Electron Diffraction Experiment

Serdar Ali Andırınlıoğlu

April 2022

1 Introduction

The purpose of this experiment is to examine the wave properties of matter in accordance with De Broglie's proposal of attributing a wavelength to a particle. With that, the planar geometry of graphite molecule is investigated.

2 Theory

After Einstein discovered the photoelectric effect, plank assumed that the electromagnetic waves are not continuous but rather discreet. The Energy associated with this single particle of light, namely a photon, is as follows

$$E = \frac{hc}{\lambda} \tag{1}$$

From this property of the photon, Louis De Broglie proposed that the opposite must have held since they were looking for a universal property. So De Broglie proposed every particle, namely, matter with definite momentum energy, has a wave associated with it and it is called the De Broglie wave and it can be stated as follows[1]

$$\lambda = \frac{h}{p} \tag{2}$$

Where p is the momentum of the particle.

This assertion should not be understood as the particle emitting some sort of classical wave. Instead, it is a hint to build the Schrödinger equation as a linear model where the linear combination of different solutions of the differential equation is also a solution. According to the classical theory of electromagnetism, the electromagnetic waves propagating in space do not affect each other and this phenomenon is called the principle of superposition. So if the wave assertion is true then some sort of interference pattern should have been observed. In fact, this is the main idea of this experiment.

From the equation (2) a particle's wave of mass m can be expressed as

$$\lambda = \frac{h}{mv}$$

In this experiment, the particle that is being investigated is the electron. So the wavelength of the electron with charge e is inversely proportional to its speed v and the speed can be obtained from basic non-relativistic kinematic equation $\frac{1}{2}mv^2$ when accelerating potential V_A is applied.

$$K_E = \frac{1}{2}mv^2 = eV$$

From the equation (2) we get the electron's wavelength as follows

$$\lambda = \frac{h}{\sqrt{2meV_A}} \tag{3}$$

Calculating the equation above with the numerical values for the electron, the wavelength of the electron can be expressed as follows

$$\lambda = \sqrt{\frac{150}{V_A}} \tag{4}$$

Where units of λ are angstrom and the units of the applied accelerating potential V_a are Volts. Since the applied potential causes the electron to low speed with respect to the speed of light the relativistic correction of the above equation is not of interest for this experiment.

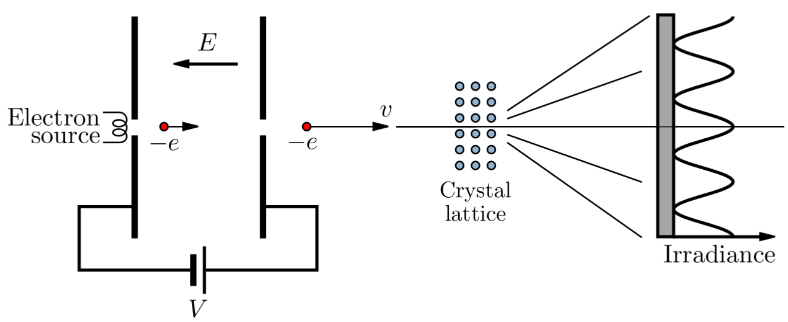
3 The Experiment

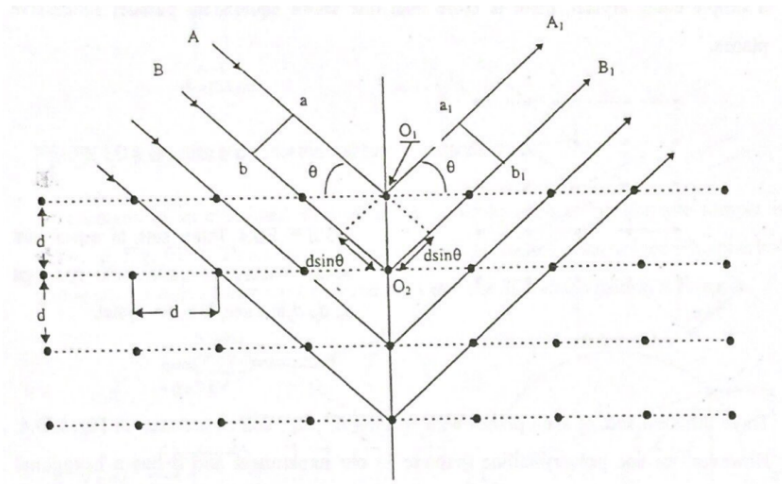
In the experiment in order to see the diffract of the electrons a thin film of randomly oriented crystals has been used. Since the crystals enjoy the property of the atoms that it's have been regular arrays, when an electron strikes the molecule, each layer of lattice diffracts the electron in all directions, and when the path difference is a multiple of the wavelength, constructive interference, or diffraction pattern, is observed. It should be pointed out that from this point we consider the wavelength associated with the matter purely.

Firstly, the electrons are being extracted from a hot filament and the electrons are being accelerated by the potential V_A between cathode and anode. Then electron collimator collimates the beam and subjects the beams to the graphite crystal. The sample of graphite crystal consists of many randomly oriented so that the bond between individual layers is broken. This causes the conical form of the scattered electron beam which exhibits rings on the observation screen. And observation screen is coated with a fluorescent material since otherwise observing the ring pattern on spherical glass wouldn't be possible.

From the figure below the condition for constructive interference, namely, the Bragg condition can be easily seen from the simple geometry

$$2d \sin\ = n\lambda \tag{5}$$





And from the figure below, it is easy to derive the relationship between the radius of the ring and the diffraction angle using the small angle approximation where $sin =$

$$\sin 2\alpha = \frac{r}{R}$$
$$\Rightarrow 2\sin\alpha = \frac{r}{R}$$

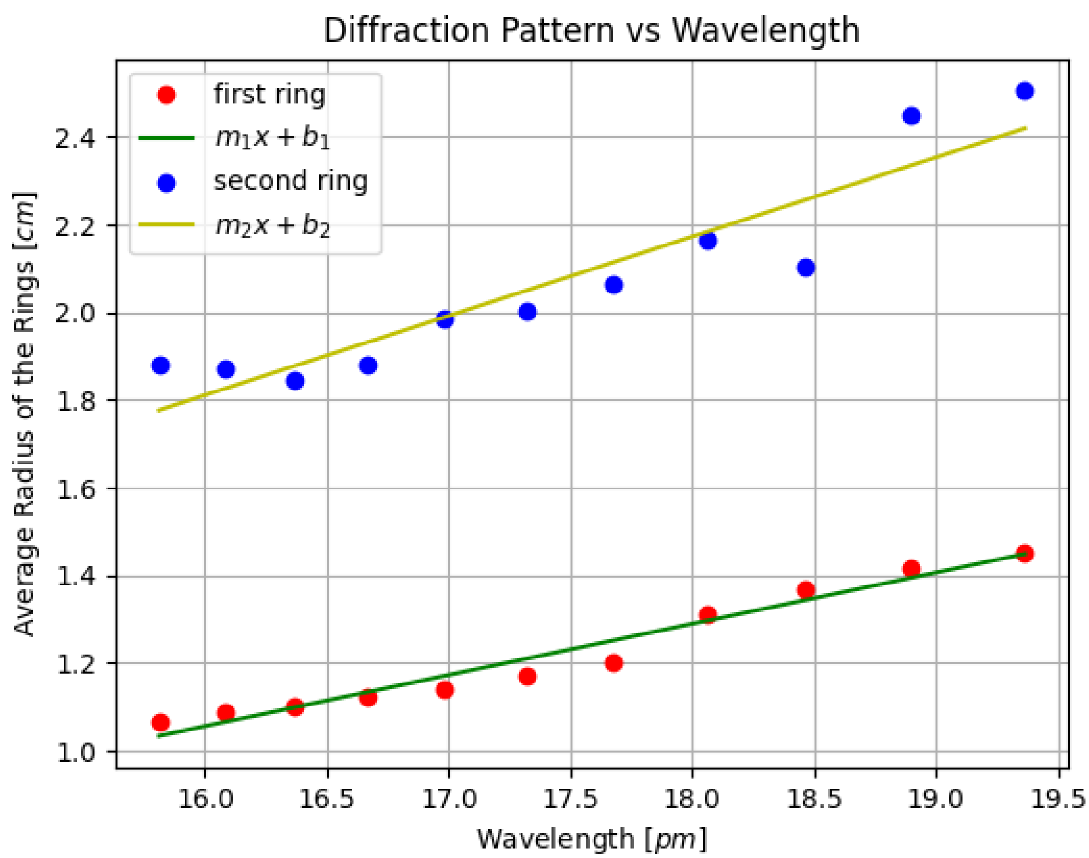
And from the figure it can be seen that $sin\alpha = sin2$ or $sin\theta = \frac{sin2\alpha}{4} = \frac{r}{4R}$ Applying this to the equation (5) we get

$$d = \frac{2R}{r}n\lambda \tag{6}$$

4 Data

Table 1: Radius of the Rings Observed

			First Ring			Second Ring		
Measured Anode Voltage [V]	Converted Anode Voltage [V]	Wavelength [$\mathring{A}$]	Inner Diameter [cm]	Outer Diameter [cm]	Average Diameter [cm]	Inner Diameter [cm]	Outer Diameter [cm]	Average Diameter [cm]
2	4000	0.1936	2.57	3.24	2.905	4.64	5.38	5.01
2.14	4200	0.1889	2.66	3.00	2.83	4.70	5.10	4.90
2.2	4400	0.1846	2.55	2.93	2.74	4.53	4.89	4.21
2.3	4600	0.1805	2.46	2.79	2.625	4.21	4.45	4.33
2.4	4800	0.1767	2.20	2.60	2.40	3.85	4.40	4.12
2.5	5000	0.1732	2.10	2.59	2.345	3.75	4.26	4.00
2.6	5200	0.1698	2.15	2.42	2.285	3.74	4.20	3.97
2.7	5400	0.1666	2.09	2.41	2.25	3.56	3.97	3.76
2.8	5600	0.1636	2.10	2.30	2.2	3.47	3.92	3.69
2.9	5800	0.1608	2.04	2.31	2.175	3.56	3.93	3.74
3.0	6000	0.1581	1.98	2.29	2.135	3.55	3.97	3.76



Where $m_1 = 0.11677$, d_1= 108.75pm and $m_2 = 0.18079$, d_2=70.24pm

$$m = \frac{r}{\lambda}$$
$$d = \frac{2R}{r}\lambda$$
$$\Rightarrow d = \frac{2R}{m}$$

5 Discussion

Comparing the slit spacing value with the known spacing value, a high percentage error has been observed. The percentage error for the known $d_1 = 123pm$ value is 42.8% and for the second spacing $d_2 = 213pm$ the percentage error is 49.1%.

The main reason for these high uncertainties in the old measurement devices. While taking data the power fluctuation has been observed thus the diffraction pattern has changed. The second reason is that the data has been recorded with a Vernier ruler and since the border of the bright rings is hard to measure there was a high random error.

With such high uncertainties, it is wrong to state that the electron's behaviour of wave is not as expected. But the diffraction pattern itself indicates that the electrons intrinsically have some sort of superposition property.

6 References

1-)$https://en.wikipedia.org/wiki/LouisdeBroglie$

A Phyton Code

```
import math

import numpy as np
import matplotlib.pyplot as plt

voltages = [4000 + (i * 200) for i in range(11)]
w_length = []
first_inner_radius = np.array([2.57, 2.66, 2.55, 2.46, 2.2, 2.1, 2.15, 2.09, 2.1, 2.04, 1.98])
first_outer_radius = np.array([3.24, 3.00, 2.93, 2.79, 2.60, 2.59, 2.42, 2.41, 2.3, 2.31, 2.29])

second_inner_radius = np.array([4.64, 4.7, 3.53, 4.21, 3.85, 3.75, 3.74, 3.56, 3.47, 3.56, 3.55])
second_outer_radius = np.array([5.38, 5.1, 4.89, 4.45, 4.40, 4.26, 4.2, 3.97, 3.92, 3.93, 3.97])
"""
for i in range(11):
    y = 4000 + (i * 200)
    voltages.append(y)
print(voltages)
"""
for i in voltages:
    y = math.sqrt(150/i)
    w_length.append(y)
print('voltages')
print(voltages)
print('wavelengths')
print(w_length)
print('wavelengths in picometer')
w_length_pico = np.array(w_length) * 100
print(w_length_pico)

first_average = (first_outer_radius + first_inner_radius) / 4
second_average = (second_outer_radius + second_inner_radius) / 4

print('average position of first ring')
print(first_average)

print('\naverage position of second ring')
print(second_average)

m,b   = np.polyfit(w_length_pico, first_average,1)
m2,b2 = np.polyfit(w_length_pico, second_average,1)
print('slope of the lines m and m2')
print(m,m2)

d1= 2*6.35/m
d2= 2*6.35/m2
print('structure values in pm')
print(d1,d2)
```

```
plt.plot(w_length_pico, first_average, 'ro', label = 'first ring')
plt.plot(w_length_pico, m * w_length_pico + b, 'g-', label = '$m_1x+b_1$' )
plt.plot(w_length_pico, second_average,'bo',label = 'second ring')
plt.plot(w_length_pico, m2 * w_length_pico +b2, 'y-', label = '$m_2x+b_2$')
plt.legend(loc ="upper left")
plt.title('Diffraction Pattern vs Wavelength')
plt.xlabel('Wavelength $[pm]$')
plt.ylabel('Average Radius of the Rings $[cm]$')
plt.grid()
plt.show()
```