

To find the minimum of the function using the Newton method, we'll first need to calculate the gradient and the Hessian of the function, then apply the Newton iteration step. Given the function:

$$f(x_1, x_2) = Ax_1^2 + Bx_2^2 - 0.25Ax_1x_2 - 0.5Ax_1 - 0.5Bx_2 + AB$$

where (  $A = 5$  ) and (  $B = 8$  ), the gradient (  $\nabla f$  ) is a vector of partial derivatives of (  $f$  ) with respect to (  $x_1$  ) and (  $x_2$  ):

$$\nabla f = \left[ \frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2} \right]$$

The partial derivatives are:

$$\frac{\partial f}{\partial x_1} = 2Ax_1 - 0.25Bx_2 - 0.5A$$

$$\frac{\partial f}{\partial x_2} = 2Bx_2 - 0.25Ax_1 - 0.5B$$

The Hessian (  $H$  ) is a matrix of second-order partial derivatives:

$$H = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} \end{bmatrix}$$

The second-order partial derivatives are:

$$\frac{\partial^2 f}{\partial x_1^2} = 2A$$

$$\frac{\partial^2 f}{\partial x_1 \partial x_2} = \frac{\partial^2 f}{\partial x_2 \partial x_1} = -0.25A$$

$$\frac{\partial^2 f}{\partial x_2^2} = 2B$$

So the Hessian is:

$$H = \begin{bmatrix} 2A & -0.25A \\ -0.25A & 2B \end{bmatrix}$$

The Newton iteration step is given by:

$$X_{k+1} = X_k - H^{-1} \nabla f(X_k)$$

Starting from the initial point (  $X_0 = (-2A, 2B)$  ), we can apply the Newton iteration until the Kuhn-Tucker condition is met, which is when the norm of the gradient is less than (  $10^{-5}$  ).

Since this is an analytical solution, you would use a calculator to compute the values of the gradient and Hessian at each iteration, and then update the values of (  $x_1$  ) and (  $x_2$  ) accordingly until convergence.

Please note that the convergence criterion is based on the norm of the gradient, not the change in the function value or the change in the variables. The norm of the gradient should be calculated as:

$$\| \nabla f \| = \sqrt{\left(\frac{\partial f}{\partial x_1}\right)^2 + \left(\frac{\partial f}{\partial x_2}\right)^2}$$

And you would check if (  $\| \nabla f \| < 10^{-5}$  ) to stop the iterations. Remember to substitute the values of (  $A$  ) and (  $B$  ) into the gradient and Hessian before starting the iterations. Good luck with your calculations!

The inverse of the Hessian (  $H^{-1}$  ) is:

$$H^{-1} = \frac{1}{\det(H)} \begin{bmatrix} 16 & 1.25 \\ 1.25 & 10 \end{bmatrix}$$

Where (  $\det(H) = 10 \times 16 - (-1.25)^2 = 160 - 1.5625 = 158.4375$  ).

So,

$$H^{-1} = \frac{1}{158.4375} \begin{bmatrix} 16 & 1.25 \\ 1.25 & 10 \end{bmatrix} = \begin{bmatrix} 0.1009 & 0.0079 \\ 0.0079 & 0.0631 \end{bmatrix}$$

Now, let's perform the first iteration. The initial point is (  $X_0 = (-10, 16)$  ). The gradient at (  $X_0$  ) is:

$$\nabla f(X_0) = [2 \times 5 \times (-10) - 0.25 \times 8 \times 16 - 0.5 \times 5.2 \times 8 \times 16 - 0.25 \times 5 \times (-10) - 0.5 \times 8]$$

$$\nabla f(X_0) = [-100 - 32 - 25.256 + 1.25 - 4]$$

$$\nabla f(X_0) = [-134.5, 253.25]$$

The Newton step is:

$$X_1 = X_0 - H^{-1} \nabla f(X_0)$$

$$X_1 = \begin{bmatrix} -10 \\ 16 \end{bmatrix} - \begin{bmatrix} 0.1009 & 0.0079 \\ 0.0079 & 0.0631 \end{bmatrix} \begin{bmatrix} -134.5 \\ 253.25 \end{bmatrix}$$

$$X_1 = \begin{bmatrix} -10 \\ 16 \end{bmatrix} - \begin{bmatrix} 0.1009 \times (-134.5) + 0.0079 \times 253.25 \\ 0.0079 \times (-134.5) + 0.0631 \times 253.25 \end{bmatrix}$$

$$X_1 = \begin{bmatrix} -10 \\ 16 \end{bmatrix} - \begin{bmatrix} -13.57 + 2.00 \\ -1.06 + 15.98 \end{bmatrix}$$

$$X_1 = \begin{bmatrix} -10 \\ 16 \end{bmatrix} - \begin{bmatrix} -11.57 \\ 14.92 \end{bmatrix}$$

$$X_1 = \begin{bmatrix} 1.57 \\ 1.08 \end{bmatrix}$$

The norm of the gradient, which you'll use as a stopping criterion for the Newton method, is calculated after each iteration. It's the square root of the sum of the squares of the components of the gradient vector. For the function (  $f(x_1, x_2)$  ), the norm of the gradient (  $\|\nabla f\|$  ) is given by:

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x_1}\right)^2 + \left(\frac{\partial f}{\partial x_2}\right)^2}$$

After the first iteration, we calculated the gradient at (  $X_0$  ) to be (  $\nabla f(X_0) = [-134.5, 253.25]$  ). To find the norm of this gradient:

$$\|\nabla f(X_0)\| = \sqrt{(-134.5)^2 + (253.25)^2}$$

Now, let's calculate it:

$$\|\nabla f(X_0)\| = \sqrt{(18090.25) + (64136.5625)}$$

$$\|\nabla f(X_0)\| = \sqrt{82226.8125}$$

$$\|\nabla f(X_0)\| \approx 286.74$$

This value is much larger than (  $10^{-5}$  ), so you would proceed with the next iteration. After each iteration, you will recalculate the gradient at the new point (  $X_k$  ), and then compute the norm of the gradient again. Once the norm of the gradient is less than (  $10^{-5}$  ), you can stop the iterations as you've reached the convergence according to the Kuhn-Tucker condition.

Remember, the norm provides a measure of how close you are to a point where the function does not increase or decrease, which in optimization problems, typically indicates a minimum or maximum if one exists. Keep iterating until the norm of the gradient is less than the threshold you've set.