Fracture design application

Description

This application helps to calculate **the optimal fracture parameters** so that the well productivity is maximum for a given mass of injected proppant for a given reservoir and proppant properties. In addition, **the actual proppant placement** in the fracture is determined depending on the technological limitations. Besides, the parameters of **treatment mode** are calculated.

The calculations in the application are based on the **Unified Fracture Design** methodology proposed by M. Economides. This methodology allows us to evaluate hydraulic fracturing parameters in high-permeability and low-permeability reservoirs.

1. Input data

M_{prop}	Proppant mass (for 2 wings), kg – the most important variable for characterizing the size of fracturing
ρ_{prop}	Specific gravity of proppant material (for water = 1)
m_{prop}	Porosity of proppant pack - depends on closure stress
k_f	Proppant pack permeability, mD - depends on closure stress and the extent of fracture contamination by residual fracture fluid
$D_{p\,max}$	Max proppant diameter, mm – depends on the mesh size
k	Formation permeability, mD
h_p	Permeable (leakoff) thickness, m – used in calculation of productivity index (and volumetric efficiency of proppant) and apparent leakage coefficient
r_w	Well radius, m – needed to calculate the pseudo skin factor
r_e	Well drainage radius, m
s_f	Pre-treatment skin factor
h_f	Fracture height, m – one of the key parameters for hydraulic fracturing planning
E'	Plane strain modulus, atm – defined by Young's modulus and Poisson's ratio
q_i	Slurry injection rate (two wings, liquid + proppant), m^3/min
K'	Rheology, $(N/m^2) \cdot$ s – consistency index of the injected fluid
n'	The index of the power law of the flow behavior

C_L	Leakoff coefficient in permeable layer, m $\cdot$ $min^{1/2}$
S_p	Spurt loss coefficient, m^3/m^2
c_{max}	Max possible added proppant concentration, kg/m^3 neat fluid – the main limitation set by the equipment
K_{long}	Multiply optimal length by factor
K_{Nolt}	Multiply Nolte pad by factor

2. Optimization of fracture parameters

Fracture length and *dimensionless fracture conductivity* are the main variables determining the productivity index of a stimulated well. The dimensionless fracture conductivity C_{fD} is the ability of the fracture to transmit reservoir fluid to the wellbore divided by the ability of the formation to transmit fluid to the fracture:

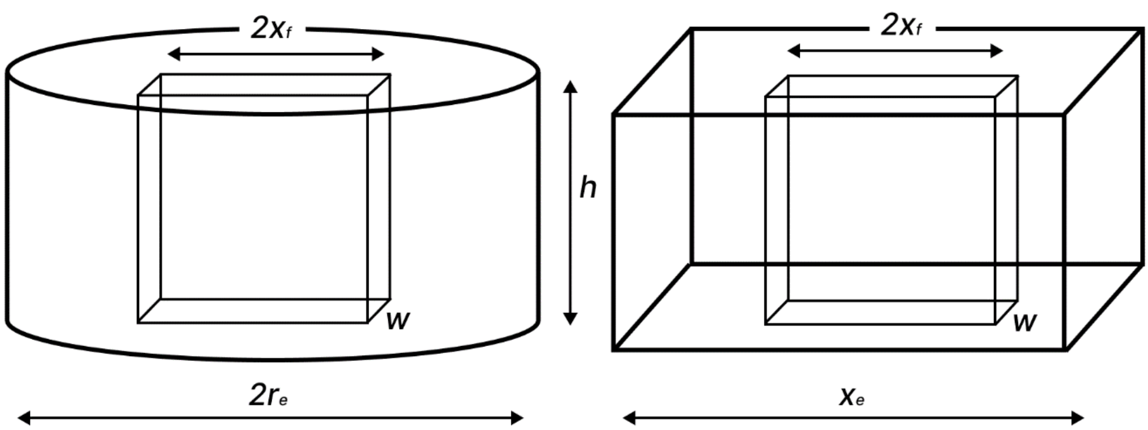
$$C_{fD} = \frac{k_f \cdot w}{k \cdot x_f} \tag{2.1}$$

where: x_f – fracture half length, w – fracture width, k – formation permeability, k_f – proppant pack permeability.

In the well-fracture-formation system, a fracture is considered to penetrate the entire pay zone. In reality, the drainage area is not circular or rectangular, but this approximation is sufficient for most of them.

The relationship between drainage area A, drainage radius r_e and lateral length x_e is given by the relation:

$$A = \pi r_e^2 = x_e^2 \tag{2.2}$$



The fracture penetration in the *x* direction is defined as:

$$I_x = \frac{2x_f}{x_e} \tag{2.3}$$

where x_f – fracture half length, x_e – side length of the rectangular drainage area.

To determine the necessary relationships between fracture length and fracture width when planning fracturing, the most important parameter is *the dimensionless proppant number* N_{prop}, which links the fracture penetration and dimensionless fracture conductivity:

$$N_{prop} = I_x^2 \cdot C_{fD} \tag{2.4}$$

$$N_{prop} = \frac{2k_f}{k} \cdot \frac{V_{prop}}{V_{res}} \tag{2.5}$$

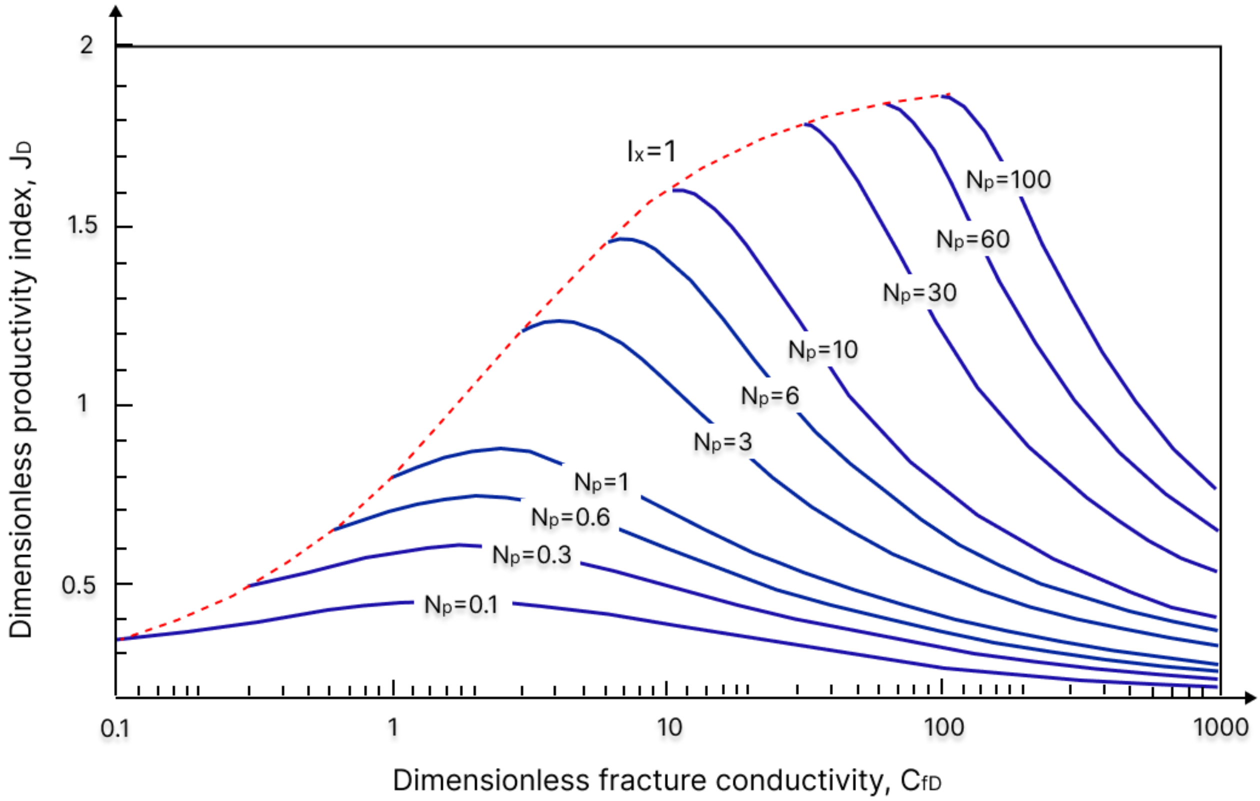
The volume of the fracture:

$$V_{prop} = \frac{M_{prop} \cdot r_p}{1000 \cdot \rho_{prop} \cdot (1 - m_{prop})} \tag{2.6}$$

$$r_p = \frac{h_p}{h_f} \tag{2.7}$$

The purpose of optimization is to determine such parameters of the fracture, so that *the dimensionless productivity index of the well* J_d at a given dimensionless proppant number N_{prop} (equivalent to the proppant mass M_{prop}) would have the maximum value.

The dimensionless productivity of the well is a function of the dimensionless proppant number and dimensionless fracture conductivity: $J_d^{opt} = f(N_{prop}, C_{fD}^{opt})$.



Optimal half length:

$$x_f = \left(\frac{V_f k_f}{C_{fD,opt} h k} \right)^{1/2} \tag{2.8}$$

where V_f is the volume of one of the fracture wings (inside the pay zone), m^3

Optimal propped width:

$$w = \left(\frac{C_{fD,opt} V_f k}{h k_f} \right)^{1/2} = \frac{V_f}{x_f h} \tag{2.9}$$

3. Fracturing efficiency evaluation

Hydraulic fracturing increases the productivity index and, accordingly, leads to a decrease in the skin effect. In hydraulic fracturing, it is more correct to talk about the pseudo-skin effect, since in addition to eliminating contamination of the bottom-hole zone, new channels of fluid flow are created and old ones are expanded.

The unified fracture design uses the dimensionless productivity index J_d as a measure of treatment efficiency. The change in this variable demonstrates the real impact of a propped fracture on well performance.

For a well located in the center of circular drainage area, the expression for the dimensionless productivity index for pseudostationary flow is as follows:

$$J_d = \frac{1}{\ln\left[\frac{0{,}472\ r_e}{r_w}\right] + s_f} = \frac{1}{\ln\left[\frac{0{,}472\ r_e}{r'_w}\right]} \tag{3.1}$$

where r_e – well drainage radius, r_w – well radius, s_f – pseudo-skin, r'_w – effective well radius ($r'_w = e^{-s_f}$).

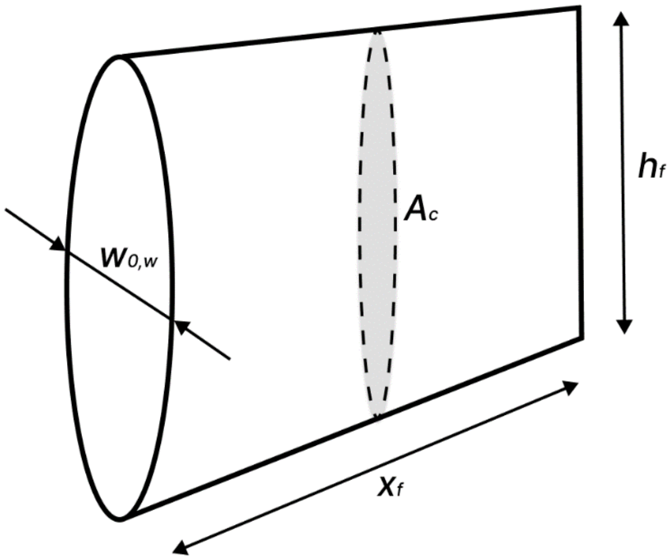
The multiplicity of well productivity increase after hydraulic fracturing is calculated through the ratio of dimensionless well productivity before and after the treatment.

4. Actual proppant placement

Technological and economic limitations often do not allow to achieve optimal proppant placement in the fracture. This is related to the capacity of pumping units, fracture fluid properties (maximum possible proppant concentration in the fluid, rheology parameters), etc.

In the fracture design application, the PKN (Perkins-Kern-Nordgren) model was used as a fracture propagation model.

The PKN model assumes that the plane deformation state is characteristic of all planes perpendicular to the fracture propagation direction. However, the stresses and pressures in these planes are not always assumed to be the same. It is assumed that the plane is vertical and runs normal to the direction of fracture propagation. The pressure component directed vertically is neglected. The net pressure p_n is considered as a function of the horizontal coordinate *x*. In the vertical direction, the pressure is assumed constant in a given coordinate *x* and defines an elliptical section.



Maximum width of the ellipse:

$$w_0 = \frac{2h_f p_n}{E'} \tag{4.1}$$

The model assumes that the net pressure is 0 at the end of the fracture, and the average linear velocity of the fluid is estimated as the ratio of the injection rate q_i to the cross-section area.

Pressure loss equation:

$$\frac{dp_n}{dx} = \frac{-4\mu q_i}{\pi w_0^3 h_f} \tag{4.2}$$

Fracture width:

$$w_0(x) = w_{w,0}\left(1 - \frac{x}{x_f}\right)^{1/4} \tag{4.3}$$

The maximum width of the ellipse in the bottomhole zone is given by the expression:

$$w_0(x) = 3{,}57\left(\frac{\mu q_i x_f}{E'}\right)^{1/4} \tag{4.4}$$

From the value of the fracture width in the bottomhole zone, we can calculate the average fracture width by multiplying it by a constant shape factor γ:

$$\bar{w} = \gamma w_{w,0} \tag{4.5}$$

where $\gamma = \frac{\pi}{4}\frac{4}{5} = \frac{\pi}{5} = 0{,}628$

The shape factor has two components. The first is equal to $\frac{\pi}{4}$ and considers the ellipticity of the vertical shape. The second is equal to $\frac{4}{5}$, considers the change in the maximum width in the horizontal direction.

Maximum proppant concentration, kg/m^3 of slurry:

$$c_{e\,max} = \frac{c_{max}}{1 + \frac{c_{max}}{1000 \cdot \rho_{prop}}} \tag{4.6}$$

Concentration of proppant in the fluid in case all proppant is placed in the fracture, kg/m^3 of slurry:

$$c_e = \frac{M_{prop}}{h_f \cdot \bar{w} \cdot 2x_f \cdot K_{long}} \tag{4.7}$$

where $\bar{w}$ - average fracture width, m; h_f - fracture height, m; x_f – fracture length, m; K_{long} - optimal length increase factor.

If $c_e > c_{e\,max}$, optimal proppant placement in the fracture is impossible. Pumped proppant mass for 2 wings of the fracture:

$$M_{act} = M_{prop} \cdot \frac{c_{e\,max}}{c_e} \tag{4.8}$$

5. Treatment mode

The actual leakoff of the fracture fluid into the formation is determined by both the filtration crust and the fluid flow conditions in the formation.

The leak rate υ_L is given by Carter's I equation:

$$\upsilon_L = \frac{C_L}{\sqrt{t}} \tag{5.1}$$

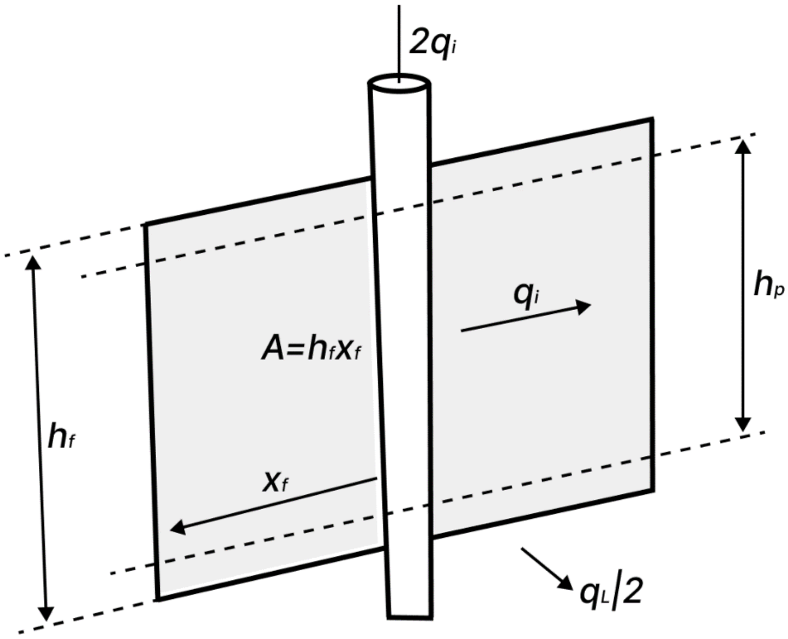
where C_L is the leakoff coefficient in permeable layer (m/s$^{1/2}$), t is the time from the beginning of the leakoff process.

Integral form of the Carter equation:

$$\frac{V_{lost}}{A_L} = 2C_L\sqrt{t} + S_p \tag{5.2}$$

where V_{lost} is the volume of fluid passing through the area A_L for the time period from 0 to t, S_p is the spurt loss coefficient.

The spurt loss coefficient can be considered as the fluid flow that goes instantly through the area at the beginning of the process of leakoff into the formation. The coefficients C_L and S_p can be calculated from laboratory experiments or from the interpretation of a calibration test.



The fluid injection time t_e is determined from the equation:

$$\frac{q_i t_i}{h_f x_f} - 2K_L C_L\sqrt{t} - (\overline{w_e} + 2S_p) = 0 \tag{5.3}$$

Injection fluid volume:

$$V_i = q_i t_e \tag{5.4}$$

Fluid efficiency:

$$\eta_i = \frac{h_f x_f \overline{w_e}}{V_i} \tag{5.5}$$

For a given injection time and slurry volume, a stepped pump schedule is required to obtain a proppant-packed fracture design. The fluid pumped without proppant at the beginning of the operation is called "pad". It initiates and widens the fracture. After the "pad" is pumped, the proppant concentration in the slurry increases in steps up to the maximum value that is reached at the end of the treatment.

Exponent of added proppant concentration of the injected proppant:

$$\varepsilon = \frac{1-\eta_e}{1+\eta_e} \tag{5.6}$$

The volume of the "pad" and the time for its pumping:

$$V_{pad} = \varepsilon V_i \tag{5.7}$$

$$t_{pad} = \varepsilon t_e \tag{5.8}$$

The required concentration (mass per unit volume of injected slurry):

$$c = c_e \left(\frac{t - t_{pad}}{t_e - t_{pad}}\right)^{\varepsilon} \tag{5.9}$$

where c_e is the maximum proppant concentration in the injected slurry at the end of the treatment.

Areal concentration of proppant after fracture closure:

$$c_s = \frac{M_{act}}{h_f \cdot 2x_f} \tag{5.10}$$