

Mechanical Bloch equations.

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February 14, 2025

Abstract

Walking through (2014, Frimmer and Novotny) “The classical Bloch equations” [1]¹ referencing (2010, Smith) “Waves and Oscillations: A Prelude to Quantum Mechanics” [2].

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1 Setup

Python

```
import pennylane as qml
import numpy as np
import matplotlib.pyplot as plt
qml.drawer.use_style("black_white")
```

Result

¹Preprint: <https://arxiv.org/abs/1410.0710>

2 Conversation

The main motion equations are:

$$\begin{aligned} m \frac{d^2 x_A}{dt^2} + m\gamma \frac{dx_A}{dt} + (k + \kappa - \Delta k(t)) x_A - \kappa x_B &= F(t) \\ m \frac{d^2 x_B}{dt^2} + m\gamma \frac{dx_B}{dt} + (k + \kappa + \Delta k(t)) x_B - \kappa x_A &= 0 \end{aligned}$$

Dividing by m .

$$\begin{aligned} \frac{d^2 x_A}{dt^2} + \gamma \frac{dx_A}{dt} + \left(\frac{k + \kappa}{m} - \frac{\Delta k(t)}{m} \right) x_A - \frac{\kappa}{m} x_B &= \frac{1}{m} F(t) \\ \frac{d^2 x_B}{dt^2} + \gamma \frac{dx_B}{dt} + \left(\frac{k + \kappa}{m} + \frac{\Delta k(t)}{m} \right) x_B - \frac{\kappa}{m} x_A &= 0 \end{aligned}$$

Introducing Ω frequencies:

$$\Omega_0^2 = [k + \kappa]/m$$

$$\Omega_d^2 = \Delta k/m$$

$$\Omega_c^2 = \kappa/m$$

becomes

$$\left(\frac{d^2}{dt^2} + \gamma \frac{d}{dt} + \Omega_0^2 \right) \begin{pmatrix} x_A \\ x_B \end{pmatrix} + \begin{pmatrix} -\Omega_d^2 & -\Omega_c^2 \\ -\Omega_c^2 & \Omega_d^2 \end{pmatrix} \begin{pmatrix} x_A \\ x_B \end{pmatrix} = \begin{pmatrix} F(t) \\ 0 \end{pmatrix}$$

User:

Please rewrite the below two equation into the matrix form. I'd like to be like $(\dots) (\mathbf{x}_A ; \mathbf{x}_B) + (\dots) (\mathbf{x}_A ; \mathbf{x}_B) =$

$$\begin{aligned} \frac{d^2 x_A}{dt^2} + \gamma \frac{dx_A}{dt} + \left(\frac{k + \kappa}{m} - \frac{\Delta k(t)}{m} \right) x_A - \frac{\kappa}{m} x_B &= \frac{1}{m} F(t) \\ \frac{d^2 x_B}{dt^2} + \gamma \frac{dx_B}{dt} + \left(\frac{k + \kappa}{m} + \frac{\Delta k(t)}{m} \right) x_B - \frac{\kappa}{m} x_A &= 0 \end{aligned}$$

Introducing Ω frequencies:

$$\Omega_0^2 = [k + \kappa]/m$$

$$\Omega_d^2 = \Delta k/m$$

$$\Omega_c^2 = \kappa/m$$

AI:

To rewrite the given equations in matrix form, we'll utilize the newly introduced frequencies Ω_0 , Ω_d , and Ω_c . We then represent the system using matrix operations on a vector composed of x_A and x_B . The two equations provided are:

$$\begin{aligned}\frac{d^2 x_A}{dt^2} + \gamma \frac{dx_A}{dt} + \left(\frac{k + \kappa}{m} - \frac{\Delta k(t)}{m} \right) x_A - \frac{k}{m} x_B &= \frac{1}{m} F(t) \\ \frac{d^2 x_B}{dt^2} + \gamma \frac{dx_B}{dt} + \left(\frac{k + \kappa}{m} + \frac{\Delta k(t)}{m} \right) x_B - \frac{k}{m} x_A &= 0\end{aligned}$$

Using the new frequency parameters, the equations become:

$$\begin{aligned}\frac{d^2 x_A}{dt^2} + \gamma \frac{dx_A}{dt} + (\Omega_0^2 - \Omega_d^2) x_A - \frac{k}{m} x_B &= \frac{1}{m} F(t) \\ \frac{d^2 x_B}{dt^2} + \gamma \frac{dx_B}{dt} + (\Omega_0^2 + \Omega_d^2) x_B - \frac{k}{m} x_A &= 0\end{aligned}$$

To express these in a matrix form:

$$\begin{pmatrix} \frac{d^2}{dt^2} + \gamma \frac{d}{dt} + (\Omega_0^2 - \Omega_d^2) & -\frac{k}{m} \\ -\frac{k}{m} & \frac{d^2}{dt^2} + \gamma \frac{d}{dt} + (\Omega_0^2 + \Omega_d^2) \end{pmatrix} \begin{pmatrix} x_A \\ x_B \end{pmatrix} = \begin{pmatrix} \frac{1}{m} F(t) \\ 0 \end{pmatrix}$$

This representation condenses the system of differential equations into a single matrix equation, allowing easier manipulation and understanding of the coupled dynamics.

References

- [1] Martin Frimmer and Lukas Novotny. "The classical Bloch equations". In: *American Journal of Physics* 82.10 (Oct. 2014), pp. 947–954. ISSN: 1943-2909. DOI: [10.1119/1.4878621](https://doi.org/10.1119/1.4878621). URL: <http://dx.doi.org/10.1119/1.4878621>.
- [2] W.F. Smith. *Waves and Oscillations: A Prelude to Quantum Mechanics*. Oxford University Press, 2010. ISBN: 9780199742127. URL: <https://api.semanticscholar.org/CorpusID:118019131>.