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Modal Analysis of Rectangular Plate with Central Hole Subjected to Various End Conditions

M.L. Pavan Kishore[a], Sreenivasulu Bezawada[b], B.C.Raghu Kumar Reddy[c] *

[a]Research Scholar,Department of Mechanical Engineering, NIT Rourkela, Odisha, INDIA
[b]Assistant Professor, Department of Mechanical Engineering, MITS Madanapalle,A.P,INDIA
[c]Assistant Professor, Department of Mechanical Engineering, Malla reddy college of engg,Hyderabad,A.P,INDIA

Abstract

A series of Eigen value analysis was performed to deal with vibrational characteristics determination of a rectangular plate having cut out at centre subjected to various end conditions. The computations performed in this work analyse the significance of existence of the importance of presence of cut out in the rectangular plate in terms of its dynamic characteristics. These calculations are carried out by using FEM based solver Ansys. A comparison has been made between the uniform rectangular plate with and without hole. The results obtained by using Ansysare evaluated and found that simulation results are in close agreement with the results obtained from the literature survey and are found in good agreement.

Keywords:Ansys, Finite element method, Mesh size, static analysis.

1. Introduction

Plates are defined as plane structural elements with a small thickness compared to planar dimensions. The thickness to width ratio of a plate being less than 0.1.During 19th century numerous plate theories are developed of which two theories are widely accepted in engineering. The Kirchhoff theory and Mindlin plate theory [FSDT]. The analysis of free vibration of plates was well documented by Leissa[1] includes a variety of boundary conditions and

* Corresponding author. Tel.: 91-7205636631
E-mail address: kishoremamdoor9@gmail.com

aspect ratios using trigonometric series. Gorman [2] solved problem on free vibration of rectangular plate with different boundary conditions aspect ratio, poisons ratio using method of super position. The computations based on these two approaches pose serious demands and have been well accepted to match with exact results. Cheng [3] made an attempt to determine the vibrational behaviour around an eccentric hole and revealed the relationship between various parameters eccentricity, hole size, boundary conditions. A closed form of solution in explicit form for mode shapes and natural frequencies was given by Wook Kang et al [4] with good accuracy. Zhou et al [5] in his study used the symmetric expansion to obtain the natural modes of the Kirchoff circular and annular plates. Research on thick plate vibration problems was provided by Liew[6] to some extent and Reisser-Mindlin assumptions are employed for development of plate elements related to thick plates. Apple et al [7] investigated the case where the thickness varied in only one direction and calculated the fundamental frequency of simply supported rectangular plate having linear thickness variation. Plunkett [8] investigated the free vibration of linearly tapered rectangular cantilever plates. Weisensel G. N[9] discussed the result of an extensive literature search and review of available sources of numerical natural frequency information for stationary circular and annular elastic plates. Gorman used method of superposition to examine free vibrational analysis of cantilever plates [10] combination with a clamped and simply supported boundary conditions for rectangular plates [11].More research has been carried out on vibration analysis of plates [12-15].Narita in 2000[15] used modified Ritz method to determine the free vibration analysis of isotropic and anisotropic rectangular thin plates subjected to general boundary conditions.

Nomenclatures

D	Bending rigidity
h	Thickness of plate
E	Young's modulus
f	Natural frequency
U(x,y)	Mode shapes
m,n	Mode numbers
ρ	Density
ν	Poisson ratio
λ	Dimensionless parameter
a,b	Dimensions of the plate
x,y	Coordinates of the plate

2. Literature Survey

An isotropic rectangular plate with a central hole under loading conditions have found applications in various fields of engineering such as civil, mechanical, aerospace and automobile. The linear vibration of rectangular plates with cut outs have been studied by several researchers by applying extended methods to isotropic plates to orthotropic plates. In general cut outs are made to lighten the structure for venting and for altering the resonant frequency and therefore it becomes important to determine the natural frequency of these plates, to deal with such type of problems discrete type methods finds its suitability. Of all these discrete methods finite element methods is the most widely used method for such type of problems [16-19]. The behavior of plates with holes has been investigated by Pickett [20]. Liu [21, 22] employed a differential quadrature element method to find out the solution of plate with discontinuities and also successfully employed for vibration of such discontinuous plates. Yang et al [23] used semi analytical methods with solution to act as an alternative to this type of problems. To analyse the irregular shaped plates and plates with curved boundaries Yang et al [23, 24] developed the semi analytical iso parametric strip distributed method to provide an alternative to these problems. For the vibrational analysis of arbitrarily shaped plates Geannakakes[25] used the semi analytical line strip method that uses the beam characteristics of orthogonal polynomials. Using optimized Rayleigh Ritz method Laura [26] suggested a semi analytical method to find out the vibrational analysis of simply supported rectangular plates with rectangular cut outs. A domain decomposition method based on Rayleigh Ritz method was carried out by Liew et al [27-30] and Liew[31] to study the vibrational characteristics of the plates with mixed edge boundary conditions and shape re entrant geometries. Even the application of boundary element method as an alternative to finite element method can be successfully implemented to analyse plates with irregularities such as cracks and holes [32, 33]

3. MATHEMATICAL FORMULATION

$$[D(\frac{\partial^4 u}{\partial x^4}+2\frac{\partial^4 u}{\partial x^2 \partial y^2}+\frac{\partial^4 u}{\partial y^4})+\rho h\frac{\partial^2 u}{\partial t^2}=0 \qquad [1]$$

Substituting

$$u(x,y,t)=U(x,y)e^{j\omega t} \qquad [2]$$

gives

$$D(\frac{\partial^4 U}{\partial x^4}+2\frac{\partial^4 U}{\partial x^2 \partial y^2}+\frac{\partial^4 U}{\partial y^4})-\rho h\omega^2 U=0 \qquad [3]$$

Where D is flexural rigidity and defined by

$$D=\frac{Eh^3}{12(1-\nu^2)} \qquad [4]$$

For Clamped plate from all edges the boundary conditions are

$$\left.\begin{array}{ll} u(0,y,t)=0 & u(x,0,t)=0 \\ u(a,y,t)=0 & u(x,b,t)=0 \\ \frac{\partial u}{\partial x}(0,y,t)=0 & \frac{\partial u}{\partial y}(x,0,t)=0 \\ \frac{\partial u}{\partial x}(a,y,t)=0 & \frac{\partial u}{\partial y}(x,b,t)=0 \end{array}\right\} \qquad [5]$$

The natural frequencies of first six mode shapes of rectangular plate with clamped free boundary conditions are considered for study. The dimensionless frequency parameter for rectangular plate is generally a function of the boundary conditions applied at the edges of the plate is given by the equation

Natural frequency

$$f_{ij}=\frac{\lambda_{ij}{}^2}{2\pi a^2}\left[\frac{Eh^3}{12\rho(1-\nu^2)}\right]^{\frac{1}{2}} \qquad [6]$$

And assumed mode shape to be:

Case 1: Clamped at x=0; &x=a;

$$X(x)=\cos\gamma_1(\frac{x}{a}-\frac{1}{2})+\frac{\sin(\gamma_1/2)}{\sinh(\gamma_1/2)}\cosh\gamma_1(\frac{x}{a}-\frac{1}{2}) \qquad [7]$$

Where m=2, 4, 6….

The values of γ_1 are obtained as roots of

$$\tan(\gamma_1/2)+\tanh(\gamma_1/2)=0 \qquad [8]$$

$$X(x) = \sin\gamma_2\left(\frac{x}{a}-\frac{1}{2}\right)+\frac{\sin(\gamma_2/2)}{\sinh(\gamma_2/2)}\sinh\gamma_2\left(\frac{x}{a}-\frac{1}{2}\right) \quad [9]$$

Where m=3,5,7….

The values of γ_2 are obtained as roots of

$$\tan(\gamma_2/2)+\tanh(\gamma_2/2)=0 \quad [10]$$

Case 2: Clamped at x=0; &free at x=a;

$$X(x)=\cos\left(\frac{\gamma_3 x}{a}\right)-\cosh\left(\frac{\gamma_3 x}{a}\right)+\left(\frac{\sin\gamma_3-\sinh\gamma_3}{\cos\gamma_3-\cosh\gamma_3}\right)+\left(\sin\frac{\gamma_3 x}{a}-\sinh\frac{\gamma_3 x}{a}\right) \quad [11]$$

m=1,2,3…..

where $\cos\gamma_3\cosh\gamma_3=-1$ [12]

4. MATERIAL&MECHANCIAL PROPERTIES

The material selected for the case is mild steel and the mechanical properties applied for the plate are given below in table.1.

Table 1: Mechanical properties of Mild steel

Material	Mild steel
Density	7850kg/m3
Poisson's Ratio	0.34
Young's Modulus	2.1e11N/m^2

5. FINITE ELEMENT MESH& BOUNDARY CONDITIONS

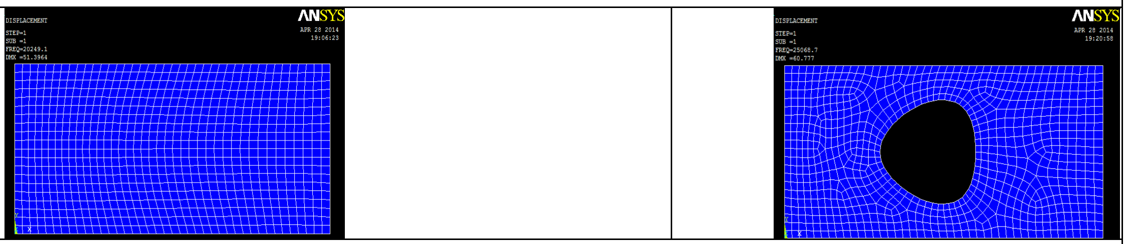
The element type selected for the mesh is triangular free mesh for the plate and the element size is about. The total number of elements created is and the associated nodes are about. Even though the problem is a two dimensional case the thickness cannot be ignored the two dimensional mesh is converted to three dimensional mesh using shell element. Since the element type takes into account of the thickness. The final 3d mesh model consists of totally number of nodes and elements are shown below in figure. Considering the rectangular plate with and without presence of hole various boundary conditions are applied as shown below in table 2&3 respectively. The end condition clamped corresponds to arresting all degrees of freedom.

Table2:uniform rec plate without hole and constraints

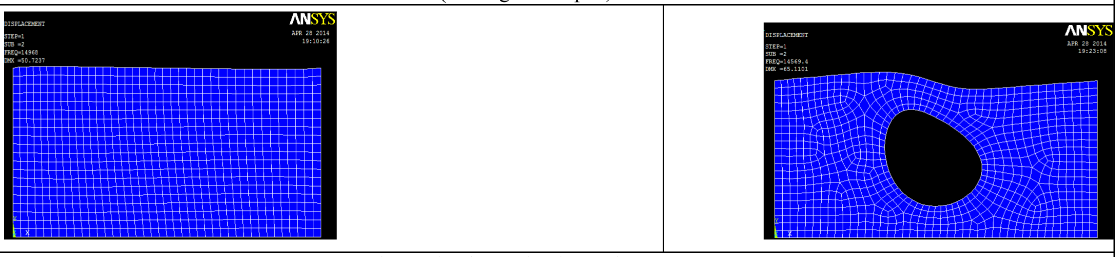
C-C-C-C	C-C-C-F	C-C-F-F	C-F-F-F	C-F-C-F	C-F-C-C	C-C-F-C	C-F-F-C
20249	14437	8988.5	1753.3	6397.9	14437	16306	8988.5
26983	14968	13448	6357.9	12593	14968	24290	13448
28424	17654	14692	6093.7	12740	17654	26033	14692
29826	23135	18467	14038	20172	23135	26845	18647
32241	26334	20507	17956	20695	26334	29277	20507
34514	26979	22924	18509	23554	26979	31247	22924

Table3:uniform rec plate with hole and constraints

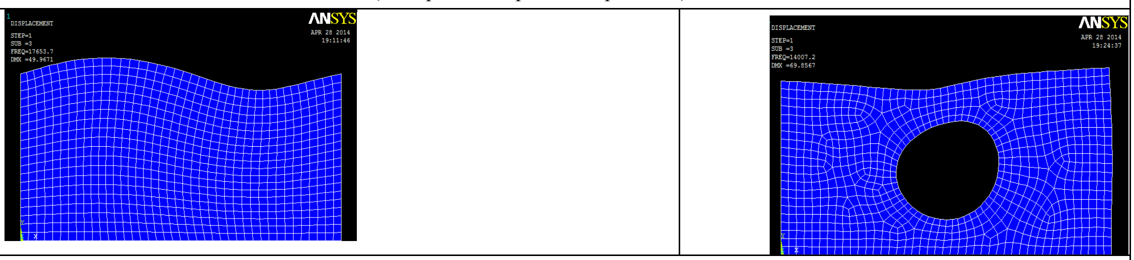
C-C-C-C	C-C-C-F	C-C-F-F	C-F-F-F	C-F-C-F	C-F-C-C	C-C-F-C	C-F-F-C
25069	12142	7830.3	1529	7556	12142	16320	7830.3
25854	14569	11480	5216.9	10085	14569	25459	11479
30342	17248	14007	7035.5	14010	17248	27216	14007
30393	19886	15691	9075.9	14851	19886	27754	15691
32279	24080	19366	13803	16471	24080	30372	19366
33769	24460	21101	17669	19674	24460	32949	21102



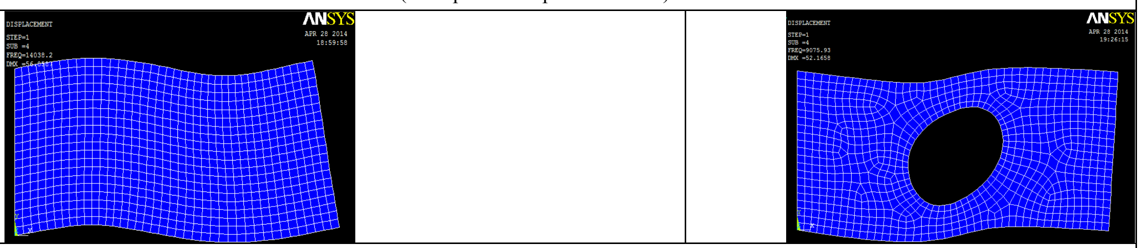
C-C-C-C(All edges clamped)



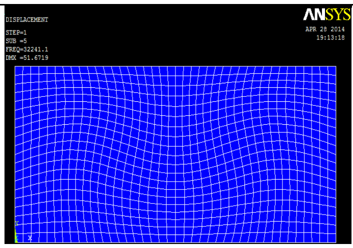
C-C-C-F(Clamped- Clamped- Clamped-Free)

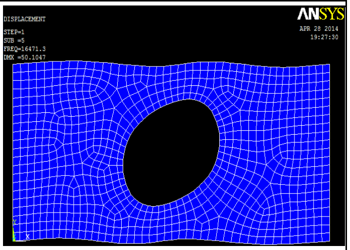


C-C-F-F(Clamped- Clamped-Free-Free)

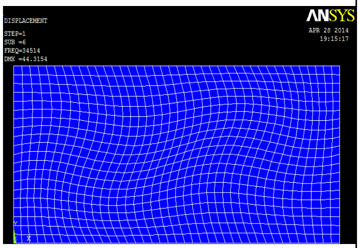


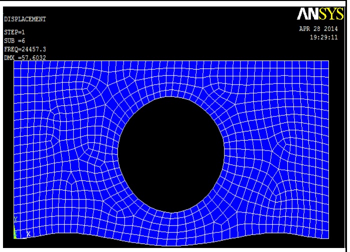
C-F-F-F(Clamped- Free- Free- Free)



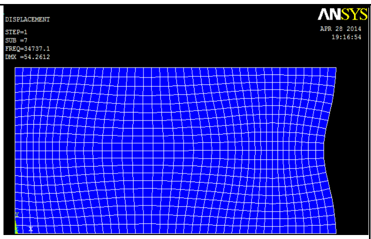


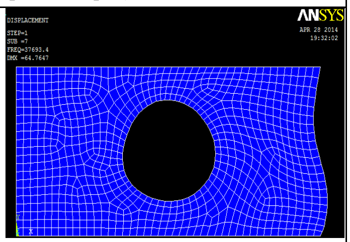
C-F-C-F(Clamped- FreeClamped- Free)



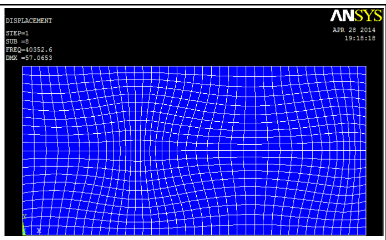


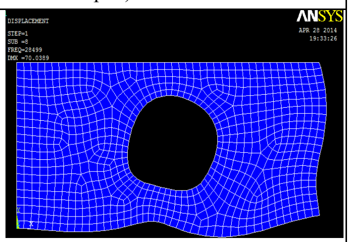
C-F-C-C (Clamped- Free- Clamped-Clamped)



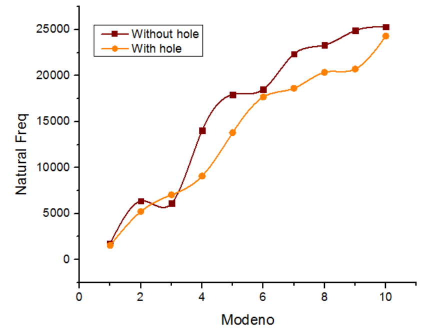


C-C-F-C (Clamped-Clamped- Free- Clamped)

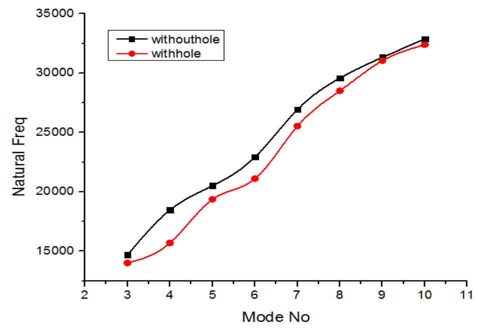




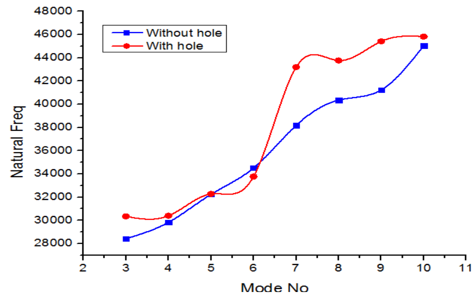
C-F-F-C(Clamped-Free- Free- Clamped)



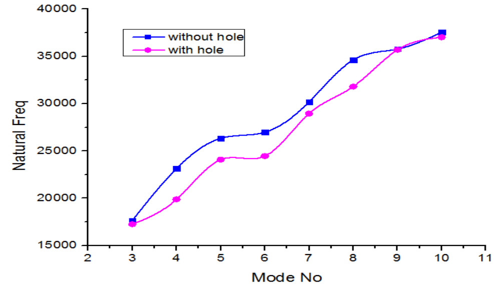
Graph 1: C-C-C-C



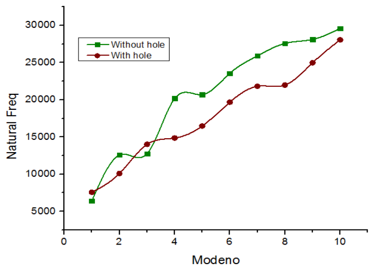
Graph 2: C-C-C-C



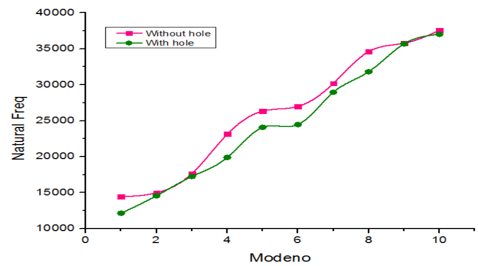
Graph 3: C-C-C-C



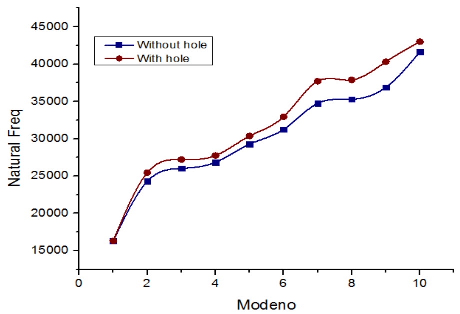
Graph 4: C-C-C-C



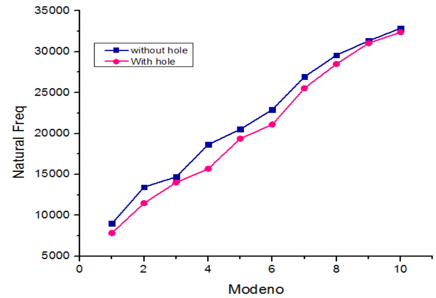
Graph 5: C-C-C-C



Graph 6: C-C-C-C



Graph 7: C-C-F-C



Graph 8: C-F-F-C

6. CONCLUSIONS

In attempt from the results tabulated for a rectangular with one of the edges of width fixed and other subjected to various boundary conditions considering width as dependent and length as independent variable the following conclusions are drawn

1. With all the edges of plate width fixed and edges of length clamped plate with hole has higher bending frequencies than plate without hole and torsional frequencies are higher for plate without hole are higher than plate with hole.
2. With all the edges of plate width fixed and any one of the edge of length set to free condition or both the edges set free both the bending modes and torsional modes for plate with hole is less than plate without hole
3. With one of the edge of width subject to free condition and length set to clamped condition both bending and torsional frequencies for plate with hole are greater than without hole.
4. With one of the edge of width subject to free condition and length set to clamped free condition or free free condition both bending and torsional frequencies for plate with hole are less than without hole.
5. From above cases it may be concluded that rectangular plate with central hole and one of its edges [width] fixed the clamped-clamped-free-clamped constraints produced high bending and torsional frequencies in comparison to other boundary conditions.

REFERENCES

[1]Leissa, A. W. The Free Vibration of Rectangular Plates. Journal of Sound and vibration, Vol.31 [3], 1973 Pp. 257 – 293
[2]Gorman, D. J. [1982]. Free Vibration Analysis of Rectangular Plates. North Holland: Elsevier.
[3] L. Cheng, et.al. Vibration analysis of annular-like plates, Journal of Sound and Vibration, vol. 262, pp. [2003],1153–1170.
[4] Wook Kang & et.al, Approximate closed form solutions for free vibration of polar orthotropic circular plates, Journal of Applied Acoustics, vol. 66 [2005], 1162–1179.
[5] Z.H.Zhou et.al, Natural vibration of circular and annular thin plates by Hamiltonian approach, Journal of Sound and Vibration, vol.330, [2011], 1005–1017.
[6] K.M. Liew[1995] Research on thick plate vibration: a literature survey, Journal of Sound and Vibration 180
[7] J F.C.Apple and N.R.Byers: 1965 .App1.Mech., 32, No.I, 163-167, Fundamental Frequency of Simply Supported Rectangular Plates with Linearly Varying Thickness.
[8] J R.Plunkett: 1963 .Mech. Eng. Sci., 5, No.2, 146-156, Natural Frequencies of Uniform and Non-Uniform Rec-tangular Cantilever Plates.
[9] Weisensel G. N, Natural Frequency Information for Circular and Annular Plates, Journal of Sound and Vibration,] vol. 133[l], [1989], 129-134.
[10] Gorman, D. J., 1976, "Free Vibration Analysis of Cantilever Plates by the Method of Super-Position," J. Sound Vib. 49, pp. 453–467.
[11] Gorman, D. J., 1977, "Free-Vibration Analysis of Rectangular Plates with Clamped-Simply Supported Edge Conditions by the Method of Super position," ASME J. Appl. Mech., 44, pp. 743–749.
[12] Pan, E., 2001, "Exact Solution for Simply Supported and Multi layered Magneto-Electro-Elastic Plates," ASME J. Appl. Mech., 68, pp. 608–618.
[13] Cheng, Z.-Q., and Reddy, J. N., 2003, "Green's Functions for Infinite and Semi-Infinite Anisotropic Thin Plates," ASME J. Appl. Mech., 70, pp. 260–267.
[14]Tong, P., and Huang, W., 2002, "Large Deflection of Thin Plates in Pressure Sensor Applications," ASME J. Appl. Mech., 69, pp. 785–789.
[15] Narita, Y., 2000, "Combinations for the Free-Vibration Behaviours of Anisotropic Rectangular Plates under General Edge Conditions," ASME J. Appl.Mech., 67, pp. 568–573.
[16]Boay CG. Free vibration of laminated plates with a central circular hole. Composite Structures 1996;35:357–68.
[17] Sabir AB, Davies TG. Natural frequencies of square plates with reinforced central holes subjected to in plane loads. Thin-Walled Structures 1997; 28:337–53.
[18] Siva kumar K, Iyengar NGR, Deb K. Free vibration of laminated composite plates with cut out. Journal of Sound and Vibration 1999; 221:443–70.
[19] Siva kumar K, Iyengar NGR, Deb K. Optimum design of laminated composite plates with cut outs undergoing large amplitude oscillations. Advanced Composite Materials 1999;8:295–316.
[20] G. Pickett 1964 Proceedings of the Second South Eastern Conference Theoretical and Applied Mechanics, Atlanta, Georgia.Bending, buckling and vibrations of plates with holes.
[21] Liu FL, Liew KM. Vibration analysis of discontinuous Mindlin plates by differential quadrature element method. Trans. ASME Journal of Vibration and Acoustics 1999; 121:204–8.

[22] Liu FL, Liew KM. Differential quadrature element method: a new approach for free vibration analysis of polar Mindlin plates having discontinuities. Computer Methods in Applied Mechanics and Engineering1999; 179:407–23.
[23]Yang B, Zhou J. Semi-analytical solution of 2-D elasticity problems by the strip distributed transfer function method. International Journal of Solids and Structures 1996; 33:3983–4005.
[24] Yang B, Park DH. Analysis of plates with curved boundaries using iso parametric strip distributed transfer functions. International Journal for Numerical Methods in Engineering 1999; 44:131–46.
[25] Geannakakes GN. Vibration analysis of arbitrarily shaped platesusingbeam characteristic orthogonal polynomials in the semi-analytical 1nite strip method. Journal of Sound and Vibration 1990; 137:283–303.
[26]Laura PAA, Romanelli E, Rossi RE. Transverse vibrations of simply supported rectangular plates with rectangular cut outs. Journal of Sound and Vibration 1997;202:275–83.
[27] Liew KM, HungKC, Lim MK. Method of domain decomposition in vibrations of mixed edge anisotropic plates. International Journal of Solids and Structures 1993;30:3281–301.
[28] Liew KM, HungKC, Lim MK. Roles of domain decomposition method in plate vibrations—treatment of mixeddiscontinuous periphery boundaries. International Journal of Mechanical Sciences 1993;35:615–32.
[29] Liew KM, Hung KC, Sum YK. Flexural vibration of polygonal plates: treatments of sharp re-entrant corners. Journal of Sound and Vibration 1995; 183:221–38.
[30] Liew KM, Sum YK. Vibration of plates having orthogonal straight edges with clamped boundaries.Journal of Engineering Mechanics, ASCE 1998; 124:184–92.
[31] Liew KM, NgTY, Kitipornchai S. A semi-analytical solution for vibration of rectangular plates with abrupt thickness variation. International Journal of Solids and Structures 2001; 38:4937–54.
[32] Pan EN. A general boundary element analysis of 2-D linear elastic fracture mechanics.International Journal of Fracture 1997; 88:41–59.
[33] Chau KT, Wang YB. A new boundary integral formulation for plane elastic bodies containing cracks and holes.International Journal of Solids and Structures 1999; 36:2041–74.