Maxwell equation simulations.

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Abstract

Simulating Maxwell equations numerically, following *(2019, Houle and Sullivan) "Electromagnetic Simulation Using the FDTD Method with Python"* [1].

Contents

1 Chapter 1

Wiki on the FDTD method.
Base formulas:

$$\frac{\partial E}{\partial t} = \frac{1}{\epsilon_0}\nabla \times H,$$
$$\frac{\partial H}{\partial t} = -\frac{1}{\mu_0}\nabla \times E.$$

Note: ∇ in Cartesian coordinates:

$$\nabla \times \mathbf{F} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\mathbf{i} + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right)\mathbf{j} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\mathbf{k}$$

where $\mathbf{F}$ is a vector field, and $\mathbf{i}, \mathbf{j}, \mathbf{k}$ are unit vectors in the x, y, and z directions, respectively.

Below are the equations of a plane wave traveling in the z direction with the electric field oriented in the x direction and the magnetic field oriented in the y direction

$$\frac{\partial E_x}{\partial t} = -\frac{1}{\epsilon_0}\frac{\partial H_y}{\partial z},$$
$$\frac{\partial H_y}{\partial t} = -\frac{1}{\mu_0}\frac{\partial E_x}{\partial z}.$$

After the central difference approximation:

$$\frac{E_x^{n+1/2}(k) - E_x^{n-1/2}(k)}{\Delta t} = -\frac{1}{\epsilon_0}\frac{H_y^n(k+1/2) - H_y^n(k-1/2)}{\Delta x},$$
$$\frac{H_y^{n+1}(k+1/2) - H_y^n(k+1/2)}{\Delta t} = -\frac{1}{\mu_0}\frac{E_x^{n+1/2}(k+1) - E_x^{n+1/2}(k)}{\Delta x}.$$

Note: a comment from [1]: It might seem more sensible to use Δz as the incremental step because in this case we are going in the z direction. However, Δx is so commonly used for a spatial increment that we will use Δx.

The formulation of the above equations assume that the E and H fields are interleaved in both space and time. H uses the arguments $k + 1/2$ and $k - 1/2$ to indicate that the H field values are assumed to be located between the E field values. Similarly, the $n + 1/2$ or $n - 1/2$ superscript indicates that it occurs slightly after or before n, respectively.

Python

```
import numpy as np
from math import exp
from matplotlib import pyplot as plt
```

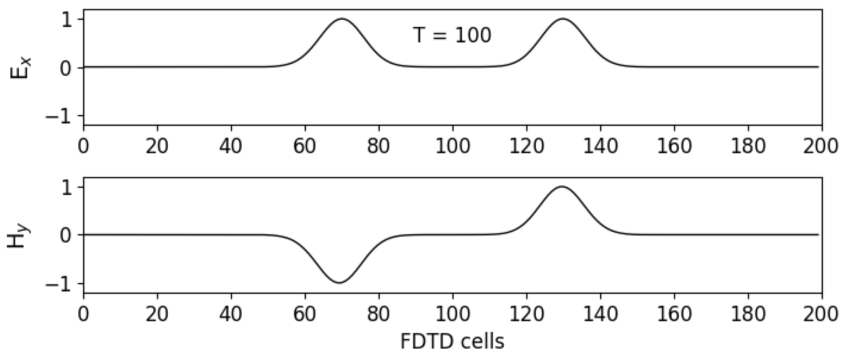
Python

```
ke = 200
ex = np.zeros(ke)
hy = np.zeros(ke)
# Pulse parameters
kc = int(ke / 2)
t0 = 40
spread = 12

nsteps = 100
# Main FDTD Loop
for time_step in range(1, nsteps + 1):
  # Calculate the Ex field
  for k in range(1, ke):
    ex[k] = ex[k] + 0.5 * (hy[k - 1] - hy[k])
  # Put a Gaussian pulse in the middle
  pulse = exp(-0.5 * ((t0 - time_step) / spread) ** 2)
  ex[kc] = pulse
  # Calculate the Hy field
  for k in range(ke - 1):
    hy[k] = hy[k] + 0.5 * (ex[k] - ex[k + 1])

# Plot the outputs as shown in Fig. 1.2
plt.rcParams['font.size'] = 12
plt.figure(figsize=(8, 3.5))
plt.subplot(211)
plt.plot(ex, color='k', linewidth=1)
plt.ylabel('E$_x$', fontsize='14')
plt.xticks(np.arange(0, 201, step=20))
plt.xlim(0, 200)
plt.yticks(np.arange(-1, 1.2, step=1))
plt.ylim(-1.2, 1.2)
plt.text(100, 0.5, 'T = {}'.format(time_step),
horizontalalignment='center')
plt.subplot(212)
```

```
plt.plot(hy, color='k', linewidth=1)
plt.ylabel('H$_y$', fontsize='14')
plt.xlabel('FDTD cells')
plt.xticks(np.arange(0, 201, step=20))
plt.xlim(0, 200)
plt.yticks(np.arange(-1, 1.2, step=1))
plt.ylim(-1.2, 1.2)
plt.subplots_adjust(bottom=0.2, hspace=0.45)
plt.savefig('img/maxwell-sim-1.png')
```



2 Discussion

User:

Hi model. Please, describe the below Maxwell's equations. What is H, what is E. Please define the ∇ operator.

$$\frac{\partial E}{\partial t} = \frac{1}{\epsilon_0}\nabla \times H,$$
$$\frac{\partial H}{\partial t} = -\frac{1}{\mu_0}\nabla \times E.$$

AI:

Hi! Let's explore the Maxwell's equations you've provided, which describe the fundamental behavior of electric and magnetic fields.

The equations are:

$$\frac{\partial E}{\partial t} = \frac{1}{\epsilon_0}\nabla \times H,$$
$$\frac{\partial H}{\partial t} = -\frac{1}{\mu_0}\nabla \times E.$$

Here's a breakdown of the components involved:

- **E (Electric Field):**
  - Denoted by E, it represents the electric field vector, a fundamental field that describes the force experienced by a charged particle in space.
  - It is measured in volts per meter (V/m) and describes the magnitude and direction of the electric force per unit charge.
- **H (Magnetic Field Intensity):**
  - Denoted by H, it represents the magnetic field intensity vector. It's related to the magnetic field B through the material's magnetic properties.
  - It is measured in amperes per meter (A/m) and is often used in disciplines involving magnetized materials.
- **The ∇ Operator (Nabla or Del):**
  - The ∇ symbol is a vector differential operator known as the del or nabla operator.
  - When applied to a vector field (like H or E) in the form $\nabla\times$, it represents the curl of the field.
  - The curl of a vector field gives a measure of the field's tendency to circulate around a point, effectively capturing the rotation or swirling of the field.
  - For example, in Cartesian coordinates:

$$\nabla \times \mathbf{F} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\mathbf{i} + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right)\mathbf{j} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\mathbf{k}$$

    where $\mathbf{F}$ is a vector field, and $\mathbf{i}, \mathbf{j}, \mathbf{k}$ are unit vectors in the x, y, and z directions, respectively.
- **Constants:**
  - ϵ_0 is the permittivity of free space, a measure of how an electric field affects and is affected by a medium. Its approximate value is 8.854×10^{-12} F/m (farads per meter).
  - μ_0 is the permeability of free space, indicating the ability of a medium to support the formation of a magnetic field. Its value is approximately $4\pi \times 10^{-7}$ H/m (henrys per meter).

These equations describe the dynamics of electromagnetic fields in space, with the curl operations indicating how changes in one type of field (electric or magnetic) can generate or be related to changes in the other field over time. This intertwined behavior of E and H is critical in explaining electromagnetic wave propagation, transmission lines, and antennas, among other phenomena.

References

[1] Jennifer E. Houle and Dennis M. Sullivan. *Electromagnetic Simulation Using the FDTD Method with Python*. John Wiley & Sons, Ltd, 2019. ISBN: 9781119565826. DOI: https://doi.org/10.1002/9781119565826. URL: https://onlinelibrary.wiley.com/doi/abs/10.1002/9781119565826.