Microwave Optics Experiment

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December 2021

1 Abstract

In this experiment, the wavelength of a standing wave pattern and microwave velocity in a dielectric medium will be measured. Also, a circularly polarized beam from a linearly polarized wave will be created.

2 Theory

Propagating electromagnetic waves have the constant velocity of c and can be represented as time-varying electric and magnetic field vectors. It is convenient to investigate only the electric field vector of an electromagnetic wave and deduce properties of light without loss of generality. The electric field vector of an electromagnetic wave is represented as follows

$$\vec{E}(x,y) = E_{max}cos(kx - \omega t + \phi)$$

Where ϕ is the phase shift ω is the angular frequency. According to Maxwell's law the time average of the intensity is as follows.

$$I\langle t\rangle = \langle S\langle t\rangle\rangle = \frac{1}{2c\mu_0}{E_0}^2 \tag{1}$$

To measure the wavelength of the microwave, the transmitter and the receiver are placed as in figure 1. The main idea behind the measurement is the fact that the receiver and the transmitter partially reflect the light in accordance with the Fresnel's equations. Since the glass has a higher refractive index, whenever the light is reflected back and forth from the setup, it does not undergo a $\pm\pi$ phase shift. It should be underlined that the frequencies of the reflected and initial waves are the same. By the linearity of electromagnetic waves, the superposition principle emerges. That is an electromagnetic distortion in a location from two sources is the summation of the effects of the individual sources, since we can not observe any non-linear effect due to our energy regime. From the exact formula for the addition of two waves with the same frequency (see Appendix A) the following result gives the phase difference

$$\cos(\alpha_2 - \alpha_1) = \cos(x_2 - x_1) \tag{2}$$

Since the detector measures the intensity of the light, from combining eq (1) and (2) it can be concluded that the difference of the positions between successive minimum intensity gives us half of the wavelength of the light λ. In an experiment from 10 successive measurements, the wavelength of the light has been calculated. So the formula for calculating the average wavelength is

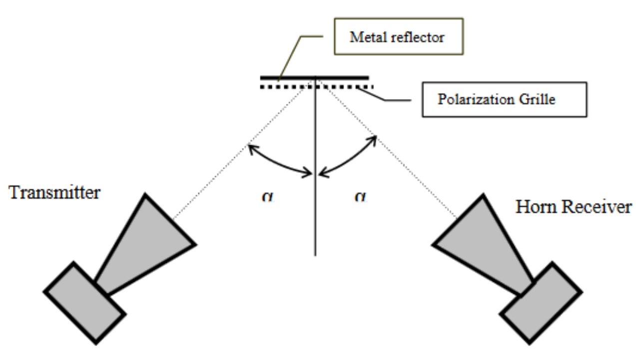
$$\bar{\lambda} = 2\frac{\sum |x_{i+1} - x_i|}{N-1} \tag{3}$$

For the second part of the experiment a dielectric material has introduced the path of the light, hence the optical path that light has to travel is increased by an amount nx (see Appendix A) First the dielectric material placed half of the emitter and the effect of the interference observed through the detector. Secondly, an identical dielectric material has been placed on the other side of the emitter with the first one and the interference has been observed. If the intensity of the light after introducing the dielectric material is zero, it means that the optical path difference is exactly half of the wavelength. Subtracting the optical path of the two waves, one with the dielectric material and the other without, we get the following equation

$$L(n-1) \approx \frac{\lambda}{2} \tag{4}$$

In the last part, a circularly polarized light has been created using the fact that the emitter emits a linearly polarized microwave light. A linearly polarized light can also be expressed as the superposition of the two circularly polarized light. First, the emitter turned 45°, and then a vertically placed grill was introduced in front of the metal reflector. This grill is polarized and only allows the horizontal component of the light to pass the metal reflector behind it and reflect the rest(see figure 1 below). The metal reflector behind then reflects the horizontally polarized light. If the optical path difference is $\frac{1}{4}$ then we get a circularly polarized light. In order to introduce this phase shift, we need to set the optical path difference to $\lambda + n\frac{\lambda}{4}$ where $n \in N$. Using just basic geometry and Pythagorean's theorem required distance between grill and metal reflector can be expressed as follows

$$d\sqrt{2} = \lambda + n\frac{\lambda}{4} \tag{5}$$



3 Results

Part A

i	1	2	3	4	5	6	7	8	9
Position of the minima	111.2 ±02cm	109.8 ±02cm	108.5 ±02cm	107.5 ±02cm	105.5 ±02cm	104.1 ±02cm	102.6 ±02cm	101.1 ±02cm	100.3 ±02cm

according to formula (3) $\bar{\lambda} = 2.725 \pm 0.07cm$. Uncertainties of each measurement were $\pm 0.2cm$ and eight successive measurement reduced it by a factor of $\sqrt{8}$ because it is assumed that the ruler has a normal error distribution, thus error interval has reduced.

Part B

From equation 4 and measured wavelength of the light refractive index of the dielectric materiel has found as follows

$$\begin{aligned} n \approx& \frac{2.725}{5} + 1 \\ =& 1.54 \\ v =& \frac{c}{n} \\ =& 194670427 \frac{m}{s} \end{aligned}$$

Part C

Receiver Angle	0	60	90	120	150	180	220	270
Intensity	2.3mW	1mW	0.9mW	1.1mW	2.4mW	2.4mW	1.9mW	0.9mW

4 Discussion

From part A the wavelength of the light has been calculated and it is found to be different from the real value of $2.85cm$. The main reason for this is the reading intensity meter is open to human error and also intensity meter has its own uncertainties. From λ the refractive index of the material has been found. In the last part In order to introduce $\frac{1}{4}$ phase shift, the distance between the metal reflector and metal grill has been arranged according to the formula (5) and the intensity readings were made. But according to the measurements instead of circularly polarized light, elliptically polarized light has been observed. The main reason is the systematic error while calculating the wavelength of the light, thus required distance to create the phase shift, and placing the metal grill in front of the metal reflector. The uncertainties while measuring the distances taken to be $\pm 0.2cm$. The propagation of error while calculating required distance d and the relatively great uncertainties while placing the metal grill gave us the wrong results above.

Overall the measured value of the wavelength of the light is not in agreement with the real value of the wavelength of the light that has been emitted. But the calculations based on that wavelength information carried the uncertainties along with it and thus creating circularly polarized light was not possible.

5 References

1) Hecht, Eugene (2002). Optics. Addison-Wesley. ISBN 978-0-321-18878-6.
2) Wave Interference, Wikipedia https://en.wikipedia.org/wiki/Wave$_interference$

A Sum of Two Waves With The Same Frequency

The electric field component of the travelling wave vector can be expressed as follows

$$
\begin{aligned}
E_1 &= E_{01}e^{(kx-\omega t+\epsilon_1)} \\
E_2 &= E_{02}e^{(kx-\omega t+\epsilon_2)} \\
E_1 + E_2 &= E_{01}e^{(i\alpha_1)}e^{(-i\omega t)} + E_{02}e^{(i\alpha_2)}e^{(-i\omega t)} = e^{(-i\omega t)}(E_{01}e^{(i\alpha_1)} + E_{02}e^{(i\alpha_2)})
\end{aligned}
$$

Where $\alpha_1 = kx + \epsilon_1$ and $\alpha_2 = kx + \epsilon_2$. Denote $\tilde{E} = E_{01}e^{(i\alpha_1)} + E_{02}e^{(i\alpha_2)}$. Then using Euler's identity in order to get the amplitude of the addition of waves, we get the following expression

$$
\begin{aligned}
\tilde{E} &= E_{01}cos\theta_1 + iE_{01}sin\alpha_1 + E_{02}cos\theta_2 + iE_{02}sin\alpha_2 \\
&= (E_{01}cos\theta_1 + E_{02}cos\theta_2) + i(E_{01}sin\alpha_1 + iE_{02}sin\alpha_2)
\end{aligned}
$$

$$
\begin{aligned}
{E_0}^2 =& (E_{01}cos\theta_1 + E_{02}cos\theta_2)^2 + (E_{01}sin\alpha_1 + iE_{02}sin\alpha_2)^2 \\
=& {E_{01}}^2 + cos^2\alpha_1 + 2E_{01}E_{02}cos\alpha_1 cos\alpha_2 + {E_{02}}^2 cos^2\alpha_2 \\
&+ {E_{01}}^2 sin^2\alpha_1 + 2E_{01}E_{02}sin\alpha_1 sin\alpha_2 + {E_{02}}^2 sin^2\alpha_2 \\
=& {E_{01}}^2(cos^2\alpha_1 + sin^2\alpha_1) + {E_{02}}^2(cos^2\alpha_2 + sin^2\alpha_2) + 2E_{01}E_{02}(cos\alpha_1 cos\alpha_2 + sin\alpha_1 sin\alpha_2) \\
=& {E_{01}}^2(cos^2\alpha_1 + sin^2\alpha_1) + {E_{02}}^2(cos^2\alpha_2 + sin^2\alpha_2) + 2E_{01}E_{02}\cos(\alpha_2 - \alpha_1) \\
=& {E_{01}}^2 + {E_{02}}^2 + 2E_{01}E_{02} + \cos(\alpha_2 - \alpha_1)
\end{aligned}
$$

Where $\cos(\alpha_2 - \alpha_1)$ is called the phase difference. Since we assumed that the frequency of this waves are identical, ϵ terms in α is zero. Writing α explicitly gives us the optical path difference $\sigma = k(x_2 - x_1)$. So when the phase difference is zero, the interfered wave's amplitude is maximum. Since the intensity detector measures the derived current, observing two minimum intensity point will indicate the half of the wavelength of the light. If two light propagating in the different media whose refractive indexes n_1 and n_2 respectively then the optical path difference is

$$\sigma = k(x_2 n_2 - x_1 n_1)$$

Here the xn terms are called the optical path.