

Universidad de
los Andes

Laniakea in a Cosmological Context

or

Detection of galaxy superclusters in
simulated cosmological structures

Sergio Daniel Hernández Charpak
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Advisor: Jaime E. Forero-Romero

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Universidad de los Andes
Facultad de Ciencias, Departamento de Física
Bogotá, Colombia

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2 Abstract

Recently Tully et al. (2014) [1] used local cosmic flow information to define our local supercluster, Laniakea. In this work we present a study on large cosmological N-body simulations aimed at establishing the significance of Laniakea in a cosmological context. We explore different algorithms to define superclusters from the dark matter velocity field in the simulation. We summarize the properties of the supercluster population by their abundance at a given total mass and its shape distributions. We find that superclusters similar in size and structure to Laniakea are relatively uncommon on a broader cosmological context.

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3 Motivation

We live in the planet Earth. Our planet orbits around the Sun in the Solar System, under the influence of the gravitational forces. Our Solar System forms, along with many other stars and systems, our home galaxy, the Milky Way. Galaxy themselves, under the same forces, form even larger structures. It is these structures we call galaxy superclusters.

Detecting these galaxy superclusters is a challenging task. Observationally there exist different algorithms to identify them based on physical criteria [2]. One possible physical criterion relates the velocity flows with the matter density.

Indeed, due to gravitational attraction, it is possible to relate velocity fields with matter density fields. A proposal to find galaxy superclusters is to analyze the velocity field of galaxies as matter tend to flow to dense region. Spacial regions where the galaxy flow is convergent represent galaxy superclusters.

Recently a team of astronomers built a galaxy velocities flow map of the local universe in a scale of hundreds of million light years [3]. In this map, the team found converging velocity flows which form a structure in the local region. They named this region Laniakea, the galaxy supercluster which includes our galaxy, the Milky Way [1]. It is the first time a galaxy supercluster is defined with this criterion. Also, these observations are difficult. As a result of these difficulties, it is not possible to build a statistical sample superclusters obtained from the observational approach.

We propose a method to detect galaxy superclusters in cosmological simulations following physical criteria similar to the one used to define Laniakea. After an interpolation of the velocity and density fields we apply a region growing algorithm based in predefined parameters. We test our method on N-body simulations and successfully detect structures we compare to Laniakea.

We aim to quantify if Laniakea can be considered as an atypical structure in the Universe.

The definition of Laniakea by Tully et al [1] is based on the peculiar velocity flow. Chon et al provide an interesting work on a different definition of galaxy superclusters, a gravitationally bound dense large scale structure [4] and [5]. On this different criteria, Laniakea and how Laniakea fits their definition for supercluster differs.

4 Introduction

4.1 The Standard Cosmological Model

The current standard cosmological model is known the Lambda - Cold Matter Model, or Λ -CDM Model. It assumes Einstein's equations of General Relativity.[6].

The letter Λ makes a reference to the cosmological constant Ω_Λ , which is associated with dark energy and the accelerated expansion of the Universe according to General Relativity. Dark energy is a fairly unknown form of energy. It is proposed to explain the accelerated rate of expansion of the Universe [7].

The dark matter is a hypothetical type of matter which is nor ordinary matter (baryonic matter), nor neutrino particles nor dark energy. It is dark as it does not seem to produce any radiation (light). There has been indirect observations of dark matter as difference between gravitational matter implied by velocities of stars and observed matter and gravitational lensing between others [8]. Cold means that the velocity of the hypothetical fundamental particles conforming the dark matter are not relativistic. A consequence of this is a bottom-up scheme. With bottom-up, we mean that small structures merge into one larger structure. The opposite would be hot matter, meaning that the hypothetical fundamental particles could achieve relativistic velocities. In this hot matter model, there is a top-down scheme, where an initial large structure breaks into smaller structures. [6]

The simplest Λ -CDM Model is based on the following 6 general parameters.

1. The Hubble parameter, H . It represents the expansion of the universe, how things are moving away from us. It is defined as

$$H = \frac{\dot{a}}{a},$$

where $a(t)$ refers to the scale factor of the Universe. For nearby galaxies, H is found to be constant and thus is defined as the Hubble constant H_0 .

$$H_0 = 100 h \text{ km s}^{-1} \text{ Mpc}^{-1} = 67 \pm 7 \text{ km s}^{-1} \text{ Mpc}^{-1}.$$

Mpc is a cosmological length unit. $1\text{Mpc} = 10^6 \text{ pc}$ where $1\text{pc} = 3.087 \times 10^{16} \text{ m}$.

h is the dimensionless parameter defined by the equation above, $H_0 = 1 \text{ km s}^{-1} \text{ Mpc}^{-1} h^{-1}$. So $h \simeq 0.67$.

In this document the length units are Mpc/h .

2. Critical density ρ_{cr} found when the cosmological constant is 0 in Einstein's equations, condition needed to have a close universe. It represents the point of change between an expending and contracting universe. $\rho_{cr} = \frac{3H^2}{8\pi G} = (8.62 \pm 0.12) \times 10^{27} \text{ kg/m}^3$.

3. Baryon density parameter Ω_b . the baryonic matter is the visible matter we interact with, composed by the quarks described by the Standard Model of Particle Physic. The density of baryonic matter expressed as a fraction of the critical density is $\Omega_b = \frac{\rho_b}{\rho_{cr}} = 0.0486 \pm 0.0010$.
4. Dark matter density parameter Ω_c . Dark matter corresponds to a type of matter which do not produced any type of visible radiation. It interacts exclusively with the gravitational force. The density of dark matter expressed as a fraction of the critical density. $\Omega_c = \frac{\rho_c}{\rho_{cr}} = 0.2589 \pm 0.0057$.
5. Matter density parameter Ω_m or Ω_0 . It corresponds to the sum of Ω_c and Ω_b , $\Omega_0 = 0.3089 \pm 0.0062$
6. Dark energy density parameter, cosmological vacuum energy density (or cosmological constant), Ω_Λ . As we stated before, it is associated to the accelerated rate of expansion of the Universe. $\Omega_\Lambda = 0.6911 \pm 0.0062$

The total density parameter, a constraint, is the sum of the density parameters and has to be equal to one. $\Omega_{tot} = \Omega_b + \Omega_c + \Omega_\Lambda = 1$ or $\Omega_{tot} = \Omega_0 + \Omega_\Lambda = 1$.

Another important concept before the rest of the work is to define the redshift. The redshift z can be defined by the following equation[7]:

$$1 + z = \frac{\lambda_{obs}}{\lambda_{em}}.$$

It is a measure of the difference between the observed wavelength λ_{obs} and the wavelength received from a distant object λ_{em} . It is a convenient way to label time, with $z = 0$ being the present time.

This is the model which represents the best the Universe as we see it. It is in this model we develop our method to find galaxy supercluster.

4.2 Cosmic flows and peculiar velocities

Our method to detect superclusters in cosmological simulations is based on the peculiar velocity field analysis in a similar fashion as Tully et al. did to find Laniakea [1] [9].

- Relation between the velocity and matter density fields

The peculiar velocity is defined as the velocity relative to a rest frame. Its radial component corresponds to the velocity of a galaxy that cannot be explained by the expansion of the Universe. Hubble's law states that nearby galaxies move away from us at a speed proportional to the distance from us (the proportion is Hubble's constant). In other words,

$$v_p = v_{obs} - \frac{7}{7} H_0 \times d_{obs}.$$

With v_p the peculiar velocity, v_{obs} and d_{obs} the observed velocity and distance, H_0 the Hubble constant as described in section 4.1.

From the linear perturbations in the Newtonian limit of the Λ CDM model presented in section 4.1 There is a linear relation between the peculiar velocity and the peculiar acceleration [10].

$$\vec{v}_p \simeq \frac{2}{3H\Omega_b} \Omega_b^{0.6} \vec{g}.$$

With H the Hubble parameter, Ω_b the baryonic density parameter and \vec{g} the peculiar acceleration with $\vec{g} = -\frac{1}{a}\nabla\phi$ with ϕ the gravitational potential and a the scale factor of the universe.

In other words we obtain an expression:

$$\vec{v}_p \propto -\nabla\phi.$$

If we take the divergence we obtain:

$$\nabla \cdot \vec{v}_p \propto -\nabla^2\phi.$$

But $\nabla^2\phi$ responds a Poisson-like equation with the density field ρ . We can finally write:

$$\nabla \cdot \vec{v}_p \propto -\rho.$$

We define the overdensity δ as the excess density in relation with the mean density of the universe. $\delta = \rho/\bar{\rho} - 1$ with $\bar{\rho}$ the mean density. δ is an adimensional quantity.

Our equation for peculiar velocity can be written as [1]:

$$\nabla \cdot \vec{v}_p = -H_0 f(\Omega_m, \Omega_\Lambda) \delta.$$

With \vec{v}_p the peculiar velocity field, H_0 the Hubble constant, Ω_m the matter density parameter, Ω_Λ the Dark matter density parameter, δ the excess density field and $f(\Omega_m, \Omega_\Lambda)$ a linear function of Ω_m and Ω_Λ .

With this relation we can directly link the peculiar velocity field and the excess density field. They will have related behaviours.

- Observational methods

We now proceed to describe some of the observational methods used Cosmicflows-2 [3]. To define Laniakea, the catalogue of peculiar velocities is detailed within 100 Mpc with data covering out to 400 Mpc from Earth. Six main methods were jointly used to estimate the galaxy distances.

1. Cepheid Period-Luminosity Relation (Cepheid PLR) applied to the Hubble Space Telescope (HST) distance scale key project. The HST data covers velocities of galaxies located with distances within 10 Mpc from Earth. Their observation can be difficult due to luminosities of neighbouring objects.
2. Tip of Red Giant Branch (TRGB). Red Giants are stars fainter than the Cepheids but are located in regions with small luminosity noise. This method, complementary to the Cepheid PRL method covers velocities of galaxies with distances within 10 Mpc from Earth.
3. Surface Brightness Fluctuation (SBF) method measures the variance in the distribution of light emitted in a galaxy. The noisy fluctuations we observe are proportional to the distance of the galaxy. This method covers velocities of galaxies located with distances within 100 Mpc from Earth.
4. Elliptical fundamental plane (EFP). EFP, is a set of correlations between properties (such as the mass or the luminosity) of elliptical galaxies. It is found as a relations between the effective radius, the central velocity dispersion and the average surface brightness. From the effective radius, it is possible to measure the angular size. The distance of the galaxy from the observer is then determined as an approximation. The quality of these measurements is not very good. Despite of that, it can be used for large samples of galaxies in a large volume.
5. Spiral luminosity-rotation. It is based on the Tully-Fisher relation, an empiric relation discovered by Tully and Fisher [11] between the mass of a spiral galaxy and its emission line width (its angular velocity). From luminosity of the spiral galaxy is extracted the distance of the galaxy. This method also provides less accurate distances, but can be applied to large sets of galaxies.
6. Supernovae Ia method. Supernovae Ia correspond to binary systems, with a white dwarf star. The white dwarf explodes and produces a very bright phenomenon which can been seen from great distances with good accuracy. The quality of these measurements is good and can cover large distances. Samples of these supernovas are small.

Finally, after the observation were made, a Wiener Filter was applied. A Wiener filtering method is a tool to reconstruct large scale structure information from some of the incomplete and noisy data [12]. It was applied by Tully et al. [1] to obtained a better Signal/Noise proportion in the peculiar velocity field from the Cosmicflows-2

observations.

Using this data of the nearby galaxies velocities, Tully et al [1] used the V-Web Algorithm to analyse where the cosmic flows converge and diverge. The boundaries of Laniakea correspond to the diverging cosmic flows.

4.3 The V-Web Algorithm

The contour of the region of Laniakea were reconstructed using the V-web Algorithm [1]. The basis of our method for the detection superclusters in cosmological simulation is the V-Web Algorithm, first introduced by Hoffman et al [13].

This algorithm is deeply connected to the velocities. Its core is the shear velocity tensor defined as:

$$\Sigma_{\alpha\beta} = -\frac{1}{2H_0} \left(\frac{\partial v_\alpha}{\partial x_\beta} + \frac{\partial v_\beta}{\partial x_\alpha} \right),$$

With v_α and x_α representing the α component the peculiar velocity and the comoving position, respectively.

$\Sigma_{\alpha\beta}$ can be represented by a 3×3 symmetric matrix with real values. This makes it possible to diagonalize it and find its 3 eigenvalues.

These eigenvalues λ_1 , λ_2 and λ_3 carry information about the motion of the fluid. By convention they are ordered such as $\lambda_1 > \lambda_2 > \lambda_3$.

the Universe present a filamentary structure with knots, filaments, voids and sheets. The V-Web scheme refers to the web classification based on how many of these eigenvalues are above a predefined threshold λ_{th} .

Respectively if 0, 1, 2 or 3 eigenvalues are above the threshold the point is classified as a void, sheet, filament or knot.

Our method uses indirectly these eigenvalues λ_1 , λ_2 and λ_3 through the Fractional Anisotropy and the trace of $\Sigma_{\alpha\beta}$.

4.4 Fractional Anisotropy (FA) and Velocity Divergence (VDH)

Our method uses on the Fractional Anisotropy (FA) and the trace of the Velocity Shear Tensor.

The Fractional Anisotropy (FA) was first conceived in the medical context by Peter Basser [14]. Libeskind et al [15] first introduced the FA in the cosmological context. The FA carries the information of how isotropic (or anisotropic) is a collapse or expan-

sion in a region. It has been used as a tracer of cosmic voids by Bustamante et al. [16].

The FA can be written as:

$$FA = \frac{1}{\sqrt{3}} \sqrt{\frac{((\lambda_1 - \lambda_3)^2 + (\lambda_2 - \lambda_3)^2 + (\lambda_1 - \lambda_2)^2)}{\lambda_1^2 + \lambda_2^2 + \lambda_3^2}},$$

With λ_1 , λ_2 and λ_3 the eigenvalues obtained from the V-Web algorithm. The FA is dimensionless and takes values between 0 and 1.

We interpret $FA = 0$ as an isotropic velocity distribution (where $\lambda_1 = \lambda_2 = \lambda_3$) and $FA = 1$ as a highly anisotropic velocity distribution. The nearer the FA is to 1, the more anisotropic a collapse or expansion and the nearer the FA is to 0, the more isotropic is the collapse or expansion.

- The trace of the Velocity Shear Tensor (VDH)

We describe in section 4.2 the divergence of the peculiar velocity field is proportional to the overdensity:

$$\nabla \cdot \vec{v}_p \propto -\delta$$

The trace of the velocity shear tensor $\Sigma_{\alpha\beta}$, $\lambda_1 + \lambda_2 + \lambda_3$ is equal to the divergence of the velocity divided by a factor H_0 , the Hubble constant.

$$\lambda_1 + \lambda_2 + \lambda_3 = \frac{-\nabla \cdot \vec{v}}{H_0}.$$

This is an dimensionless quantity, proportional to the overdensity δ .

We name the trace of $\Sigma_{\alpha\beta}$ or $\frac{-\nabla \cdot \vec{v}}{H_0}$ as VDH which states as Velocity Divergence divided by the Hubble constant.

If the VDH has a positive value, it means there is a collapse. The velocity flows are then going inwards converging. And if the VDH has a negative value, it means there is an expansion. The velocity flows are then going outwards, diverging.

From the results of Tully et al [1] we find that Laniakea is a structure with a VDH positive and an FA lesser than 1 as it is a structure where the velocity flows are converging and they are not converging anisotropically. We can then use the FA and VDH to find structures with these properties.

In the next section we describe how we use the FA and VDH to find superclusters in cosmological simulations.

5 Methods

We run our own cosmological N-body simulations. We proceed to compute the FA and VDH from the simulations data in a series of intermediate steps. Finally, our algorithm uses the FA and VDH computed from N-body simulations to detect superclusters.

Our code and algorithm are available for the public in Github <https://github.com/sercharpk/Monografia/>

5.1 Cosmological Simulations

To make our own cosmological simulations we use the N-Body simulation software Gadget-2 [17] widely used in the scientific community to generate different cubic boxes of dark matter particles (DM).

By DM particle we mean a computational particle which only interacts through gravity interaction. In the simulation each particle represents a fluid element. The DM particles are placed in a 3D grid layout first and then are perturbed following a Gaussian distribution before the simulation start, during the initial conditions generation. The perturbation is Gaussian because the distribution observed in the Cosmic Microwave Background (the CMB) is Gaussian.

Once the simulation starts, the particles interact only with gravity. Gadget-2 simulates this fluid. The simulations ran on the HPC cluster at UNIANDES. The main properties (boxsize, number of particles, number of CPUs used and time of the simulation) are described in table 1.

We are looking for structures of the same order of magnitude of Laniakea (radius of 80 Mpc/h [1]). Our simulations need to be of at least the same order of magnitude. We also need a sufficient number of DM particles to be able to describe the density field on the smaller scales probed by observations (around 5 Mpc/h). From these constraints we decided to run three simulations as listed in table 1.

Name	Boxsize [Mpc/h]	DM particles	# CPU	Time [hours]
Medium	500	512^3	48	10
Small (Dense)	250	512^3	48	6
Medium (Dense)	500	1024^3	48	80

Table 1: Different N-body Simulations

The initial conditions were generated using Volker Springel's N-Genic software. For the initial conditions we specify the cosmological constants described in section 4.1. They are described in table 2

Ω_0	Ω_Λ	Ω_b	H
0.3	0.7	0	0.7

Table 2: Cosmological Constants in the N-body Simulations. The different Ω are dimensionless quantities and H is in units of $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

As we specify in section 4.1, Ω_0 corresponds to the Cosmological matter density parameter in units of the critical density at red shift $z = 0$. Ω_Λ is the cosmological vacuum energy density (cosmological constant) in units of the critical density at red shift $z = 0$ (present). For the standard cosmological model, $\Omega_0 + \Omega_\Lambda = 1$. Ω_b is the baryon density in units of the critical density at red shift $z = 0$. Finally, H denotes the Hubble constant at red shift $z=0$ in units of $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

We then use different algorithms to identify the superclusters within the simulation. But first we must apply some intermediate steps.

5.2 Intermediate Steps

Our algorithm uses the FA and VDH computed from the N- body simulations¹. There are three steps involved before. First, the interpolation of the particle data informations into a grid. Second, the V-web computation on this grid. Finally, the computation of the FA and VDH on this grid.

In summary, our approach follows the steps:

1. Calculate the density field using interpolation on the simulation data.
2. Calculate the velocity field using interpolation on the simulation data.
3. Use finite differences to compute the velocity shear tensor $\sum_{\alpha\beta}$.
4. Diagonalize $\sum_{\alpha\beta}$ and find its eigenvalues λ_i and eigenvectors \hat{u}_i
5. Compute the FA and VDH grids as described in section 4.4

We first describe our interpolation method.

- The Cloud In Cell (CIC) interpolation method

¹ We also explored an approach of the magnitude of the velocity in the different simulations. We work with all the particles and points given by the simulation. As our final results are not based in this naive approach we leave it as an appendix here. It is explained in further detail in Appendix A.1

The Cloud In Cell (CIC) is a method to interpolate a quantity such as the density in a grid. The (CIC) method is a derivation of the Particle in Cell (PIC) method, a Particle Mesh (PM) method, first introduced by Francis Harlow in the 1950's in the field of computational fluid dynamics [18]. CIC is very popular among the scientific community [19].

We form, in our case, a N^3 grid. This grid is superposed with the cubic box with the results of our simulations. We then form particle clouds (cubes in our case) around each particle. For each particle cloud, each cell where the particle cloud superposes is assigned a weighted part of the physical quantity we look to mesh proportional to the superposed volume. A function can then be applied to smooth the obtained field.

We perform this CIC for the density field and the velocity field. In figure 1 a graphical explanation is provided for the 2-D case.

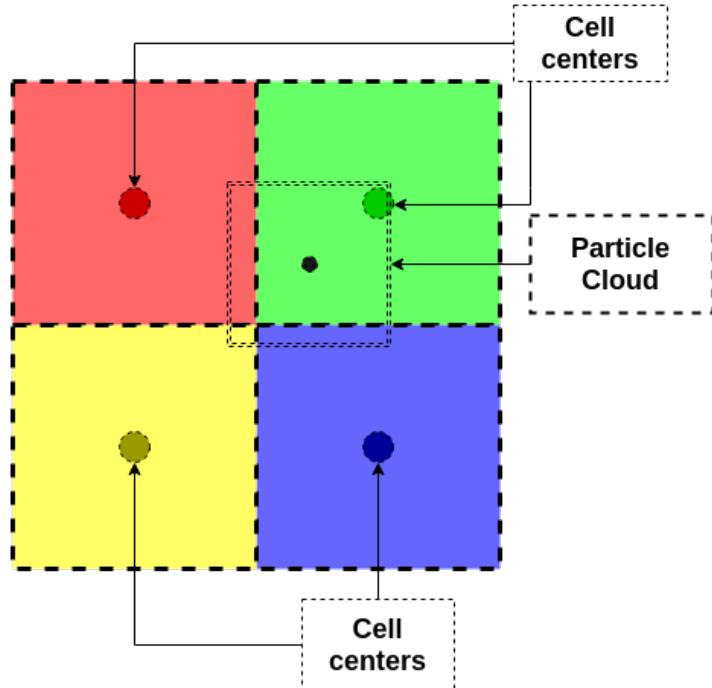


Figure 1: Visualization of the Cloud In Cell Method (CIC). The particle is the black dot. Each color represents a different cell of the grid. We form a particle cloud around the particle. Each cell is assigned a weight of the quantity meshed proportional to the area of the cloud superposed in each cell. So in this example most of the meshed quantity of the particle is assigned to the green cell (upper right), then equivalent parts are assigned to the red (upper left) and blue (down right) cells. And finally a smaller part is assigned to the yellow cell (down left)

- The V-Web scheme, the FA and the VDH

We describe the V-Web scheme in section 4.3 and the FA and VDH in section 4.4. The FA gives us a dimensionless quantity to analyse isotropy and the VDH gives us a also another dimensionless quantity which behaves in a similar fashion as the matter overdensity δ .

This methodology is very close with the approach used to define Laniakea [1] which is important to validate our later comparison of Laniakea with our detected structures.

Once we have completed these intermediate steps we can proceed to find the super-clusters in our data.

5.3 Superclusters Finding Algorithms

Our main algorithm is a region growing algorithm as described by Gonzalez [20] mainly used in image segmentation.

5.3.1 Region Growing Algorithm: Basic Description

The region growing algorithm is used to identify regions sharing common properties. First we identify points, as few as possible, which we know for sure are contained in these regions. These points are the seeds of our regions. We then define a certain growth predicate, if the neighbours of a seed validate this predicate they are considered as members of the group. These new members are then treated as seeds and ask their own neighbours if they validate the predicate. This goes on until all the points have been evaluated or until a limit defined by the user is reached.

We can write in a more formal way:

$f(x, y, z)$ denotes the input data. $S(x, y, z)$ denotes a seed array, with value 1 where the particle can be define as a seed and 0 where not. $S(x, y, z)$ is the same size of $f(x, y, z)$. Q denotes a predicate which is to be apply to the input data and determines if the region which starts at the seeds grows or not.

1. We search for seeds from $f(x, y, z)$ based in a criterion
2. For each detected seed, we change the values of $S(x, y, z)$ to 1 at the seed's coordinates.
3. For each detected seed, we iterate its neighbors if they validate the predicate Q .
4. If the particle c at x_c, y_c, z_c satisfies Q , it is marked as part of the region, $S(x_c, y_c, z_c) = 1$
5. Apply the algorithm to c , and so on (Recursion).

The algorithm is based on a connectivity concept (definition of neighbours). Here we are working on a 4-connectivity. 4 connectivity means that no diagonal particles are considered as neighbours. The following matrix shows this principle in a 2 dimensional case. Only the elements labelled as 1 are considered as neighbours of the center C .

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & C & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

So, if a particle is in coordinates (x, y, z) its neighbours are $(x - 1, y, z)$, $(x + 1, y, z)$, $(x, y - 1, z)$, $(x, y + 1, z)$, $(x, y, z - 1)$ and $(x, y, z + 1)$.

Another widely used connectivity is the 8-connectivity. As it is intuitive, in 8-connectivity, the diagonal particles are also considered as neighbours. We decided to use 4-connectivity to simplify our own implementation.

5.3.2 Region Growing Algorithm with the FA and the trace of the velocity shear tensor

Our implementation is based in the FA and the VDH. It uses 4-connectivity as described before.²

- Seeds

We are looking for overdense structures. Therefore we will look for particles with $VDH \geq 1.0$. We also want to analyse how the FA fits. We know that the centres of our structures should be fairly isotropic so we will explore seeds with FA lesser than a threshold. We will perform several runs of the algorithm, so we can explore the different criterion for the choice of the seeds. These can be seen in table 3.

FA_{seed}	VDH_{seed}
< 0.5	> 1.0
< 0.6	> 1.0
< 0.7	> 1.0
< 0.8	> 1.0
< 0.9	> 1.0

Table 3: Different values of the FA and VDH for the choice of the seeds

- Labelling the regions

We label groups as a function of its respective seed to extract properties (volume, shape, mass) for each one of them.

² A simplified version code can be seen in appendix A.2.

- Predicate Q

We define, as we did for the choice of the seeds, thresholds for the FA and for VDH. If the neighbour cell validates these thresholds, and hasn't been labelled yet, it is labelled as the group of the seed particle. So in other words:

$$Q : (FA(x, y, z) \leq FA_{growth}) \& (VDH(x, y, z) \geq VDH_{growth})$$

As we are looking for overdense regions (no voids) we define the threshold for VDH as 0.0. As we are exploring the different values for the FA. each run will follow a row of the table 4.

FA_{growth}	VDH_{growth}
< 0.5	> 0.0
< 0.6	> 0.0
< 0.7	> 0.0
< 0.8	> 0.0
< 0.9	> 0.0

Table 4: Different values of the FA and VDH for the growth of the regions

Finally to find the groups we used the Friends-of-Friends (FoF) algorithm.

The FoF algorithm looks at the distances between points in a given set. If the distance between a point a and a point b is less than a predefined threshold, a and b now form a region. For a region to be created by the FoF algorithm it also needs to have a predefined minimum number of members.

We choose as set of points all the particles which validate the predicate Q (following table 4, as maximum distance the distance between one particle and its neighbours as described by 4-connectivity. And we use the FoF algorithm to form the groups.

We search for the seeds following table 3 and obtain a list of the seeds. We then proceed to validate which of the groups obtained by the FoF is a valid group. To be considered as a valid group, it needs to contain at least one seed particle. This procedure allow us to take care of the case where two seeds form two groups which eventually get together. Now we can make sure these groups are merged into one and there is no need to search for this case individually.

Once we find the superclusters using this algorithm we proceed to do the following:

1. Extract the properties (volume, shape, mass) from the superclusters
2. Compare the superclusters detected in the simulation to Laniakea

6 Results

The following results correspond to our **Small (Dense)** described in table 1.

6.1 Fractional Anisotropy and Divergence in the Simulation

6.1.1 Fractional Anisotropy

We compute the FA after interpolating the density and velocity fields on a 256^3 grid using the CIC method and applying the V-Web algorithm. Figure 2 shows the FA histogram. Figure 3 shows a 2-D cut of the FA grid.

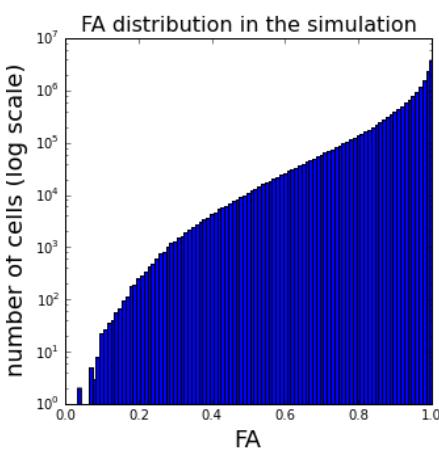


Figure 2: FA Histogram in the simulation

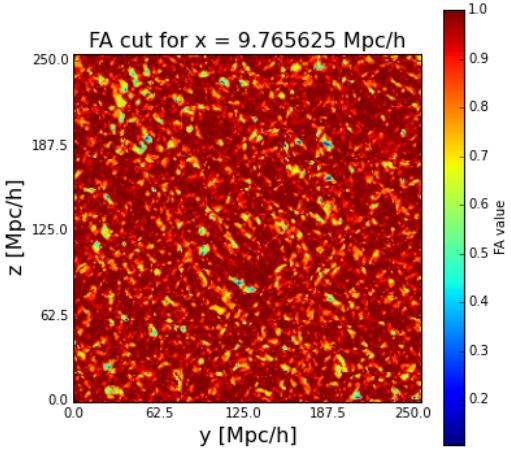


Figure 3: A cut of the FA in the x axis

As we saw in section 4.4, we expect to have few highly isotropic particles. The histogram in figure 2 we confirms this expectation.

The 2-D cut of the FA in figure 3 shows the filamentary structures we would expect of the Cosmic Web. This cut is very similar to the ones presented by Bustamante et al. [16] in their work. This is a good indicator that our simulation, CIC procedure V-Web application and FA forming are correctly executed. Bustamante et. al explain that void regions should have a $FA < 0.95$. We expect superclusters to have a similar isotropic behaviour as voids. Therefore we explore similar FA values in our method. To make sure we do not find voids we make use of the VDH.

6.1.2 VDH, Velocity Divergence

As we did for the FA, we visualize the histogram (Figure 4) and a 2-D cut (Figure 5) of the VDH.

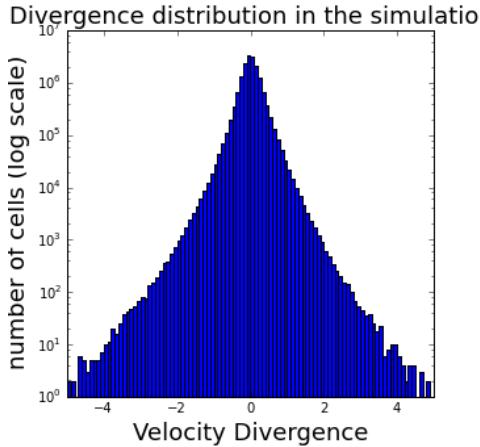


Figure 4: VDH Histogram in the simulation

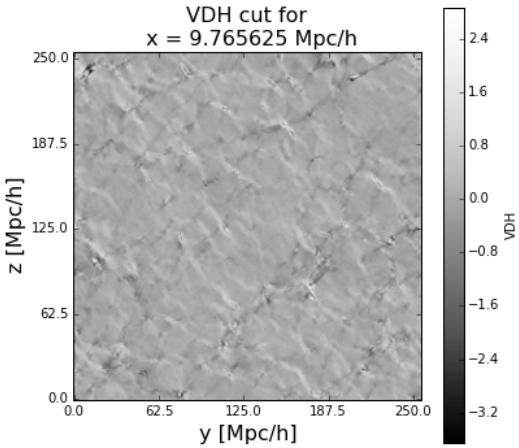


Figure 5: A cut of the VDH in the x axis

The VDH distribution is symmetric around 0.0. As we are searching for overdense structures (superclusters) we can then diminish our search space to half of it only by selecting the points with $VDH > 0.0$.

Observing a 2D cut of the Velocity Divergence as shown in figure 5 also gives a good idea of good idea of the Cosmic Web as we can easily recognize filaments (highly underdense, $VDH \leq -1.0$) and highly overdense regions ($VDH \geq 1.0$). Our candidates for seeds of supercluster structures are the highly overdense regions with $VDH \geq 1.0$.

6.2 Region Growing Algorithm and FoF with the FA and VDH

We apply all the combinations for thresholds in table 3 (seeds) and table 4 (growth).

Our algorithms run successfully on a regular laptop. Two examples of detected superclusters are shown individually on figure 6 and at the scale of the simulation (of size 250 Mpc/h) on figure 7. Finally we can see all the detected regions on the simulation with one selection of thresholds in figure 8.

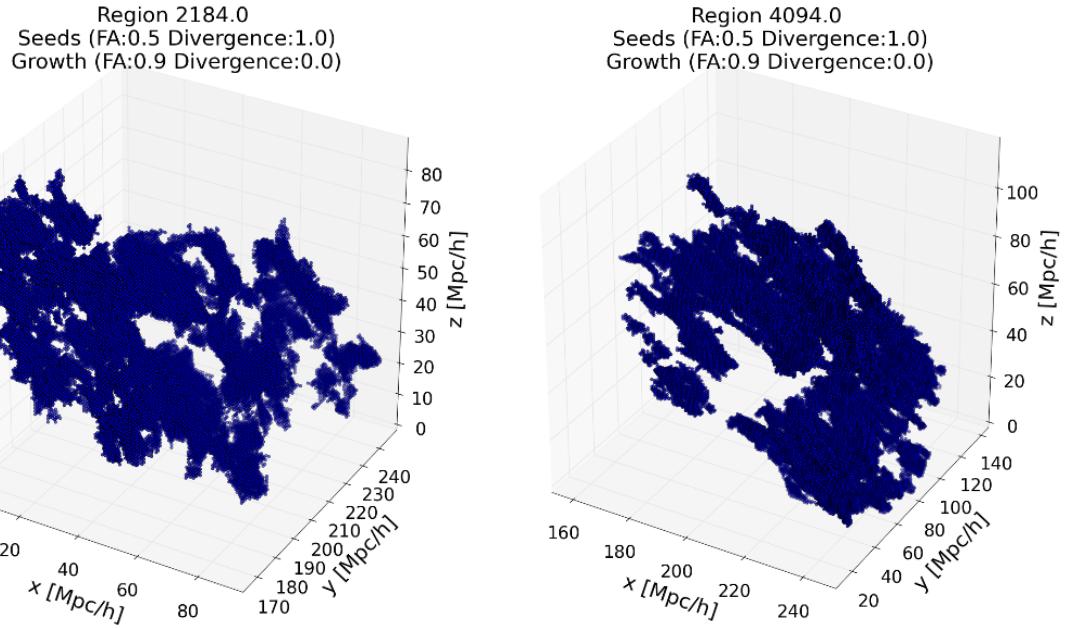


Figure 6: Two detected superclusters in the Small (Dense) simulation with thresholds: Seed (FA: 0.5, Div: 1.0) and Growth (FA: 9.0, Div:0.0).

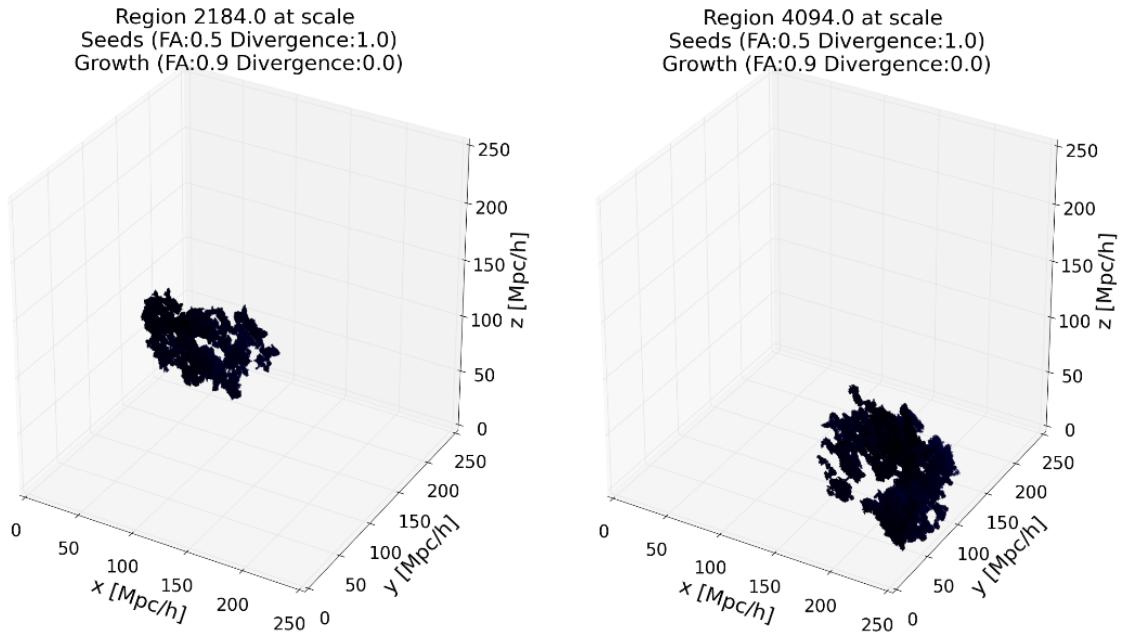


Figure 7: Two detected superclusters in the Small (Dense) simulation at scale with thresholds: Seed (FA: 0.5, Div: 1.0) and Growth (FA: 9.0, Div:0.0).

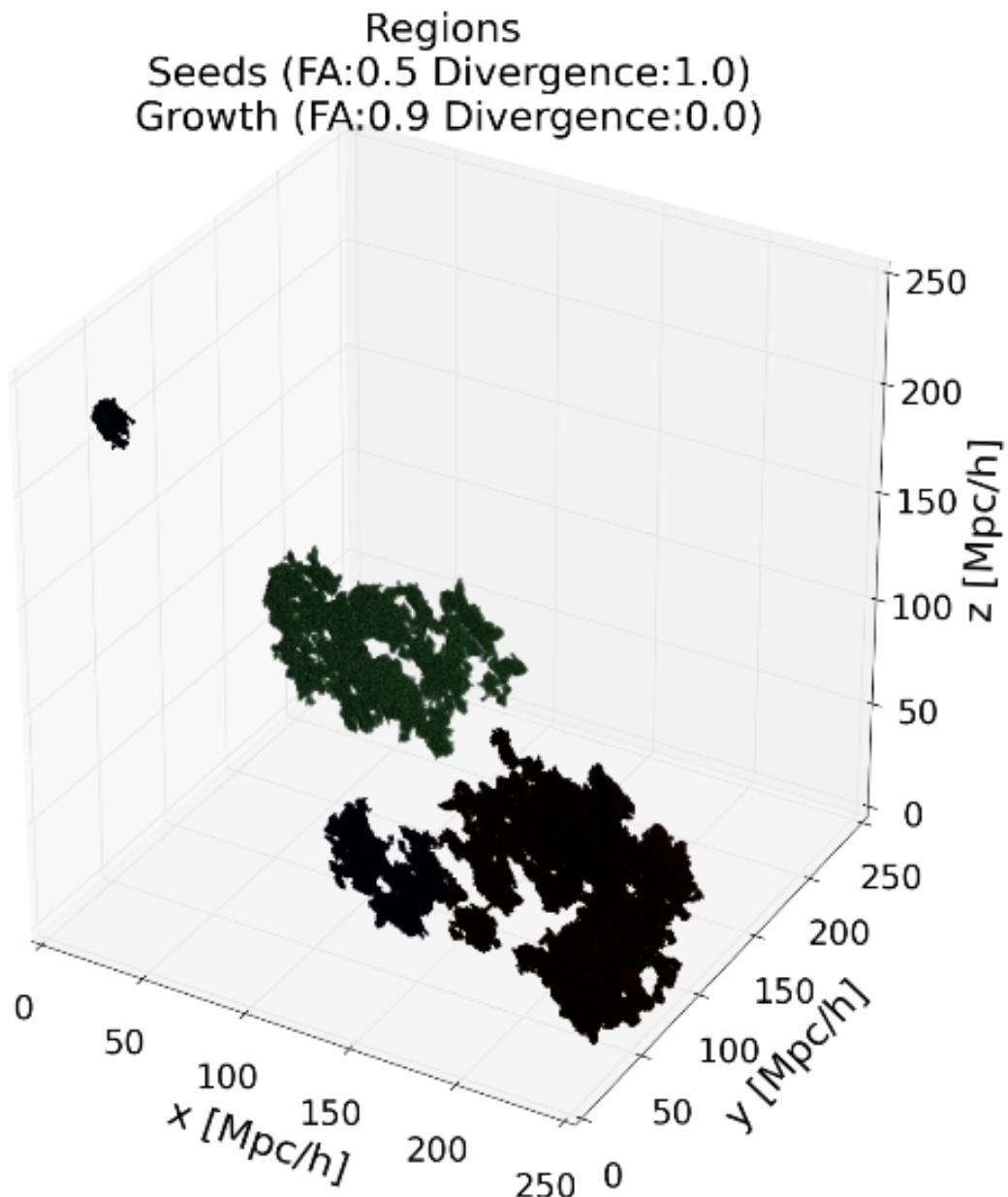


Figure 8: All the superclusters detected in the Small (Dense) simulation with thresholds: Seed (FA: 0.5, Div: 1.0) and Growth (FA: 9.0, Div:0.0). Different colors indicate different regions.

We can observe from figures 8, 7 and 6 that the structures have different shapes and sizes. There are no hints for a specific shape and size. In the next section we analyse with more detail how FA thresholds influence these results.

6.2.1 Influence of the FA in the seeds

We proceed to observe how changing the FA threshold for the seed selection process described in section 5.3 influences the detected structures.

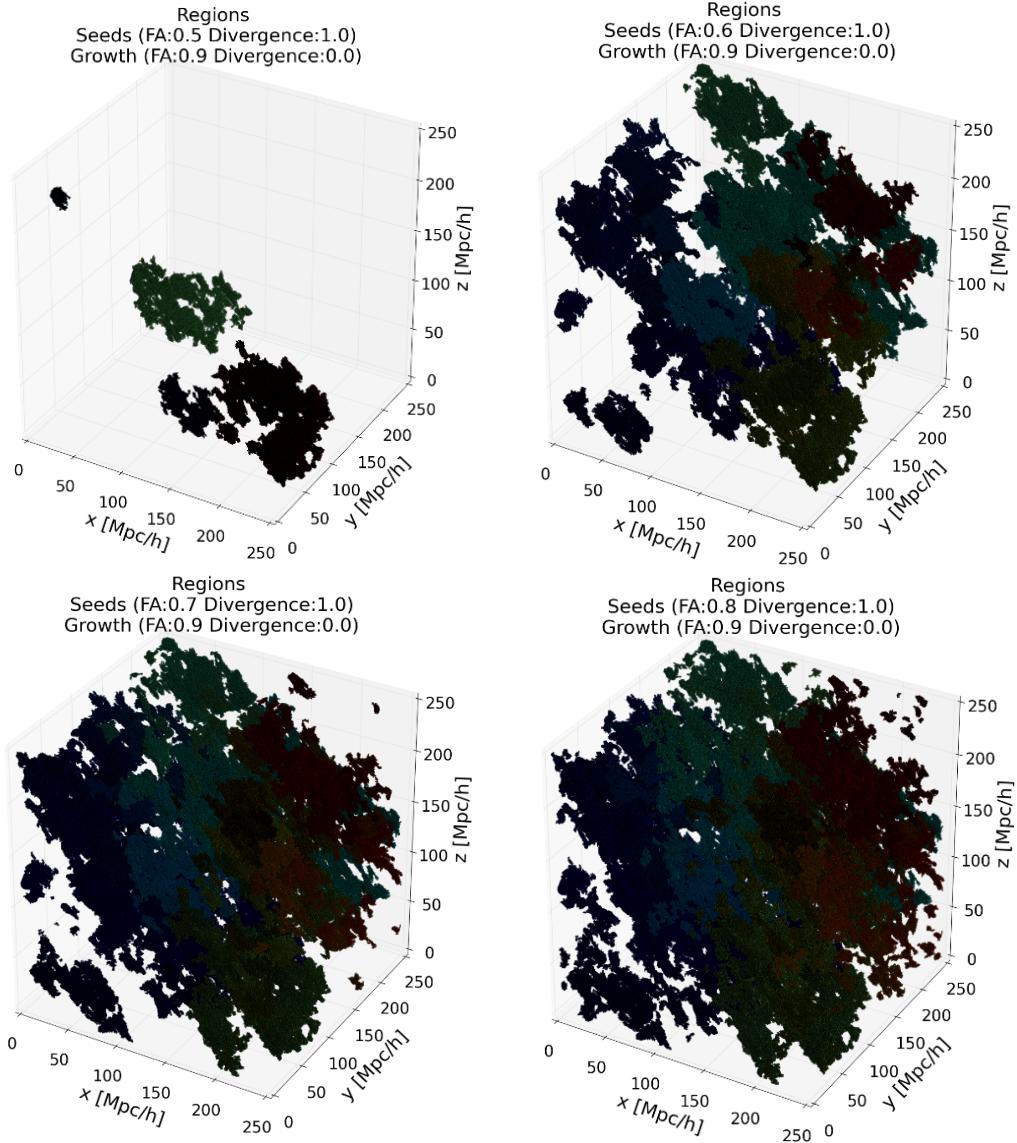


Figure 9: Effect of increasing the FA threshold in the seed selection. From upper left to down right the FA for the seeds are: 0.5, 0.6, 0.7 and 0.8. The growth FA is 0.9 and $\text{tr}(\sum_{\alpha\beta})$ is 0.0. The $\text{tr}(\sum_{\alpha\beta})$ for the seeds is 1.0.

In figure 9, we observe qualitatively increasing the FA threshold in the seed selection increases strongly the number of regions detected. This indicates that the number of seeds for the algorithm increases as well with the FA threshold in the seed selection. The FA histogram (figure 2) shows this as well as most part of the particles have a high value of FA.

6.2.2 Influence of the FA in the growth

We now observe how changing the FA threshold for the growth space affects our results.

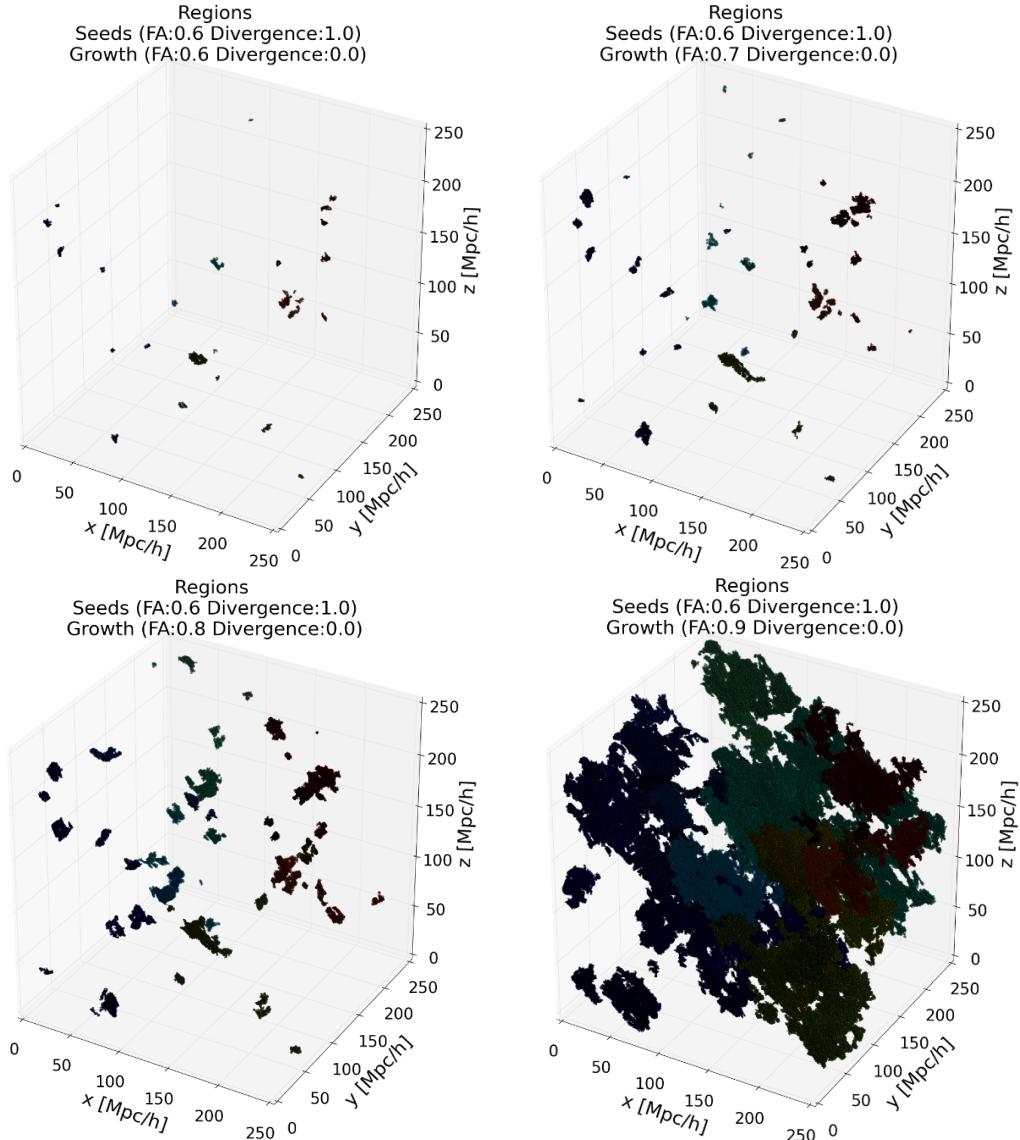


Figure 10: Effect of increasing the FA threshold in the growth. From upper left to down right the FA for the growth is: 0.6, 0.7, 0.8 and 0.9. The seed FA is 0.6 and VDH is 1.0. The VDH for the growth is 0.0.

Figure 10 shows qualitatively how augmenting the FA thresholds for the growth changes drastically the volumes of the regions detected, specially from the transition form FA threshold 0.8 to 0.9.

6.3 Region properties and comparisons with Laniakea

Let us remember the properties of Laniakea we can compare our detected structures with are described in table 5

Radius (Mpc)	80
Mass (Solar Masses)	1×10^{17}
Shape	Oblate top

Table 5: Properties of Laniakea we can compare with as described by Tully et al. [1]

6.3.1 Shape of the regions

To study the shape of the regions in a quantitative way we proceed, for each region, to form the Inertia Tensor, calculate its three eigenvalues λ_a , λ_b and λ_c and compare them. We order the eigenvalues from the smallest to the largest eigenvalue in order to get $\lambda_a < \lambda_b < \lambda_c$ and later observe how these eigenvalues differ from each other.

For this we plot the difference between λ_a and λ_b versus the difference between λ_b and λ_c . We obtain plots as the ones in figures 12 and 13 and we find that part of the regions we detected have the same shape as Laniakea (oblate top).

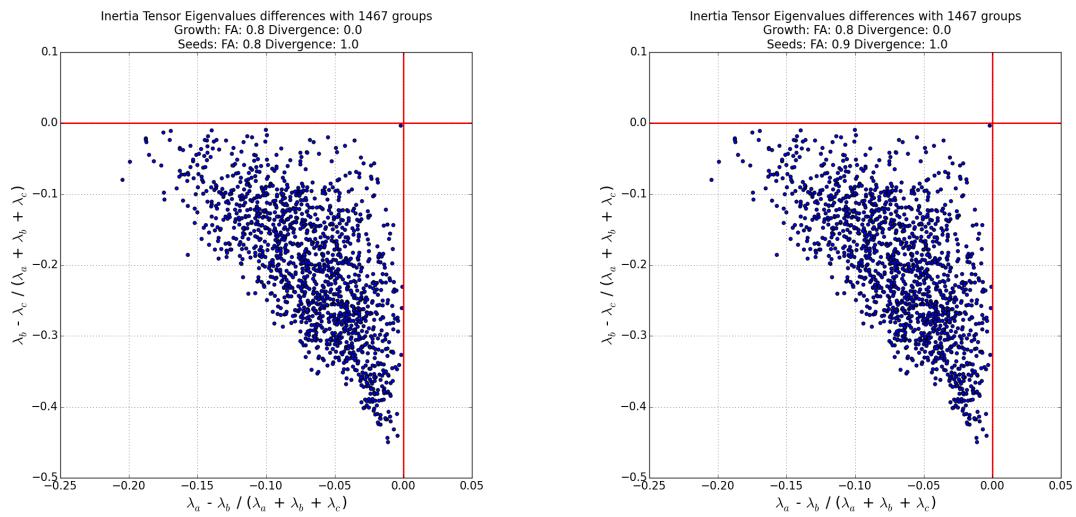


Figure 11: Inertia Tensor eigenvalues differences (normalized) for two different cases. All the graphs are alike even for different values of FA and VDH.

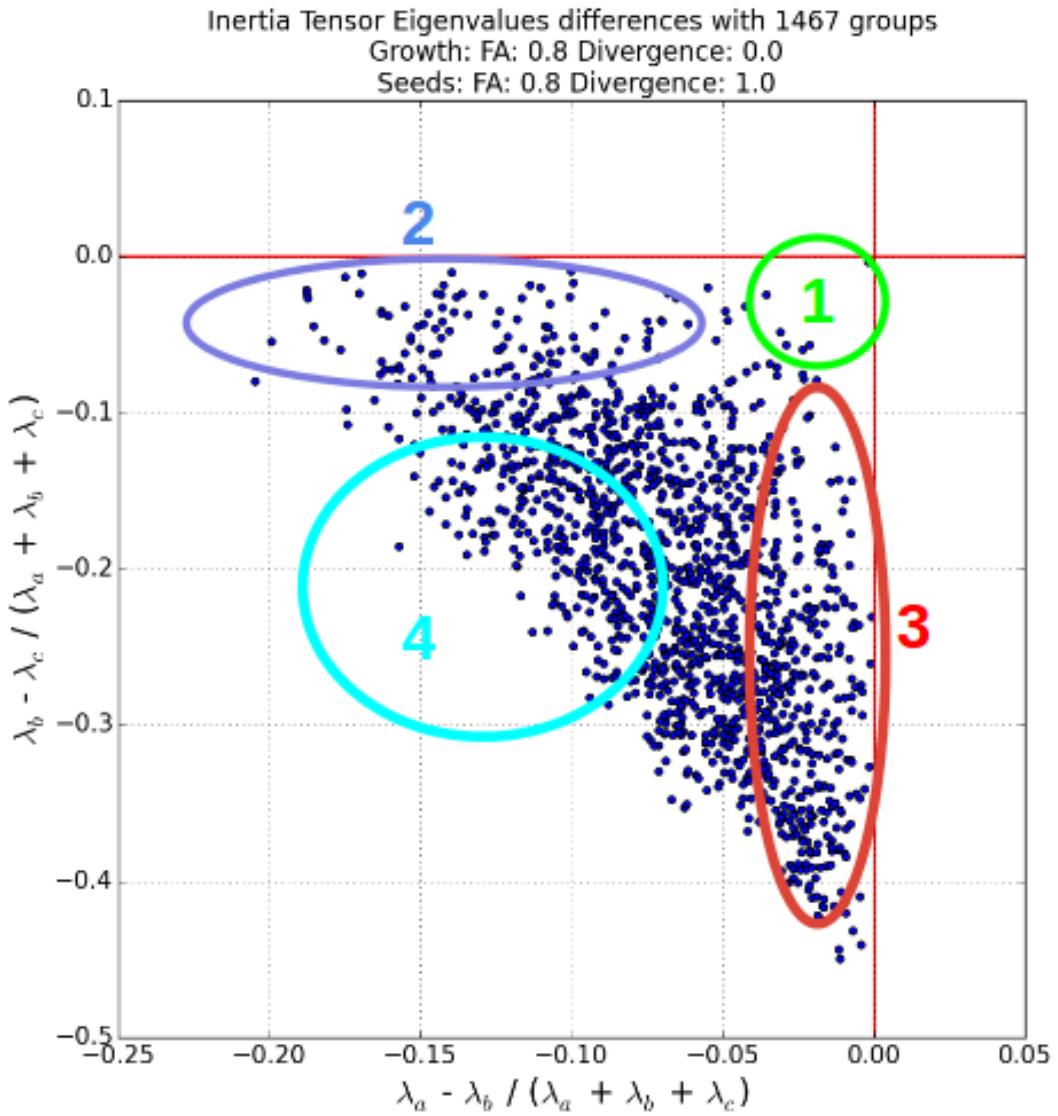


Figure 12: Inertia Tensor eigenvalues differences. We identify the 4 different cases. The case 1 corresponds to a sphere, where all the three eigenvalues are equal. The case 2 corresponds to a prolate top (an elongated sphere), where the two larger eigenvalues are equal. The case 3 corresponds to an oblate top (a flattened sphere), where the two smaller eigenvalues are equal. And the case 4 corresponds to an asymmetric top where all three eigenvalues are different. Laniakea corresponds to an oblate top case.

We have compared the shape of our regions with Laniakea. We now proceed to compare the volume (size) of our regions with Laniakea.

6.3.2 Volume distribution

Laniakea is a spheroid of radius 80 Mpc [1] which corresponds to a radius of 56 Mpc/h. We can then state it has approximatively a volume of $7.36 \times 10^5 [Mpc/h]^3$ which is smaller than the total volume of our simulation $1.56 \times 10^7 [Mpc/h]^3$.

The detected structures vary in volume and shape. The number and size of the detected structures strongly depend on the choice for FA thresholds (seed and growth) as it is presented in sections 6.2.1 and 6.2.2. We know that if we choose as FA thresholds 1.0, all the simulation volume is detected as one region. This result is of no interest here as we know it does not corresponds to reality. We proceed to visualize the distribution of detected structures volume for FA thresholds where we have a significant number of detected structures.

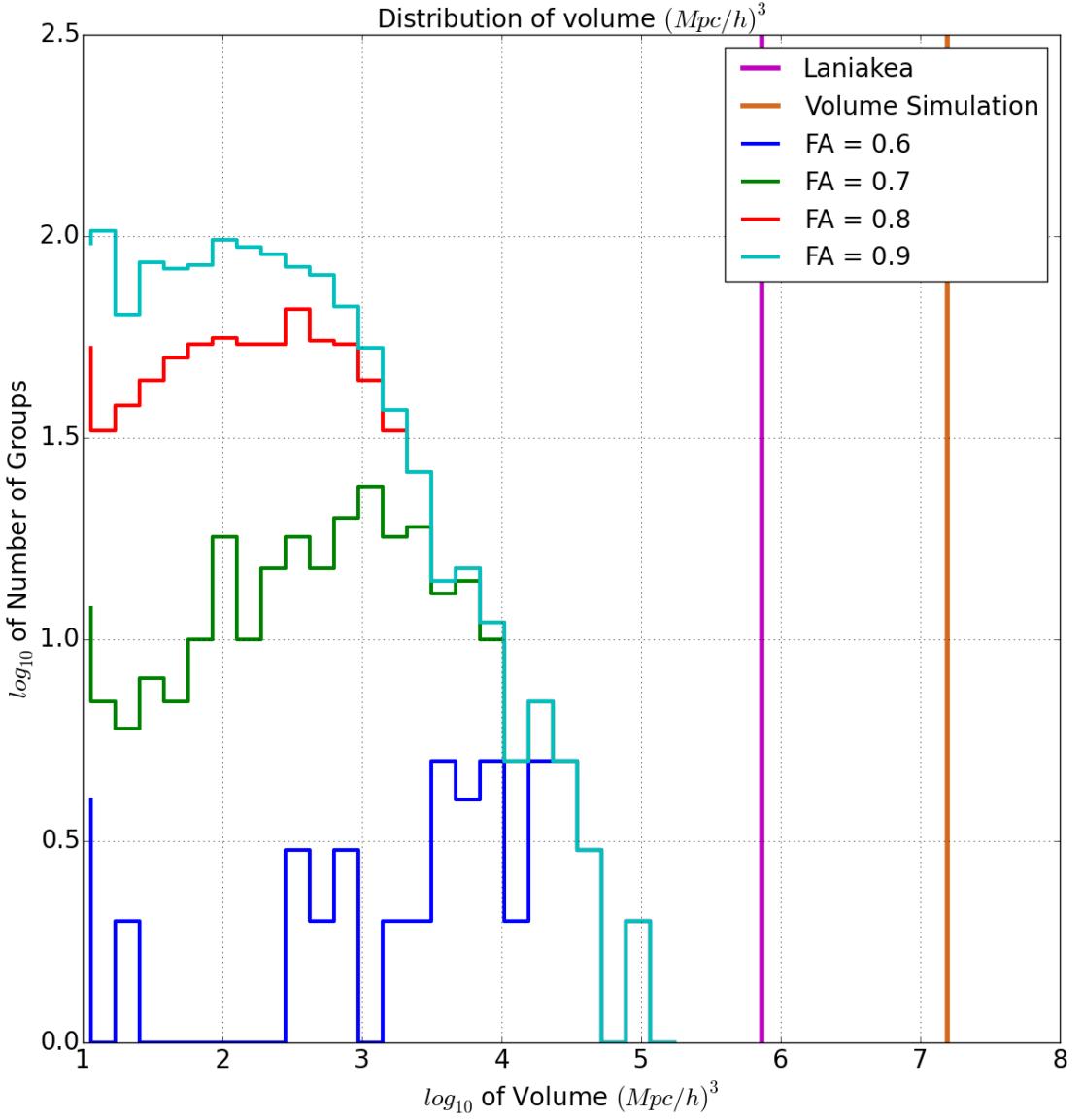


Figure 13: Distributions of volumes for different seed FA thresholds. The growth FA threshold is 0.9. The growth VDH is 1.0. The VDH for the seeds is 0.0. We can see how Laniakea is larger than the detected structures and smaller than the total simulation volume. We can see that as we increase the seeds FA threshold there are more regions of small volume detected and that the larger regions detected are always detected independently of the seed FA threshold.

In figure 14 we show how Laniakea is always larger than the detected superclusters in our simulation and smaller than the total simulation volume. We can see how the larger regions are detected independently of the seed FA threshold in place. This shows that our method is robust to this parameter.

7 Conclusions and Future Work

Our method, based on the Fractional Anisotropy (FA) and the Velocity Divergence, successfully finds structures in cosmological simulations.

At first it seems very sensitive to its parameters (FA and Divergence thresholds). An exhaustive calibration can must be performed before interpreting the results. Nevertheless, We find that the larger structures (which are the ones we are interested in) are detected independently of the seed FA threshold parameter. This shows that our method is robust.

We state that Laniakea is, in the Cosmological context, atypically larger. We visualise that, as long as the growth FA threshold is not the maximal FA value (1.0) and that the volume of our detected structure is not equal to the available growth space, Laniakea's volume is always larger than the detected structures volumes.

However, we only provide here results for the Small (Dense) simulation. We generated data from the two other simulations exposed in table 1. A detailed analysis of how the detected structures behave for the different simulations is to be done.

A more exhaustive FA thresholds analysis is to be done similar to the one performed in this work. The behaviour for Growth FA thresholds for values between 0.9 and 1.0 is to be examined in more detail.

In order to have a larger sample of detected large structures, a bigger simulation is to be performed with parameters Boxsize: 1000 Mpc/h and 2048^3 DM particles.

Finally, to have a deeper perspective of Laniakea, it is of great interest to apply our method to the observational data which was used to define Laniakea in order to validate it. We could then obtained the seed and growth FA and VDH thresholds which corresponds to Laniakea.

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A Appendix

A.1 Approach by the Speed

A.1.1 Region Growing Algorithm with the Speed

We present here a naive way to approach to the problem.

1. We calculate the magnitude of the velocity (the speed) of each particle.
2. We look for regions where the center has the highest speed and from it the speed decreases while the particle is further away from the center.
3. We define the limits of this region the regions where the speed begins to increase with the distance from the center.

We use a region growing algorithm, a simple image segmentation algorithm used to identify different regions in a image [20].

$f(x, y, z)$ denotes the input data. $S(x, y, z)$ denotes a seed array, with value 1 where the particle can be define as a seed and 0 where not. It is the same size of $f(x, y, z)$. Q denotes a predicate which is to be apply to the input data and determines if the region which starts at the seeds grows or not. Here Q denotes: "if the speed of the close particle is lower than the marked one but greater than a threshold, mark that particle and continue."

1. We choose seeds to begin the growth. In this case we choose the particles with high velocities. Here

$$|v| > th_{high}$$

2. We open a window of particles to look for 2 close particles from the seed s which do not form part of $S(x, y, z)$. Here:

$$window = 20000$$

3. Once we found the 2 close particles c we apply the predicate Q . Here:

$$Q := |v_s| > |v_c| > |th_{low}|$$

4. If the particle c satisfies Q , it is marked, $S(c) = 1$
5. Apply the algorithm to c , and so on (Recursion).

Here we defined empirically different thresholds in order to observe the results.

First we used:

$$th_{low} = v_{min} + \frac{\sigma_v}{2} \text{ and } th_{high} = v_{max} - \sigma_v .$$

Secondly we used:

$$th_{low} = \vec{v} + 2\sigma_v \text{ and } th_{high} = \vec{v} + 8\sigma_v .$$

The first version of the algorithm was written in python using the module pyGadgetReader [21] to read the data and transform it to NumPy arrays in Python. In Appendix A.1.3 we attach the source code used.

A.1.2 Results of the Speed Approach

This approach was tested on the simulation of boxsize 500 Mpc/h and 512^3 dark matter particles (DM) described in table 1.

We first visualize the speed histogram in figure 15.

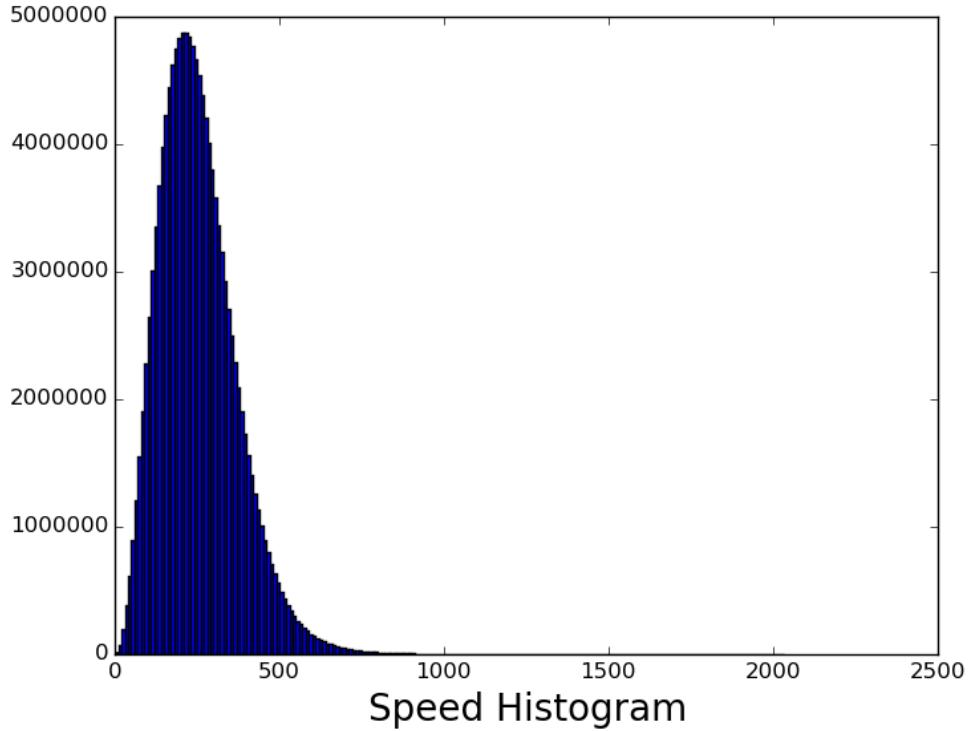


Figure 14: Speed Histogram of the simulation

The histogram resembles a Poisson distribution. This is a consequence of the perturbations generated in the initial conditions generation.

v_{max}	2023.87
v_{min}	0.34
σ_v	117.75

Table 6: Properties of the speed distribution

Our hypothesis is based on the most direct implication of gravity. The lower speed particles will mainly be in the border regions and the higher speed particles will be in the center regions.

For this we must choose a threshold for low velocities and a threshold for high velocities. We obtain: $th_{low} = v_{min} + \frac{\sigma_v}{2} = 59.2$ and $th_{high} = v_{max} - \sigma_v = 1906.12$

We first have a look at the particle distribution of particles with speed higher than the threshold for low velocities.

Magnitudes de Velocidades menores a 59.216707265

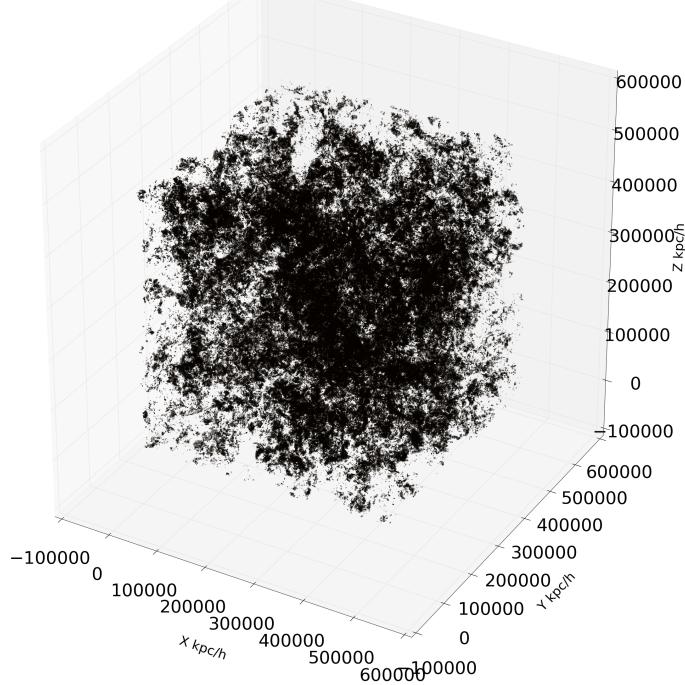


Figure 15: DM particles with $|v| < 59.2$

There seem to be some structures, but it is not clear enough. This is due to the high number of DM particles used in this simulation (512^3).

We produce cuts in the z-direction to visualize in 2-D the speed distribution.

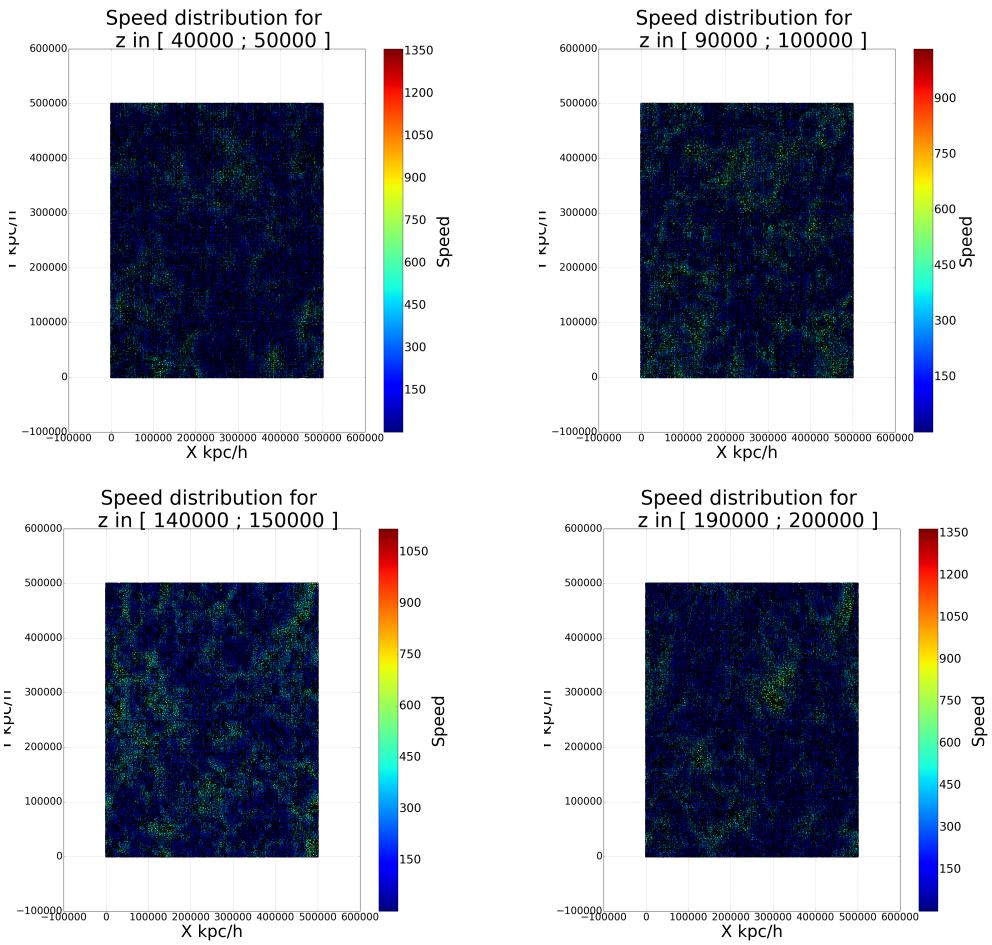


Figure 16: 2D slices of the speed distribution

We can observe in figure 17 intuitive signs of structures related to the speed distribution. We can observe filament structures

It is more clear if we observe from one side the particles with speed greater than a th_{high} and the particles with speed lesser than a th_{low} .

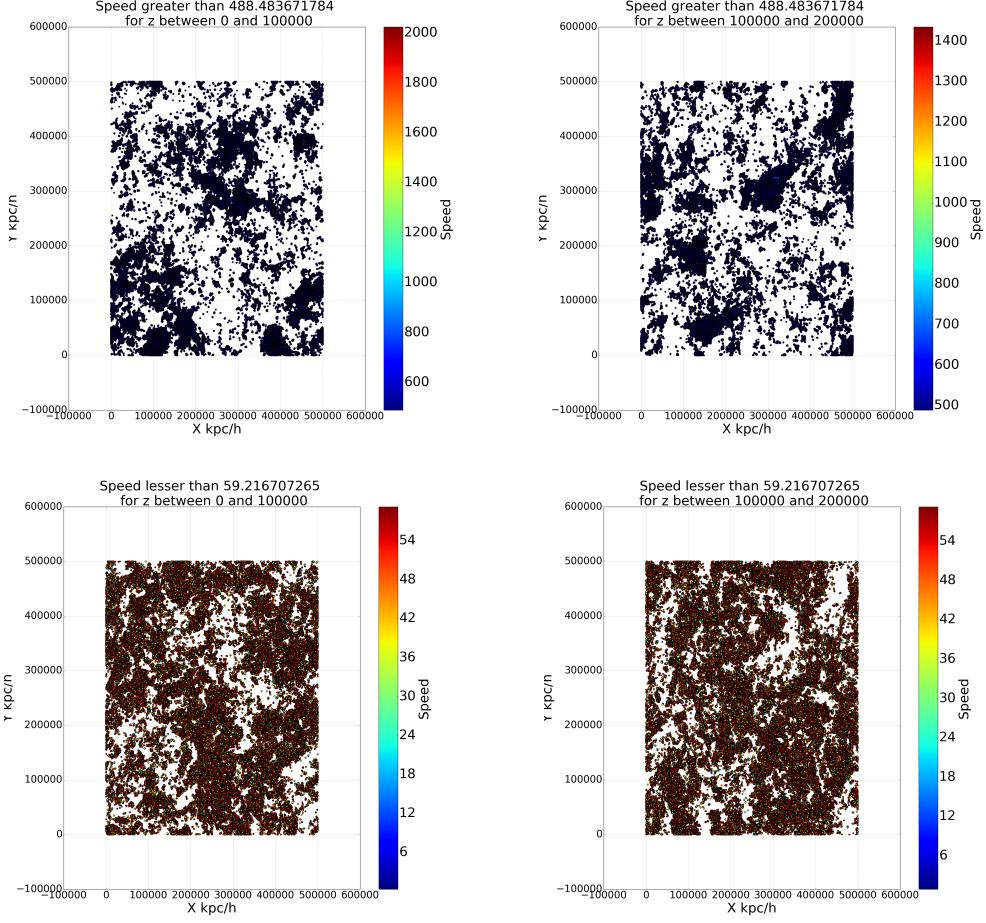


Figure 17: 2D slices of the speed distribution. The upper slices with $|\vec{v}| > th_{high} = 488$ and the two lower slices with $|\vec{v}| < th_{low} = 59$ for the same cuts, $z = \{100, 200\} Mpc/h$

As we can see in figure 18 the cuts with low and high thresholds are complementary. It is evident that structures are present.

Finally we apply our region growing algorithm and we obtain the following results.

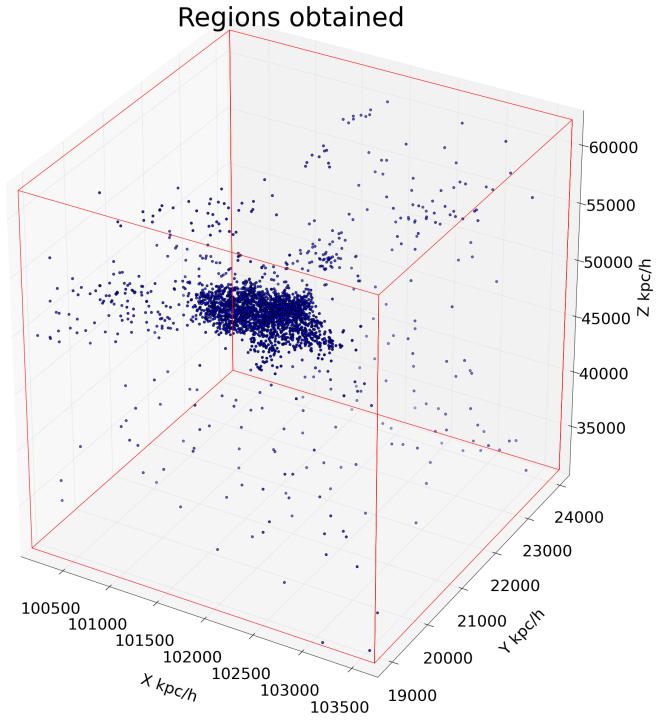


Figure 18: Region identified by the algorithm

And, changing the thresholds to: $th_{low} = \bar{v} + 2\sigma_v = 488.48$ and $th_{high} = \bar{v} + 8\sigma_v = 1195.0$

We obtain:

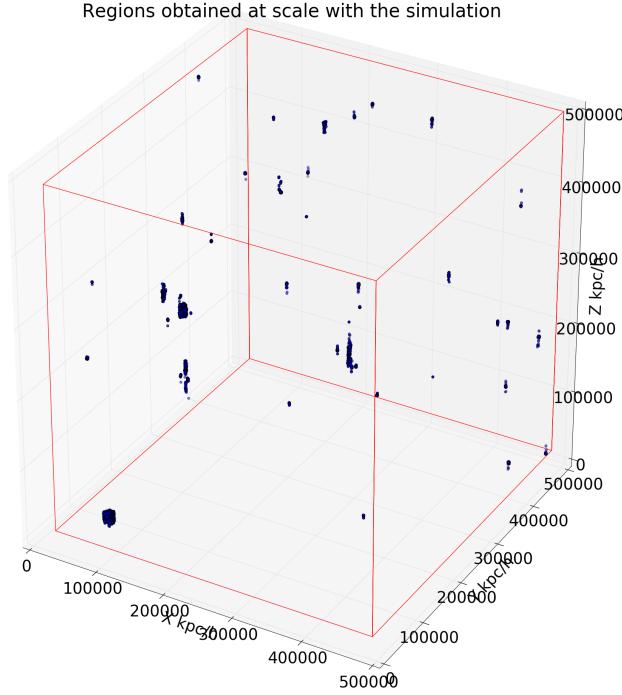


Figure 19: Region identified by the algorithm with the second thresholds

We have different observations:

1. The details of the regions in figures 19 and 20 seem accurate with what we would expect. It is more dense in the center and less dense in the borders.
2. The regions obtained have a dimension of only a few Mpc/h (Laniakea's dimension is two orders of magnitude higher). This could mean that Laniakea is atypical.
3. There is first only one region identified. This indicates than all the high velocity particles are congregated in only one region (the detected region). If we set lower th_{high} we can see we get more regions.
4. This is certainly a good naive approach to the problem. Nevertheless for more accurate results it needs to be drastically changed.

The script in Appendix A.1.3 was executed in a student's laptop and later on the HPC cluster. For a significant number of particles, the laptop limited memory (8GB) crashes. Running in the HPC solves this issue.

A.1.3 Code example for Region Growing Algorithm - Approach by the Speed

Listing 1: Region Growing Algorithm Code - Speed

```

from pygadgetreader import *
import matplotlib
matplotlib.use('Agg')
from mpl_toolkits.mplot3d import Axes3D
from matplotlib import cm
import matplotlib.pyplot as plt
import numpy as np

snap_file_sim='/lustre/home/ciencias/fisica/pregrado/sd.hernandez204/Outputs/28_02_2016/snapshot_005'
requested='dmcount'
n_dm=readheader(snap_file_sim,requested)
print "Number_of_Dark_Matter_particles"
print n_dm

array_pos=readsnap(snap_file_sim,'pos','dm')
array_vel=readsnap(snap_file_sim,'vel','dm')

array_magnitud_vel=np.zeros(n_dm)
for i in range(0,n_dm):
    array_magnitud_vel[i]=np.linalg.norm(array_vel[i,:])
print "The speed array has been calculated"

longitud = (int)(array_pos.shape[0])
print "The length of the position array should be the same number of particles"
print str(longitud)

array_copy=array_pos.copy()
array_results=np.zeros((longitud,4))
array_results[:,:-1] = array_copy
print "The results array has been created."

vel_max = np.max(array_magnitud_vel)
vel_min = np.min(array_magnitud_vel)
vel_std = np.std(array_magnitud_vel)
vel_mean = np.mean(array_magnitud_vel)

print "Vel_max, -Vel_min, -std_vel, -vel_mean"
print vel_max, vel_min, vel_std, vel_mean
thresh_v_hig = vel_mean + 8.0 * vel_std
thresh_v_low = vel_mean + 2.0 * vel_std
print "Thresh_high, -Thresh_low"
print thresh_v_hig, thresh_v_low

#Gets the indexes of the seeds
indexes = np.where(array_magnitud_vel > (thresh_v_hig))[0]
print 'There are: '+str(indexes.shape[0])+' seeds'

def region_accretion(l):
    #Resize the window if necessary
    size_window_up = n_window
    if(((l+size_window_up) > longitud)):
        size_window_up = longitud - l
    size_window_down = n_window
    if(((l-size_window_down) < 0)):
        size_window_down = 0 + l
    distancias_l_up = np.zeros(size_window_up)
    distancias_l_down = np.zeros(size_window_down)
    distancias_l_up[:] = 1000000
    distancias_l_down[:] = 1000000
    if(size_window_up == size.window_down):
        for k in range(0,size.window_up):
            if(((l+k) < longitud)):
                if((array_results[l+k,3] == 0)):
                    distancias_l_up[k]=np.sqrt(np.sum((array_results[l,0:2]-array_results[l+k,0:2])**2))
                if(((l-k) > 0)):
                    if((array_results[l-k,3] == 0)):
                        distancias_l_down[k]=np.sqrt(np.sum((array_results[l,0:2]-array_results[l-k,0:2])**2))
    else:
        for k in range(0,size.window_up):

```

```

    if(((l+k) < longitud)):
        if((array_results[l+k,3] == 0)):
            distancias_l_up[k]=np.sqrt(np.sum((array_results[1,0:2]-array_results[1+k,0:2])**2))
    for k in range(0,size_window_up):
        if(((l-k) > 0)):
            if((array_results[l-k,3] == 0)):
                distancias_l_down[k]=np.sqrt(np.sum((array_results[1,0:2]-array_results[1-k,0:2])**2))
    distancias_l_up[0]=1000000
    distancias_l_down[0]=1000000
    closest_l_up = np.min(distancias_l_up)
    index_closest_up = np.argmin(distancias_l_up)
    closest_l_down = np.min(distancias_l_down)
    index_closest_down = np.argmin(distancias_l_down)
    #Now si if these validate the condition
    #It has to see if it got any new candidates (not marked particles)
    if ((closest_l_up != 1000000)):
        vel_l = array_magnitud_vel[1]
        vel_closest_up = array_magnitud_vel[1 + index_closest_up]
        if( (vel_l>= vel_closest_up>=thresh_v_low)&(array_results[1+index_closest_up ,3]==0)):
            #it is marked
            array_results[1 + index_closest_up ,3] = 1
            #Recursion
            l_up = 1 + index_closest_up
            region_accretion(l_up)
    if ((closest_l_down != 1000000)):
        vel_l = array_magnitud_vel[1]
        vel_closest_down = array_magnitud_vel[1 + index_closest_down]
        if( (vel_l>= vel_closest_down>=thresh_v_low)&(array_results[1-index_closest_down ,3]==0)):
            #it is marked
            array_results[1 - index_closest_down ,3] = 1
            #Recursion
            l_down = 1 - index_closest_down
            region_accretion(l_down)

    #Define the default size of the search window
    n_window=10000
    #Now mark the seeds
    for i in indexes:
        array_results[i,3] = 1
    #Now do the recursion
    for i in indexes:
        #Recursion
        region_accretion(i)

    print 'The_regions_have:' + str(len(array_results[array_results[:,3] == 1,:])) +'galaxies.'
    print 'Starting_with_points_with_V>' + str(thresh_v_hig)
    print 'Ending_at_points_with_V<' + str(thresh_v_low)

    #Now it gets the points of the regions
    array_pos_region=array_results[(array_results[:,3] == 1),:]

```

A.2 Code example for Region Growing Algorithm - Approach by the FA and VDH

Listing 2: Region Growing Algorithm Code - FA & VDH

```

from ast import literal_eval
from struct import *
import numpy as np
import matplotlib.pyplot as plt
from pylab import *

# A Region Accretion Algorithm using the FA and the VDH

#Predicate to be part of the group
def predicate(i,j,k, seed_FA):
    if(results[i,j,k]>=1):
        return False #There is no need to re-evaluate the cell
    elif( (FA[i,j,k] <= thresh_FA) & (Trace[i,j,k] >= thresh_Trace)):
        #elif((Trace[i,j,k] >= thresh_Trace)):
            #It validates the predicate
        return True
    else:
        #It fails the predicate
        return False

#The algorithm , it evaluates the special cases
def region_accretion_4_neigboors(i,j,k):
    present_FA = FA[i,j,k]
    present_region = results[i,j,k]
    #Finds if it is on the border of the grid
    case = 0

```

```

#Cases
if ( i==0):
    case+=1
if ( j==0):
    case+=2
if ( k==0):
    case+=4
if ( i==n_grid ):
    case+=8
if ( j==n_grid ):
    case+=16
if ( k==n_grid ):
    case+=32
#General case
if (case==0):
    if( predicate(i+1,j,k,present_FA)):
        results[i+1,j,k] = present_region
        region_accretion_4_neigboors(i+1,j,k)
    if( predicate(i,j+1,k,present_FA)):
        results[i,j+1,k] = present_region
        region_accretion_4_neigboors(i,j+1,k)
    if( predicate(i,j,k+1,present_FA)):
        results[i,j,k+1] = present_region
        region_accretion_4_neigboors(i,j,k+1)
    if( predicate(i,j-1,k,present_FA)):
        results[i,j-1,k] = present_region
        region_accretion_4_neigboors(i,j-1,k)
    if( predicate(i,j,k-1,present_FA)):
        results[i,j,k-1] = present_region
        region_accretion_4_neigboors(i,j,k-1)
    if( predicate(i-1,j,k,present_FA)):
        results[i-1,j,k] = present_region
        region_accretion_4_neigboors(i-1,j,k)
#Special cases
#... Border cases

inputfolder = 'simulation_folder/'
inputfile_1 = 'snapshot_005.eigen_1'
inputfile_2 = 'snapshot_005.eigen_2'
inputfile_3 = 'snapshot_005.eigen_3'

# Formation of the Shear Tensor eigenvalues grids , FA and VDH
grid_1 = read_CIC_scalar(inputfolder+inputfile_1)
grid_2 = read_CIC_scalar(inputfolder+inputfile_2)
grid_3 = read_CIC_scalar(inputfolder+inputfile_3)

FA = (grid_1-grid_3)**2 + (grid_2-grid_3)**2 + (grid_1-grid_2)**2
FA = FA/(grid_1**2 + grid_2**2 + grid_3**2)
FA = np.sqrt(FA)/np.sqrt(3.0)

Trace = grid_1 + grid_2 + grid_3

#Seed selection
n_x,n_y,n_z = shape(FA)
n_grid = n_x-1 # the size of box -1, the maximum index
results = np.zeros(shape(FA)) #preps the results matrix
thresh_FA = 0.8
thresh_Trace = 0.0
thresh_FA_seeds = 0.6
thresh_Trace_seeds = 1.0

#Needs to pick the seeds.
seed_list = []

counter = 0
for i in range (n_x):
    for j in range (n_y):
        for k in range (n_z):
            if((FA[i,j,k]<thresh_FA_seeds) & (Trace[i,j,k]>thresh_Trace_seeds)):
                counter +=1
                results[i,j,k] = counter
                seed_list.append([i,j,k])

#Growth thresholds selection
thresh_FA = 0.8
thresh_Trace = 0.0

#Runs the Algorithm
for (i,j,k) in seed_list:
    region_accretion_4_neigboors(i,j,k)

```