

# Atomic Spectra Experiment

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## 1 Introduction

In this experiment the atomic spectra of the sodium atom (Na) and it's fine structure will be examined with grating spectroscopy method.

## 2 Theory

Atomic spectra is the spectrum of the electromagnetic wave that has been observed from an atom either as emission or absorption. When an atom is excited, in our case by applying potential difference, the electrons are excited into higher energy states. These states are the variables in the expansion of the solution of Schrödinger's equation in a given potential configuration. The Schrödinger Equation in spherical coordinates is the following[1]:

$$\hat{H}(r, \theta, \varphi)\Psi(r, \theta, \varphi) = E\Psi(r, \theta, \varphi) \quad (1)$$

Where  $\hat{H}$  is the Hamiltonian operator and E is the energy operator.

Solving the time-independent Schrödinger equation for the hydrogen atom, namely one proton one-electron system, according to the spherical geometry of the problem and using the method of separation of variables for solving the partial differential equation (1) two ordinary differential equations for radial and angular parts are obtained. After putting together all the necessary conditions for Schrödinger's equation, the solution is specified by three variables. Physical attributes to those variables are as follows, firstly the principal quantum number governs the possibility to find the electron at a point around the nucleus, the second variable is the azimuthal quantum number  $l$ , which describes the shape of the space that the electron can be found and the third one is the magnetic quantum number  $m_l$ , which describes the orientation of the electron with respect to an applied magnetic field.

In the hydrogen case, the potential configuration for the quantum mechanical system is  $\frac{-e}{r}$ . And the energy eigenstates can be expressed as  $E_n = -\frac{1}{8} \frac{Z^2 e^4 m}{\epsilon_0^2 h^2} \frac{1}{n^2}$ . But the experiment has been conducted with the sodium atom, which has 11 electrons, One of which is at the outermost shell and thus sees a potential like a hydrogen's. Other effects can be neglected so energy eigenstates reduce to hydrogen's case. But in the other cases, the potential configuration and thus the solution of the Schrödinger equation are quite different. The potential configuration for the other electrons can be modified as  $-\frac{Ze}{r} + C$ , where  $C$  accounts for the potential, that has been created because of other electrons

When an electron is excited it changes its energy eigenstate. This transition is discrete and thus the electron must obtain the exact amount of energy that suffices to make the transition. But this excited state is achieved only for a small time (orders of nano-second), that is because the naive solution for the Schrödinger equation of the hydrogen system does not account for the other interactions in the system. When the eigenstate is again disturbed, the electron lowers its energy

eigenstate and during this transition, the energy difference between gaps (eigenstates) creates a photon of the wavelength in accordance with Planck's formula for the energy of a photon  $E = h\nu$ .

Observing the emission of the hydrogen atom we get an empirical equation for the observed wavelengths known as Balmer Series

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

Generalising this representation we get

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \quad (2)$$

These equations are in agreement with the energy eigenstates of the solution of the naive quantum system above. For the sodium case, this empirical equation has a deficiency that should be modified. The modified version is

$$\frac{1}{\lambda} = R\left(\frac{1}{(n_f - \mu_f)^2} - \frac{1}{(n_i - \mu_i)^2}\right) \quad (3)$$

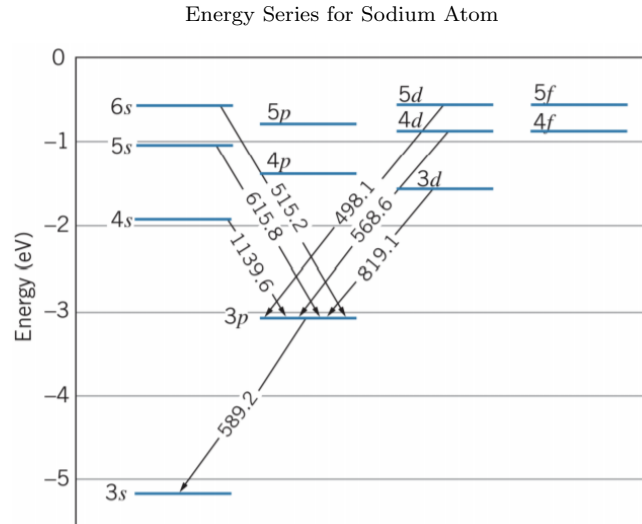
Where  $\mu$  accounts for the quantum deficiencies and the numerical values for the  $\mu$  is

$n/l$	0	1	2	3
3	1.37	0.88	0.01	0.001
4	-1.37	-0.88	-0.01	-

Energy eigenvalues for the sodium case can also be modified as the following

$$E_{n,l} = -\frac{mZ^2e^4}{8\epsilon_0^2h^2} \frac{1}{(n - \mu)^2} \quad (4)$$

The spectrum of the atoms that belong to the first group of the periodic table consists of four main series namely the principle, diffuse, sharp, and Bergman (fundamental), and the corresponding energy transitions for the sodium atom can be seen from the plot below[2]:



Interpreting this and the Schrödinger equation, it can be seen that the highest possible transition emits a photon of a wavelength that is yellow. The color of the sodium lamp is therefore yellow although there are also other photos with different wavelengths.

### 3 Experiment Procedure

For observing the spectrum and the fine structure of the sodium atom, first, a potential difference between the ends of the sodium lamp is applied. That causes the transition of the electrons inside of the tube and thus their eigenstates changes. After the initial distribution, the electron returns its original position by emitting a photon and thus the potential of the system lowers. This process happens simultaneously throughout the tube and consequently, photon emissions are observed.

To see the spectrum of the sodium atom, i.e the transition its electrons make, the wavelength of the photons that have been emitted must be investigated. Since emitted light is not collimated, a collimator is introduced to the system to make the light rays parallel.

After the photons pass through the collimator, in order to split them into their wavelength a grating must be done. A grating of 5900 lines per centimeter is introduced between the telescope, which is focused on a far object to focus the parallel light, and the collimator. This causes phase differences throughout the grating and causes destructive and constructive interference. When the path difference is exactly the integer multiple of the wavelength, constructive interference has been observed.

$$n\lambda = \sin(\theta)d \quad (5)$$

Where  $\theta$  is the angle between the telescope and the normal plane of the grating.

Finally, the telescope is placed on a vernier scale to keep track of the angular displacement of the telescope as the spectral lines are observed.

In the second part of the experiment, the fine structure of the sodium atom should be observed. This corresponds to the transitions between two close energy states. In the first part, the resolution of the grating,  $R = nN$  where  $N$  number of grating, is not enough to observe these different states and it has been observed as if they are on the same line. To increase the resolution of the system the grater is tilted 30 degrees. This causes another path difference and the eq.5 has become

$$n\lambda = (\sin(\theta) + \sin(30))d \quad (6)$$

Now it must be pointed out that observing the higher-order terms in the spectrum is harder as opposed to the lower terms since the amplitude, or the number of photons diffracted at that angle is low and since in this experiment there are no photon detectors except the human eye. That is why the measurements include high random errors.

## 4 Measurements

### 4.1 Part I

Spectrum of the Sodium Atom

Colour of the lines observed	Order number, n	Angular position of the image left, $\theta_1$	Angular position of the image right, $\theta_2$	Average angular position of image, $\theta$	$\sin \theta$	Wave-length $\lambda(\text{\AA})$	% error	$\delta\lambda$
Violet	1	160, 12'	196°, 20'	17.06°	0.2933	4972.24	2.956%	146.9
Green-Blue	1	162, 20'	197, 22'	17.51	0.3	5099.5	2.956%	150.0
Yellow	1	160, 01'	200, 00'	20	0.342	5796.95	2.954%	171.2
Red	1	158, 20'	202, 06'	21.84	0.372	6305.35	2.954%	186.2

Where

$\Delta d$ : error in diffraction constant ( $\approx 500\text{\AA}$ )

$\Delta\theta$ : reading error in diffraction angle ( $\approx 2'$ )

$\Delta\lambda$ : error in measured wavelength.

The error propagation is calculated according to the each variables contribution to the function with respect to their partial derivatives. This calculation has been done with an online error propagation calculator[3]

### 4.2 Part II

Fine Structure of the Sodium Atom

Colour Observed	Order [n]	$\theta_{d1}$	$\theta_{d2}$	$\lambda_1(\text{\AA})$	$\lambda_2(\text{\AA})$	Experimental $\Delta\lambda$ ( $\text{\AA}$ )	Theoretical $\Delta\lambda$ ( $\text{\AA}$ )	Percentage Error $\Delta\lambda$
Yellow	2	245°.05'	245°.10'	5464.91	5470.96	6.05	6	2.95 %
Green	2	221°.15'	221°.19'	2446.24	2459.54	13.3	5.6	2.95%
	3	242°.05'	242°.10'	5242.89	5249.41	6.52	5.6	2.95 %

## 5 Results

In the first part of the experiment, the measurements with their confidence interval are in agreement with the theoretical expectations given (see appendix). For the second part of the experiment, the fine structure is agreement for just one case. The reason for that is the measurement for the third-order diffraction for yellow light is not gathered. Instead, measuring the first two orders of diffraction's angle caused incomplete measurements. The second reason why the second and third-order of the green light is not in agreement with the theoretical data is firstly the angles that have been recorded are open to human error since the experiment is not conducted in a fully dark area, and even if it was, for aligning the telescope to the spectral line, the cross-hair of the telescope must be placed middle of the spectral line and it is not possible in very dark environment. And

the second reason for the error is recording the angle for the second order of diffraction for the green light is simply done wrong. An incomplete usage of Vernier's scale has been exhibited.

And while adjusting the grating, with respect to the 0'th diffraction. A ruler has been placed on the vernier scale to deduce the reference angle. This also contains an error and is open to the human misalignment of the ruler. That is why it may cause wrong calculations.

For lowering these errors, a photon detector and an electronic angular distance meter could be used. Such precautions would prevent the inconvenient and wrong measurement of higher-order diffraction and subsequently would give a better understanding of an atom's fine structure.

## 6 References

- 1- Gasiorowicz, S. (2003) Quantum Mechanics. 3rd Edition, John Wiley, New York.
- 2- <http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/sodium.html>
- 3-<http://www.julianibus.de>

## A Theoretical Values for Sodium Atom

**Table 1 Theoretical emission wavelengths of Sodium for Part1**

Wavelength, $\lambda$ [Å]	Relative Intensity	Color Observed
6160.7	500	Red
6154.2	1500	Red
5895.9	5000	Yellow
5889.9	9000	Yellow
5688.2	300	Green
5682.6	80	Green
5675.2	150	Green
5670.1	100	Green
5153.6	600	Green-Blue
5149.0	400	Green-Blue
4982.8	200	Violet
4978.5	15	Violet
4668.5	200	Violet
4664.8	50	Violet
4498.7	70	Violet
4494.2	60	Violet

**Table 2 Theoretical  $\Delta\lambda = \lambda_1 - \lambda_2$  values for Part 1**

Color Observed	Order	Theoretical $\Delta\lambda = \lambda_1 - \lambda_2$ [Å]
Red	2	6.5
	3	
Yellow	2	6.0
	3	
Green	2	5.6
	3	