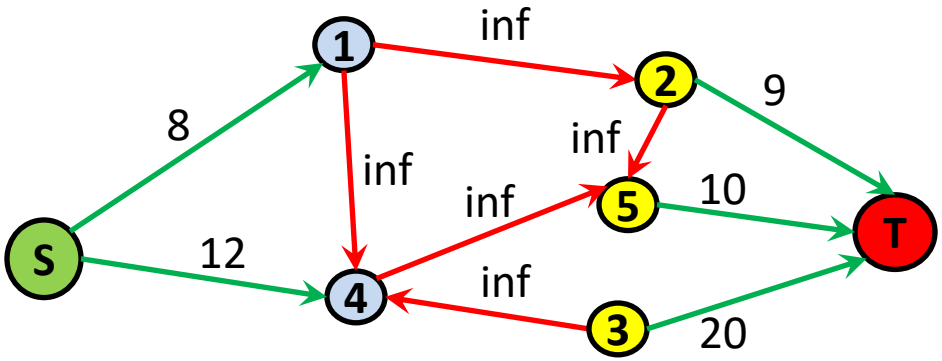
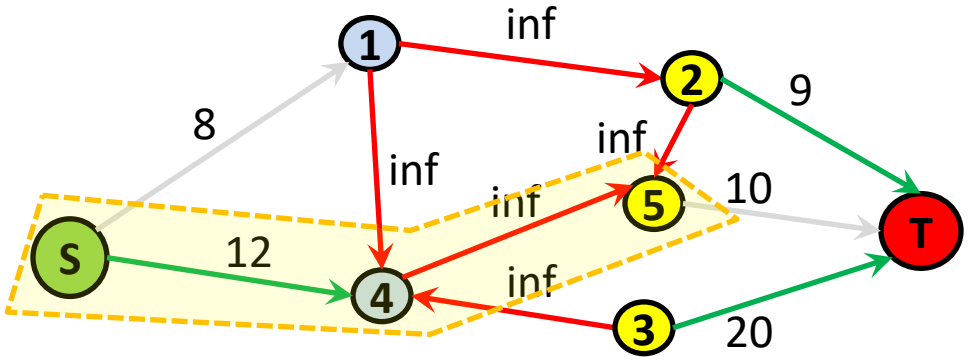
Problem 377 - firing

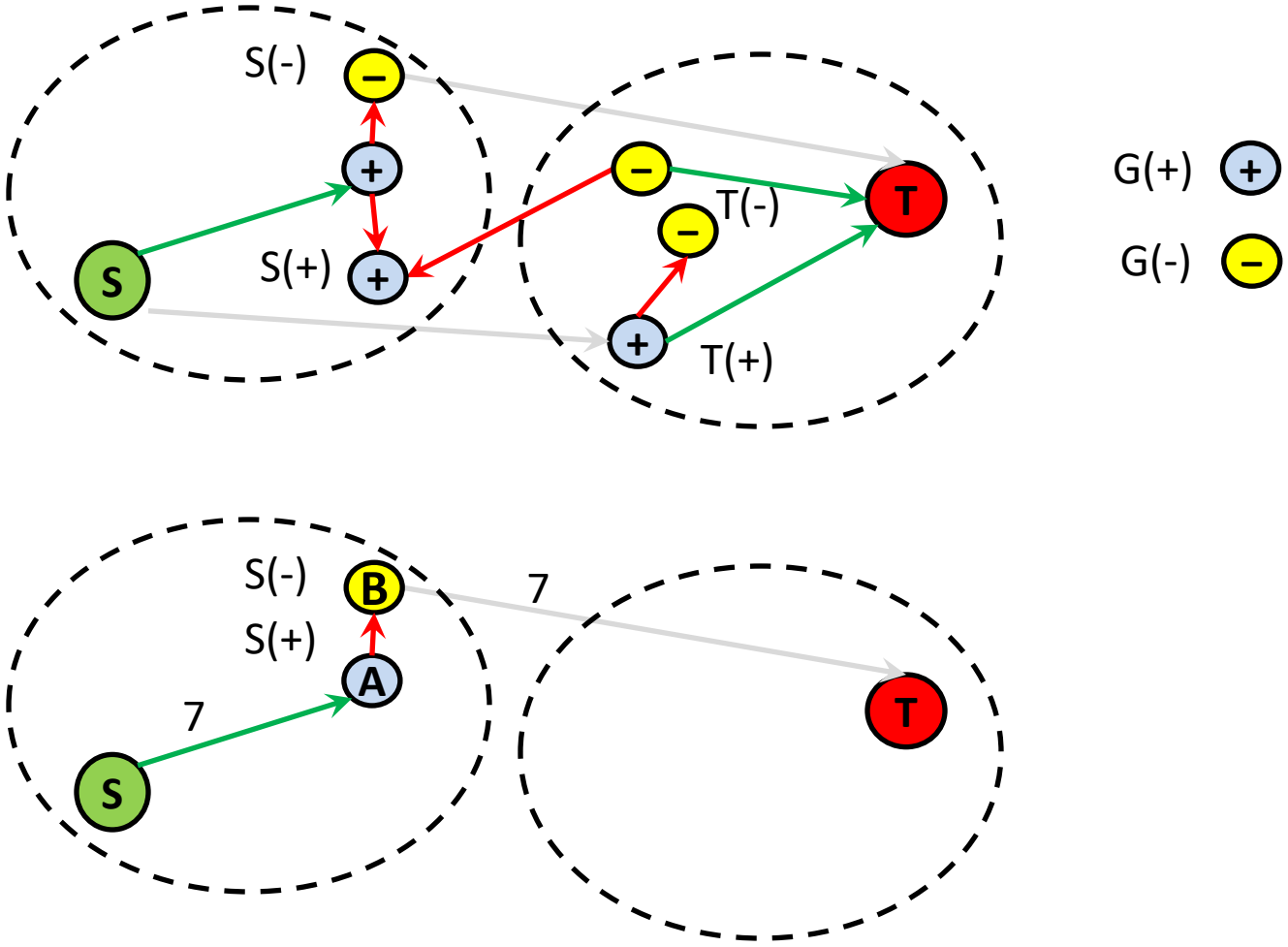
example



5 5
8
-9
-20
12
-10
1 2
2 5
1 4
3 4
4 5



Build graph



- The employee are divided into positive group G(+) and negative group G(−).
- Build graph:
  - for each node i in G(+), add S -> i,
  - for each node i in G(−), add i -> T,

  (absolute value of profit as the weight of the edge)
  - for any i is the boss of j, add i -> j, edge weight = inf.
- Run maxFlow algorithm on the graph.

- The min cut will separate the graph into two parts, i.e. deleting the edges in the min cut set, there will be no more flow from S to T
- Some nodes in G(−) are separated into left part(connected by source), while some nodes in G(+) are separated into right part.
- we denote S(−) as the nodes connected by source and belongs to G(−). resp. S(+),T(−),T(+)

- We regard nodes in S(-) and S(+) are the employees to be fired.
- Conclusion: the edge in the min cut set only contains edges from S to T(+) and edges from S(-) to T.
- Because, the maxflow cannot be infinity, there exists no edge from {S(+),S(-)} to {T(+),T(-)}. Otherwise, the edge belongs to min cut set, then the maxflow is infinity. Therefore, for all nodes in {S(+),S(-)}, if one fired, then his underlings are all fired. There might be edges from from {T(+),T(-)} to {S(+),S(-)}.
- Cost = $\sum S(+) - \sum S(-) = \sum G(+) - \sum T(+) - \sum S(-)$.
- $\sum G(+)$ is a fixed value, to maximize Cost is to minimize $\sum\{T(+),S(-)\}$, which is the total edge weight in min cut set. Therefore, maxflow is correct.

- For a subset of points in {S(-),S(+)}, if we do not fire them and still get the same value of maxflow. That means, SA=BT, seen in the fig. So, after running max flow algorithm, the flow capacity of SA is zero.
- Then, we do not select the node A, since zero.
- By using BFS, we search all the nodes connected by the source, following the edges with positive remaining flow capacity.

- reference: http://www.cnblogs.com/oyking/p/3249174.html
- the codes in the reference is not correct!