Calculus II: Project 1

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Problem 1

Choose a profile picture, which can be scanned from a drawing or desinged entirely on a computer. Each drawing must be unique and must be approved before proceeding to continue the project. The drawing must have an axis of symmetry. The design must be such that an application of Simpson's rule can be used to approximate the several integrals. Pixelated desings are also possible. The boundary must be a simple closed curve with (no holes, no self-crossing).

Solution

We chose an figure that when revolved, results in an innovative approach to lifesavers, as it provides a conforable space for placing one's arms and safely placing items that may be useful to one's survival. Our basic design is the following:

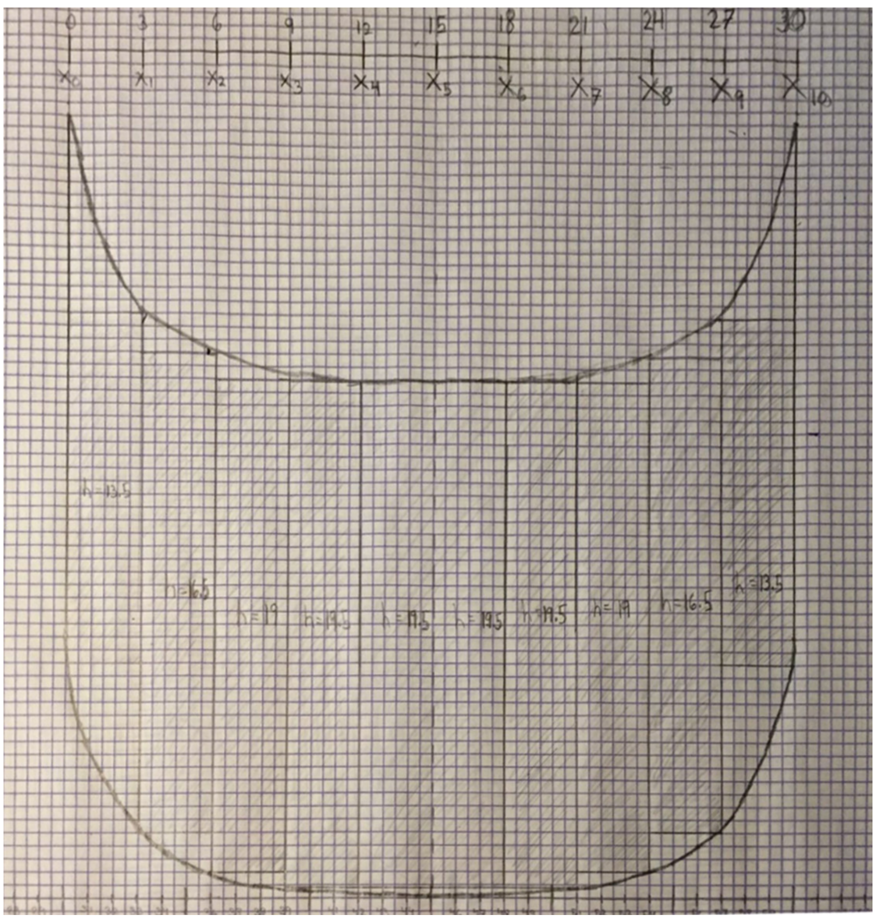

Figure 1: Cross section design

Problem 2

Choose a particular scale, origin and coordinate axes for your drawing.

Solution Our coordinate axis is the following:

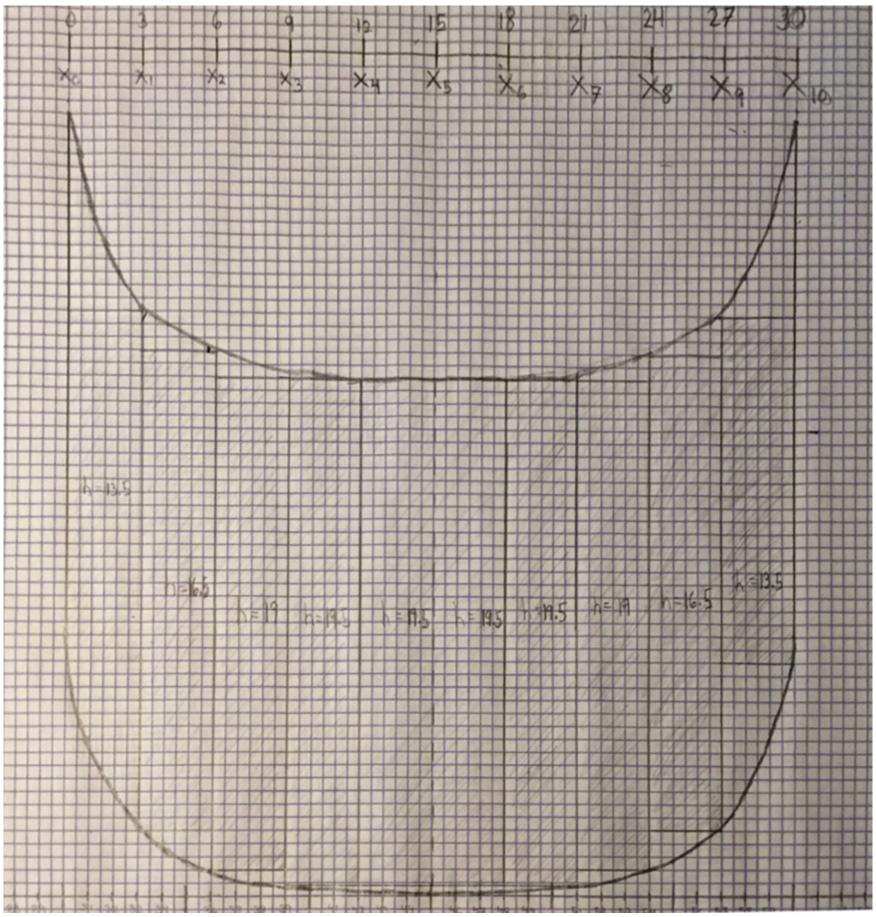

Figure 2: Cross section with axis

Problem 3

Numerically, and an approximation to the area of your drawing using Simpson's rule and the Trapezoidal rule. If your drawing allows it, and an exact answer also.

Solution First we present the samples and their respective heights:

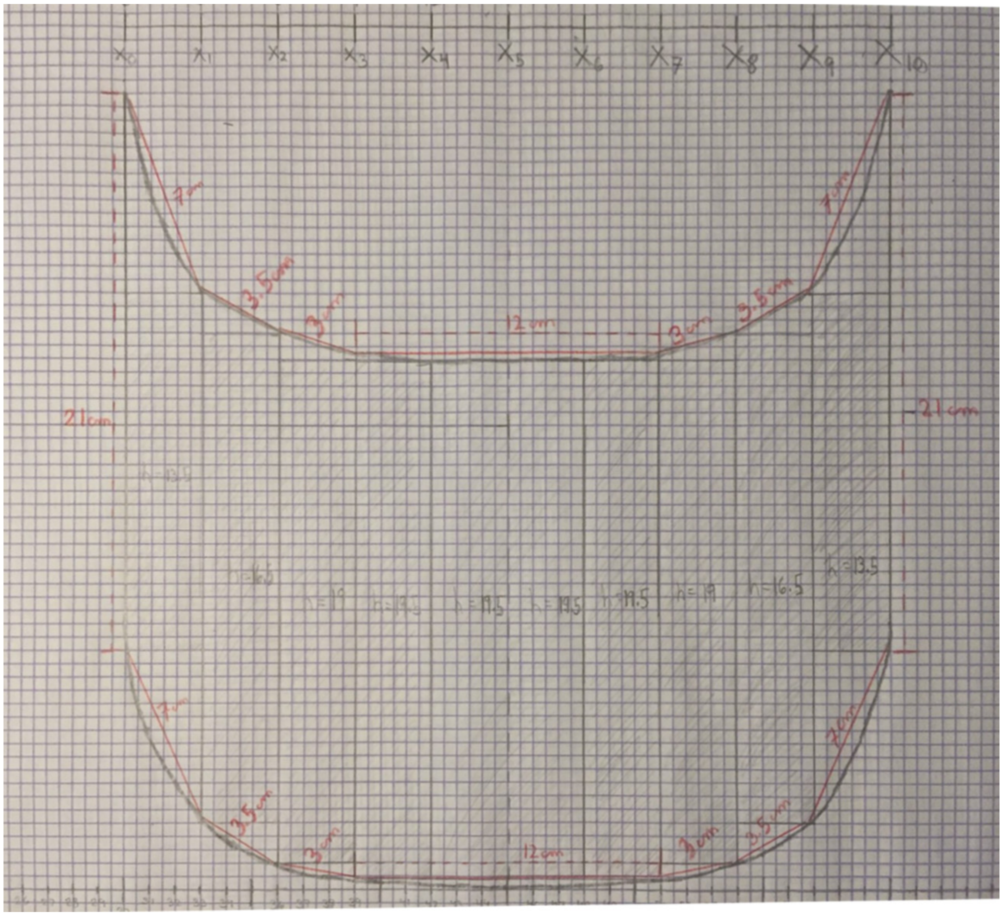

Figure 3: Cross section with measurements

Trapezoidal rule

By definition, we know that the Trapezoid rule has the following form:

$$\int_a^b f(x)dx \approx \frac{h}{2}[f(x_0) + 2f(x_1) + f(x_2) + fx(_3) + \cdots + 2f(x_{n-1}) + f(x_n)] \tag{1}$$

Thus, substituting our measurements we get:

$$\begin{aligned}\int_a^b f(x)dx &\approx \frac{3}{2}[21 + 2 \times 16.5 + 19 + 2 \times 19.5 + 19.5 + \\ &\quad 2 \times 19.5 + 19.5 + 2 \times 19.5 + 19 + 2 \times 16.5 + 21] \\ &\approx \frac{3}{2}[302] \\ &\approx 453\end{aligned}$$

This the approximation of the area via the trapezoid rule is $Area \approx 453cm^2$

Simpon's method

By definition, we know that Simpon's method has the following form:

$$\int_a^b f(x)dx \approx \frac{3}{h}[f(x_0)+4f(x_1)+2f(x_2)+4fx(_3)+\cdots+4f(x_{n-3})+2f(x_{n-2})+4f(x_{n-1})+f(x_n)] \tag{2}$$

And substituting with our measurements:

$$\begin{aligned}\int_a^b f(x)dx &\approx \frac{3}{3}[21+4\times 16.5+2\times 19+4\times 19.5+2\times 19.5\\ &\approx +4\times 19.5+2\times 19.5+4\times 19.5+2\times 19+4\times 16.5+21]\\ &\approx \frac{3}{3}[562]\\ &\approx 562\end{aligned}$$

This the approximation of the area via Simpon's method is $Area \approx 562cm^2$

Problem 4

Numerically, and an approximation of the perimeter of your drawing approximating it using straight lines and adding the length.

Solution

From Figure 4, we can see that from our 16 measurements, we can approximate the perimeter as the sum:

$$P = 21+7+3.5+3+12+3+3.5+7+21+7+3.5+3+12+3+3.5+7$$

Which gives us a result of: $P \approx 120cm$

Problem 5

Choose an axis parallel to the axis of symmetry of your figure on which you will revolve your figure to produce a life saver with your figure as profile.

Solution

We have choosen the axis as the axis of revolution $x = 0$, therefore according to our diagram we have an inner radius of 30cm and an outer radius of 60cm.

Problem 6

Investigate Pappus' Theorem for solids of Revolution, and use it to

a) Compute the volume of the resulting solid of revolution.

b) Compute the surface area of the resulting solid of revolution

Solution
Volume of solid of revolution
From Pappus, we know that:

$$V = 2\pi r A$$

Where V is the volume, r is the distance from the axis to the centroid and A is the area of the region. Therefore, substituting, we get

$$\begin{aligned} V &\approx 2\pi \times 45 \times 562 \\ &\approx 158901.756 \end{aligned}$$

Thus the volume is approximately equal to $158901.8cm^3$
Surface area
From Pappus, we know that

$$S = 2\pi r L$$

Where S is the surface area, r is the distance from the axis to the centroid and L is the perimeter of the region. Therefore, substituting, we get

$$\begin{aligned} S &\approx 2\pi 45 \times 120 \\ &\approx 33929.2 \end{aligned}$$

Thus the surface is approximately equal to $33929.2cm^2$

Problem 7

Investigate Arquimedes' Theorem to And the buoyance force of the life saver assuming it is completely submerged in water, and is filled with air.

Solution According to Archimedes' buoyancy principle, the buoyant force is defined as:

$$\begin{aligned} \text{Buoyant Force} &= \text{weight of displaced liquid} \\ F = mg &= \rho V g \end{aligned}$$

Where m is the mass of the displaced liquid, g is the force of gravity, V is the volume of the displaced liquid and $\rho$is the density of the liquid.

Problem 7 continued on next page...

Therefore, substituting (assuming a density of seawater of 1.025 kg/L and a value for g of 9.81) we have

$$\begin{aligned} F &\approx 1.05 \times 562 \times 9.81 \\ &5788.881 \end{aligned}$$

Thus the buoyant force is equal to 5788.881 Newtons.

Problem 8

Using Simpson's rule, and the volume of the solid of revolution without using Pappus' Theorem.

Solution In order to find the volume of revolution using Simpon's rule, we are going to use the shell method.
First we find the volume of a single cylinder (at x_3):

$$V = 2\pi(x)(f(u) - f(b))\Delta x$$

Where x is the radius, u is the upper bound, b is the lower bound and Δx the thickness of the shell.
Substituting we have

$$\begin{aligned} V &\approx 2\ pi(39)(19.5)(3) \\ &\approx 14335.08 \end{aligned}$$

And an approximation to the volume would be:

$$V \approx \sum_{j=0}^{10} 2\pi(x_j)(f(u_j) - f(b_j))\Delta x$$

Which would give us a result of:

$$\begin{aligned} V \approx & 2\pi \times 3 \times (33 \times 16.5 + 36 \times 19 + 39 \times 19.5 + 42 \times 19.5 + 45 \times 19.5 + 48 \times 19.5 + \\ & 51 \times 19.5 + 54 \times 19 + 57 \times 16.5 + 60 \times 21) \\ \approx & 166677.198 \end{aligned}$$

Thus our approximations yields a volume of $166677.198 cm^3$

Problem 9

Make a real design of your life saver. You may 3d print it if you wish.

Problem 9 continued on next page...

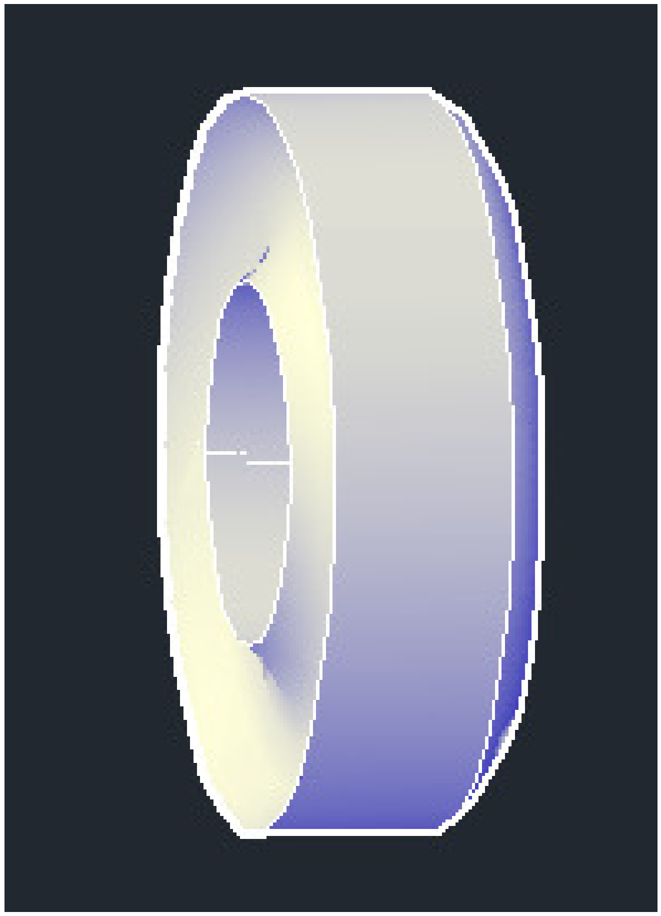

Figure 4: 3D Design