Emergency Maneuver Game

Sercan Albut
sercan.albut@studenti.unipd.it

Marina Vicini
marina.vicini@studenti.unipd.it

Abstract

In this report, Game theory will be used to solve several types of emergency maneuver problems regarding autonomous vehicles. Four different environments have been considered, starting from a base game and extending it. Through the game theory models we would like to show how the programmed autonomous systems solve always more complex situations. We considered two autonomous robotic cars, a road obstacle (that in this case is a deer) and choices of swerving to the right lane or remaining in the same lane. The two actions can lead to different outcomes according if the car swerves off of road, keep driving on an empty lane, hit the obstacle or hit the other car. We will consider a static game and an extension of it (considering another choice of stopping the car) and Bayesian games (considering three different types of how the deer can move). The respective calculations will include computing the Nash Equilibrium, supporting representations of outcome matrix and visual representation of the situations that these cars are in.

1. Introduction

Autonomous vehicles, also called self-driving cars, are one of the most important innovation in the field of transportation. The benefits of the technology are several. First of all, from an environmental perspective, the adoption of autonomous cars would reduce traffic congestion, significantly reducing the CO_2 emission produced by cars each year [5]. Moreover, this system would decrease the time spent commuting. Another important aspect is simplicity: according to the National Highway Traffic Safety Administration of the Department of Transportation of the USA, autonomous driving would improve access to transportation to the elderly and to people with disabilities [6]. Overall, this technology has the potential to increase productivity and quality of life. Lastly, an important factor of autonomous driving is safety. The key is that the vehicles would increasingly help drivers avoid crashes that lead to fatalities and injuries. Around 94% of crashes in the USA in 2015 were a result of human error [6], that shows the importance of automated systems. In this report we will model a situation of Emergency Maneuver through Game Theory, focusing on safety. In an emergency maneuver game, two cars have to take a simultaneous decision on how to respond to a sudden obstacle on the road.
In section 1.1, we will show why Game Theory is an appropriate tool for this situation and in section 2, we will go into the details of the game.

1.1. Why Game Theory?

Game Theory is developed for creative decision making between two or more players who have multiple options. There are many different types of games with different types of complexity. So Game Theory is a valid tool to create strategies for players balancing their outcome. It helps players and inspectors to visualize the possible outcomes regarding the game flow. Games are categorized according to specifications of players, environment and conditions. How the game is defined, what are the actions and the environment game types can change the outcome of the game. There might also be unknown variables, which make outcomes hard to compile, but eventually through game theory the most rational choices for the players are computed.

The biggest distinction in game theory is based of cooperativeness and non-cooperativeness of the the players. Cooperative games are referred to mutual outcome and rather beneficial for both players to consider if it is possible. Non-cooperative games usually consider eliminating the other player and goes on the worst outcome path for individual profit. So any decision making, action-reaction, transaction including more than two players can be implemented in game theory. In today's world, technological advancements are rapidly developing new ways to use Artificial Intelligence. The companies are increasing their investments on the AI guided technologies. AI is code-based and can be implemented by Machine Learning algorithms. The idea revolves around minimizing the human error. Mechanised and machine driven systems are supposedly more efficient, punctual and in the long run low cost.

Autonomous cars can not rely on trial-error process since the whole idea is to eliminate the error before it costs human lives. So there needs to be a new way of decision making. That is where game theory comes in. It tells the machine what to do in exact situation. What the machine needs is to have a data flow through its system. For the autonomous cars data flow starts with description of the AI and definition of itself then continues with inputs which are provided by sensors of the vehicle. After that information flow continues non-stop until the machine shuts down. The sensors are eyes and ears of the vehicle. Through them information flows and it creates a situation for algorithm to solve and use the outcome to perform an act. So in this situation the process of decision making comes in and that is exactly where game theory is involved.

Other then the obvious choices AI encounters, there are also uncertainty and decision making process. Game theory is a theoretical framework for conceiving social situations among competing players. In some respects, game theory is the science of strategy, or at least the optimal decision-making of independent and competing actors in a strategic setting. In this case, the self driving cars are identified as players. In this context, the goal is to reduce the damage, that means avoiding incidents. Since traffic continuously flows and requires decisions in milliseconds, choices and outcomes usually depend on guessing other vehicles or objects behaviour. This situation can be modelled through static games. In game theory, a simultaneous game or static game is a game where each player chooses their action without knowledge of the actions chosen by other players. So taking players as cars we will end up with a precise simulation of traffic. Traffic simulation happens between two rational player who their first two priority are transportation and safety. There exist a part of traffic which is unknown to both players. Among the examples of unknown is an animal crossing, that is the case considered in this paper. Other possibilities could be the weather or a pedestrian. For this last case, a lot of research can be found [1, 2]. These objects or items can be called as Nature's choice which the behaviour of the object at the time of encounter is unknown. So does the outcome. This part of the question can be solvable by Bayesian games. In Game Theory, a Bayesian game is a game that models the outcome of player interactions using aspects of Bayesian probability. Bayesian games are designed for the specification of the solutions to games with incomplete information. Giving the data flow, autonomous cars will have the best choice regarding the out come of a situation even though the information is not complete.

So, in this report we would like to explain how important Game Theory is for autonomous cars and how valuable it will be in the future for other technologies whose work depends on the decision making. We will start with the explanation of situations, notation then continue with different aspects of the situations that AI can encounter. The situations that will be considered are includes static games and Bayesian games. Weighted cost values of the actions taken are not real money but a simplifications for the algorithm to understand and make a decision. By the end of the research which action should be taken by cars will be made clear so that the algorithm works accordingly.

2. Game Model

The base game describes a situation of two autonomous cars driving on a two lane road side by side to the same direction, proposed in [8].This is a static game of complete information. We will call them "Car 1" which is located at the left side of the road and "Car 2" which is located right side of the road. Then there is the obstacle what we call "deer". This obstacle will be still during base game and it will appear in front of the Car 1. Later on we will consider be other possibilities for the obstacle movement on further games. Car 1 has two choices, swerving right or continuing the road. Car 2 has same choices but if Car 2 swerves, it will swerve off road while Car 1 swerves right lane which is current Car 2's lane. On the following games these cars will have other options to consider. Costs are weighted the

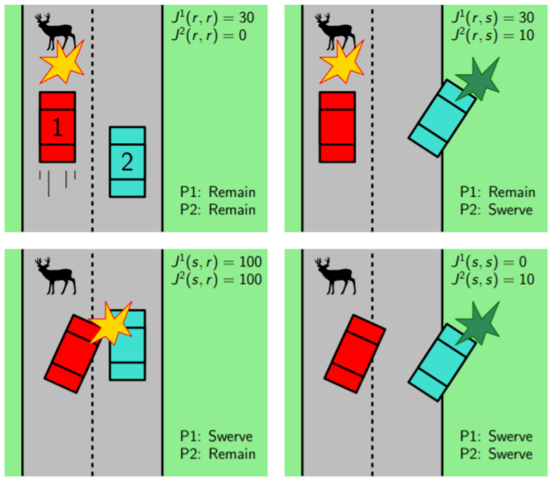


Figure 1: Representation of the game [8]. The red car is car 1 and the light blue car is car 2.

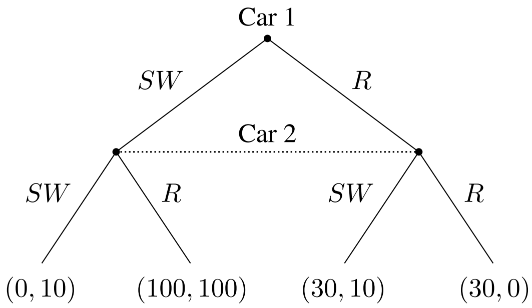
actions respectively. If a car crashes to another car, it will have more damage and if it crashes to obstacle, will have less damage then crashing to another car. If a car goes off road, it will have also a small amount of damage. Only continuing empty road and swerving an empty lane will have cost of zero. The effective cost of each action can be found in Table 1.

		Car 2 SW	Car 2 R
Car 1	SW	0, 10	100, 100
	R	30, 10	30, 0

Table 1: (Static) base game with costs values.

So to understand the content of the table, we can see that:

- If car 1 remains on the same lane (action "R"), it will hit the deer causing a cost of 30. The action of car 2 has no influence on the payoff of car 1.
  - If car 2 swerves off the road, it will have a cost of 10. This action for car 2 has always the same cost, because it is not affected nor affect the other car.
  - While if it stays on the same lane, it will have a cost 0, since it is not affect by the other car.
- If car 1 swerves to the right lane to not hit the deer, its cost depends on the action of car 2.
  - If car 2 continues off the road, then the two cars will crash, and the accident will give a cost of 100 to both of them. This is the worst outcome that could happen.
  - However, if car 2 swerves off the road, car 1 will find an empty lane on the right and it can continue without problems getting a cost of 0. Given that car 2 has swerved off the road, it will pay a cost of 10, as before.



This game can be referred to as the Chicken Game, already used in Autonomous vehicle environment in [4, 3]. The goal of the game is individuating actions to minimize the cost.

We can solve this game finding two Nash Equilibra in the pure strategies, that are respectively (SW, SW) and (R, R). Now we can look for a NE into the mixed strategies, applying the Indifference Theorem [7]. Denoting α the probability of car 1 playing "SW", and by β the probability of car 2 playing "SW", we obtain:

$$\alpha = \frac{3}{10} \text{ and } \beta = \frac{1}{10}$$

The presence of multiple NEs implies that the game does not have a single operation point. Hence, Game Theory does not provide clear, singular prediction, but this fact could be exploited for the development of engineering protocols to select a choice through the NEs.

2.1. Adding Stop

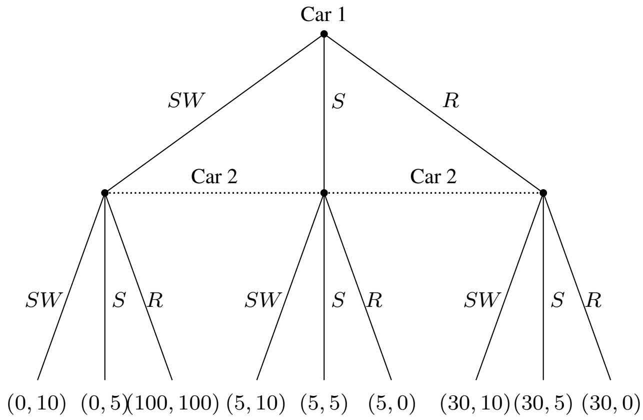
Now, we can extend the previous game by considering an additional action: Stop ("S"). When the car plays this action, the car stops. The cost for this action is always the same and it corresponds to 5. This action gives less damage than swerving off the road, but more than continuing driving without hitting anything. This is because, when stopping the car there is energy loss, and if it is done suddenly it provides damage to the tyres. We assume that when this action is performed, the car will not hit the deer. The graphical representation of this game is a 3×3 table:

Car 1 \ Car 2	S	SW	R
S	5, 5	5, 10	5, 0
SW	0, 5	0, 10	100, 100
R	30, 5	30, 10	30, 0

Table 2: Static game with actions "S", "SW" and "R".

The 2×2 bottom right table represents the previous game, while the first row and the first column represent the cost of playing this action respectively by car 1 and by car 2.

- If car 1 stops, its cost will always be the same, equal to 5. This action has no effect on the second car, and so the cost for the second car depends on its action. If it stops, it has a cost of 5. If it swerves off the road, it has a cost of 10 and if it continues driving it has no cost since the lane is empty.
- If car 2 stops, its cost will always be the same, equal to 5. Car 1 has the possibility of achieving no cost swerving to the other lane, that is free since car 1 stopped to make the road available for car 2. If car 1 remains on the same lane, it will hit the deer causing a cost of 30.



Here we can notice that the action "R" of Car 1 is stricly dominated by the action "S", while for Car 2, the action "SW" is strictly dominated by "S". Hence, through Iterated Elimination of Strictly Dominated Strategies (IESDS) we remain with a 2×2 table, that can be seen in Table 3.

Car 1 \ Car 2	S	R
S	5, 5	5, 0
SW	0, 5	100, 100

Table 3: Static game of Table 2 after removing strictly dominated strategies.

From Table 3, two NEs can be found in the pure strategies: (SW, S) and (S, R). In both these situations, there are no incidents (car-car or deer-car) and no cars goes off the road. No car has an incentive to deviate from these strategies. Following the same procedure as before, we can look for a NE in the mixed strategies in the 2×2 table, since any mixed strategy where a strictly dominated strategy is played with positive probability is strictly dominated. So we can obtain

$$\alpha = \frac{95}{100} \text{ and } \beta = \frac{95}{100}$$

where α is the probability of car 1 of playing "S" and β is the probability of car 2 of playing "S".

2.2. Deer Movement

We can add complexity to the game considering that the deer when it seems the car approaching can decide to move. This game is more realistic, since the deer is a living animal and seeing the cars approaching may decide to move. However, the car cannot know in advance what the action of the deer it will be, but they know that the deer can move. So we have a game of imperfect information. This situation will be modeled as a Bayesian game. We consider three actions for the deer:

(i) The deer can stay in lane (as the situation model above). Let us call $p \in [0, 1]$ the probability of this happening.

(ii) The deer moves the right lane, that is the one where there is car 2. Let us call $q \in [0, 1]$ the probability of this event.

(iii) The deer moves out of the road. This event happens with probability $1 - p - q \in [0, 1]$

For simplicity, we will consider the three events as equiprobable, so $p = q = \frac{1}{3}$.

Let us consider the cost (payoff) of the actions of car 1 and car 2 in case of the deer moving to the other lane:

		Car 2		
		S	SW	R
Car 1	S	5, 5	5, 10	5, 30
	SW	30, 5	30, 10	130, 130
	R	0, 5	0, 10	0, 30

Table 4: Costs values for the case of the deer moving to the other lane.

In Table 4, we can see the different costs for the different actions taken by the cars:

- If car 1 or car 2 stops, their cost is always 5.
- If car 1 swerves, it will hit the deer that has moved to the other lane, so its cost will be 30. In the case that car 2 remains on the road, the two cars will hit each other and hit the deer reaching a cost of 130 (the sum of the cost of the accident and of hitting the deer). The same reasoning is for action "R" of car 2, if car 2 remains on the same lane without stopping or swerving, it will hit the deer (and also car 1 if it swerves).
- If car 2 remains on its lane, it will have a cost of 0, since the road will be always empty. While, if car 2 swerves, its cost will be always 10.

Now let us consider the costs of the different strategies in the case of the deer moving out of the road:

		Car 2		
		S	SW	R
Car 1	S	5, 5	5, 10	5, 0
	SW	0, 5	0, 10	100, 100
	R	0, 5	0, 10	0, 0

Table 5: Cost values for the case that the deer moves out of the road.

In Table 5, we can notice that:

- The action "S" always lead to a cost of 5 for both cars. The action "SW" of car 2 gives a cost of 10 in any case, and the action "R" of car 1 has no cost.
- The action "SW" of car 1 has no cost if car 1 stops or swerves making the lane free for the other car. An incident happens if car 1 swerves and car 2 remains in the same lane.
- Since the deer moves out of the road, no car will have the cost of 30 of hitting the deer.

The cost of the different actions if the player stays in the same lane can be found in Table 2. Now we can compute the final cost of the actions. Since both cars do not know the deer will do, their actions are the same for the previous game ("S", "SW" and "R"), what changes are the costs, that are computing averaging the different situations (since they are equiprobable). To compute the NE, we can proceed through the IESDS. We can notice that the actions "SW" and "R" of car 2 are strictly dominated by the action "S" of car 2. While for car 1, the actions "SW" and "R" are strictly dominated by the action "S". After eliminating strictly eliminated strategy, the NE is immediately found: (S, S). Therefore, there are no NE in the mixed strategies.

		Car 2		
		S	SW	R
Car 1	S	5, 5	5, 10	5, 10
	SW	0, 5	10, 10	110, 110
	R	30, 5	10, 10	10, 10

Table 6: Bayesian game: the deer moves to another lane, out of the road or stays in the same lane with probability $\frac{1}{3}$.

2.3. Deer's Movement seen by Car 1

Let us consider another Bayesian game. Let us start again with the Base Game, where each car has two actions each: "SW" and "R". For simplicity, we do not consider in this game the action of stopping: "S". As the previous game, we take into account that the deer can move (considering the same 3 different types as before) We still assume that each of the three is equiprobable, so that the computations can be simplified. Moreover, since the deer starts in the same lane as car 1, let us consider a game where car 1 sees the intentions of the deer and so can act on it. Car 2 does see the deer but not its intention (like in game considered in Section 2.2). The strategies for car 1 consists on a triple of actions, one for each type of the deer movement, while the strategy of car 2 consists in a single action, given that it does not have extra information. The strategy for car 1 corresponds to a specific choice for each movement of the deer that are respectively considered in this order: deer stays in its lane, deer moves to the right lane and deer moves out of the road. In Table 7, we can see the cost for each strategy of car 1 and of car 2.

We can notice that the action "R" of car 2 is dominated by the action "SW", however, it is not strictly dominated. Through IESDS, we can remove the strictly dominated strategies so for car 1 there are: (SW, SW, R), (R, SW, SW), and (SW, SW, SW) are strictly dominated by (SW, R, SW). Moreover, (R, SW, R), (R, R, SW) are strictly dominated by (SW, R, R). The remaining strategies can be found in Table 8.

Here we can compute the Bayesian Nash Equilibria in the pure strategies, obtaining three values: ((SW, R, SW), SW), ((SW, R, R), SW) and ((R, R, R), R). No NEs were found in the mixed strategies.

Car 1	Car 2: SW	Car 2: R
(SW, SW, SW)	10, 10	110, 110
(SW, SW, R)	10, 10	76.6, 76.6
(SW, R, SW)	0, 10	66.6, 76.6
(R, SW, SW)	20, 10	86.6, 76.6
(SW, R, R)	0, 10	33.3, 43.3
(R, SW, R)	20, 10	53.3, 43.3
(R, R, SW)	10, 10	43.3, 43.3
(R, R, R)	10, 10	10, 10

Table 7: Bayesian game where the car 1 sees the intentions of the deer moving and the deer can move to another lane, out of the road or stays in the same lane with probability $\frac{1}{3}$.

Car 1	Car 2: SW	Car 2: R
(SW, R, SW)	0, 10	66.6, 76.6
(SW, R, R)	0, 10	33.3, 43.3
(R, R, R)	10, 10	10, 10

Table 8: Bayesian game of Table 2.2 after removing the strictly dominated strategies.

3. Conclusion

It is obvious that traffic requires decision making. As the decision complexity increases, so does the choices to be considered. Until 21st century, traffic and drivers would rely on their instincts and reflexes, and in some situations they could be not enough to overwhelm the complexity. Decision making in traffic requires fast response in short time interval, showing the importance of applying Game Theory in this situations, since it can compute a decision more rationally. Even the most complex situations can be solved via applying science. Elimination of uncertainty and fast processing of the information is the future not only for traffic but for a lot of fields. That is precisely why we as humans need self driving cars and their technology. In the formalization process, Game Theory is useful. As described in the process developed in this report, situations can increasingly increase in difficulty but the solutions will come with one step at a time.

We considered a context of two autonomous vehicles driving side by side in a two-lane road. A deer is located on the left lane. The cars can choose to swerve (the car on the right lane swerving off the road) or remain in their road. We formalized this situation through Game Theory, obtaining a static game of complete information. Here we looked for NEs. This game was then extended considering a third action: stopping the car. The NEs values were looked for also in this static game.

Furthermore, we consider an extension of the game to a Bayesian game, where the deer can move to the other lane, out of road or remain in the same lane. For simplicity, we considered these three possibilities as equiprobable. This game is of complete information, but the cars do not know how the deer will move, so it is imperfect. A Bayesian Nash Equilibrium was found.

A final game was considered: a Bayesian game where the deer intention of movement was seen by the car 1 (since they are on the same lane). The other car sees the deer but not its movement, but it is aware that that the deer can move. BNEs were found also in this game.

Future research could be extending this last Bayesian game (of Section 2.3) to include the third action "S". Another possibility for further research would be considering the types of the deer movement with different probabilities (using zoological insights of the deer's behavior).

In the outcome of multiple NEs in a decision making process what would be the best choice for the company to make? As the problems escalate the equations will bring more complex structures and eventually finding all NEs won't solve the decision making process of the cars. At that time Companies who creates self driving cars will go with the safest option for their customers. It can require manually implementing an emergency stop function to brake the code circle, until the people behind the coding will figure out the best weights of the decision they make. They will modify the algorithms of the car according to best out come for the safety of their customer. By data collection and sampling they will continue to modify artificial intelligence codes and most of the process will use the game theory and its equations. The codes and situations will evolve, included games will change, weights of the decisions will switch, outcomes will vary according to situations but the game theory part and its equations will stay intact.

References

[1] Fanta Camara, Nicola Bellotto, Serhan Cosar, Dimitris Nathanael, Matthias Althoff, Jingyuan Wu, Johannes Ruenz, Andre Dietrich, and Charles W. Fox. Pedestrian models for autonomous driving part i: Low-level models, from sensing to tracking. *IEEE Transactions on Intelligent Transportation Systems*, 22(10):6131–6151, Oct 2021.

[2] Fanta Camara, Nicola Bellotto, Serhan Cosar, Florian Weber, Dimitris Nathanael, Matthias Althoff, Jingyuan Wu, Johannes Ruenz, Andre Dietrich, Gustav Markkula, and et al. Pedestrian models for autonomous driving part ii: High-level models of human behavior. *IEEE Transactions on Intelligent Transportation Systems*, 22(9):5453–5472, Sep 2021.

[3] Fanta Camara and Charles Fox. Game theory for self-driving cars. 03 2020.

[4] C. Fox, Fanta Camara, Gustav Markkula, Richard Romano, Ruth Madigan, and Natasha Merat. When should the chicken cross the road? - game theory for autonomous vehicle - human interactions. 01 2018.

[5] James Manyika, Michael Chui, Jacques Bughin, Richard Dobbs, Peter Bisson, and Alex Marrs. Disruptive technologies: Advances that will transform life, business, and the global economy. 2013.

[6] NHTSA. Automated driving systems 2.0, a vision for safety.

[7] Steven Tadelis. *Game Theory: An Introduction*. Number 10001 in Economics Books. Princeton University Press, 2012.

[8] Alessandro Zanardi, Saverio Bolognani, Andrea Censi, and Emilio Frazzoli. Game theoretical motion planning. 2021.