q_x & p_x \\ n_y & o_y & q_y & p_y \\ n_z & o_z & q_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
  

$$n_x = C_{23} C_1 \quad o_x = -S_{23} C_1$$

$$n_y = C_{23} S_1 \quad o_y = -S_{23} S_1$$

$$n_z = S_{23} \quad o_z = C_{23}$$

$$q_x = S_1 \quad p_y = 0.7582 C_{23} C_1 + 1.1582 C_2 C_1$$

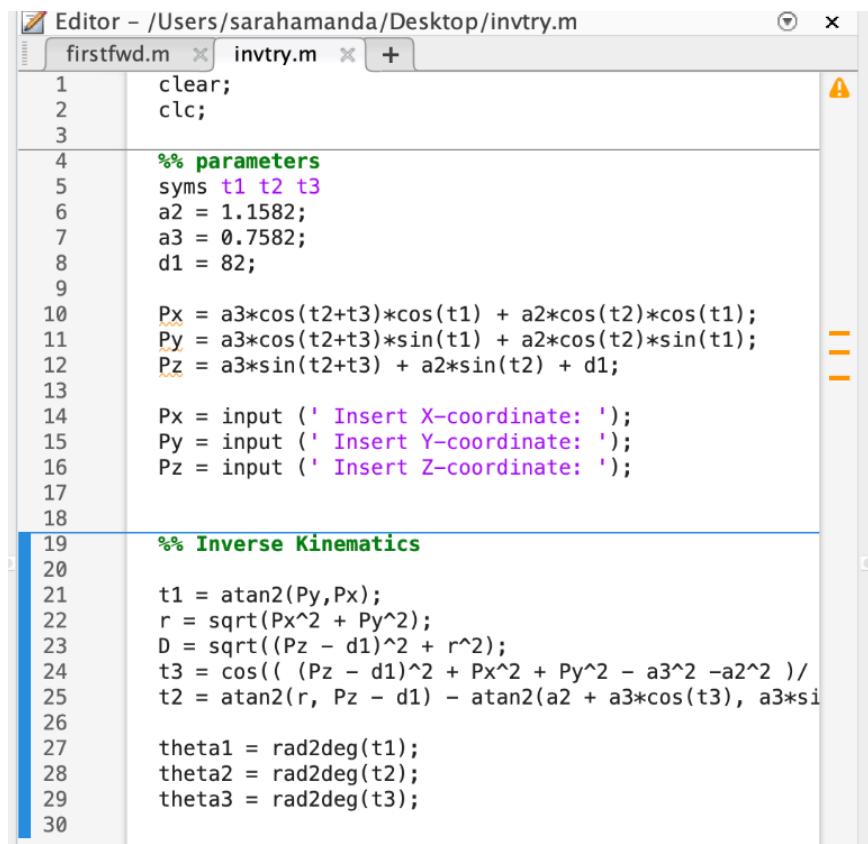
$$q_y = -C_1 \quad p_y = 0.7582 C_{23} S_1 + 1.1582 C_2 S_1$$

$$q_z = 0 \quad p_z = 0.7582 S_{23} + 1.1582 S_2 + 82$$

$${}^3_0 A = \begin{bmatrix} C_{23}C_1 & -S_{23}C_1 & S_1 & 0.7582C_{23}C_1 + 1.1582C_2C_1 \\ C_{23}S_1 & -S_{23}S_1 & -C_1 & 0.7582C_{23}S_1 + 1.1582C_2S_1 \\ S_{23} & C_{23} & 0 & 0.7582S_{23} + 1.1582S_2 + 82 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

## INVERSE KINEMATICS

The figure below shows the Matlab coding to develop Inverse Kinematics of the robot. The program will prompt the user for X-coordinate, Y-coordinate, and Z-coordinate. It will then calculate using the Px, Py, and Pz formula that we had gotten from Forward Kinematics. The output will show the value of theta in degrees.



```

Editor - /Users/sarahamanda/Desktop/invtry.m
firstfwd.m x invtry.m x +
1 clear;
2 clc;
3
4 %% parameters
5 syms t1 t2 t3
6 a2 = 1.1582;
7 a3 = 0.7582;
8 d1 = 82;
9
10 Px = a3*cos(t2+t3)*cos(t1) + a2*cos(t2)*cos(t1);
11 Py = a3*cos(t2+t3)*sin(t1) + a2*cos(t2)*sin(t1);
12 Pz = a3*sin(t2+t3) + a2*sin(t2) + d1;
13
14 Px = input (' Insert X-coordinate: ');
15 Py = input (' Insert Y-coordinate: ');
16 Pz = input (' Insert Z-coordinate: ');
17
18
19 %% Inverse Kinematics
20
21 t1 = atan2(Py,Px);
22 r = sqrt(Px^2 + Py^2);
23 D = sqrt((Pz - d1)^2 + r^2);
24 t3 = cos(( (Pz - d1)^2 + Px^2 + Py^2 - a3^2 - a2^2 )/
25 t2 = atan2(r, Pz - d1) - atan2(a2 + a3*cos(t3), a3*si
26
27 theta1 = rad2deg(t1);
28 theta2 = rad2deg(t2);
29 theta3 = rad2deg(t3);
30

```

## OUTPUT

### Command Window

```
Insert X-coordinate: 5
Insert Y-coordinate: 6
Insert Z-coordinate: 8
>> t1

t1 =
0.8761

>> theta1

theta1 =
50.1944

>> t2

t2 =
1.1342

>> theta2

theta2 =
64.9872
```

```
--  
t3 =
-0.8516

>> theta3

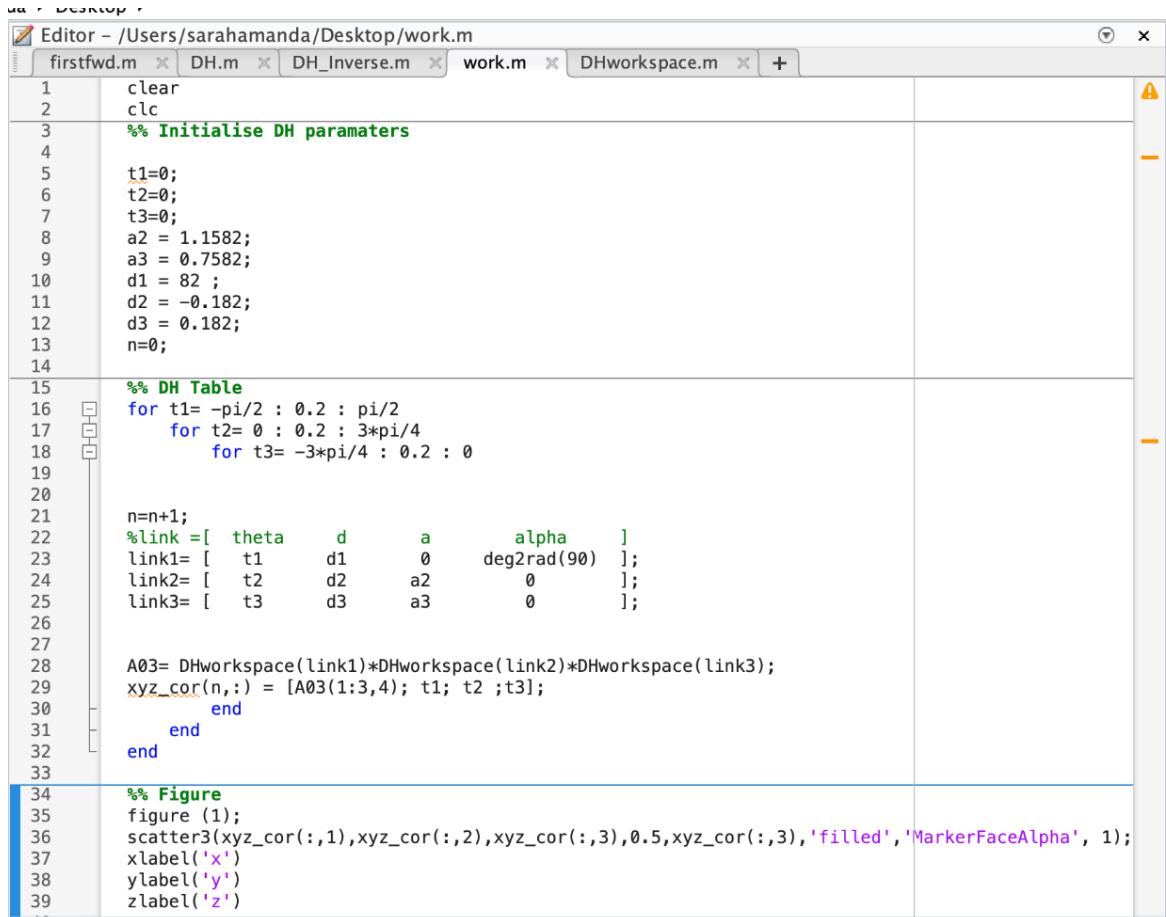
theta3 =
-48.7910
```

f<sub>x</sub> >>

### 3.0 ROBOT WORKSPACE

Next we will be simulating the work envelope of the robot. A robot's work envelope otherwise known as its workspace is defined to be its range of motion when the manipulator moves forward, backwards, up and down. The shape created is called the workspace and it is highly dependent and determined by the length of the robot's arm and its axis design. A robot usually performs and is limited to moving in this work envelope. Hence, a work envelope of a robot is considered important when selecting a robot for a particular task or application.

The work envelope of the robot is simulated in Matlab. Figure 6 shows the code where we first initialise the parameters for d, a, theta, and alpha. Then, we will call the function for DHworkspace shown at figure 7.

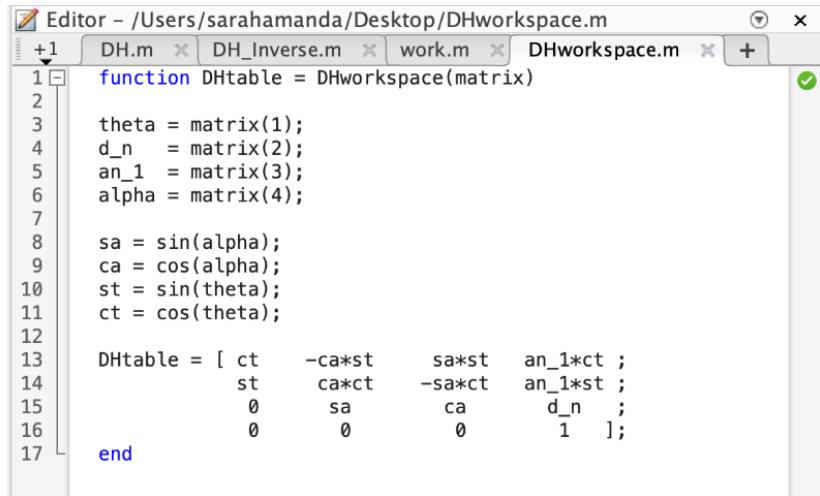


The screenshot shows a Matlab editor window titled 'Editor - /Users/sarahamanda/Desktop/work.m'. The window contains the following code:

```
1 clear
2 clc
3 %% Initialise DH parameters
4
5 t1=0;
6 t2=0;
7 t3=0;
8 a2 = 1.1582;
9 a3 = 0.7582;
10 d1 = 82 ;
11 d2 = -0.182;
12 d3 = 0.182;
13 n=0;
14
15 %% DH Table
16 for t1= -pi/2 : 0.2 : pi/2
17     for t2= 0 : 0.2 : 3*pi/4
18         for t3= -3*pi/4 : 0.2 : 0
19
20             n=n+1;
21             %link =[ theta      d       a       alpha    ];
22             link1= [ t1      d1      0       deg2rad(90)    ];
23             link2= [ t2      d2      a2      0           ];
24             link3= [ t3      d3      a3      0           ];
25
26
27             A03= DHworkspace(link1)*DHworkspace(link2)*DHworkspace(link3);
28             xyz_cor(n,:)= [A03(1:3,4); t1; t2 ;t3];
29         end
30     end
31 end
32
33 %% Figure
34 figure (1);
35 scatter3(xyz_cor(:,1),xyz_cor(:,2),xyz_cor(:,3),0.5,xyz_cor(:,3),'filled','MarkerFaceAlpha', 1);
36 xlabel('x')
37 ylabel('y')
38 zlabel('z')
```

Figure 6: Work Envelope Matlab

Figure 7 shows the DHworkspace function that will be called into the main work envelope simulation.



```

Editor - /Users/sarahamanda/Desktop/DHworkspace.m
+1 DH.m × DH_Inverse.m × work.m × DHworkspace.m × + ✓
1 function DHtable = DHworkspace(matrix)
2
3 theta = matrix(1);
4 d_n = matrix(2);
5 an_1 = matrix(3);
6 alpha = matrix(4);
7
8 sa = sin(alpha);
9 ca = cos(alpha);
10 st = sin(theta);
11 ct = cos(theta);
12
13 DHtable = [ ct -ca*st sa*st an_1*ct ;
14 st ca*ct -sa*ct an_1*st ;
15 0 sa ca d_n ;
16 0 0 0 1 ];
17 end

```

Figure 7: DHworkspace Function

Next, we will write the scatter plot coding to show the work envelope of robot in X-Y plane, Y-Z plane, X-Z plane and 3D space.



```

33
34 %% Figure
35 figure (1);
36 scatter3(xyz_cor(:,1),xyz_cor(:,2),xyz_cor(:,3),0.5,xyz_cor(:,3),'filled','MarkerFaceAlpha', 1);
37 xlabel('x')
38 ylabel('y')
39 zlabel('z')
40
41 figure (2)
42 scatter3(xyz_cor(:,1),xyz_cor(:,2),xyz_cor(:,3),0.5,xyz_cor(:,3),'filled','MarkerFaceAlpha', 1);
43 view(90,90);
44 xlabel('x')
45 ylabel('y')
46 zlabel('z')
47 title('Workspace in X-Y plane');
48
49 figure (3)
50 scatter3(xyz_cor(:,1),xyz_cor(:,2),xyz_cor(:,3),0.5,xyz_cor(:,3),'filled','MarkerFaceAlpha', 1);
51 view(90,0);
52 xlabel('x')
53 ylabel('y')
54 zlabel('z')
55 title('Workspace in Y-Z plane')
56
57 figure (4)
58 scatter3(xyz_cor(:,1),xyz_cor(:,2),xyz_cor(:,3),0.5,xyz_cor(:,3),'filled','MarkerFaceAlpha', 1);
59 view(0,0);
60 xlabel('x')
61 ylabel('y')
62 zlabel('z')
63 title('Workspace in X-Z plane')
64

```

Figure 8: Figure Coding

The scatter plot is shown in Figure 9-12 for the 3D plane, X-Y plane, Y-Z plane, and X-Z plane respectively.

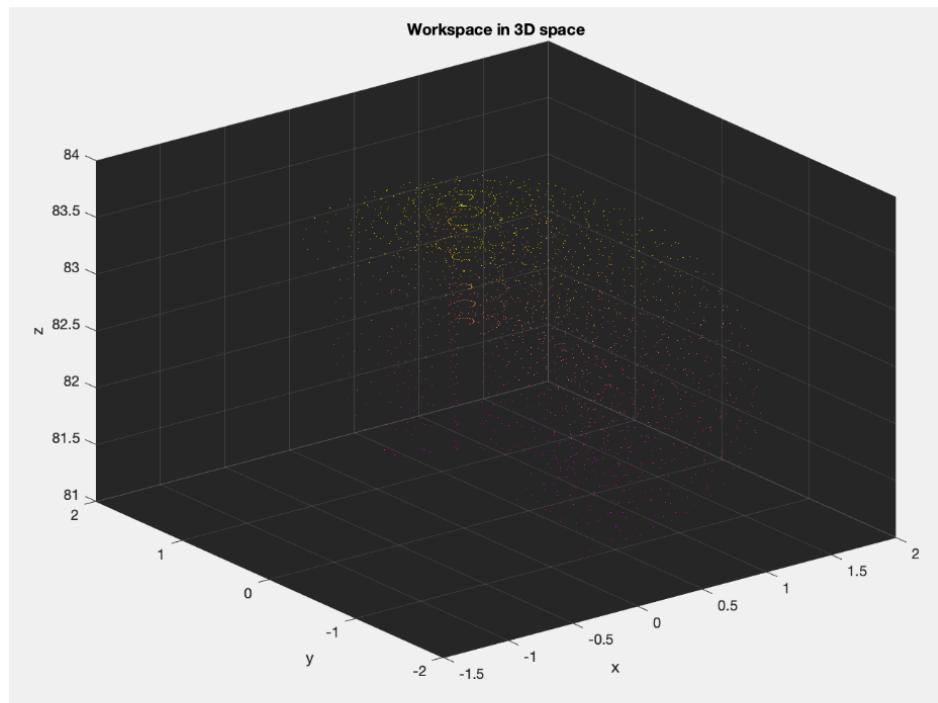


Figure 9: Work Envelope in 3D plane

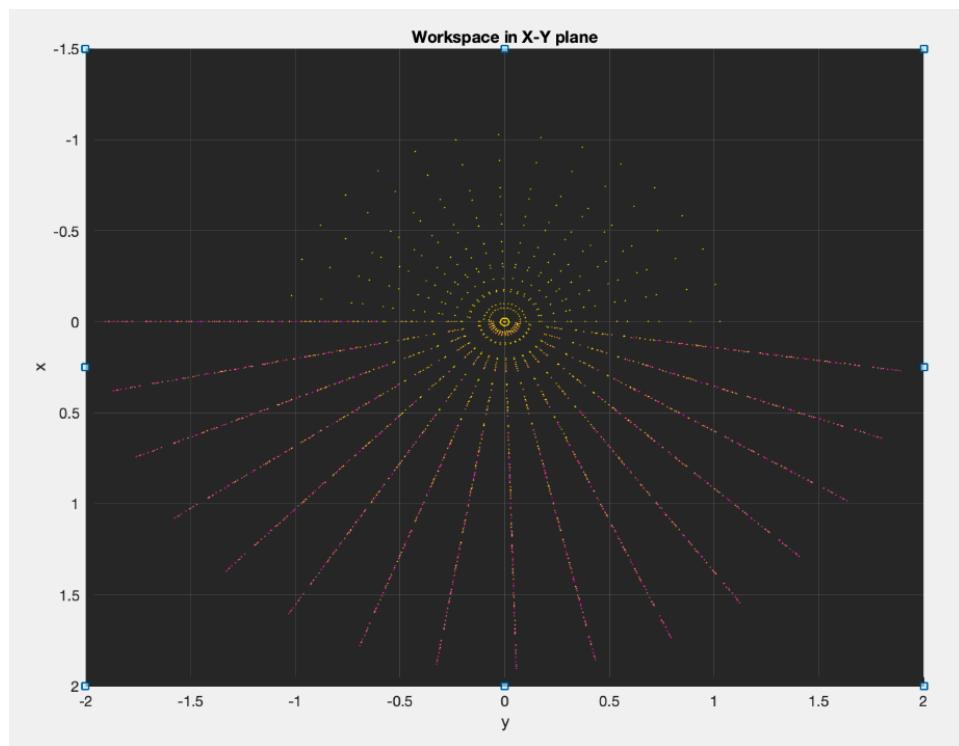


Figure 10: Work Envelope in X-Y plane

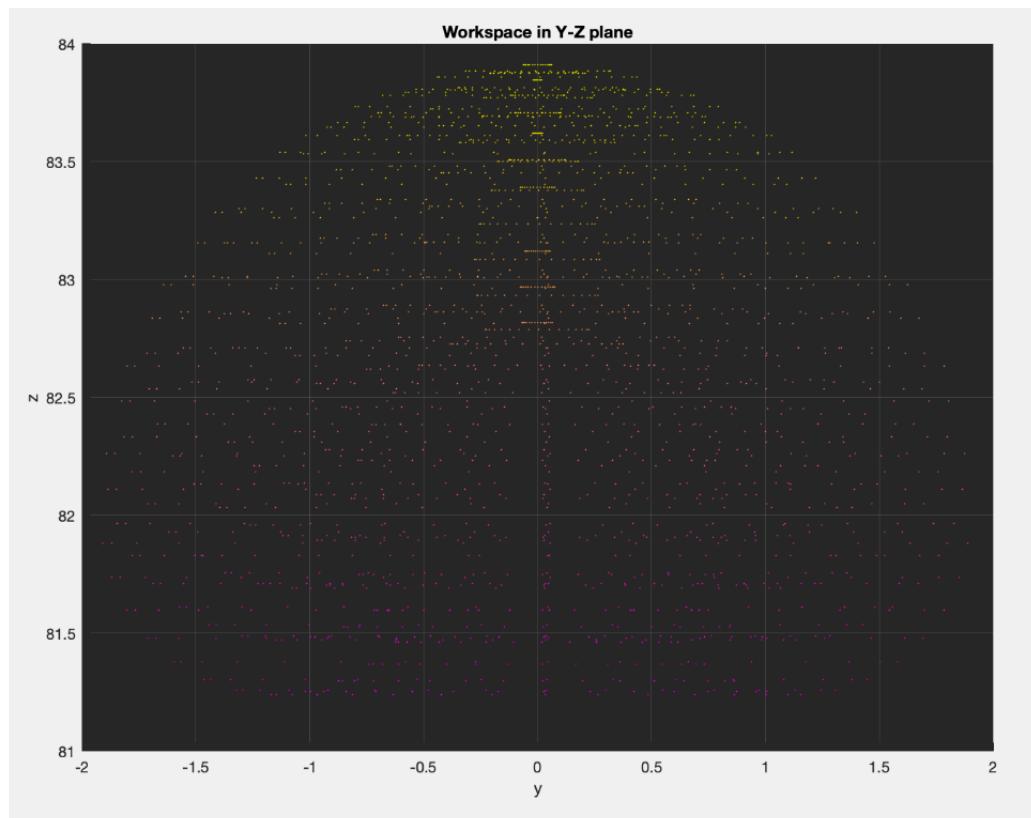


Figure 11: Work Envelope in Y-Z plane

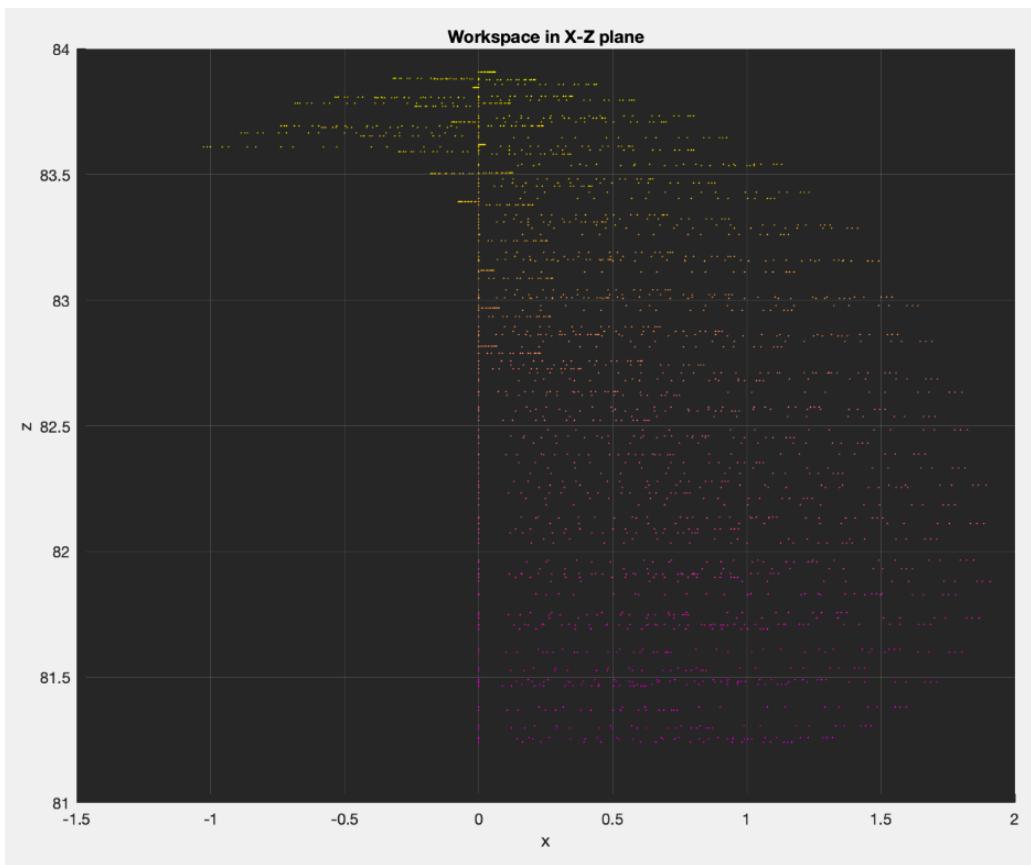


Figure 12: Work Envelope in X-Z plane