Prism Spectrometer Experiment

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1 Introduction

In this experiment the relationship between the refractive index n and wavelength λ has been observed and the Cauchy Equation coefficients for sample prism has been approximated using mercury plasma as light source and a thin lens to observe parallel beam rays emitted through the collimator.

2 Theory

The refractive index n is a dimensionless parameter that indicates how the path of light bends when propagating into a different medium[1]. For an observer in the main medium the bending of the path manifest itself as the decrease (or rarely, increase) in the speed of light with respect to the speed of light in vacuum. This refraction process happens in the prism also, therefor it is possible to measure the refractive index n for a prism. With the Snell's law it possible to calculate n knowing the angle of deviation. The derivation of this formula,with given diagram below (see fig.2, is the following[2]:

$$\begin{aligned}
&A = r_1 + r_2 = 2r \\
&r = \frac{A}{2} \\
&A + \sigma = i_1 + i_2 \\
&A = \sigma_m = 2i \\
&i = \frac{A + \sigma_m}{2} \\
&n = \frac{sin(i)}{sin(r)} \quad (Snell'sLaw) \\
&n = \frac{sin(\frac{A+\sigma_m}{2})}{sin(\frac{A}{2})}
\end{aligned} \tag{1}$$

In the case Prism placed in vacuum and minimum deviation ($i_1 = i_2 = i, \sigma = \sigma_m$) The variables are :

OP = Incidence Ray
i_1 = Incidence Angle
i_2 = Refraction Angle
A = Angle of Apex
r_1 = r_2

It should be pointed out that the refractive index n is a function dominated by the wavelength of the light. When the wavelength of the light decreases, the refractive index, thus refraction increases. The refractive index of a subject material thus depends on the wavelength that has been emitted.Cauchy equation for a sample crystal empirical relation between the refractive index n and λ as follows:

$$n(\lambda) = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + ... \tag{2}$$

For practical purposes and for relatively closed wavelength considering just two terms of the equation, namely A and B, is enough. The Cauchy equation is valid in the visible light wavelength regime where normal dispersion is observable[3].

The apex angle of the prism is calculated with total reflection phenomena, deviation angle of the reflected image of the light source has been measured from both ends to find apex angle.

In the experiment the light source is the mercury plasma. Applying voltage to the tube of mercury plasma, several different wavelength of light in the visible light regime is observed. This is caused by the specific energy gap between two discreet energy levels of the electrons in the mercury atom. Applied Voltage causes electrons in the atom to gain energy and increase their energy levels. Due to the random collisions in the plasma or some other effects, excited electron returns to its original position by emitting a photon that has energy of a band gap. Since the refractive index of the prism depends on the wavelength, the emitted light from the plasma will be separated with different angles for different wavelength. The light source's emission values are given, thus measuring the deviation for each different wavelength it is possible to approximate the Cauchy Equation coefficients. In this experiment only the firs two constant has been calculated.

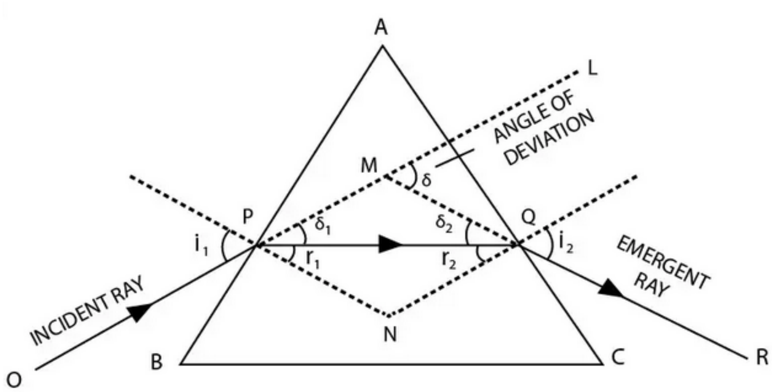


fig.2

3 Results

Results below calculated with base error of $\pm 0.5°$. It is assumed that the error of the measurement devices exhibits normal distribution, thus successive measurements decreased the error by $\sqrt{n}. Fortheerrorpropagation, anerrorpropagationcalculaterwasused[4]$.

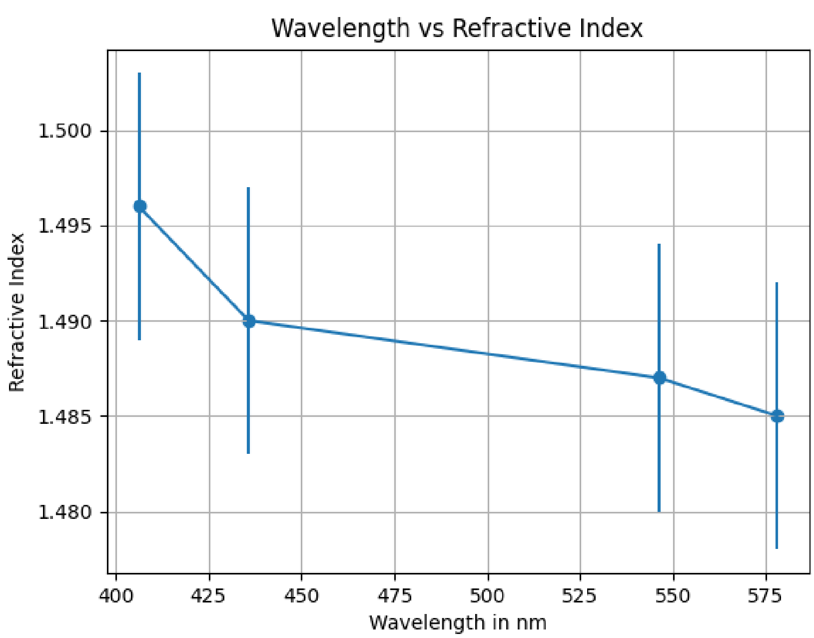
Result Table For The Apex Angle of The Prism

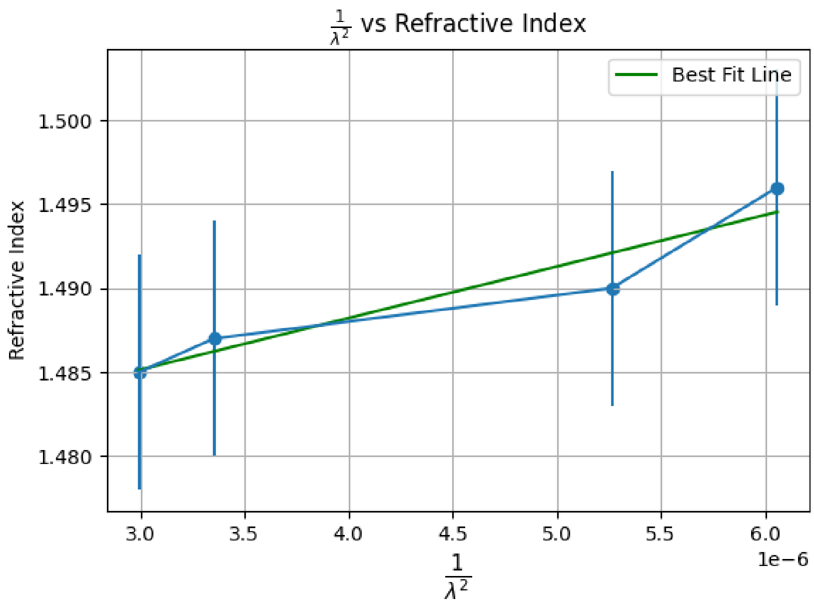
Position 1	Position 2	Aphex Angle α_i
47°	74°	60.5°
47°	73.5°	60.25°
49.5°	70°	59.75°

The average apex angle, $\overline{\alpha_i} = 60.16° \pm 0.28°$

Result Table For Refractive Index of Different Wavelength

λ	Position 1	Position2	σ_{min}	n
406.2nm	0°	37°	37°	1.496 $\pm$0.007
435.8nm	0°	36.5°	36.5°	1.490 $\pm$0.007
546.1nm	0°	36.2°	36.2°	1.487 $\pm$0.007
578.05nm	0°	36°	36°	1.485 $\pm$0.007





Slope of the best fit line, thus B in Cauchy Equation is 3065.6552.
Line intersect y-axis at 1.47and this equals A in Cauchy Equation

4 Discussion

While observing the deviation angles through lens the recorded measurements were highly open to systematic human error. The sensitivity of the vernier scale was not good enough to measure the differences for closely separated light. In the case of naming the violet light from the given emission chart of the mercury plasma, there were two discreet violet light but from the lens just one violet light has been observed. It is due to the low resolution of our optical system, namely bad grating. That is why it is assumed the observed violet light is the average of those values. The obtained Cauchy coefficients may be enough for the calculation of the refractive index n of the prism for visible light regime but even if we have calculated the higher order terms of the sequence, due to error of our measurements and systematical error of the experiment design, the estimated refraction index for high frequency ,thus low wavelength, light would not be in agreement with empirically obtained data.

5 References

1)Hecht, Eugene (2002). Optics. Addison-Wesley. ISBN 978-0-321-18878-6.
2)https://www.toppr.com/ask/question/derive-the-expression-for-refractive-index-of-the-material
3)F.A. Jenkins and H.E. White, Fundamentals of Optics, 4th ed., McGraw-Hill, Inc. (1981).
4)https://nicoco007.github.io/Propagation-of-Uncertainty-Calculator/