

1. given two $n \times n$ matrices $A = (a_{ij})$ and $B = (b_{uv})$. QAP asks for a permutation π of $\{1, \dots, n\}$ minimizing

$$\sum_{i=1}^n \sum_{j=1}^n a_{ij} b_{\pi(i) \pi(j)}.$$

2. • let A be the adjacency matrix of your graph:

$$a_{ij} = \begin{cases} 1, & \text{if } (i, j) \in E, \\ 0, & \text{otherwise.} \end{cases}$$

- let B encode label-products:

$$b_{uv} = u \times v.$$

3. assign labels $\ell(0), \dots, \ell(n-1)$ is the same as choosing a permutation π , and we want

$$\max_{\pi} \sum_{(i,j) \in E} \ell(i) \ell(j) = \max_{\pi} \sum_{i < j} a_{ij} \pi(i) \pi(j).$$

$$\max_{\pi} \sum_{i < j} a_{ij} \pi(i) \pi(j) = - \min_{\pi} \sum_{i < j} a_{ij} \pi(i) \pi(j),$$

which (up to the usual factor of 2 in symmetric sums) is exactly

$$\min_{\pi} \sum_{i=1}^n \sum_{j=1}^n a_{ij} b_{\pi(i) \pi(j)}.$$