

#4.12 Compute the permutation products.

- (a) $(1, 5, 2, 4)(1, 5, 2, 3)$
- (b) $(1, 5, 3)(1, 2, 3, 4, 5, 6)(1, 5, 3)^{-1}$
- (c) $[(1, 6, 7, 2)^2(4, 5, 2, 6)^{-1}(1, 7, 3)]^{-1}$
- (d) $(1, 6)(1, 5)(1, 4)(1, 3)(1, 2)$

Ans:

- (a) We start with 3:

$$\sigma(3) = 5, \sigma(5) = 4, \sigma(4) = 1, \sigma(1) = 2, \sigma(2) = 3$$

Thus, $(1, 5, 2, 4)(1, 5, 2, 3) = (3, 5, 4, 1, 2) = (1, 2, 3, 5, 4)$

- (b) First calculate $(1, 5, 3)^{-1} = (1, 3, 5)$. Then we can calculate the product of $(1, 5, 3)(1, 2, 3, 4, 5, 6)(1, 3, 5)$:

$$\sigma(1) = 4, \sigma(4) = 3, \sigma(3) = 6, \sigma(6) = 5, \sigma(5) = 2, \sigma(2) = 1$$

Thus $(1, 5, 3)(1, 2, 3, 4, 5, 6)(1, 3, 5) = (1, 4, 3, 6, 5, 2)$

- (c) First calculate $[(1, 6, 7, 2)^2(4, 5, 2, 6)^{-1}(1, 7, 3)]^{-1}$
 $= (2, 7, 6, 1)(2, 7, 6, 1)(4, 5, 2, 6)(3, 7, 1)$ Then:

$$\sigma(1) = 3, \sigma(3) = 1, \sigma(2) = 2, \sigma(4) = 5, \sigma(5) = 6, \sigma(6) = 4, \sigma(7) = 7,$$

Thus $[(1, 6, 7, 2)^2(4, 5, 2, 6)^{-1}(1, 7, 3)]^{-1} = (1, 3)(2)(4, 5, 6)(7)$

- (d) $(1, 6)(1, 5)(1, 4)(1, 3)(1, 2)$

$$\sigma(1) = 2, \sigma(2) = 3, \sigma(3) = 4, \sigma(4) = 5, \sigma(5) = 6, \sigma(6) = 1$$

Thus $(1, 6)(1, 5)(1, 4)(1, 3)(1, 2) = (1, 2, 3, 4, 5, 6)$

#4.26 $f_3 : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f_3(x) = -x^3$ **Ans:**

Recall that a permutation is one-to-one and onto function. Thus, we check these two properties.

- (a) f_3 is one-to-one:

Assume $f_3(x) = f_3(y)$, then $-x^3 = -y^3$ implies $x = y$. Thus, f_3 is one-to-one.

- (b) f_3 is onto:

For any $y \in \mathbb{R}$, we can find $x = -\sqrt[3]{y} \in \mathbb{R}$ such that $f_3(x) = -x^3 = -(-\sqrt[3]{y})^3 = y$. Thus, f_3 is onto.

So since f_3 is one-to-one and onto, it is a permutation.

#4.35

#5.8

#5.22

#5.30

#5.56

#5.61