Miniproject 2: Modeling and Calculus

Overview: In this miniproject you will use technological tools to turn data and into models of real-world quantitative phenomena, then apply the principles of the derivative to them to extract information about how the quantitative relationship changes.

Prerequisites: Sections 1.1–1.6 in *Active Calculus*, specifically the concept of the derivative and how to construct estimates of the derivative using forward, backward and central differences. Also basic knowledge of how to use Desmos.

1. A settlement starts out with a population of 1000. Each year the population increases by 10%. Let $P(t)$ be the function that gives the population in the settlement after t years.

   (a) Find the missing values in the table below.(Create your own table.)

t	0	1	2	3	4	5	6	7
$P(t)$	1000							

   (b) Find a formula for $P(t)$. You can reason it out directly or you can have Desmos find it for you by creating the table of values above (using x_1 and y_1 as the column labels) and noting that the exponential growth of the data should be modeled using an exponential model of the form

   $$y_1 \sim a \cdot b^{x_1} + c$$

   (c) What will the population be after 100 years under this model?

   (d) Use a central difference to estimate the values of $P'(t)$ in the table below. What is the interpretation of the value $P'(5)$?

t	1	2	3	4	5	6
$P'(t)$						

   (e) Use a central difference to estimate the values of $P''(3)$. What is the interpretation of this value?

   (f) Place the table from (d) in a table in Desmos with column labels x_1 and y_1. The data also looks like it follows an exponential growth model, so we enter the code below in a Desmos cell to get a formula for $P'(t)$.

   $$y_1 \sim a \cdot b^{x_1} + c$$

   (Note that Desmos won't let you use the notation $P'(t)$.) The functions $P(t)$ and $P'(t)$ are multiples of each other. Find k so that $P'(t) = k \cdot P(t)$.

2. The dosage recommendations for a certain drug are based on weight.

Weight (lbs)	20	40	60	80	100	120	140	160	180
Dosage (mg)	10	30	70	130	210	310	430	570	730

(a) Find a function D(x) that approximates the dosage when you input the weight of the individual. (Make a table in Desmos using x_1 and y_1 as the column labels and you will see that the points seem to form a parabola. Use Desmos to find a model of the form

$$y_1 \sim ax_1^2 + bx_1 + c$$

and define $D(x) = ax^2 + bx + c$.)

(b) Find the proper dosage for a 128 lb individual.

(c) What is the interpretation of the value $D'(128)$.

(d) Estimate the value of $D'(128)$ using viable techniques from our calculus class. Be sure to explain how you came up with your estimate.

(e) Given the value $D'(130) = 6$, find an equation of the tangent line to the curve $y = D(x)$ at the point where $x = 130$ lbs.

(f) Find the point on the tangent line in the previous part that has x-coordinate $x = 128$. Does the output value on the tangent line for $x = 128$ lbs give a good estimate for the dosage for a 128 lb individual?

Submission instructions: The writeup that you prepare is to be saved as a PDF file and submitted using Canvas. (You may use any program you want to write the writeup but the submission *must* be a PDF, or your work will be marked at Novice level and returned without comment. You may important screenshots to show the Desmos output for each step. Just remember to be neat with your work.)