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MECHANICS

Janibekov's Effect and the Laws of Mechanics

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INTRODUCTION

In the course of his space flight of 1985, two-time Hero of the USSR flier-cosmonaut V.A. Janibekov performed an important observation. He observed that under zero-gravity conditions, a twisted and left free nut began rectilinear motion accompanied by rotation about the nut axis. Then, suddenly and without an explicit reason, the nut axis changed its direction by 180°. These overturns repeated in identical intervals of time. The phenomenon seemed exciting and puzzling [1].

At that time, the scientific community found nothing new in this phenomenon, insofar as this was a simple case of the body's motion near an immobile point in the absence of an external moment of force. This case was well known under the name of the Eulerian case. It served as the best evident experiment confirming the theory. However, in newspapers, on TV, and on the Internet, there appeared various (including pseudo-scientific) explanations of the phenomenon under consideration. A significant splash of interest occurred in 2012, when the end of world was expected, and various versions for the development of events were analyzed. On May 13, 2012, on the official site of Roscosmos (Federal Space Agency) on the occasion of the 70th birthday of V.A. Janibekov, there appeared an article [2] containing the following statement: "In 1985, this cosmonaut made the unexpected discovery called Janibekov's effect."

Janibekov's rotating nut has caused astonishment and simultaneous danger to a certain part of the scientific world. A hypothesis was proposed that our planet in the course of its orbital motion can execute the same overturn. "For 10 years, Janibekov's effect was considered to be secret. However, the discovery of Janibekov became a push to the development of the quantum study of the macroscopic world." Soon, these words were repeated on the TV news.

There are many internet sites with similar discussions. However, there are also serious scientific considerations, e.g., numerical simulations of the nut motion, which are based on well-known Eulerian equations [3]. The complete solution to the problem is described in terms of elliptic functions, which, apparently, hampers understanding of this phenomenon. Below, we present a simple geometric interpretation of the motion observed by V.A. Janibekov. For understanding this theory, it is sufficient to know the corresponding equations within the limits of the initial course of mechanics, which is available for students of the first and second years of education at a technical university.

KINEMATIC AND DYNAMIC CHARACTERISTICS OF A SOLID

In Janibekov's experiment, no external forces acted on the body (nut). Therefore, the body's center of mass C moved uniformly and rectilinearly. We connect the origin of the inertial reference system to the center of mass. In this coordinate system, the body rotates about immobile point C. In addition to the inertial reference system, we introduce a mobile coordinate system ξ, η, ζ fixed to the body. Both systems have the same basis unit vectors $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$. Axis η and the corresponding basis vector $\mathbf{e}_2$ are directed along the nut axis. Axes ξ, η, ζ are the principal axes of the body's inertia tensor (Fig. 1).

We now present the general expressions for the angular-velocity $\boldsymbol{\omega}$, vector $\mathbf{K}$ of the angular momentum, and kinetic energy T:

$$\omega = p\mathbf{e}_1 + q\mathbf{e}_2 + r\mathbf{e}_3, \quad \mathbf{K} = X\mathbf{e}_1 + Y\mathbf{e}_2 + Z\mathbf{e}_3, \quad 2T = Xp + Yq + Zr, \tag{1}$$

where $X = Ap$, $Y = Bq$, and $Z = Cr$ are the components of $\mathbf{K}$, and A, B, C ($A > B > C$) are the body's moments of inertia.

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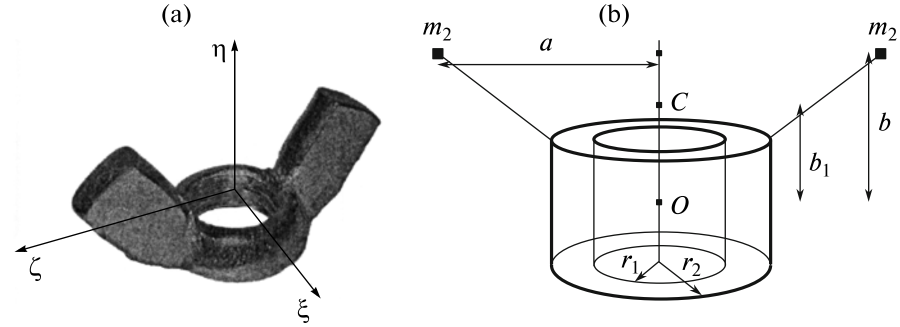

Fig. 1. Wing nut in Janibekov's experiment and simplified model for calculating moments of inertia.

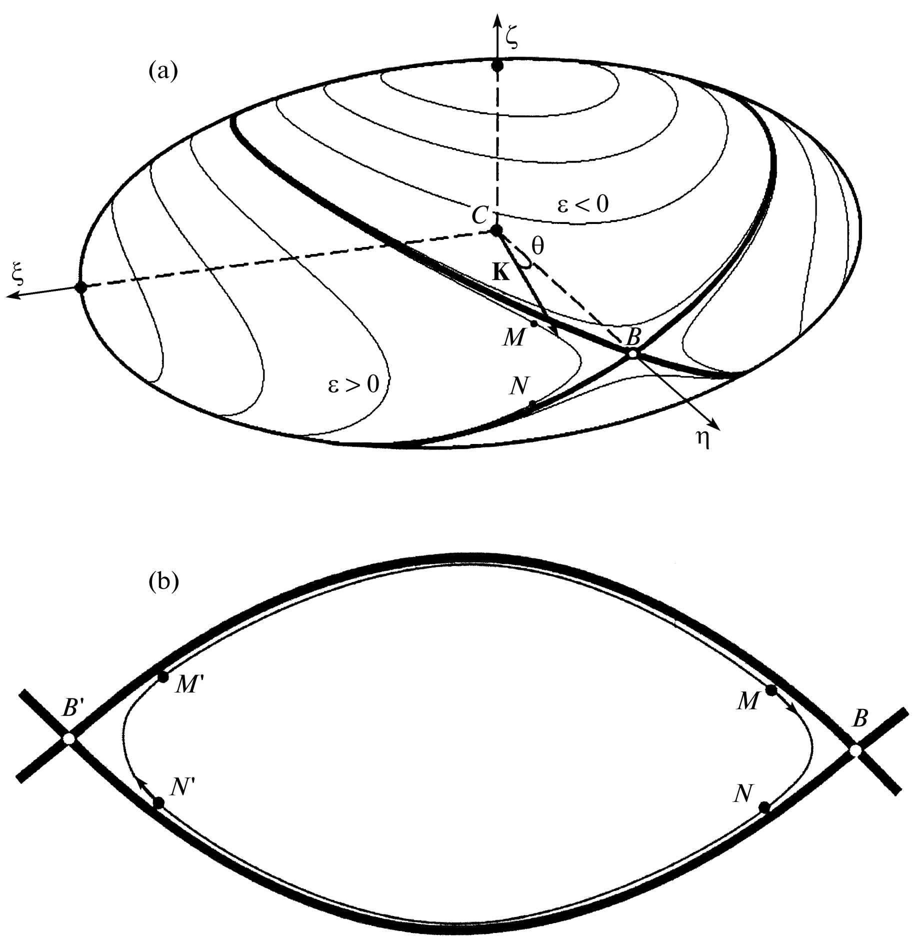

Fig. 2. MacCullagh ellipsoid and lines of its intersection with spheres of various radii. Separatrices are marked by thick solid lines. N' and M' (not shown in the figure) are the corresponding points on the other side of the ellipsoid. $MNN'M'$ is the trajectory of vector $\mathbf{K}$ for the angular momentum in mobile axes.

In the case of Janibekov's experiment, it is of interest to find the variation of the nutation angle θ between the immobile vector $\mathbf{K}$ and the $\mathbf{e}_2$ axis (Fig. 2a). Upon projecting vector $\mathbf{e}_2$ onto the vector $\mathbf{K}$, we arrive at the expression

$$\cos\theta = \frac{Y}{K}, \quad K = \sqrt{X^2 + Y^2 + Z^2}. \tag{2}$$

LAWS OF CONSERVATION

In the absence of external forces, vector $\mathbf{K}$ of the angular momentum is quiescent in space. At the same time, axes $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$ of the body are mobile. Therefore, the components of vector $\mathbf{K}$ in the mobile axes vary with time, but the length squared of the vector is conserved. In addition, the body's kinetic energy T is also

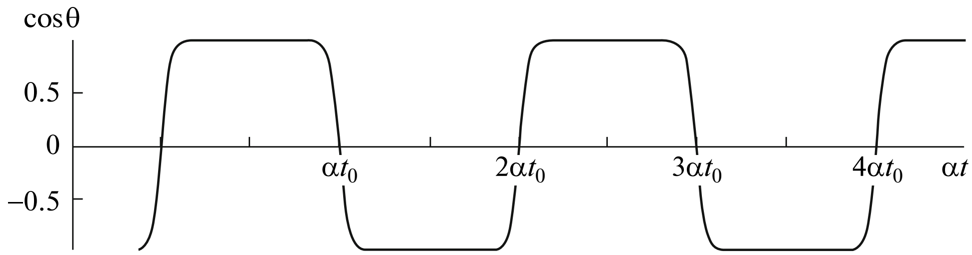

Fig. 3. Time variation of the angle θ between the mean axis and the immobile vector **K**.

conserved. These two laws of conservation are of the form

$$X^2 + Y^2 + Z^2 = K^2, \quad \frac{X^2}{A} + \frac{Y^2}{B} + \frac{Z^2}{C} = 2T. \qquad (3)$$

As follows from the equations, the trajectory being described by the vector **K** in mobile axes is the intersection of a sphere and an ellipsoid. This representation of the trajectory is called the MacCullagh geometric interpretation [4].

In Fig. 2, a family of trajectories for vector **K** on the MacCullagh ellipsoid is shown. As can be seen, if at the initial instant of time, vector **K** is located in the axis η with the mean value of the moment of inertia, then the trajectories of this vector are separatrices (thick solid lines). The motion occurs from pole B to opposite pole B'. The nutation angle varies from 0 to 180°. This is the motion that explains the overturn in the experiment executed by V.A. Janibekov.

If, at the initial instant of time, we impose vector **K** onto axes ξ or ζ corresponding to the large or the small axes of inertia, respectively, then vector **K** will not be displaced. In the case of small deviation of vector **K** from axes ξ or ζ, the vector describes a closed trajectory of a small radius. This effect was also demonstrated by V.A. Janibekov. He hit with a striker along the axis of the rotating gyroscope, and its axis virtually did not deviate. Our Earth rotates about its axis at the maximum value of the moment of inertia. Therefore, there is no danger for the Earth to execute an overturn.

Nevertheless, it is impossible to find the time dependence of the nutation angle solely on the basis of laws of conservation (3). To this end, we should allow for the equation governing the variation for one of the components of vector **K**.

THE MOTION ALONG THE SEPARATRIX

Let the angular velocity at the initial instant of time be directed along the $\mathbf{e}_2$ axis, $Y = K$, and $\frac{Y^2}{B} = 2T$ which implies $K^2 = 2TB$. We now multiply the second equation of (3) by B and then subtract from the first equation. As a result, we arrive at the equation describing two planes:

$$\left(1 - \frac{B}{X}\right)X^2 - \left(\frac{B}{C} - 1\right)Z^2 = 0$$

$$\Rightarrow X\sqrt{1 - \frac{B}{A}} \pm Z\sqrt{\frac{B}{C} - 1} = 0.$$

Thus, when the condition $K^2 = 2TB$ is valid, trajectories of vector **K** are intersections of the planes with the MacCullagh ellipsoid. These are four semicircles located between ellipsoid poles B and B', which are called separatrices. They are shown by thick lines in Fig. 2: (a) on the MacCullagh ellipsoid and (b) as a scan of the separatrices.

Upon projecting the law of conservation $\frac{d\mathbf{K}}{dt} + \omega \times K = 0$ for the angular momentum in mobile axes onto the $\mathbf{e}_2$ axis, we arrive at the expression

$$\frac{dY}{dt} = -(A - C)\frac{XZ}{AC}. \qquad (4)$$

We now substitute instead of X and Z, the expressions for components of the angular-momentum vector, which were found for laws of conservation (3). Thus, we have

$$X^2 = \frac{B - C}{A - C}\frac{A}{B}(K^2 - Y^2), \quad Z^2 = \frac{A - B}{A - C}\frac{C}{B}(K^2 - Y^2), \qquad (5)$$

from which follows the equation for the vector Y

$$\frac{dY}{dt} = \pm\frac{\alpha}{K}(K^2 - Y^2), \quad \alpha = \frac{K}{B}\sqrt{\frac{(A - B)(B - C)}{AC}}.$$

The solution to this equation for Y is expressed in terms of the hyperbolic tangent, and X, Y, and cosθ can be found from expressions (5) and (2) [5]:

$$X = \pm K\sqrt{\frac{(B - C)A}{B(A - C)}}\frac{1}{\cosh\alpha t}, \quad Z = \pm K\sqrt{\frac{(A - B)C}{B(A - C)}}\frac{1}{\cosh\alpha t},$$

$$Y = \pm K\tanh\alpha t, \quad \cos\theta = \pm\tanh\alpha t. \qquad (6)$$

As is seen from (6), the motion of vector **K** from pole B' to pole B occurs along one of four semicircles, with the nutation angle θ varying within the range $(0, \pi)$. In the case of the rigorous coincidence with a separatrix, this motion lasts an infinitely long time.

VARIATION OF THE NUTATION ANGLE IN THE CASE OF A SMALL DEVIATION FROM THE SEPARATRIX

In the case of a small deviation from the separatrix, which can be characterized by the parameter $\varepsilon = K^2 - 2TB$, and for finite time, vector $\mathbf{K}$ describes the closed trajectory $MNN'M'$ close to separatrices BB' (Fig. 2). Half of this time, i.e., the half-period t_0 of the body's motion, can be found in the manner described below.

Based on the laws (3) of conservation, we express X and Z in terms of Y,

$$X^2 = \frac{B-C}{A-C}\frac{A}{B}(K^2 - Y^2) + \frac{AC}{B(A-C)}\varepsilon,$$

$$Z^2 = \frac{A-B}{A-C}\frac{C}{B}(K^2 - Y^2) - \frac{AC}{B(A-C)}\varepsilon,$$

and substitute them into Eq. (4). Then, from Eq. (4), we find the half-period t_0 by the quadrature

$$t_0 = 2\frac{AC}{A-C}\int_0^{Y_0}\frac{dY}{X(Y)Z(Y)}. \tag{7}$$

Depending on the sign of ε, the upper limit of the integral Y_0 is found as a root of equations $X(Y) = 0$ or $Z(Y) = 0$. In order to calculate the half-period for $\varepsilon \to 0$, it is convenient to use the asymptotic relationship (see Appendix):

$$\alpha t_0 = \ln\frac{16}{|\mu|} + O(\mu\ln|\mu|),$$
$$\mu = \frac{\varepsilon(A-C)B}{K^2(A-B)(B-C)} + o(\varepsilon). \tag{8}$$

The time dependence of $\cos\theta$ can be combined on the basis of function (6) with allowance for the found half-period t_0, insofar as far from poles B and B', the motion depends weakly on ε. Within the range $t \in (-t_0, 5t_0)$, this dependence is of the form (see Fig. 3):

$$\cos\theta = \tanh\alpha t - \tanh\alpha(t - t_0) + \tanh\alpha(t - 2t_0)$$
$$- \tanh\alpha(t - 3t_0) + \tanh\alpha(t - 4t_0).$$

An observer residing in a quiescent space will see the periodic change in the direction of the body's mean axis to the opposite direction. Insofar as the velocity of the end of vector K near the poles is close to zero, the mean axis is delayed for a sufficiently long time near the pole (arc MN in Fig. 2).

In the course of passing by arc NN', the overturn occurs and $\cos\theta$ varies from the value close to 1 to that close to -1. Further, we consider conditionally that $\cos\theta = 0.9$ at point N and $\cos\theta = -0.9$ at the symmetric point N'. The time of the overturn τ in the motion along the arc NN' when $\cos\theta$ varies from 0.9 to -0.9 can be found from the solution to the equation $\tanh\frac{\alpha\tau}{2} = 0.9$. This value is approximately equal to $\tau = \frac{3}{\alpha}$. For small μ, the ratio of τ to the half-period of the motion attains the value $\frac{\tau}{t_0} = 3\left(\ln\frac{16}{|\mu|}\right)^{-1}$.

Let, at the initial instant of time, the body be twisted about the mean principal axis at an angular velocity q_0 along the axis η with small random deviations p_0, r_0 ($p_0^2 + r_0^2 \ll q_0^2$) with respect to two other axes. In this case, for the angular-momentum modulus and for parameters ε and μ, we have

$$K = Bq_0, \quad \varepsilon = A(A-B)p_0^2 - C(B-C)r_0^2,$$

$$\mu = \frac{A-C}{B}\left(\frac{A}{B-C}\frac{p_0^2}{q_0^2} - \frac{C}{A-B}\frac{r_0^2}{q_0^2}\right).$$

In accordance with the observed value $\frac{\tau}{t_0}$, we can estimate the initial deviation of the angular-velocity direction from the principal axis. For example, if the ratio of the overturn time to the half-period is $\frac{\tau}{t_0} = 0.4$, then using the calculated moments of inertia (see Appendix) and under the assumption $r_0 = 0$, we obtain $\mu \approx 0.009$ and $\frac{p_0}{q_0} \approx 0.08$.

APPENDIX

Calculation of principal moments of inertia. For calculating the principal moments of inertia approximately, we replace the nut under study with a hollow cylinder of external and internal radii r_1 and r_2, respectively. The height of the cylinder is h, and its mass is m_1. In addition, the cylinder is supplied with two point masses m_2 located at a distance a from the cylinder axis symmetrically with respect to it. The base of the perpendicular to the cylinder axis, which originates at point mass m_2, is at a distance b from the cylinder center of mass. Masses m_2 are fixed to the cylinder by rods whose masses are negligibly small (Fig. 1b).

The center of mass C of this mechanical system is located at a distance b_1 from the cylinder center of mass O:

$$b_1 = OC = \frac{2m_2}{2m_2 + m_1} b,$$

where $m_1 = \rho\pi h(r_2^2 - r_1^2)$. We also place the origin of the coordinate system ξ, η, ζ into the cylinder center of mass. The axis η is directed along the cylinder axis, whereas the axis ζ is located in the plane formed by both the cylinder axis and the point masses m_2.

Upon determining the moments of inertia with respect to the ξ, η, ζ axes of the cylinder

$$A_1 = C_1 = \frac{m_1}{4}\left(r_1^2 + r_2^2 + \frac{h^2}{3} + 4b_1^2\right),$$

$$B_1 = \frac{m_1}{2}(r_1^2 + r_2^2)$$

and to two masses m_2

$$A_2 = 2m_2(a^2 + (b - b_1)^2), \quad B_2 = 2m_2 a^2,$$

$$C_2 = 2m_2(b - b_1)^2,$$

we find the principal moments of inertia for the entire system:

$$A = A_1 + A_2, \quad B = B_1 + B_2, \quad C = C_1 + C_2.$$

In our calculations, we take $\rho = 7800$ kg m^{-3}, $r_1 = 5 \times 10^{-3}$ m, $r_2 = 7 \times 10^{-3}$ m, $h = 5 \times 10^{-3}$ m, $a = 10^{-2}$ m, $b = 8 \times 10^{-3}$ m. Thus, we find $b_1 = 5 \times 10^{-3}$ m and $m_1 = 3 \times 10^{-3}$ kg. We also assume that $m_2 = m_1$. Upon calculating, we arrive at $A = 8 \times 10^{-7}$ kg m^2, $B = 7 \times 10^{-7}$ kg m^2, $C = 2 \times 10^{-7}$ kg m^2.

Calculation of the period. The sign of ε determines the shape of the curve described by the vector **K**. For the positive and negative signs, this curve is closed around the axes ξ and ζ, respectively. Therefore, in the case of $\varepsilon > 0$ and $\varepsilon < 0$, time attains a quarter of period for $Z(Y) = 0$ and $X(Y) = 0$, respectively. We now consider the first case $\varepsilon > 0$. By changing the variable $Y = x\sqrt{K^2 - \frac{A}{A-B}\varepsilon}$, integral (7) for the half-period is reduced to the form

$$\alpha t_0 = 2\sqrt{1 + \frac{B-C}{A-C}\mu}\int_0^1 \frac{dx}{\sqrt{(1-x^2)(1+\mu-x^2)}},$$

where μ is expressed in terms of parameter ε:

$$\mu = \frac{(A-C)B\varepsilon}{(B-C)((A-B)K^2 - A\varepsilon)}.$$

For this integral, we can also obtain the asymptotic expansion over the small parameter $\mu \to +0$ and find on this basis the asymptotic expansion for the half-period

$$\alpha t_0 = \ln\frac{16}{\mu}\left[1 + \left(\frac{1}{2\ln\frac{16}{\mu}} - \frac{A - 2B + C}{4(A-C)}\right)\mu + O(\mu^2)\right].$$

The basic asymptotic term in this two-term expression coincides with (8), whereas the second term in brackets determines the relative error. Similarly, by changing $Y = x\sqrt{K^2 + \frac{C}{B-C}\varepsilon}$, we find the asymptotic form of (8) for $\varepsilon < 0$.

Thus, the motion accompanied by periodic overturns is described by Eulerian equations obtained and analyzed by Leonard Euler more than 250 years ago. In the conditions of zero gravity, this effect is manifested most explicitly, which attracted the attention of cosmonaut V.A. Janibekov. Here, we encounter no new effect and no new physical laws are required to explain this form of motion. As far as an Earth's overturn is concerned, the danger does not exist. The Earth's` axial rotation occurs with the largest moment of inertia and is stable.

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