Nonconvexity certificate in $b_i = 0$ case

Cases

Let $f\colon \mathbb{R}^n \to \mathbb{R}^m$ be a quadratic map: $f_i(x) = x^T A_i x$, $A_i = A_i^T$.
Consider $c \in \mathbb{R}^m$ and boundary points ∂F_c:

$$c \cdot f(x) \to \min_c$$

Where $A = \sum c_i A_i$. Assuming $A \geqslant 0$.
Minimization leads to $Ax = 0$. The following cases hold:

1. $RgA = n$. $x = 0$ is a unique solution
2. $RgA = n - 1$. $x = \alpha e$, $f(x) = \alpha^2 f(e)$
3. $RgA = n - 2$. In this case $x = \alpha_1 e^1 + \alpha_2 e^2$. Consider $f(x) = \alpha_1^2 f_{11} + 2\alpha_1\alpha_2 f_{12} + \alpha_2^2 f_{22}$.
   (a) f_{11}, f_{22}, f_{12} are linearly independent. In this case ∂F_c is nonconvex
   (b) $Rg||f_{11} f_{12} f_{22}|| = 1$. ∂F_c convex.

Result

If exist $c \in \mathbb{R}^m$:

1. $RgA = n - 2$
2. $A \geqslant 0$
3. f_{ij} are linearly independent

Then $F = \mathrm{Im} f$ is nonconvex.