Problem1:

consider the nonhomogeneous linear recurrence relation $a_n = 3a_{n-1} + 2^n$

a) Show that whether $a_n = -2^{n+1}$ is a solution of the given recurrence relation or not. Show your work step by step.

answer:

a) I assume that $a_n = -2^{n+1}$ and plug into the recurrence relation.

$$-2^{n+1} = 3a_{n-1} + 2^n$$

$$-2^{n+1} = 3(-2^{n+1-1}) + 2^n$$

$$-2^{n+1} = 3(-2^n) + 2^n$$

$$3(-2^n) = 3(-2^n)$$

$a_n = -2^{n+1}$ is a solution of the given recurrence relation.

b) Find the solution with $a_0 = 1$.

1) $a_n^{(g)} = a_n^{(h)} + a_n^{(p)}$

2) I find characteristic polynomial:

$$a_n^{(h)} \Rightarrow \underset{r}{a_n} = 3\underset{r^0=1}{a_{n-1}} \Rightarrow r - 3 = 0$$

$$r = 3$$

$\boxed{a_n^{(h)} = c_1 . 3^n}$ → I find the homogeneous part.

3) $a_n^{(p)} = A.2^n$ → must be 2^n on both sides of the equation.

$a_{n-1} = A.2^{n-1}$

$$\Rightarrow A.2^n = 3.A.2^{n-1} + 2^n, \quad a_0 = 1$$

$$A = \frac{3A}{2} + 1 \Rightarrow \boxed{A = -2}$$

$\boxed{a_n^{(p)} = -2.2^n}$ → I find the particular part

4) $a_n = c_1 . 3^n - 2.2^n, \quad a_0 = 1$

$$1 = c_1 - 2 \Rightarrow \boxed{c_1 = 3} \Rightarrow \boxed{a_n = 3^{n+1} - 2^{n+1}}$$

Problem 2: Solve the recurrence relation $f(n) = 4f(n-1) - 4f(n-2) + n^2$ for $f(0)=2$ and $f(1)=5$.

answer: $f(n) - 4f(n-1) + 4f(n-2) = n^2$

$a_n^p = 2^n (n^2) \rightarrow (r-2)^3$

$a_n^p = A + Bn + Cn^2$

homogenous part:

I assume that $f(n-2) = r^0$

so $r^n - 4r + 4 = 0$

$(r-2)^2 = 0 \Rightarrow r = 2$

$a_n^h = D2^n + En2^n$

$$a_n = \underbrace{A + Bn + Cn^2}_{\text{non-homogeneous part}} + \underbrace{D2^n + En2^n}_{\text{homogeneous part}}$$

I plug the non-homogeneous part into first equation.

$A + Bn + Cn^2 - 4(A + B(n-1) + C(n-1)^2) + 4(A + B(n-2) + C(n-2)^2) = n^2$

$Cn^2 - 4(C(n^2 - 2n + 1)) + 4(C(n^2 - 4n + 4)) = n^2$

$Cn^2 = n^2 \Rightarrow C = 1$

$Bn - \cancel{4Bn} + 4B + \cancel{4Bn} - 8B + 8nC - 16Cn = n^2$

$Bn - 8B + 4 - 8Cn = n^2$

$Bn - 8n = n^2$

$n(B-8) \Rightarrow B = 8$

$A - 4A + 4A + 4B - 8B - 4C + 16C = n^2$

$= A - 4B + 12C = n^2$

$= A - 4.8 + 12.1 = n^2 \Rightarrow A = 20$

non-homogeneous part $= 20 + 8n + n^2$

for $f(0) = 2 \Rightarrow 20 + D = 2 \Rightarrow \boxed{D = -18}$

$a_n = n^2 + 8n + 20 + 2^n(D + En)$ for $f(1) = 5 \Rightarrow 1 + 8 + 20 + 2(-18 + E) = 5$

$29 - 36 + 2E = 5$

$2E = 12$

$\boxed{E = 6}$

Result $= \boxed{a_n = n^2 + 8n + 20 + 2^n(-18 + 6n)}$

Problem 3: Consider the linear homogeneous recurrence relation
$a_n = 2a_{n-1} - 2a_{n-2}$

a) Find the characteristic roots of the recurrence relation.

$a_n - 2a_{n-1} + 2a_{n-2} = 0$

I assume that $a_{n-2} = r^0 = 1$

$r^2 - 2r + 2 = 0 \rightarrow$ characteristic polynomial

$\Delta = (-2)^2 - 4 \times 1 \times 2 = -4$

$r_1 = \dfrac{-(-2) + \sqrt{-4}}{2} = 1 + i$

$r_2 = \dfrac{-(-2) - \sqrt{-4}}{2} = 1 - i$

b) Find the solution of the recurrence relation with $a_0 = 1$ and $a_1 = 2$.

$a_n = \alpha (1+i)^n + \beta (1-i)^n$

$a_0 = \alpha (1+i)^0 + \beta (1-i)^0$

$\boxed{a_0 = \alpha + \beta = 1}$

$a_1 = \alpha (1+i)^1 + \beta (1-i)^1$

$a_1 = 2 = \alpha (1+i) + \beta (1-i) \rightarrow \alpha + \alpha i + \beta - \beta i = 2$

$\alpha i - \beta i + 1 = 2$

$\alpha i - \beta i = 1$

$i(\alpha - \beta) = 1 \Rightarrow (\alpha - \beta) = \dfrac{1}{i} = -i$

$\alpha + \beta = 1$

$\alpha - \beta = -i$

$2\alpha = 1 - i$

$= \boxed{\alpha = \dfrac{1-i}{2}}$ $\boxed{\beta = \dfrac{1+i}{2}}$

$$\boxed{a_n = \frac{1-i}{2}(1+i)^n + \frac{1+i}{2}(1-i)^n}$$

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