Solutions

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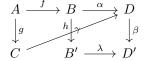
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1 Introduction

The book (2013, Spivak) "Category theory for scientists" [1].



Chapter 2. The Category of Sets

2.4 Products and coproducts

2.4.1.4

How many elements does the $\{a,b,c,d\}*\{1,2,3\}$ have?

Answer: 12

2.4.1.8

Answer:

- (a) No, because $a(b+c) \neq (a+b)c$.
- (b) No, because $x * 0 \neq x$.
- (c) Yes.

2.4.1.15

(a) Let X and Y be sets.. construct the "swap map" $s:(X\times Y)\to (Y\times X)$

Answer: $s: (X \times Y) \to (Y \times X) = (,) \circ \langle \pi_2, \pi_1 \rangle$ Note: we used angle brackets. Is it really correct?

(b) Can you prove that s is a isomorphism using only the universal property for product?

Note: $(f: X \to Y)$ is an isomorphism if $\exists (g: Y \to X) : g \circ f = id_X \land f \circ g = id_Y$

Note: diagram (f, g, h) commutes if $f \circ g = h$

Answer:

In universal property of products, put A equal to $Y \times X$ and get $\exists !g: (Y \times X) \to (X \times Y)$. In a similar way, we have $\exists !s: (X \times Y) \to (Y \times X)$. We need to show that $g \circ s = id_{(X \times Y)}$ and $s \circ g = id_{(Y \times X)}$. TODO

References

[1] David I. Spivak. "Category theory for scientists". In: (2013). URL: https://api.semanticscholar.org/CorpusID:126379073.