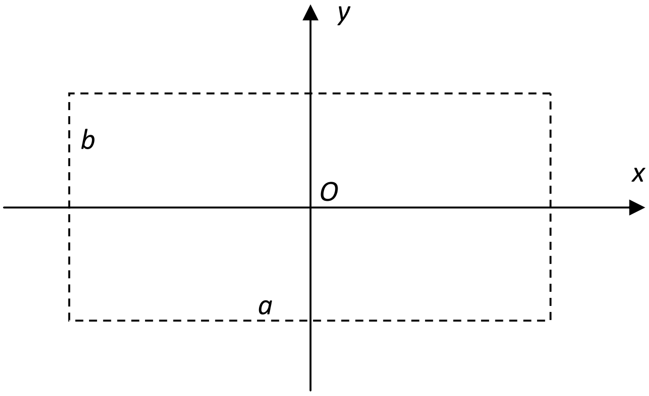
Geometry of the Waveguide:



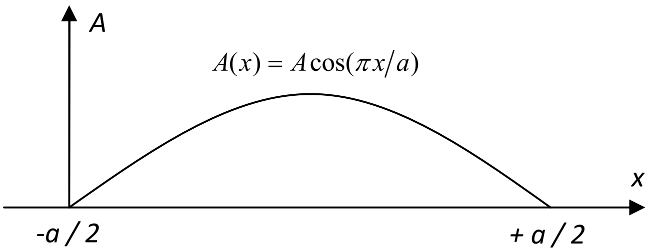
Suppose: $A = 1$

The spectral density G at the Fraunhofer region is:

$$G(w_1, w_2) = \int_{-\frac{a}{2}}^{\frac{a}{2}} \int_{-\frac{b}{2}}^{\frac{b}{2}} \cos\left(\frac{\pi x}{a}\right) \exp\left[-jw_1 x - jw_2 y\right] dxdy.$$

$$G(w_1, w_2) = \int_{-\frac{a}{2}}^{\frac{a}{2}} \cos\left(\frac{\pi x}{a}\right) \exp\left[-jw_1 x\right] dx \int_{-\frac{b}{2}}^{\frac{b}{2}} \exp\left[-jw_2 y\right] dy$$

Distribution of EM Field:



Distribution of EM Field along y axis is uniform:
A(*y*) = 1

$$\int_{-\frac{a}{2}}^{\frac{a}{2}} \cos\left(\frac{\pi}{a}x\right) \exp\left[-jw_1 x\right] dx = \frac{1}{-w_1^2 + \left(\frac{\pi}{a}\right)^2} \exp\left[-jw_1 x\right]\left(-jw_1 \cos\left(\frac{\pi}{a}x\right) + \frac{\pi}{a}\sin\left(\frac{\pi}{a}x\right)\right)\Bigg|_{-\frac{a}{2}}^{\frac{a}{2}} =$$

$$\frac{1}{-w_1^2 + \left(\frac{\pi}{a}\right)^2} \exp\left[-jw_1 \frac{a}{2}\right]\left(-jw_1 \cos\left(\frac{\pi}{2}\right) + \frac{\pi}{a}\sin\left(\frac{\pi}{2}\right)\right) - \frac{1}{-w_1^2 + \left(\frac{\pi}{a}\right)^2} \exp\left[jw_1 \frac{a}{2}\right]\left(-jw_1 \cos\left(-\frac{\pi}{2}\right) + \frac{\pi}{a}\sin\left(-\frac{\pi}{2}\right)\right) =$$

$$\frac{1}{-w_1^2 + \left(\frac{\pi}{a}\right)^2} \exp\left[-jw_1 \frac{a}{2}\right]\frac{\pi}{a} + \frac{1}{-w_1^2 + \left(\frac{\pi}{a}\right)^2} \exp\left[jw_1 \frac{a}{2}\right]\frac{\pi}{a} = \frac{1}{-w_1^2 + \left(\frac{\pi}{a}\right)^2} \frac{\pi}{a}\left(\exp\left[-jw_1 \frac{a}{2}\right] + \exp\left[jw_1 \frac{a}{2}\right]\right) =$$

$$\frac{1}{-w_1^2 + \left(\frac{\pi}{a}\right)^2} \frac{\pi}{a} 2 \frac{\exp\left[-jw_1 \frac{a}{2}\right] + \exp\left[jw_1 \frac{a}{2}\right]}{2} = \frac{2}{-w_1^2 + \left(\frac{\pi}{a}\right)^2} \frac{\pi}{a} \cos\left(w_1 \frac{a}{2}\right)$$

$$\int_{-\frac{b}{2}}^{\frac{b}{2}} \exp\left[-jw_2 y\right] dy = \frac{1}{-jw_2} \exp\left[-jw_2 y\right]\Bigg|_{-\frac{b}{2}}^{\frac{b}{2}} = \frac{2}{-w_2} \frac{\exp\left[-jw_2 \frac{b}{2}\right] - \exp\left[jw_2 \frac{b}{2}\right]}{2j} = \frac{-2}{-w_2}\sin\left(w_2 \frac{b}{2}\right) = \frac{2}{w_2}\sin\left(w_2 \frac{b}{2}\right)$$

$$\boxed{G(w_1, w_2) = \int_{-\frac{a}{2}}^{\frac{a}{2}} \cos\left(\frac{\pi x}{a}\right) \exp\left[-jw_1 x\right] dx \int_{-\frac{b}{2}}^{\frac{b}{2}} \exp\left[-jw_2 y\right] dy \quad \rightarrow \quad G(w_1, w_2) = \frac{4\pi a}{\left(\pi^2 - a^2 w_1^2\right) w_2} \cos\left(w_1 \frac{a}{2}\right) \sin\left(w_2 \frac{b}{2}\right)}$$

$$w_1 = k\frac{x_2}{L}; \quad w_2 = k\frac{y_2}{L} \quad \Rightarrow \quad G(w_1, w_2) = \frac{4\pi a L^3}{\left((\pi L)^2 - (akx_2)^2\right) ky_2} \cos\left(\frac{ka}{2L}x_2\right) \sin\left(\frac{kb}{2L}y_2\right)$$