

ECS132, homework1

Group 14: Xin Jin, Serena Tan, Davis Foster, Wendy Nguyen

January 19th 2018

1 Problem A

Let three cars be C_1, C_2, C_3 , let $C_1 = n$ be event that car 1 took n minutes to arrive.

1.1 1

We wish to find $P(\text{first arrive is 4 min})$,
 $P(\text{first arrive is 4 min}) = P(\text{no car arrives at 3 and exist a car arrives at 4})$
 $= P(\text{no 3 min})P(\text{exist 4 min} \mid \text{no 3 min})$ (book 2.7, not independent)
 $= P(C_1 \neq 3 \text{ and } C_2 \neq 3 \text{ and } C_3 \neq 3)P(\text{exist 4 min} \mid \text{no 3 min})$
 $= P(C_1 \neq 3)P(C_2 \neq 3)P(C_3 \neq 3)P(\text{exist 4 min} \mid \text{no 3 min})$ (book 2.6, independent)
 $= 0.5 * 0.5 * 0.5 * P(\text{exist 4 min} \mid \text{no 3 min})$
 $= 0.125 * P(\text{exist 4 min} \mid \text{no 3 min})$
 $= 0.125 * (1 - P(\text{no 4 min} \mid \text{no 3 min}))$
 $= 0.125 * (1 - P(C_1 = 4 \text{ and } C_2 = 4 \text{ and } C_3 = 4 \mid \text{no 3 min}))$
 $= 0.125 * (1 - P(C_1 = 4 \mid \text{no 3 min})P(C_2 = 4 \mid \text{no 3 min})P(C_3 = 4 \mid \text{no 3 min}))$ (book 2.6, independent)
 $= 0.125 * (1 - 0.5 * 0.5 * 0.5)$
 $= 0.109$

1.2 2

We wish to find $P(\text{sum is 10})$,
 $P(\text{sum is 10}) = P(C_1 = 4 \text{ and } C_2 = 3 \text{ and } C_3 = 3 \text{ or } C_1 = 3 \text{ and } C_2 = 3 \text{ and } C_3 = 4 \text{ or } C_1 = 3 \text{ and } C_2 = 4 \text{ and } C_3 = 3)$
 $= P(C_1 = 4 \text{ and } C_2 = 3 \text{ and } C_3 = 3) + P(C_1 = 3 \text{ and } C_2 = 4 \text{ and } C_3 = 3) + P(C_1 = 3 \text{ and } C_2 = 3 \text{ and } C_3 = 4)$ (book 2.4)
 $= P(C_1 = 4)P(C_2 = 3)P(C_3 = 3) + P(C_1 = 3)P(C_2 = 4)P(C_3 = 3) + P(C_1 = 3)P(C_2 = 3)P(C_3 = 4)$ (book 2.6)
 $= 0.25 * 0.5 * 0.5 + 0.5 * 0.25 * 0.5 + 0.5 * 0.5 * 0.25$
 $= 0.188$

1.3 2

We wish to find $P(\text{each is 3 min} \text{ --- arrive at same time})$,

$P(\text{each is 3 min} \text{ --- same time}) = P(\text{each is 3 min and same time}) / P(\text{same time})$ (2.8, Mailing Tubes)

$= P(\text{each is 3 min}) / P(\text{same time})$

$= P(C_1 = 3 \text{ and } C_2 = 3 \text{ and } C_3 = 3) / P(\text{same time})$

$= P(C_1 = 3)P(C_2 = 3)P(C_3 = 3) / P(\text{same time})$ (2.6)

$= 0.5 * 0.5 * 0.5 / P(\text{same time})$

$= 0.125 / P(\text{same time})$

$= 0.125 / P(\text{each is 3 or each is 4 or each is 5})$

$= 0.125 / (P(\text{each is 3}) + P(\text{each is 4}) + P(\text{each is 5}))$ (2.4)

... (we just compute $P(\text{each is } n)$ using same methods as computing $P(\text{each is 3})$ above)

$= 0.125 / (0.5^3 + 0.25^3 + 0.125^3)$

$= 0.800$

2 Problem B

Let $X_m = n$ denote n attempts happens in m epoch.

We wish to find $P(X_2 \geq 1 \mid 2 \text{ attempts})$.

$P(2 \text{ attempts}) = P(X_1 = 0 \text{ and } X_2 = 2 \text{ or } X_1 = 1 \text{ and } X_2 = 1 \text{ or } X_1 = 2 \text{ and } X_2 = 0)$

$= P(X_1 = 0 \text{ and } X_2 = 2) + P(X_1 = 1 \text{ and } X_2 = 1) + P(X_1 = 2 \text{ and } X_2 = 0)$ (2.2)

$= P(X_1 = 0)P(X_2 = 2 \mid X_1 = 0) + P(X_1 = 1)P(X_2 = 1 \mid X_1 = 1) + P(X_1 = 2)P(X_2 = 0 \mid X_1 = 2)$ (2.7)

$= (1 - p)^2 * p^2 + 2 * p * (1 - p) * p + p^2 * (1 - p)^2$

take $p = 0.6$

$= 0.403$

$P(X_2 \geq 1 \text{ and } 2 \text{ attempts}) = P(X_1 = 0 \text{ and } X_2 = 2 \text{ or } X_1 = 1 \text{ and } X_2 = 1)$

$= P(X_1 = 0 \text{ and } X_2 = 2) + P(X_1 = 1 \text{ and } X_2 = 1)$ (2.2)

$= (1 - p)^2 * p^2 + 2 * p * (1 - p) * p$

take $p = 0.6$

$= 0.346$

$P(X_2 \geq 1 \mid 2 \text{ attempts}) = P(X_2 \geq 1 \text{ and } 2 \text{ attempts}) / P(2 \text{ attempts})$ (2.8, Mailing Tubes)

$= 0.346 / 0.403$

$= 0.859$

3 Problem C

No written details as required. Let code tells.