Quantum lock in PennyLane

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Abstract

Walking through the quantum lock problem described in one of PennyLane tutorials.

Contents

1 Setup

We are walking through the Building a quantum lock using phase kickback tutorial by Danial Motlagh.

Python

```
import pennylane as qml
import numpy as np
import matplotlib.pyplot as plt
qml.drawer.use_style("black_white")
```

Result

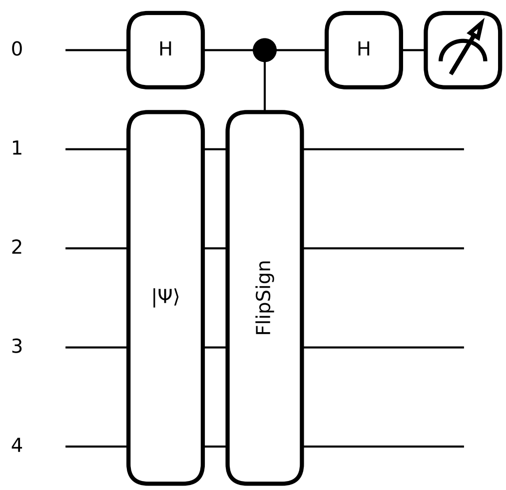
2 Implementation

Python

```
import pennylane as qml
import numpy as np

num_wires = 5
dev = qml.device("default.qubit", wires=num_wires, shots=1)
```

```python
def quantum_lock(secret_key):
  return qml.FlipSign(secret_key, wires=list(range(1, num_wires)))

def build_key(key):
  return qml.BasisState(key, wires=list(range(1, num_wires)))

@qml.qnode(dev)
def quantum_locking_mechanism(lock, key):
  build_key(key)
  qml.Hadamard(wires=0)  # Hadamard on ancilla qubit
  qml.ctrl(lock, control=0)  # Controlled unitary operation
  qml.Hadamard(wires=0)  # Hadamard again on ancilla qubit
  return qml.sample(wires=0)

def check_key(lock, key):
  if quantum_locking_mechanism(lock, key) == 1:
    print("Great job, you have uncovered the mysteries of the quantum universe!")
  else:
    print("Nice try, but that's not the right key!")

qml.draw_mpl(quantum_locking_mechanism)(lock, secret_key)
plt.savefig('img/fig.pdf')
```

Result



Running the experiment

Python

```python
secret_key = np.array([0, 1, 1, 1])
lock = quantum_lock(secret_key)
```

```
check_key(lock, secret_key)
```

```
Result

Great job, you have uncovered the mysteries of the quantum universe!
```

3 Discussion

User:

Hi Ai. Lets discuss this quantum program in PennyLane. It illustrates the Phase kickback.

```python
num_wires = 5
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  return qml.sample(wires=0)
```

Can you try to generate an equation corresponding to this circuit?

AI:

Sure, let's analyze the quantum program you've presented and express its operations as an equation. This code is implementing a variant of the phase kickback mechanism using controlled operations. We'll break down the steps:

1. **Initialization**: It initializes the key via the `BasisState` operation.
2. **Hadamard Gate**: It applies a Hadamard gate to the first qubit, putting it into superposition.
3. **Controlled Operation**: It applies a controlled operation (`ctrl`) which depends on the secret lock function.
4. **Final Hadamard Gate**: It applies another Hadamard gate to the first qubit before sampling it.

The circuit can be described by the following sequence of operations:

$$\text{Hadamard} \rightarrow \text{Controlled-Lock} \rightarrow \text{Hadamard}$$

Given that the *quantum lock* is represented by a `FlipSign` operation, we can express the effect on the initial state of the system. Using $|\psi\rangle$ to denote the initial state of the ancillary qubits representing the key, and assuming the control qubit is initially in the state $|0\rangle$, the transformation is given by:

$$\text{Initial State: } |0\rangle \otimes |\psi\rangle$$

Applying the Hadamard on the control qubit:

$$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

Controlled operation with `FlipSign`, dependent on whether the control qubit is $|1\rangle$:

$$\frac{1}{\sqrt{2}}(|0\rangle \otimes |\psi\rangle + |1\rangle \otimes \text{FlipSign}(|\psi\rangle))$$

Finally, applying the Hadamard on the control qubit to complete the phase kickback process:

$$H\left(\frac{1}{\sqrt{2}}(|0\rangle \otimes |\psi\rangle + |1\rangle \otimes \text{FlipSign}(|\psi\rangle))\right)$$

When we sample, we effectively measure the final state of the control qubit, which gives us insight into the key.

This equation outlines the process described by the program in PennyLane, depicting how the initial state is manipulated to exhibit phase kickback.

User:

Can you break down the last equation, namely:

$$H\left(\frac{1}{\sqrt{2}}(|0\rangle \otimes |\psi\rangle + |1\rangle \otimes \text{FlipSign}(|\psi\rangle))\right)$$

I'd like you to apply this Hadamard to the first qubit and write down the result

AI:

Certainly! Let's break down the equation step by step by applying the Hadamard gate to the control qubit component inside the expression:

Our initial expression is:

$$H\left(\frac{1}{\sqrt{2}}\left(|0\rangle \otimes |\psi\rangle + |1\rangle \otimes \text{FlipSign}(|\psi\rangle)\right)\right)$$

The Hadamard gate H applied on a single qubit can be represented as:

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Applying this Hadamard gate to our expression means we are operating on the control qubit states $|0\rangle$ and $|1\rangle$:

$$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

Substituting these transformations into our original expression, we get:

$$= \frac{1}{\sqrt{2}} \left(H|0\rangle \otimes |\psi\rangle + H|1\rangle \otimes \text{FlipSign}(|\psi\rangle) \right)$$

Applying the transformations:

$$= \frac{1}{\sqrt{2}} \left(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |\psi\rangle + \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \otimes \text{FlipSign}(|\psi\rangle) \right)$$

Simplifying further:

$$= \frac{1}{2} \left(|0\rangle \otimes |\psi\rangle + |1\rangle \otimes |\psi\rangle + |0\rangle \otimes \text{FlipSign}(|\psi\rangle) - |1\rangle \otimes \text{FlipSign}(|\psi\rangle) \right)$$

Finally, collecting like terms:

$$= \frac{1}{2} \left(|0\rangle \otimes (|\psi\rangle + \text{FlipSign}(|\psi\rangle)) + |1\rangle \otimes (|\psi\rangle - \text{FlipSign}(|\psi\rangle)) \right)$$

This is the result after applying the Hadamard to the first qubit in your equation. The outcome will depend on the nature of the *FlipSign* operation applied to $|\psi\rangle$.

3.1 The case of matching key

User:

Great! Now consider that *FlipSign* changes the sign of the state to the opposite: $|\psi\rangle \mapsto -|\psi\rangle$.

AI:

If the *FlipSign* operation changes the sign of the state, i.e.,

$$\text{FlipSign}(|\psi\rangle) \mapsto -|\psi\rangle$$

we can substitute this into our expression that we derived:

The modified expression becomes:

$$= \frac{1}{2} \left(|0\rangle \otimes (|\psi\rangle + (-|\psi\rangle)) + |1\rangle \otimes (|\psi\rangle - (-|\psi\rangle)) \right)$$

Simplifying each term:
- The first term simplifies to:

$$|0\rangle \otimes (|\psi\rangle - |\psi\rangle) = |0\rangle \otimes 0$$

- The second term simplifies to:

$$|1\rangle \otimes (|\psi\rangle + |\psi\rangle) = |1\rangle \otimes 2|\psi\rangle$$

The entire expression becomes:

$$= \frac{1}{2} (|0\rangle \otimes 0 + |1\rangle \otimes 2|\psi\rangle)$$

$$= |1\rangle \otimes |\psi\rangle$$

Thus, after applying the Hadamard to the first qubit and considering the *FlipSign* operation, the final state is $|1\rangle \otimes |\psi\rangle$. This indicates that the effect of the *FlipSign* operation is crucial in determining the final measured state.

3.2 The case of mismatching key

User:

Great! Now lets take a step back and consider that $FlipSign$ just returns the original state.

AI:

If the *FlipSign* operation does not alter the state, meaning:

$$\text{FlipSign}(|\psi\rangle) \mapsto |\psi\rangle$$

we can substitute this into our previously derived expression:

The original expression becomes:

$$= \frac{1}{2} (|0\rangle \otimes (|\psi\rangle + |\psi\rangle) + |1\rangle \otimes (|\psi\rangle - |\psi\rangle))$$

Simplifying each term:
- The first term simplifies to:

$$|0\rangle \otimes (|\psi\rangle + |\psi\rangle) = |0\rangle \otimes 2|\psi\rangle$$

- The second term simplifies to:

$$|1\rangle \otimes (|\psi\rangle - |\psi\rangle) = |1\rangle \otimes 0$$

The entire expression becomes:

$$= \frac{1}{2} (|0\rangle \otimes 2|\psi\rangle + |1\rangle \otimes 0)$$

$$= |0\rangle \otimes |\psi\rangle$$

Thus, when *FlipSign* returns the original state, the final state after applying the Hadamard to the first qubit is $|0\rangle \otimes |\psi\rangle$. This demonstrates how the absence of sign change affects the resulting quantum state.