NORMAL DISTRIBUTION (GAUSSIAN DISTRIBUTION)

Presented by Group 6

OUR TEAM

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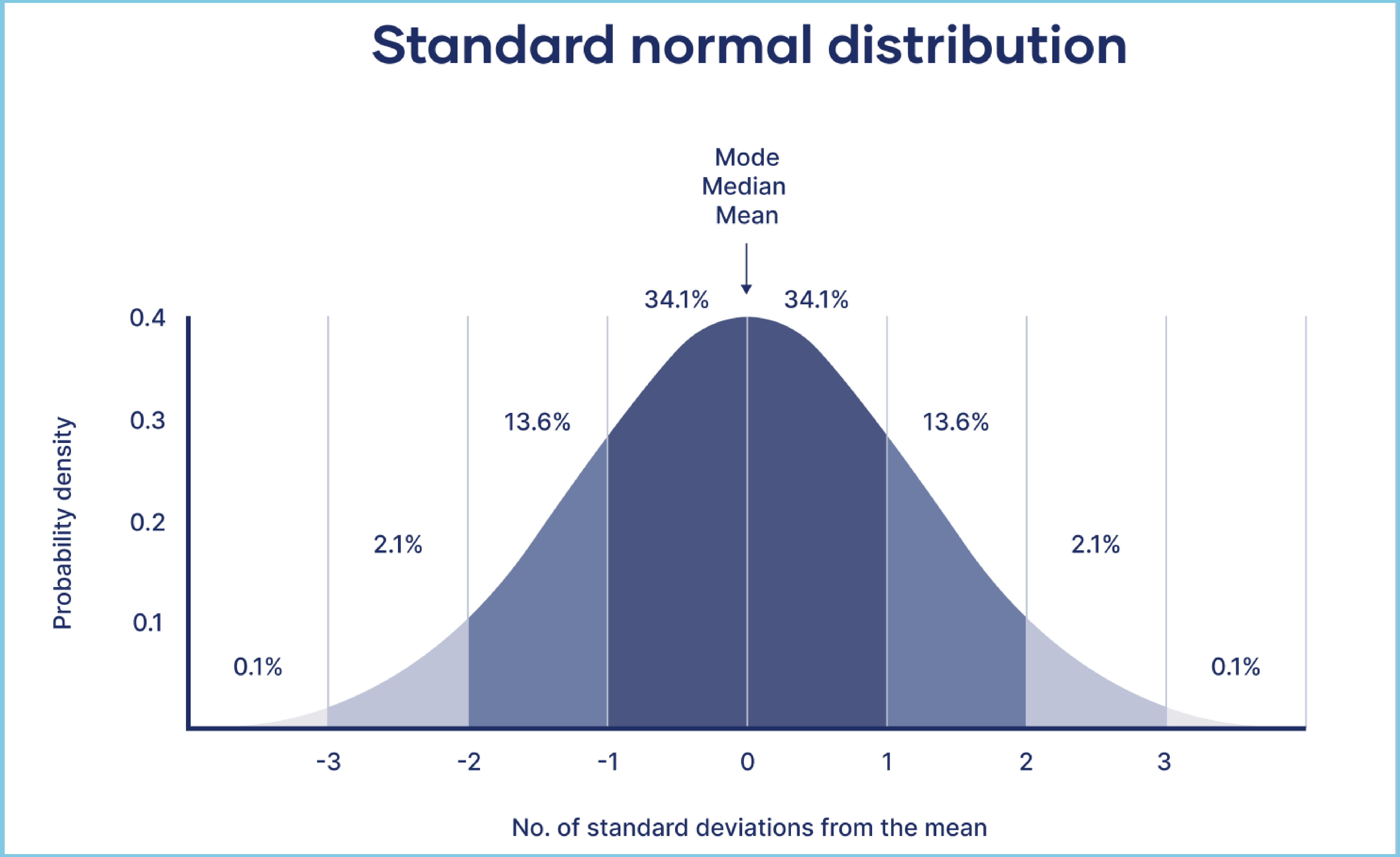
WHAT IS GAUSSIAN DISTRIBUTION ?

The Normal distribution, also known as the Gaussian distribution, is a continuous probability distribution characterized by its bell-shaped curve. The Normal distribution is significant in statistics due to the Central Limit Theorem,

Gaussian Distribution diagram



RELEVANT FORMULA

The probability density function (PDF) of the Gaussian distribution is:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} .$$

Where:

- f(x) = probability density at value xxx,
- μ = mean (center of the distribution),
- σ = standard deviation (spread of the distribution),
- e = Euler's number (approximately 2.71828),
- π = Pi (approximately 3.14159).

EXAMPLE APPLICATION: MACHINE LEARNING COURSE GRADES

Suppose the grades of machine learning students at ALU are normally distributed with a mean (μ) of 75 and a standard deviation (σ) of 10. The university wants to understand how their students are performing by determining the percentage of students who scored above 85 and those who scored below 65.

Steps for calculations

1. $X \sim N(\mu = 75, \sigma = 10)$

where :

$\mu = 75$ is the mean (average grade)

$\sigma = 10$ is the standard deviation

2. Calculating cumulative distribution

$$P(X < x) = \frac{1}{2}\left[1 + erf\left(\frac{x - \mu}{\sigma\sqrt{2}}\right)\right]$$

where : erf = error function.

Calculating $P(X < 85)$

Standardizing the value

$$z = \frac{x - \mu}{\sigma} = \frac{85 - 75}{10} = \frac{10}{10} = 1$$

For $z = 1$, error function value is approximately : 0,8427

$$erf(1) \approx 0{,}8427$$

⇒ CDF

$$P(X < 85) = \tfrac{1}{2}[1 + 0{,}8427]$$
$$\approx \tfrac{1}{2} \times 1{,}8427$$
$$\approx 0{,}92135$$

⇒ Calculating Probability of scoring above 85

$$P(X > 85) = 1 - P(X < 85)$$
$$= 1 - 0{,}92135$$
$$\approx 0{,}07865$$

This implies that approximately 7,87% of students scored above 85.

4. Calculating $P(X < 65)$

1. z-score for 65

$$z = \frac{65 - 75}{10} = \frac{-10}{10} = -1$$

2. Using CDF for $z = -1$

$$erf(-1) \approx -0{,}8427$$

3. Calculating the CDF

$$P(X < 65) = \tfrac{1}{2}[1 + erf(-1)]$$
$$= \tfrac{1}{2}[1 - 0{,}8427]$$
$$\approx \tfrac{1}{2} \times 0{,}1573$$
$$\approx 0{,}07865$$

Final Probabilities

1. The probability of scoring above 85

$P(X > 85) \approx 0{,}07865$

$P(X < 65) \approx 0{,}15866$

CONCLUSION

The Normal distribution plays a key role in statistics, supported by the Central Limit Theorem, which shows sample means tend to follow a normal distribution.

references

- normal distribution explaination