Miniproject 7: The Shape of a Graph

Overview: In this miniproject you will be using the techniques of calculus to find the behavior of a graph.

Prerequisites: The project draws heavily from the ideas of Chapter 1 and 2.8 together with ideas and techniques of the first and second derivative tests from 3.1.

1. We are given the functions

$$f(x) = \frac{12x^2 - 16}{x^3}, \qquad f'(x) = -\frac{12(x^2 - 4)}{x^4}, \qquad f''(x) = \frac{24(x^2 - 8)}{x^5}.$$

The questions below are about the function $f(x)$. Answer parts (a) through (j) below. If the requested feature is missing, then explain why. Be sure to include the work/test that you used to rigorously reach your conclusion. It is not sufficient to refer to the graph.

(a) State the function's domain.

(b) Find all x and y-intercepts.

(c) Find all equations of horizontal asymptotes.

(d) Find all equations of vertical asymptotes.

(e) Find the interval(s) where f is increasing.

(f) Find the x-value(s) of all local maxima. (Find exact values, and not decimal representations)

(g) Find the x-value(s) of all local minima. (Find exact values, and not decimal representations)

(h) Find the interval(s) on which the graph is concave downward.

(i) State the x-value(s) of all inflection points. (Find exact values, and not decimal representations)

(j) Include a sketch of the graph of $y = f(x)$. Plot the different segments of the graph using the color code below.

blue: $f' > 0$ and $f'' > 0$
red: $f' < 0$ and $f'' > 0$
black: $f' > 0$ and $f'' < 0$
gold: $f' < 0$ and $f'' < 0$

(In Desmos you could plot $y = f(x)$ on the interval $[2,3]$ by typing $y = f(x)\{2 \le x \le 3\}$.) Be sure to set the bounds on the graph so that the features of the graph that you listed above are easy to see.

Submission instructions: The writeup that you prepare is to be saved as a PDF file and submitted using Canvas. (You may use any program you want to write the writeup but the submission *must* be a PDF, or your work will be marked as Unsatifactory and returned without comment. You may important screenshots to show the Desmos output for any steps. Just remember to be neat with your work.)