#0.06 $\{n \in \mathbb{Z} \mid n^2 < 0\}$
Ans: $\varnothing$

#0.12 Let $A = \{1, 2, 3\}$ and $B = \{2, 4, 6\}$. For each relation between A and B given as a subset of $A \times B$, decide whether it is a function mapping A into B. If it is a function, decide whether it is one-to-one and whether it is onto B.

a. $\{(1, 2), (2, 6), (3, 4)\}$ ***Ans:*** Yes, a one-to-one and onto function
b. $\{(1, 3)\}$ and $\{(5, 7)\}$ ***Ans:*** Not a function
c. $\{(1, 6), (1, 2), (1, 4)\}$ ***Ans:*** Not a function
d. $\{(2, 2), (3, 6), (1, 6)\}$ ***Ans:*** Yes, onto function
e. $\{(1, 6), (2, 6), (3, 6)\}$ ***Ans:*** Yes, onto functgion
f. $\{(1, 2), (2, 6)\}$ ***Ans:*** Not a function

#0.16 List the elements of the power set of the given set and give the cardinality of the power set:

a. $\varnothing$ ***Ans:*** $\{\varnothing\}$: cardinality 1
b. $\{a\}$ ***Ans:*** $\{\varnothing, a\}$: cardinality 2
c. $\{a, b\}$ ***Ans:*** $\{\varnothing, a, b, ab\}$: cardinality 4
d. $\{a, b, c\}$ ***Ans:*** $\{\varnothing, a, b, c, ab, ac, bc, abc\}$: cardinality 8

#0.30 • $x\mathscr{R}y$ in $\mathbb{R}$ if $x \geq y$
Determine whether the given relation is an equivalence relation on the set. Describe the partition arising from each equivalence relation.
Ans: This is NOT an equivalence relation because it does not satisfy symmetric properities. Example: $2 \geq 1$ but $1 \ngeq 2$.

#1.03 Compute $(b * d) * c$ and $b * (d * c)$. Can you say on the basis of this computation whether $*$ is associative?
Ans: No, because $(b * d) * c = e * c = a$, but $b * (d * c) = b * b = c$

#1.10 Let $*$ be defined on $\mathbb{Z}^+$ by letting $a * b = 2^{ab}$.
Determine whether the operation $*$ is associative, whether the operation is commutative, and whether the set has an identity element.
Ans: It is commutative because $a * b = b * a = 2^{ab}$, but it is not associative because $(a * b) * c = 2^{2^{ab}c}$, but $a * (b * c) = 2^{2^{bc}a}$. There is no identity element for this set.

#1.27 Let H be the subset of $M_2(\mathbb{R})$ consisting of all matrices of the form

$$\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

for $a, b \in \mathbb{R}$. Is H closed under:

(a) matrix addition?
Ans: Yes, it is closed under addition
$$\begin{bmatrix} a & -b \\ b & a \end{bmatrix} + \begin{bmatrix} c & -d \\ d & c \end{bmatrix} = \begin{bmatrix} a+c & -(b+d) \\ b+d & a+c \end{bmatrix}$$

(b) matrix multiplication?
Ans: Yes, it is closed under multiplication
$$\begin{bmatrix} a & -b \\ b & a \end{bmatrix} \begin{bmatrix} c & -d \\ d & c \end{bmatrix} = \begin{bmatrix} ac-bd & -(ad+bc) \\ ad+bc & ac-bd \end{bmatrix}$$

#2.02 Let $*$ be defined on $2\mathbb{Z} = \{2n \mid n \in \mathbb{Z}\}$ by letting $a * b = a + b$. Determine whether the binary operation $*$ gives a group structure on the given set.
Ans: Yes, all three axioms, associativity, indentity element and inverse holds for this set.