American University of Armenia, CSE
CS 111 - Discrete Mathematics (B)
Homework Assignment 6 FALL 2019

Due Date: Friday October 25 in class by 9:35*
*Late or Electronic submissions will **NOT** be accepted

1. Prove each of the following statements.

   (a) Let a, b and n be positive integers such that $a \equiv b \pmod{n}$. Give an inductive proof that $a^k \equiv b^k \mod n$ for all $k \geq 0$.

   (b) If $a,\ b \in \mathbb{Z}$ and $c \in \mathbb{Z}^+$, then $a \equiv b \pmod{n}$ if and only if $ca \equiv cb \pmod{cn}$.

   (c) Let $a, b \in \mathbb{Z}$ and $d,\ n \in \mathbb{Z}^+$. If $a \equiv b \pmod{n}$ and $d|n$ then $a \equiv b \pmod{d}$.

   (d) Let $a, b \in \mathbb{Z}$. If $gcd(a, n) = 1$ for positive integer n, then $ax \equiv b \pmod{n}$ has a solution $x = c$ for $c \in \mathbb{Z}$.

2. Let R be the following relation defined on the set $\{a, b, c, d\}$:

$$R = \{(a, a), (a, c), (a, d), (b, a), (b, b), (b, c), (b, d), (c, b), (c, c), (d, b), (d, d)\}$$

   Determine whether R is:

   (a) reflexive
   (b) irreflexive
   (c) symmetric
   (d) asymmetric
   (e) antisymmetric
   (f) transitive

   Explain your answer in one line of correct mathematical form.

3. The relation R on the set A is **assymetric**, if for every $a, b \in A$ if $(a, b) \in R$ implies $(b, a) \notin R$. List all the assymetric relatiosn on the set $\{1, 2\}$.

4. Consider the relation R defined on the set of integers:

$$a\,R\,b \text{ if and only if } 5|(a + 4b).$$

   (a) Prove that R is an equivalence relation.
   (b) What are the equivalence classes of R?

5. Suppose that R and S are reflexive relations on a set A. Prove or disprove each of the following statements.

   (a) $R \cup S$ is reflexive.

   (b) $R \cap S$ is reflexive.

   (c) $S \circ R$ is reflexive.

   (d) $R - S$ is irreflexive.

   (e) $\overline{R}$ is irreflexive.

   (f) $R \triangle S$ is irreflexive.

6. Let R be a relation from set A to B

   (a) Let $R_1 = \{(a,b) | a < b\}$ on the set of integers. Find R_1^{-1}.

   (b) Let $R_2 = \{(a,b) | a$ divides $b\}$ on the set of positive integers. Find R_2^{-1}.

   (c) Show that a relation R on a set A is symmetric if and only if $R = R^{-1}$.

   (d) Show that a relation R on a set A is antisymmetric if and only if $R \cap R^{-1} \subseteq \{(a,a) | a \in A\}$.

7. Let S be a relation $(\mathbb{R} - \{0\}) \times (\mathbb{R} - \{0\})$ defined as $S = \Big\{ \big((a,b),(c,d)\big) : b/a = d/c \Big\}$.

   (a) Prove S is an equivalence relation.

   (b) Give the equivealence classes of S.

   (c) Give a geometric interpretation of the equivalence classes, assuming an element of $(\mathbb{R} - \{0\}) \times (\mathbb{R} - \{0\})$ is a point on the plane.