ECS132, homework1

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1 Problem A

Let three cars be C_1, C_2, C_3, let $C_1 = n$ be event that car 1 took n minutes to arrive.

1.1 1

We wish to find P(first arrive is 4 min),
P(first arrive is 4 min) = P(no car arrives at 3 and exist a car arrives at 4)
= P(no 3 min)P(exist 4 min | no 3 min) (book 2.7, not independent)
= P($C_1! = 3$ and $C_2! = 3$ and $C_3! = 3$)P(exist 4 min | no 3 min)
= P($C_1! = 3$)P($C_2! = 3$)P($C_3! = 3$)P(exist 4 min | no 3 min) (book 2.6, independent)
= 0.5 * 0.5 * 0.5 * P(exist 4 min | no 3 min)
= 0.125 * P(exist 4 min | no 3 min)
= 0.125 * (1 - P(no 4 min | no 3 min))
= 0.125 * (1 - P($C_1! = 4$ and $C_2! = 4$ and $C_3! = 4$ | no 3 min))
= 0.125 * (1 - P($C_1! = 4$ | no 3 min)P($C_2! = 4$ | no 3 min)P($C_3! = 4$ | no 3 min)) (book 2.6, independent)
= 0.125 * (1 - 0.5 * 0.5 * 0.5)
= 0.109

1.2 2

We wish to find P(sum is 10),
P(sum is 10) = P($C_1 = 4$ and $C_2 = 3$ and $C_3 = 3$ or $C_1 = 3$ and $C_2 = 3$ and $C_3 = 4$ or $C_1 = 3$ and $C_2 = 3$ and $C_3 = 4$)
= P($C_1 = 4$ and $C_2 = 3$ and $C_3 = 3$) + P($C_1 = 3$ and $C_2 = 4$ and $C_3 = 3$) + P($C_1 = 3$ and $C_2 = 3$ and $C_3 = 4$) (book 2.4)
= P($C_1 = 4$)P($C_2 = 3$)P($C_3 = 3$) + P($C_1 = 3$)P($C_2 = 4$)P($C_3 = 3$) + P($C_1 = 3$)P($C_2 = 3$)P($C_3 = 4$) (book 2.6)
= 0.25 * 0.5 * 0.5 + 0.5 * 0.25 * 0.5 + 0.5 * 0.5 * 0.25
= 0.188

1.3 3

We wish to find P(each is 3 min — arrive at same time),
P(each is 3 min — same time) = P(each is 3 min and same time) / P(same time) (2.8, Mailing Tubes)
= P(each is 3 min) / P(same time)
= P($C_1 = 3$ and $C_2 = 3$ and $C_3 = 3$) / P(same time)
= P($C_1 = 3$)P($C_2 = 3$)P($C_3 = 3$) / P(same time) (2.6)
= 0.5 * 0.5 * 0.5 / P(same time)
= 0.125 / P(same time)
= 0.125 / P(each is 3 or each is 4 or each is 5)
= 0.125 / (P(each is 3) + P(each is 4) + P(each is 5)) (2.4)
... (we just compute P(each is n) using same methods as computing P(each is 3) above)
= $0.125/(0.5^3 + 0.25^3 + 0.25^3)$
= 0.800

2 Problem B

Let $X_m = n$ denote n attempts happens in m epoch.
We wish to find P($X_2 >= 1$ | 2 attempts).

P(2 attempts) = P($X_1 = 0$ and $X_2 = 2$ or $X_1 = 1$ and $X_2 = 1$ or $X_1 = 2$ and $X_2 = 0$)
= P($X_1 = 0$ and $X_2 = 2$) + P($X_1 = 1$ and $X_2 = 1$) + P($X_1 = 2$ and $X_2 = 0$) (2.2)
= P($X_1 = 0$)P($X_2 = 2|X_1 = 0$) + P($X_1 = 1$)P($X_2 = 1|X_1 = 1$) + P($X_1 = 2$)P($X_2 = 0|X_1 = 2$) (2.7)
$= (1-p)^2 * p^2 + 2 * p * (1-p) * p + p^2 * (1-p)^2$
take p = 0.6
= 0.403

P($X_2 >= 1$ and 2 attempts) = P($X_1 = 0$ and $X_2 = 2$ or $X_1 = 1$ and $X_2 = 1$)
= P($X_1 = 0$ and $X_2 = 2$) + P($X_1 = 1$ and $X_2 = 1$) (2.2)
$= (1-p)^2 * p^2 + 2 * p * (1-p) * p$
take p = 0.6
= 0.346

P($X_2 >= 1$ | 2 attempts) = P($X_2 >= 1$ and 2 attempts) / P(2 attempts) (2.8, Mailing Tubes)
= 0.346 / 0.403
= 0.859

Thus, P($X_2 >= 1$ | 2 attempts) = 0.859

3 Problem C

No written details as required. Let code tells.