Problem 1: conditional Statements:

a) If it snows tonight, then I will stay at home.

converse: If I stay at home, then it will snow tonight.

contrapositive: If I don't stay at home, then it won't snow tonight.

Inverse: If it doesn't snow tonight, then I won't stay at home.

b) I go to the beach whenever it is a sunny summer day.

converse: It is a sunny summer day whenever I go to the beach.

contrapositive: It is not a sunny summer day whenever I don't go to the beach.

Inverse: I don't go to the beach whenever it is not a sunny summer day.

c) If I stay up late, then I sleep until noon.

converse: If I sleep until noon, then I stay up late.

contrapositive: If I don't sleep until noon, then I don't stay up late.

Inverse: If I don't stay up late, then I don't sleep until noon.

Problem 2 : Truth Tables For Logic Operators!

"exclusive or"

a) $(p \oplus \neg q)$

solution:

$1 \oplus 1 \equiv 0$

$1 \oplus 0 \equiv 1$

$0 \oplus 1 \equiv 1$

$0 \oplus 0 \equiv 0$

p	q	$\neg q$	$p \oplus \neg q$
1	1	0	1
1	0	1	0
0	1	0	0
0	0	1	1

b) $(p \Leftrightarrow q) \oplus (\neg p \Leftrightarrow \neg r)$

$1 \Leftrightarrow 1 \equiv 1$ $\quad 1 \Leftrightarrow 0 \equiv 0$

$0 \Leftrightarrow 0 \equiv 1$ $\quad 0 \Leftrightarrow 1 \equiv 1$

solution:

$\neg p$	$\neg r$	p	q	r	$(p \Leftrightarrow q)$	$(\neg p \Leftrightarrow \neg r)$	$((p \Leftrightarrow q) \oplus (\neg p \Leftrightarrow \neg r))$
							0
						1	
0	0	1	1	1	1		1
0	1	1	1	0	1	0	1
0	0	1	0	1	0	1	0
0	1	1	0	0	0	0	0
1	0	0	1	1	0	0	1
1	1	0	1	0	0	1	1
1	0	0	0	1	1	0	1
1	1	0	0	0	1	1	0

c) $(p \oplus q) \Rightarrow (p \oplus \neg q)$

(solution)

p	q	$\neg q$	$p \oplus q$	$p \oplus \neg q$	$(p \oplus q) \Rightarrow (p \oplus \neg q)$
1	1	0	0	1	1
1	0	1	1	0	0
0	1	0	1	0	0
0	0	1	0	1	1

Problem 3 : Predicates and Quantifiers :

- $P(x)$: "x can speak English."
- $Q(x)$: "x knows python."
- $H(x)$: "x is happy."

a) There is a student at the university who can speak English and who knows python.

(solution •) $\exists x(P(x) \land Q(x))$

b) There is a student at the university who can speak English but who doesn't know python.

(solution •) $\exists x((P(x) \land \neg(Q(x))$

c) Every student at university either can speak English or knows python.

(solution •) $\forall x(P(x) \lor Q(x))$

d) No student at the University can speak English or knows python.

(solution •) $\forall x(\neg P(x) \lor \neg Q(x))$

e) If there is a student at the university who can speak English and know python, then she /he is happy.

$\exists x(P(x) \land Q(x) \Rightarrow H(x))$

f) $(\exists x\, H(x)) \land (\exists y\, H(y))$

g) $\neg \forall x((Q(x)) \land P(x))$ → solution → Not student at University know python and speak English.

Problem 4 : Mathematical Induction: prove that $3+3.5+3.5^2 \_\_\_ +3.5^n = \frac{3(5^{n+1}-1)}{4}$ whenever n is a nonnegative integer.

1.step:

$n=0$ is true?

$3 = 3\frac{(5^1-1)}{\cancel{4}}^{1} \Rightarrow 3=3$ ✓ → correct

2.step: we assume that for $n=k$ is true.

$3+3.5+3.5^2+\_\_\_+3.5^k = \frac{3.(5^{k+1}-1)}{4}$

3.step: is $n=k+1$ true? ✓

$3+3.5+3.5^2+\_\_\_+3.5^k + 3.5^{(k+1)} = \frac{3.(5^{k+2}-1)}{4}$

we find yet in 2.step

$\frac{3.(5^{k+1}-1)}{4} + 3.5^{(k+1)} = \frac{3.(5^{k+2}-1)}{4} =$

$\Rightarrow \frac{3.5^{k+1}-3+12.5^{(k+1)}}{\cancel{4}} = \frac{3.(5^{k+2}-1)}{\cancel{4}} =$

$\Rightarrow 5^{k+1}(15)-3 = 3(5.5^{k+1}-3)$

$\Rightarrow 5^{k+1}.(15)-3 = 15.5^{(k+1)}-3$

✓

for $n=k+1$ is true so the above equality is true.

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Problem 5: Mathematical Induction:

- Prove that n^2-1 is divisible by 8 whenever n is an odd positive integer.

1.step: $n=1$ is true? → I give the least value for positive integer.

$1^2-1=0$

$0:8=0$ → correct ✓

2.step: we suppose that $n=k$ is true.

$k^2-1=8p$ $\quad p \in Z^+$

→ because k^2-1 is divisible by 8.

3.step: $n=k+1$ is true?

$(k+1)^2-1 = k^2+2k+1-1 = \underline{k^2+2k}$

→ $k^2=8p+1$, so $k=\sqrt{8p+1}$

$\Rightarrow k^2+2k = 8p+1+2\sqrt{8p+1}$

and finally we give the positive integer and prove that.

for $n=3$ so $n=k+1$, $k=2$ → I give the 3 to n because n is odd number.

$k^2=8p+1=4$

$\Rightarrow \underbrace{(8p+1)}_{4} + \underbrace{2\sqrt{8p+1}}_{\underbrace{2\times 2}_{4}} = 8:8=1$ ✓

↓

correct ✓

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problem 6: sets:

which of the following sets are equal? Show your work step by step

a) $\{t : t$ is a root of $x^2-6x+8=0\}$

b) $\{y : y$ is a real number in the closed interval $[2,3]\}$

c) $\{4,2,5,4\}$

d) $\{4,5,7,2\}-\{5,7\}$

e) $\{q : q$ is either the number of sides of a rectangle or the number of digits in any integer between 11 and 99$\}$

solutions:

a) $t = x^2-6x+8=0 \Rightarrow (x-4).(x-2)=0$

$x \quad -4$

$x \quad -2$

so $x=4$, $x=2$) roots → I find the roots $\{4,2\}$ for a)

b) y is a real number in the closed interval $[2,3]$.

c) $\{4,2,5,4\}$

d) $\{4,\cancel{5},\cancel{7},2\}-\{\cancel{5},\cancel{7}\} \Rightarrow \{4,2\}$

e) $\{q$: q is number of sides of a rectangle $=\{4\}$

or the number of digits in any integer between 11 and 99$\}$

$\{2\}$

$\{4\} \vee \{2\} = \{4,2\}$ → becouse "or" means $\vee =$

so Sets in option a,d and e are equal

Problem Bonus: Logic in Algorithms!

a) The output is given as "1" so we'll take the bottom case and the result is: True because the output is "1".

b)
```
#Include <stdio.h>
int main(){
    int q;
    int p;
    int q';
    if((p && q) || (p || q'))
    return 0;
}
```

=

input	output
1	0
0	1

so q→q'