Franck Hertz Experiment

Serdar Ali Andırınlıoğlu

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1 Introduction

In this experiment, the discreet energy levels of the atoms are investigated according to Bohr's suggestion and the first excitation energy of a Mercury atom (Hg) is determined.

2 Theory

In a stable atom, the model of an electron being in a well-defined circular orbit led to the problem of energy loss. Since the classical theory of electromagnetism predicts that an accelerating charged particle emits electromagnetic waves, Bohr suggested that the angular momentum transfer[1] (and subsequently the transfer of other basic quantities of physics) manifest itself not in a continuous manner but rather in a discreet way. Applying this idea in a 2 charge particle atom one can obtain the possible energies of the electron can be multiple of the following

$$E_n = -\frac{mZ^2e^4}{8\epsilon_0^2h^2}\frac{1}{n^2}$$

Where n is a positive integer, h is Planck's constant. For more complicated atoms the formula above changes slightly for the outermost electrons because the more the possibility of finding the electron further from the nucleus, the more negligible the other charged particle's effects. Considering Bohr's suggestion, the collision of energetic particles with the Mercury atom does not excite the electrons of the atom unless the energy of the particle that has been sent is enough to excite the electrons in the atom. In the experiment, electrons are accelerated via a potential and sent to the mercury vapor. At the end they close the circuit and cause a current to thrive. According to Bohr's suggestion, a sharp drop in the current intensity must be observed since exciting the Mercury atom corresponds to the inelastic collision, where after the collision the kinetic energy of the system is not conserved. In the following derivation, the kinetic energy of an electron after the elastic collision with the Mercury atom can be easily derived. Considering a head-on collision of a Mercury atom initially at rest with mass m_A with an electron of mass m_e and kinetic energy K_0 and momentum p_0

$$\vec{p_0} + 0 = \vec{p'} + \vec{p_A} \tag{1}$$

$$K_0 + 0 = K' + K_A \tag{2}$$

$$mv_0\hat{i} + 0 = mv'\hat{i} + m_Av_A\hat{i} \tag{3}$$

$$m(v_0 - v') = m_Av_A \tag{4}$$

$$\frac{1}{2}mv_0^2 = \frac{1}{2}m'^2 + \frac{1}{2}m_Av_A^2 \tag{5}$$

Where prime symbols represents the quantities after the collision. The energy loss of an electron in this scenario from (3) is

$$\Delta K = K_0 - K' = K_A \tag{6}$$

the equation (6) can be rewritten and divided by equation (5)

$$m(v_0^2 - v'^2) = m_A v_A^2 \tag{7}$$

$$\frac{m(v_0 - v')(v_0 + v')}{m(v_0 - v')} = \frac{m_A v_A^2}{m_A v_A} \tag{8}$$

$$v_0 + v' = v_A \tag{9}$$

$$v' = v_A - v_0 \tag{10}$$

When Eq. (10) is put into Eq.(4);

$$m v_0 - m\left(v_A - v_0\right) = m_A v_A$$

$$v_A = \frac{2 m v_0}{m + m_A}$$

$$K_A = \frac{1}{2} m_A v_A^2 = \frac{1}{2} m_A \frac{4 m^2 {v_0}^2}{\left(m + m_A\right)^2}$$

$$K_A = \underbrace{\frac{1}{2} m v_0^2}_{K_0} \frac{4 m m_A}{\left(m + m_A\right)^2}$$

$$\Delta K = K_A = K_0 \frac{4 \frac{m}{m_A}}{\left(1 + \frac{m}{m_A}\right)^2}$$

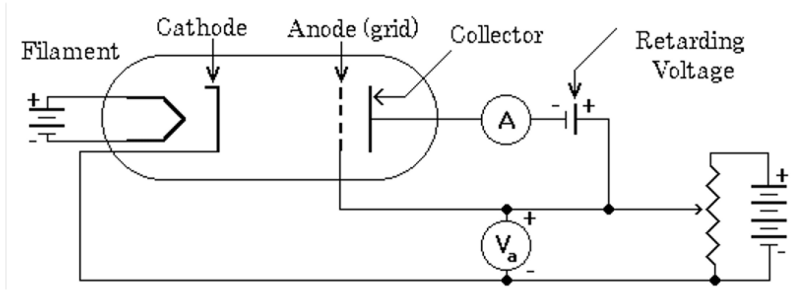
So it can be seen that the loss of kinetic energy due to the elastic collision is negligibly low. So one might expect that the sharp drops in the current caused by inelastic collisions and those inelastic collisions correspond to the exaction of Mercury atoms.

In the experiment, the electron source is the heated filament. The material is heated and thus the electrons on the surface of the filament have enough energy to be unbounded from the nucleus. Obtaining free electrons in such a way is called the *thermionic emission*.

3 Experiment Procedure

The experiment equipment consists of a mercury tube, an electric oven, power supplies and DC amplifier, a digital thermometer, and an oscilloscope.

The oven provides heat to mercury atoms to be evaporated. This way after ejecting the electrons from the filament, the interaction of the electrons with the mercury atoms can be observed as current in the circuit. It should be pointed out that to increase of possibility of collisions of electrons with the Mercury atom, the temperature of Mercury vapor must be held between 160°-200°. The scheme of the experimental setup is as below figure



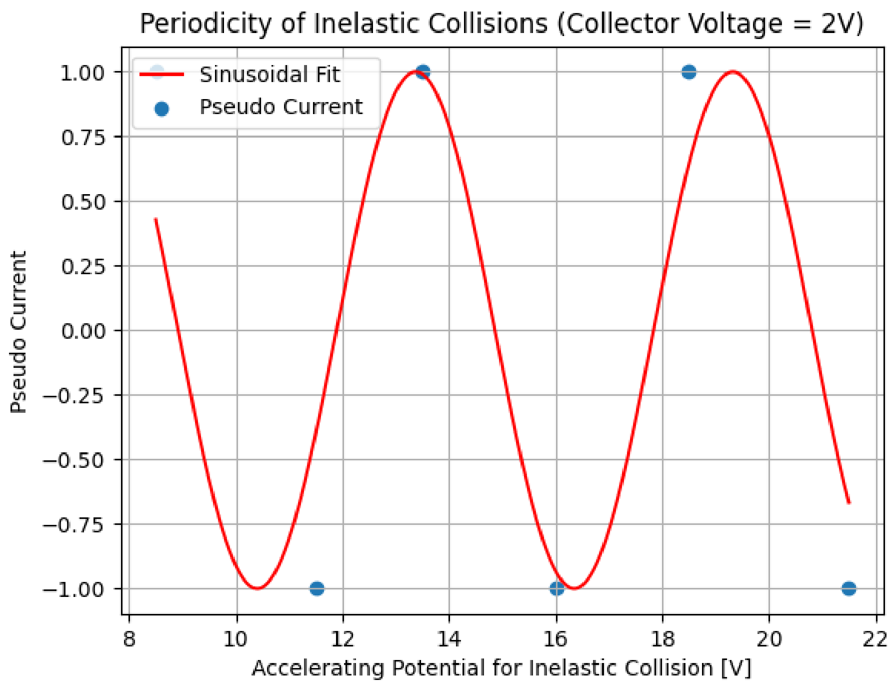
After heating the filament, the electrons are accelerated in the tube between the cathode and anode. The anode is in a grid form so that the accelerated and collated electrons can be observed in the collector. There is also a potential difference between the anode grid and the collector, this low potential difference stops the electrons that have low energies so that the shape observed on the oscilloscope is sharper.

Between the cathode and anode, there is a potential difference that enlarges the interval between the first successive maxima/minima points and the origin in the oscilloscope so that the observed excitation energy of the Mercury atom is slightly higher than the theoretical value.

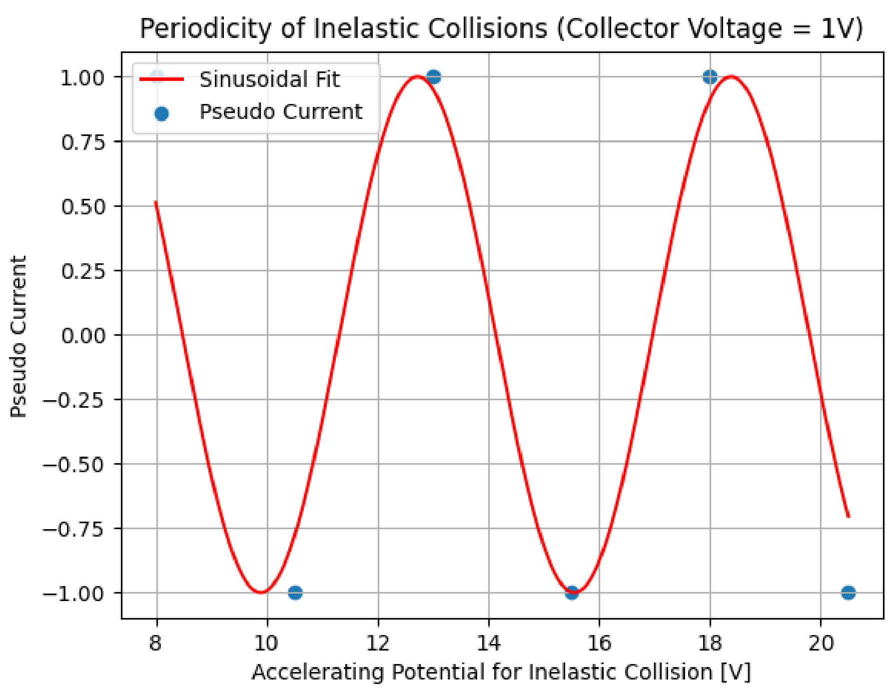
4 Data

Table 1: Data for different filament and collector voltages at instant oven temperature

	Filament Voltage = 6V Collector Voltage = 2V Oven Temperature = 180-190		**Filament Voltage = 6V Collector Voltage = 1V Oven Temperature =180-190**	
Voltage at second minima/maxima [V]	11.5	8.5	10.5	8
Voltage at third minima/maxima [V]	16	13.5	15.5	13
Voltage at fourth minima/maxima [V]	21.5	18.5	20.5	18
Voltage difference between the second and the third minima/maxima [V]	4.5	5	5	5
Voltage difference between the third and the fourth minima/maxima [V]	5.5	5	5	5
Mean of voltage difference between the minima/maxima [V]	5	5	5	5



the wave number k_2, in the sin fit is 1.056830



the wave number k_1, in the sin fit is 1.110924

Focusing on the table of the data set for the first setting, namely when the collector voltage is 1V, it is observed that the mean of the measurements is 5V and the confidence interval or the standard deviation, $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{N}} = 0.1178$.

For the second data set, when the collector voltage is 1V. The mean of the measurements is 5 and no standard deviation is observed *from the data* set. Knowing the theoretical value of the excitation energy of Mercury atom is $4.9eV$, the percentage error is 2.04%.

Since each interval indicates the potential barrier between two possible energy states for the most outer electron of the Mercury atom, multiplying the interval with the charge of the electron would yield the excitation energy of the Mercury atom, so it can be easily shown that the excitation energy E_{Hg} is

$$E_{Hg} = V \times e = 5 \times 1.602 \times 10^{-19} = 8.0108 \times 10^{-19} \pm 0.0943 \times 10^{-19} J \tag{11}$$

Where the confidence interval is calculated by accounting the standard deviation on the both data sets.

To obtain the contact potential, the potential between the anode and cathode V_a, $4V$ must be subtracted from the corresponding voltage value of the first maximum. Applying that for both collector potential configurations it is obtained that

$$8V - 4.9V = 3.1V$$
$$8.5V - 4.9V = 3.6V$$

$$\Rightarrow V_a = 3.35V$$

The other method for finding the excitation energy of the Mercury atom is looking at the periodicity of the graphs that have been presented above. Using the nonlinear fitting method, the wavenumbers k of each sin function have been calculated, remembering that the period of the sin function is $\frac{2\pi}{k}$ it is obtained

$$E_{Hg} = \frac{2\pi}{k_2} 1.602 \times 10^{-19} = 9.524 \times 10^-19J$$
$$E_{Hg} = \frac{2\pi}{k_1} 1.602 \times 10^{-19} = 9.060 \times 10^-19J$$

$$\bar{E}_{Hg} = 9.292 \times 10^-19$$

So the error is 18.37 %

5 Discussion

While measuring the intervals from the oscilloscope, since the measurement devices are analog, the data that has been obtained is highly open to human error. Measuring the intervals between successive minima/maxima is made with the naked eye and even making simultaneous measurements, the observation between laboratory partners differs. There are also other error sources. Changing the reverse bias caused a slight shift between successive minima, and changing oven temperature decreased the quality of the sharpness of minimas/maximas on the oscilloscope, first minima/maxima couldn't be observed due to low sharpness resolution on the oscilloscope.

The other sharp drops that have been observed are caused also by the first exaction of the Mercury atom, higher-level excitation is not responsible for the inelastic collision because electrons haven't been supplied with enough energy to make such transitions and the intervals between peaks have remained unchanged. Observing the 4.9V difference between sharp drops indicates that an electron excites more than one Mercury atom.

To conclude, although the percentage error for the first excitation level is not negligibly low, in the oscilloscope sharp drops are observed and thus Bohr's suggestion of discreet energy levels is in agreement with the experiment.

6 References

1-)Kragh, Helge (2012). Niels Bohr and the Quantum Atom: The Bohr Model of Atomic Structure 1913-1925. OUP Oxford. p. 18. ISBN 978-0-19-163046-0.

A Phyton Code

The periodicity of the occurrence of the peaks is found using a non-linear fit from scipy library.

```
import matplotlib.pyplot as plt
import numpy as np
from scipy.optimize import curve_fit
import pylab
col_voltage_1 = [8, 10.5, 13, 15.5, 18, 20.5]
col_voltage_2 = [8.5, 11.5, 13.5, 16, 18.5, 21.5]
pseudo_array = [1, -1, 1, -1, 1, -1, ]

def objective(x,a):
        return np.sin(a*x)

popt, lak = curve_fit(objective, col_voltage_1,pseudo_array)
a = popt
print(popt)

x_line = np.arange(min(col_voltage_1), max(col_voltage_1), 0.001)
y_line = objective(x_line, a)
plt.plot(x_line, y_line, color ='red', label='Sinusoidal Fit')
plt.scatter(col_voltage_1, pseudo_array, label = 'Pseudo Current ')
plt.xlabel('Accelerating Potential for Inelastic Collision [V]')
plt.ylabel('Pseudo Current')
plt.title('Periodicity of Inelastic Collisions (Collector Voltage = 1V)')
plt.legend()
plt.grid()
plt.show()
```