Solutions

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1 Introduction

The book *(2013, Spivak) "Category theory for scientists"* [1].

$$\begin{tikzcd}
A \arrow[r, "f"] \arrow[d, "g"] & B \arrow[r, "\alpha"] \arrow[d, "h"'] & D \arrow[d, "\beta"] \\
C \arrow[urr, "\gamma"'] & B' \arrow[r, "\lambda"] & D'
\end{tikzcd}$$

Chapter 2. The Category of Sets

2.4 Products and coproducts

2.4.1.4

How many elements does the $\{a, b, c, d\} * \{1, 2, 3\}$ have?

Answer: 12

2.4.1.8

Answer:

(a) No, because $a(b + c) \neq (a + b)c$.

(b) No, because $x * 0 \neq x$.

(c) Yes.

2.4.1.15

(a) Let X and Y be sets.. construct the "swap map" $s : (X \times Y) \to (Y \times X)$

Answer: $s : (X \times Y) \to (Y \times X) = (,) \circ \langle \pi_2, \pi_1 \rangle$
Note: we used angle brackets. Is it really correct?

(b) Can you prove that s is a isomorphism using only the universal property for product?
Note: $(f : X \to Y)$ is an isomorphism if $\exists (g : Y \to X) : g \circ f = id_X \wedge f \circ g = id_Y$
Note: diagram (f, g, h) commutes if $f \circ g = h$

Answer:
In universal property of products, put A equal to $Y \times X$ and get $\exists! g : (Y \times X) \to (X \times Y)$. In a similar way, we have $\exists! s : (X \times Y) \to (Y \times X)$. We need to show that $g \circ s = id_{(X \times Y)}$ and $s \circ g = id_{(Y \times X)}$.
TODO

References

[1] David I. Spivak. "Category theory for scientists". In: (2013). URL: https://api.semanticscholar.org/CorpusID:126379073.