Questions

Abstract

Stupid obstacles I can't overcome.

1 c_{bad} and function $z(c)$

Definition of c_{bad} contains two conditions. Why $z(c)$ is finite only for $c \in c_{bad}$? It seems that $z(c)$ is finite when pseudo-inverse matrix can be calculated. It means that $z(c)$ is defined for a larger set rather than c_{bad}. Is it true?

2 Case $\mathbb{R}^3 \to \mathbb{R}^3$ with zero vector n

Consider artificial toy example of the mapping

$$y_i(x) = x^T A_i x + 2x^T b_i,$$

$$A_1 = \sqrt{2}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}, \quad b_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad A_2 = \sqrt{2}\begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -0.5 \end{bmatrix}, \quad b_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},$$
$$A_3 = \sqrt{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}, \quad b_3 = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}. \tag{1}$$

For $c_+ = \frac{1}{\sqrt{2}}[1;0;1]$ we have $A_{c+} = I$, $b_{c+} = 0$, so we are in the general framework.

The matrix $A_c = \sum c_i A_i$ is positive semidefinite for all $c(t) = [1;t;t^2]$ and the set of vectors c_{bad} is parameterized as:

$$c_{bad} = \frac{1}{\sqrt{1+t^2+t^4}} \begin{bmatrix} 1 \\ t \\ t^2 \end{bmatrix} \tag{2}$$

This example allows us to validate formulas for $\nabla z \doteq \dfrac{dz}{dc}$, $\dot{A}^{-1}$ e t.c.

The set c_{bad} is plotted on the sphere, red line corresponds to the direction c_+.

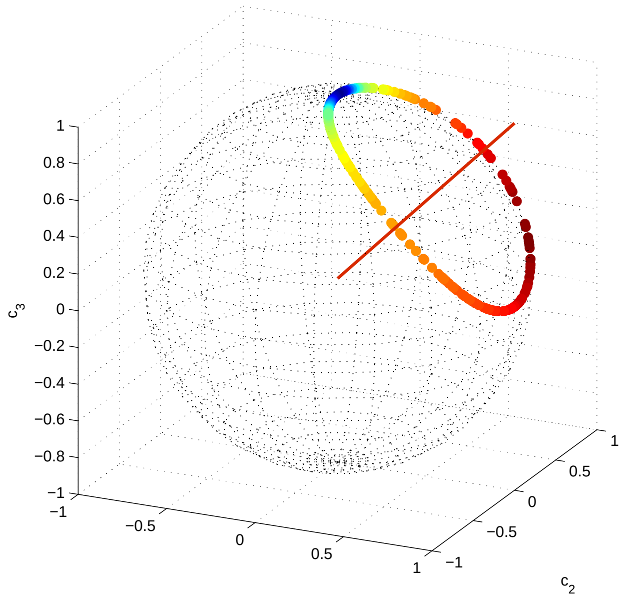

Figure 1: c_{bad} for $\mathbb{R}^3 \to \mathbb{R}^3$ example.

Vectors c_{bad} are not in the plane orthogonal to c_+. I do not understand why $n = 0$, n being the projection direction (0.14) from your draft? What's wrong with this example? Where does «degeneracy» of the $\mathbb{R}^3 \to \mathbb{R}^3$ case appear?

3 ∇z is not orthogonal to c_+

You mention that it must be but it is not. Why it must be?

In the example above the gradient of $z(c)$ is

$$\nabla z = \frac{\sqrt{t^4+t^2+1}}{(t^2-t+1)^3} \begin{bmatrix} t(-2t^3+t^2+2t-1) \\ (t^2-1)^2 \\ -t^3+2t^2+t-2 \end{bmatrix},$$

that is evidently is not orthogonal to $c_+ = [1; 0; 1]$.

4 Manual projection

I try to implement the procedure of making $c(\lambda) = c' + \lambda n$ to belong to c_{bad}, i.e. find λ such that

$$A \cdot c' + \lambda A \cdot n \succeq 0$$
$$(x, b \cdot c' + \lambda b \cdot n) = 0$$

The solutions of the first equations are among the generalized eigenvalues and can be calculated straightforward. But these λ do not satisfy the second equation. Then it looks like there are no c_{bad} in the direction n. Correct me if I'm wrong.