Diffraction and Interference Experiment

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1 Abstract

In this experiment the interference and diffraction of the light, which is caused by the wave nature of the light, will be investigated. Deductions from the wave nature of the light will be used to calculate the laser source wavelength by its diffraction pattern.

2 Theory

In order to explain interference and diffraction phenomena, the wave character of light must be investigated. The propagation of light has been described by the *Huygens-Fresnel* principle. This principle states that every point of the propagating wavefront of the source can be taught as a secondary spherical point source and any point in the field is the superposition of those wave sources. Being superposition of the preceding waves and the superposition of different wave sources at a point is simply interference. Diffraction can be explained simply by stating that it is any deviation from geometrical optics[1]. The main distinction between interference and diffraction is that in interference phenomena the interfering beams are originating from different discrete point sources whereas in the diffraction phenomena the interfering beams originate from continuous distribution of sources.

If the observation screen, light source, and diffraction apparatus are effectively far from each other then it is often considered that the wavefront of the light is plane. Under this assumption, the theory as been developed is called *Fraunhofer* or *far-field diffraction*. If this assumption can not be made then the curvature of the wavefront of the light must be considered as well, this situation is called *near-field diffraction*.

2.1 Single Slit Diffraction

Firstly the single slit diffraction is being investigated. In order to do calculations about the diffraction pattern, a couple of arrangements must be made. First of which is that the waves coming to the slit must have plane wavefronts. This is accomplished by using a laser with a small divergence angle. The second arrangement is that the observation screen must be sufficiently far enough so that it can be assumed that the light coming to the screen has plane wavefronts. It can be seen from the figure that the waves from a single slit do not arrive at a specific point P in phase. Essentially this is the cause of the diffraction pattern. From simple 2-D Euclidean geometry it can be seen that the wavefront coming from the center of the slit has $\triangle$ shorter optical path length than the wave coming from the upper beginning of the slit. Considering a "*tiny*" opening of an interval with length ds as a source and then calculating its effect on point P and integrating over the slit region gave us the exact information of diffraction pattern under the assumptions discussed above. Each interval contributes a spherical wavelet at P whose magnitude is directly proportional to the infinitesimal length ds

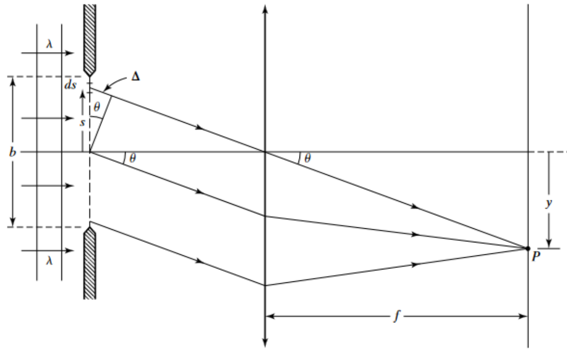


Figure 1 Construction for determining irradiance on a screen due to Fraunhofer diffraction by a single slit.

$$dE_p = \frac{E_L ds}{r} e^{i(kr-\omega t)}$$

Where r is the optical-path length from interval to point P. And amplitude term has a $1/r$ dependence because the area of a sphere increases with r^2 and thus irradiance decreases with square law. Now before the integration, the distance r is fixed such that it is at the origin of the slit. Then to represent any other wave at distance s, taking the phase into account the following expression is obtained

$$dE_p = \frac{E_L ds}{r_0 + \triangle} e^{i[k(r_0+\triangle)-\omega t]} = \frac{E_L ds}{r_0 + \triangle} e^{i(kr_0-\omega t)} e^{ik\triangle} \tag{1}$$

Since it is assumed that the r_0 being the distance from the diffraction device to the observation screen, much greater than the slit width, the $\triangle$ term in the amplitude expression can be ignored because it is much smaller than the r_0. But the $\triangle$ term in the phase term can not be ignored because $k\triangle = (2\pi/\lambda)\triangle$, so as $\triangle$ varies by one wavelength, the phase $k\triangle$ varies over 2π. This will give the necessary information for the diffraction pattern once the integration is done. In the figure above and from simple geometry clearly $\triangle = s\sin\theta$.So the equation (1) becomes

$$dE_p = \frac{E_L ds}{r_0} e^{i(kr_0-\omega t)} e^{iks\sin\theta} \tag{2}$$

Now integrating this over the slits region

$$\int_{slit} dE_p = \frac{E_L}{r_0} e^{i(kr_0-\omega t)} \int_{-b/2}^{b/2} e^{iks\sin\theta} ds = \frac{E_L}{r_0} e^{i(kr_0-\omega t)} \left(\frac{e^{iks\sin\theta}}{ik\sin\theta}\right)_{-b/2}^{b/2} \tag{3}$$

Inserting the limits of the integration

$$E_P = \frac{E_L}{r_0} e^{i(kr_0-\omega t)} \frac{e^{(ikb\sin\theta)/2} - e^{-(ikb\sin\theta)/2}}{ik\sin\theta} \tag{4}$$

To make the exponential terms more clear we make the following substitution

$$\beta = \frac{1}{2} kb\sin\theta$$

Then using Euler's identity,

$$\begin{aligned} E_P &= \frac{E_L}{r_0} e^{i(kr_0-\omega t)} \frac{b(e^{i\beta} - e^{-i\beta})}{2i\beta} = \frac{E_L}{r_0} e^{i(kr_0-\omega t)} \frac{b(2i\sin\beta)}{2i\beta} \\ &= \frac{E_L b}{r_0} \frac{\sin\beta}{\beta} e^{i(kr_0-\omega t)} \end{aligned} \tag{5}$$

The equation (5) includes the *sinc* function $\sin\beta/\beta$,where β varies with θ. The irradiance I at P is proportional to the square of amplitude of the electric field at P. Then with equation (5) the amplitude of the electric field at point P is

$$E_0 = \frac{E_L b}{r_0}\frac{\sin\beta}{\beta} \tag{6}$$

The exact equation for irradiance is,

$$E = \frac{1}{2}\epsilon_0 c E_0^2 \tag{7}$$

Using the equation (7),

$$I = \frac{\epsilon_0 c}{2}E_0^2 = \frac{\epsilon_0 c}{2}\frac{E_L b}{r_0}^2\frac{\sin\beta^2}{\beta^2} = I_0 \sin \tag{8}$$

making I_0 all the constant terms we get

$$I = I_0\frac{(\sin\beta)^2}{\beta^2} = I_0 sinc^2\beta \tag{9}$$

As the argument of the *sinc* function goes to 0 the function itself approaches 1. To find the minimum irradiance in equation (9) the argument β must be zero, that is $\beta = \frac{1}{2}kbsin\theta = m\pi$ where $m = \pm 1 \pm 2, ...$ settin $k = 2\pi/\lambda$ we get the condition for zeros of the *sinc* function

$$m\lambda = b\sin\theta \qquad m = \pm 1 \pm 2, ... \tag{10}$$

Observing the figure above with the small angle approximation, y the distance from the center of the screen is approximately $y \cong f\sin\theta$ so according to equation (10) the irradiance is maximum at $\theta = 0$ and drops 0 at y_m such that

$$y_m \cong \frac{m\lambda f}{b} \tag{11}$$

2.2 Double Slit Diffraction

The formula for obtaining the diffraction pattern for two narrow slits with *far field* assumptions is essentially the same except now there are two integral two evaluate. Considering the same figure above except another slit with separation a ,

$$E_p = \frac{E_L}{r_0}e^{i(kr_0-\omega t)}\int_{-(1/2)(a-b)}^{-(1/2)(a+b)} e^{iks\sin\theta}ds + \frac{E_L}{r_0}e^{i(kr_0-\omega t)}\int_{(1/2)(a-b)}^{(1/2)(a+b)} e^{iks\sin\theta}ds \tag{12}$$

Integrating this and inserting the integration limits leads us

$$E_p = \frac{E_L}{r_0}e^{i(kr_0-\omega t)}\frac{b}{2i\beta}[e^{i\alpha}(e^{i\beta} - e^{-i\beta}) + e^{-i\alpha}(e^{i\beta} - e^{-i\beta})] \tag{13}$$

Where $\beta = \frac{1}{2}kb\sin\theta$ and $\alpha = \frac{1}{2}ka\sin\theta$. Using the Euler's Identity we get

$$E_p = \frac{E_L}{r_0}e^{i(kr_0-\omega t)}\frac{b}{2i\beta}(2i\sin\beta) \tag{14}$$

Finally,

$$E_p = \frac{E_L}{r_0}e^{i(kr_0-\omega t)}\frac{2b\sin\beta}{\beta}\cos\alpha \tag{15}$$

The amplitude of this electric field is,

$$E_0 = \frac{E_L}{r_0}\frac{2b\sin\beta}{\beta}\cos\alpha \tag{16}$$

According to equation (7) the irradiance at point P is,

$$I = 4I_0(\frac{\sin\beta}{\beta})^2\cos\alpha^2 \tag{17}$$

But this equation(17) is the modulated version of the equation (9) for single slit by cos^2 term.

Then finding the zeros of this modifier term will give the zero intensity points at the observation screen. Writing the cosine term using the substitution α and k,

$$cos^2\alpha = cos^2[\frac{\pi a(\sin\theta)}{\lambda}] \tag{18}$$

The diffraction envelope has a minimum when $\beta = m\pi$, with $m = \pm1, \pm2, ...$ And intereference maxima is given $p\lambda = a\sin\theta$ with $p = 0, \pm1, \pm2, ...$ dividing these equations we get the following expression for missing order

$$a = \frac{p}{m}b \tag{19}$$

Substituting b from above and using small angle approximation $\sin\theta = \frac{y}{f}$ we get,

$$\lambda = a\frac{\bar{y}}{f} \tag{20}$$

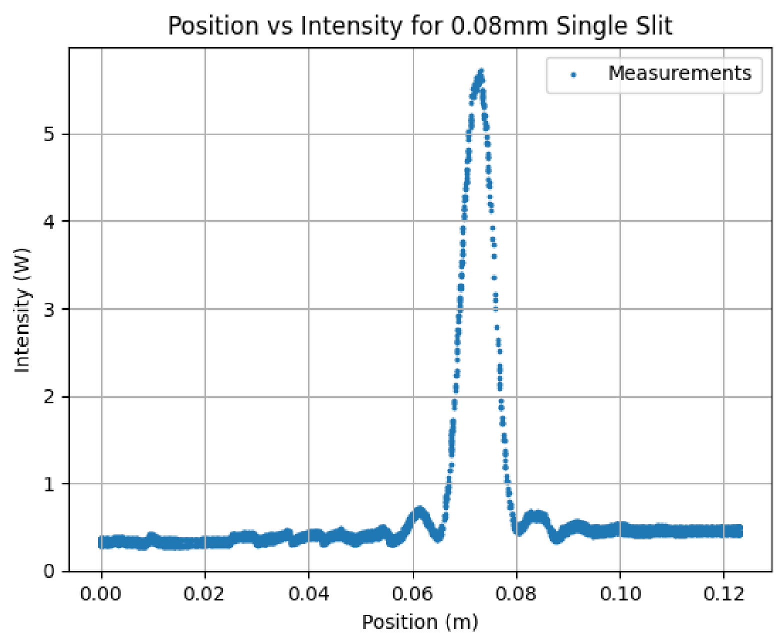
Where $\bar{y}$ is the distance between two envelope minima.

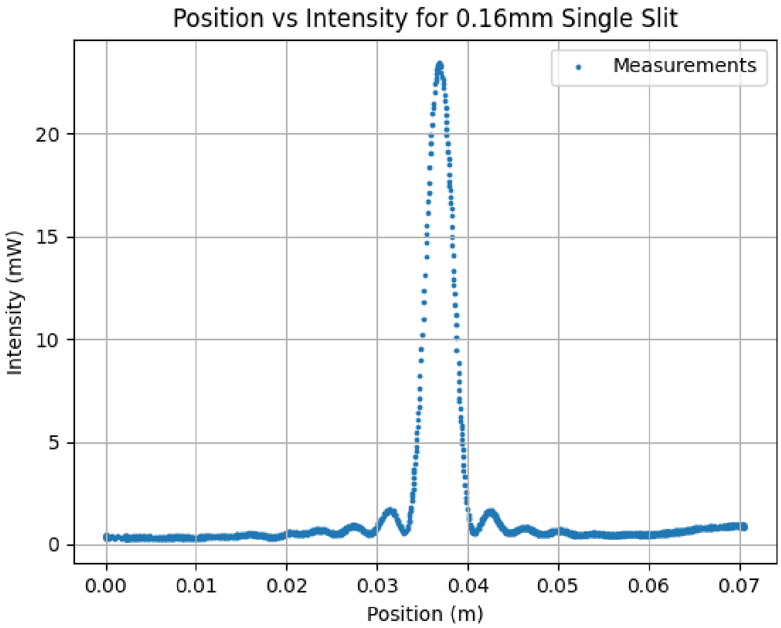
3 Results

In the experiment, a laser source of known wavelength was placed in front of the diffraction device and the observation screen placed *relatively* far from the diffraction device so that the formulas obtained in the theory section can be applied. The following results show that our data is in agreement with the theory for far field diffraction. In the table, the properties of the setup and results can be found and from the graphs, the diffraction pattern for single and double slit setup can be seen. Error propagation is calculated with an online error propagation calculator[2].

3.1 Part A: Single Slit

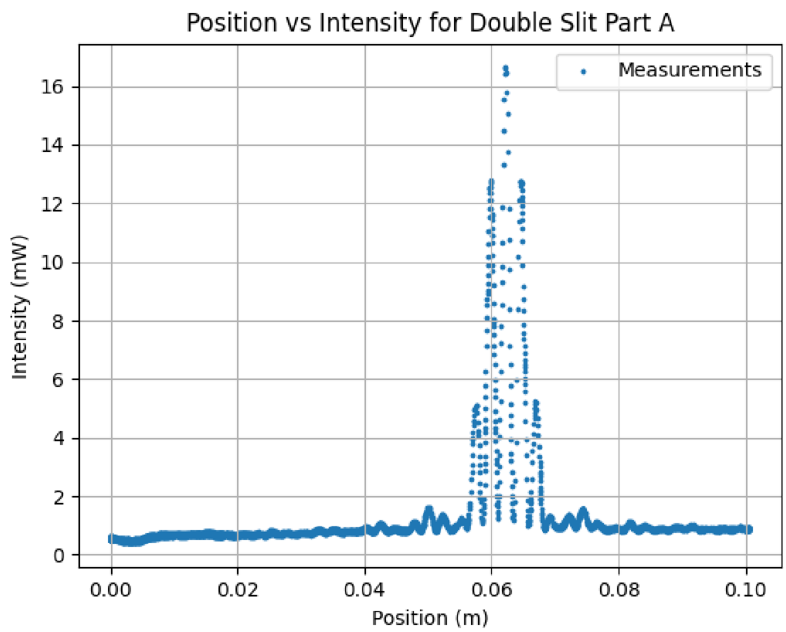
Pattern	A	B
Width of the slit, a	$0.08mm \pm 0.005mm$	$0.16mm \pm 0.005mm$
Distance from the slit to the screen, L	$940mm \pm 15mm$	$940mm \pm 15mm$
Average distance between minima, y	$7.75mm$	$3.7mm$
$\overline{\lambda} = \frac{a\overline{y}}{L}$	$659nm$	$639nm$
Error $\triangle$ y on $\overline{y}$	$0.1mm$	$0.01mm$
Error on $\triangle\lambda$ on $\overline{\lambda} = \frac{a}{L}\triangle y$	$43.14nm$	$27.89nm$
$\lambda = \overline{\lambda} \pm \triangle\lambda$	$659nm \pm 43.4nm$	$639nm \pm 27.89nm$



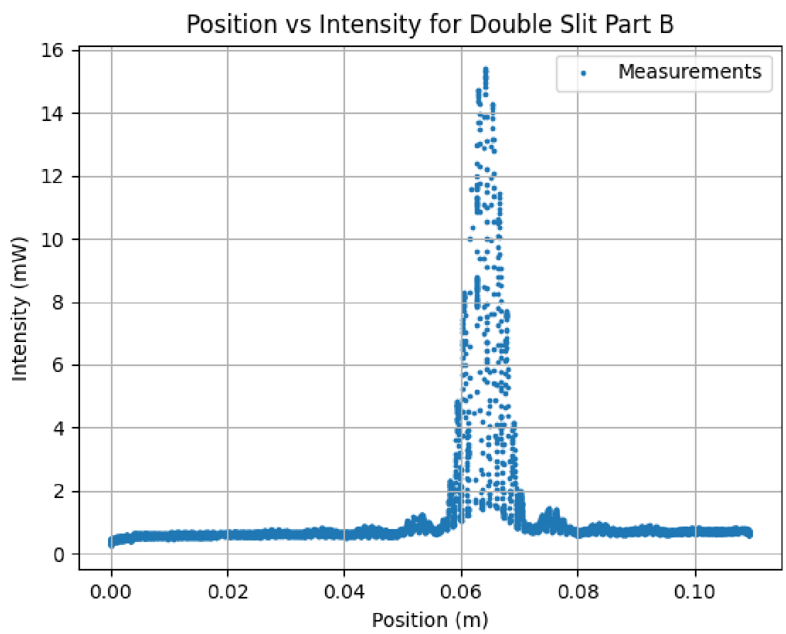


3.2 Part B: Double Slit

Pattern	D	E
Distance between the center of the slits, d	$0.25mm$	$0.5mm$
Distance from the slit to the screen, L	$940mm$	$940mm$
Average distance between minima, $\overline{y}$	$2.466mm$	$1.25mm$
$\overline{\lambda} = \frac{d\overline{y}}{L}$	655.85	$664.89nm$
Error $\triangle$ y on $\overline{y}$	$0.1mm$	$0.1mm$
Error on $\triangle\lambda$ on $\overline{\lambda} = \frac{d}{L}\triangle y$	$31.44nm$	$55.28nm$
$\lambda = \overline{\lambda} \pm \triangle\lambda$	$655.84 \pm 31.44nm$	$664.89 \pm 55.28nm$



Position vs Intensity for Double Slit Part A
Measurements
Intensity (mW)
Position (m)
0
2
4
6
8
10
12
14
16
0.00
0.02
0.04
0.06
0.08
0.10



Position vs Intensity for Double Slit Part B
Measurements
Intensity (mW)
Position (m)
0
2
4
6
8
10
12
14
16
0.00
0.02
0.04
0.06
0.08
0.10

4 Discussion

From *far field* diffraction theory the λ value is calculated. And from the data that has been collected in the laboratory, with the uncertainties, is in agreement with the manufacturer's value of the wavelength of the light $\lambda = 650nm$. It must be pointed out that the device for which the diffraction effect was obtained has high uncertainties and even if we ignored the other uncertainties, such as caused by the power meters uncertainty, thus the values we obtained for λ has approximately 8%.

5 References

1) Hecht, Eugene (2002). Optics. Addison-Wesley. ISBN 978-0-321-18878-6.
2)Online Error Propagation Calculator, http://www.julianibus.de/

A Python Code

For finding the average distances between two adjacent minima values and graph of part A has been calculated with my python codes.

```
import matplotlib.pyplot as plt
import numpy as np
import csv

x = []
y = []
local_minimums = []
deneme = [1, 2, 3, 4, 3, 5, 6, 7, 5, 8, 9, 7, 11, 12, 15, 18, 17, 19, 20]

with open('Single_Slit0.08mm_deneme1.csv', 'r') as csvfile:
    plots = csv.reader(csvfile, delimiter=',')
    for row in plots:  # using for loop i put the values is the empty arrays x,y
        x.append(float(row[0]))
        y.append(float(row[1]))

y_new = y[0:round(len(y)/2)]   #verinin ilk yarısına bakıyorum
x_new = x[0:round(len(x)/2)]   #çünkü en parlak olan kısımdaki aralık değerleri berbat eder.

for i in range(1, len(y_new) - 1):   #local minimum olanları bir matrix'e atıyorum
    if y_new[i - 1] > y_new[i] and y_new[i + 1] > y_new[i]: #local min olma koşulu
        local_minimums.append(x_new[i])     #y de karşılık gelen x değerlerini alıyorum

clean_localmin = []


for e in local_minimums:
    if not clean_localmin.__contains__(e):
        clean_localmin.append(e)

print('local minimum values (on x)')
print(local_minimums)
print('non repating local min values')
print(clean_localmin)

differences_x = []
for i in range(1, len(clean_localmin)): # iki minimum arasındaki farkı buluyorum
    differences_x.append(clean_localmin[i] - clean_localmin[i-1])

print('differences:')
print(differences_x)
avarage_x = sum(differences_x)/len(differences_x) # farkların ortalamasını alıyorum
print('avarage of differences')
print(avarage_x)
plt.scatter(x, y, s=3, label='Measurements')
plt.xlabel('Position (m)')
plt.ylabel('Intensity (W)')
```

```python
plt.grid()
plt.title('Position vs Intensity for 0.08mm Single Slit')
plt.legend()
plt.show()
```