Particle Filter

Computer Vision (CS0029)

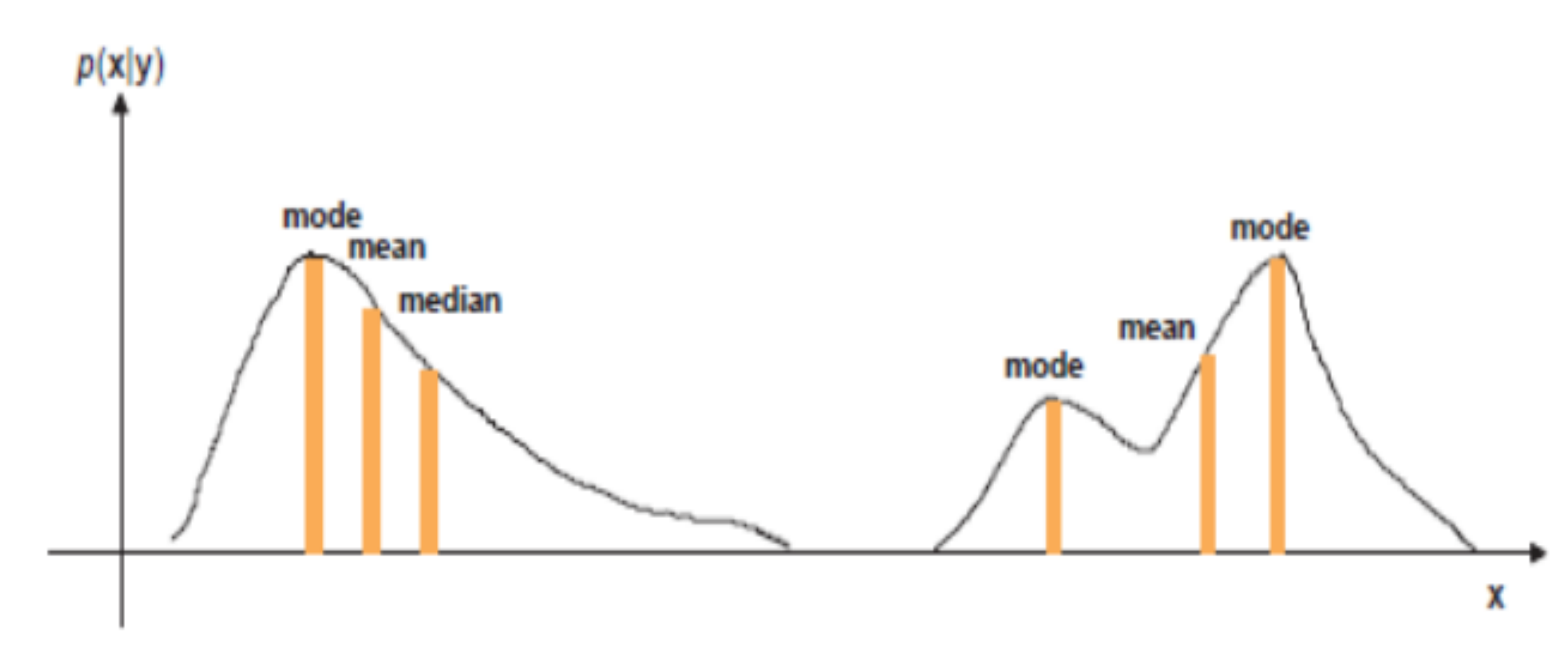
Why Particle Filter?

- Kalman filter
  - Kalman filter is limited to linear system with Gaussian noise
  - Extended KF and Unscented KF are limited to Gaussian distribution
  - Many practical problem show non linear dynamics and non Gaussian noises
  - We often find multi-model distributions

- Find an approximate solution using a complex model rather than a exact solution using a simplified model

Multi-model distribution

- A Gaussian distribution is unimodal
- For a multimodal distribution, the mode, mean and median do not coincide
- Kalman filter is not suitable for multi-model distribution

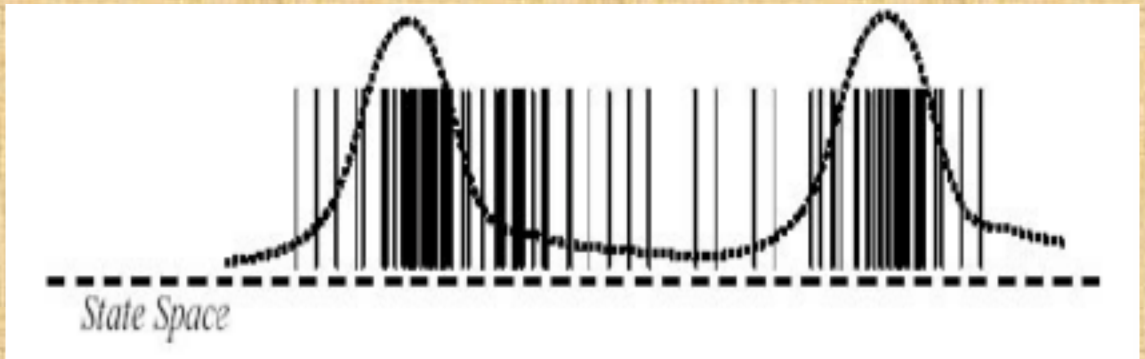


Particle Filter

- Particle filter is often called:
  - Condensation algorithm
  - Sequential Monte Carlo methods
  - Bootstrap filters …

- Particle filter is based on representing the posterior PDF by a set of randomly chosen weighted samples
- “Randomly chosen” = “Monte carlo”

Basic Principle



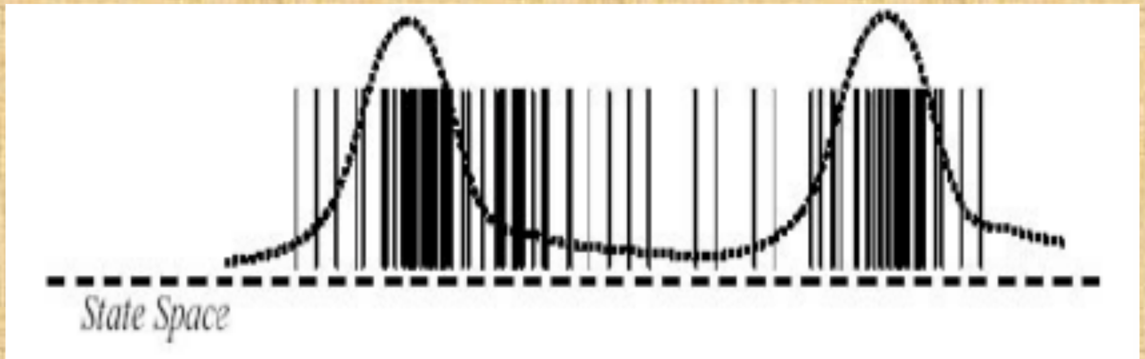
- Represent the PDF as a set of weighted samples
- For $N \to \infty$, PF converges to the true PDF
- For $N \to \infty$, PF approaches optimal Bayesian estimate
- Region of high density
  - Many particle
  - Large weight particle

$\{x_{0:k}^i\}$: set of particles

$\{w_k^i\}$: associated weights, normalized to $\sum_i w_k^i = 1$

$p(x|Z_{1:k})=\sum_{i=1}^{N} w_k^i \delta\,(\mathrm{x}_{0:\mathrm{k}} - x_{0:k}^i)$

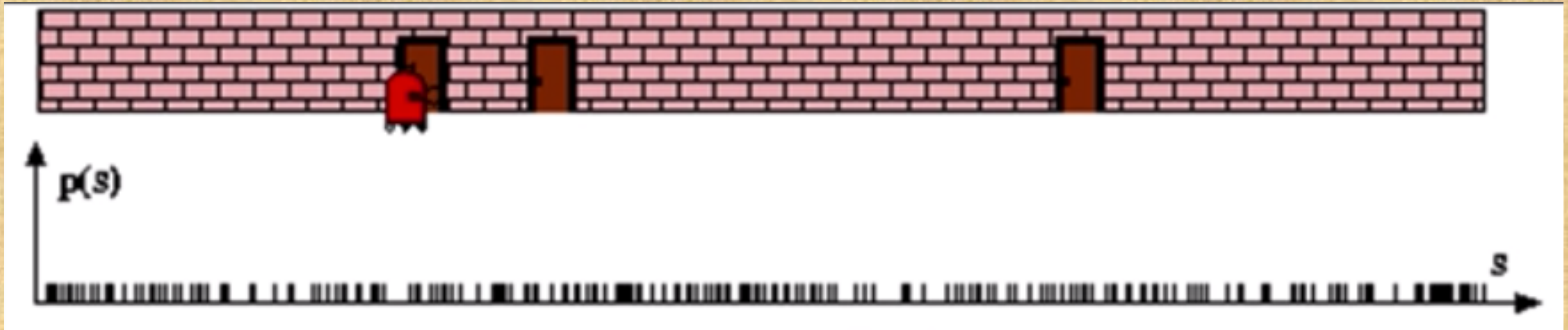
Basic Principle



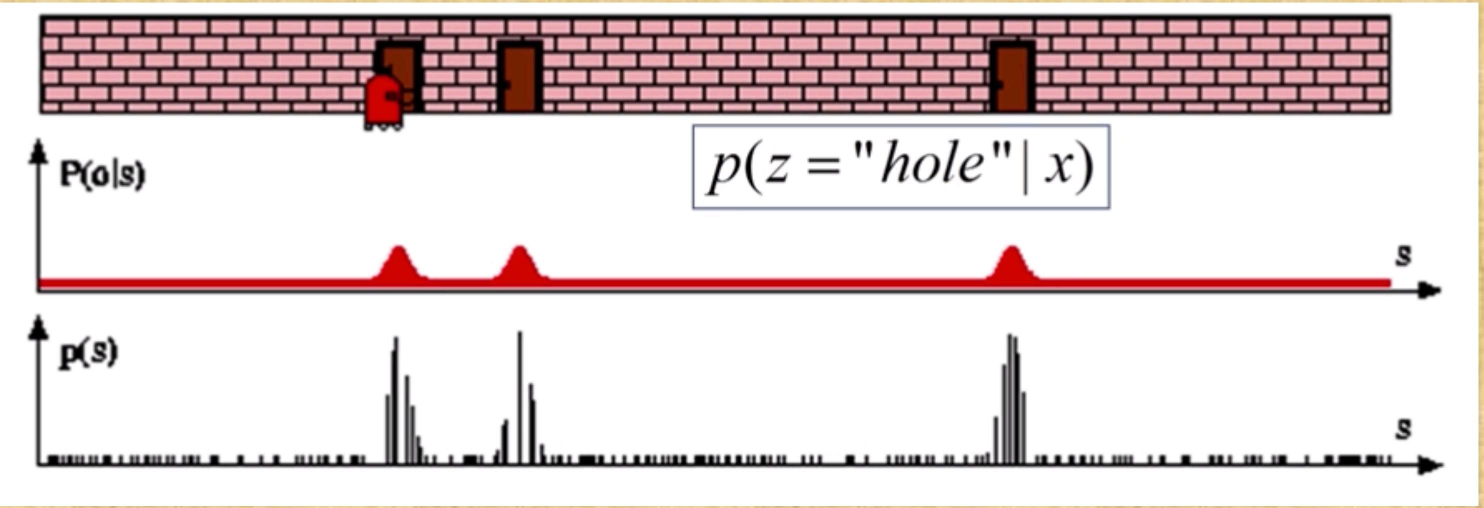
- Density is represented by both where the particles are and their weight
- $p(x = x_0)$ is now probability of drawing an x with value (really close to) x_0

Robot Localization Example

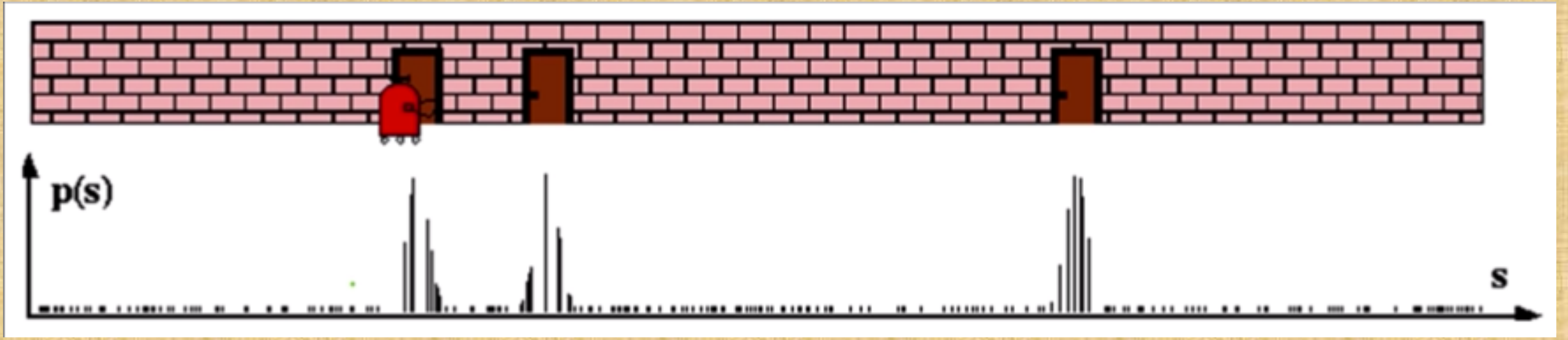
Prior density



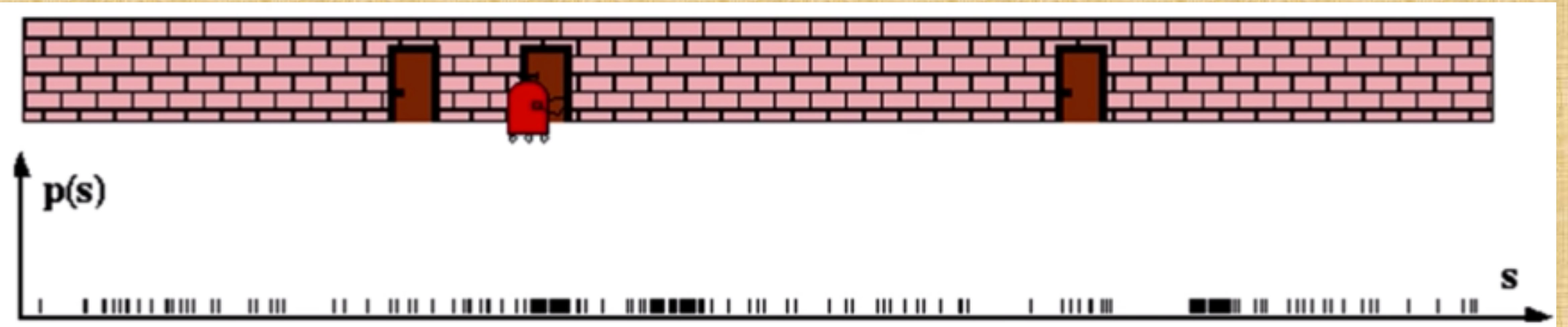
Add sensor information



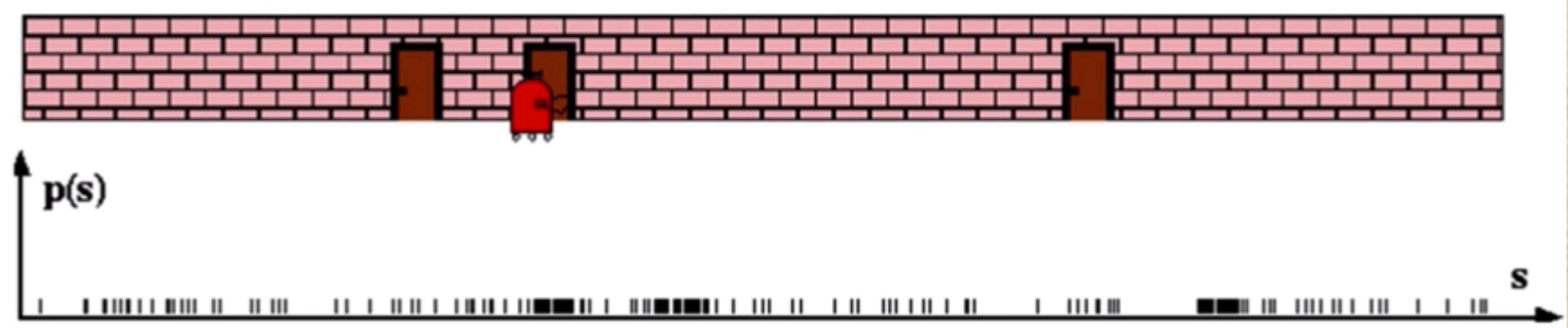
Resample and Move the Robot



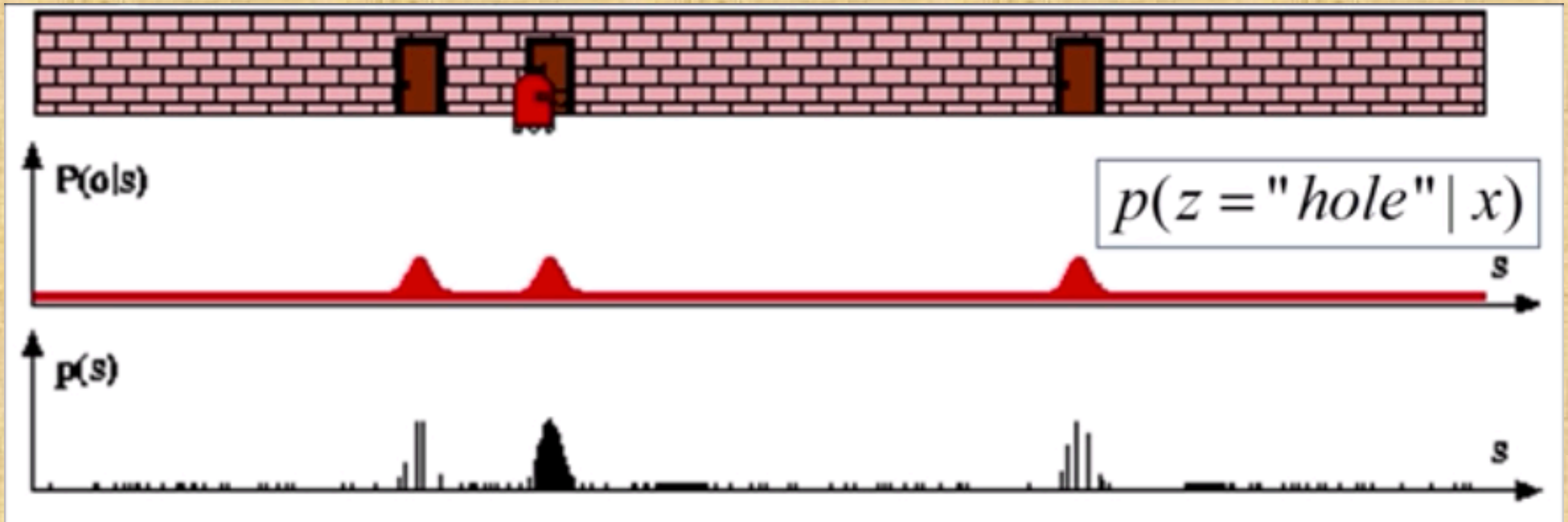
Resample



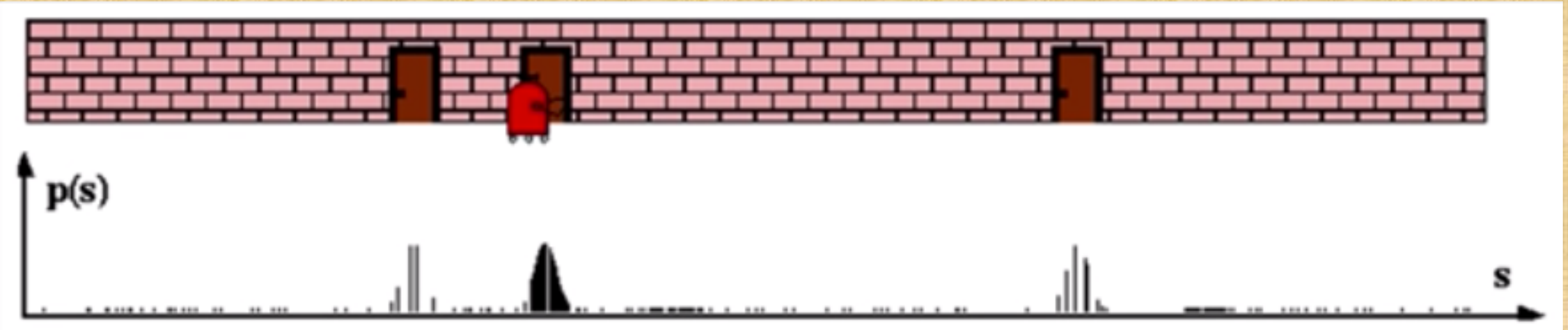
Next Sensor Reading



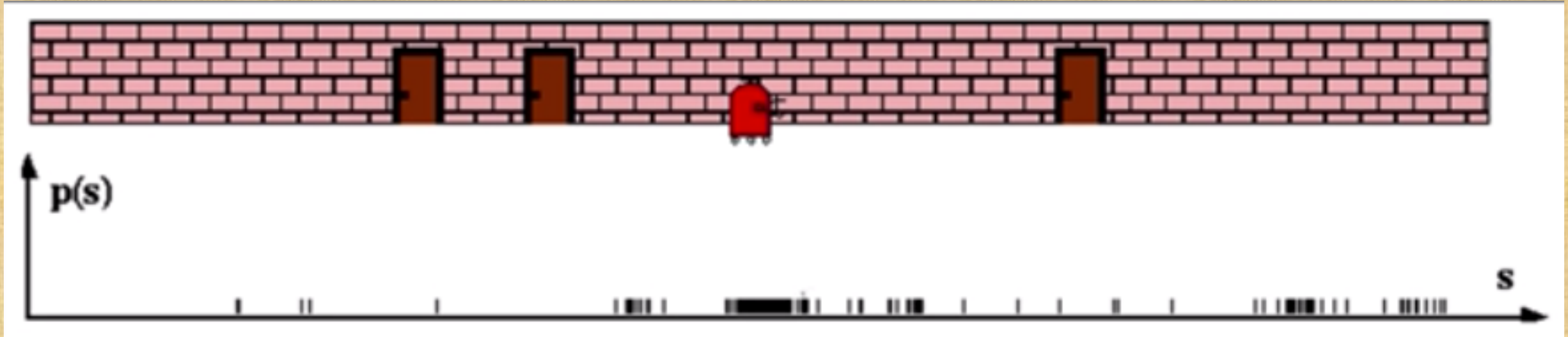
Robot see a hole again



Move Again



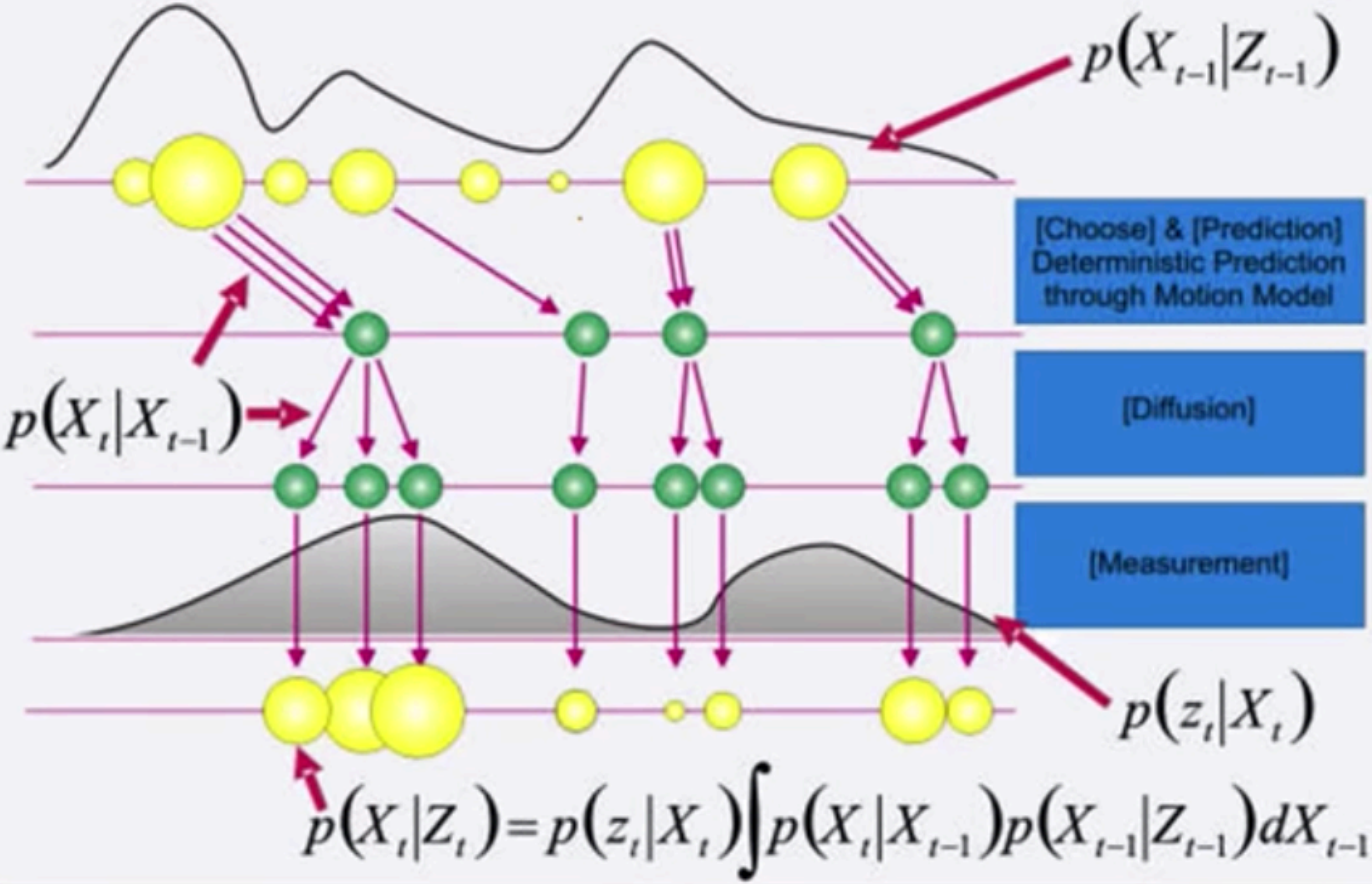
Move and Resample



Algorithm of Particle Filter

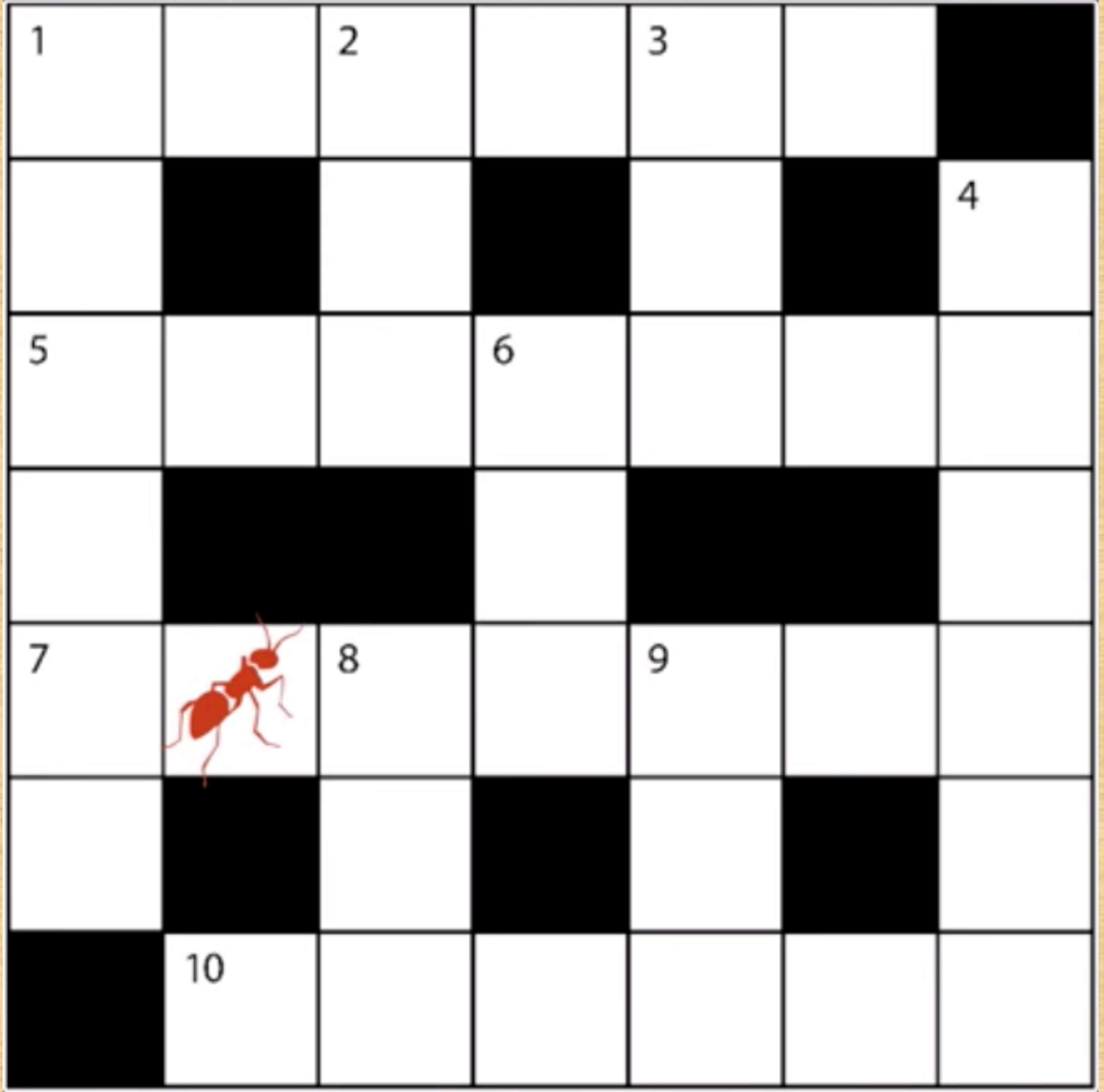
Particles from previous time step: $\{S_{t-1} \ =< x_{t-1}^{j}, w_{t-1}^{j} >, u_t, z_t\}$

1. $S_t = \emptyset, \eta = 0$
2. $For\ i = 1 \ldots n$ (Resample, generate n new samples)
3. Sample index j(i) from the discrete distribution give by w_{t-1}
4. Sample x_t^i from $p(x_t|x_{t-1}, u_t)$ using $x_{t-1}^{j(i)}$ and u_t Control
5. $w_t^i = p\left(z_t \middle| x_t^i\right)$ (Compute importance weight - reweight)
6. $\eta = \eta + w_t^i$ (Update normalization factor)
7. $S_t = S_t \cup \{< x_t^i, w_t^i >\}$
8. $For\ i = 1 \ldots n$
9. $w_t^i = \frac{w_t^i}{\eta}.$ (Normalize weights)


$p(X_{t-1}|Z_{t-1})$
[Choose] & [Prediction) Deterministic Prediction through Motion Model
[Diffusion]
[Measurement]
$p(X_t|X_{t-1})$
$p(z_t|X_t)$
$p(X_t|Z_t) = p(z_t|X_t)\int p(X_t|X_{t-1})p(X_{t-1}|Z_{t-1})dX_{t-1}$

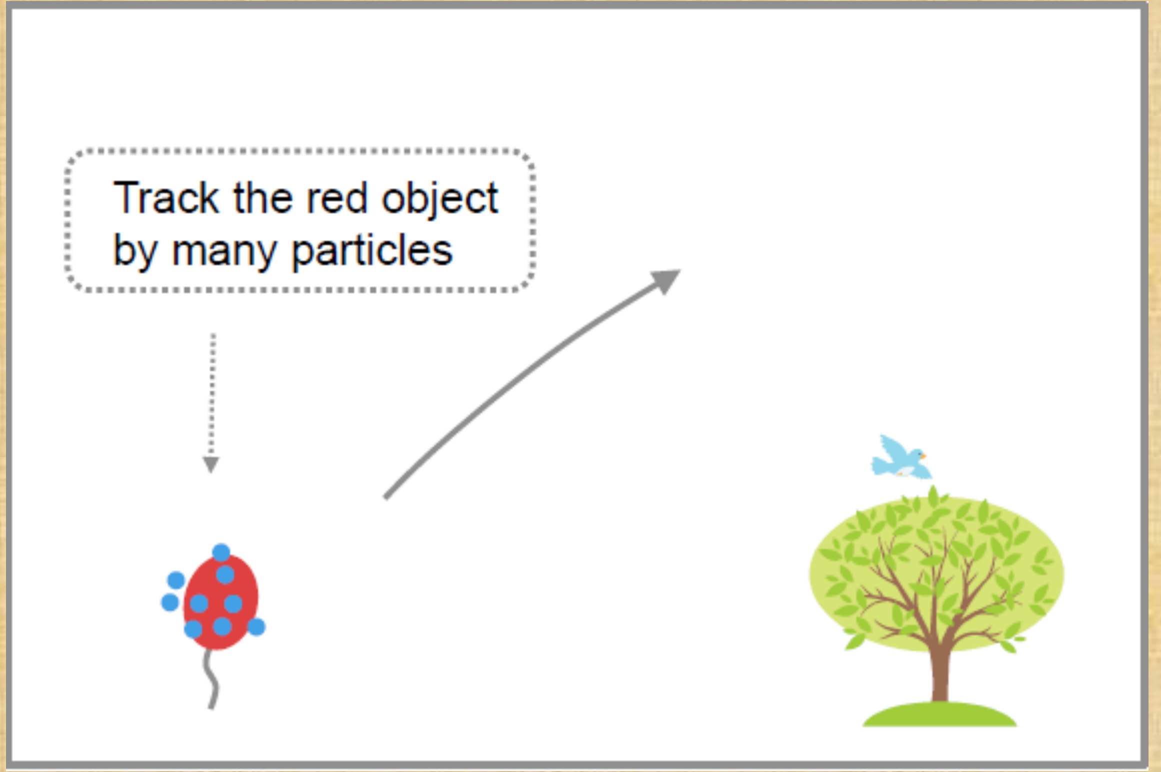
Practice

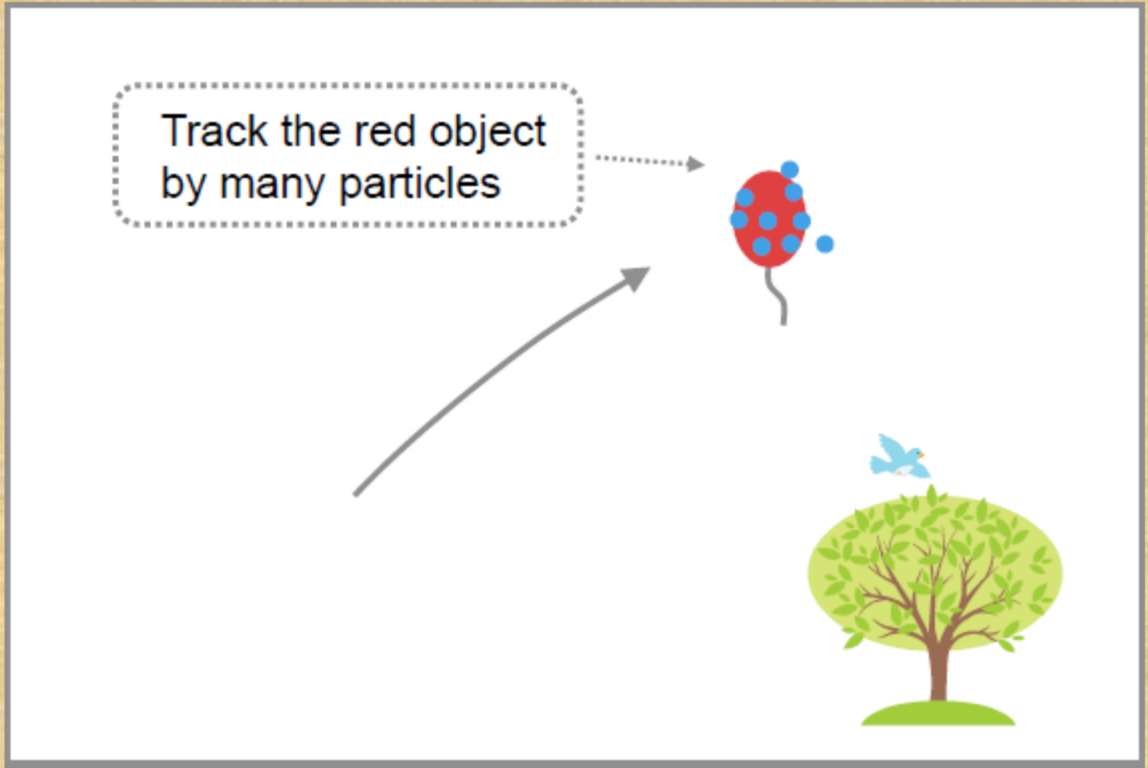
Time (t)	Observation (z)	Control (u)
0	1	↑
1	0	←
2	1	↑
3	1	↑
4	0	→
5	1	



Simple Example in Image Tracking

- Track the redshirt in a video





Simple Example in Image Tracking

- Track the redshirt in a video

- State Vector?

- Observation: color of pixel which particle exist
  - Likelihood: $P\left(Z_n \middle| X_n^k\right) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{d^2}{2\sigma^2}\right)$
  - $d = \sqrt{(r-255)^2 + g^2 + b^2}$

Summary

- A implementation of Bayes filter
- Use particles (Monte Carlo method) to present a distribution
  - Arbitrary distribution

- Steps
  - Resample by each particles' weight from previous time step
  - Predict particles' locations at time t from t-1 $P(X_t)$
  - Likelihood of measurements from each particles' location $P(Z_t|X_t)$
  - Update the weight of each particle by the likelihood of particle's measurement
  - Normalize particle's weights : $\sum_{i=0}^{n} w_i = 1$