Draft.pdf

Equation 0.19

Consider $A = S\Lambda S^T$, $S^T S = E$, x_0 is a simple zero eigenvector of A: $Ax_0 = 0$, $||x_0|| = 1$.
$A^{-1} = S\Lambda_1 S^T$, $\Lambda = (\lambda_1, ..., \lambda_{n-1}, 0)$, $\Lambda_1 = (\lambda_1^{-1}, ..., \lambda_{n-1}^{-1}, 0)$.
Equation (0.19):

$$\dot{x_0} = -A^{-1}\dot{A}x_0$$

Consider $A^{-1}\dot{A}x_0 = S\Lambda_1 S^T(\dot{S}\Lambda S^T + S\dot{\Lambda}S^T + S\Lambda\dot{S}^T)x_0\boxed{=}$.
Consider $x_0 = Sy_0$, where $y_0 = (0, 0, ..., 1)$. Therefore, $0 = \dot{y}_0 = \dot{S}^T x_0 + S^T \dot{x}_0$
Going back to 0.19, the part $\Lambda S^T x_0 = \Lambda y_0 = 0$, another part $\dot{\Lambda} S^T x_0 = 0$. Consequently,
$\boxed{=}S\Lambda_1 S^T S\Lambda\dot{S}^T x_0 = S\Lambda_1\Lambda\dot{S}^T x_0 = -S\Lambda_1\Lambda S^T\dot{x}_0 = -\sum_{i=1}^{n-1} s_i s_i^T \dot{x}_0 = -(E - x_0 x_0^T)\dot{x}_0 = -\dot{x}_0 + x_0 x_0^T \dot{x}_0\boxed{=}$.
Taking a derivative $||x_0|| = 1$, we get $x_0^T \dot{x_0} = 0$, therefore,
$\boxed{=} - \dot{x}_0$