STABILIZATION PRINCIPLES

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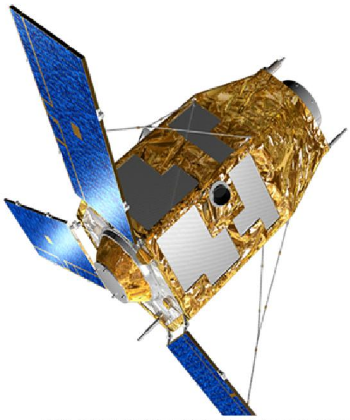
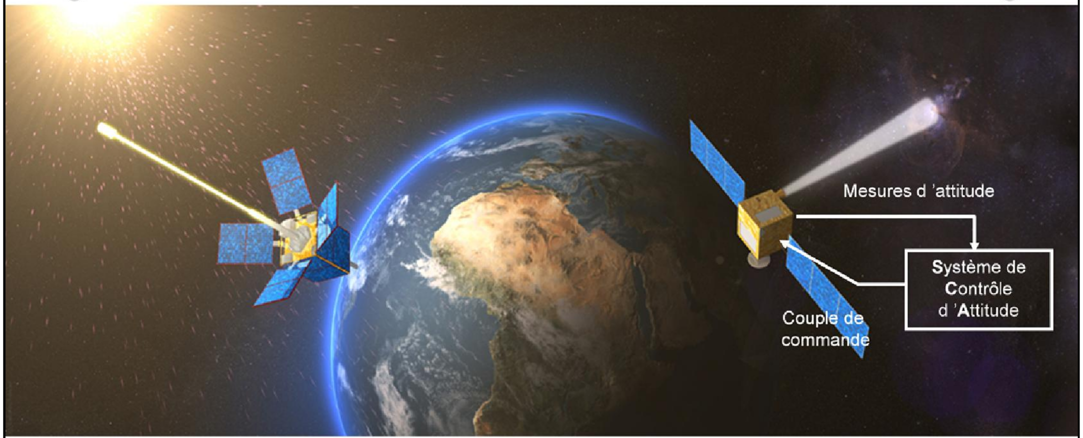


CONTROL/STABILIZATION METHODS



Passive control

When the control accuracy is not a major requirement

⇒ We take advantage of the environment properties to get a naturally stable system

Active control

When the system is unstable by design or when accurate pointing is required

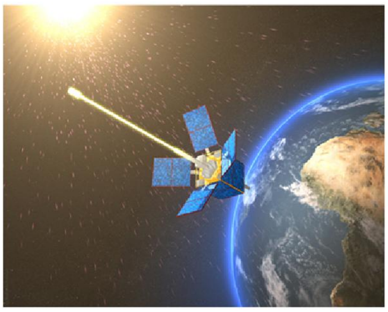
⇒ An attitude control system is implemented

Agenda

- ❖ **Passive stabilization**
- ❖ **Semi-passive stabilization**
- ❖ **Active stabilization**

- Passive stabilization
  - **Panorama**
  - Gravity gradient
  - Gyroscopic stiffness
- Semi-passive stabilization
- Active stabilization

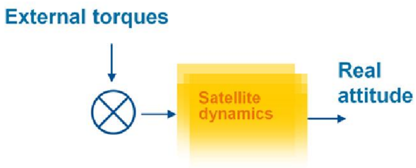
PASSIVE STABILIZATION (1/7)



Passive control

Main points :

- Use of the environment natural properties to drive the satellite back to some specific attitude in presence of others disturbing torques
- Damping of the oscillations around the equilibrium configuration
- Control of less than 3 axes
- Coarse control $\Rightarrow$ control accuracy has not to be a major requirement



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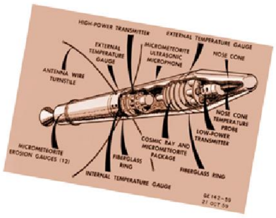
PASSIVE STABILIZATION (2/7)

Environment natural properties:

- ❑ Gravity
- ❑ Magnetic field
- ❑ Solar pressure
- ❑ Aerodynamic pressure
- ❑ Dynamic equation

Methods :

- ❑ Gravity gradient for an elongated body
- ❑ Earth magnetic field for a magnetized body
- ❑ Solar sail
- ❑ Aerodynamic drag at low altitude
- ❑ Gyroscopic stiffness for a spinning body



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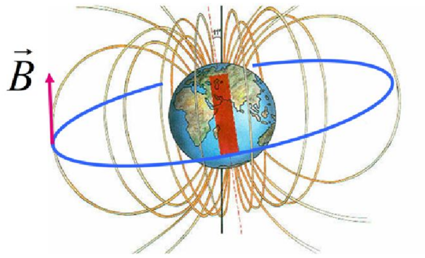
PASSIVE STABILIZATION (3/7)
Magnetic

Principle

- Use of fixed magnetic bars to follow the Earth magnetic field
- Stable position when the magnetic moment M is aligned with the magnetic field lines



Note

- Maximal efficiency on near-equatorial orbits with a North-South pointing since field direction is quasi constant
- Magnetic field magnitude depends on orbit altitude $\Rightarrow$ application to LEO missions

$$\vec{C} = \vec{M} \wedge \vec{B}$$

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PASSIVE STABILIZATION (4/7)
Aerodynamic

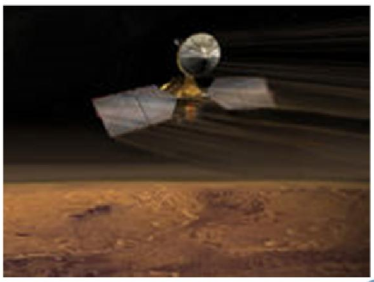
Principle

- Installation of fixed flaps or tail to provide the vehicule with a particular orientation w.r.t. the velocity vector

GOCE Satellite (orbit 260 km)

Application

- in Low Earth Orbit (LEO) where the effect is predominant
- orbital insertion by a unique pass (aerocapture) or periodic ones (aerobraking) through the atmosphere



Aerobraking (Mars Global Surveyor)

PASSIVE STABILIZATION (5/7)
Gravity Gradient

Principle

➢ Utilization of vehicle inertia properties to maintain it pointed to the Earth

Note

➢ Presence of oscillations around the satellite/Earth direction (nadir) due to initial biases and perturbations
$\Rightarrow$ damping is required

➢ Two axes are usually controlled, rotation around nadir is often free (but may be also controlled)

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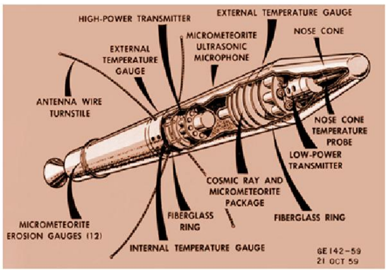
PASSIVE STABILIZATION (6/7)
Gyroscopic Stiffness

Principle

➢ Use of gyroscopic stiffness properties by giving the vehicle a high angular momentum through the rotation of the whole satellite or a part of it



Explorer III (1958) : spin stabilised NASA satellite

Note

➢Two axes are controlled, rate around rotation axis is free

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PASSIVE STABILIZATION (7/7)
Performances

Stabilization type	Pointing	Typical accuracy	Applications
Gravity gradient	Local vertical (nadir)	> 5° (2 axes)	UOSAT
Magnetic	North / South	> 5° (2 axes)	In addition to active stabilization method
Gyroscopic stiffness	Any inertial direction	0.1 à 1° (2 axes)	METEOSAT SatCom during transfer

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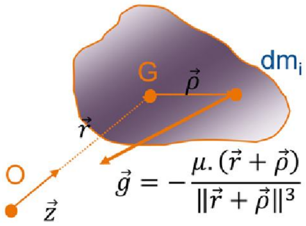
- Passive stabilization
  - Panorama
  - Gravity gradient
  - Gyroscopic stiffness
- Semi-passive stabilization
- Active stabilization

GRAVITY GRADIENT

Torque expression

Gravity field in 1/r² $\Rightarrow$ Gravity force barycenter ≠ center of mass

LAGRANGE
(1736-1813)



Torque associated to a mass element

$$d\vec{C}_i = \vec{\rho} \wedge d\vec{F}_i = -\vec{\rho} \wedge \frac{\mu.(\vec{r}+\vec{\rho})dm_i}{\|\vec{r}+\vec{\rho}\|^3}$$

Resulting torque computation

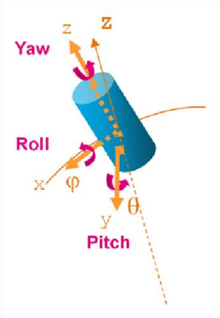
$$\rho \ll r \quad \rightarrow \quad (\rho + r)^{-3} = r^{-3}\left(1 - 3.\frac{\rho}{r}\right)$$

$$G\ mass\ center \ \rightarrow \iiint \vec{\rho}.dm = \vec{0}$$

$$\Rightarrow \boxed{\vec{C} = -\iiint \vec{\rho} \wedge \frac{\mu.(\vec{r}+\vec{\rho})dm_i}{\|\vec{r}+\vec{\rho}\|^3} \approx 3\frac{\mu}{r^3}\vec{z} \wedge J_G\vec{z}}$$

GRAVITY GRADIENT
Stability analysis for small angles (1/9)

Hypotheses



- The satellite axes are the inertia main axes :

$$J_G = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & Izz \end{bmatrix}$$

- Z axis is aligned with the radial direction
- A small pointing error is considered (θ, φ)

  ⇒ Expression of the Earth center of mass / satellite vector in the satellite frame :

$$\vec{z} = \begin{bmatrix} -\sin\theta \\ \cos\theta.\sin\varphi \\ \cos\theta.\cos\varphi \end{bmatrix}$$

- Circular orbit : $\frac{\mu}{R^3} = \omega_0^2$
- Gravity gradient torque approximation : $\vec{C}_{gdg} = 3.\frac{\mu}{R^3}.\vec{z} \wedge J_G.\vec{z}$

Is the position stable or unstable ?

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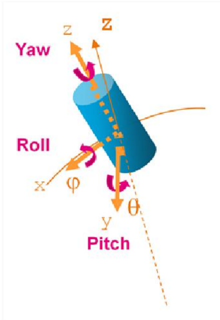
GRAVITY GRADIENT
Stability analysis for small angles (2/9)

- **Gravity gradient torque**:

$$C = 3.\omega_0^2.\begin{pmatrix} (I_{zz} - I_{yy}).\varphi \\ (I_{zz} - I_{xx}).\theta \\ 0 \end{pmatrix}$$



- **Stable behavior if the pulling torque is negative** :

$$C_x = -K_x.\varphi \quad with \quad K_x = -3\omega_0^2.(I_{zz} - I_{yy}) > 0 \rightarrow I_{yy} > I_{zz}$$

$$C_y = -K_y.\theta \quad with \quad K_y = -3\omega_0^2.(I_{zz} - I_{xx}) > 0 \rightarrow I_{xx} > I_{zz}$$

⇒ Inertia along the yaw axis must be the smallest (expected result!)

⇒ What can we say for the relation between Ixx and Iyy ?? ... The analysis based only on the torques expression is not sufficient

⇒ **The study of the dynamics has to be done**

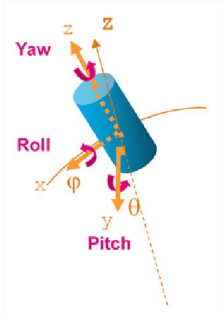
GRAVITY GRADIENT

Stability analysis for small angles (3/9)

Dynamics equation in the orbital frame (Euler equation)

$$\dot{\overrightarrow{\mathrm{H}_{G}}}_{/Rv} + \vec{\Omega} \wedge \overrightarrow{\mathrm{H}_{G}} = \vec{\mathrm{C}}$$

With $\vec{\Omega} = \begin{bmatrix} p \\ q \\ r \end{bmatrix}$ and $\vec{\mathrm{H}} = J_G.\vec{\Omega} = \begin{bmatrix} I_{xx}.p \\ I_{yy}.q \\ I_{zz}.r \end{bmatrix}$



The dynamics equation can be written :

$$I_{xx}.\dot{p} + (I_{zz} - I_{yy}).q.r = 3.\omega_0^2(I_{zz} - I_{yy})\varphi \qquad (1)\ roll$$

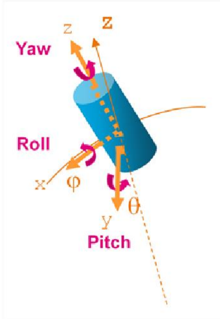
$$I_{yy}.\dot{q} + (I_{xx} - I_{zz}).p.r = 3.\omega_0^2(I_{zz} - I_{xx})\theta \qquad (2)\ pitch$$

$$I_{zz}.\dot{r} + (I_{yy} - I_{xx}).p.q = 0 \qquad (3)\ yaw$$

GRAVITY GRADIENT
Stability analysis for small angles (4/9)

Kinematics equation (with Euler angles) :

$$p = \dot{\varphi} - \omega_0.\psi \qquad (1)\ \text{roll}$$
$$q = \dot{\theta} - \omega_0 \qquad (2)\ \text{pitch}$$
$$r = \dot{\psi} + \omega_0.\varphi \qquad (3)\ \text{yaw}$$



Motion equation (small angles approximation):

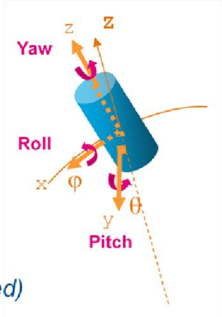
$$I_{xx}.(\ddot{\varphi} - \omega_0.\dot{\psi}) - \omega_0(I_{zz} - I_{yy})(\dot{\psi} + \omega_0.\varphi) = 3\omega_0^2.(I_{zz} - I_{yy})\varphi \quad (1)\ \text{roll}$$
$$I_{yy}.\ddot{\theta} = 3\omega_0^2.(I_{zz} - I_{xx})\theta \quad (2)\ \text{pitch}$$
$$I_{zz}.(\ddot{\psi} + \omega_0.\dot{\varphi}) - \omega_0(I_{yy} - I_{xx})(\dot{\varphi} - \omega_0.\psi) = 0 \quad (3)\ \text{yaw}$$

GRAVITY GRADIENT
Stability analysis for small angles (5/9)

Pitch equation : $I_{yy}.\ddot{\theta} = 3\omega_0^2.(I_{zz} - I_{xx})\theta$

$$k_\theta = \frac{I_{xx} - I_{zz}}{I_{yy}}$$

$$\boxed{\ddot{\theta} + 3.k_\theta.\omega_0^2 = 0}$$



The motion is stable for $k_\theta > 0 \Rightarrow$ ***I_{xx} > I_{zz}*** *(condition already identified)*

⇒ Harmonic motion :

$$\omega_{pitch} = \omega_0\sqrt{\frac{3.(I_{xx} - I_{zz})}{I_{yy}}}$$

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GRAVITY GRADIENT
Stability analysis for small angles (6/9)

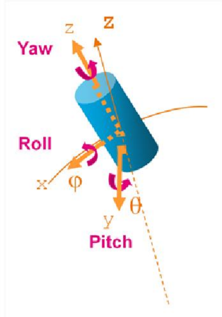
Roll/Yaw equations :

$$I_{xx}.\left(\ddot{\varphi}-\omega_0.\dot{\psi}\right)-\omega_0\left(I_{zz}-I_{yy}\right)\left(\dot{\psi}+\omega_0.\varphi\right)=3\omega_0^2.\left(I_{zz}-I_{yy}\right)\varphi$$
$$I_{zz}.\left(\ddot{\psi}+\omega_0.\dot{\varphi}\right)-\omega_0\left(I_{yy}-I_{xx}\right)\left(\dot{\varphi}-\omega_0.\psi\right)=0$$

$$k_\varphi=\frac{I_{yy}-I_{zz}}{I_{xx}},\quad k_\psi=\frac{I_{yy}-I_{xx}}{I_{zz}}$$

$$\ddot{\varphi}-\left(1-k_\varphi\right).\omega_0.\dot{\psi}+4\omega_0^2.k_\varphi.\varphi=0$$
$$\ddot{\psi}+\left(1-k_\psi\right).\omega_0.\dot{\varphi}+\omega_0^2.k_\psi.\psi=0$$

$$\begin{bmatrix}\dot{\psi}\\ \ddot{\psi}\\ \dot{\varphi}\\ \ddot{\varphi}\end{bmatrix}=\begin{bmatrix}0 & 1 & 0 & 0\\ -\omega_0^2k_\psi & 0 & 0 & -(1-k_\psi).\omega_0\\ 0 & 0 & 0 & 1\\ 0 & (1-k_\varphi).\omega_0 & -4\omega_0^2k_\varphi & 0\end{bmatrix}.\begin{bmatrix}\psi\\ \dot{\psi}\\ \varphi\\ \dot{\varphi}\end{bmatrix}$$



System : $\dot{X}=A.X$

⇒ Computation of the determinant of the matrix : (sI – A)

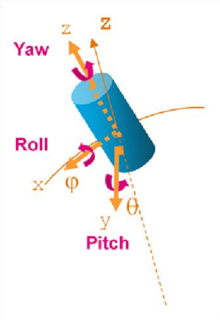
GRAVITY GRADIENT

Stability analysis for small angles (7/9)

Roll/Yaw equation :

$$k_\varphi = \frac{I_{yy}-I_{zz}}{I_{xx}}, \quad k_\psi = \frac{I_{yy}-I_{xx}}{I_{zz}}$$

$$\begin{bmatrix}\dot{\psi}\\ \ddot{\psi}\\ \dot{\varphi}\\ \ddot{\varphi}\end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\omega_0^2 k_\psi & 0 & 0 & -(1-k_\psi).\omega_0 \\ 0 & 0 & 0 & 1 \\ 0 & (1-k_\varphi).\omega_0 & -4\omega_0^2 k_\varphi & 0 \end{bmatrix} . \begin{bmatrix}\psi\\ \dot{\psi}\\ \varphi\\ \dot{\varphi}\end{bmatrix}$$



$$\Delta = \det(sI - A)$$

$$\Delta = s^2(s^2 + 4\omega_0^2 k_\varphi) + s^2(1-k_\varphi)(1-k_\psi).\omega_0^2 + \omega_0^2 k_\psi (s^2 + 4\omega_0^2 k_\varphi)$$

$$\Delta = s^4 + \omega_0^2 s^2 (1 + 3k_\varphi + k_\varphi k_\psi) + 4\omega_0^4 k_\varphi k_\psi$$

$$\boxed{\Delta = s'^2 + b_1.s + b_0 \quad (s' = s^2)}$$

Polynom in ‘s²’

$\Rightarrow$ the motion is stable if the eigenvalues are all pure imaginary

$\Leftrightarrow$ if the s² solutions are real negative

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GRAVITY GRADIENT

Stability analysis for small angles (8/9)

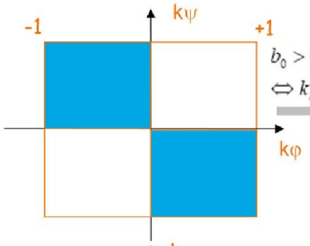
$\Delta = s'^2 + b_1.s + b_0 \quad (s' = s^2)$

Must have real roots < 0

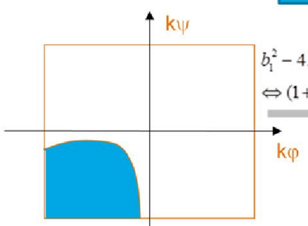
$k_\varphi = \frac{I_{yy} - I_{zz}}{I_{xx}}, \quad k_\psi = \frac{I_{yy} - I_{xx}}{I_{zz}}$

Conditions to satisfy to ensure the system stability

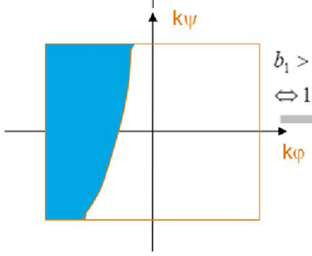
Forbidden region



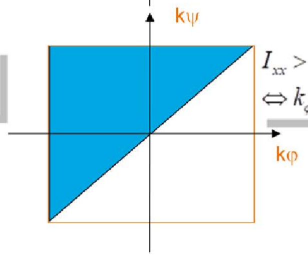
$b_0 > 0$
$\Leftrightarrow k_\varphi . k_\psi > 0$



$b_1^2 - 4.b_0 > 0$
$\Leftrightarrow (1 + 3k_\varphi + k_\varphi . k_\psi)^2 - 16.k_\varphi . k_\psi > 0$



$b_1 > 0$
$\Leftrightarrow 1 + 3k_\varphi + k_\varphi . k_\psi > 0$



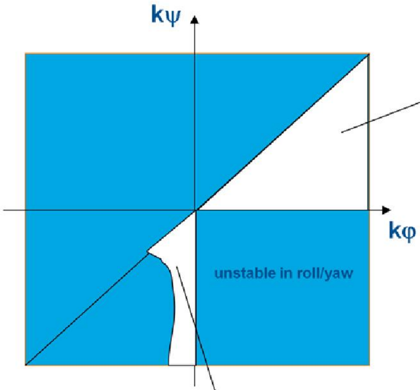
$I_{xx} > I_{zz}$
$\Leftrightarrow k_\varphi > k_\psi$

Pitch relation

GRAVITY GRADIENT

Stability analysis for small angles (9/9)

Satbility analysis summary



$$k_\varphi > k_\psi \Leftrightarrow I_{XX} > I_{ZZ}$$
$$k_\varphi > 0,\ k_\psi > 0 \Rightarrow I_{YY} > I_{ZZ} \ \ and \ I_{YY} > I_{XX}$$

Stable zone for $I_{yy} > I_{xx} > I_{zz}$

To ensure stability

- ✓ The yaw inertia is the smallest
- ✓ The pitch inertia is the largest

Stable zone $I_{yy} < I_{zz} < I_{xx}$

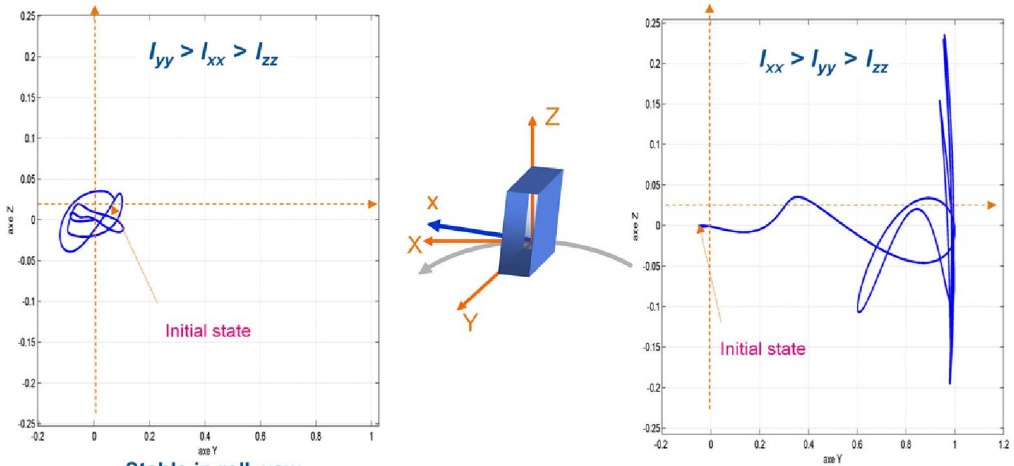
bound by $1 + 3\,k\varphi + k\varphi\, k\psi > 4(k\varphi\, k\psi)^{1/2}$

Not robust

GRAVITY GRADIENT
Illustration

Simulation over 3 orbits (h=700 km)

Projection of the X satellite axis in the Y,Z plane of the orbital frame



Stable in roll, yaw

Unstable in roll, yaw

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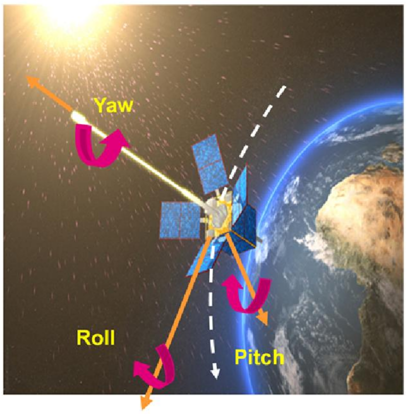
GRAVITY GRADIENT

Synthesis



- Pointing with low accuracy: 1 to 10°
- Low altitude circular orbit (h < 1500 km)
- High constraint on the moments of inertia

$I_{yaw} < I_{roll} < I_{pitch}$

- Satellite behaves as a pure oscillator around the equilibrium configuration

⇒ Damping si required

Possible damping techniques :

- ✓ Fuel or other liquids sloshing (passive)
- ✓ Magnetic bars
- ✓ Thrusters

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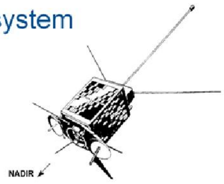
GRAVITY GRADIENT
Examples

- Space stations
- Space shuttle
- Low performance satellites or as a complement for an active system



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- **Passive stabilization**
  - **Panorama**
  - **Gravity gradient**
  - **Gyroscopic stiffness**
- **Semi-passive stabilization**
- **Active stabilization**

GYROSCOPIC STIFFNESS

Two techniques

- **Spin stabilization**
  - ⇒ **to be considered now**
- Stabilization with angular momentum bias
  - ⇒ Semi-passive method (see next part)

SPIN STABILIZATION

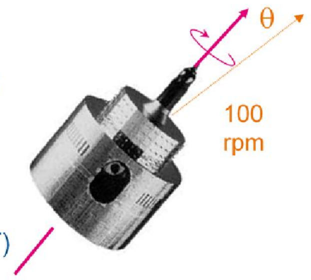
rpm = rotation per minute

Principle

- Spacecraft is spun around one of its axes
- Rotation rate depends on desired efficiency
  - 10 rpm for a launcher,
  - 15 to 60 rpm for a geo. satellite in transfer phase,
  - 100 rpm for some geo. satellites during nominal mission.

A little bit of history

- Technique used from the beginning of the space era:
  - Simple (passive and intuitive – gyro principle)
  - Cheap
- Technique still in use on GEO orbits (ex : METEOSAT)



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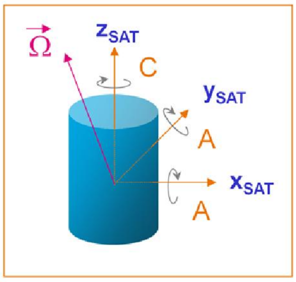
SPIN STABILIZATION
Poinsot motion

Analysis of the motion without perturbing torque

⇒ Poinsot motion



Hypotheses : spinning satellite around z_{sat} with axial symetry

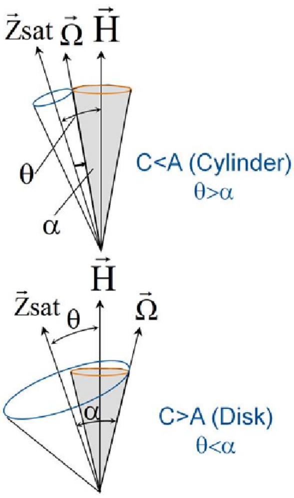
$$J_{sat} = \begin{bmatrix} A & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & C \end{bmatrix} \quad , \quad \vec{\Omega} = \begin{pmatrix} p \\ q \\ r \end{pmatrix}_{/Rv} \quad , \quad \vec{H} = \begin{pmatrix} A.p \\ A.q \\ C.r \end{pmatrix}_{/Rv}$$

Dynamic equations in the satellite frame:

$$\dot{\vec{H}}_{/Rv} + \vec{\Omega} \wedge \vec{H} = 0 \iff \begin{cases} A.\dot{p} - (A-C)qr = 0 \\ A.\dot{q} + (A-C)pr = 0 \\ C.\dot{r} = 0 \end{cases} \iff \begin{cases} p = k.cos\left(\omega \dfrac{|A-C|}{A} t + \varphi\right) \\ q = k.sin\left(\omega \dfrac{|A-C|}{A} t + \varphi\right) \\ r = cste = \omega \end{cases}$$

SPIN STABILIZATION
Poinsot motion

Analysis of the motion <u>without perturbing torque</u>



$\vec{H}$ is fixed in inertial frame (angular momentum conservation)

$\vec{H}$, $\vec{\Omega}$, $\overrightarrow{Z_{sat}}$ are coplanar since $\vec{H} = A.\vec{\Omega} + (C - A)\omega.\overrightarrow{Z_{sat}}$

α, θ (nutation angle) such that :

$$\tan(\alpha) = \tan(\Omega, Z_{sat}) = \frac{k}{r} = \text{cste}$$

$$\tan(\theta) = \tan(H, Z_{sat}) = \frac{A}{C}.\frac{k}{r} = \text{cste}$$

$\overrightarrow{Z_{sat}}$ and $\vec{\Omega}$ are rotating around $\vec{H}$ at $\frac{|A-C|}{A}\omega$ rate

It looks as if the body has a conic shape (white cone) and rolls on the virtual grey cone (either inside or outside)

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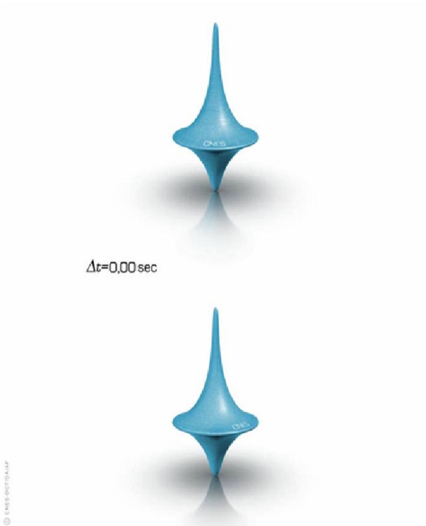
SPIN STABILIZATION
with perturbing torque

Analysis of the motion <u>with perturbing torque</u>

Dynamic equation : $\dot{\vec{H}}_{/Ri} = \vec{C} \rightarrow \Delta\vec{H} = \vec{C}.\Delta t$



- No initial spin : $\boldsymbol{H_0 = 0}$

$\vec{\boldsymbol{H}} \; \vec{\boldsymbol{C}}$ $\quad \dot{\vec{H}} = J.\ddot{\theta} = C \Rightarrow \boxed{\Delta\theta = \frac{C}{2.J}\Delta t^2}$

- Torque application effect: attitude evolution in t^2

- Presence of initial angular momentum: $\boldsymbol{H_0 \neq 0}$

$\Rightarrow \boxed{\theta = \frac{C.\Delta t}{H_0}}$

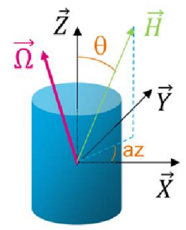
- Motion magnitude inversly proportionnal to H_0

The direction of the angular momentum varies slowly

Gyroscopic stiffness

SPIN STABILIZATION
with perturbing torque

Analysis of the motion with perturbing torque



- ✓ Perturbing torques ⇒ evolution of the nutation angle
- ✓ Possible damping through passive dissipation (liquid sloshing)
- ✓ Stability condition: dissipation leads to the minimum energy state (natural evolution)

Kinetic energy : $E_c = \frac{1}{2}.J.\Omega^2 \Rightarrow 2.E_c = J.\Omega^2 = A.p^2 + A.q^2 + C.r^2$

$$H = \begin{bmatrix} Ap \\ Aq \\ Cr \end{bmatrix} = \begin{bmatrix} H.\sin\theta.\cos(az) \\ H.\sin\theta.\sin(az) \\ H.\cos\theta \end{bmatrix} \Rightarrow \begin{cases} A^2.p^2 + A^2.q^2 = H^2.\sin^2\theta \\ C^2.r^2 = H^2.\cos^2\theta \end{cases}$$

$$\Rightarrow \begin{cases} A.p^2 + A.q^2 = \frac{H^2}{A}.\sin^2\theta \\ C.r^2 = \frac{H^2}{C}.\cos^2\theta \end{cases}$$

$$\Rightarrow 2.E_c = A.p^2 + A.q^2 + C.r^2 = \frac{H^2}{A}.\sin^2\theta + \frac{H^2}{C}.\cos^2\theta$$

$$\boxed{\Rightarrow 2.E_c = \frac{H^2}{C}.(\sin^2\theta + 1)\left(\frac{C-A}{A}\right)}$$

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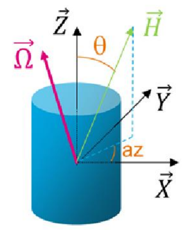
SPIN STABILIZATION
with perturbing torque

Analysis of the motion <u>with perturbing torque</u>



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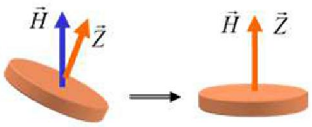
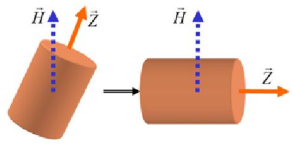
C > A ⇒ θ_{min} = 0



θ → 0

Spin axis is stable

C < A ⇒ θ_{min} = π/2



θ → π/2 = « flat spin »

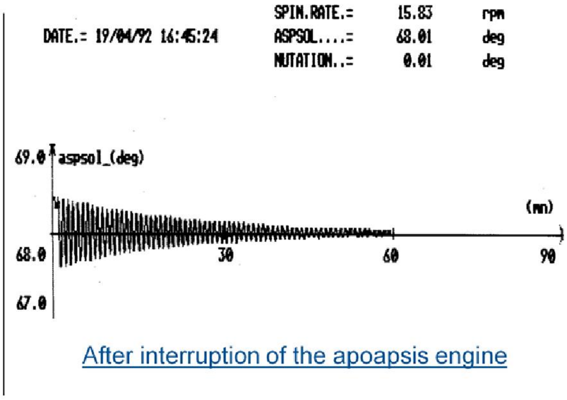
Spin axis unstable

=> Nutation active control is required

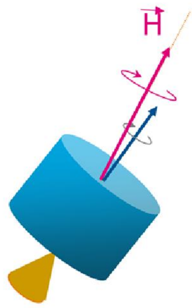
SPIN STABILIZATION
Illustration

Example of nutation damping by liquid sloshing

➔ TC2B geo transfer (spin phase).



After interruption of the apoapsis engine



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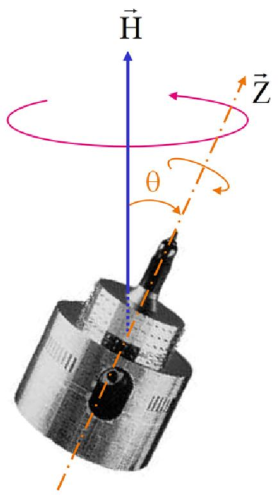
SPIN STABILIZATION
Illustration



METEOSAT example:

- C > A ⇒ spin axis is stable
  presence of a dissipating device by liquid sloshing
- $\vec{H}$ integrates the external torques
  ⇒ loses its North/South orientation imposed by the mission
  ⇒ maneuvres to maintain the direction of the angular momentum are required
  ⇒ This is called **spin active control** ….

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SPIN STABILIZATION
Nutation Active Control

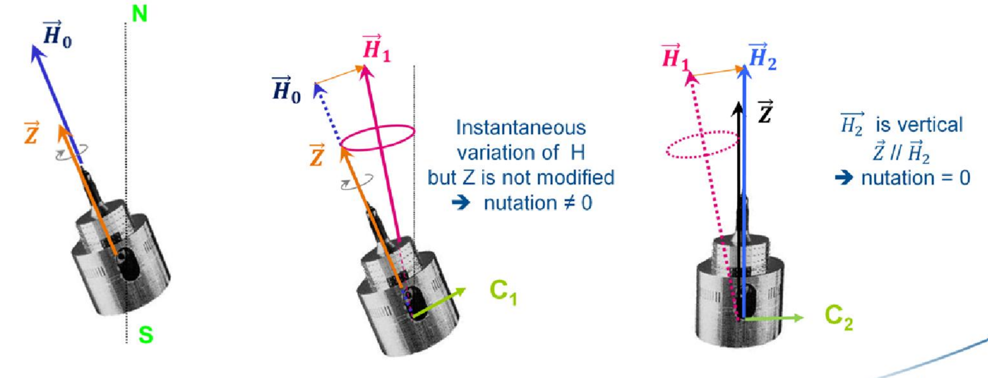
Objective : steering H back to vertical

How ? : 2 maneuvers (propulsion torques) of equal magnitude are required
The maneuvers are saparated by a **half nutation period** (modulo 1 period).

Initial state:
No nutation (Z // H)
but H is not vertical

Maneuver M1 :
Creation of a momentum increment ΔH1 but also some nutation

Maneuver M2 :
Creation of a momentum increment ΔH2 when Z is vertical



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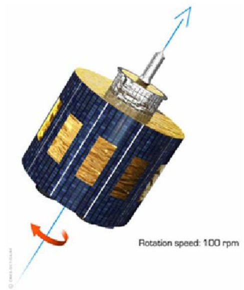
SPIN STABILIZATION
Synthesis

Properties provided by the gyroscopic stiffness

- Stable spin stable if the moment of inertia around the rotation axis is the largest
- Two axes control, angular rate around the rotation axis is free
- Limitation of the effect of the perturbations acting in the plane normal to the rotation axis
- Coarse pointing accuracy (2 axes) : 0.1° to 1°



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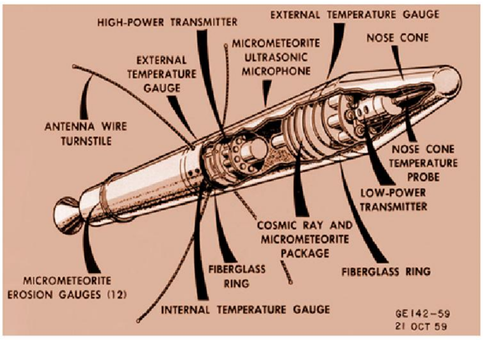
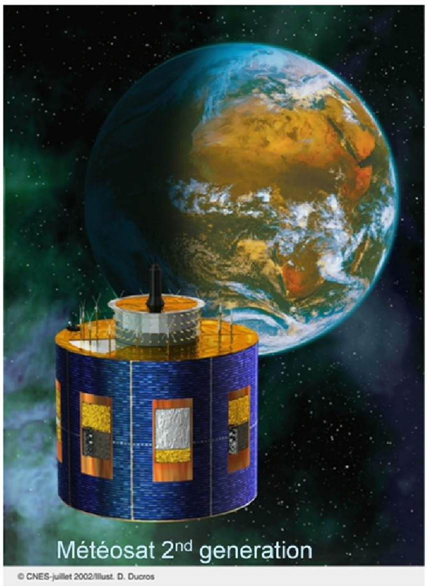
SPIN STABILIZATION
Synthesis

Applications

- Coarse pointing performance
- GEO orbit
- GEO orbital transfer



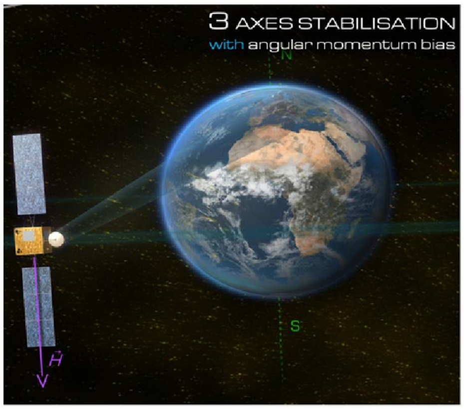
Explorer III (1958) : NASA satellite



- Passive stabilization
- **Semi-passive stabilization**
- Active stabilization

SEMI-PASSIVE STABILIZATION
with angular momentum bias

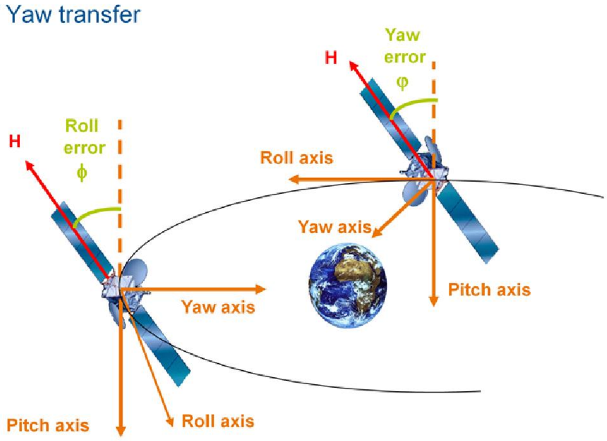
- A wheel is spun at high speed (several tr/s) to provide the satellite with gyroscopic stiffness
- The momentum wheel axis is generally normal to the orbital plane
- Typical use on the geostationnary orbits where the angular momentum can be parallel to the N/S axis



STABILIZATION with angular momentum bias
Roll / Yaw transfer (1/2)

Geostationnary satellite with on board angular momentum on the pitch axis

1) Roll / Yaw transfer



Any yaw error becomes a roll error with a similar magnitude 1/4 orbit later

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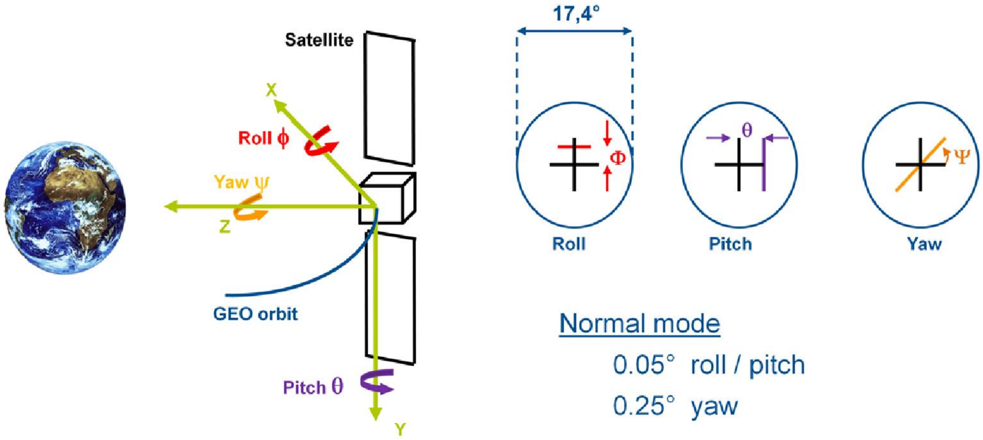
STABILIZATION with angular momentum bias

Roll / Yaw transfer (2/2)

Geostationnary satellite with on board angular momentum on the pitch axis

1) Roll / Yaw transfer

⇒ Measurement of the pointing errors on a GEO satellite



Normal mode

0.05° roll / pitch

0.25° yaw

The yaw error measurement is deduced from the roll measurement

Control possible with less sensors and actuators (2 axes control)

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STABILIZATION with angular momentum bias

Motion analysis for small angles (1/4)

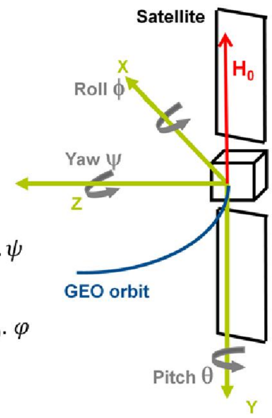
Geostationnary satellite with on board angular momentum on the pitch axis

2) Motion analysis

Dynamics equation : $\dot{\vec{H}}_{/Rv} + \vec{\Omega} \wedge \vec{H} = \vec{C}$

$H_0 \gg I_{xx}.p, I_{yy}.q \,, I_{zz}.r$

$$\vec{H} = J.\vec{\Omega} + \vec{H}_0 = \begin{bmatrix} I_{xx}.p \\ I_{yy}.q + H_0 \\ I_{zz}.r \end{bmatrix} \Rightarrow \begin{cases} I_{xx}.\dot{p} - H_0.r = C_x \\ I_{zz}.\dot{r} + H_0.p = C_z \\ I_{yy}.\dot{q} = C_y \end{cases}$$



Kinematics equation (with cardan angles) : $\Rightarrow \begin{cases} p = \dot{\varphi} - \omega_0.\psi \\ q = \dot{\theta} - \omega_0 \\ r = \dot{\psi} + \omega_0.\varphi \end{cases}$

Motion equation:

$$\Rightarrow \begin{cases} I_{xx}.\ddot{\varphi} - (I_{xx}.\omega_0 + H_0).\dot{\psi} - H_0.\omega_0.\ \varphi = C_x \\ I_{zz}.\ddot{\psi} + (I_{zz}.\omega_0 + H_0)\dot{\varphi} - H_0.\omega_0.\psi = C_z \\ I_{yy}.\ddot{\theta} = C_y \end{cases}$$

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STABILIZATION with angular momentum bias

Motion analysis for small angles (2/4)

Geostationnary satellite with on board angular momentum on the pitch axis

2) Motion analysis

- Roll / Yaw motion

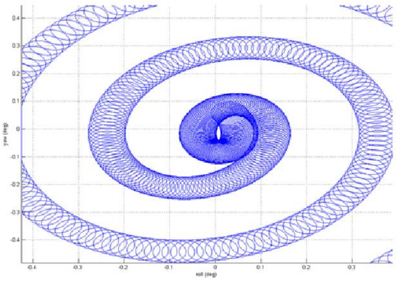
$$I_{xx}.\ddot{\varphi} - (I_{xx}.\omega_0 + H_0).\dot{\psi} - H_0.\omega_0.\ \varphi = C_x$$
$$I_{zz}.\ddot{\psi} + (I_{zz}.\omega_0 + H_0)\dot{\varphi} - H_0.\omega_0.\psi = C_z$$

- System eigen values:

$$\omega^4 - \left(\omega_0^2 + \frac{H_0}{I_{xx}.I_{zz}}\right).\omega^2 + \frac{H_0}{I_{xx}.I_{zz}}.\omega_0^2 = 0$$

$\omega_1 = \omega_0$: Long term motion (precession)

$\omega_2 = \frac{H_0}{\sqrt{I_{xx}.I_{zz}}}$: Short term motion (nutation)



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STABILIZATION with angular momentum bias
Motion analysis for small angles (3/4)

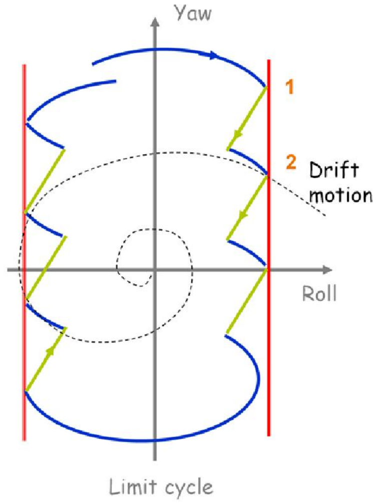
Long term control

➢ Objective :

to maintain the satellite angular momentum aligned with the N-S axis

➢ How ?

In applying corrective thrust when the roll error exceeds a given threshold



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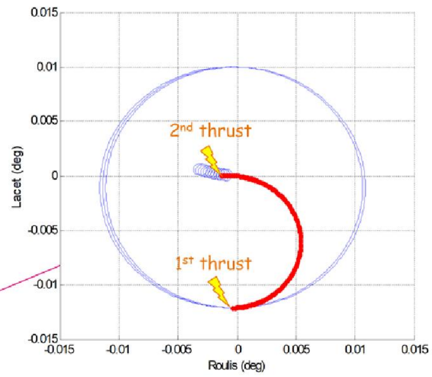
STABILIZATION with angular momentum bias
Motion analysis for small angles (4/4)

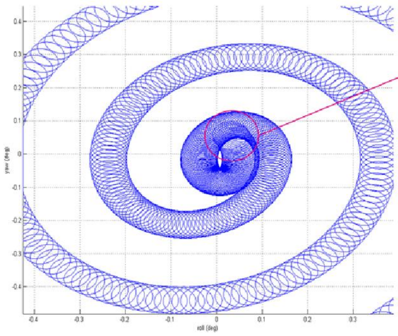
Short term control

= nutation control

How ?

Thrust application at 2 successive instants separated by $T_{nut}/2$ and based on the roll rate measurement





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STABILIZATION with angular momentum bias

Advantages of momentum wheel(s) utilization

- Slow secular drift of attitude in presence of perturbing torques
- Motion coupling on the transversal axes (roll/yaw axes)
  - possibility to control with few sensors (no yaw measurement for instance)

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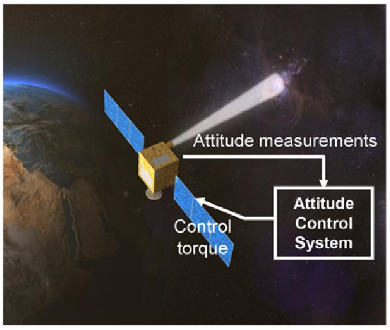
- Passive stabilization
- Semi-passive stabilization
- **Active stabilization**

ACTIVE STABILIZATION



Active control

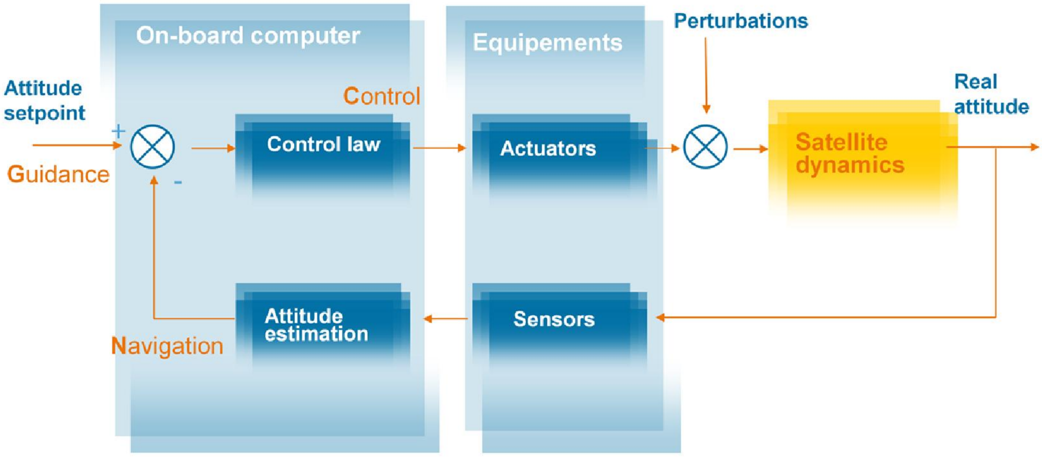
When :

- the system is unstable by design,
- the pointing / stability requirements are stringent.

AOCS loop :

- Satellite dynamics detection with sensors
- State estimation
- Computation of corrective torques
- Torque application with actuators

ACTIVE STABILIZATION
AOCS loop



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ACTIVE STABILIZATION
Sensors

Available references :

- External : Sun, stars, Earth, magnetic field, moon, planets…
  - Sun sensors (2 axes)
  - Eath sensors (2 axes)
  - Star tracker (3 axes)
  - Magnetometer (2 axes)
- Inertial :
  - Gyrometer
  - Accelerometer
- Others vehicles (ex: Rendezvous, Formation flying)
  - Camera,
  - RF sensor

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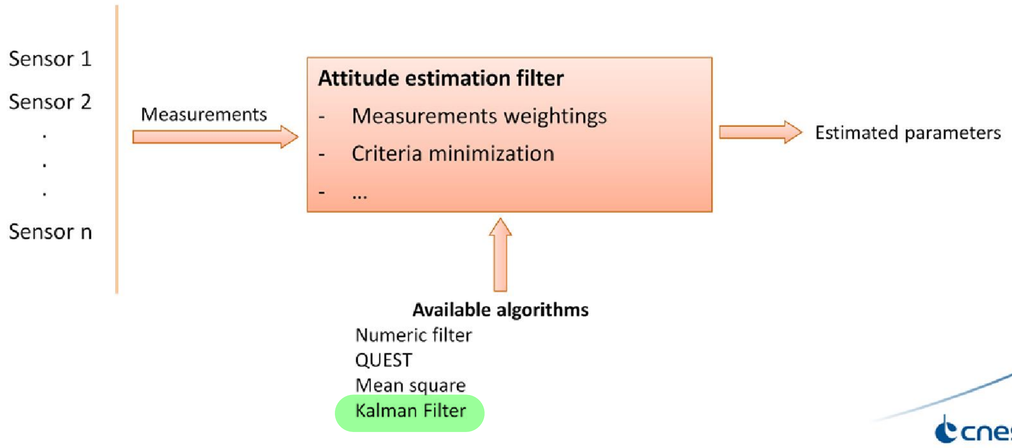
ACTIVE STABILIZATION
Attitude estimation

Attitude estimation :

1) on board : input data for the attitude control function
2) on ground : input data for attitude restitution and image processing



ACTIVE STABILIZATION
Control

Objective :

- To control the satellite attitude on the target,
- To deal with the performance/robustness compromise

Required peformance and stability:

- ✓ Pointing accuracy
- ✓ Pointing stabilization
- ✓ Response time (bandwith)
- ✓ Perturbations rejection
- ✓ No divergence

→ Controller Synthesis

Methods :

- Modal control : PID, …
- Optimal control (criteria minimization) : LQ, LQG (H2)
- Robust control : H_∞, H_{inf_struct}

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ACTIVE STABILIZATION
Actuators

Objective :

Apply the forces and torques computed by the control function

Principle :

- Angular momentum exchange (Action / Reaction)
  - Inertial actuators : Wheels, Control Moment Gyros
- Utilization of the magnetic field
  - Magneto-torquer bars
- Mass ejection
  - Thrusters
- Others
  - Solar sails, aerodynamic flaps, ...

ACTIVE STABILIZATION
Reaction wheels

Principle

➢ Use the action / reaction principle to generate a torque:

$$\vec{H}_{tot} = \vec{H}_{sat} + \vec{H}_{wheel} = J_{sat}.\vec{\Omega} + J_{wheel}.\vec{\omega}_{wheel/sat}$$

If $\vec{C}_{\text{ext}} = 0$,

$$\dot{\vec{H}}_{tot} + \vec{\Omega} \wedge \vec{H}_{tot} = 0 \Longrightarrow \dot{\vec{H}}_{sat} = -\dot{\vec{H}}_{wheel} - \underbrace{\vec{\Omega} \wedge \vec{H}_{tot}}_{\text{Gyroscopic torques}}$$

Without gyroscopic torques

$$\Delta\vec{H}_{sat} = -\Delta\vec{H}_{wheel}$$

$$\Delta\vec{\Omega} = -\frac{J_{wheel}}{J_{sat}}\Delta\vec{\omega}_{wheel}$$

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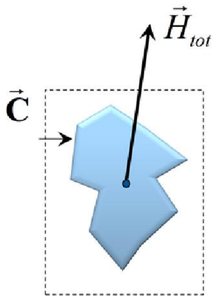
Principle

- Why the actuator angular momentum needs to be dumped ?
  - **Exchange** of kinetic momentum between the satellite and the actuators

⇒ the total angular momentum keeps integrating the external torques
⇒ divergence of H_{total}

$$\dot{\vec{H}}_{sat} + \vec{\Omega} \wedge \vec{H}_{tot} = \vec{C}_{ext} - \dot{\vec{H}}_{wheel}$$



Σ : system = sat + wheels

⇒ As H_{sat} is imposed, H_{wheel} diverges

It is necessary to create external torques (magnetic or propulsive) to reduce the RW rate

ACTIVE STABILIZATION
Reaction wheels

Using actuators based on angular momentum exchange, the control must satisfy two objectives:

- ✓ Satisfy the platform pointing and stability requirements
- ✓ Maintain the actuators in their nominal operational domain

➔ Prevent the saturation of the actuators by the application of external torques

Impact on the mission depends on the type of actuator used for dumping:

- ✓ **Thrusters**: periodic action with mission interruption
- ✓ **Magnetic torquers**: continuous action without mission interruption

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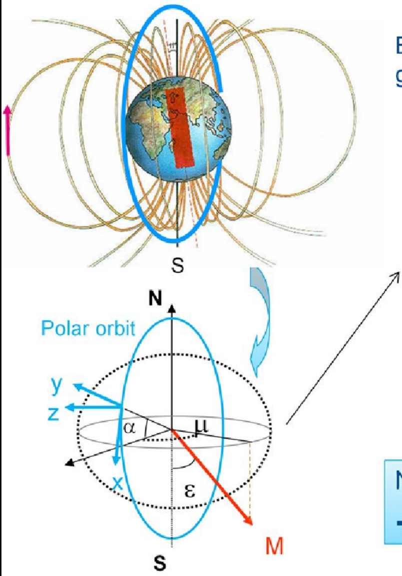
ACTIVE STABILIZATION
Reaction wheels / Momentum dumping

Utilization of magneto-torquers bars



By controlling the current in the coils, a magnetic moment **M** is generated and it interacts with the Earth magnetic field

$$\vec{C} = \vec{M} \wedge \vec{B}_{earth}$$

Magnetic field expression in the local orbital frame (satellite):

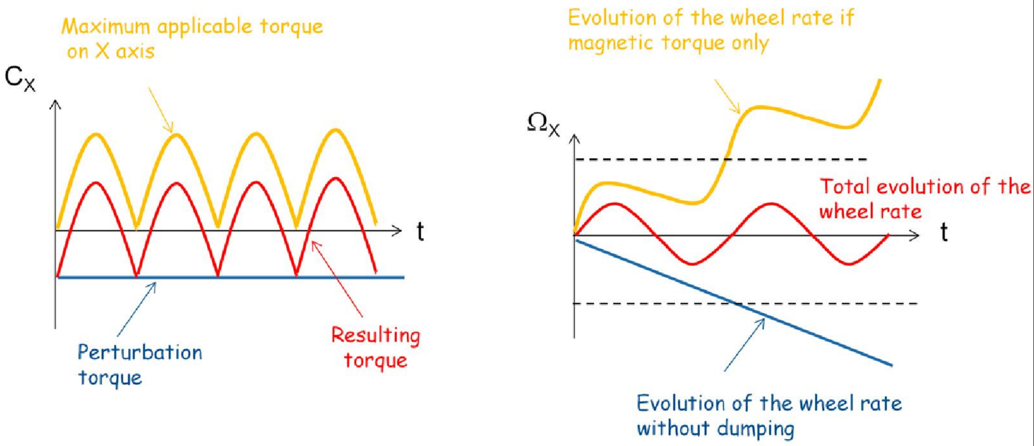
$$B = \frac{M}{R^3} . \begin{pmatrix} \cos\varepsilon.\cos\mu + \sin\varepsilon.\sin\alpha.\cos\mu \\ -2(\sin\varepsilon.\cos\alpha.\cos\mu - \cos\varepsilon.\sin\alpha) \\ -\sin\varepsilon.\sin\mu \end{pmatrix}$$

No torque in B direction

➔ 3 axes controllability guaranteed on a complete orbit

ACTIVE STABILIZATION
Reaction wheels / Momentum dumping

Utilization of magneto-torquers bars



The contribution of the magnetic torquers combined with the external perturbations allows to

- **produce a total perturbation which zero average value**
- **limit the rate of the reaction wheels**

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TO GO FURTHER …

- James WERTZ, *Spacecraft attitude determination and control*
- Peter C. HUGHES, *Spacecraft attitude dynamics, John Wiley & sons, Inc.*
- CNES, *Cours de techniques et technologie des véhicules spatiaux, Para Graphic Editions*
- O. ZARROUATI, *Trajectoires spatiales, CNES, Cépaduès Editions*
- Attitude & Orbit Control, Jeanette LINDSTRÖM