#4.12 Compute the permutation products.

(a) $(1, 5, 2, 4)(1, 5, 2, 3)$

(b) $(1, 5, 3)(1, 2, 3, 4, 5, 6)(1, 5, 3)^{-1}$

(c) $[(1, 6, 7, 2)^2(4, 5, 2, 6)^{-1}(1, 7, 3)]^{-1}$

(d) $(1, 6)(1, 5)(1, 4)(1, 3)(1, 2)$

Ans:

(a) We start with 3:

$$\sigma(3) = 5, \sigma(5) = 4, \sigma(4) = 1, \sigma(1) = 2, \sigma(2) = 3$$

Thus, $(1, 5, 2, 4)(1, 5, 2, 3) = (3, 5, 4, 1, 2) = (1, 2, 3, 5, 4)$

(b) First calculate $(1, 5, 3)^{-1} = (1, 3, 5)$. Then we can calculate the product of $(1, 5, 3)(1, 2, 3, 4, 5, 6)(1, 3, 5)$:

$$\sigma(1) = 4, \sigma(4) = 3, \sigma(3) = 6, \sigma(6) = 5, \sigma(5) = 2, \sigma(2) = 1$$

Thus $(1, 5, 3)(1, 2, 3, 4, 5, 6)(1, 3, 5) = (1, 4, 3, 6, 5, 2)$

(c) First calculate $[(1, 6, 7, 2)^2(4, 5, 2, 6)^{-1}(1, 7, 3)]^{-1}$
$= (2, 7, 6, 1)(2, 7, 6, 1)(4, 5, 2, 6)(3, 7, 1)$ Then:

$$\sigma(1) = 3, \sigma(3) = 1, \sigma(2) = 2, \sigma(4) = 5, \sigma(5) = 6, \sigma(6) = 4, \sigma(7) = 7,$$

Thus $[(1, 6, 7, 2)^2(4, 5, 2, 6)^{-1}(1, 7, 3)]^{-1} = (1, 3)(2)(4, 5, 6)(7)$

(d) $(1, 6)(1, 5)(1, 4)(1, 3)(1, 2)$

$$\sigma(1) = 2, \sigma(2) = 3, \sigma(3) = 4, \sigma(4) = 5, \sigma(5) = 6, \sigma(6) = 1$$

Thus $(1, 6)(1, 5)(1, 4)(1, 3)(1, 2) = (1, 2, 3, 4, 5, 6)$

#4.26 $f_3 : \mathbb{R} \to \mathbb{R}$ defined by $f_3(x) = -x^3$ ***Ans:***
Recall that a permutation is one-to-one and onto function. Thus, we check these two properties.

(a) f_3 is one-to-one:
Assume $f_3(x) = f_3(y)$, then $-x^3 = -y^3$ implies $x = y$. Thus, f_3 is one-to-one.

(b) f_3 is onto:
For any $y \in \mathbb{R}$, we can find $x = -\sqrt[3]{y} \in \mathbb{R}$ such that
$f_3(x) = -x^3 = -(-\sqrt[3]{y})^3 = y$. Thus, f_3 is onto.

So since f_3 is one-to-one and onto, it is a permutation.

#4.35 Give a careful proof using the definition of isomorphism that if G and G' are both groups with G abelian and G' not abelian, then G and G' are not isomorphic.

Ans:

Recall that a group isomorphism needs to satisfy the following:

(a) ϕ is a homomorphism, i.e., $\forall a, b \in G, \phi(a *_1 b) = \phi(a) *_2 \phi(b)$.

(b) ϕ is bijective, i.e., ϕ is both injective (one-to-one) and surjective (onto).

Since G' is not abelian, there exists $a, b \in G'$ such that $a * b \neq b * a$. Assume there exists an isomorphism $\phi : G \to G'$. Then:

#5.8

#5.22

#5.30

#5.56

#5.61