Bayes Filter

Computer Vision (CS0029)

Monty Hall Problem

- Not change: 33.3%
- Change:
  - Your first choice is car (33.3%), but you change the choice: 0%
  - Your first choice is goat (66.6%), but you change the choice: 100%
  - 33.3%*0%+66.6+100% = 66.6% to get the car

Bayes' Theorem

- $P(A|B) = \frac{P(B|A)P(A)}{P(B)}$
- P(A) - prior
- P(B|A) - evidence - event B occurring given that A is true
- P(A|B) - posterior - event A occurring given that B is true
- P(B) - normalization

Example

- $P(A|B) = \frac{P(B|A)P(A)}{P(B)}$

- A test for using particular drug 99% accuracy (this is, the test will produce 99% true positive result for drug users and 99% true negative result for non-drug users). We also know that our country has 0.5% people use the drug.
- *If we randomly select a person and run this test, and the result is positive, how much chance the person actually use the drug?*

- $P(User|+) = \frac{P(+|User)P(User)}{P(+)} = \frac{P(+|User)P(User)}{P(+|User)P(User)+P(+|Non-user)P(non-user)} = \frac{0.99*0.005}{0.99*0.005+0.01*0.995} = 33.2\%$

Bayes' Theorem (Generalized)

- $P(A|B,C) = \frac{P(B|A,C)P(A|C)}{P(B|C)}$

- X_i: you select door i
- H_j: door j is opened
- C_k : door k has the car
- A: C_2; B: H_3 ; C:X_1
- $P(H_3|C_2,X_1)$: 1
- $P(C_2|X_1)$: $\frac{1}{3}$
- $P(H_3|X_1)$: $\frac{1}{2}$
- $P(C_2|H_3,X_1)$=$\frac{P(H_3|C_2,X_1)P(C_2|X_1)}{P(H_3|X_1)} = \frac{\left(1 * \frac{1}{3}\right)}{\frac{1}{2}} = \frac{2}{3}$

Practice

- $P(A|B) \ = \frac{P(B|A)P(A)}{P(B)}$

- A factory has three machine A, B, C. The three machines account for 20%(A), 30%(B) and 50%(C) of the factory output.
- The fraction of defective items produced is 5% for A, 3% for B and 1% for C
- *If a randomly select item is defective, what is the probability that is was produce by machine A?*

Tracking with Dynamics

- Idea: given a model of expected motion, predict where objects will occur in next frame, even before seeing the image

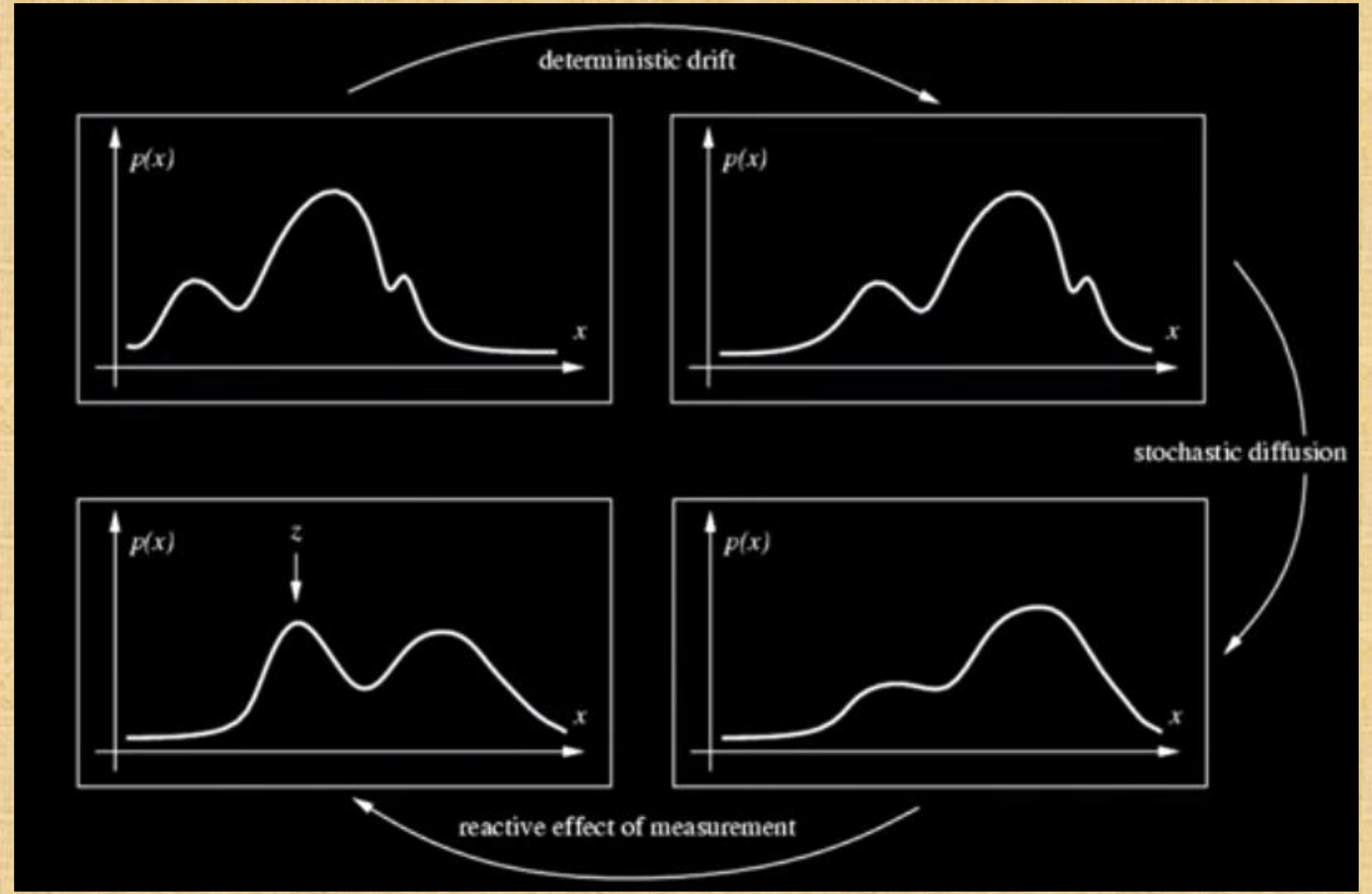
- Goal
  - Do less work looking for the object, restrict the search
  - Get improved estimates since measurement noise is tempered by smoothness, dynamics priors

- https://www.youtube.com/watch?v=M76tXEO_yGQ

Assumptions

- Continuous (modeled) motion patterns
  - Object do not disappear and reappear in different places in the scene
  - Camera is not moving instantly to new viewpoint
  - Gradual change in pose between camera and scene
- Everything is **“uncertain”**

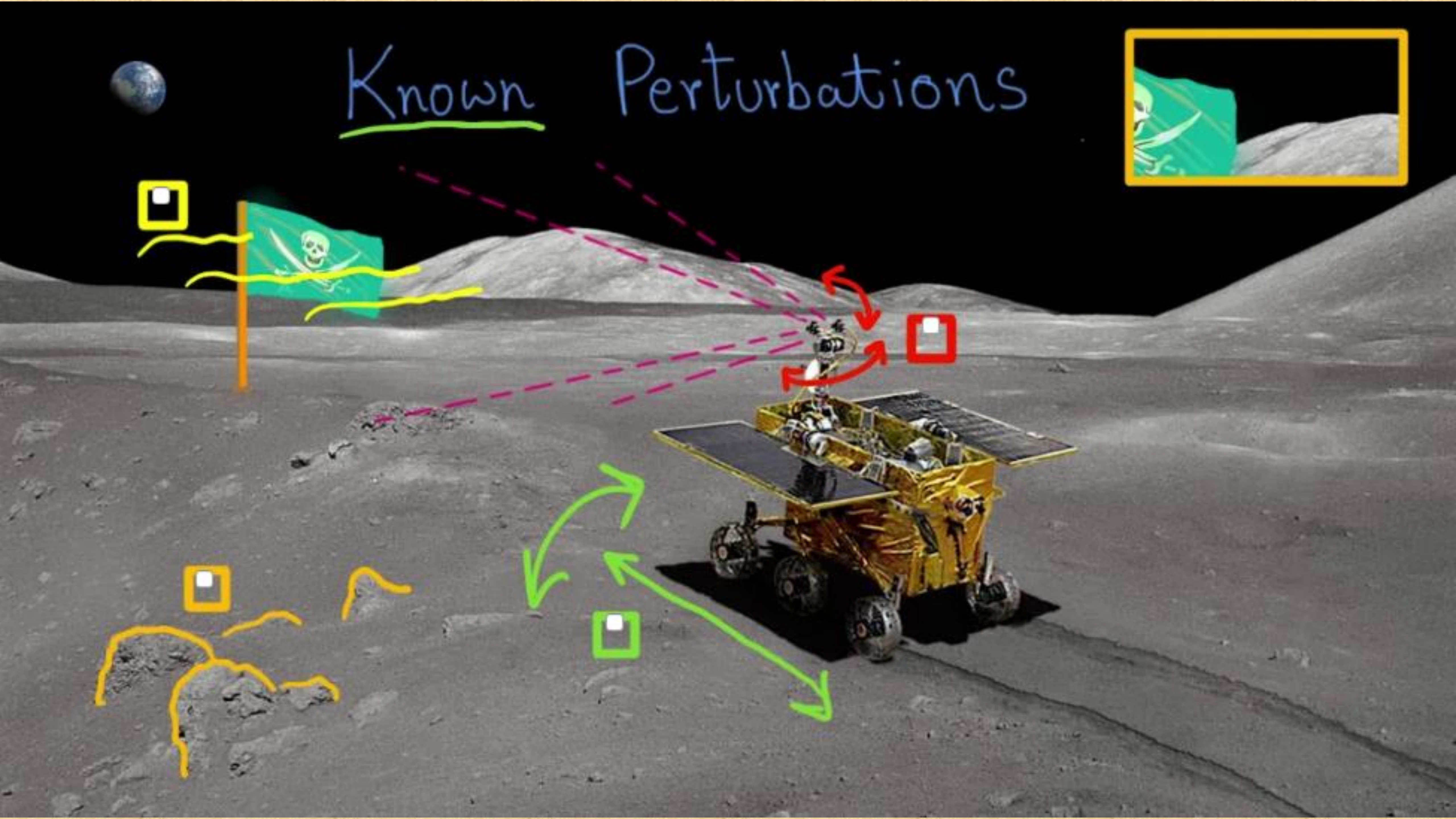
Propagation of General Densities



Perturbation

- An object moves from t to t+1

- Having a dynamics model that says how an object is likely to move
  - Perturbation: how it move and the uncertainty (noise)


Known Perturbations

Bayes Filter Framework

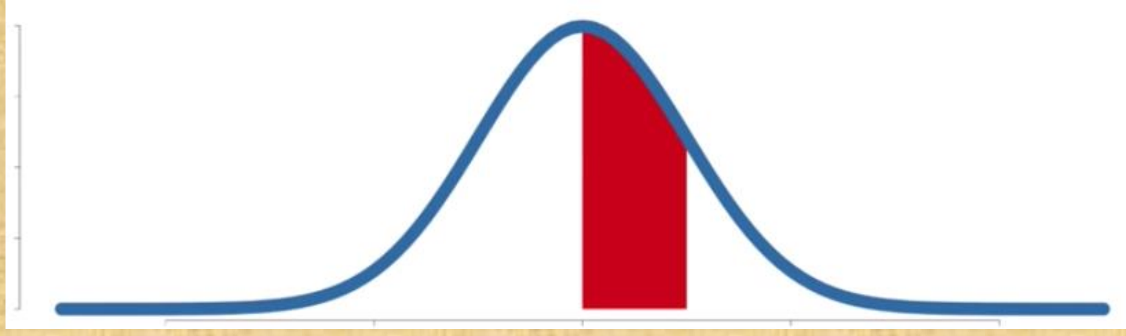
- Given
  - Prior: probability of the system state $P(x)$
  - Action (dynamics | system) model
    - $P(x_t|u_{t-1}, x_{t-1})$

  - Senor model (**likelihood**): $p(z|x)$
  - Stream of observation z and action data u
    - $Data_t = \{u_1, z_2, \dots, u_{t-1}, z_t\}$

- What do we want?
  - Estimate of the state X at time t
  - The posterior of the state is also called “belief”
    - $Bel(x_t) = P(x_t|u_1, z_2, \dots, u_{t-1}, z_t)$

Likelihood (function) and Probability (function)

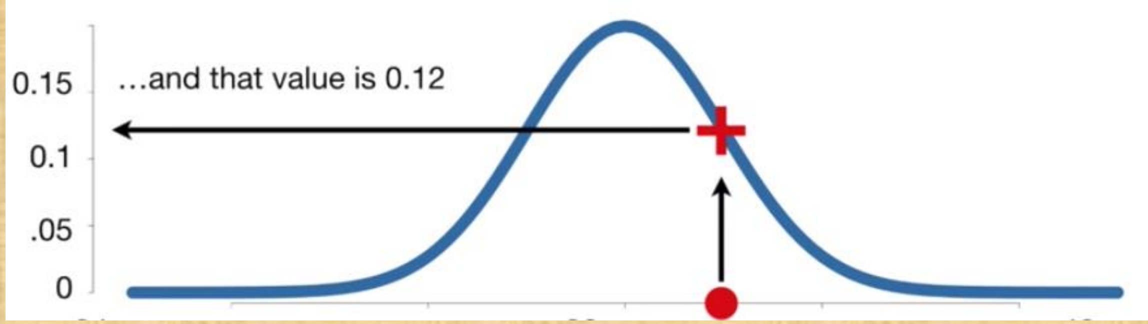
Likelihood? Probability?
The scores of a class



40 points 60 points 80 points

The chance of score is between 60 and 65 when a distribution with $\mu = 60\ and\ \sigma = 3.5$ given

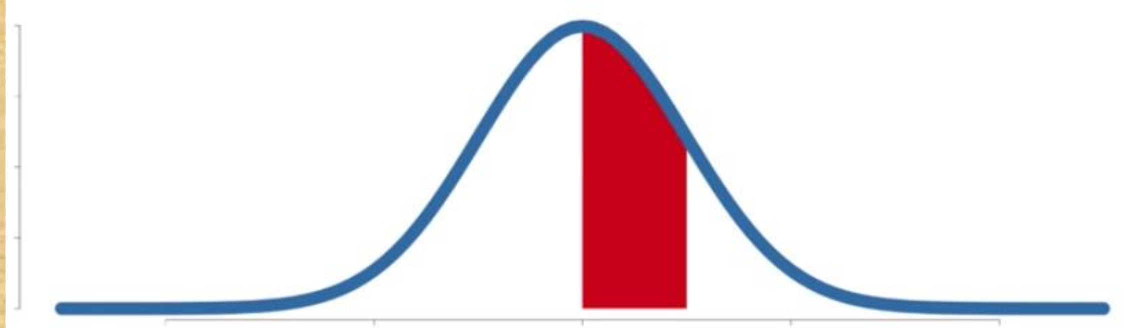
Likelihood? Probability?
The scores of a class



The chance of a distribution (parameter) is true when an observed score 65,60,55,60, … is given

Likelihood (function) and Probability (function)

Probability density function
The scores of a class

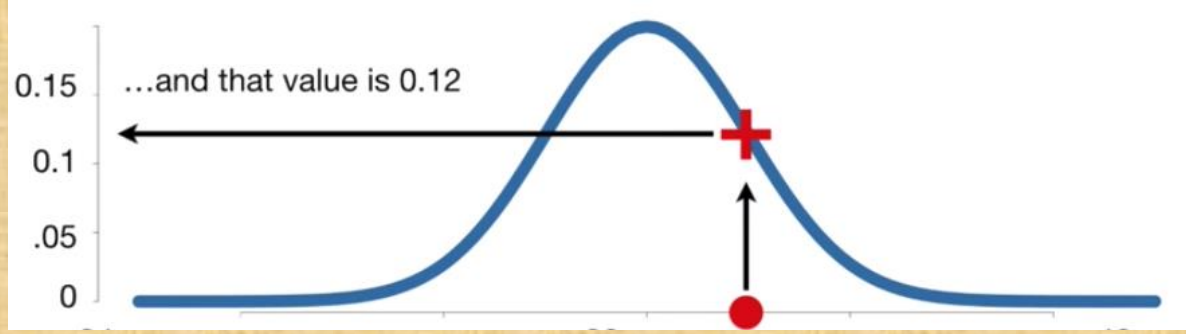


40 points 60 points 80 points

$$P(60 < score < 65 \mid \mu = 60 \; and \; \sigma = 3.5)$$
$$= red \; region = 0.27$$

Probability is the areas under a fixed distribution
$P(\, data \mid distribution)$
(the right hand side is given and we can change the left hand side)

Likelihood function
The scores of a class



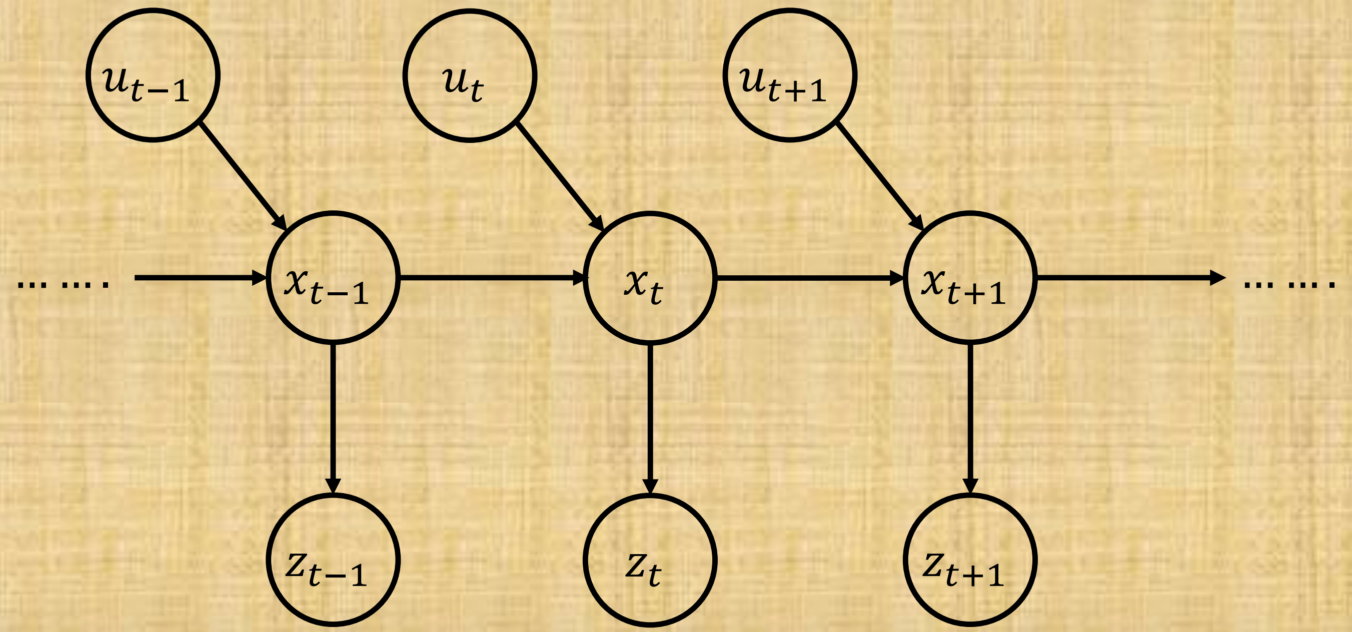
$$\mathrm{L}(\mu = 60 \; and \; \sigma = 3.5 \mid score = 65{,}60{,}55{,}60, \ldots) = 0.12$$

Likelihood is the chance of a distribution(parameters) is true distribution when measurements(scores) is given

Question?

- If the object were really at someplace, what is the chance of my measurement?
    - Probability? Likelihood?

Graphical Model Representation



- x_t(state) is only impacted by $x_{t-1}$and the perturbation u_t
- z_t (measurement) is only a function of the current state x_t
- $p(z_t | x_{0:t}, z_{1:t}, u_{1:t}) = p(z_t | x_t)$ sensor independent!
- $p(x_t | x_{1:t-1}, z_{1:t-1}, u_{1:t}) = p(x_t | x_{t-1}, u_t)$ Movkovian property!

Bayes Filters

- $Bel(x_t) = P(x_t | u_1, z_2, \dots, u_{t-1}, z_t)$
  - z: observation, u: action, x:state

Likelihood **Prior**

- By Bayes rule: $Bel(x_t) = p(z_t | x_t, u_1, z_2, \dots u_{t-1}) p(x_t | u_1, z_2, \dots, u_{t-1})$
- By sensor independent: $Bel(x_t) = p(z_t | x_t) p(x_t | u_1, z_2, \dots, u_{t-1})$
- Total probability of the prior:
  $Bel(x_t) = p(z_t | x_t) \int p(x_t | u_1, z_2, \dots, u_{t-1}, x_{t-1})\, p(x_{t-1} | u_1, z_2, \dots, u_{t-1}) dx_{t-1}$
- Markov: $Bel(x_t) = p(z_t | x_t) \int p(x_t | u_{t-1}, x_{t-1})\, p(x_{t-1} | u_1, z_2, \dots, u_{t-1}) dx_{t-1}$

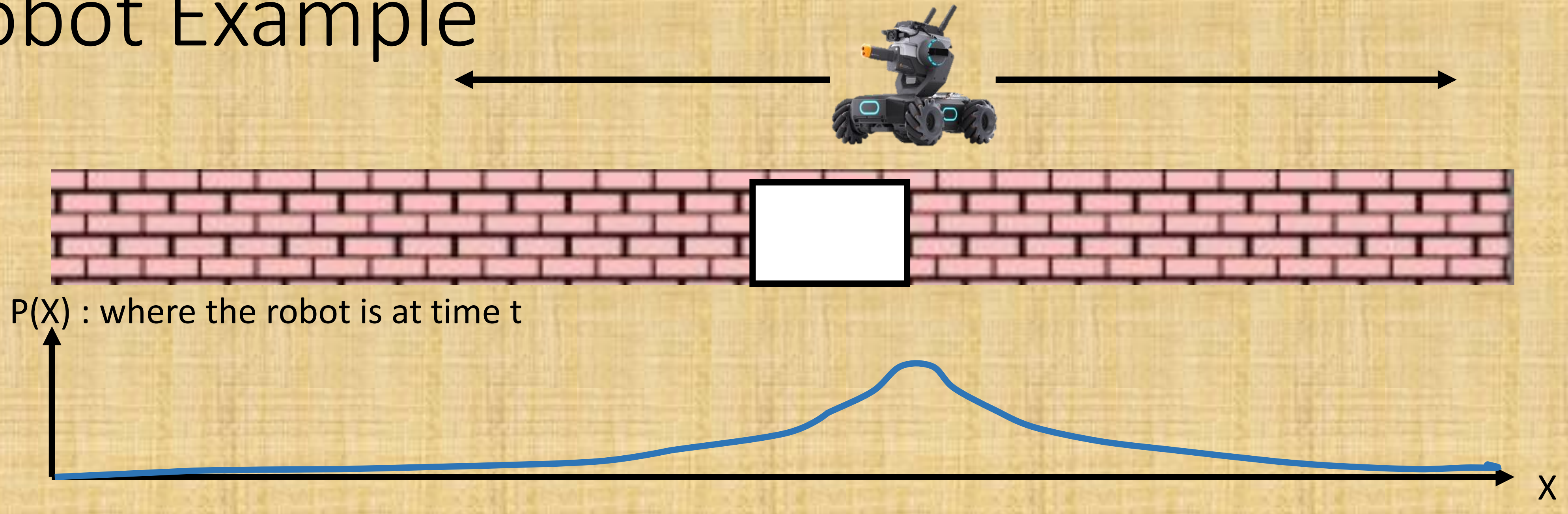
- Recursive algorithm: $Bel(x_t) = p(z_t | x_t) \int p(x_t | u_{t-1}, x_{t-1}) Bel(x_{t-1}) dx_{t-1}$

Likelihood of measurement

Prediction before taking measurement

$$Bel(x_t) = \overset{\textbf{Likelihood}}{p(z_t|x_t)} \overset{\textbf{Prior}}{\int p(x_t|u_{t-1}, x_{t-1}) Bel(x_{t-1}) dx_{t-1}}$$

Robot Example



Likelihood **Prior**

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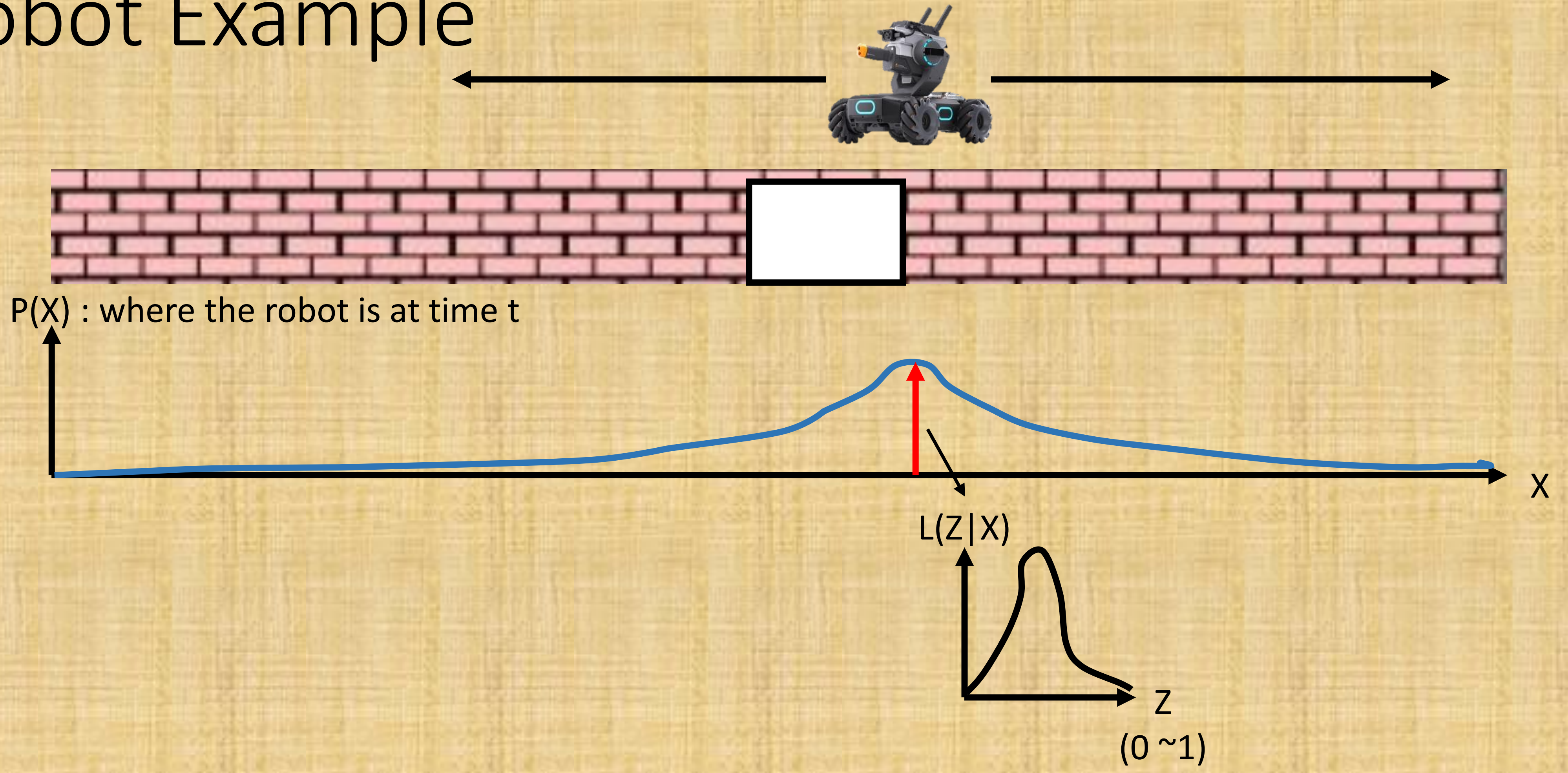

Robot Example

Likelihood **Prior**

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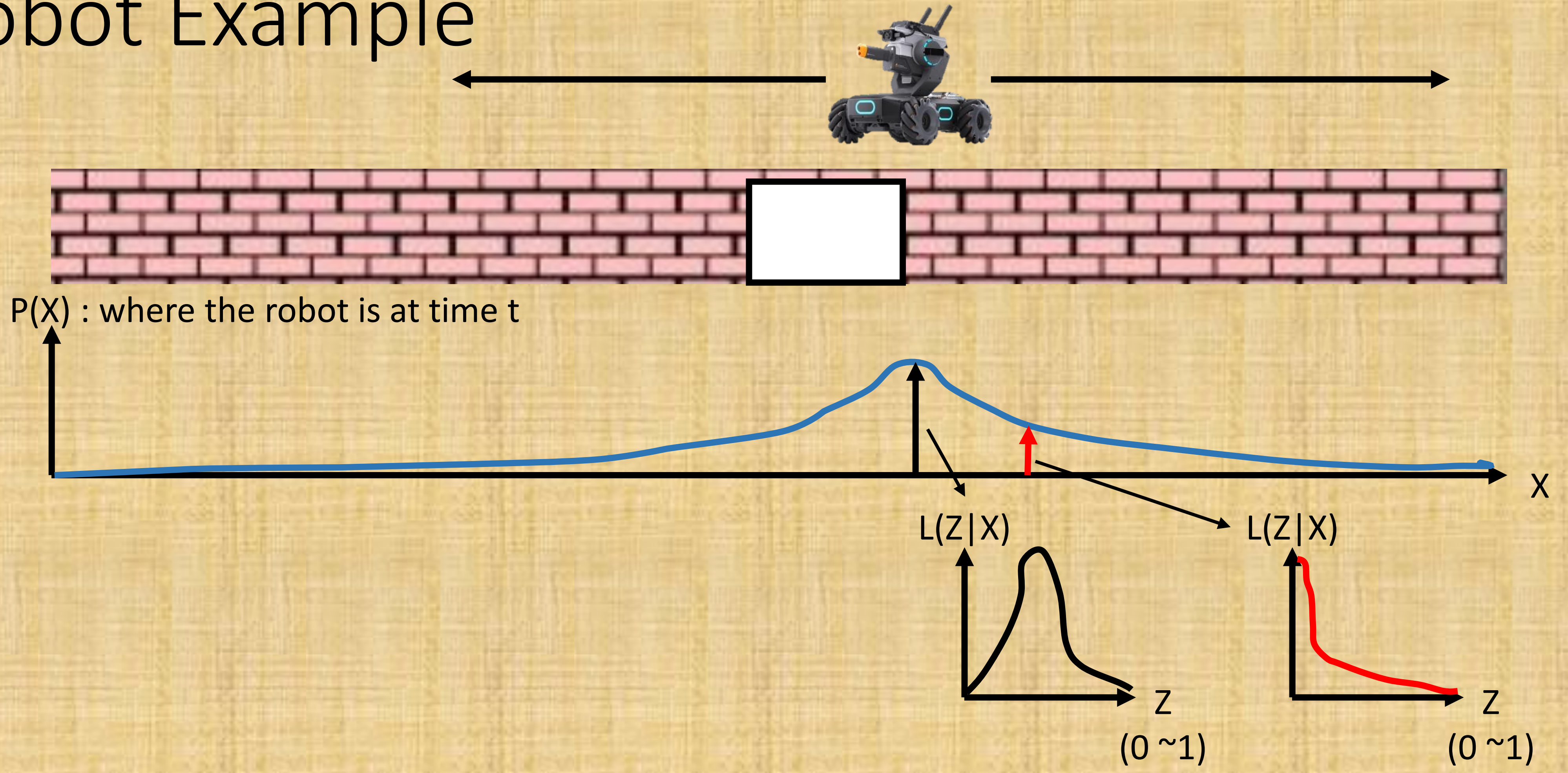
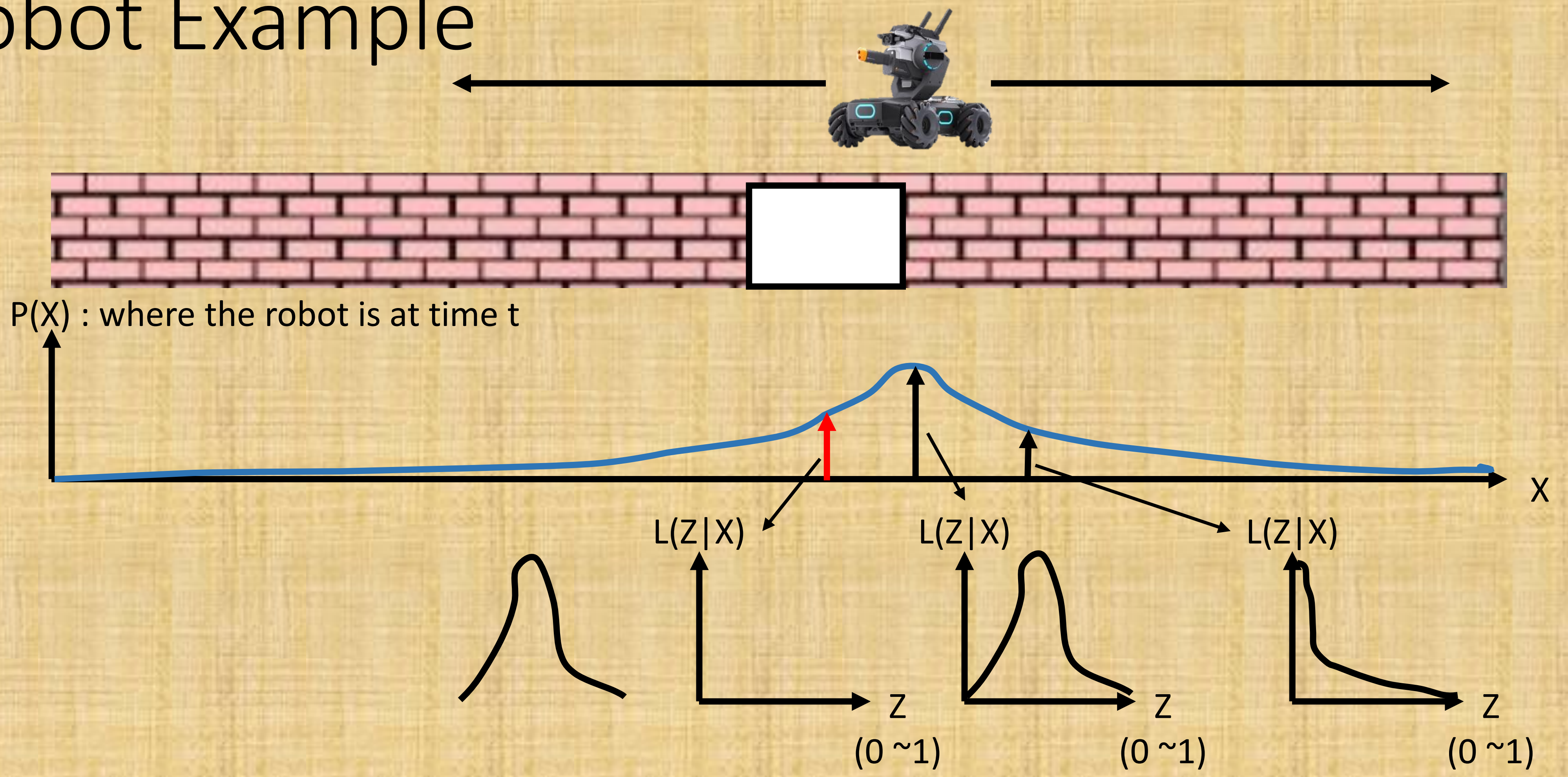


$$Bel(x_t) = \overset{\textbf{Likelihood}}{p(z_t|x_t)} \int p(x_t|u_{t-1}, x_{t-1}) \overset{\textbf{Prior}}{Bel(x_{t-1})} dx_{t-1}$$

Robot Example



Likelihood Prior

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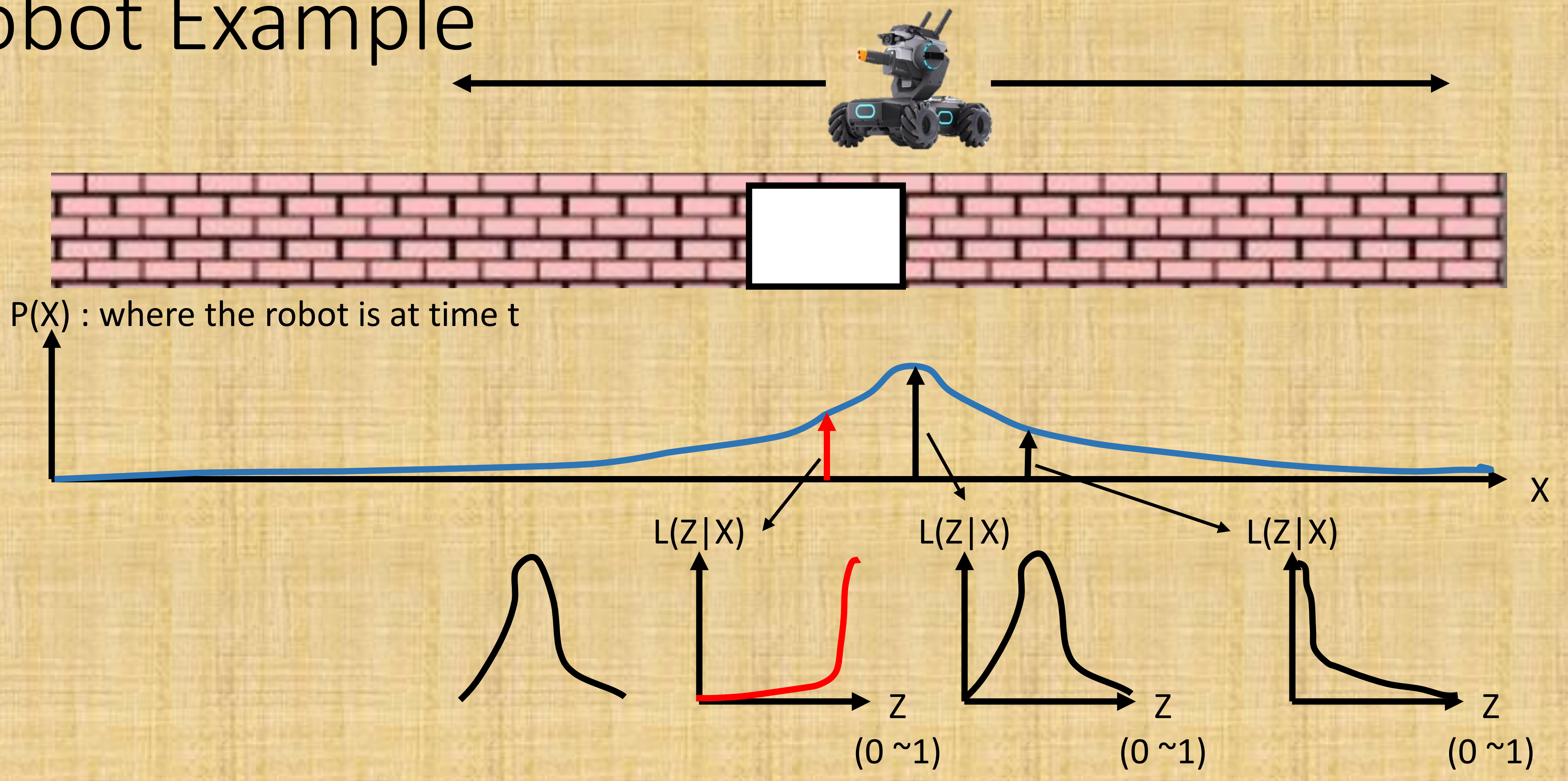

$$\overset{\textbf{Likelihood}}{p(z_t|x_t)}$$ $$\overset{\textbf{Prior}}{\int p(x_t|u_{t-1}, x_{t-1}) Bel(x_{t-1}) dx_{t-1}}$$

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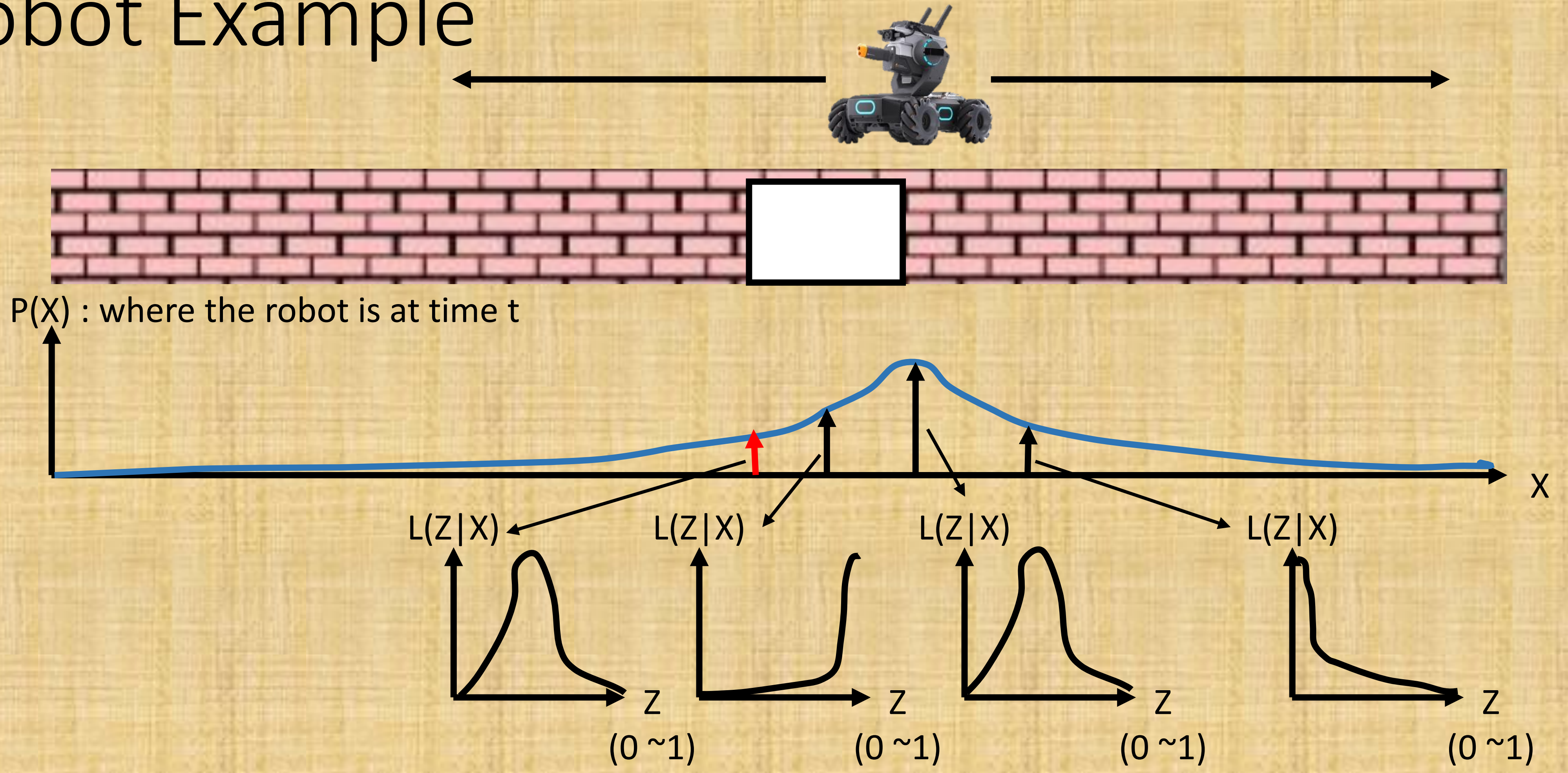

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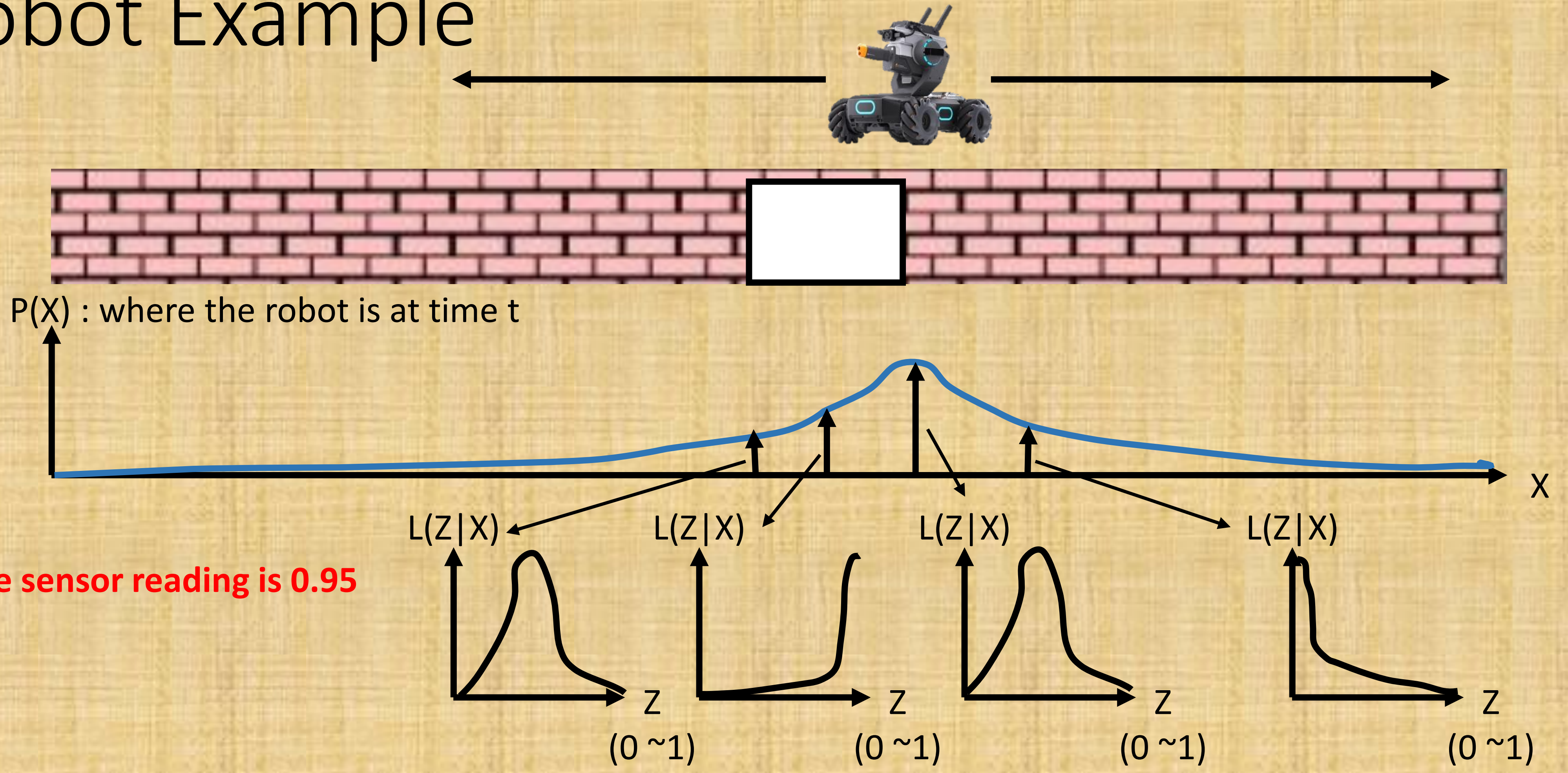

The sensor reading is 0.95

Likelihood **Prior**

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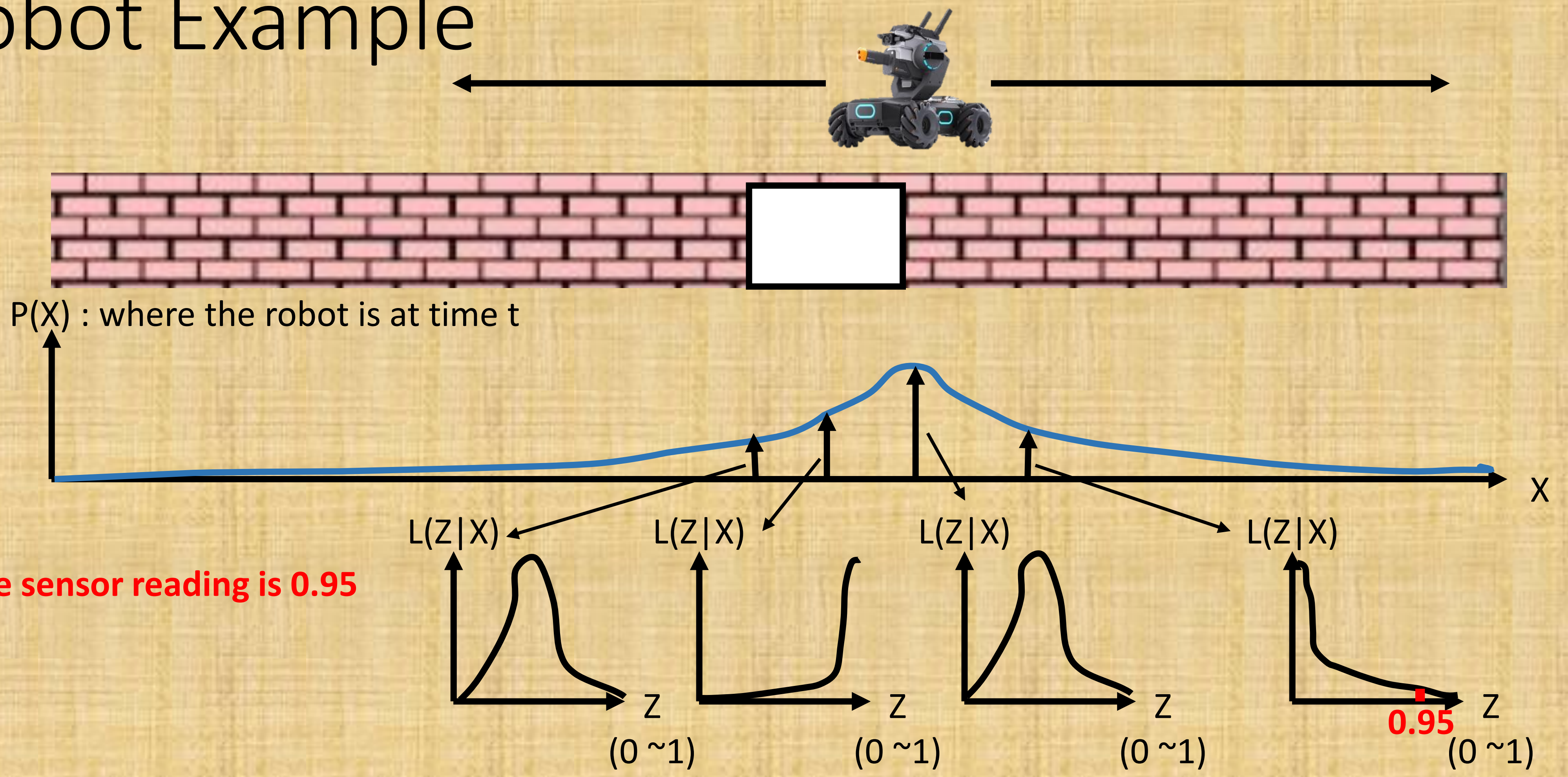

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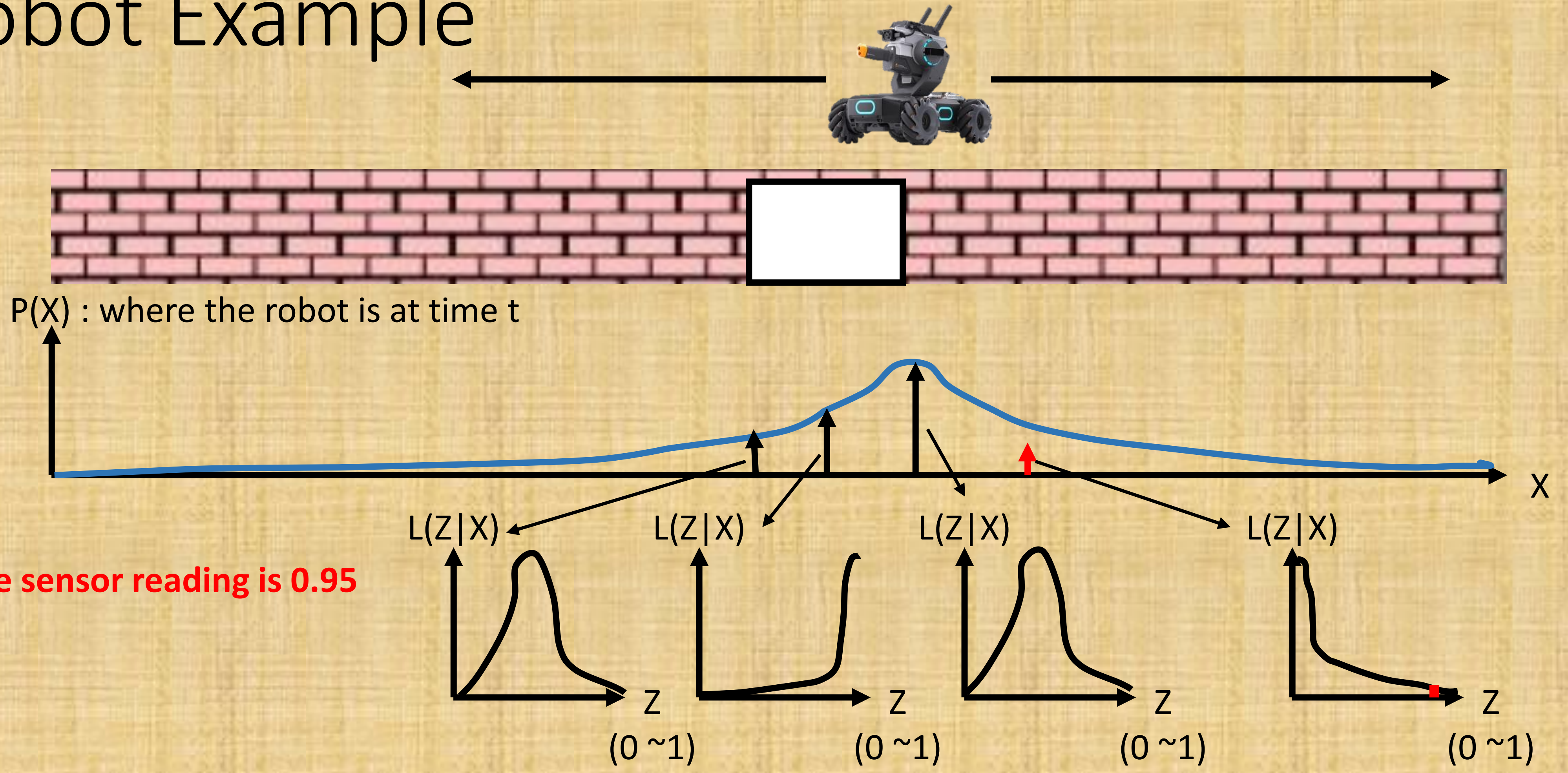

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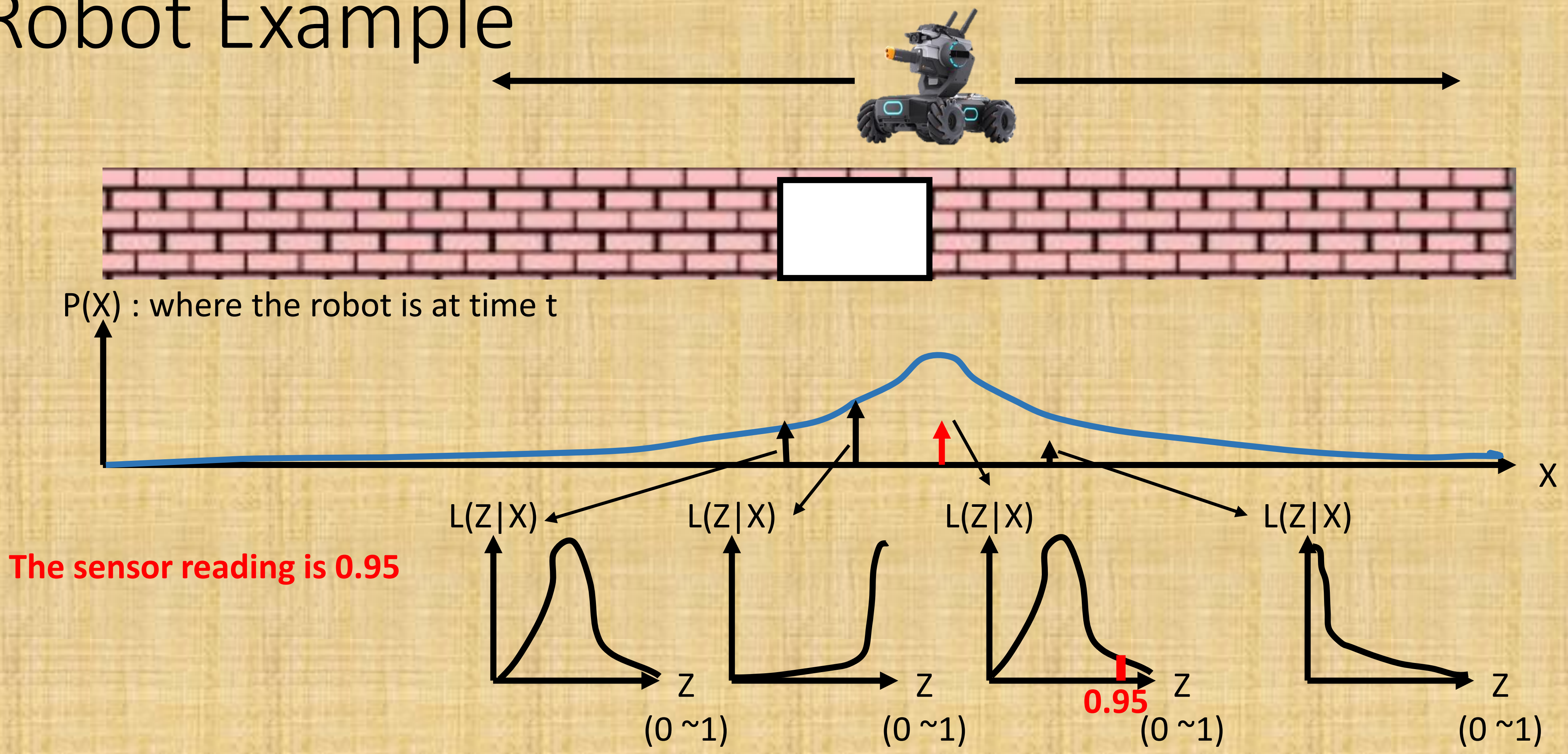
Likelihood **Prior**

$$Bel(x_t) = \underbrace{p(z_t|x_t)}_{\text{Likelihood}} \underbrace{\int p(x_t|u_{t-1}, x_{t-1}) Bel(x_{t-1}) dx_{t-1}}_{\text{Prior}}$$

Robot Example



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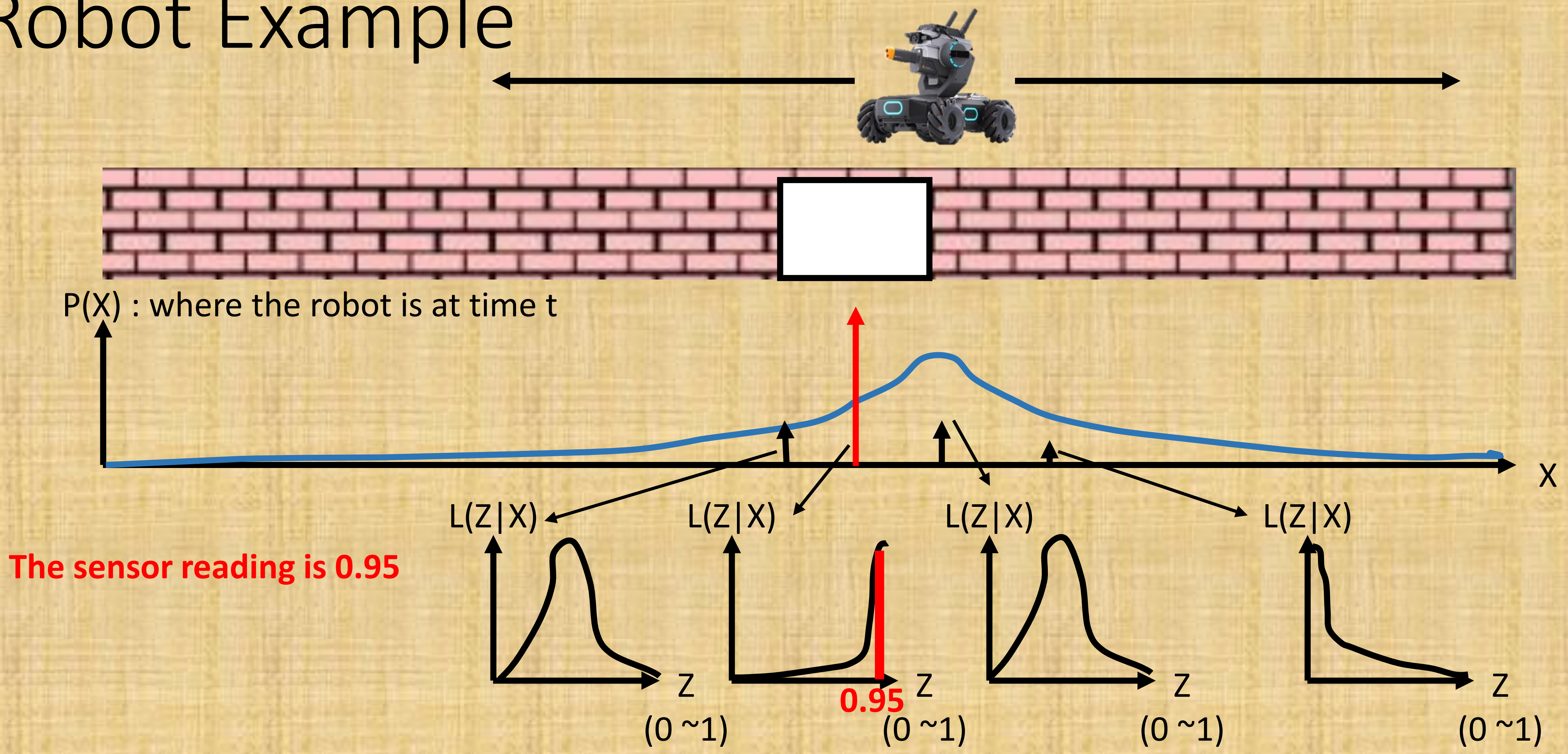

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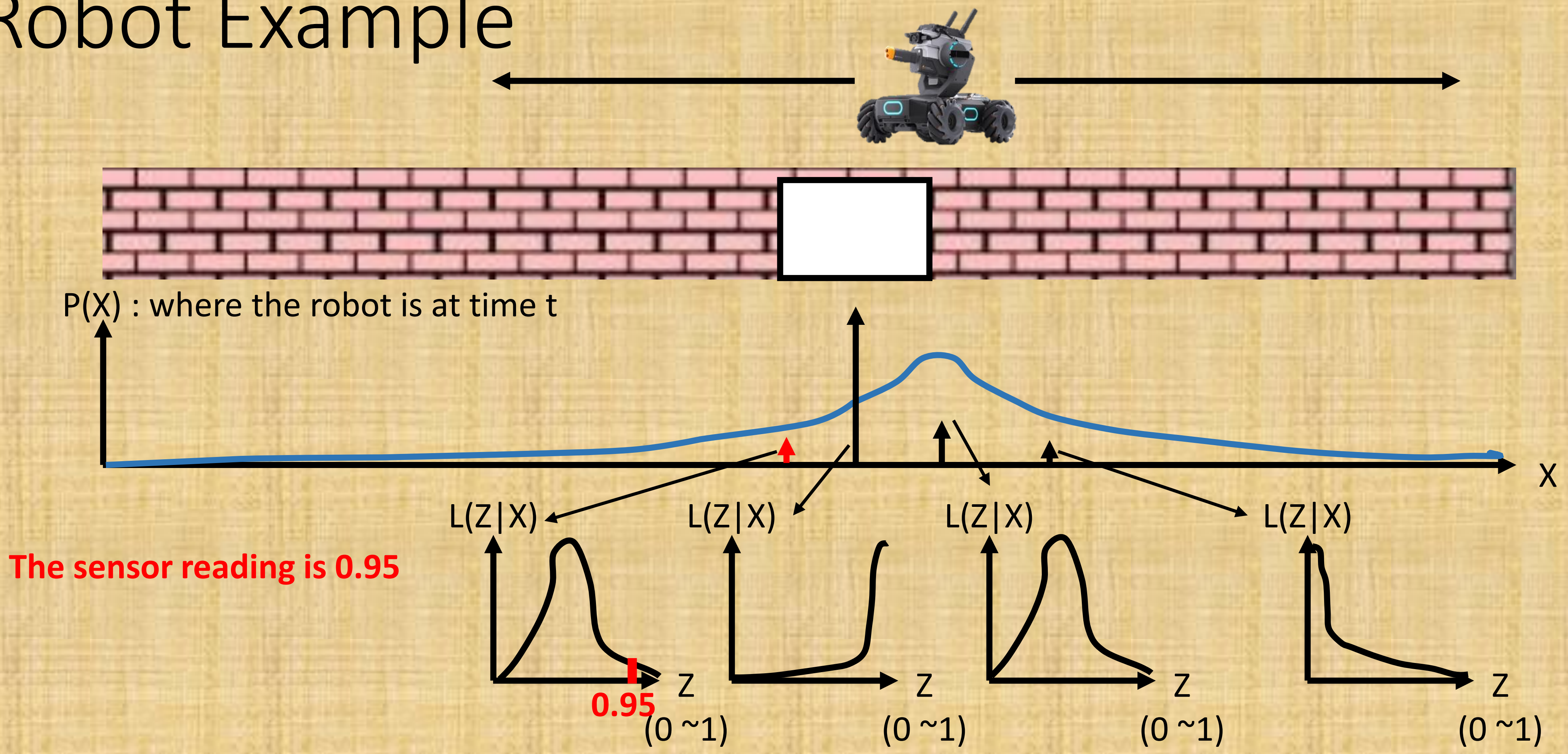

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Robot Example

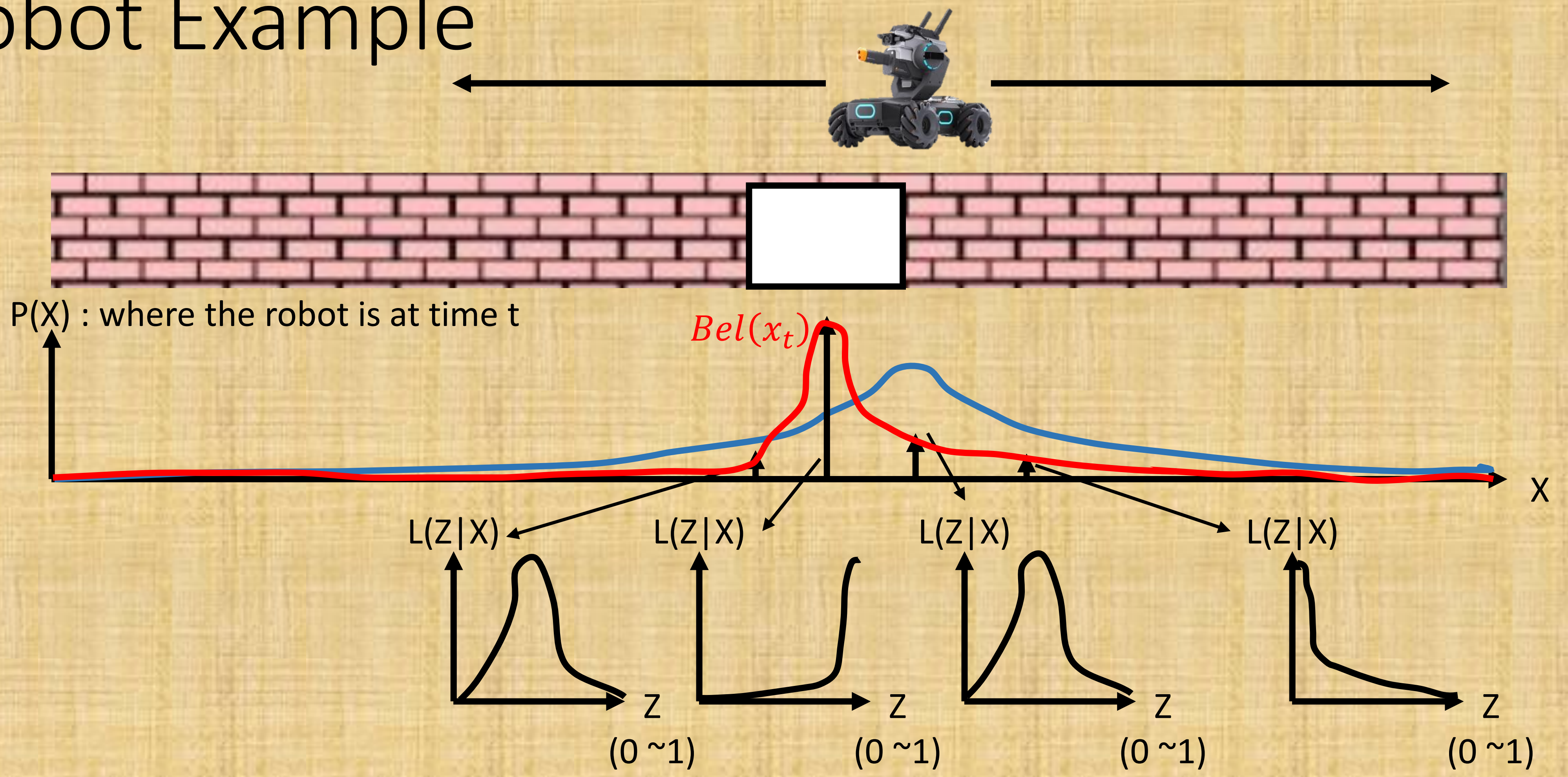

Image Example

- https://www.youtube.com/watch?v=RP4MKPI6I-U