On the feasibility for the system of quadratic equations

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Outline

1. The Power Flow feasibility problem
2. The algorithm
3. An example
4. Further plans

Power Flow Feasibility problem:

1. Large-scale power grids
2. Need to know if a regime is «normal», «safe»
3. Ohm's law $\Rightarrow$ quadratic equations:

$$y_i = f_i(x) = x^T A_i x + 2b_i^T x$$

   y (regime) known, x is not
4. Determine if $\exists x\colon y = f(x)$ (means «safe»)

The image $f(\mathbb{R}^n)$ must be examined

The solution

Given: the map $f : \mathbb{R}^n \to \mathbb{R}^m$, $f_i(x) = x^T A_i x + 2b_i^T x$, $A_i^T = A_i$

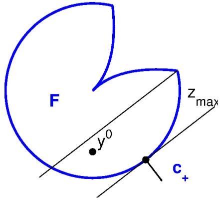
Proposed algorithm for examining $F = f(\mathbb{R}^n)$:

- **Input:** $y^0 \in F$, direction $c_+ : c_+ \cdot A > 0$
- **Output:** value $z_{\max}$ s.t. the cut $Q(c_+, z_{\max}, F)$ is convex

Solution overview:

1. Discovering boundary nonconvexities $\{F_i\}$ close to y^0
2. Projecting to c_+: (F_i, c_+)
3. Calculating $z_{\max} = \inf\limits_i \inf\limits_{y \in F_i} (c_+, y)$

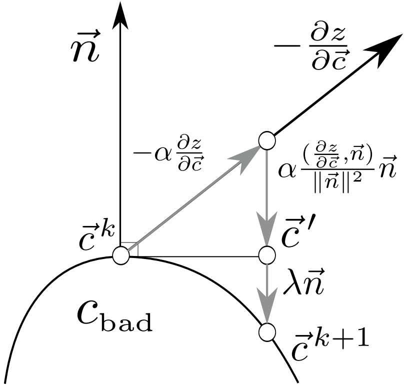
$Q(c_+, z, F) = \{y \big| (y - y^*, c_+) \in [0, z]\} \cap F$, y^* — touching point of hyperplane c_+



The solution

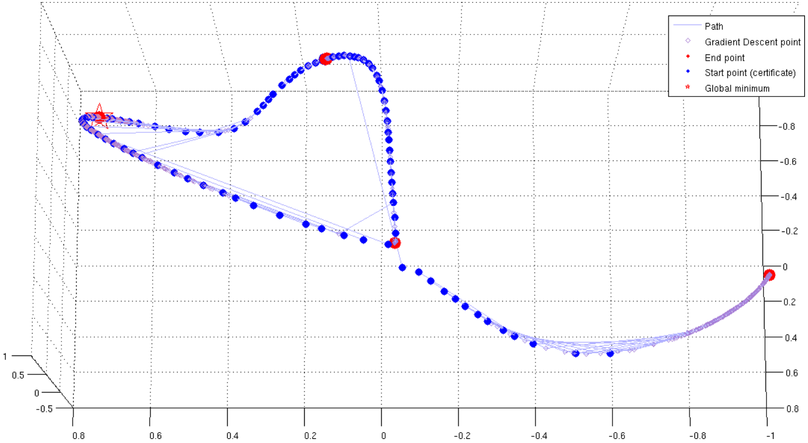
Infinite number of nonconvexities $\Rightarrow$

1. $z_{\max} = \inf\limits_{c \in c_{\mathrm{bad}}} z(c)$ — constrained optimization
2. Gradient projection method
3. Projection using $\vec{n} \perp c_{\mathrm{bad}}$



An example: $f : \mathbb{R}^4 \to \mathbb{R}^4$. Looking for $z_{\max} = \inf_{c \in c_{\text{bad}}} z(c)$

- 4 local minima
- Global minimum found



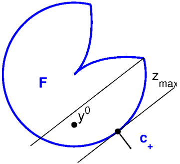
Results

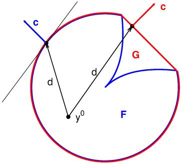
Summary

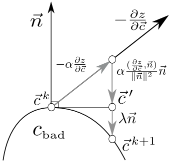
1. Power Flow feasibility
2. Algorithm cuts convex parts
3. Gradient projection method
4. Algorithm was tested on a number of maps f

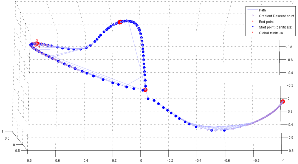
Plan: practical application

1. $\mathbb{C}$ case
2. Testing for higher dimensions









Thank you!
Questions?