1. find the value of $T(2)$ for the recurrence relation $T(n) = 3T(n-1) + 12n$, given that $T(0) = 5$

$T(2) = 3T(1) + 24$

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$T(1) = 3T(0) + 12$

$T(0) = 5$

$T(2) = (3 \times 5) + 12 = (27 \times 3) + 24$

$T(2) = 105$

2. Given a recurrence relation, solve it using substitution method

(a) $T(n) = T(n-1) + c$

$T(n-1) = T(n-2) + c + c$

$T(n-2) = T(n-3) + 3c$

$T(n) = T(n-3) + 3c$

$\downarrow$ k times

$T(n) = T(n-k) + kc$

$T(n) = T(0) + kc$

$n-k = 0$

$n = k$

where k = number of recursions

n = input size

Time complexity $O(n)$

(b) $T(n) = 2T(\frac{n}{2}) + n \rightarrow ①$

$T(\frac{n}{2}) = 2T(\frac{n}{2^2}) + \frac{n}{2}$

$2[2T(\frac{n}{2^2}) + \frac{n}{2}] + n$

$\downarrow$

$T(n) = 2^2 T(\frac{n}{2^2}) + n + n \rightarrow ②$

$2^2[2T(\frac{n}{2^3}) + \frac{n}{2^2}] + 2n$

$T(n) = 2^3 T(\frac{n}{2^3}) + 3n \rightarrow ③$

$\downarrow$ k times

$T(n) = 2^k T(\frac{n}{2^k}) + kn$

$T(\frac{n}{2^2}) = 2T(\frac{n}{2^3}) + \frac{n}{2^2}$

$T(\frac{n}{2^k}) = T(1) \quad \frac{n}{2^k} = 1$

$k = \log_2 n$

$k = \log n$

$T(n) = 2^k T(1) + kn$

$2^k = n$

$= n \times 1 + n\log n$ highest term

$O(n \log n)$

(c) $T(n) = 2T(\frac{n}{2}) + C$

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$T(\frac{n}{2^2}) = 2T(\frac{n}{2^3}) + C$

$T(n) = 2[2T(\frac{n}{2^2}) + C] + C$

$T(n) = 2^2 T(\frac{n}{2^2}) + 3C$

$T(n) = 2^2 [2T(\frac{n}{2^3}) + C] + C$

$T(n) = 2^3 T(\frac{n}{2^3}) + 7C$

$\downarrow$ k times

$T(n) = 2^k T(\frac{n}{2^k}) + 7C$

Assume base case is 1

$\frac{n}{2^k} = 1 \quad n = 2^k$

$= n \times 1 = n$

$O(n)$ Ans

(d) $T(n) = T(\frac{n}{2}) + C$

$T(\frac{n}{2}) = T(\frac{n}{2^2}) + C$

$T(\frac{n}{2^2}) = T(\frac{n}{2^3}) + C$

$T(n) = T(\frac{n}{2^2}) + 2C$

$T(n) = T(\frac{n}{2^3}) + 3C$

$\downarrow$ k times

$T(n) = T(\frac{n}{2^k}) + kC$

Assuming base case = 1

$\frac{n}{2^k} = 1 \quad n = 2^k$

$k = \log_2 n$

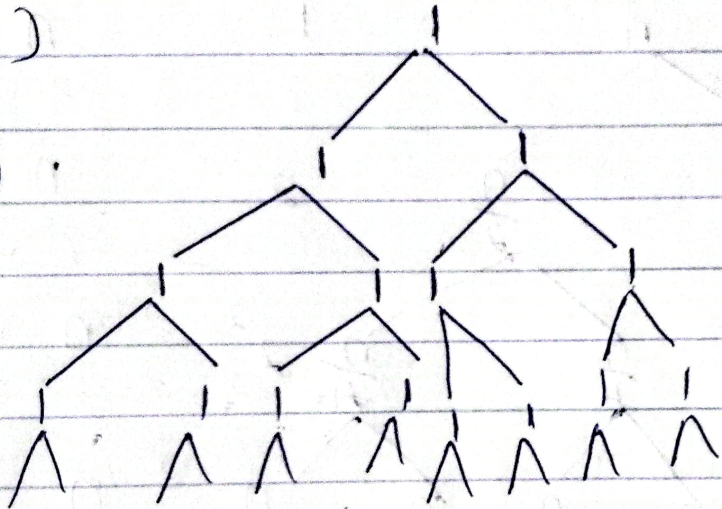
$k = \log n$

$1 \times \log n \cancel{C}$

$O(\log n)$ Ans

3(a) $T(n) = 2T(n-1) + 1$

recursive call | Tree | row sum



recursive call	row sum
$T(n)$	2^0
	2^1
$T(n-1)$	2^2
$T(n-2)$	2^3
$T(n-3)$	2^k

$\downarrow$ k times

base case when $n=0$

$n-k=0$

$n=k$, $k=n$

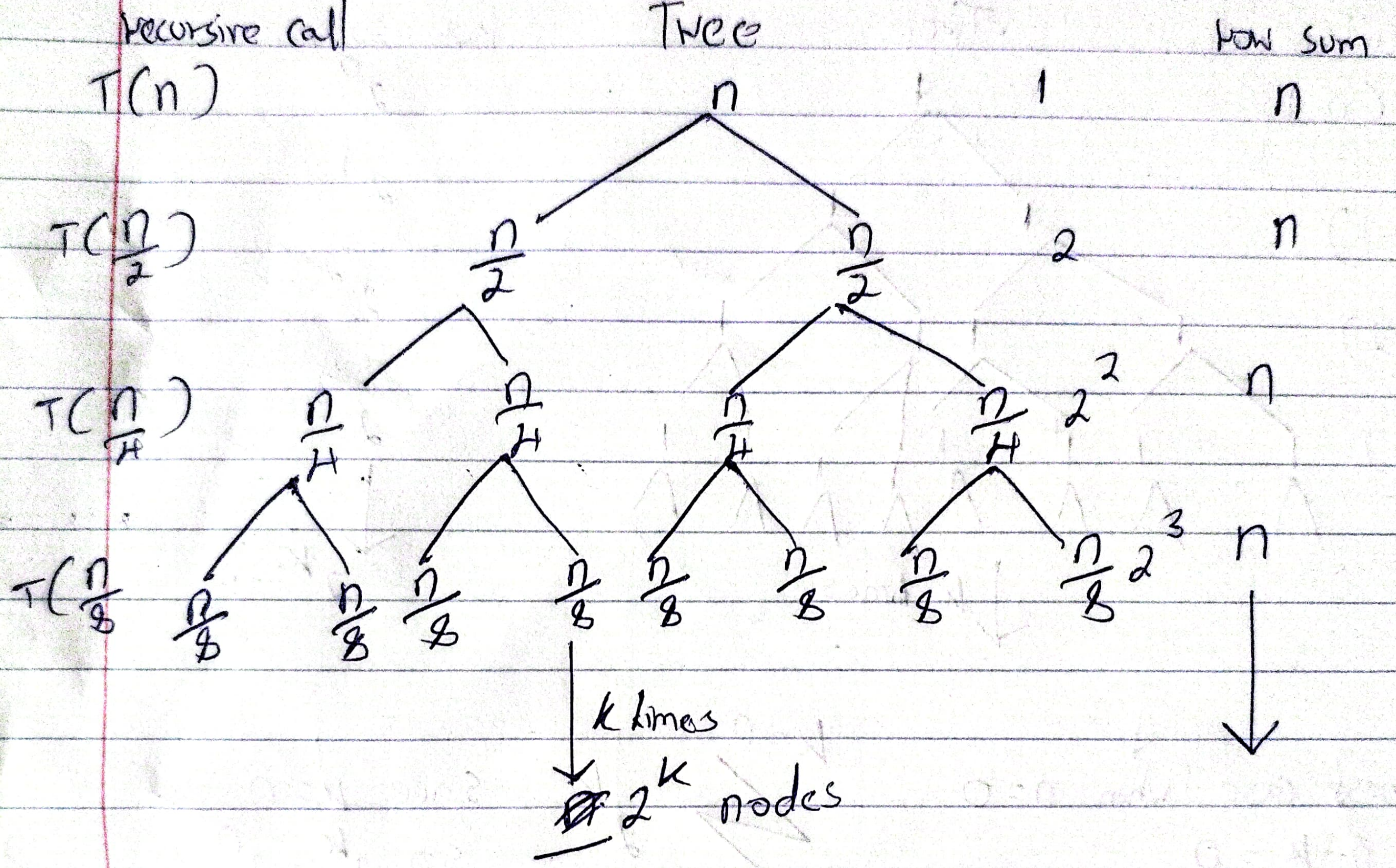
$$\sum_{k=0}^{k=n} 2^k$$

Since $k=n$

so $2^k = 2^n$

$\Theta(2^n)$

(b) $T(n) = 2T\left(\frac{n}{2}\right) + n$



base Case

when $n = 0$

$n - 2^k = 0$

$k = \log_2 n$

$$\sum_{k=0}^{k=\log_2 n} n$$

$n(\log_2 n + 1)$

$n\log_2 n + n$

grows faster

$n\log_2 n$ drop constant

$O(n\log n)$ Ans