

Chernoff Bound VS Quantum Threshold Theorem

Sergei Mironov

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Abstract

Foo Baaar

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1 Setup

AI

/reset

You are a helpful assistant. You generate responses using Latex formatting by default. Thus, whenever you want to create a Python code, you wrap it into a "python" Latex environment (i.e `\verb|begin{<LANG>}|` end `\verb|{\<LANG>}|` tags where LANG is "python"). Rather than the default Markdown formatting. And don't wrap it into equation `\verb|\\[|` or `\verb|\\]|` tags! When you generate a plain text, escape any undersores character with a backslash, like this: `\verb|_|`. For verbatim text, Instead of ``text`` use `\\verb|text|`. For bold text, instead of `\verb|**text**|` use `\textbf{text}`. For italic text, instead of `\verb|*text*|` use `\\textit{text}`. For better paragraph separation you can use `\verb|\\vsp|` tag (vertical space). For numbered lists, use `\verb|\\begin{enumerate}| ... \\verb|\\item| .. \\verb|\\item| ... \\verb|\\end{enumerate}|`. For lists, use `\verb|\\begin{itemize}| ... \\verb|\\item| .. \\verb|\\item| ... \\verb|\\end{itemize}|`. `\verb|\\problem|` and `\verb|\\ans|` just print these words in bold. Always consider that one should always be able to paste the text you generate between `\verb|\\begin{document}|` and `\verb|\\end{document}|`. For tikz pictures, use just `\verb|\\begin{...}| .. \\verb|\\end{...}|` without additional verbatim markup. Never use `\verb|````|` as in Markdown! For Python code, use 2-space indentation.

/read model prompt

/echo Done

Result

Done

Hello! How can I assist you today? If you have any questions or need help with something specific, feel free to let me know.

2 Conversation

User:

Hi model. Please tell compare the **Chernoff bound** and **Quantum Threshold Theorem**. How do they justify the sampling protocol used in quantum computations?

AI:

The Chernoff bound and the Quantum Threshold Theorem are important concepts in probability theory and quantum computing, respectively. Let's explore each one and how they are related to quantum sampling protocols.

1. Chernoff Bound:

The Chernoff bound is a probabilistic bound that provides an exponentially decreasing bound on the tail distributions of sum of independent random variables. It is particularly useful in analyzing scenarios where you need to understand how unlikely it is for the sum of random variables to deviate significantly from its expected value. The bound states that for independent random variables X_1, X_2, \dots, X_n , if X_i takes values in $[0, 1]$, then:

$$\Pr\left(\frac{1}{n} \sum_{i=1}^n X_i \geq (1 + \delta)\mu\right) \leq e^{-\delta^2 \mu n / 3}$$

for $\delta > 0$, where μ is the expected value of the sum. This bound justifies that with sufficient samples, the empirical mean will be close to the expected mean.

2. Quantum Threshold Theorem:

The Quantum Threshold Theorem is fundamental to quantum error correction. It states that as long as the error rate per operation is below a certain threshold, quantum computations can be made arbitrarily reliable by applying quantum error correcting codes. Specifically, it indicates that:

- Quantum errors can be corrected if they are sufficiently rare.
- There is a known threshold p_{th} such that, if the probability of error per gate is less than p_{th} , the computation can be made resilient to errors using error correction.

Relation to Sampling in Quantum Computations:

In quantum computations, sampling is often used to estimate the expected outcomes of quantum operations and verify the fidelity of quantum processes.

- **Chernoff Bound in Quantum Sampling:** When measuring quantum states, outcomes are probabilistic, similar to flipping a biased coin. The Chernoff bound provides confidence that the sample mean (or the observed distribution of measurement outcomes) closely approximates the

true distribution after a sufficient number of measurements, under the assumption of independent noise.

- **Quantum Threshold Theorem in Error Correction:** When implementing any real-world quantum computation, errors occur. The Quantum Threshold Theorem assures us that these sampling-based protocols yield reliable results as long as errors are corrected efficiently, maintaining fault-tolerance beneath the threshold.

Together, these concepts justify the reliability and feasibility of using sampling protocols in quantum computations, even in the presence of noise and errors, as long as appropriate error-correction and sufficient sample sizes are ensured.