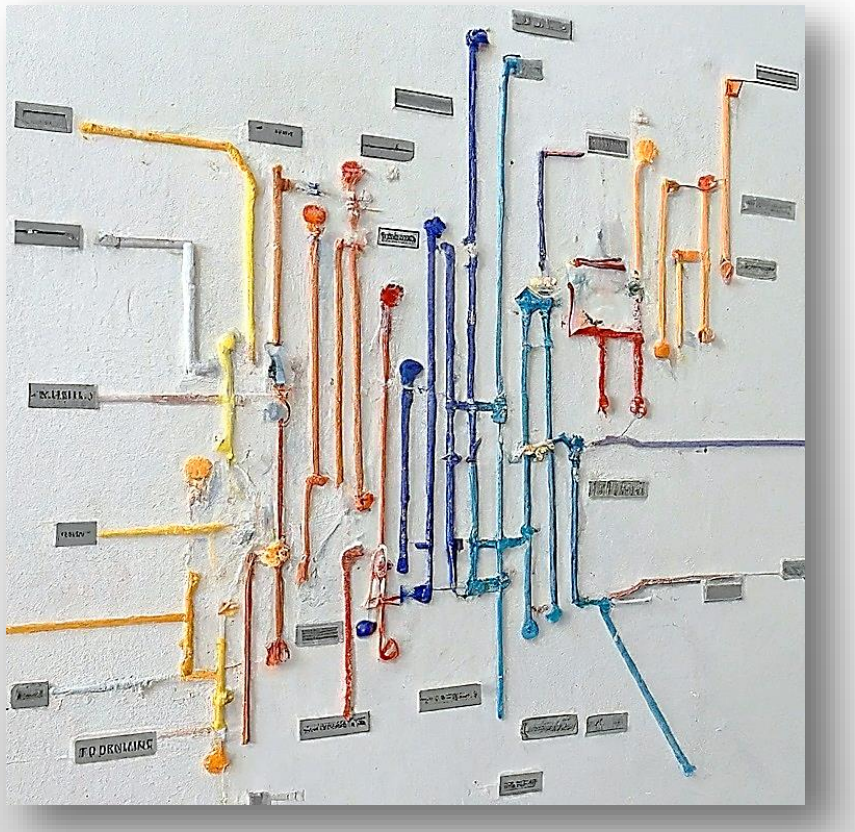
Bölüm 4: Çizge Algoritmaları

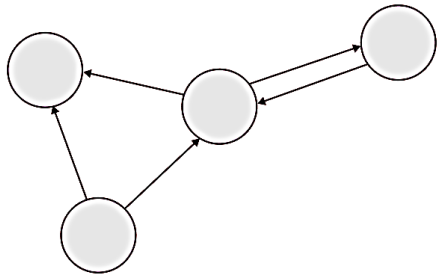
Algoritmalar

Çizge Algoritmaları

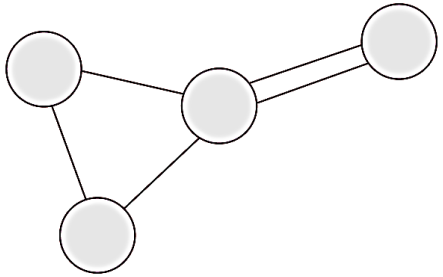
- Dünya aslında bir ağ gibidir.
  - Şehirler yollarla,
  - İnsanlar ilişkilerle,
  - Bilgisayarlar kablolarla birbirine bağlıdır.
- Çizge algoritmaları bu ağları inceler ve anlamlandırır.



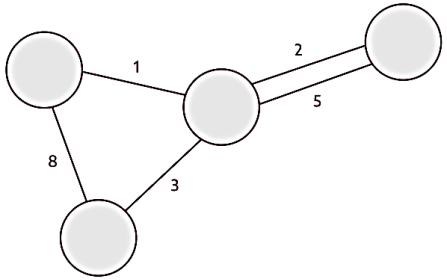
Çizge Türleri



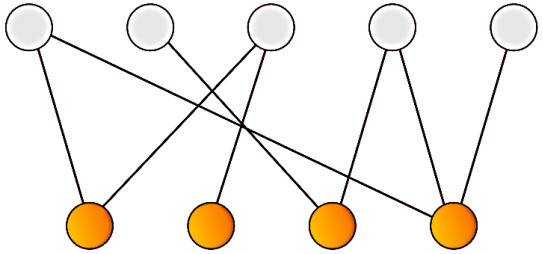
Directed graph



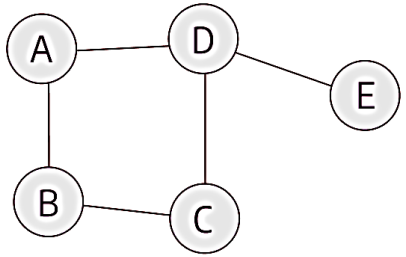
Undirected



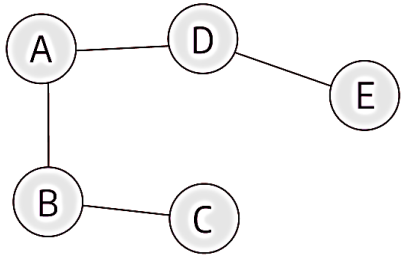
Weighted



Bipartite graph



Cyclic graph



Acyclic graph

Çizge Algoritmaları

- Birbirine bağlı noktalar (düğüm) ve bu noktaları birleştiren çizgiler (kenar) ile temsil edilen ağ yapılarını inceler.
- Ağlarda en kısa yolu hesaplama, gruplama gibi işlemleri gerçekleştirir.
- Sosyal ağlar, harita uygulamaları, navigasyon gibi birçok alanda kullanılır.

Çizge Algoritmalarının Çeşitleri

- Farklı çizge algoritmaları, farklı işlemler için kullanılır.
- Derinlik Öncelikli Arama (DFS):
  - Bir düğümden başlar, dallanarak tüm ağı gezer.
- Genişlik Öncelikli Arama (BFS):
  - Bir düğümden başlar, katman katman tüm ağı gezer.
- Dijkstra Algoritması:
  - Başlangıç düğümünden diğer düğümlere en kısa yolları bulur.
- Kruskal Algoritması:
  - Bir ağı minimum maliyetle birbirine bağlayan kenarları seçer.

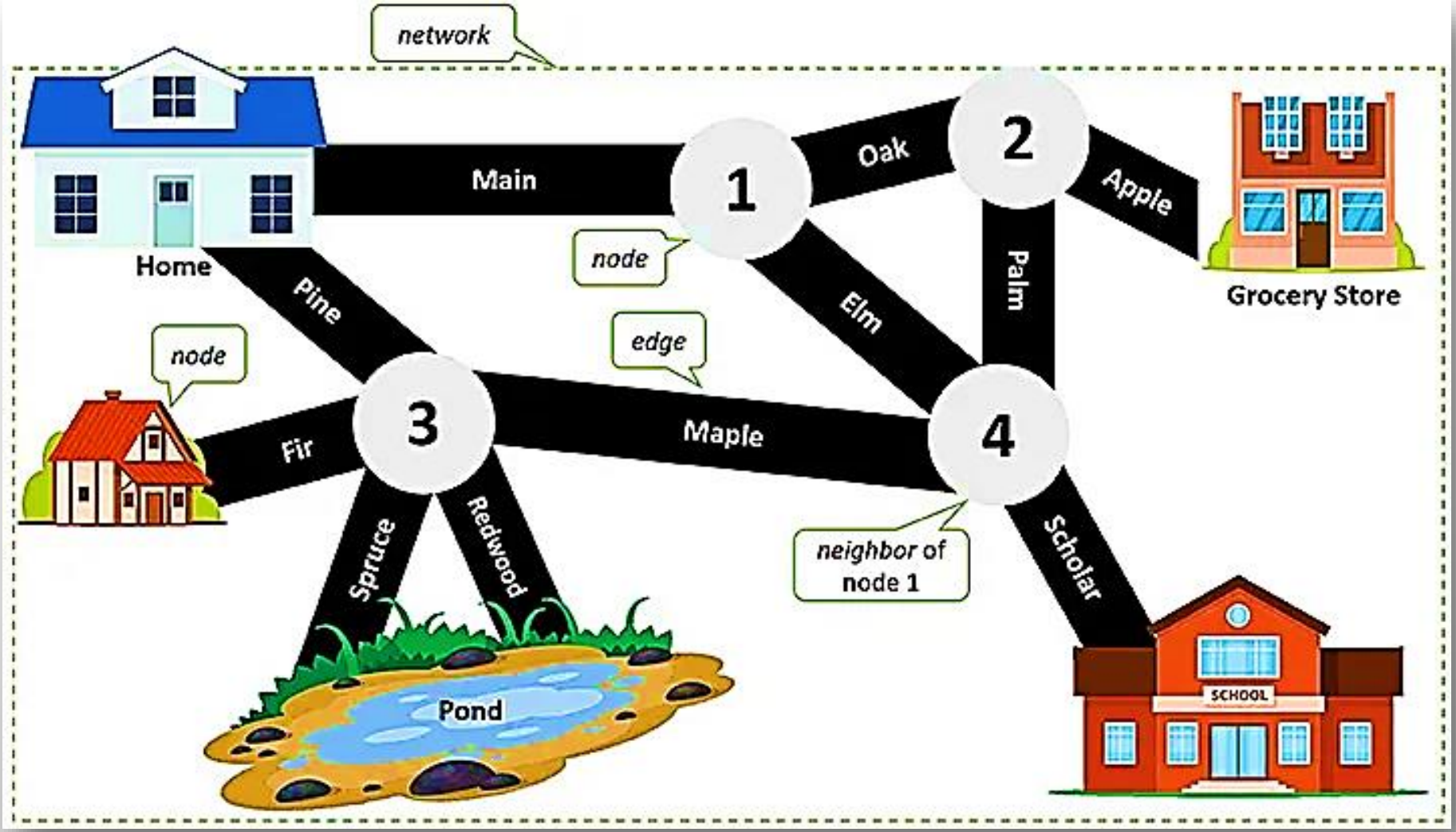
Çizge Algoritmaları

- DFS bir labirentten çıkış yolu ararken kullanılabilir.
- BFS bir haberin tüm şehre yayılma sürecini modelleyebilir.
- Dijkstra en kısa sürede teslimat yapmak için kullanılabilir.

Çizge Algoritmaları

- Çizge gezinme algoritmaları (*Graph traversal*)
- En kısa yol algoritmaları (*Shortest path*)
- Minimum yayılan ağaç algoritmaları (*Minimum spanning tree*)
- Ağ akış algoritmaları (*Network flow*)

Çizge



En Kısa Yol Algoritmaları (Shortest Path)

- Başlangıç düğümünden hedef düğüme giden en kısa yolu bulur.
- *Dijkstra*:
  - Başlangıç düğümünden diğer tüm düğümlere olan en kısa yolları bulur.
  - Ağırlıklar pozitif değer olmalıdır.
- *Bellman-Ford*:
  - Negatif ağırlıklı kenar içeren çizgelerde kullanılabilir.
  - Dijkstra algoritmasından yavaştır.
- Floyd-Warshall:
  - Tüm çiftler arasındaki en kısa yolları bulur.
  - Negatif ağırlıklı çizgelerde kullanılabilir.

En Kısa Yol Algoritmaları (Shortest Path)

- *A* Arama*:
  - Sezgisel bilgiler kullanılarak aramayı hızlandırır.
  - Hedef düğüme olan tahmini mesafeyi hesaba katar.
- BFS:
  - Ağırlıksız çizgeler üzerinde çalışır.

Dijkstra

- Tek kaynaktan diğer tüm düğümlere olan en kısa yolu bulur.
- 1956 yılında Edsger W. Dijkstra tarafından geliştirilmiştir.
- Pozitif ağırlıklı kenarlardan oluşan çizgelerde çalışır.

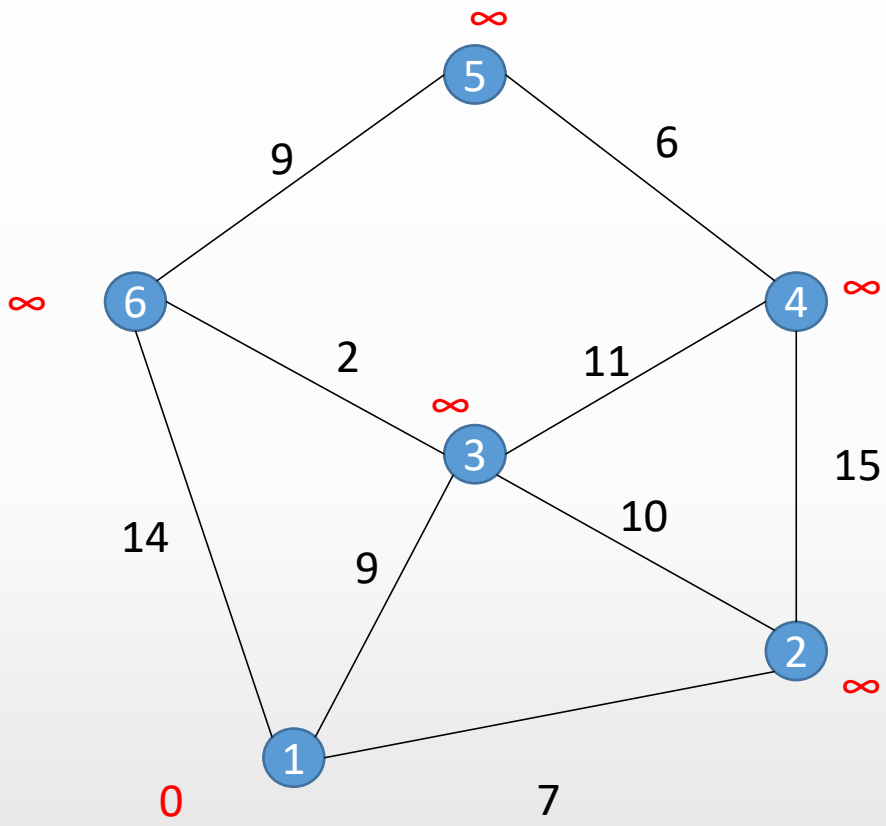
Algoritma Adımları

- Adım 1: Başlangıç düğümü seçilir ve uzaklık değeri 0 atanır. Diğer düğümlere sonsuz uzaklık atanır.
- Adım 2: Başlangıç düğümünden başlayarak, henüz işlenmemiş komşu düğümlere olan uzaklıklar hesaplanır.
- Adım 3: Daha kısa bir yol varsa, düğümün uzaklığı güncellenir.
- Adım 4: İşlenen düğümler işaretlenir ve bir sonraki düğüm seçilir.
- Adım 5: Tüm düğümler işlenene kadar Adım 2'den 4'e kadar tekrarlanır.

Algoritma Karmaşıklığı

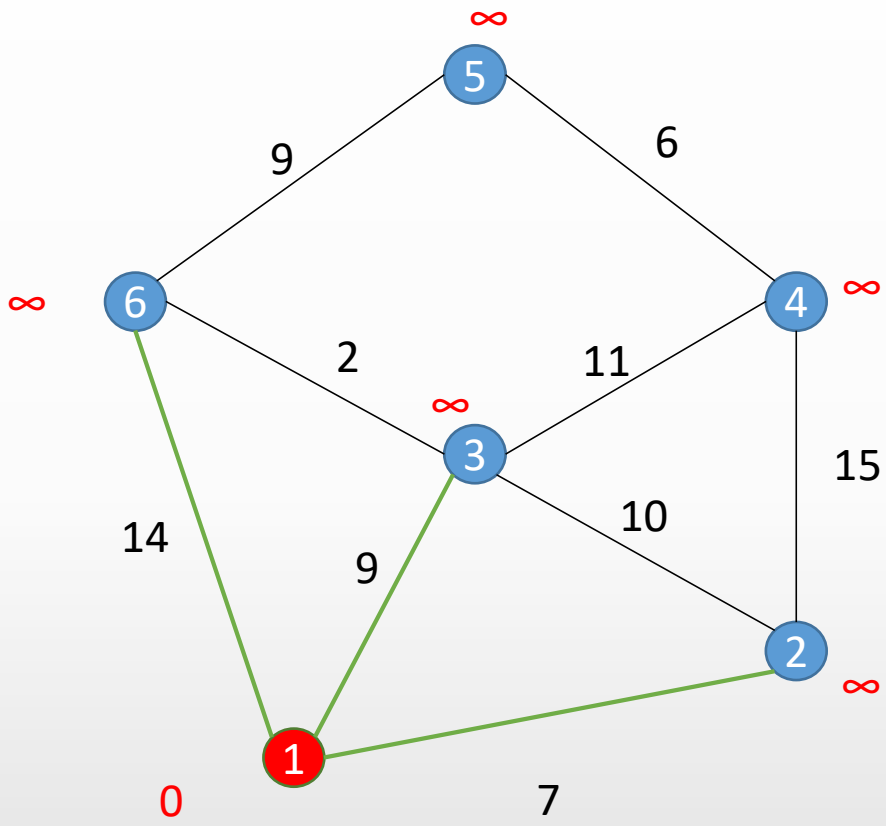
- En kısa mesafeli düğümü seçmek için;
  - Dizi temsili kullanılırsa;
    - $O(V^2)$ karmaşıklığına sahiptir.
  - Öncelik kuyruğu (*Priority Queue*) kullanılırsa;
    - karmaşıklık $O((V + E)logV)$ olur.

Dijkstra



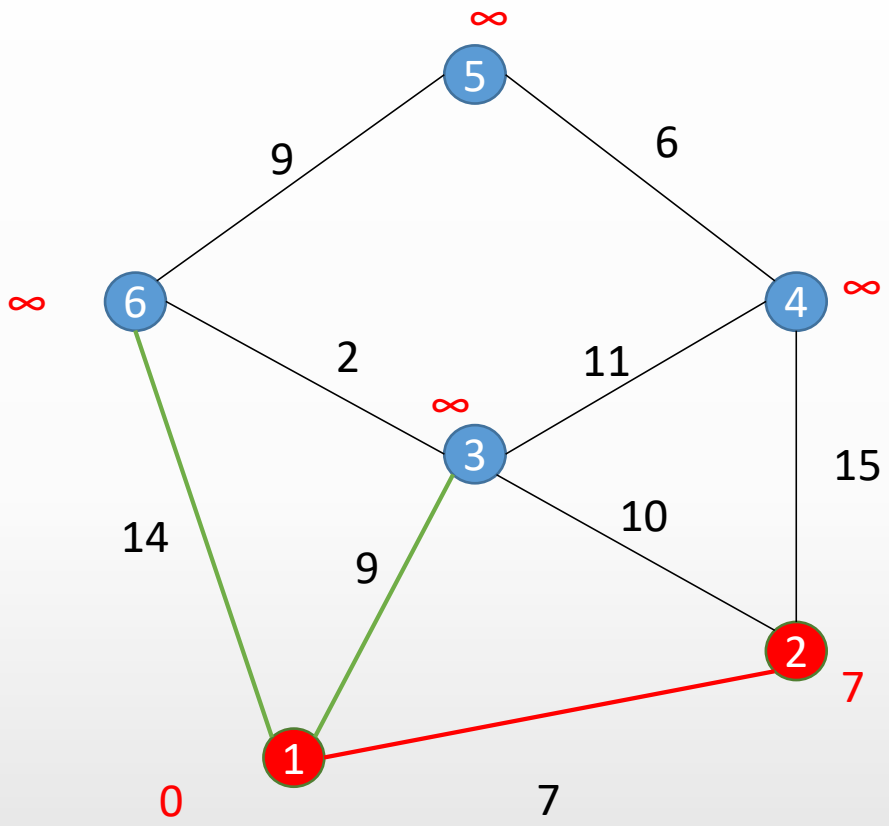
	1
1	0
2	∞
3	∞
4	∞
5	∞
6	∞

Dijkstra



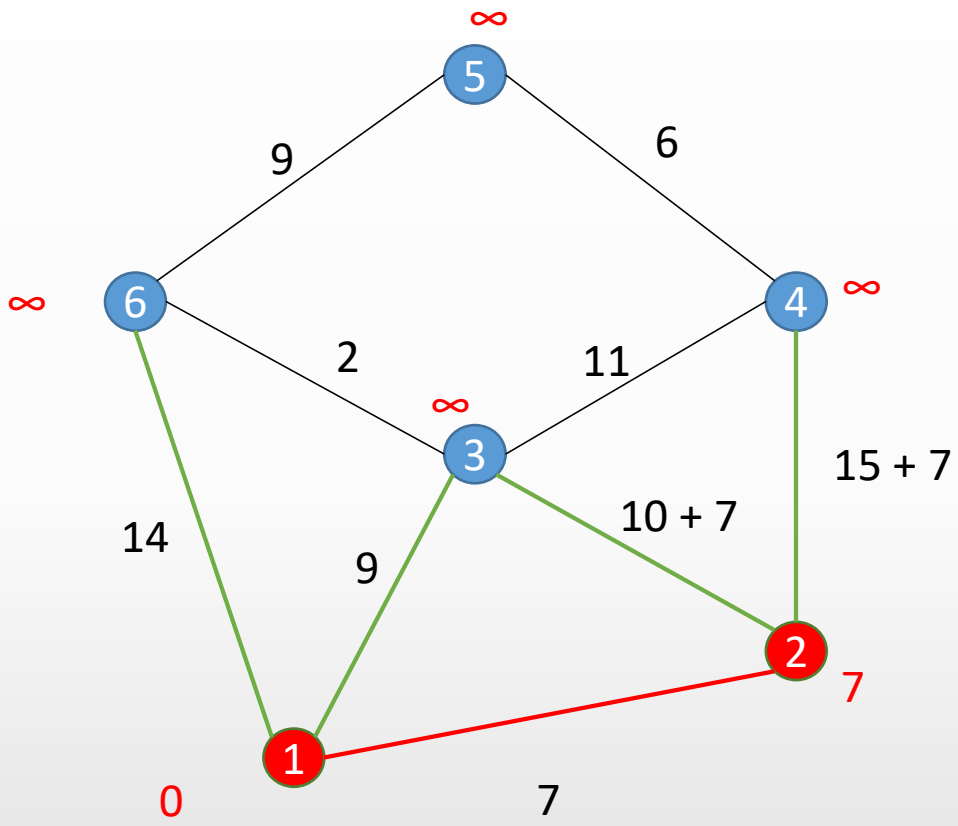
	1
1	0
2	∞
3	∞
4	∞
5	∞
6	∞

Dijkstra



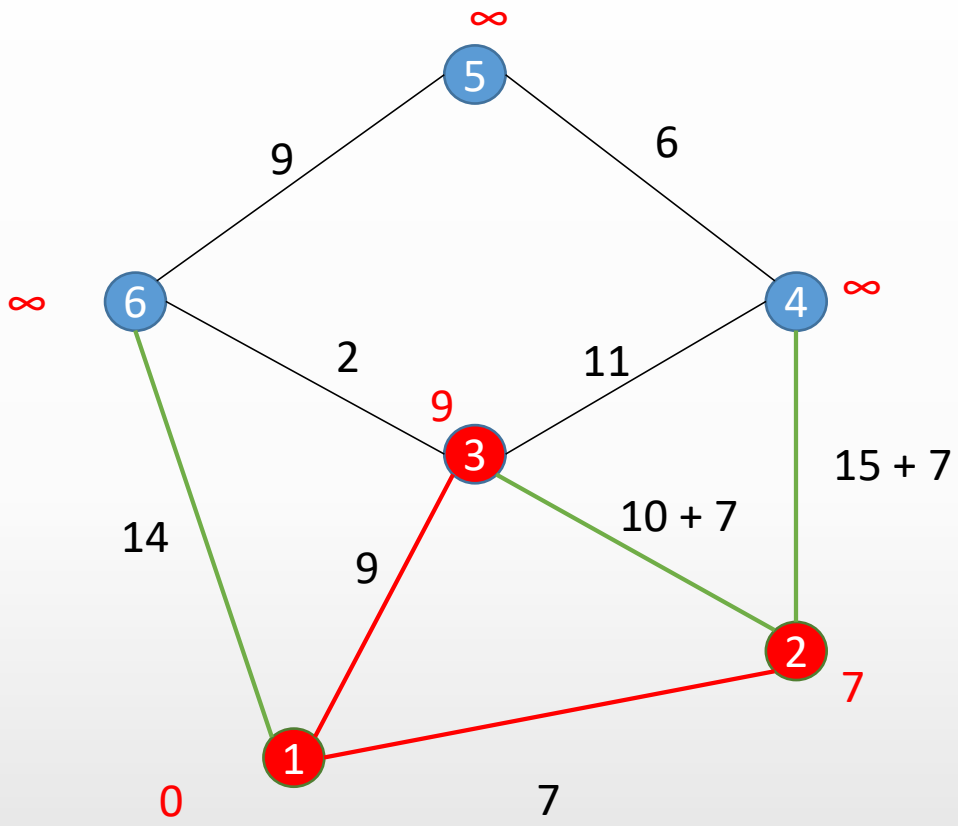
	1
1	0
2	7
3	∞
4	∞
5	∞
6	∞

Dijkstra



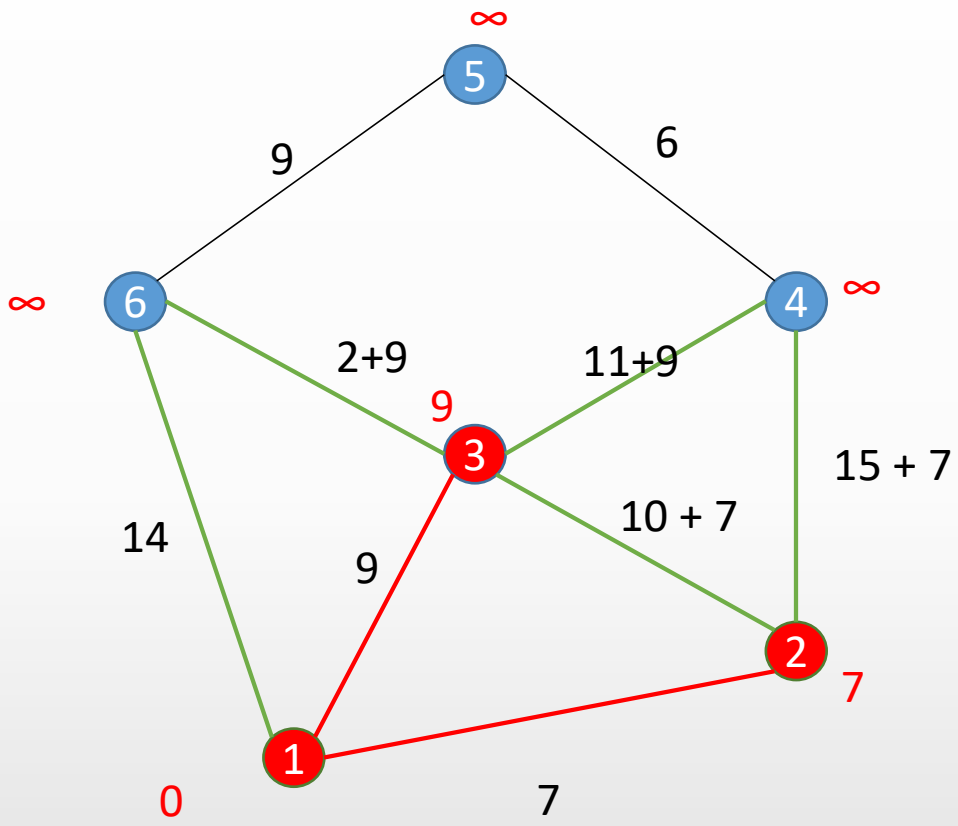
	1
1	0
2	7
3	∞
4	∞
5	∞
6	∞

Dijkstra



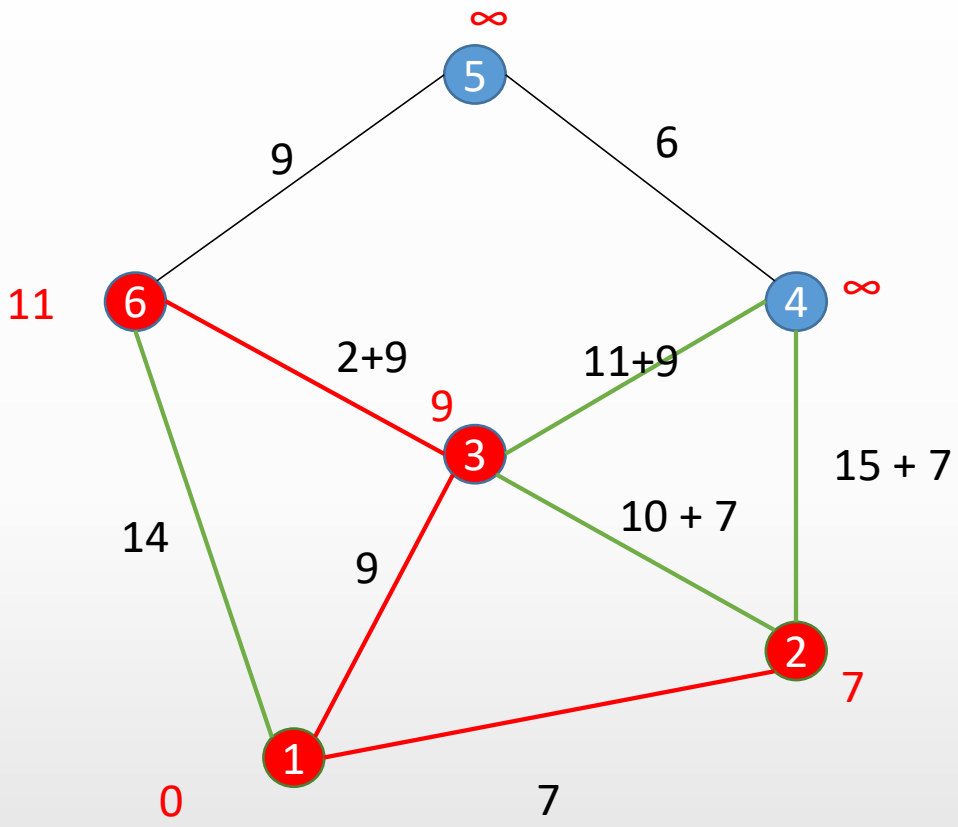
	1
1	0
2	7
3	9
4	∞
5	∞
6	∞

Dijkstra



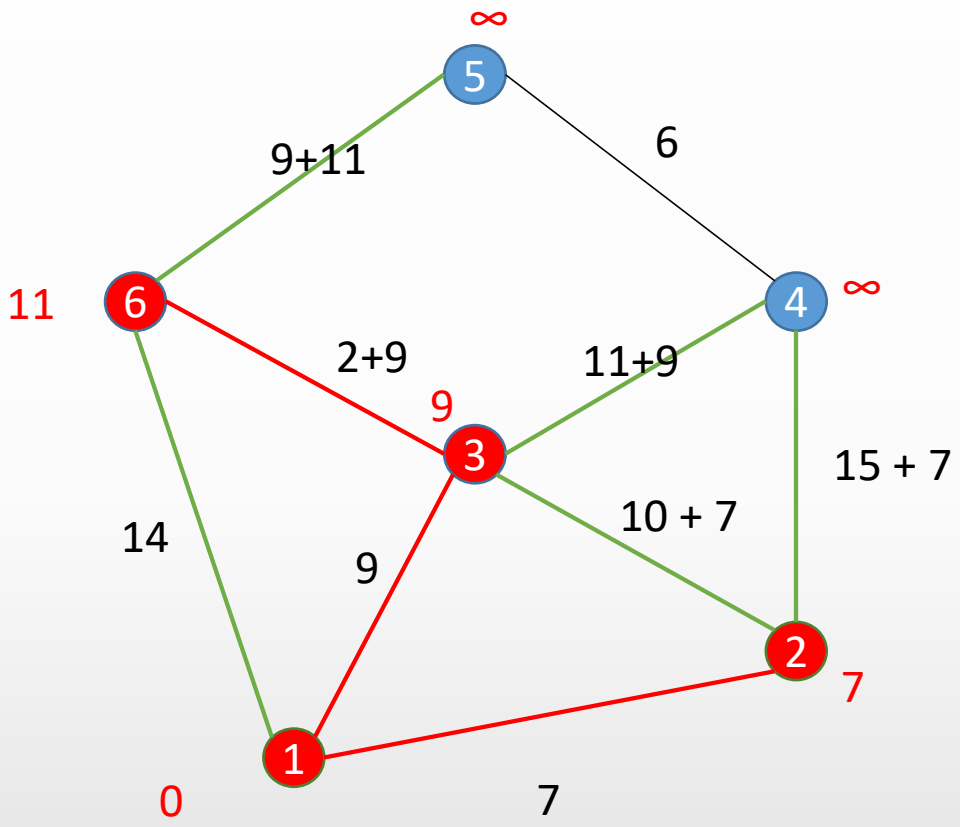
	1
1	0
2	7
3	9
4	∞
5	∞
6	∞

Dijkstra



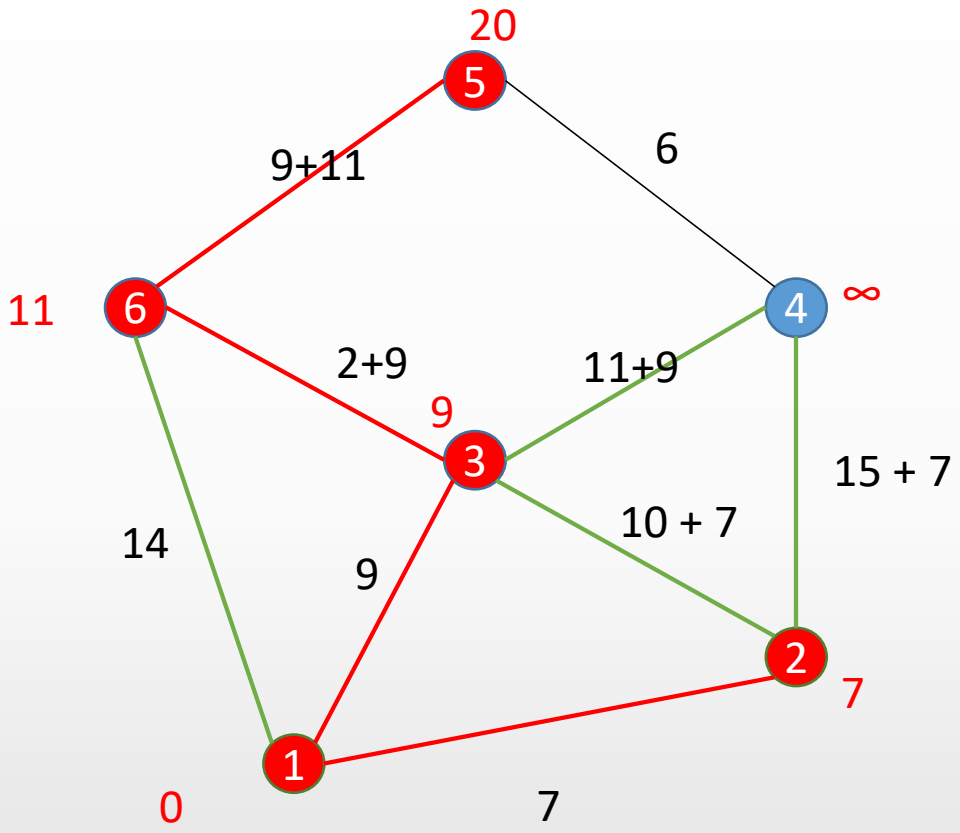
	1
1	0
2	7
3	9
4	∞
5	∞
6	11

Dijkstra



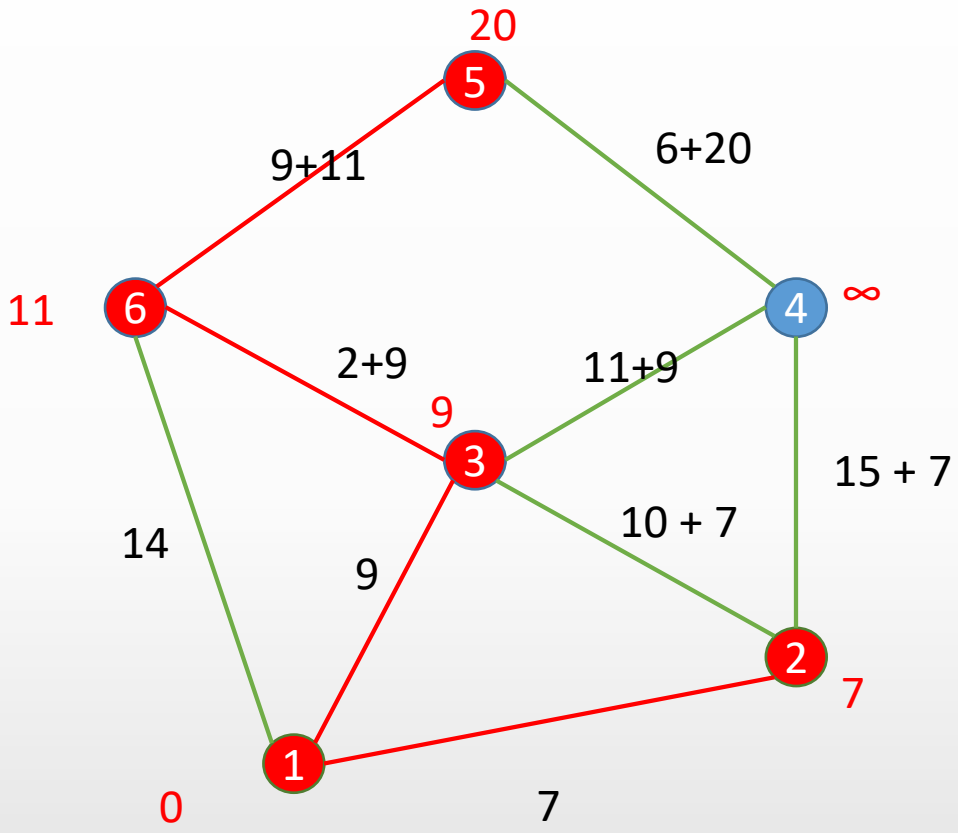
	1
1	0
2	7
3	9
4	∞
5	∞
6	11

Dijkstra



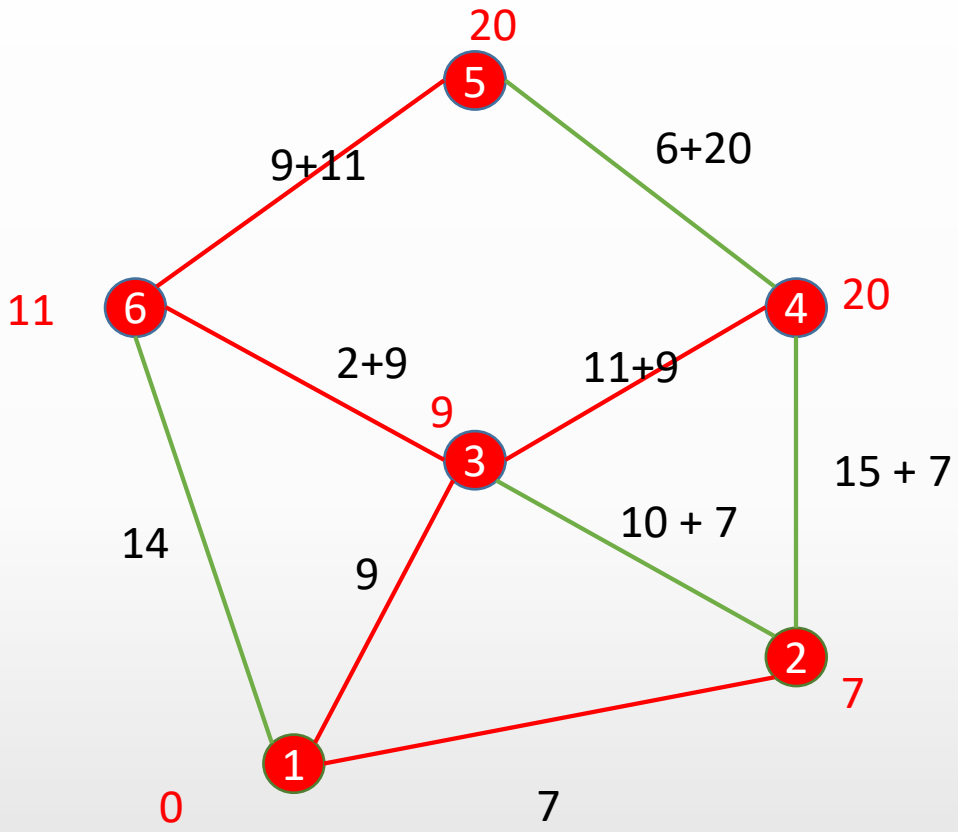
	1
1	0
2	7
3	9
4	∞
5	20
6	11

Dijkstra



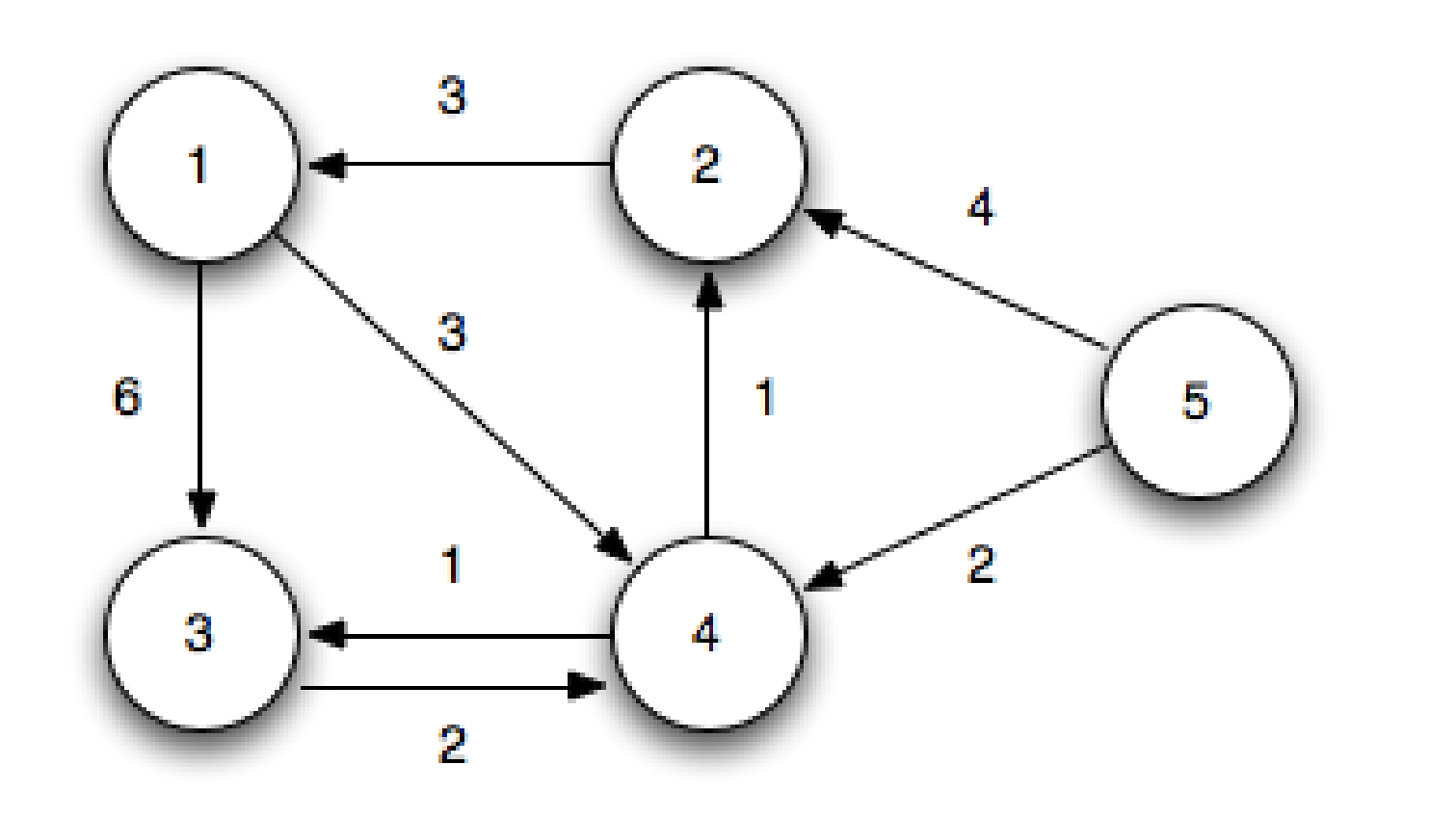
	1
1	0
2	7
3	9
4	∞
5	20
6	11

Dijkstra



	1
1	0
2	7
3	9
4	20
5	20
6	11

Örnek



- Using vertex 5 as the source (setting its distance to 0), we initialize all the other distances to ∞, set S = ∅, and place all the vertices in the queue (resolving ties by lowest vertex number)

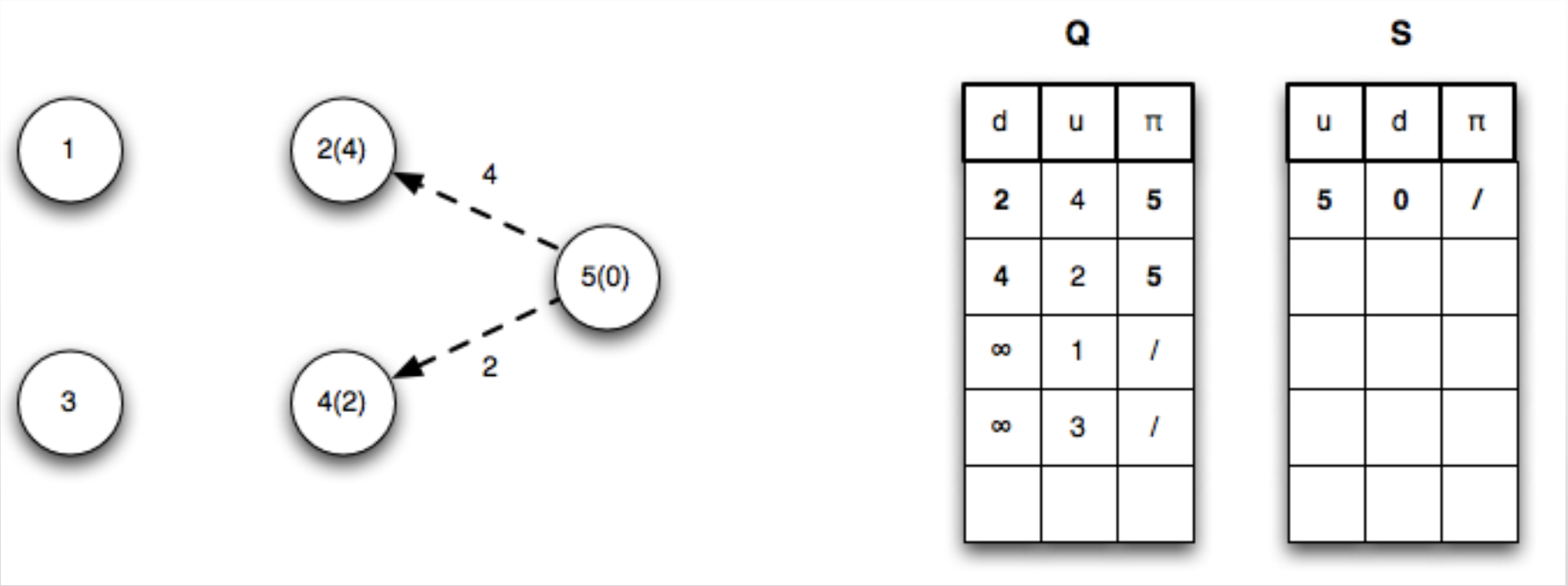
Q

d	u	π
0	5	/
∞	1	/
∞	2	/
∞	3	/
∞	4	/

S

u	d	π

- Iteration 1: Dequeue vertex 5 placing it in S (with a distance 0) and relaxing edges (u5,u2) and (u5,u4) then reprioritizing the queue



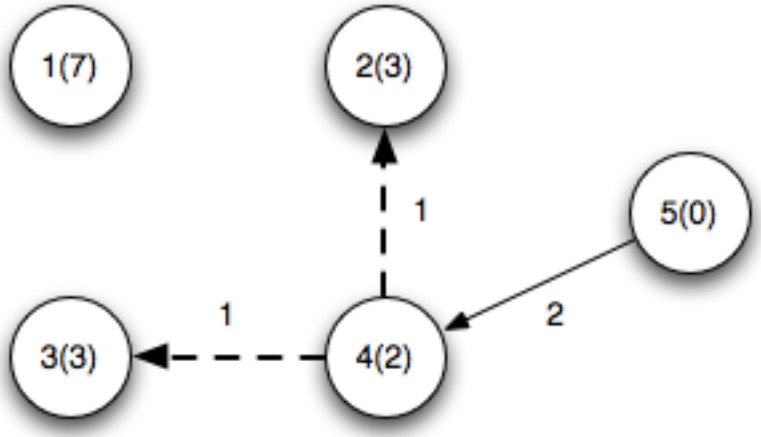
Q

d	u	π
2	4	**5**
4	2	**5**
∞	1	/
∞	3	/

S

u	d	π
5	**0**	/

- Iteration 2: Dequeue vertex 4 placing it in S (with a distance 2) and relaxing edges (u4,u2) and (u4,u3) then reprioritizing the queue. Note edge (u4,u2) finds a shorter path to vertex 2 by going through vertex 4



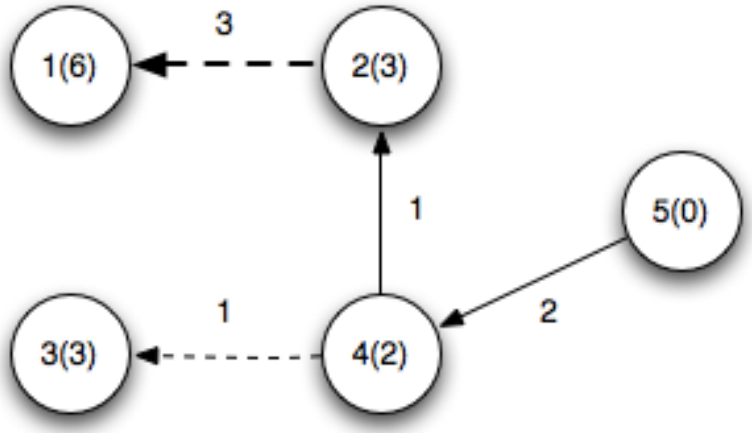
Q

d	u	π
3	2	**4**
3	3	**4**
∞	1	/

S

u	d	π
5	0	/
4	**2**	**5**

- Iteration 3: Dequeue vertex 2 placing it in S (with a distance 3) and relaxing edge (u2,u1) then reprioritizing the queue.



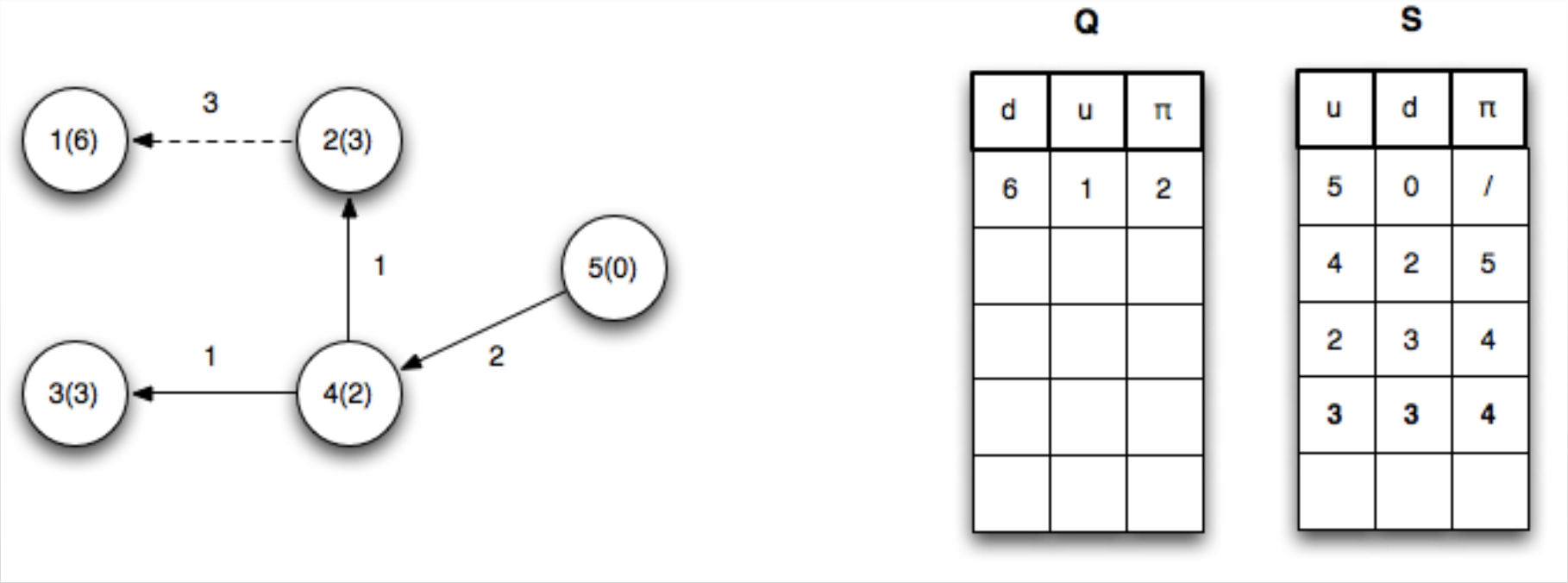
Q

d	u	π
3	3	4
6	1	**2**

S

u	d	π
5	0	/
4	2	5
2	**3**	**4**

- Iteration 4: Dequeue vertex 3 placing it in S (with a distance 3) and relaxing no edges then reprioritizing the queue.



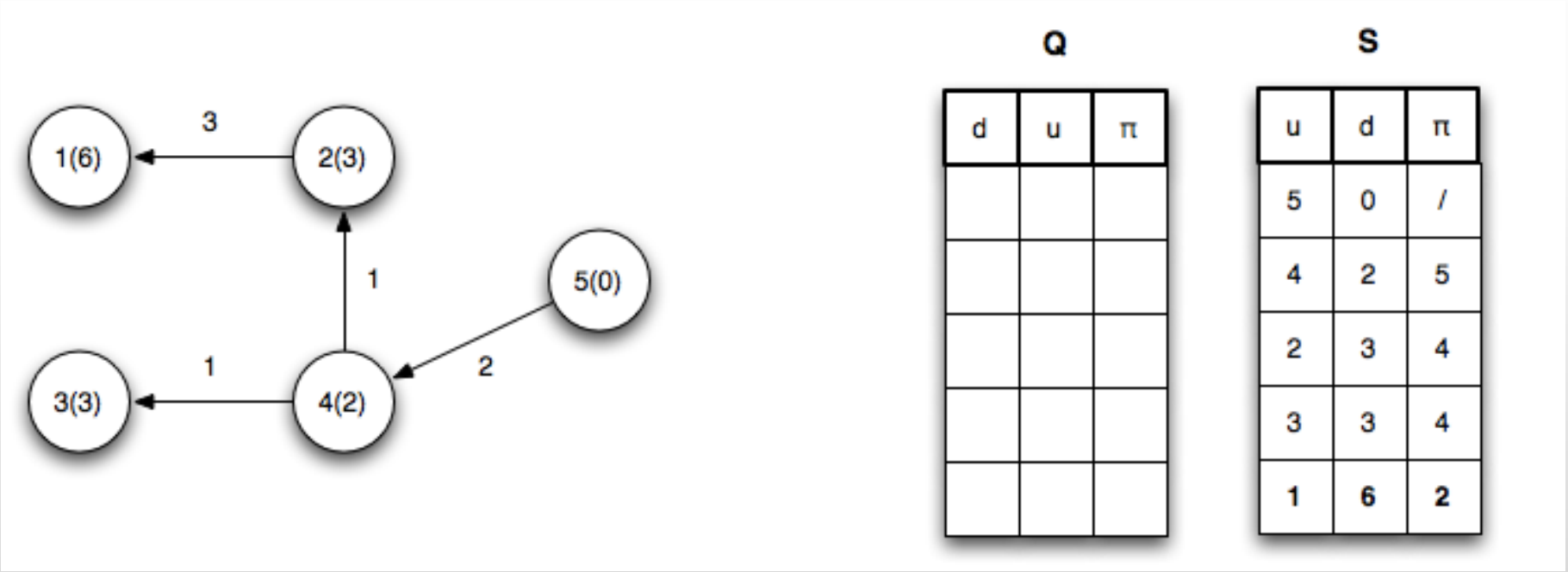
Q

d	u	π
6	1	2

S

u	d	π
5	0	/
4	2	5
2	3	4
3	**3**	**4**

- Iteration 5: Dequeue vertex 1 placing it in S (with a distance 6) and relaxing no edges.



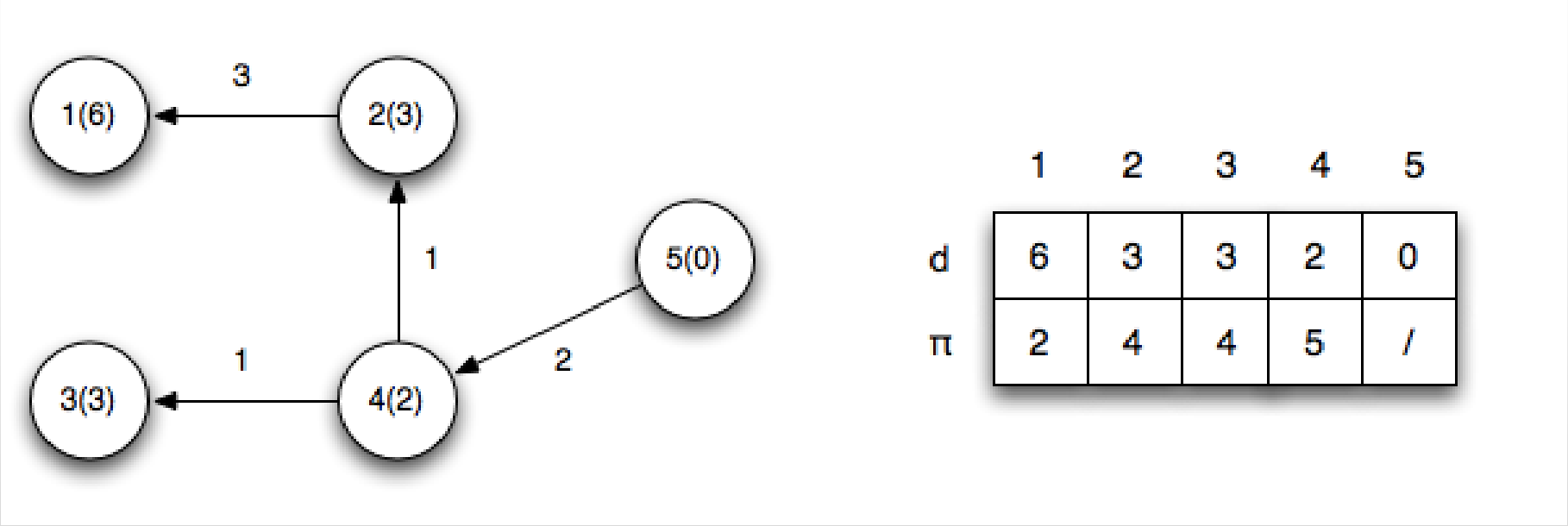
Q

d	u	π

S

u	d	π
5	0	/
4	2	5
2	3	4
3	3	4
1	**6**	**2**

- The final shortest paths from vertex 5 with corresponding distances is



	1	2	3	4	5
d	6	3	3	2	0
π	2	4	4	5	/

Bellman Ford

- Tek bir kaynaktan diğer tüm düğümlere en kısa yolu bulmak için kullanılır.
- 1958 yılında Richard Bellman ve Lester Ford Jr. tarafından geliştirilmiştir.
- Negatif ağırlıklı kenarları işleyebilir.
- Negatif ağırlıklı döngüleri bulabilir.

Algoritma İlkeleri

- Her düğüm için en kısa yol tahminlerini tutan bir dizi kullanır.
- Başlangıçta tüm düğümlerin en kısa yol tahminleri sonsuz atar.
- Çizge üzerindeki tüm kenarlar teker teker incelenir ve
  - her bir düğüm için en kısa yol tahminleri güncellenir.

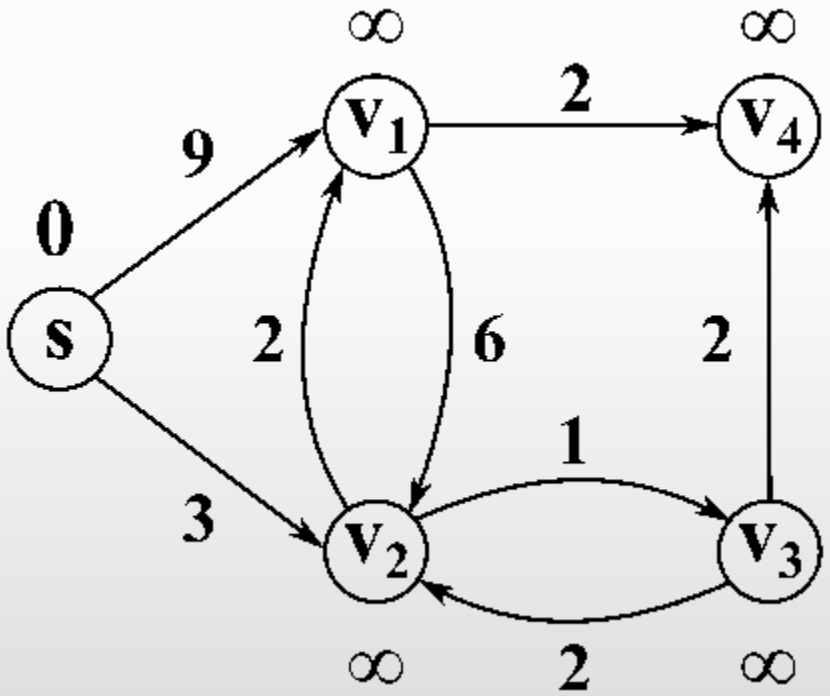
Algoritma Adımları

- Adım 1: Başlangıç düğümü seçilir ve bu düğüme uzaklık 0 atanır. Diğer düğümlere sonsuz uzaklık atanır.
- Adım 2: Tüm kenarlar tek tek incelenir ve düğümler arasındaki uzaklıklar güncellenir. Negatif döngü kontrolü için tüm kenarlar bir kez daha incelenir.
- Adım 3: Eğer bir düğümün uzaklığı güncellenirse, bu düğümün komşularının uzaklıkları da güncellenir.
- Adım 4: Eğer negatif döngü bulunursa, algoritma bu döngüyü tespit eder.

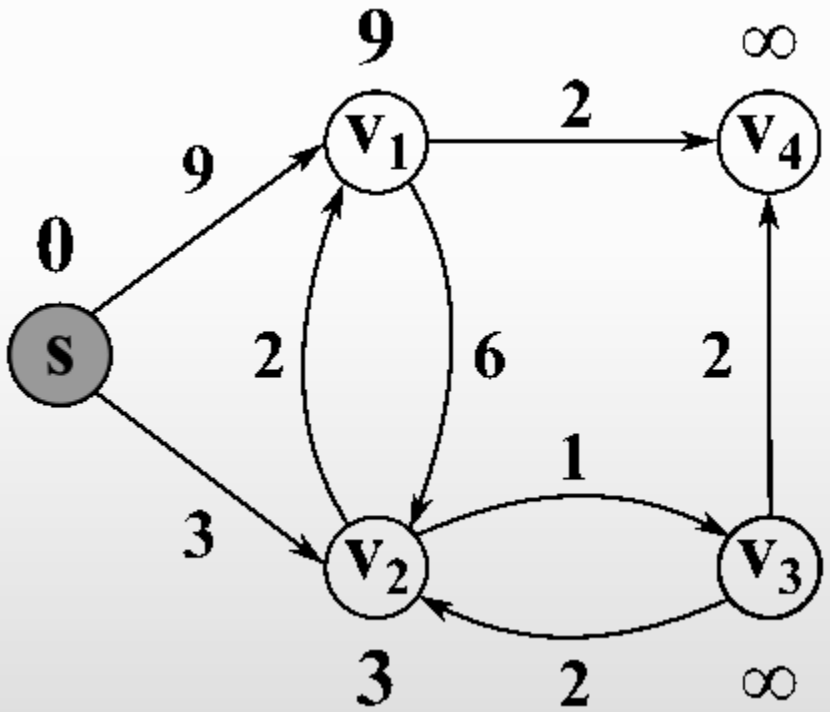
Karmaşıklık Analizi

- Çizge üzerindeki tüm kenarları V-1 kez inceler.
- *O(V*E)* karmaşıklığına sahiptir.

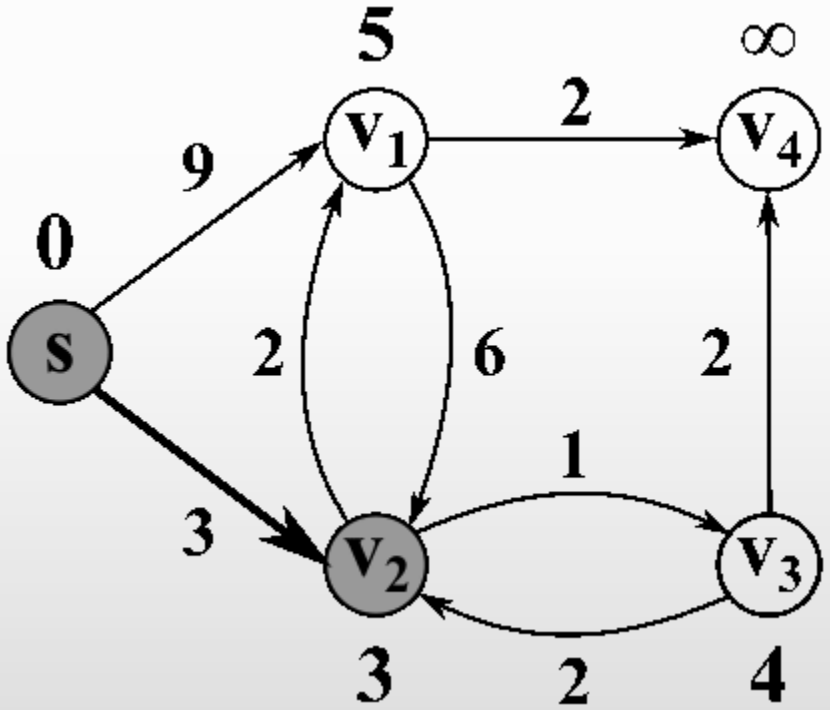
Bellman Ford



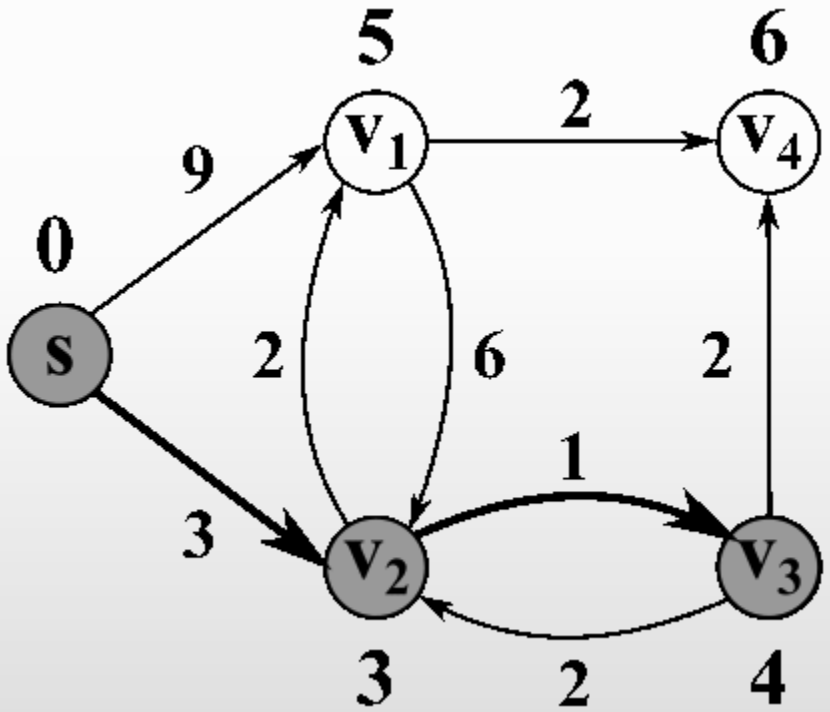
Bellman Ford



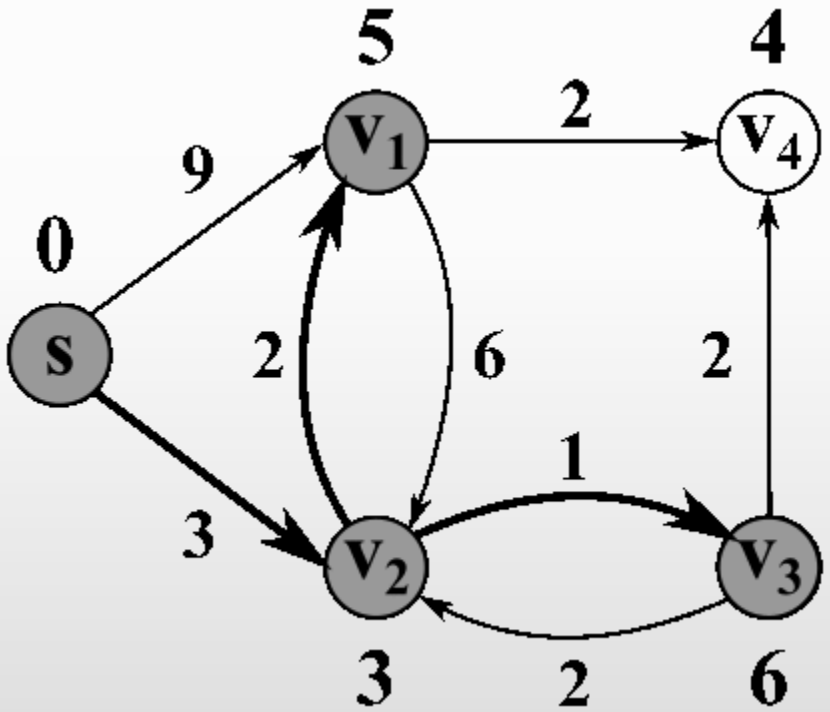
Bellman Ford



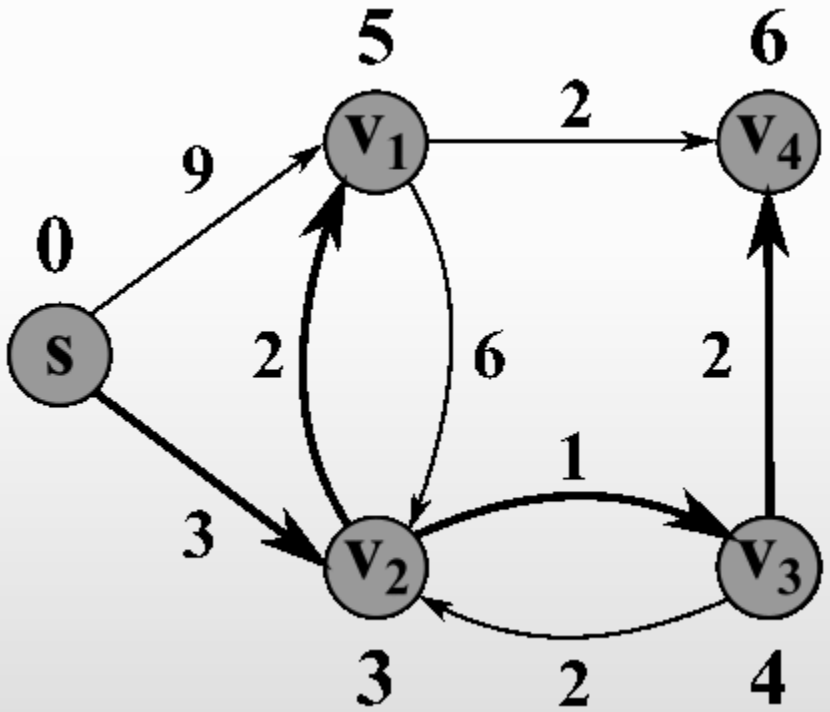
Bellman Ford



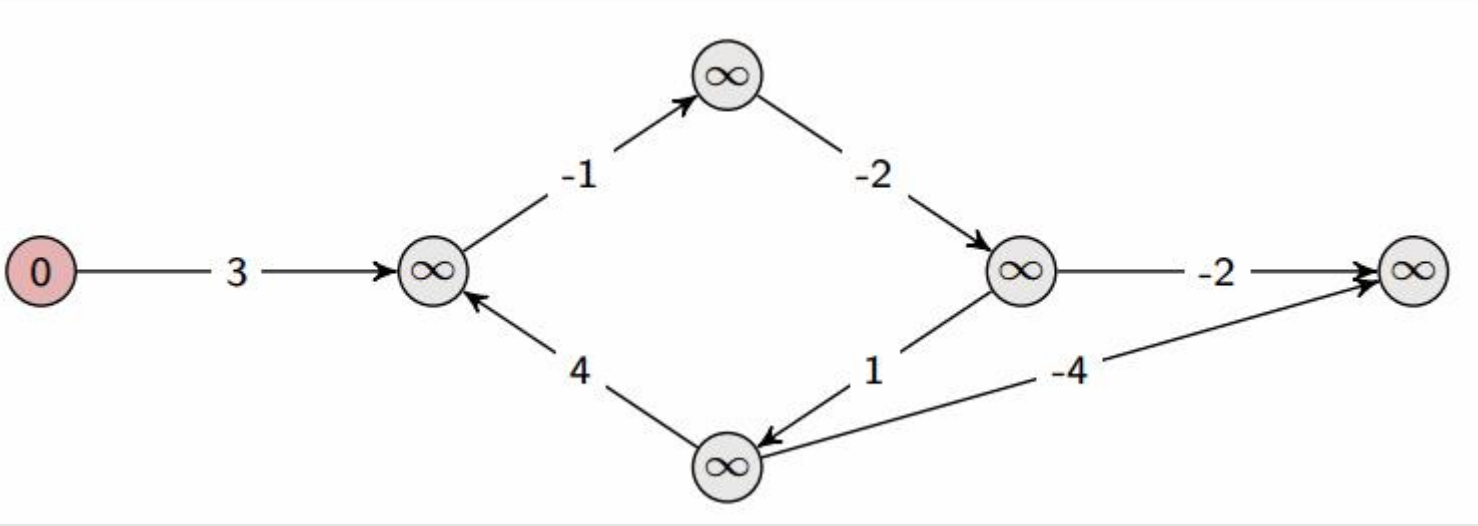
Bellman Ford



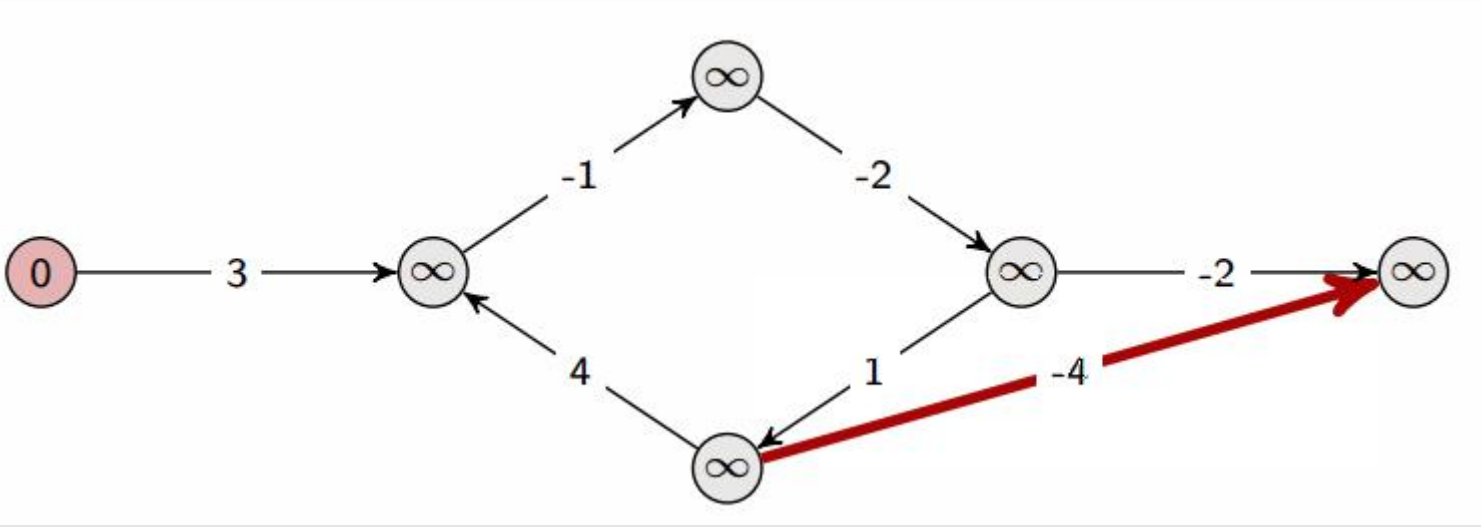
Bellman Ford



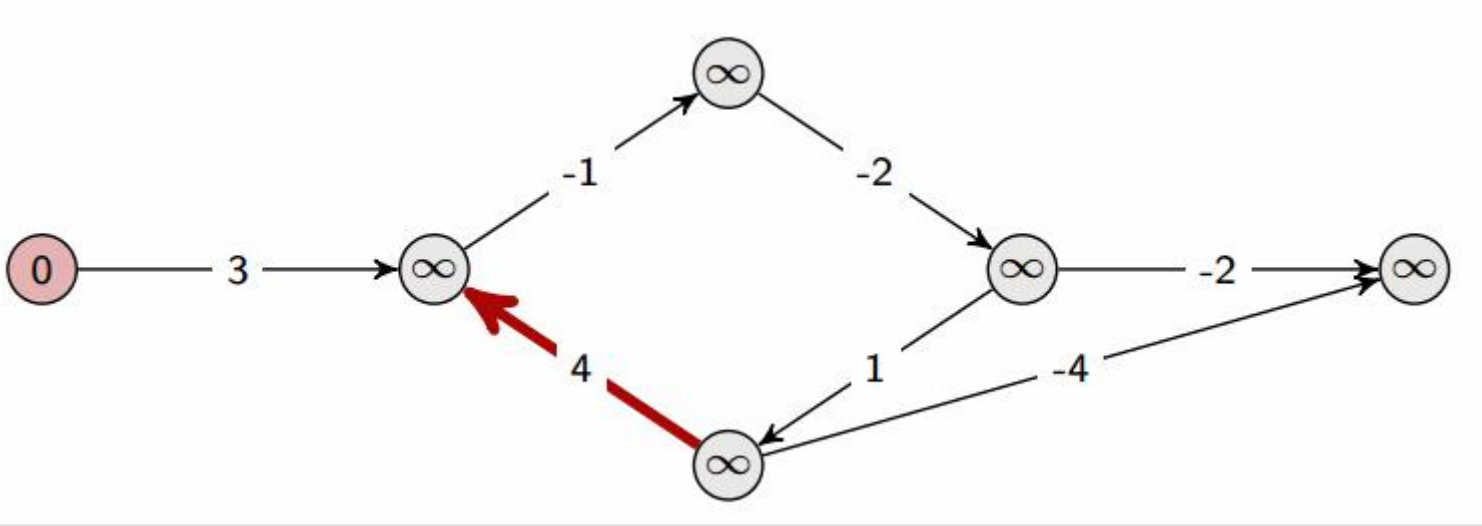
Bellman Ford



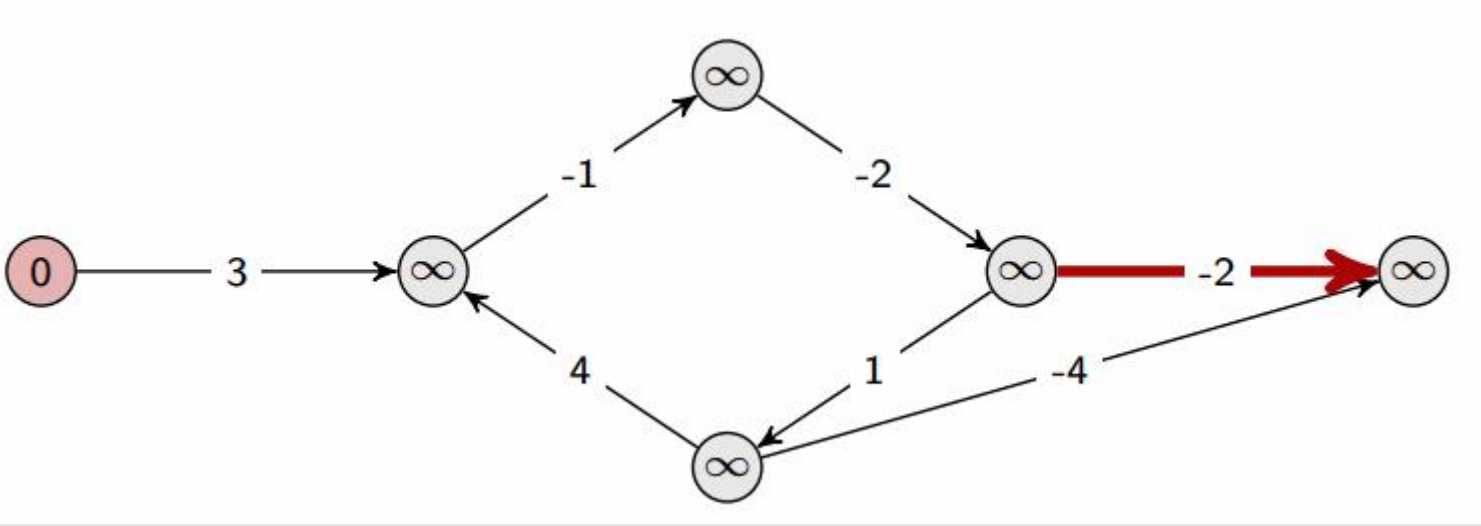
Bellman Ford



Bellman Ford



Bellman Ford



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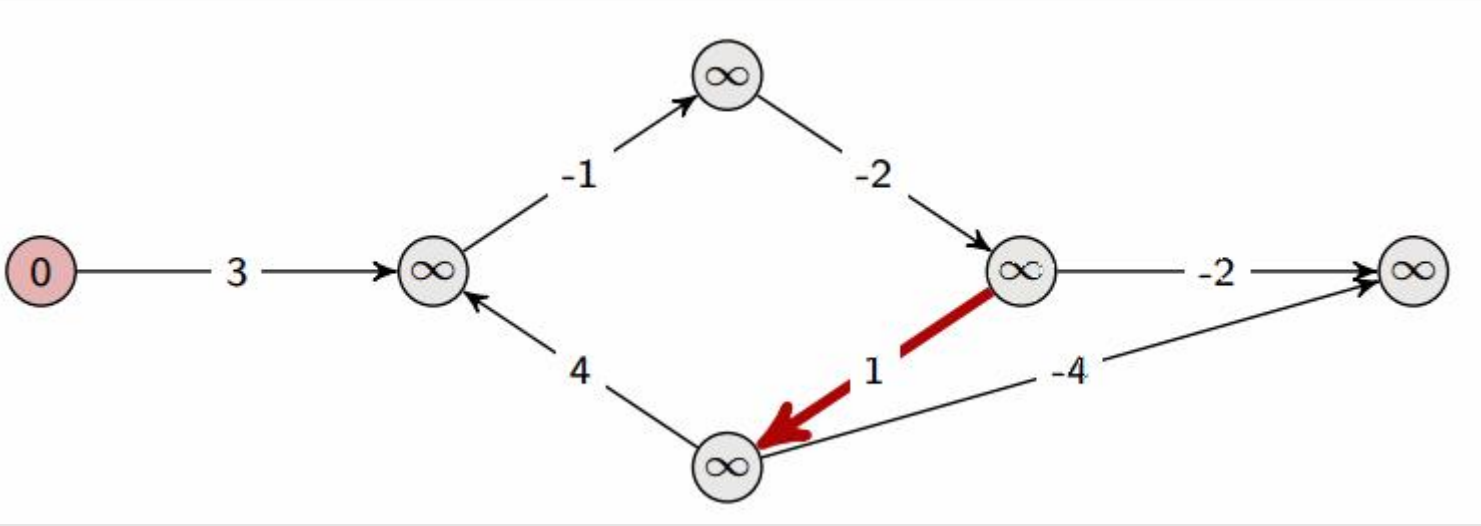

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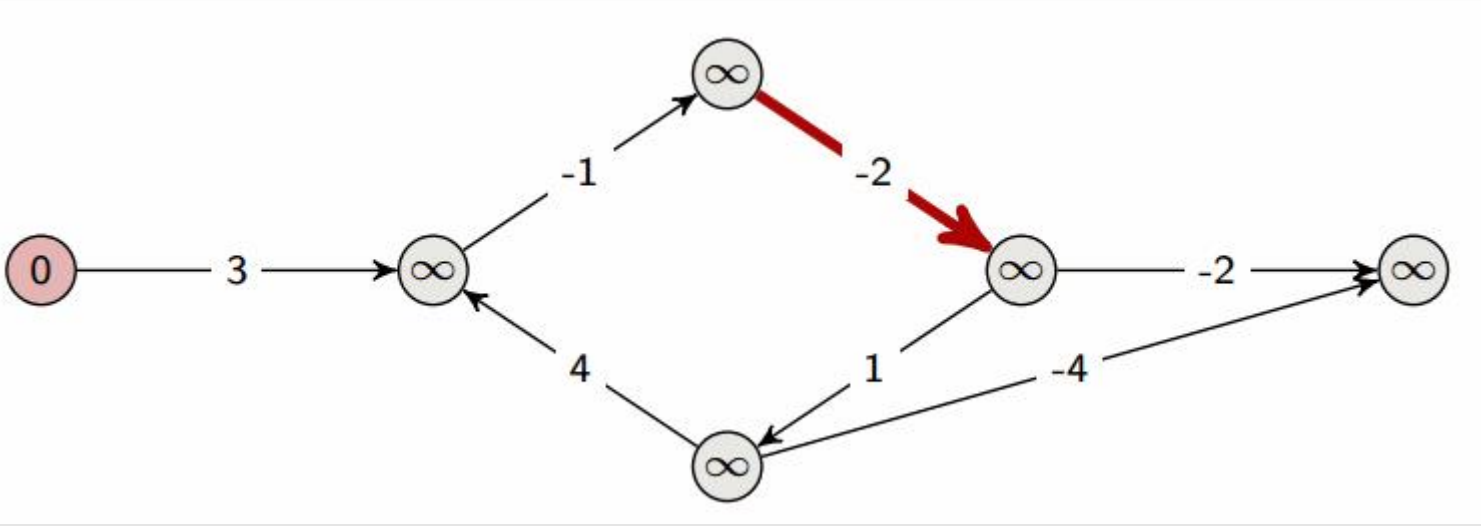

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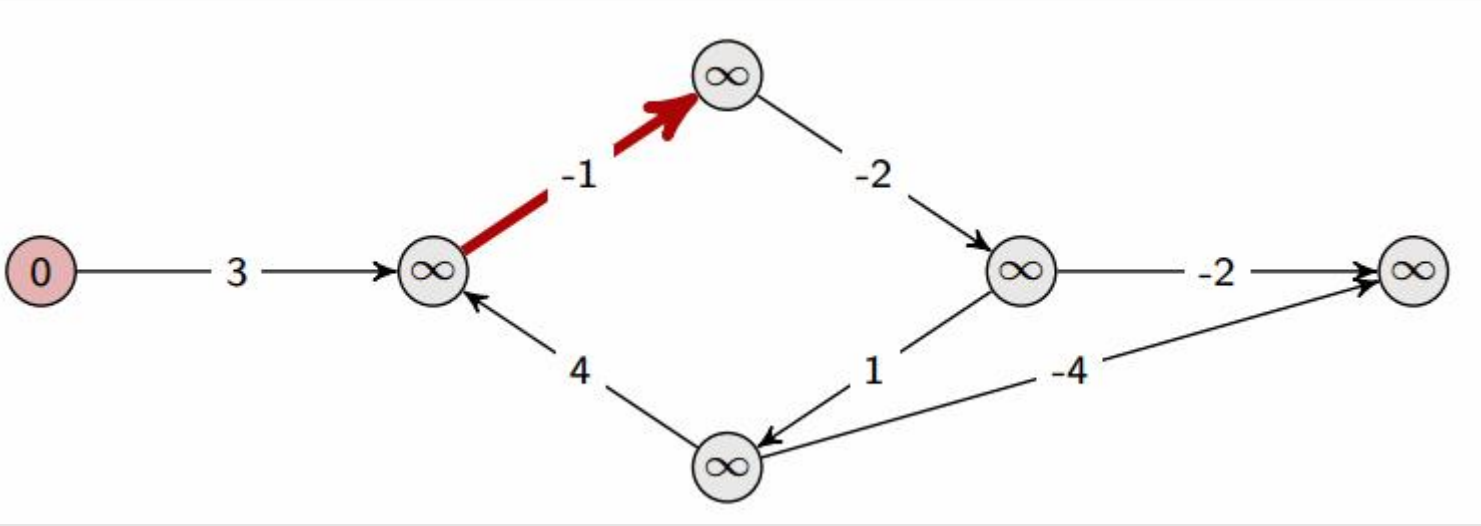

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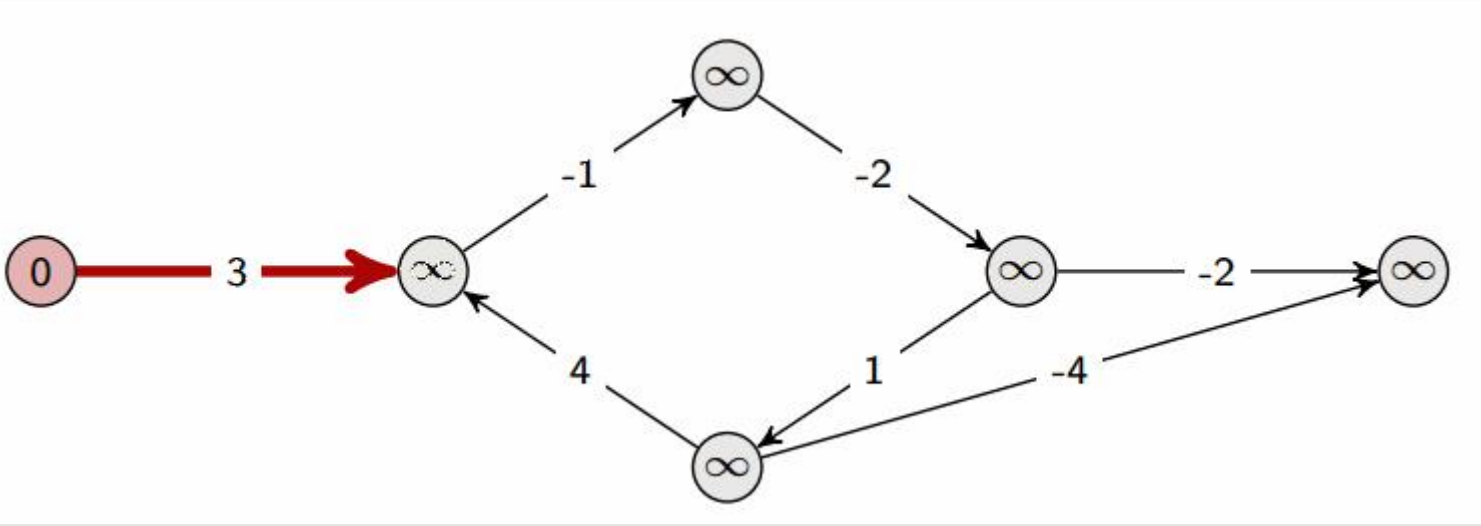

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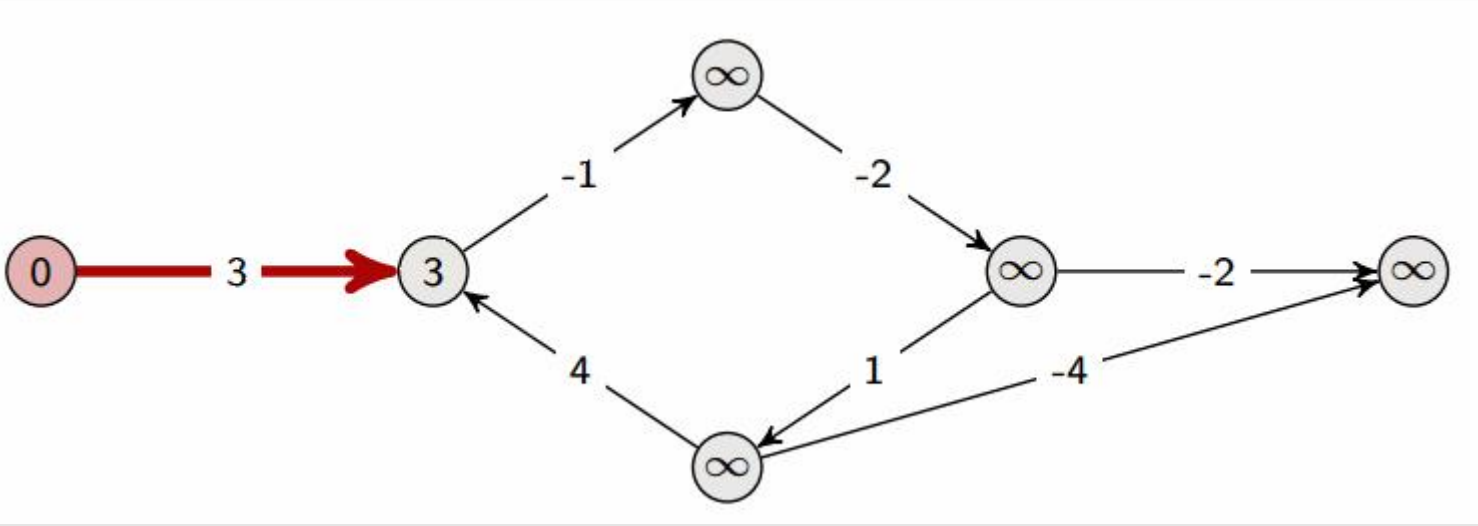

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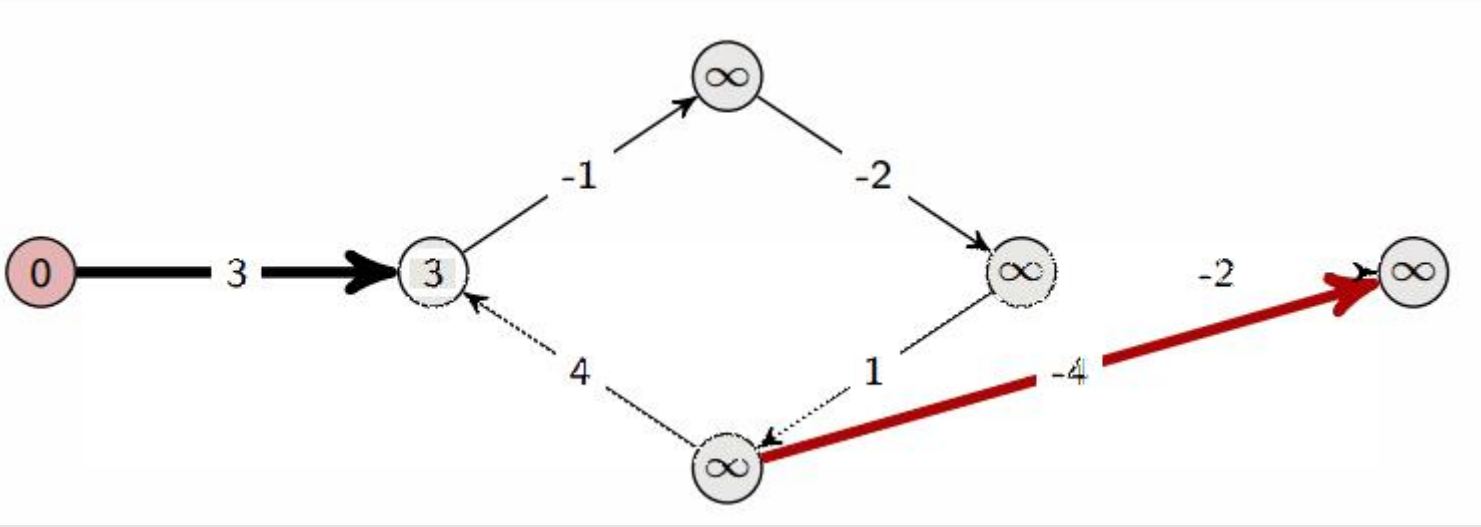

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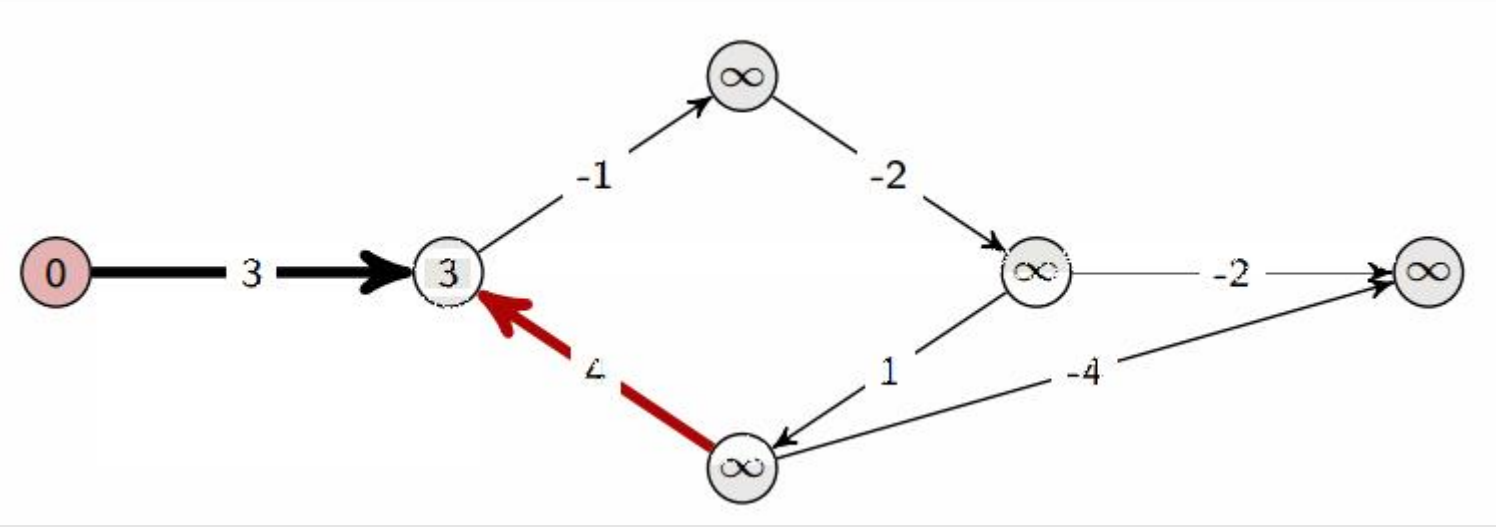

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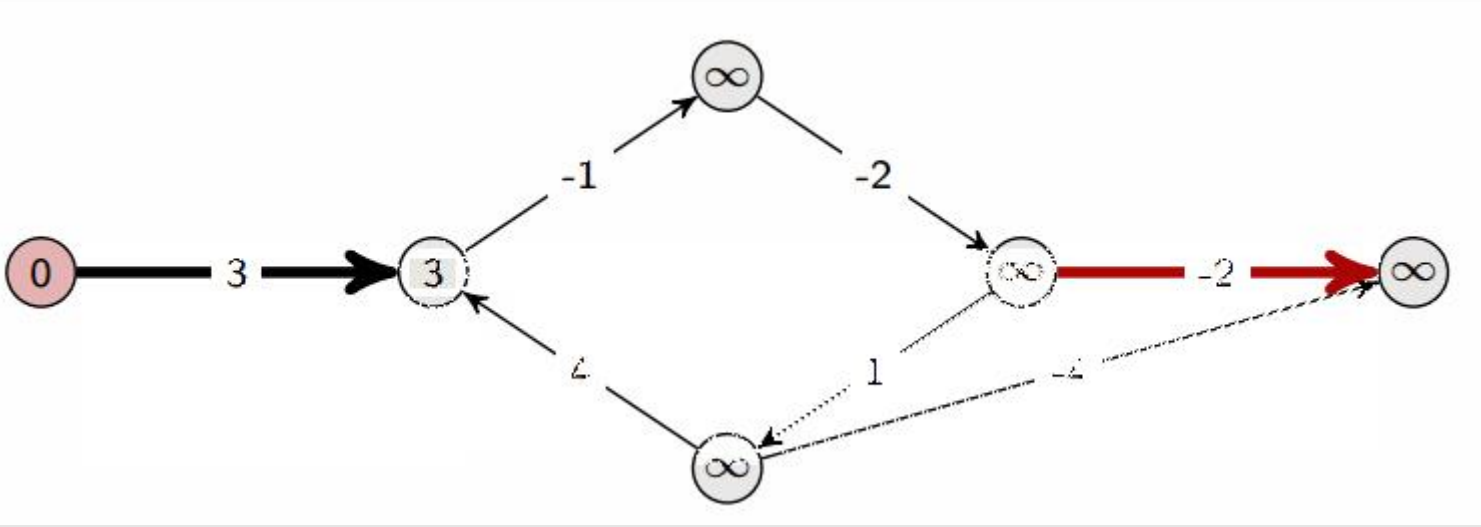

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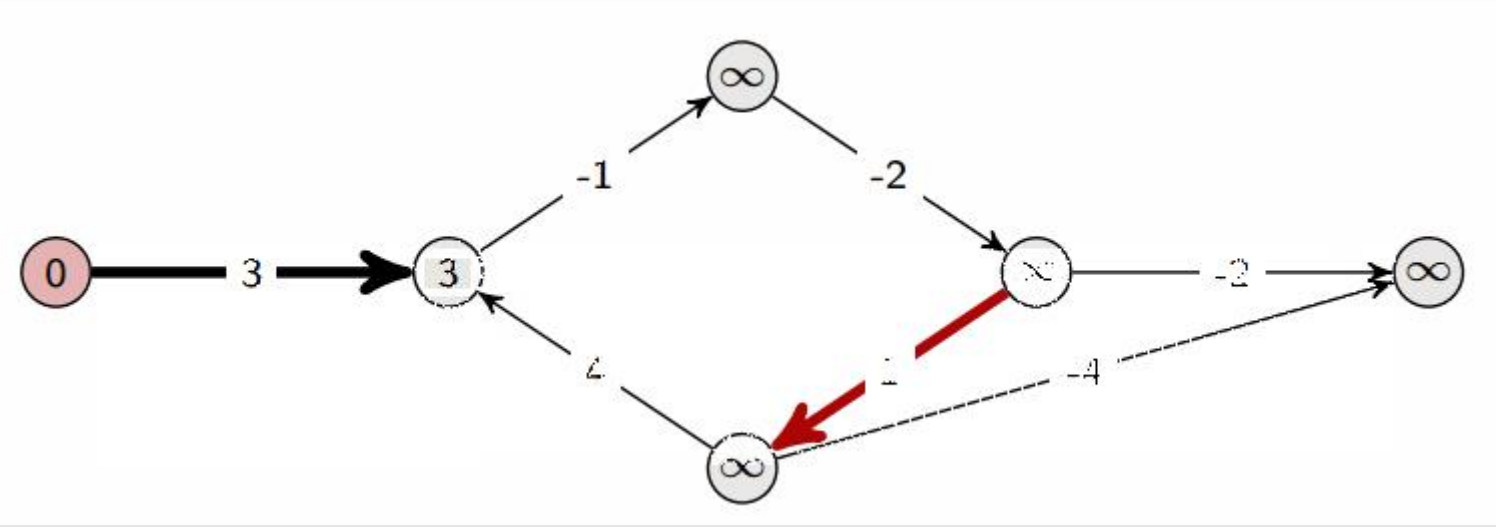

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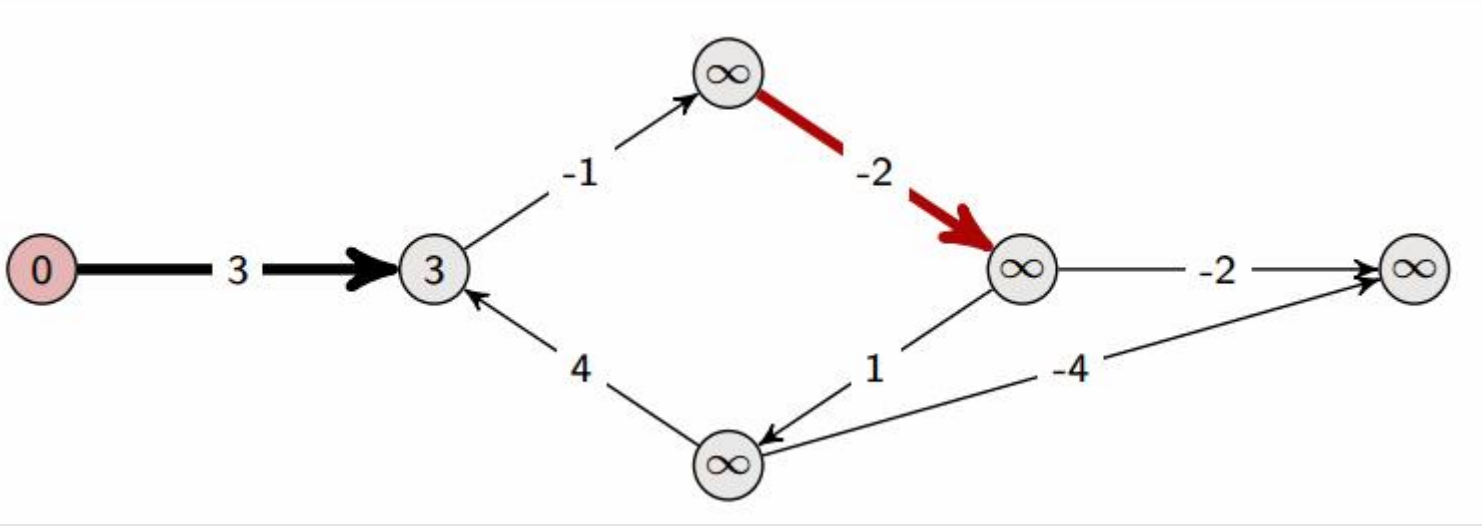

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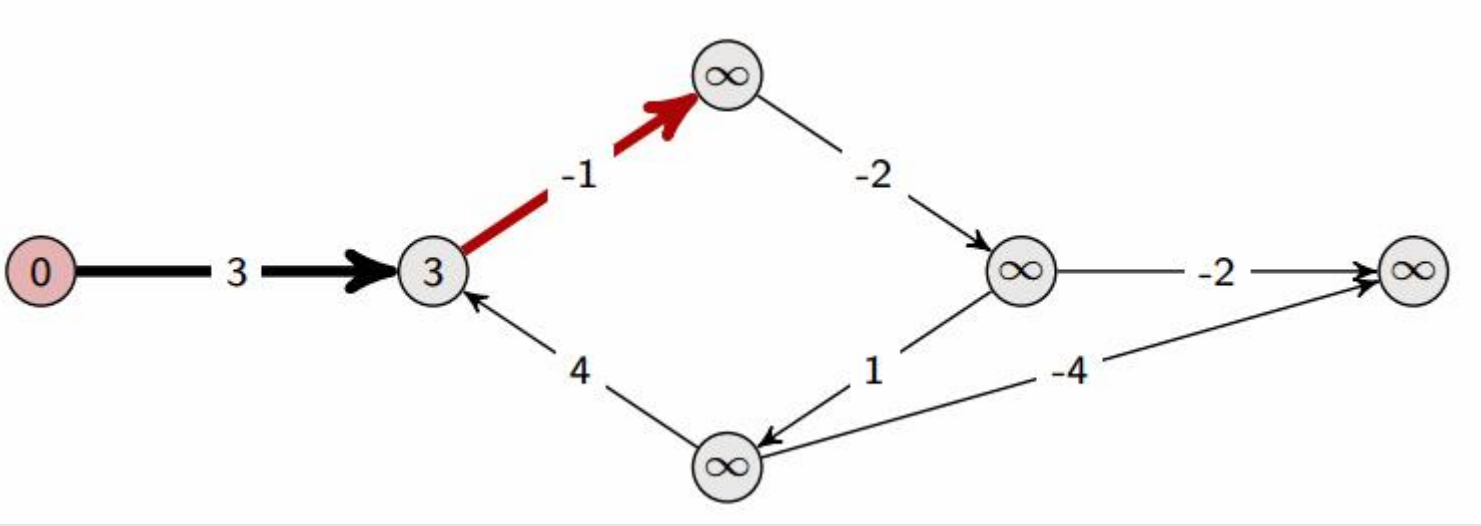

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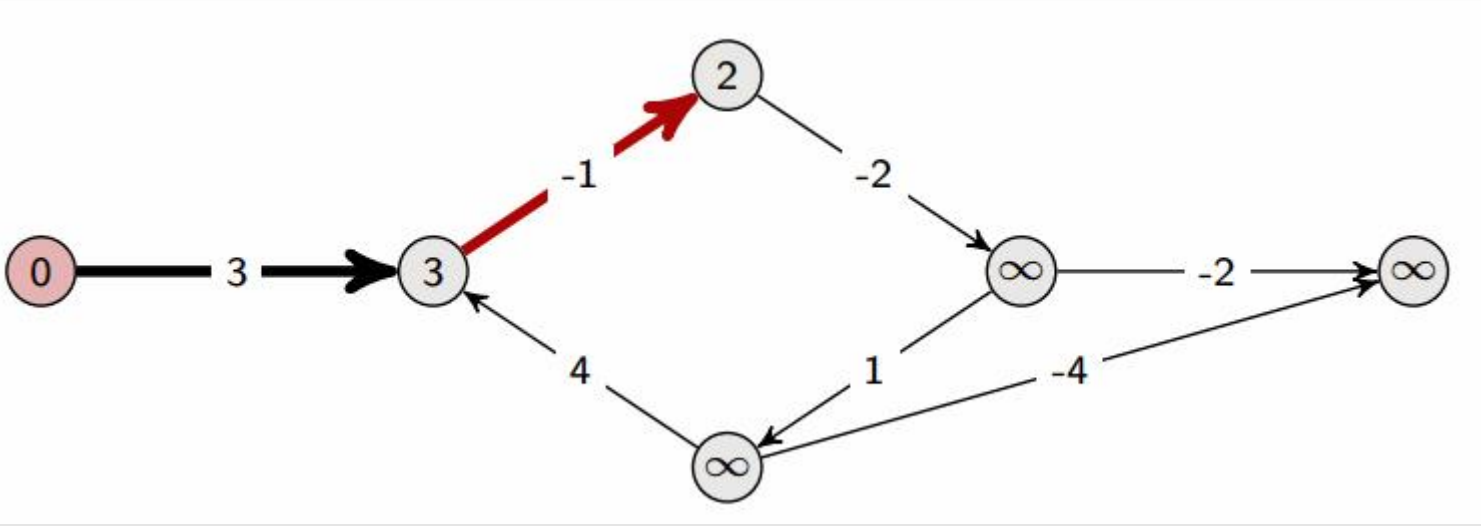

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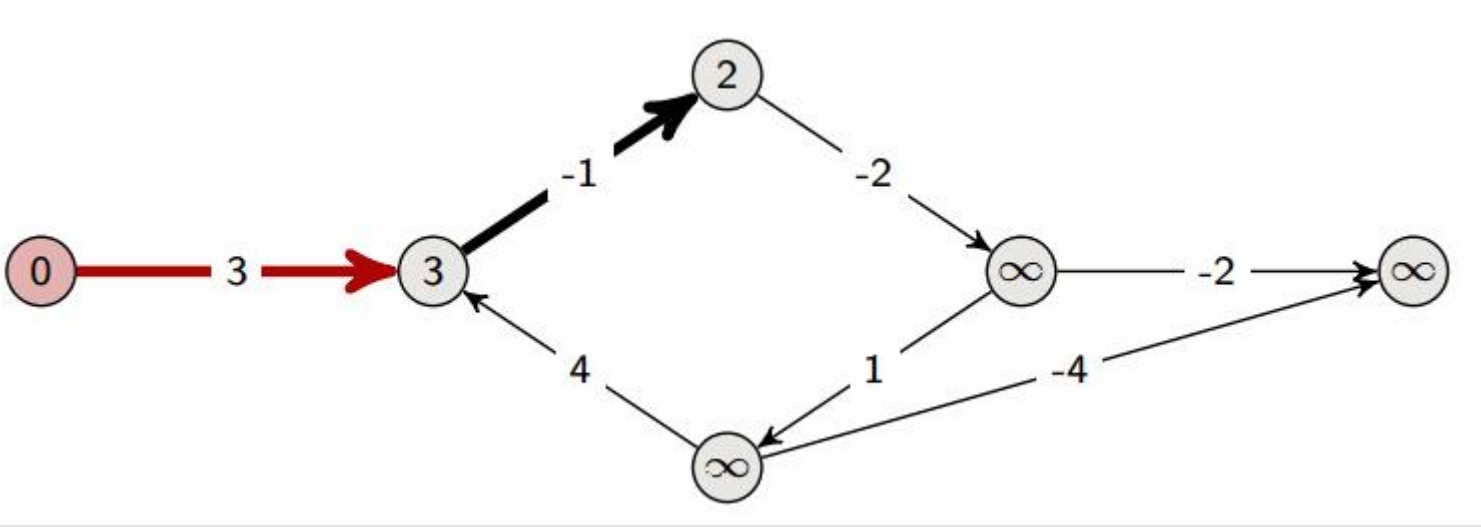

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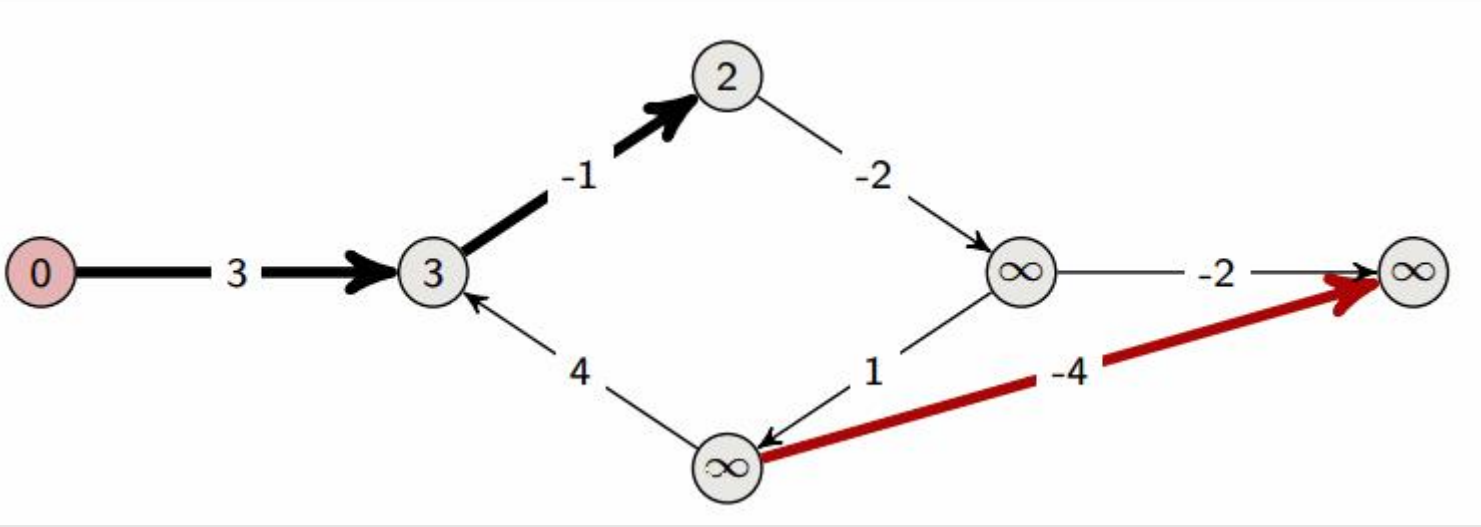

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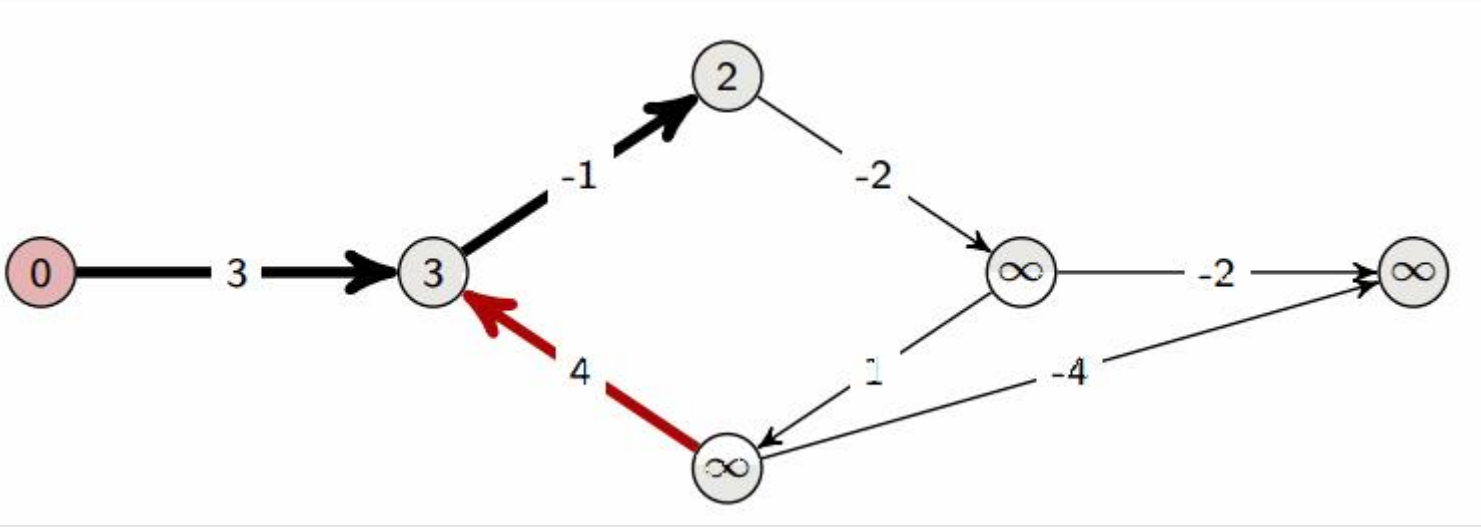

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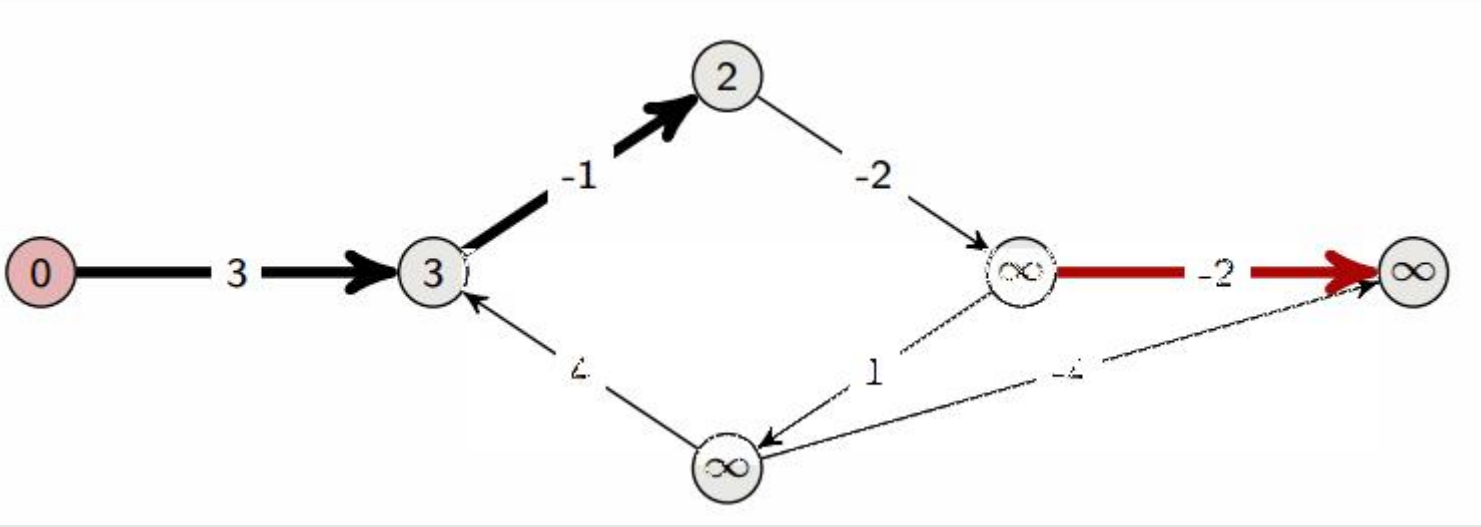

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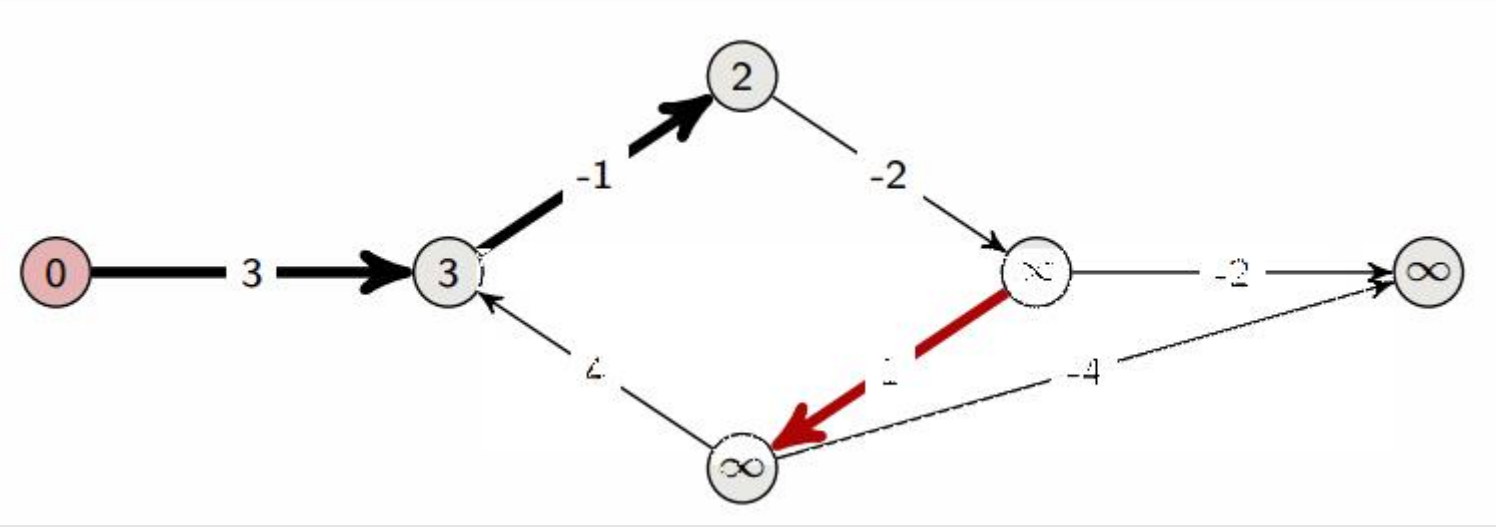

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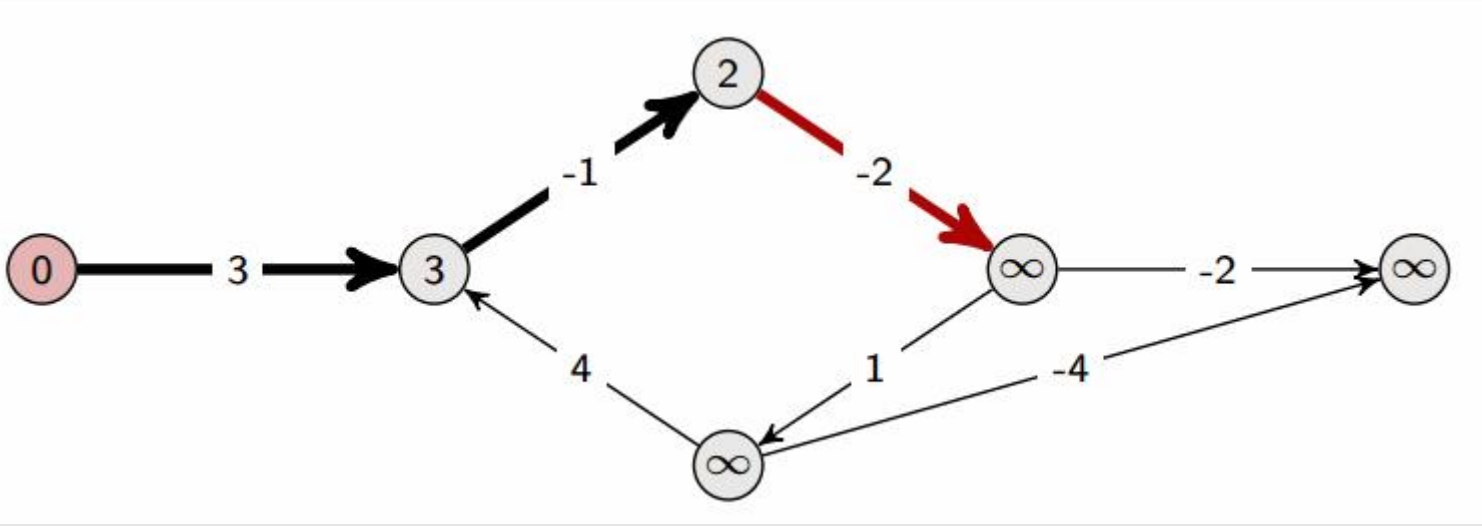

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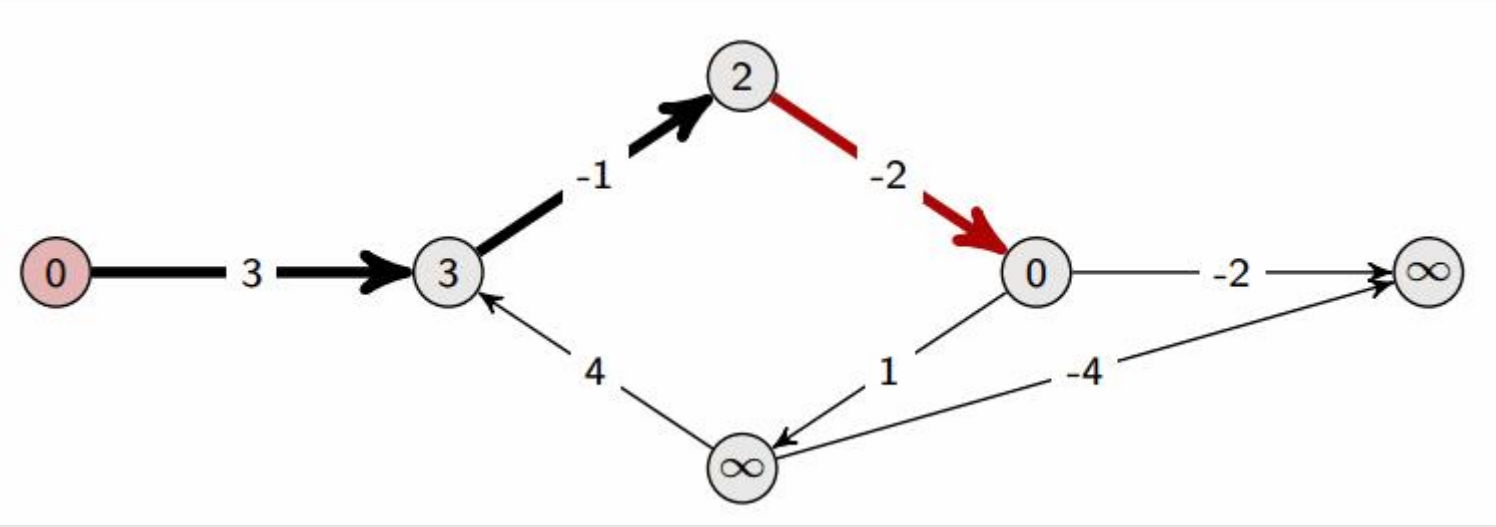

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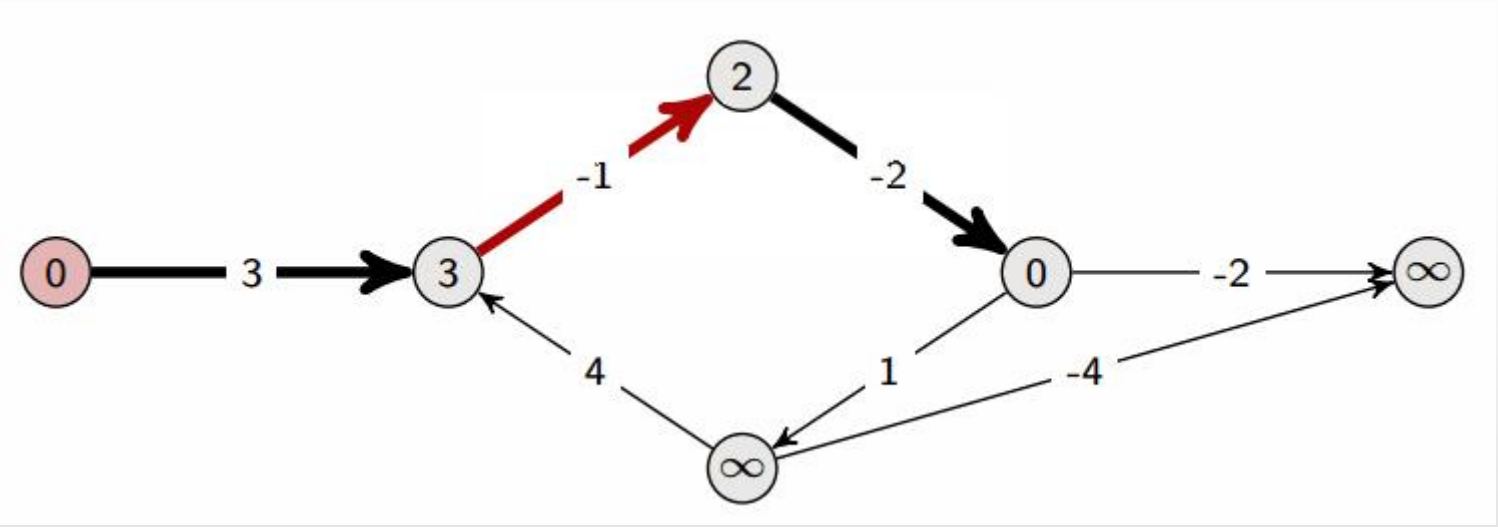

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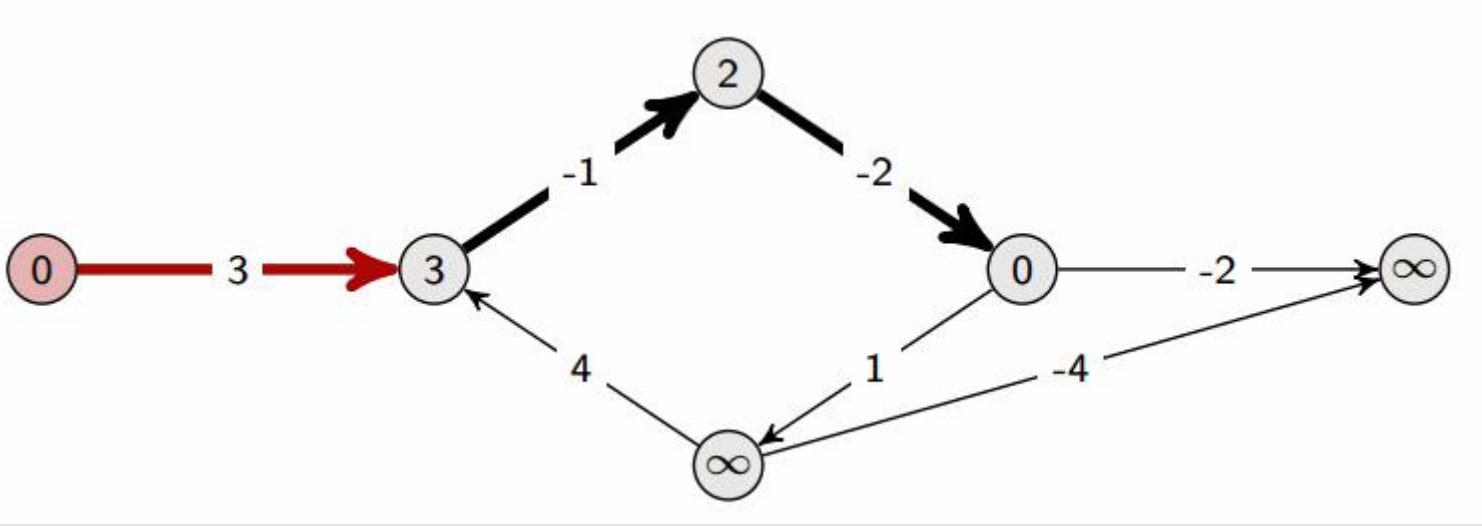

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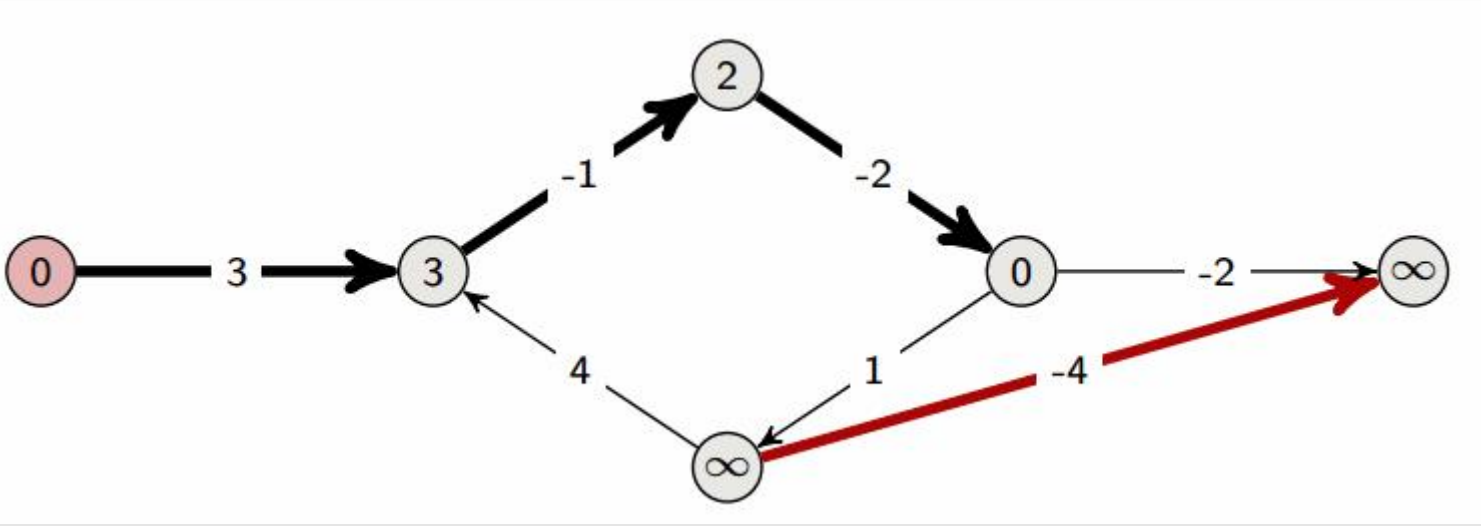

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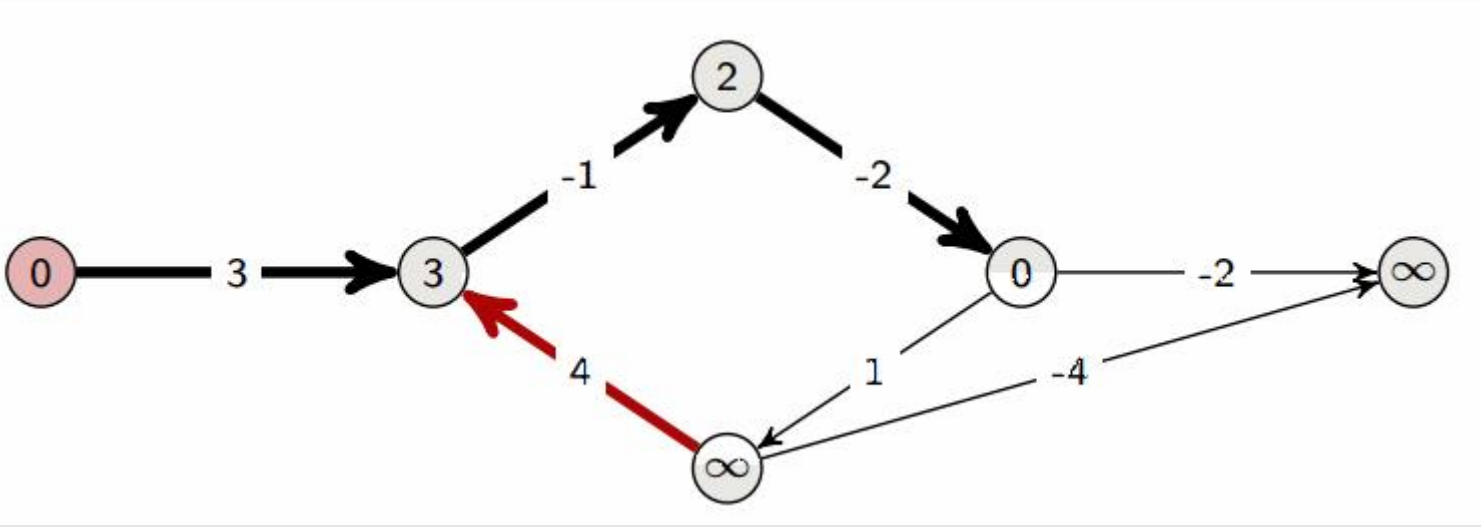

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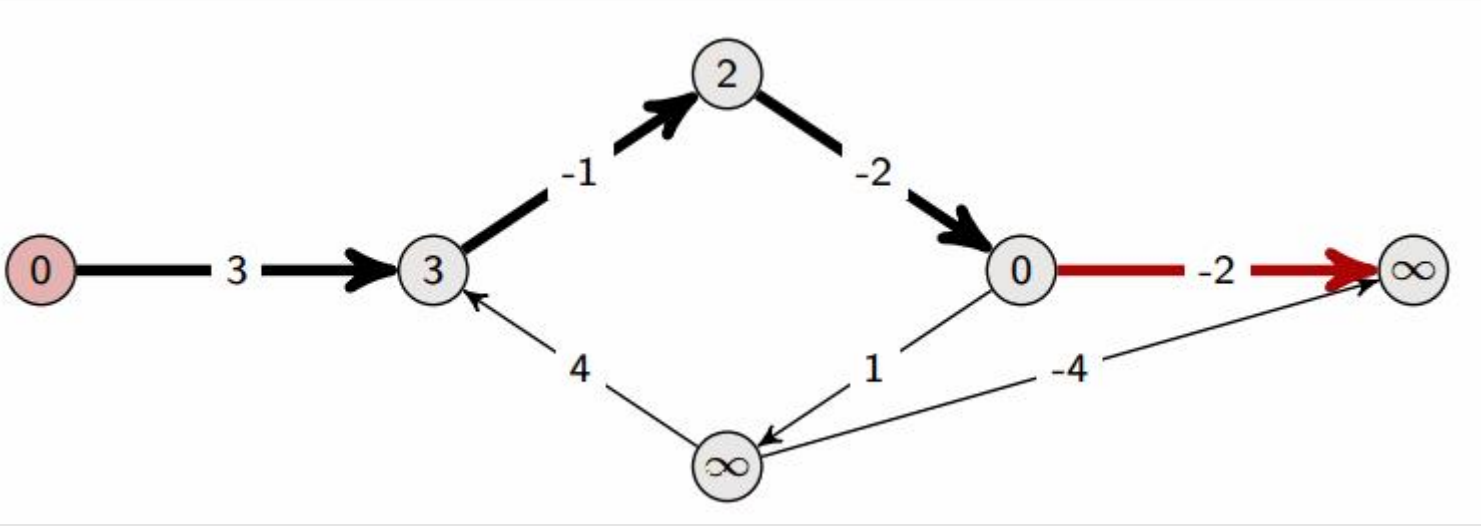

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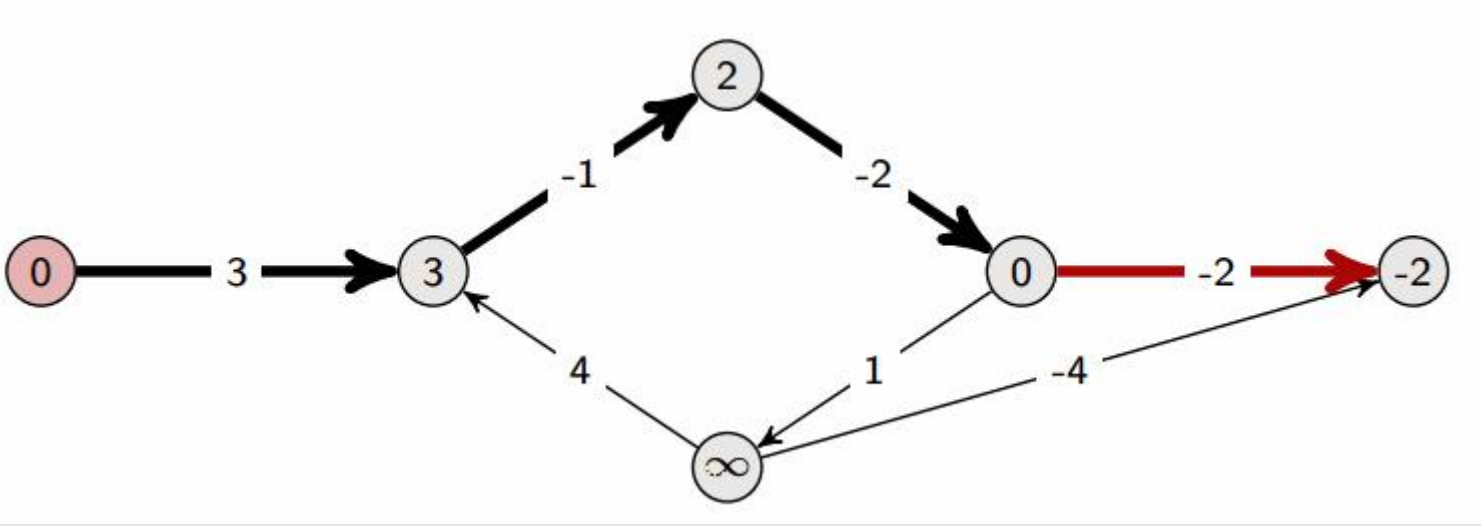

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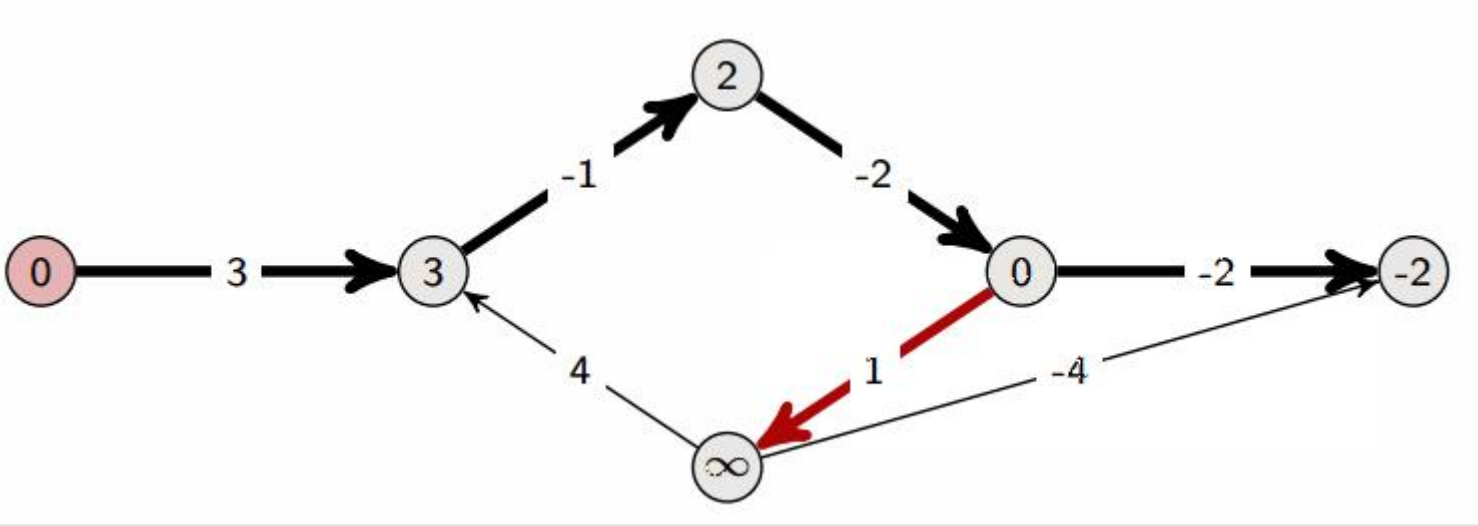

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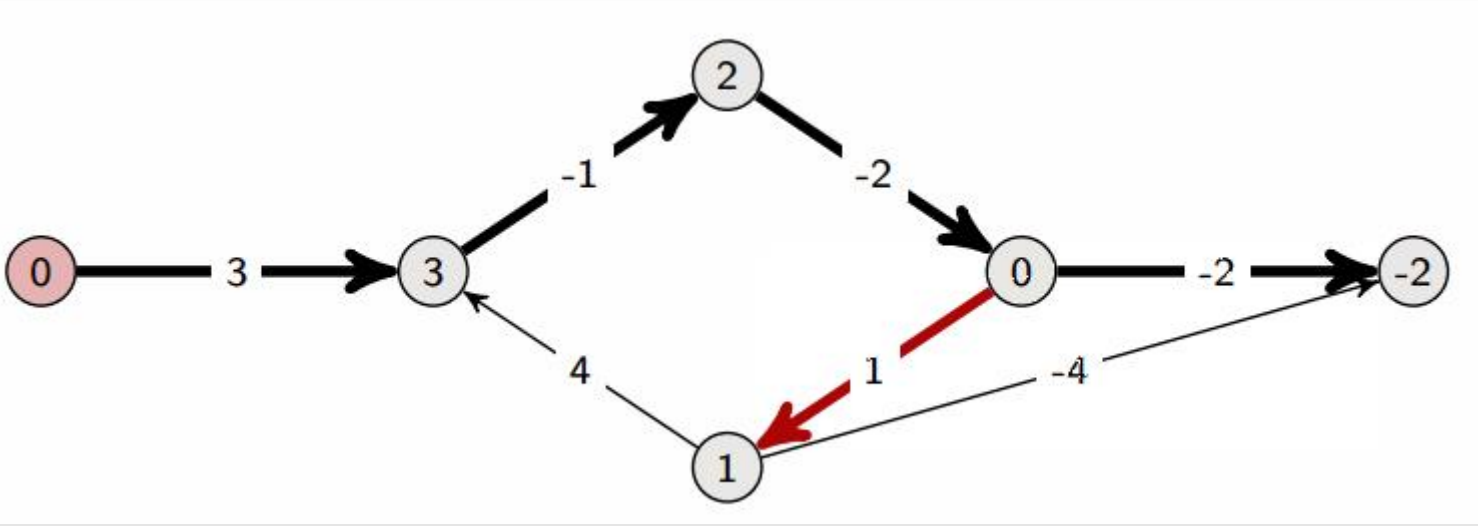

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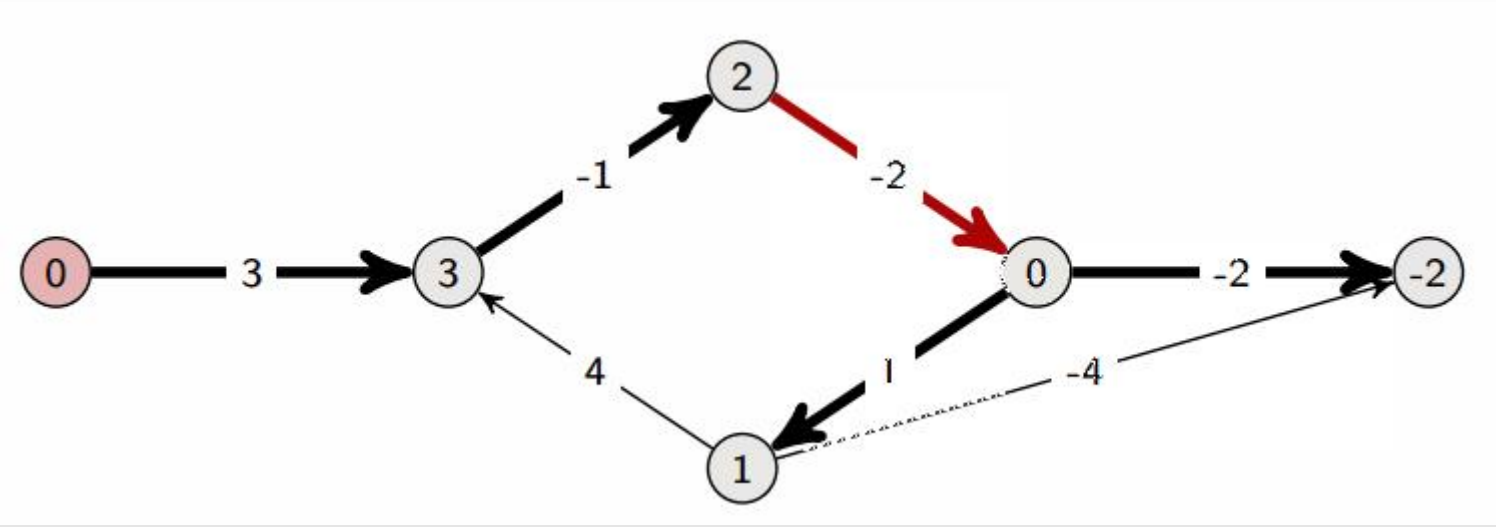

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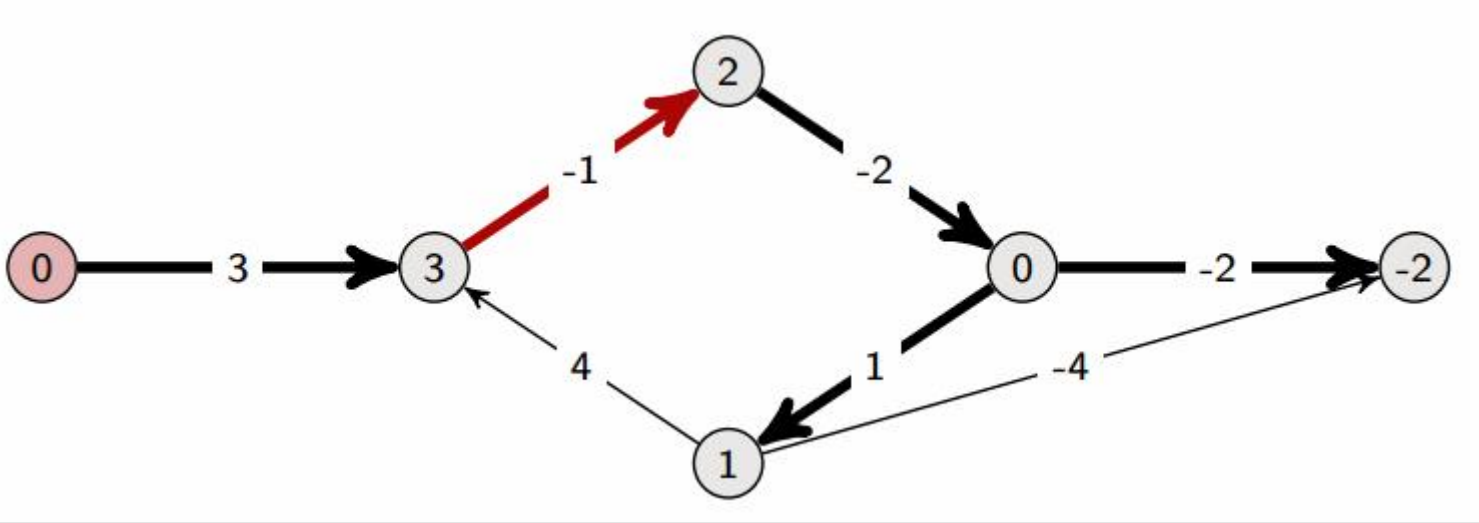

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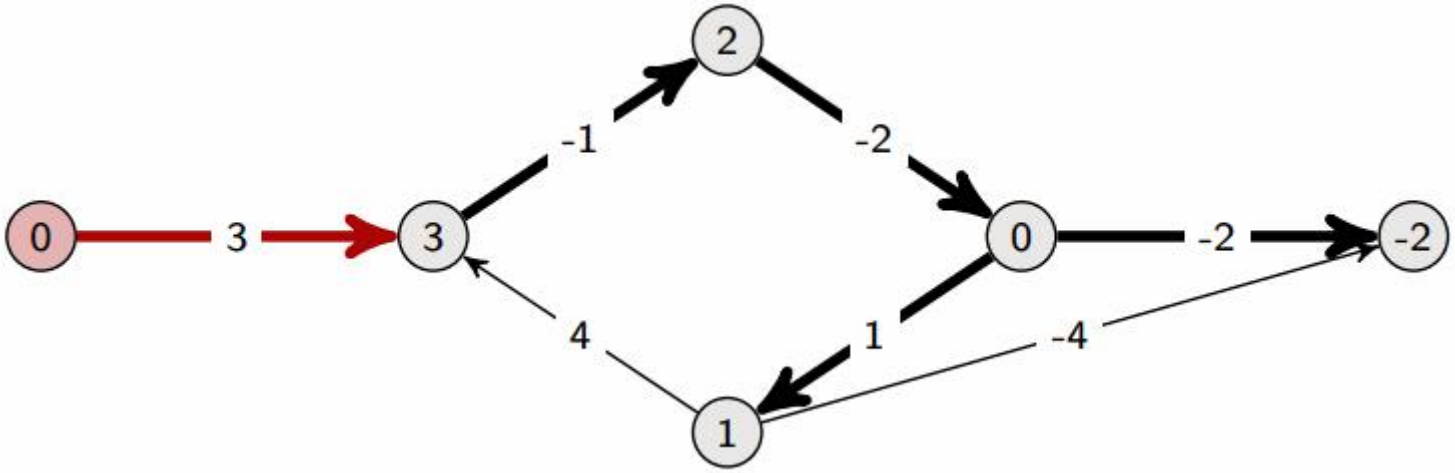

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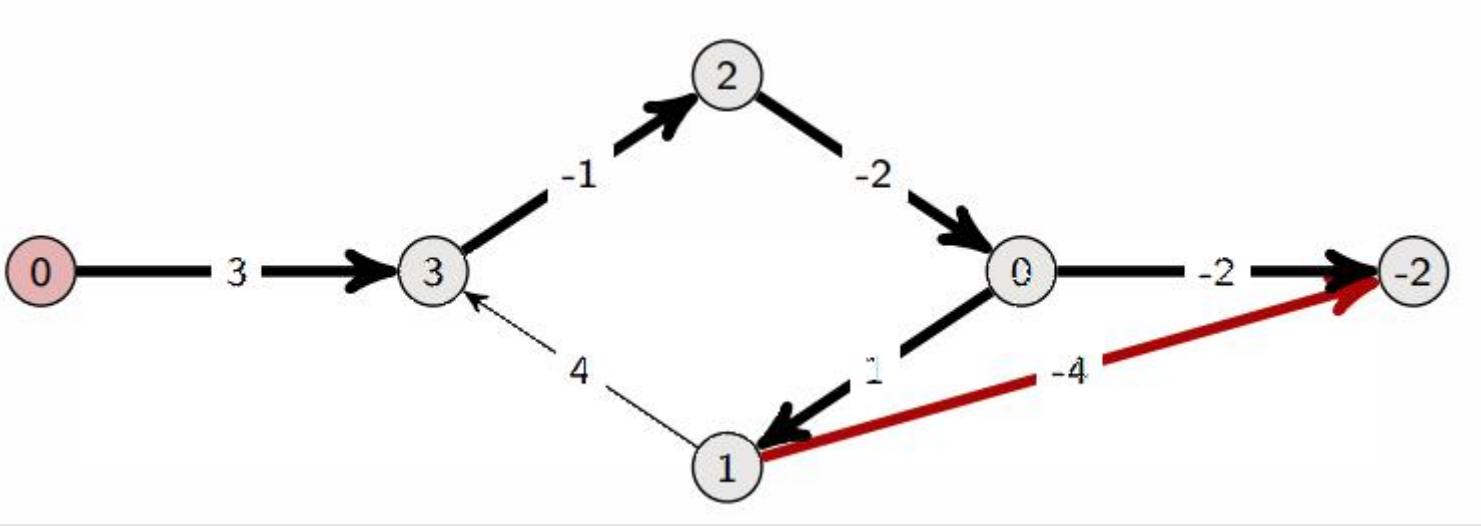

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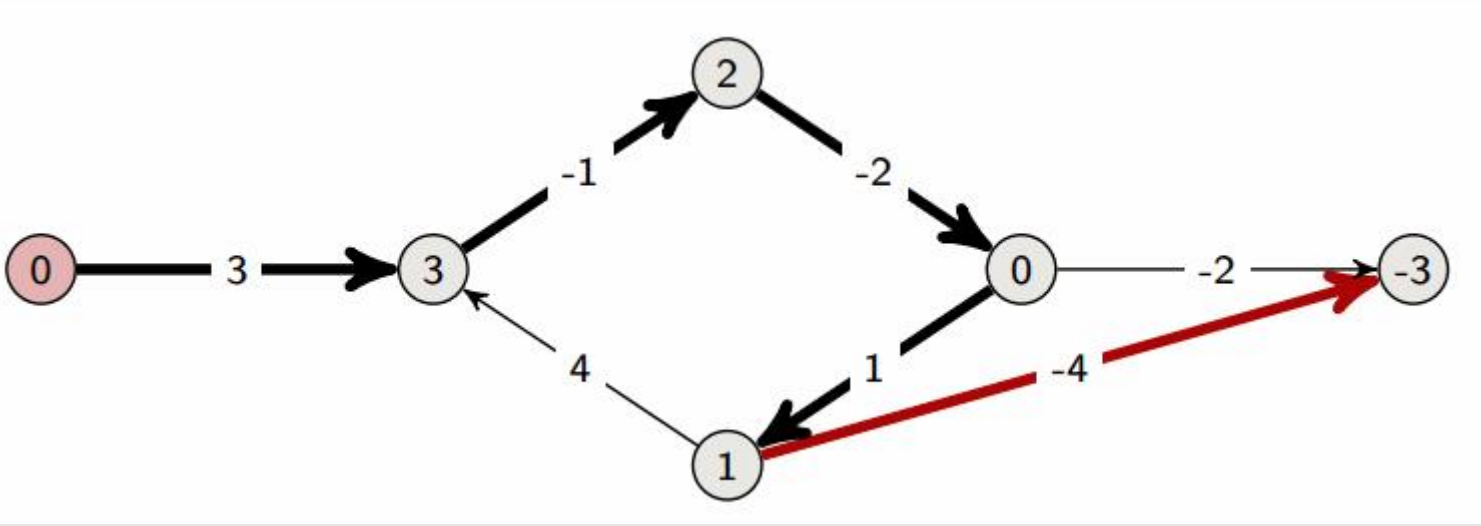

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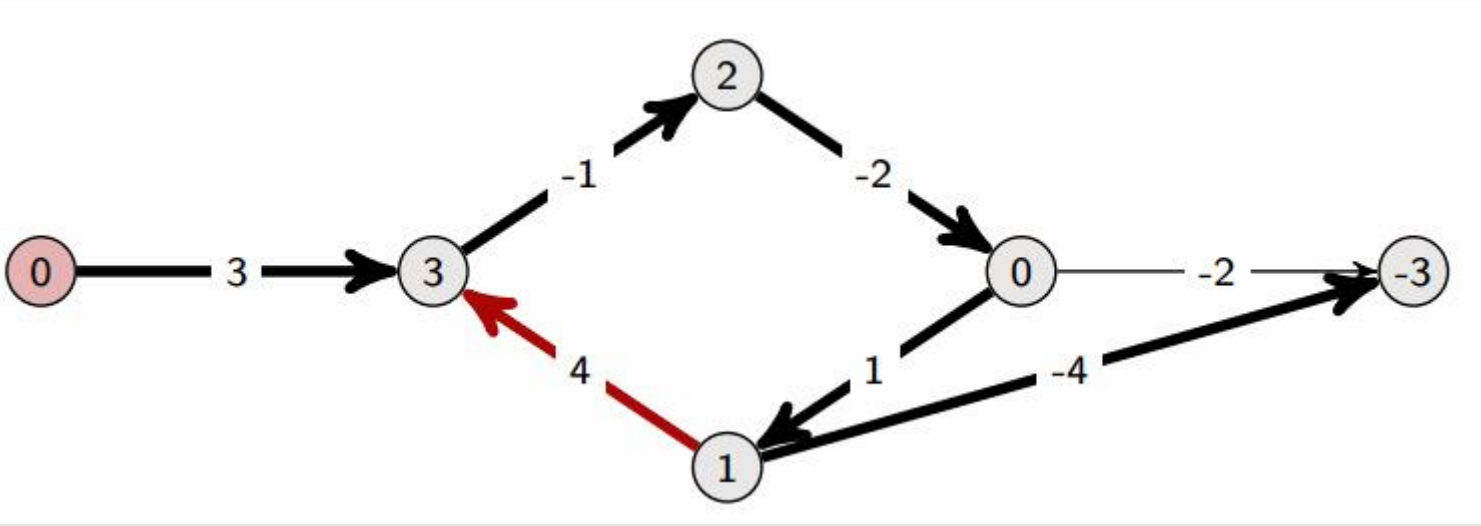

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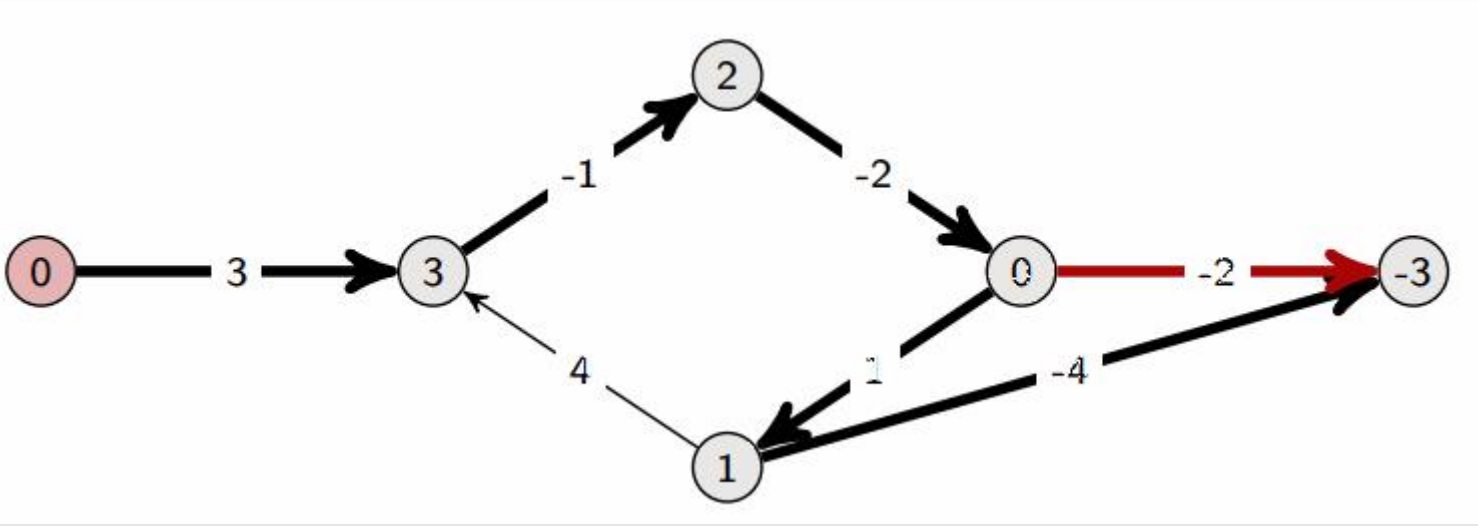

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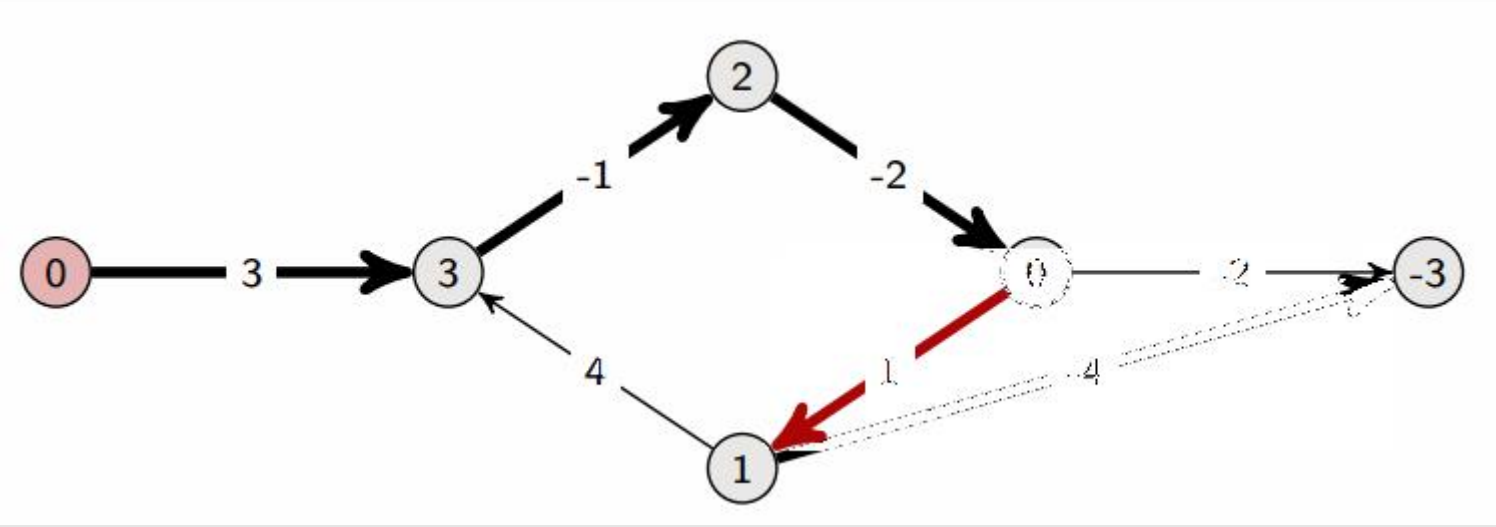

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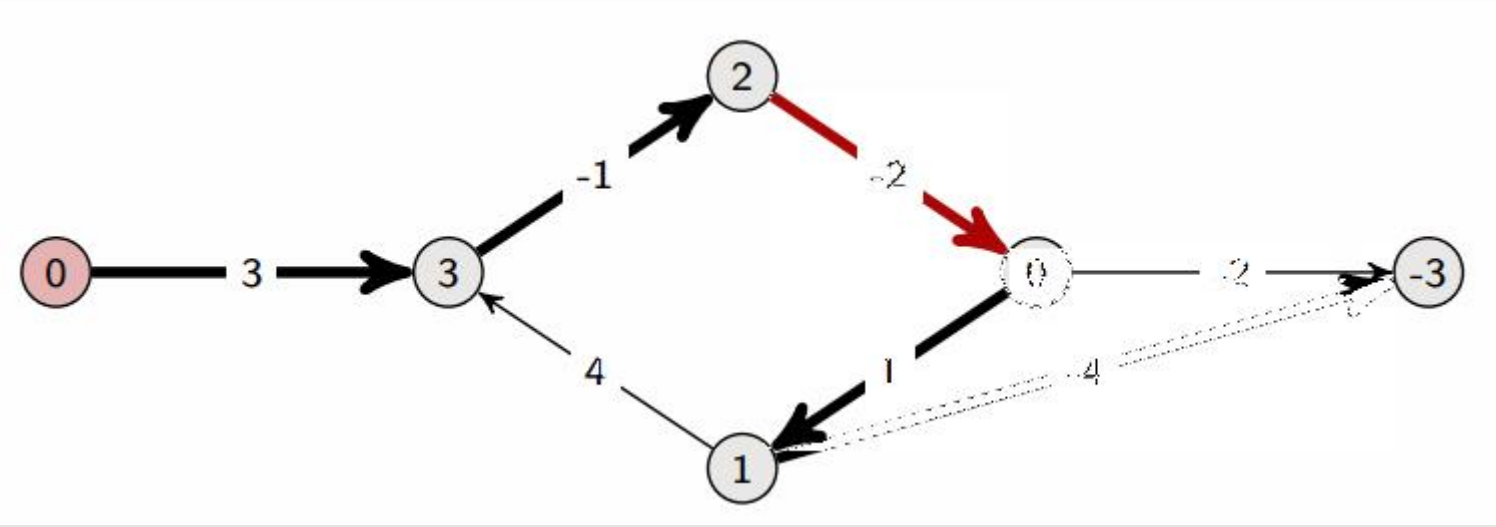

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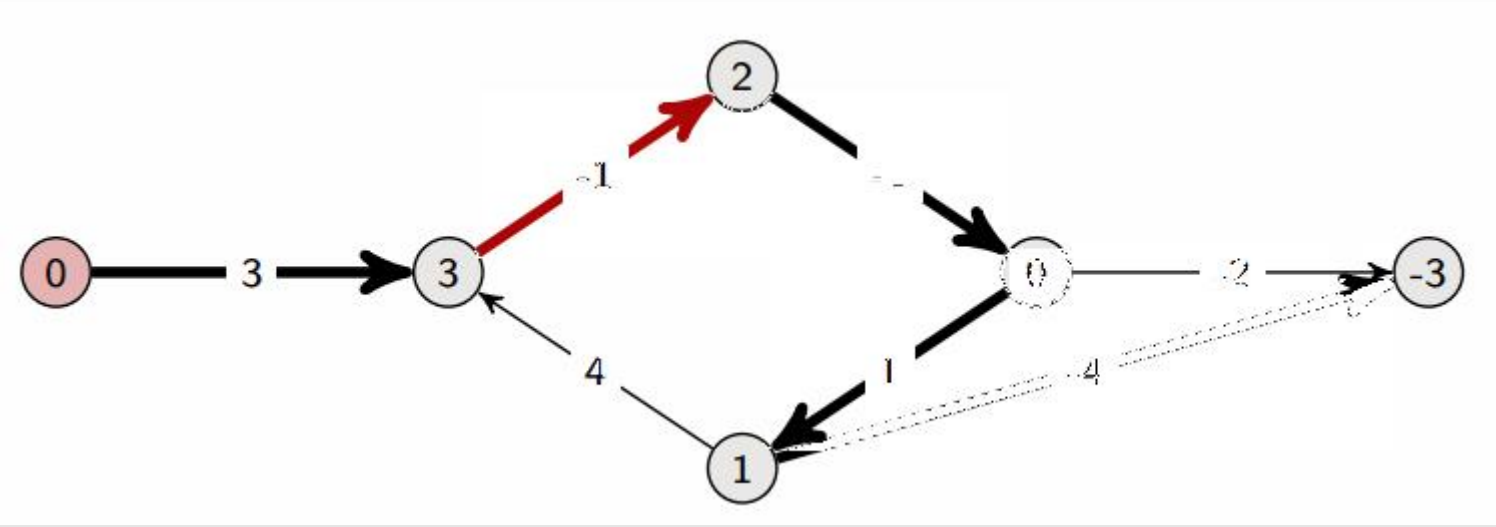

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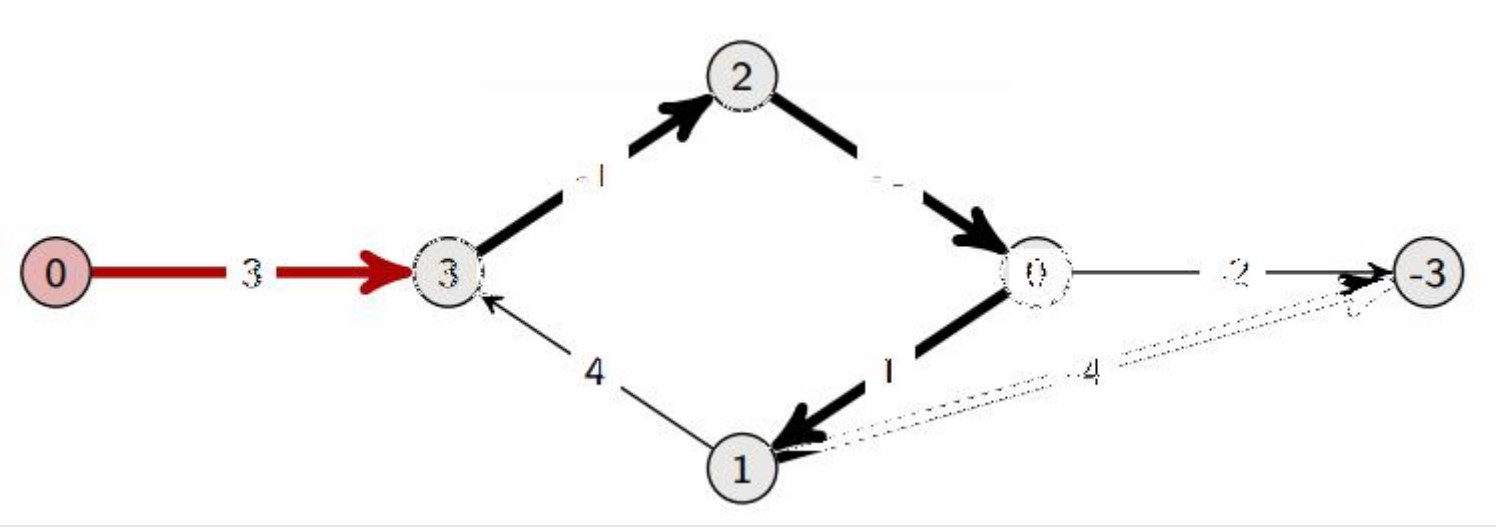

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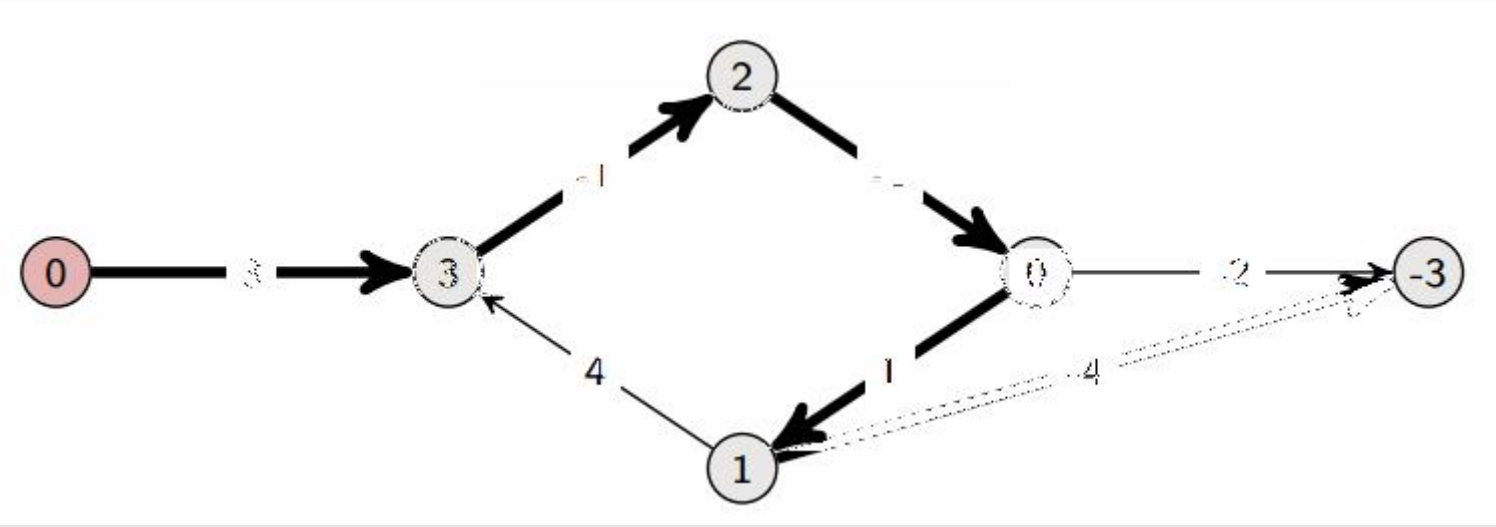
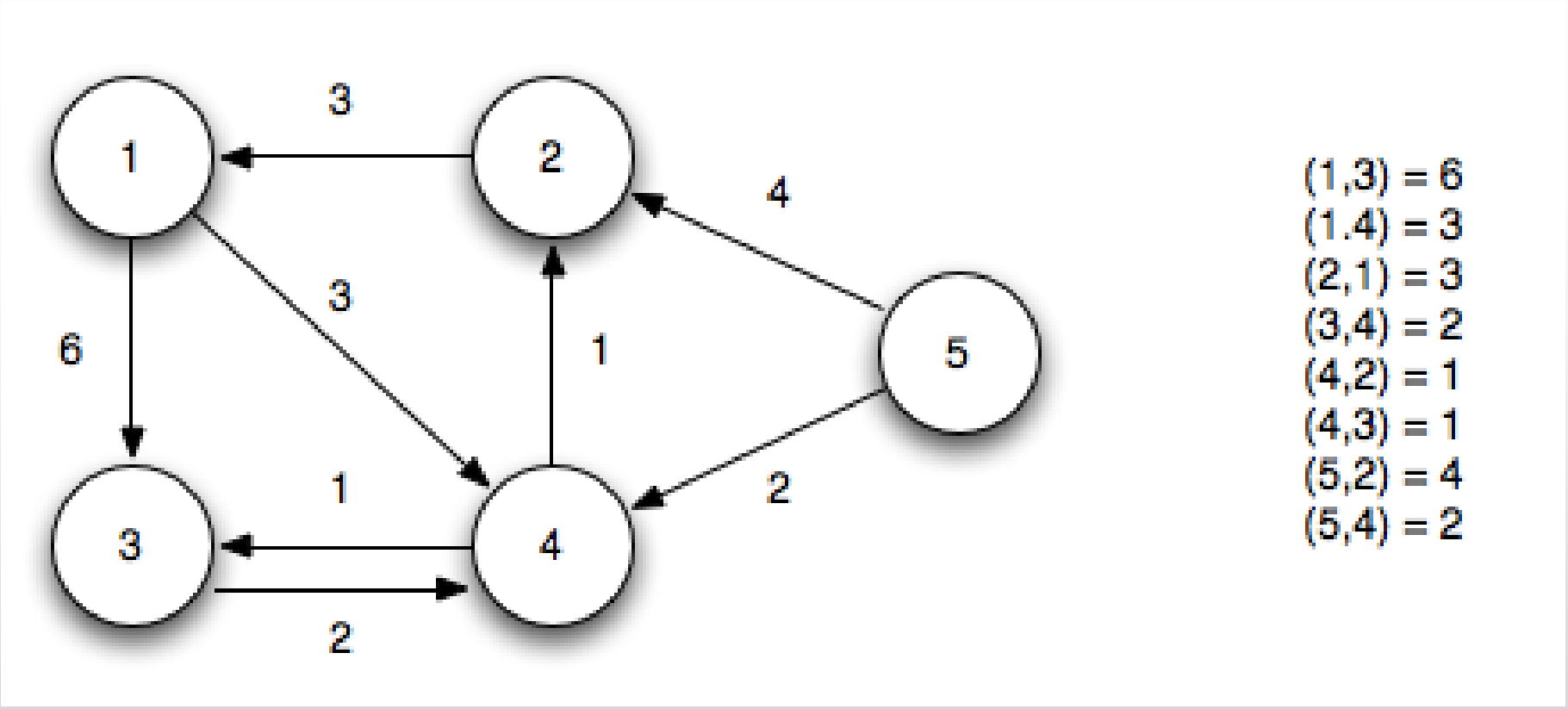
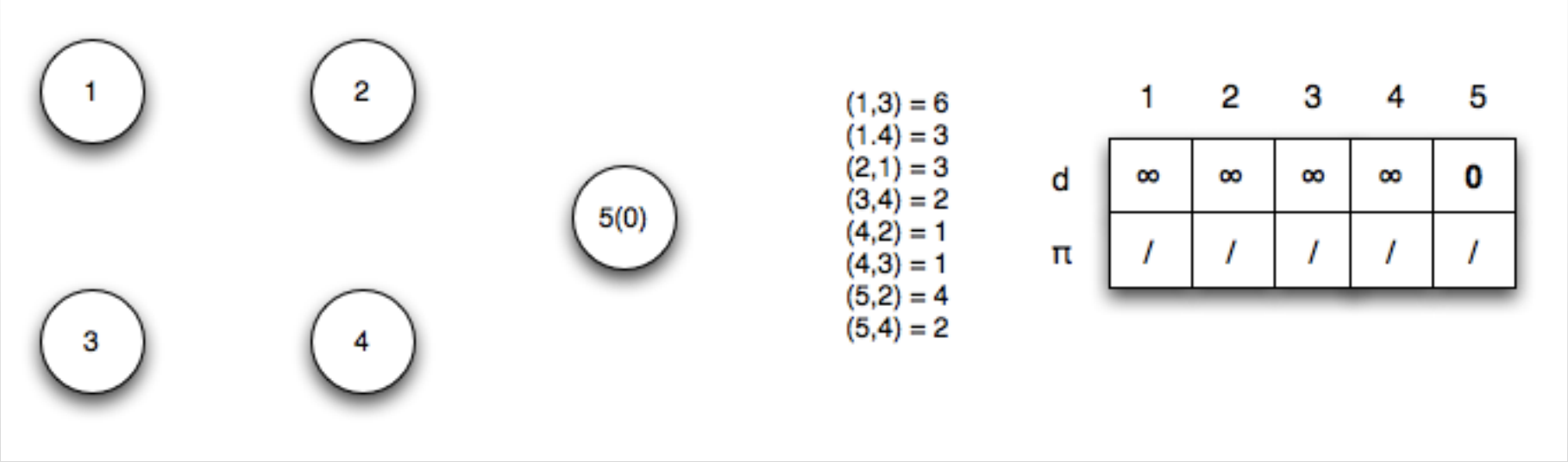


Örnek

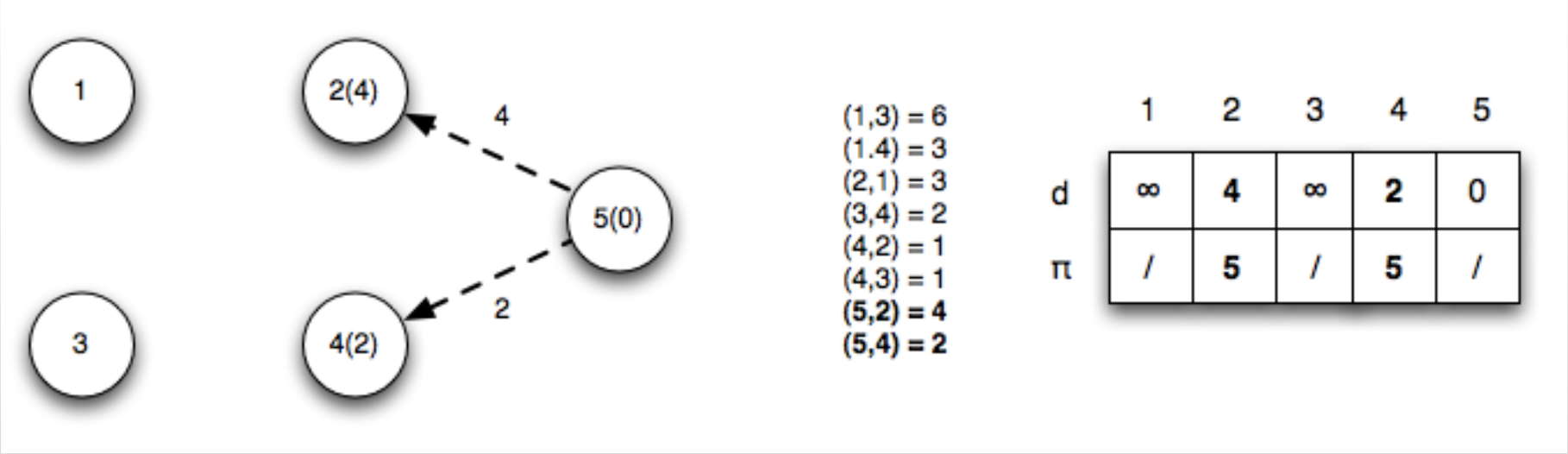


- Using vertex 5 as the source (setting its distance to 0), we initialize all the other distances to ∞.



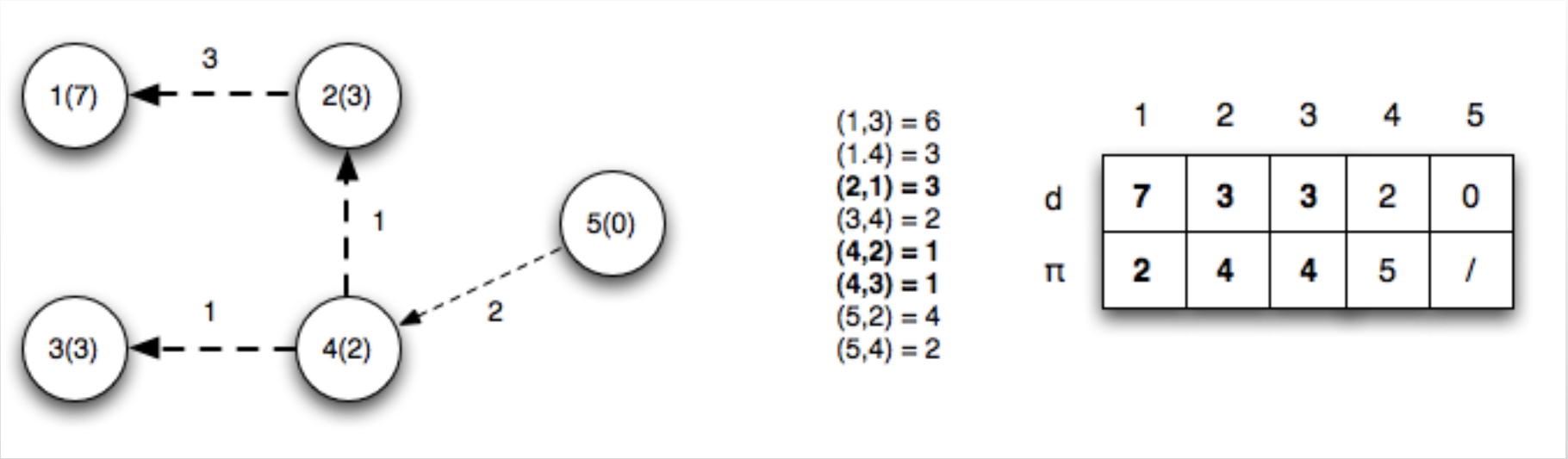
	1	2	3	4	5
d	∞	∞	∞	∞	**0**
π	/	/	/	/	/

- Iteration 1: Edges (u5,u2) and (u5,u4) relax updating the distances to 2 and 4

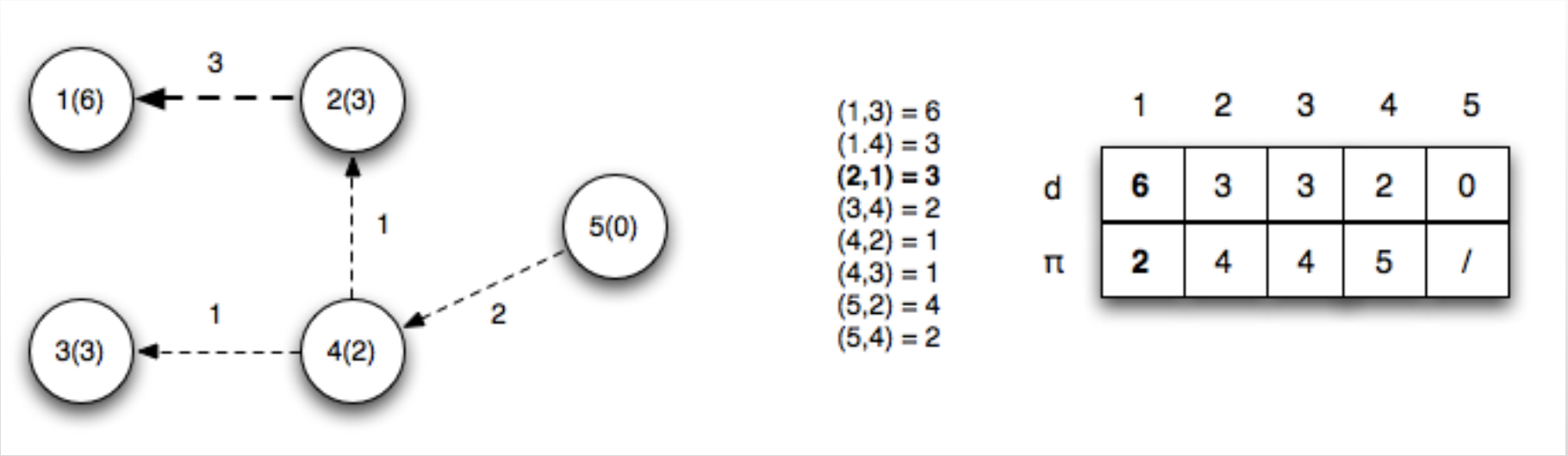


	1	2	3	4	5
d	∞	**4**	∞	**2**	0
π	/	**5**	/	**5**	/

- Iteration 2: Edges (u2,u1), (u4,u2) and (u4,u3) relax updating the distances to 1, 2, and 4 respectively. Note edge (u4,u2) finds a shorter path to vertex 2 by going through vertex 4

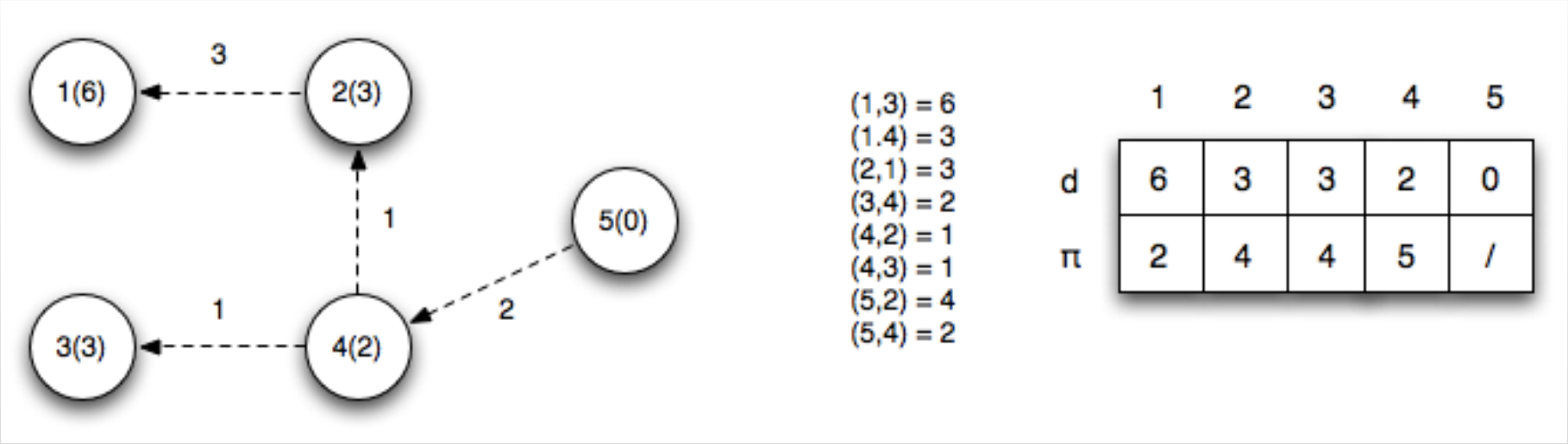


- Iteration 3: Edge (u2,u1) relaxes (since a shorter path to vertex 2 was found in the previous iteration) updating the distance to 1



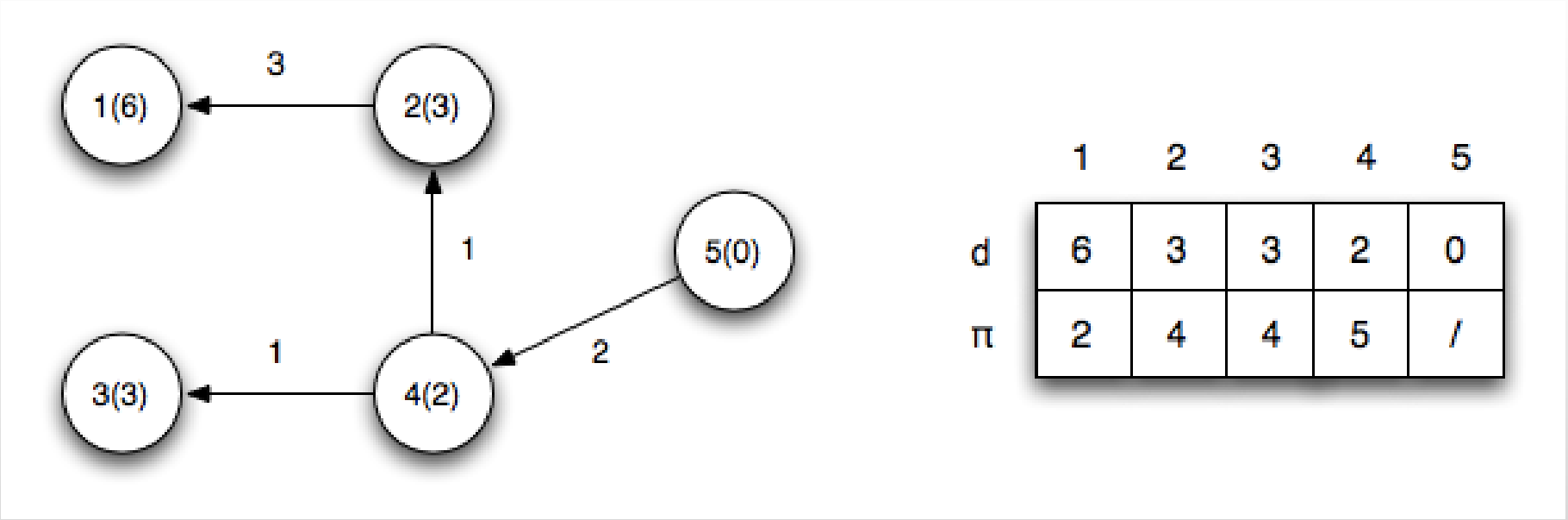
	1	2	3	4	5
d	**6**	3	3	2	0
π	**2**	4	4	5	/

- Iteration 4: No edges relax



	1	2	3	4	5
d	6	3	3	2	0
π	2	4	4	5	/

- The final shortest paths from vertex 5 with corresponding distances is



	1	2	3	4	5
d	6	3	3	2	0
π	2	4	4	5	/

- Negative cycle checks: We now check the relaxation condition one additional time for each edge. If any of the checks pass then there exists a negative weight cycle in the graph.

$$v_3.d > u_1.d + w(1,3) \Rightarrow 4 \not> 6 + 6 = 12 \checkmark$$

$$v_4.d > u_1.d + w(1,4) \Rightarrow 2 \not> 6 + 3 = 9 \checkmark$$

$$v_1.d > u_2.d + w(2,1) \Rightarrow 6 \not> 3 + 3 = 6 \checkmark$$

$$v_4.d > u_3.d + w(3,4) \Rightarrow 2 \not> 3 + 2 = 5 \checkmark$$

$$v_2.d > u_4.d + w(4,2) \Rightarrow 3 \not> 2 + 1 = 3 \checkmark$$

$$v_3.d > u_4.d + w(4,3) \Rightarrow 3 \not> 2 + 1 = 3 \checkmark$$

$$v_2.d > u_5.d + w(5,2) \Rightarrow 3 \not> 0 + 4 = 4 \checkmark$$

$$v_4.d > u_5.d + w(5,4) \Rightarrow 2 \not> 0 + 2 = 2 \checkmark$$

Floyd Warshall

- Tüm düğüm çiftleri arasındaki en kısa yolları bulur.
- 1959'da Robert Floyd tarafından bulunmuştur.
- 1962'de Stephen Warshall tarafından geliştirilmiştir.
- Negatif ağırlıklı kenarlar ve döngülerle başa çıkabilir.

Algoritma İlkeleri

- Dinamik programlama yöntemini kullanır.
- Bir matris kullanarak tüm düğümler arasındaki en kısa mesafeleri bulur.
- Bellman-Ford ve Dijkstra tek kaynaktan düğümlere en kısa yolları bulur.
- Floyd Warshall, tüm çiftler arasındaki en kısa yolları hesaplar.

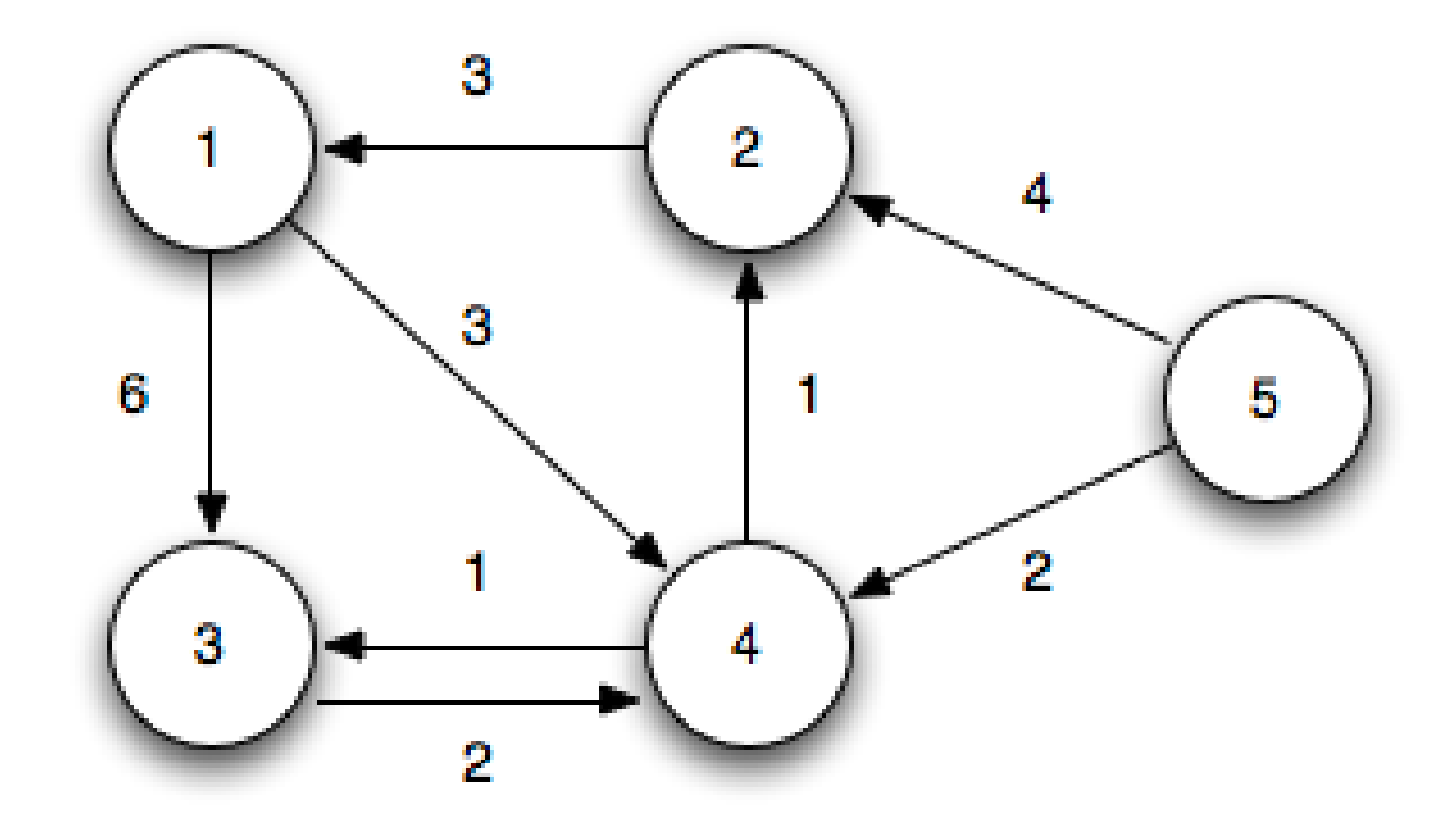
Algoritma Adımları

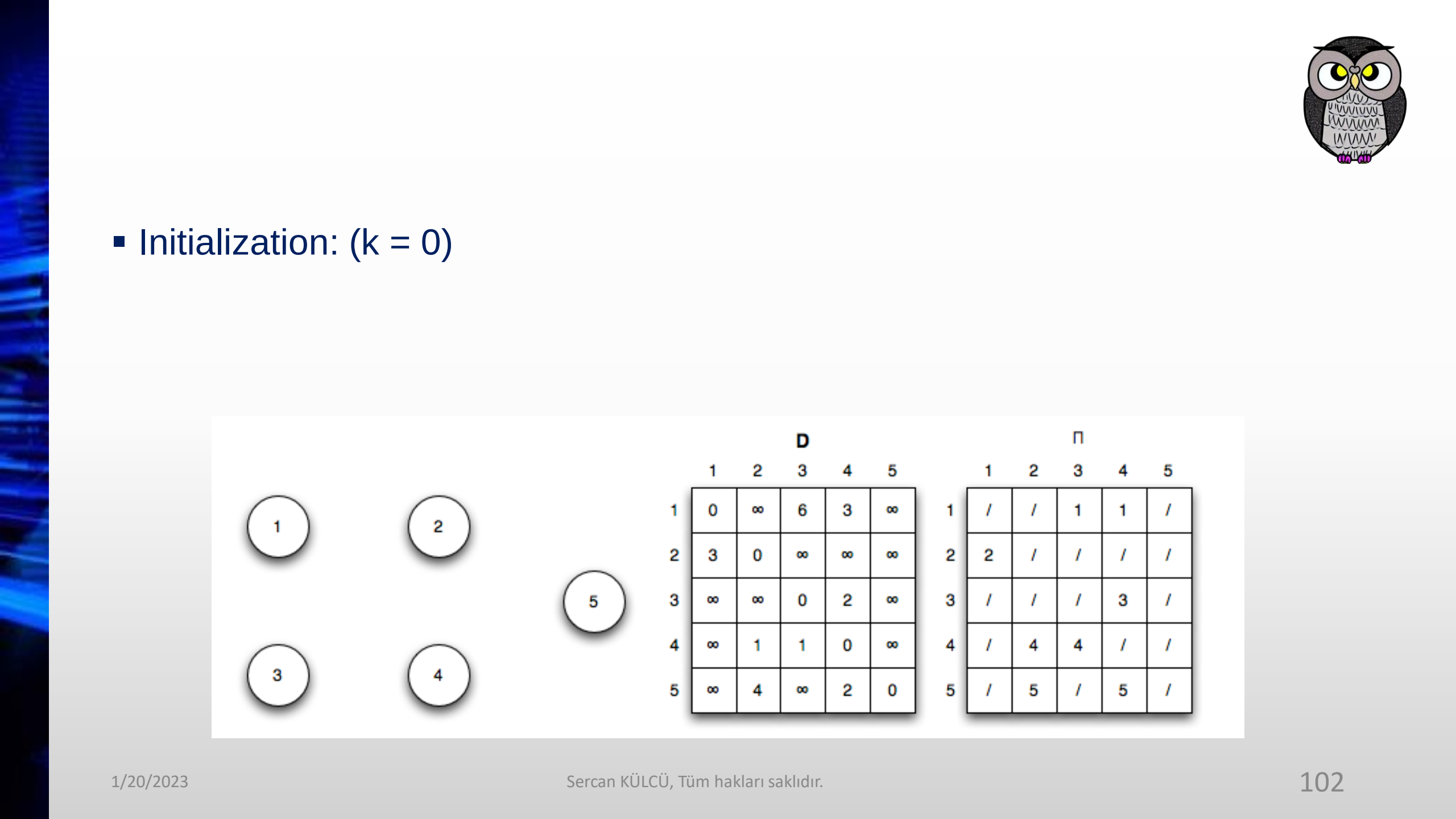
- Adım 1: Her bir çift düğüm arasındaki ağırlıklar, doğrudan kenarlarla belirtilir. Eğer iki düğüm arasında doğrudan bir kenar yoksa, uzaklık sonsuz kabul edilir.
- Adım 2: Her bir düğüm çifti için, tüm ara düğümler sırayla incelenir.
- Adım 3: Ara düğümler üzerinden geçerek, yeni yolun uzunluğu hesaplanır ve mevcut en kısa yol uzunluğu ile karşılaştırılır.
- Adım 4: Yeni bulunan en kısa yollar, matrise kaydedilir.

Karmaşıklık Analizi

- Floyd-Warshall Algoritması'nın karmaşıklığı $O(V^3)$ şeklindedir.
- V düğüm sayısını temsil eder.
- 3 adet iç içe for döngüsü kullanılır.
  - ilk döngü tüm ara düğümleri gezer
  - ikinci döngü tüm kaynak düğümleri gezer.
  - son döngü tüm hedef düğümleri gezer.

Örnek





- Initialization: (k = 0)

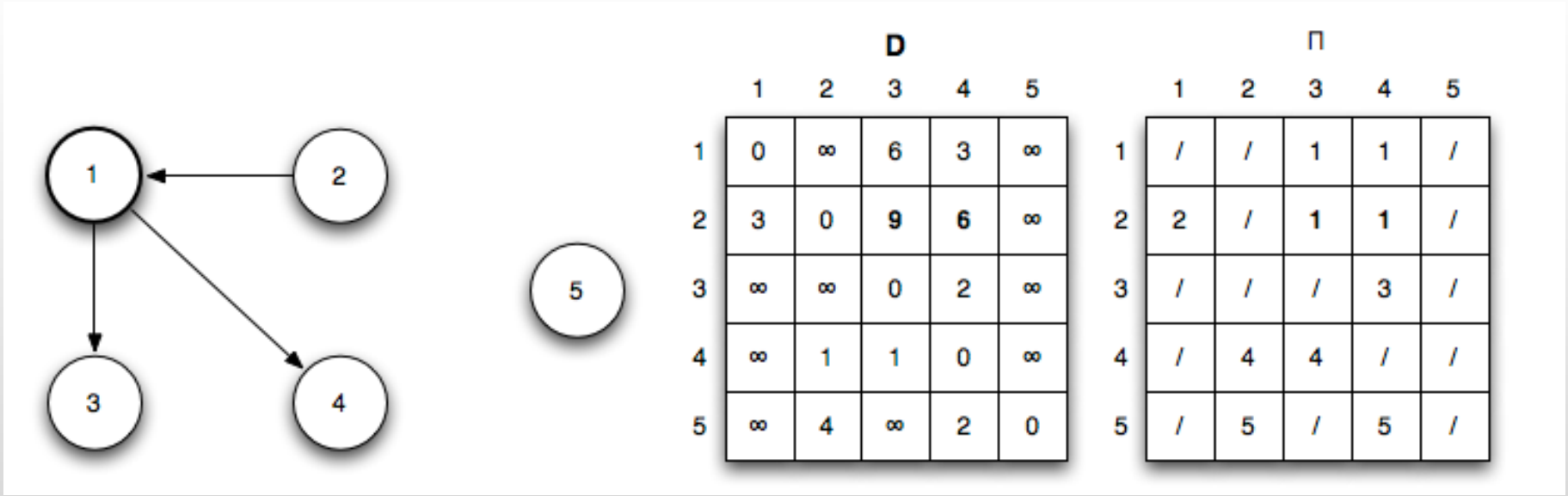
D

	1	2	3	4	5
1	0	∞	6	3	∞
2	3	0	∞	∞	∞
3	∞	∞	0	2	∞
4	∞	1	1	0	∞
5	∞	4	∞	2	0

Π

	1	2	3	4	5
1	/	/	1	1	/
2	2	/	/	/	/
3	/	/	/	3	/
4	/	4	4	/	/
5	/	5	/	5	/

- Iteration 1: (k = 1) Shorter paths from 2 ⇝ 3 and 2 ⇝ 4 are found through vertex 1



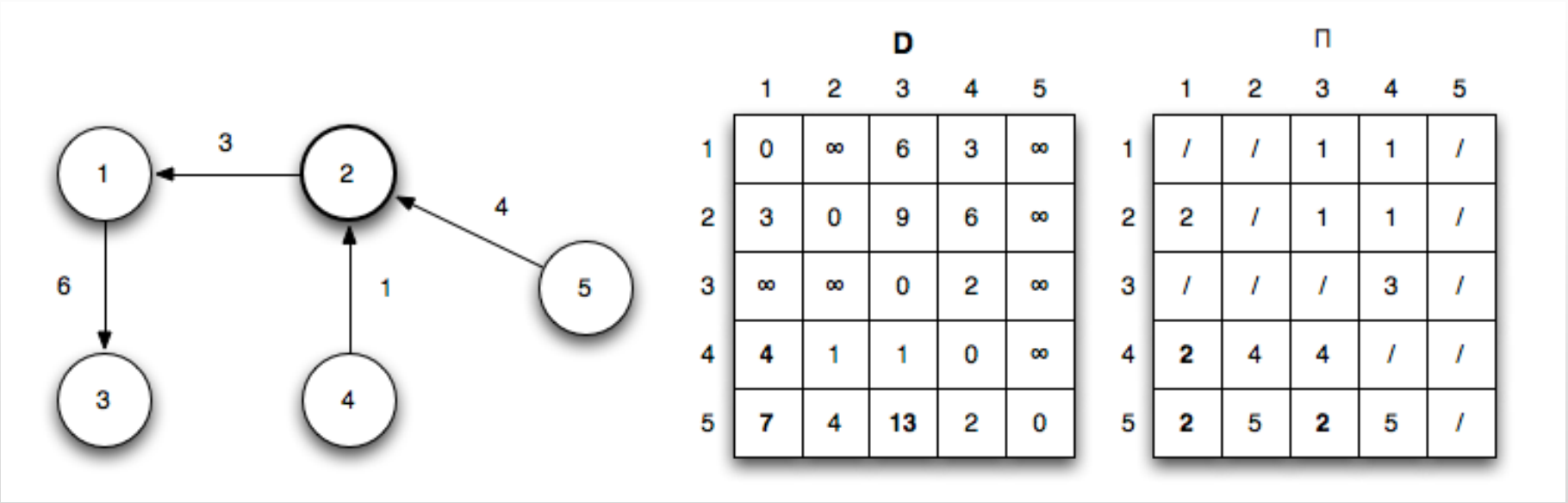
D

	1	2	3	4	5
1	0	∞	6	3	∞
2	3	0	**9**	**6**	∞
3	∞	∞	0	2	∞
4	∞	1	1	0	∞
5	∞	4	∞	2	0

Π

	1	2	3	4	5
1	/	/	1	1	/
2	2	/	**1**	**1**	/
3	/	/	/	3	/
4	/	4	4	/	/
5	/	5	/	5	/

- Iteration 2: (k = 2) Shorter paths from 4 ⇝ 1, 5 ⇝ 1, and 5 ⇝ 3 are found through vertex 2



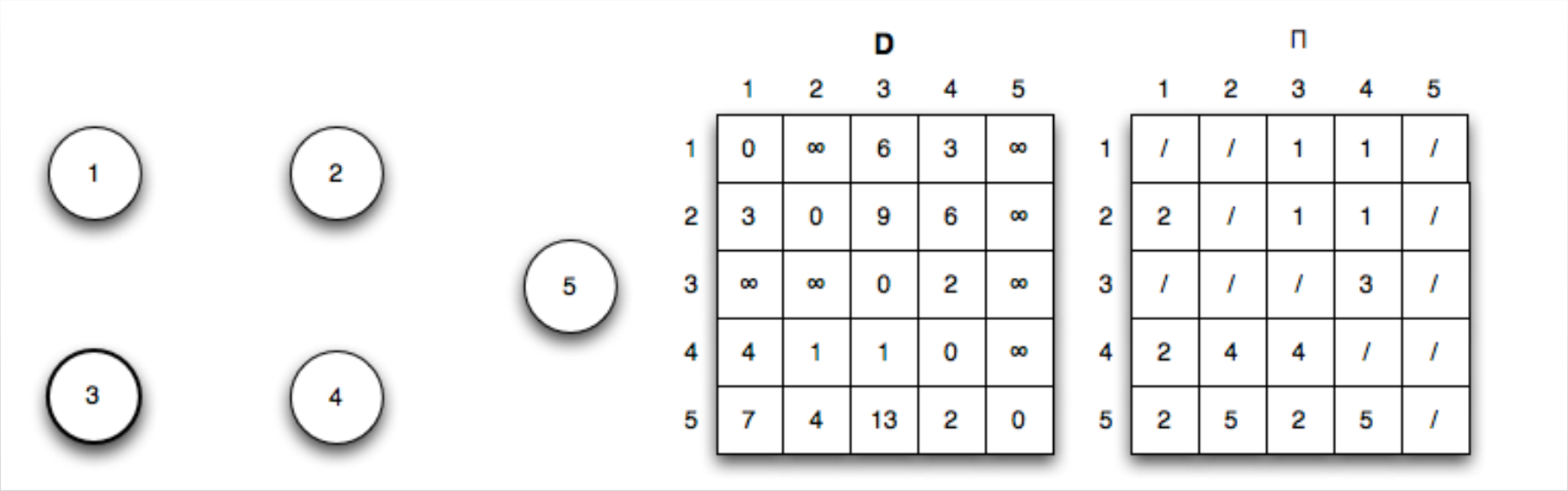
D

	1	2	3	4	5
1	0	∞	6	3	∞
2	3	0	9	6	∞
3	∞	∞	0	2	∞
4	**4**	1	1	0	∞
5	**7**	4	**13**	2	0

Π

	1	2	3	4	5
1	/	/	1	1	/
2	2	/	1	1	/
3	/	/	/	3	/
4	**2**	4	4	/	/
5	**2**	5	**2**	5	/

- Iteration 3: (k = 3) No shorter paths are found through vertex 3



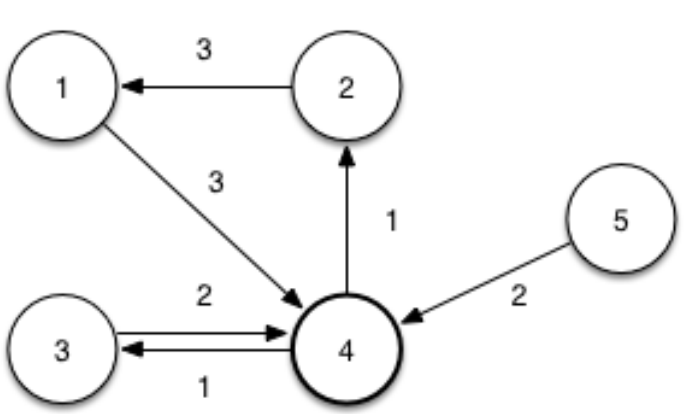
D

	1	2	3	4	5
1	0	∞	6	3	∞
2	3	0	9	6	∞
3	∞	∞	0	2	∞
4	4	1	1	0	∞
5	7	4	13	2	0

Π

	1	2	3	4	5
1	/	/	1	1	/
2	2	/	1	1	/
3	/	/	/	3	/
4	2	4	4	/	/
5	2	5	2	5	/

- Iteration 4: (k = 4) Shorter paths from 1 ⇝ 2, 1 ⇝ 3, 2 ⇝ 3, 3 ⇝ 1, 3 ⇝ 2, 5 ⇝ 1, 5 ⇝ 2, 5 ⇝ 3, and 5 ⇝ 4 are found through vertex 4



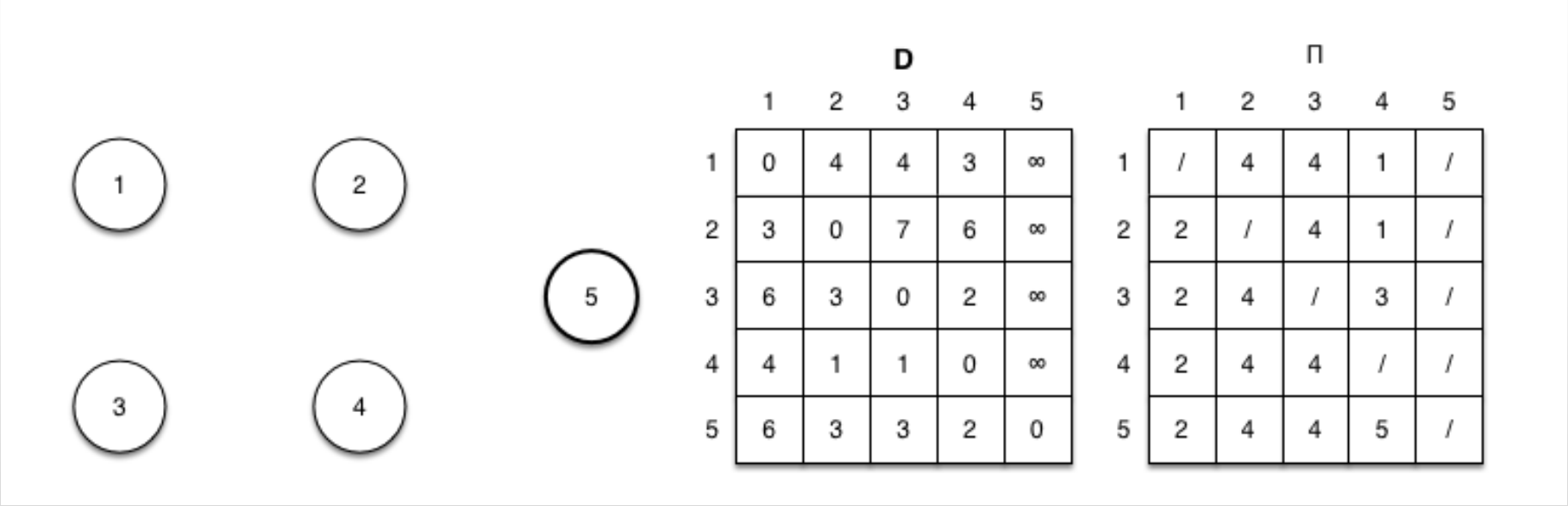
D

	1	2	3	4	5
1	0	**4**	**4**	3	∞
2	3	0	**7**	6	∞
3	**6**	**3**	0	2	∞
4	4	1	1	0	∞
5	**6**	**3**	**3**	2	0

Π

	1	2	3	4	5
1	/	**4**	**4**	1	/
2	2	/	**4**	1	/
3	**2**	**4**	/	3	/
4	2	4	4	/	/
5	2	**4**	**4**	5	/

- Iteration 5: (k = 5) No shorter paths are found through vertex 5



D

	1	2	3	4	5
1	0	4	4	3	∞
2	3	0	7	6	∞
3	6	3	0	2	∞
4	4	1	1	0	∞
5	6	3	3	2	0

Π

	1	2	3	4	5
1	/	4	4	1	/
2	2	/	4	1	/
3	2	4	/	3	/
4	2	4	4	/	/
5	2	4	4	5	/

- The final shortest paths for all pairs is given by

D

	1	2	3	4	5
1	0	4	4	3	∞
2	3	0	7	6	∞
3	6	3	0	2	∞
4	4	1	1	0	∞
5	6	3	3	2	0

Π

	1	2	3	4	5
1	/	4	4	1	/
2	2	/	4	1	/
3	2	4	/	3	/
4	2	4	4	/	/
5	2	4	4	5	/

A* (A Star) Algoritması

- İki nokta arasındaki en kısa yolu bulan bir bilgi arama algoritmasıdır.
- 1968'de Peter Hart, Nils Nilsson, Bertram Raphael tarafından geliştirildi.
- Genişlik öncelikli arama (Breadth-First Search) ile en iyi ilk arama (Best-First Search) algoritmalarının kombinasyonunu kullanır.
- Düzgün çalışması için doğru bir tahmin fonksiyonu gereklidir.
- Düğümlerin sayısı arttıkça karmaşıklığı artar.

Algoritma İlkeleri

- Her bir düğüm için tahmin (heuristic) değeri kullanır.
- Bu tahmin, düğümün hedefe olan tahmini mesafesini belirtir.
- Her adımda,
  - komşu düğümler arasından,
  - hedef ile arasındaki gerçek maliyet ve tahmini maliyet toplamı en küçük olan seçilir.
- Bu özellik sayesinde, algoritma hedefe doğru hareket ederken, aynı zamanda en az maliyetli yolu seçmeye çalışır.

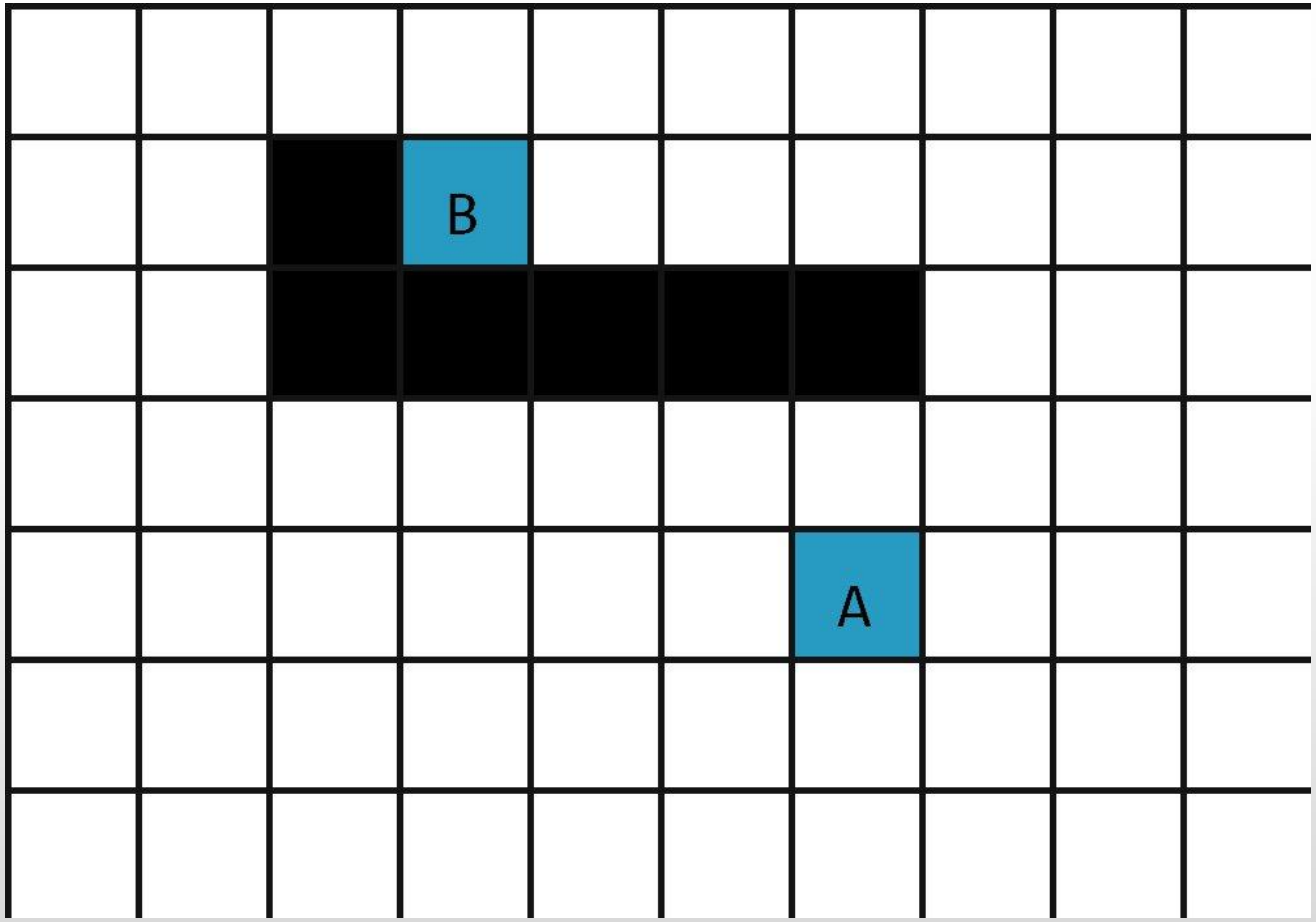
Algoritma Adımları

- Adım 1: Başlangıç düğümü seçilir ve bu düğüme uzaklık 0 atanır. Diğer düğümlere sonsuz uzaklık atanır.
- Adım 2: Mevcut düğümün komşuları incelenir ve her birinin tahmini maliyeti hesaplanır.
- Adım 3: Komşu düğümler arasından, gerçek maliyet ve tahmini maliyetin toplamı en küçük olan düğüm seçilir.
- Adım 4: Seçilen düğüm, şu ana kadar bulunan en uygun yolun bir parçası olarak kaydedilir.

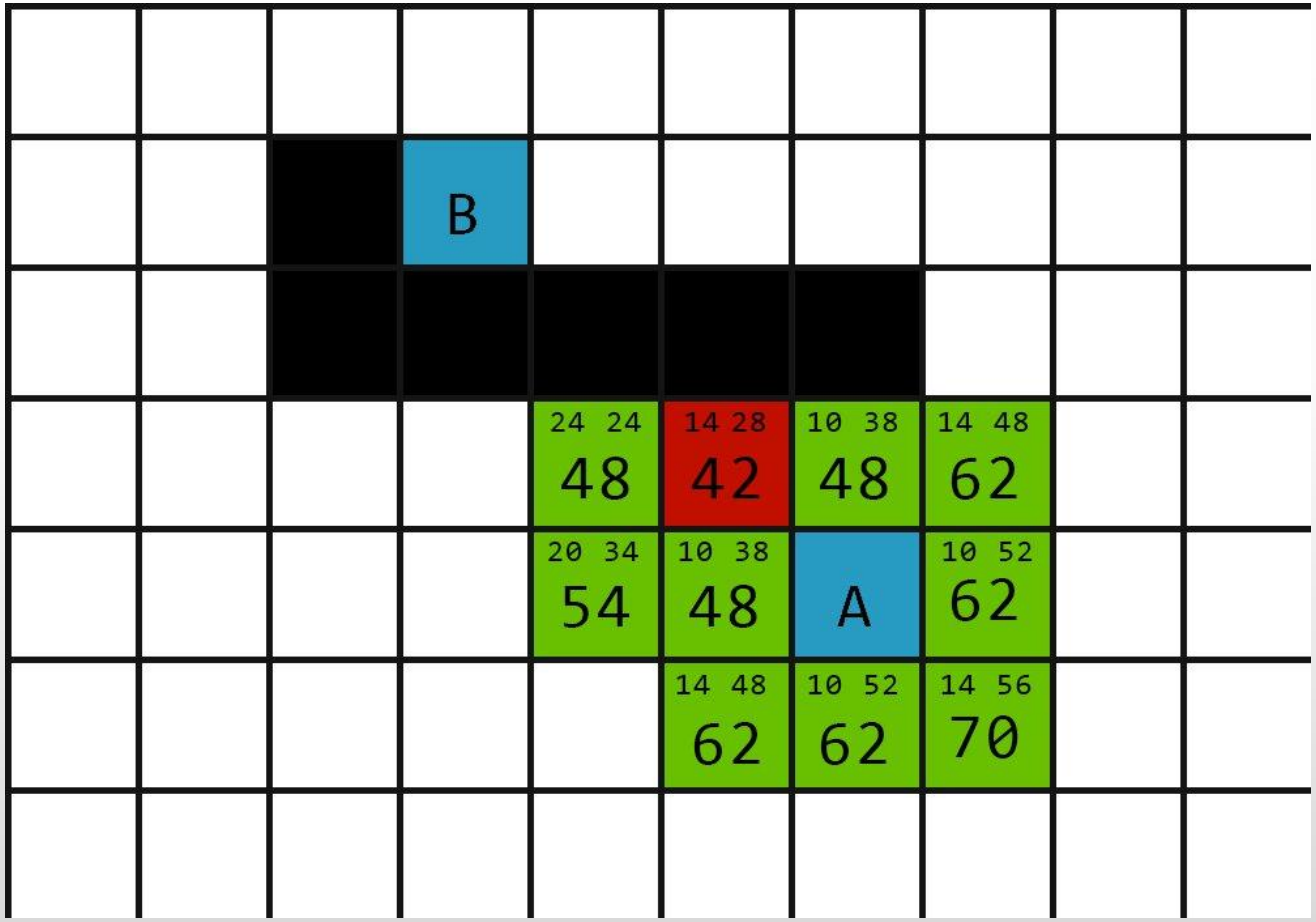
Algoritma Karmaşıklığı

- Eğer tahmin fonksiyonu gerçek maliyeti tam olarak tahmin ediyorsa,
  - karmaşıklık $O(b^d)$ şeklinde ifade edilir.
  - b çizgenin dallanma faktörünü,
  - d ise hedef düğüme olan maksimum derinliği temsil eder.

A Star



A Star



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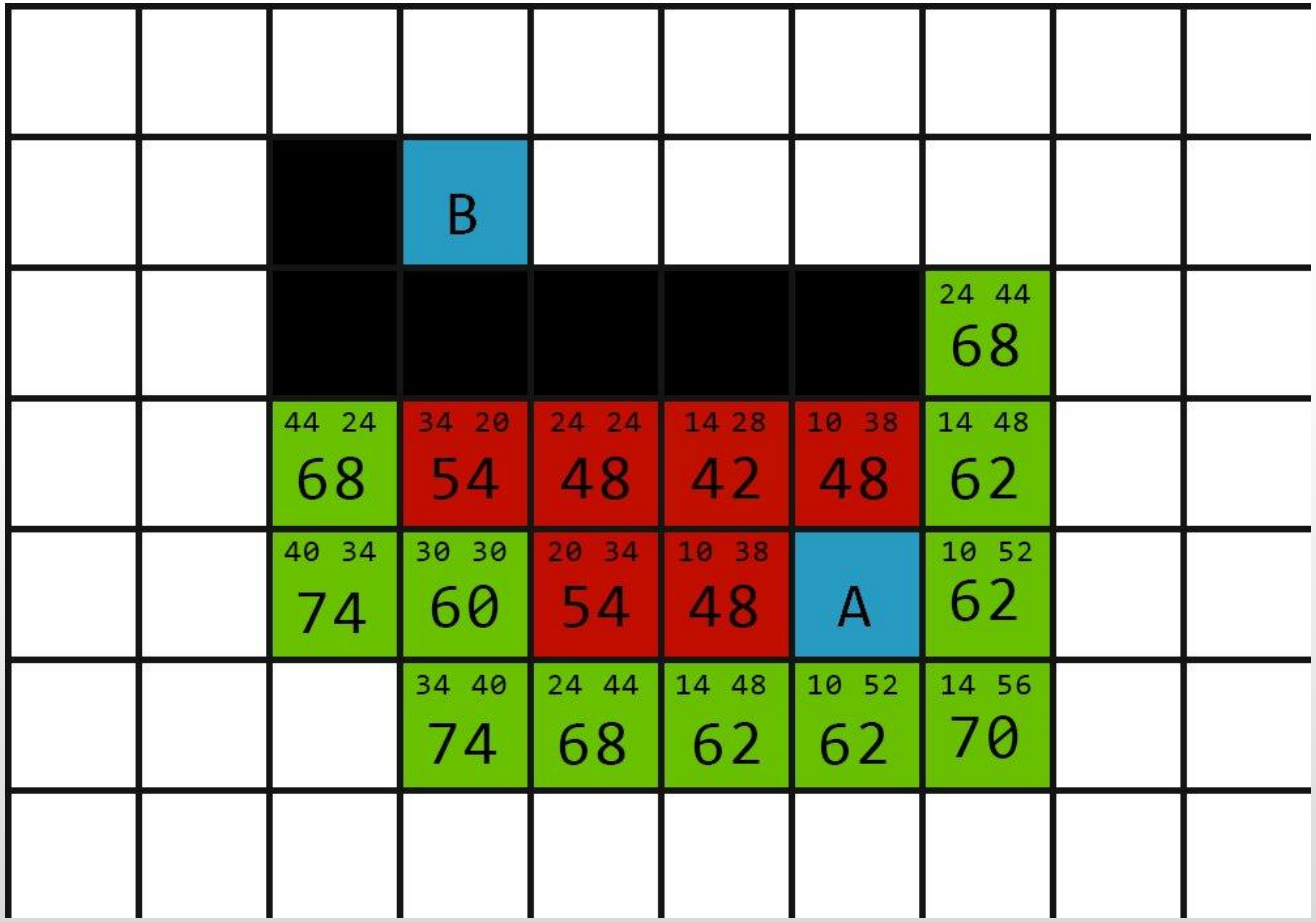

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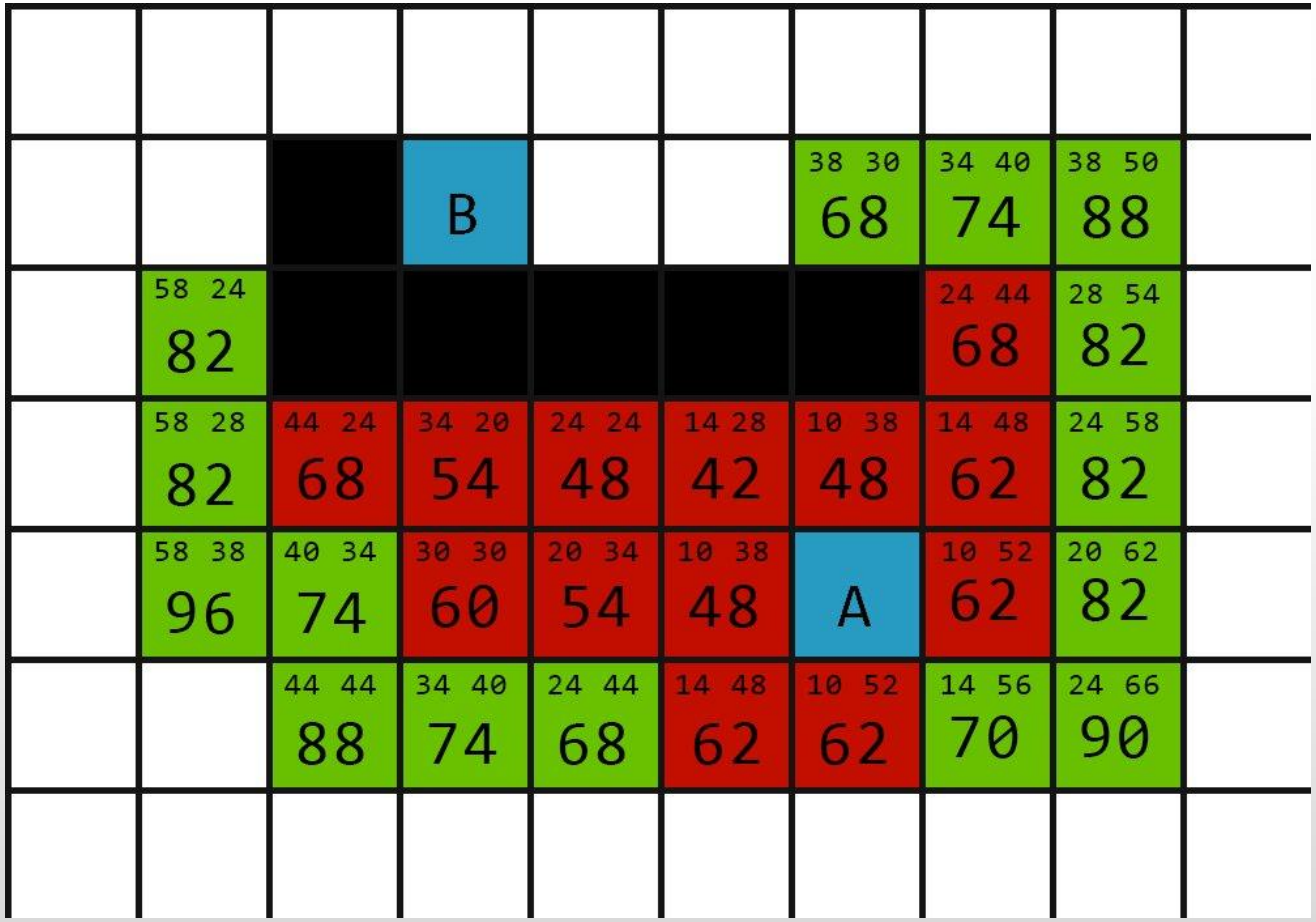
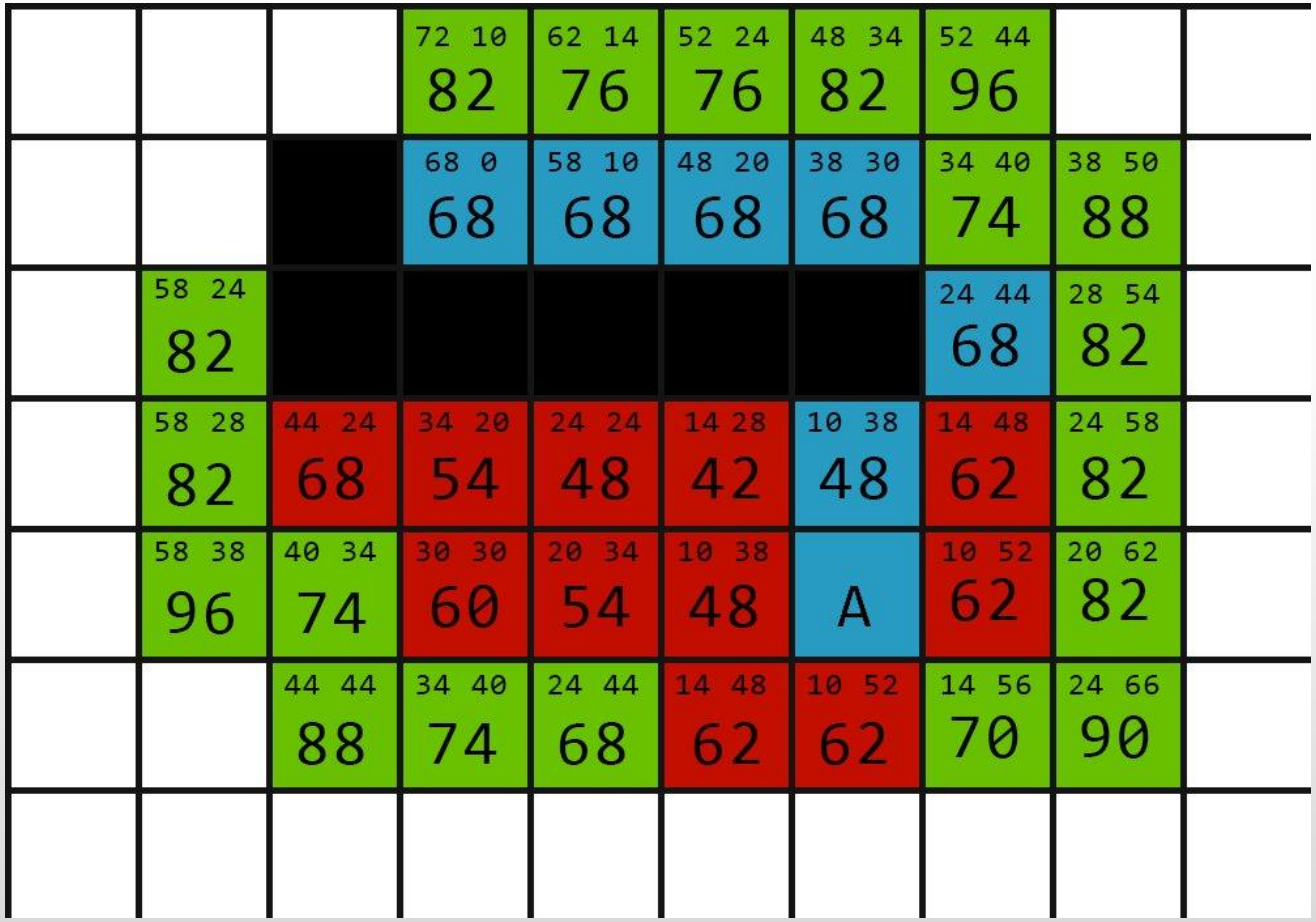


A Star



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