MATLAB Exercise #1. Return to OPTIMA before 30.9.

Upload your solutions to the return box in OPTIMA as ZIPPED file containing everything needed to run your programs. The return box in OPTIMA will open well before deadline. Use filename MATLAB1_studentnumber1_studentnumber2, where studentnumber1 and 2 are student numbers of your team members. In addition to the well-commented program, write a short report explaining how your program works and answering the problems posted (including for example figures obtained by running the simulation program). Please note that the tasks include also the theoretical parts, which need to be addressed in the report in addition to the simulations. Include the student numbers and names of your team members (two members per team) on the top of the first page of the report.

Plagiarism is strictly forbidden and will lead to immediate rejection! You do not need to be able to solve all the problems completely. If you can explain clearly how did you approach the problem and why you did not reach a solution, you may still get a portion of the points. Please note that since this exercise is MANDATORY, <u>make sure to return your solutions</u> even if you could not solve all the tasks.

Task 1: The data $x[n] = Ar^n + w[n]$ for n=0,1,…,N-1 are observed, where w[n] is zero-mean WGN with variance σ^2 and r>0 is known. See the answer to exercise 1 question in OPTIMA for the efficient estimator and its variance (the Cramer Rao lower bound). Implement Monte Carlo simulation of this problem in MATLAB and in each loop generate the data x[n] according to the given model. Evaluate the output of the efficient estimator. Finally, compare the variance of the efficient estimator to the Cramer Rao lower bound. Does your estimator reach the CRLB? Verify that your estimator is unbiased. Use for example these values

```
A = 3;
r = 0.5;
N = 100;
sigma_squared = 0.5;
MC = 10000;
```

Task 2: We observe two samples of a DC level in correlated zero-mean Gaussian noise

x[0]=A+w[0]

x[1]=A+w[1]

where **w** = [w[0] w[1]]^T is zero-mean Gaussian random vector with covariance matrix

$$C = \sigma^2 \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$$

The parameter $-1 \leq \rho \leq 1$ is the correlation coefficient between w[0] and w[1].

Find an efficient estimator and its variance (for help, see the exercises where this problem has been solved, you just need to simplify the expressions given therein for this special correlation structure). You are allowed to use MATLAB symbolic toolbox for simplifying the expressions (for help, see solutions to exercise 1).

Create a Monte-Carlo simulation program for the problem. Use A=3 as the true value. Perform MC (a sufficiently large number to get smooth plots) Monte Carlo loops. In each loop generate random variables x[0] and x[1] with the correct correlation structure (value of ρ) and estimate A using the efficient estimator. Plot the simulated probability density function and theoretical probability density in the same plot and make sure that they agree! Run the simulation for different values of ρ such as -1, 0, and +1. What is special about plot with $\rho = -1$ and why is it like this?

Task 3: For *N* IID observations of U[0, θ] PDF (uniform distribution) find the maximum likelihood estimator of θ. **Create a Monte-Carlo simulation program for the problem.** Perform MC Monte Carlo loops. In each loop, generate *N* random variables following the given uniform distribution. Estimate θ (use 3 as the true value), using the maximum likelihood estimate. Also estimate θ using the estimator A_est_mean = sample mean*2. Plot the empirical histogram obtained by the simulations for both estimators. Which estimator would you say is better and why? Are both of them unbiased? If not, which one is not?

Find out the theoretical PDF for both estimators and plot against simulated histograms. For theoretical result for MLE estimator, results in

https://en.wikipedia.org/wiki/Order_statistic#Probability_distributions_of_order_statistics

should be very helpful. You may find it easy to first consider $\theta = 1$ before proceeding to the value that you need actually use here ($\theta = 3$), with the help of theory for affine transformation of random variables.