GAMMA TRANSITION JUMP SCHEMES

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Abstract

Beam intensity loss and longitudinal emittance blow-up of high intensity proton beams at transition can be avoided if the speed of transition crossing is artificially increased by a so-called *gamma transition jump*, in which the value of γ_t is changed using fast pulsed quadrupoles. In order to improve the analysis of existing jump schemes and the preparation of new designs this lecture derives a simple expression for $\Delta\gamma_t$ as a function only of the *local unperturbed* betatron and dispersion parameters at the jump quadrupoles and their integrated normalized gradients. The zero tune shift requirement has important consequences for the quadrupole layout and the number of families. The rules to be obeyed in the design of γ_t jump schemes are discussed, using the CERN PS and SPS machines as examples.

1 INTRODUCTION

In many medium and high energy (typically $E > 5$ GeV) proton accelerators the beams have to cross *transition* , i.e. in the energy range of the machine there is a beam energy at which the derivative of the revolution frequency with respect to the momentum error crosses zero. The longitudinal beam behaviour in the neighbourhood of transition, and the radiofrequency manipulations required to accelerate the beams through transition, have been described in previous lectures [1]. The present lecture will deal with the problems caused to high intensity beams by longitudinal space charge forces, and more particularly with the most successful remedy, namely a fast γ_t jump. This solution has in the past twenty years allowed several machines to considerably raise the beam intensities that can be accelerated without intensity loss or emittance blow-up at transition.

The ratio between the beam energy at transition and the rest energy of the particles, called γ_t (gamma at transition), is independent of the particle mass. Its value depends only on the machine optics and geometry, and is close to the horizontal tune value in the case of a regular lattice. At constant magnetic field the velocity βc, the circumference C, and thus the revolution frequency $f = \beta c/C$ of each particle depend on its relative momentum error $\delta = \Delta p/p$. The frequency spread in the beam is related [2] to its momentum spread by the parameter η:

$$\eta = \frac{1}{f_o}\frac{df}{d\delta} = \frac{1}{\gamma^2} - \frac{1}{C_o}\frac{dC}{d\delta} \tag{1}$$

where C_o and f_o are the circumference and the revolution frequency of a particle with nominal momentum on the reference orbit. At transition $\eta = 0$ and the value of γ may be found from

$$\frac{1}{\gamma_t^2} = \frac{1}{C_o}\frac{dC}{d\delta} \tag{2}$$

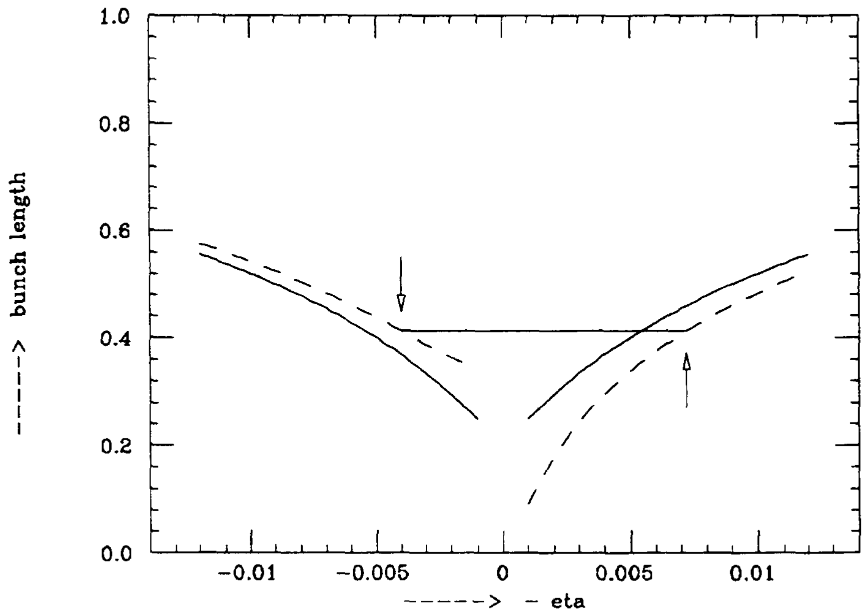

Figure 1: Schematic plot of equilibrium bunch length (in arbitrary units) before and after transition, without (solid) and with (dashed) space charge. The horizontal straight line between the arrows indicates the bunch length during a fast γ_t jump.

In the vicinity of transition η is approximately proportional to $\gamma_t - \gamma$:

$$\eta = \frac{1}{\gamma^2} - \frac{1}{\gamma_t^2} = \frac{\gamma_t^2 - \gamma^2}{\gamma^2 \gamma_t^2} \approx \left(\frac{2}{\gamma_t^3}\right) (\gamma_t - \gamma) \tag{3}$$

If the optics, and thus γ_t, is kept constant the speed at which the transition is crossed is determined only by the rate of acceleration $d\gamma/dt$. Beam intensity and emittances are in this case exposed to several intensity dependent space charge phenomena:

- The space charge effect on the bunch length changes sign at transition (figure 1), causing a sudden change of the equilibrium bunch length at a time when the motion in longitudinal phase space is slow [3, 4]. If the bunch shape oscillations resulting from this mismatch are not damped they will eventually result in filamentation and longitudinal emittance increase.

- Both the energy spread (Johnsen effect [5], intensity independent) and the space charge tune spread (Umstätter effect [4]) produce a spread in the γ_t value of the particles, which consequently do not cross transition simultaneously.

- Above transition the space charge forces make the bunches longitudinally unstable (negative mass instability [3, 4, 6, 7]). The growth rate of this instability increases rapidly with $df/d\delta$ and thus with the time elapsed since transition. After some time it slows down and eventually the instability disappears when the Landau damping due to the increasing frequency spread in the bunches becomes large enough.

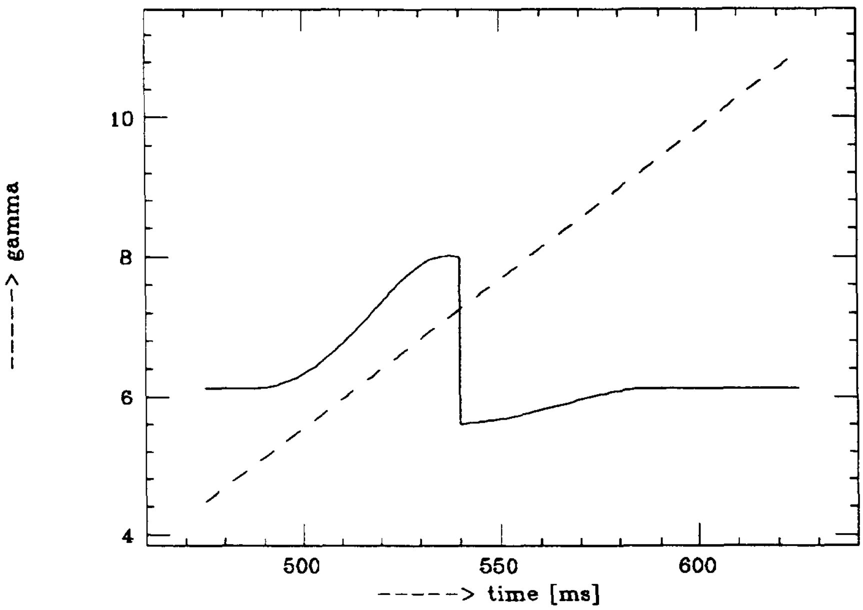

Figure 2: γ of beam (dashed) and γ_t (solid) at transition in the CERN PS. The jump is made in 0.4 ms.

In the design of accelerators it is attempted wherever possible to locate the transition energy outside the energy range of the machine, and some machines have even been built (for example the CERN LEAR) or proposed [8] with an *imaginary* γ_t value. On the other hand, the γ_t value of *existing* machines cannot be changed by a large amount without changing the ring geometry. If transition crossing cannot be avoided, the best solution to overcome the intensity limitations resulting from the above mentioned phenomena is an artificial increase of the transition crossing speed by means of fast pulsed quadrupoles. Such a γ_t jump scheme makes it possible to keep the beam at a safe distance from transition, except for the very short time during which the transition region is crossed (figure 2) at a speed increased by one or two orders of magnitude. The required jump amplitude and speed depend on the beam intensity [6, 9]:

- The distance between γ and γ_t just before and just after the jump should ideally be large enough for the space charge effect on the bunch length to be small compared to the unperturbed bunch length. With very high intensity beams this is not always possible, and it is therefore advantageous to use an asymmetric jump (figure 2), where the starting point of the jump is closer to transition than the end point. The equilibrium bunch length at the start and at the end (arrows in figure 1) can thus be made equal. The optimum asymmetry depends on the beam intensity.

- The negative mass unstable region just after transition should be crossed fast enough for the emittance blow-up to remain small. An asymmetric jump as mentioned above is better centered with respect to the unstable region than a symmetric jump.

For these reasons the optical design of a jump scheme, which is the subject of this lecture, should aim at a large $\Delta\gamma_t$, while keeping the maximum dispersion and β values below reasonable values. The tune shift should be as small as possible. The use of a general purpose beam optics program does generally not allow an exhaustive study of a large number of possibilities (locations, polarities, families, etc.). Therefore we will first establish an analytical expression of the jump amplitude as a function of the quadrupole strengths, which will allow a preselection of a limited number of solutions. The performances (jump amplitude, maximum values of β and the dispersion, tune shift, etc.) of the selected candidate schemes may then be more precisely evaluated using a beam optics program.

2 EFFECT OF QUADRUPOLES ON γ_t

The modification of γ_t^{-2} due to the presence of non-nominal quadrupole gradients may be found, according to equation (2), by calculating the corresponding circumference increase of the closed orbit of a particle with momentum error δ. To this end we consider a machine containing a set of N special quadrupoles which are not used in the nominal optics configuration, but may be powered to modify the horizontal dispersion function, for example at transition crossing. The nominal (unperturbed) machine has central orbit momentum p, magnetic rigidity of the beam $B\rho = p/e$, horizontal tune Q, and the following parameters valid at the special quadrupoles:

s_i the longitudinal position at quadrupole i $\quad i = 1,\ N$
D_i the nominal horizontal dispersion
β_i the nominal horizontal beta
μ_i the nominal horizontal betatron phase

These parameters are here assumed to be independent of δ. The machine optics may be modified by powering the special quadrupoles (treated here as thin lenses), according to the parameters:

K_i the strength $\int Gds/B\rho$ (integrated normalized gradient) of quadrupole i
D_i^* the modified dispersion at quadrupole i in presence of strengths K_1, ... K_N
β_i^* the modified horizontal beta

The nominal optics is characterized by the Courant - Snyder 'closed orbit distortion' function written here as

$$m(s_1, s) = \frac{\sqrt{\beta(s_1)\ \beta(s)}}{2\sin(\pi Q)}\ \cos(\pi Q - |\mu(s_1) - \mu(s)|) \tag{4}$$

which is the effect of a unit kick at location s_1 on the beam position at location s (figure 3). The horizontal closed orbit change due to a set of N non-nominal short horizontal orbit kicks θ_i located at s_i may be written:

$$\Delta x(s) = \sum_{i=1}^{N} \theta_i\ m(s_i, s) \tag{5}$$

if $\theta = \int Bds/(B\rho)$ is taken positive for a kick towards more positive x values. The same function m relates the nominal dispersion to the local bending radius $\rho(s)$ in the bending magnets [10]:

$$D_i = \int_0^C \frac{m(s, s_i)}{\rho(s)} ds \tag{6}$$

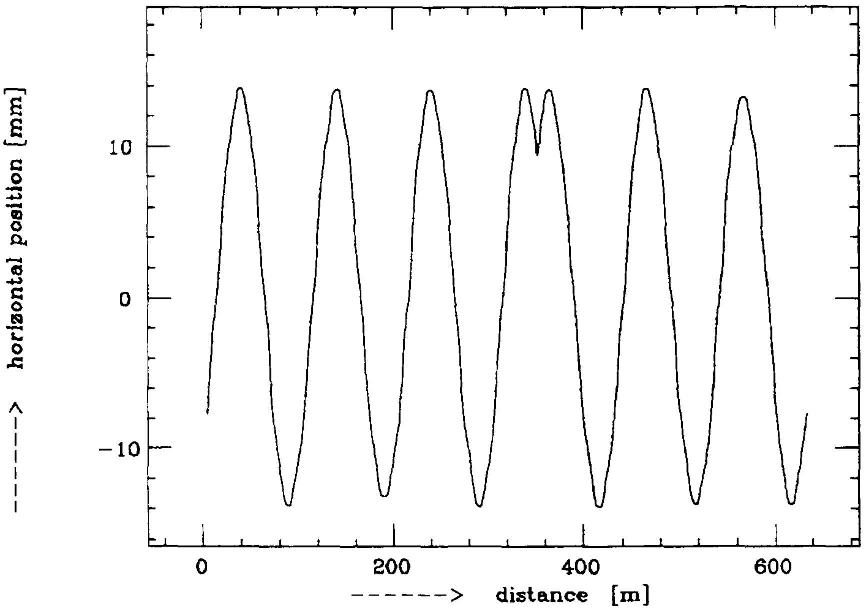

Figure 3: Closed orbit in the presence of a single kick (sampled at locations with equal β)

The parameter s denotes the projection of the longitudinal position onto the central orbit. To first order in x the ratio between a path length on an orbit $x(s)$ and its projection is equal to $1 + x(s)/\rho(s)$ in the bending magnets and equal to 1 elsewhere [10]. The circumference increase of an orbit in the presence of kicks θ_i, calculated from the path length increase in the bending magnets, may thus be written

$$\Delta C = \int_0^C \frac{\Delta x(s)}{\rho(s)}\, ds \tag{7}$$

Using (5) and (6) this may be related to the dispersion:

$$\Delta C = \sum_{i=1}^{N} \theta_i \int_0^C \frac{m(s_i, s)}{\rho(s)}\, ds = \sum_{i=1}^{N} \theta_i D_i \tag{8}$$

Since we are only looking at the properties of one orbit at a time it is irrelevant whether a kick is caused by a *dipole* field or by a *quadrupole* field, provided both have the same field value at the radial position of the orbit. The special quadrupoles may thus be treated as kicks, and their effect on the orbit and the circumference may be calculated using the dispersion, β values and phases of the *nominal* optics. On the other hand, the value of the orbit kick resulting from the strength K_i of (thin) quadrupole number i depends on the orbit position in the quadrupole, which is equal to the *perturbed* local dispersion D_i^* multiplied by the momentum error δ:

$$\theta_i = -x(s_i)\; K_i = -\delta D_i^* K_i \tag{9}$$

if K is taken positive for a horizontally focussing quadrupole. According to (8) the effect of a set of N quadrupole strengths K_i on the circumference is then

$$\Delta C = -\delta \sum_{i=1}^{N} K_i D_i^* D_i \tag{10}$$

The dispersion functions are here assumed to be independent of δ. Differentiation with respect to δ yields the γ_t^{-2} change

$$C_o\Delta(\gamma_t^{-2}) = \Delta\left(\frac{dC}{d\delta}\right) = -\sum_{i=1}^{N} K_i D_i^* D_i \tag{11}$$

which thus depends only on the *local* values of the perturbed and unperturbed dispersion functions at the N quadrupoles.

The nominal and perturbed dispersion functions are related by the closed orbit distortion function m. The closed orbit $x(s_j)$ at quadrupole j of a particle with relative momentum error δ in the presence of the N special quadrupole strengths K_i is:

$$x(s_j) = \delta D_j - \sum_{i=1}^{N} \delta K_i D_i^* m(s_i, s_j) \tag{12}$$

Differentiation with respect to δ yields

$$\vec{D^*} = \vec{D} + M.\vec{D^*} \tag{13}$$

where vector $\vec{D}$ is defined as $\vec{D} = (D_1, D_2, \ \ .. \ \ D_N)$ and matrix $\mathbf{M}$ as:

$$M_{ij} = -K_j \ m_{ij} = -K_j \ m(s_i, s_j) \qquad i,j = 1, N \tag{14}$$

valid for the nominal unperturbed machine. The modified dispersion values may be found by

$$\vec{D^*} = (1 - M)^{-1}.\vec{D} = (1 + M + M^2 + ...).\vec{D} \tag{15}$$

3 ZERO TUNE SHIFT AND NON-ZERO $\Delta(\gamma_t)$

Equations (11) and (15) have a general validity and may be applied to any γ_t modification system, DC or pulsed. However, a pulsed jump scheme for transition crossing has to modify γ_t temporarily without creating additional instabilities or beam loss, and the optical design must therefore meet three requirements simultaneously:

- produce a large γ_t change
- leave the betatron tunes unchanged
- keep D_{max} and β_{max} below reasonable values

The first two requirements may a priori seem incompatible, as γ_t is usually close to the horizontal tune. However, they may be met by locating the quadrupoles in such a way that they act strongly on the dispersion and little on β, since γ_t is related to the dispersion and Q to the β function. This is possible by exploiting the difference in periodicity of the functions ΔD and $\Delta\beta$, as will be shown below.

3.1 How to obtain $\Delta Q = 0$

To first order in K_i the tune shift is given by [10]

$$\Delta Q = \frac{1}{4\pi} \sum_{i=1}^{N} \beta_i K_i \tag{16}$$

Thus $\sum \beta_i K_i = 0$ is a first requirement to keep the horizontal tune constant. An equivalent condition should be respected in the vertical plane. Unfortunately, if no special precautions are taken the β functions are modified by the jump quadrupoles, and the expression for the tune shift contains higher order terms in K_i. However, it is possible to keep the tunes constant during the entire transition process if the quadrupole locations are chosen with care. This is illustrated by a short review of early γ_t jump schemes:

- A non-zero ΔQ was tolerated at the CERN PS between 1969 and 1973 with the so-called Q-jump scheme [11, 12], using a set of 6 quadrupoles more or less regularly spaced around the ring with *identical* strengths and polarities. A tune change of 0.25 was required to obtain a $\Delta\gamma_t$ of 0.3.
- In 1970 a scheme was proposed for the FNAL Booster using 12 regularly spaced quadrupoles with equal strengths but *alternating* polarities [13], yielding $\Delta\gamma_t = 1$ at the expense of only $\Delta Q = 0.1$.
- At the PS a large γ_t jump of 2.7 *without* tune shift is obtained [6] since 1973 by grouping 16 quadrupoles together in doublets $(i, i+1)$ for $i = 1, 3, ..$ with $K_{i+1} = -K_i$ and $\mu_{i+1} - \mu_i = \pi$.

The last example is the only one of the three cases where the β values are kept constant at the jump quadrupoles. This can be obtained in any machine using the simple and yet extremely flexible solution of grouping the quadrupoles together as doublets in such a way that for each *individual* doublet the betatron transfer matrix (from the entrance of the first lens to the exit of the second) does not depend on the excitation level. It can be shown that this is equivalent to the two conditions

$$\beta_1 K_1 + \beta_2 K_2 = 0 \qquad\qquad \sin(\mu_2 - \mu_1) = 0 \tag{17}$$

The second condition implies $\Delta\mu = n\pi$ with integer n. Such doublets leave the β values strictly unchanged outside the doublet interval, and thus at *all* quadrupole locations in the machine. The doublets must not overlap. Any scheme consisting of such doublets will then yield $\Delta Q = 0$ even for large K values. Due to the difference in periodicity of the ΔD and $\Delta\beta$ functions the effect on $\Delta(\gamma_t^{-2})$ is zero or small for *even* n, and large for *odd* n values. In the following we will therefore assume a π phase advance in all doublets.

3.2 Consequences of the use of π-doublets

A π phase advance in a zero tune shift doublet eliminates all powers > 1 of matrix $\mathbf{M}$ in equation (15). This may be shown as follows:

$$(M^2)_{ij} = \sum_k K_k m_{ik} K_j m_{kj} = K_j \sqrt{\beta_i \beta_j} \sum_k K_k \beta_k \cos_{i,k} \cos_{k,j} \tag{18}$$

where

$$\cos_{i,j} = \frac{1}{2\sin(\pi Q)} \cos(\pi Q - |\mu(s_i) - \mu(s_j)|) \tag{19}$$

The contribution of doublet $(k, k+1)$ to each element $(M^2)_{ij}$ is proportional to $K_k\beta_k + K_{k+1}\beta_{k+1}$, since $\cos_{i,k} = -\cos_{i,k+1}$ and $\cos_{k,j} = -\cos_{k+1,j}$. The zero tune shift condition cancels this contribution and equation (15) has only 2 terms:

$$\vec{D^*} = (1 + M).\vec{D} \tag{20}$$

Equation (11) then reduces to:

$$C_o\Delta(\gamma_t^{-2}) = -\sum_{i,j=1}^{N} K_i\ (1+M)_{ij}D_iD_j = -\sum_{i=1}^{N} K_iD_i{}^2 + \sum_{i,j=1}^{N} K_iK_jm_{ij}D_iD_j \tag{21}$$

valid for a π-doublet scheme with zero tune shift.

The effects of a π-doublet with $\beta_1K_1 + \beta_2K_2 = 0$ may thus be summarized as follows:

- Q is *unchanged*
- β is left *unchanged* at all jump quadrupoles
- the dispersion change is *linear* in K
- the jump amplitude has only *linear and quadratic* terms in K

The maximum β value inside the doublet interval can easily be found in most cases. For example in a regular lattice where all horizontally focussing quadrupoles are located at $\beta_i = \beta_{max}$ the maximum perturbed β^* inside doublet $(i, i+1)$ may be found [6] by

$$\frac{\beta^*_{max}}{\beta_{max}} = \frac{1+\sin\psi}{1-\sin\psi} \tag{22}$$

where ψ is defined by $\tan(\psi) = |K_i|\ \beta_{max}/2$. On the other hand, the dispersion function modification is generally non-local and the maximum dispersion value cannot be estimated a priori in a simple way.

3.3 Schemes with a constant normalized dispersion

In schemes where the horizontal normalized dispersion $D/\sqrt{\beta}$ has the same value at all quadrupoles the $\sum \beta_iK_i = 0$ requirement cancels the linear terms $-\sum K_iD_i{}^2$ of equation (21), and $\Delta(\gamma_t^{-2})$ will only have quadratic and higher order terms. If the scheme consists of π-doublets the higher order terms cancel and the γ_t change is given by the quadratic terms of (21):

$$\Delta(\gamma_t^{-2}) = \frac{1}{2C_o\sin(\pi Q)} \sum_{i,j=1}^{N} D_i^2\beta_jK_iK_j\ \cos(\pi Q - |\mu_i - \mu_j|) \tag{23}$$

valid for a π-doublet scheme with zero tune shift and identical nominal normalized dispersion values $D/\sqrt{\beta}$ at all N quadrupoles.

4 CELLS, SUPERPERIODS AND FAMILIES

As shown above, the expression for $\Delta(\gamma_t^{-2})$ contains only first and second order terms in K in π-doublet schemes. In other schemes the higher order terms are usually small and $\Delta(\gamma_t^{-2})$ is therefore in all cases dominated either by linear or by quadratic terms in K:

- In machines where the normalized dispersion function has large variations from one potential quadrupole location to another the quadrupoles of a doublet may be located at $(D/\sqrt{\beta})_{max}$ and $(D/\sqrt{\beta})_{min}$ respectively, in order to maximize the linear term $-\sum K_i D_i{}^2$ of equation (21) which dominates $\Delta(\gamma_t^{-2})$ in this case. In regular lattice structures these locations are separated by a π betatron phase advance.

- In machines with an *equal* normalized horizontal dispersion $D/\sqrt{\beta}$ at all quadrupoles the value of $\Delta(\gamma_t^{-2})$ depends quadratically on the quadrupole strengths K_i for zero tune shift schemes.

Most machines are made up of repetitive cells, each containing a small number of quadrupoles, for example (QF, QD). The vacuum chamber aperture is used in the most efficient way if the β and dispersion functions are matched to these cells, i.e. have the same periodicity. Free space availability and the preference for cell phase advances of 45°, 60° or 90° require quadrupoles of a π-doublet to occupy *equivalent* locations in different cells, resulting in identical D and β values (example of the PS). On the other hand, the presence of straight sections with zero dispersion, as required in some machines, induces $D/\sqrt{\beta}$ variations around the ring (example of the SPS).

4.1 Example of matched normalized dispersion: the CERN PS

At the CERN PS ($Q = 6.25$, $E_{min} = 2$ GeV, $E_{max} = 26$ GeV, $\gamma_t = 6.1$, $d\gamma/dt = 40\ \mathrm{s}^{-1}$) D and β have the same value at all potential jump quadrupole locations and consequently $\Delta(\gamma_t^{-2})$ depends quadratically on the K values. This means that, with one power supply feeding all quadrupoles, it would only be possible to change γ_t in one direction, depending on the sign of $\sin(\pi Q)$. However in this case the maximum change of γ_t, and thus the maximum excursions of the dispersion and β functions with respect to their nominal values, would be much larger than with a *symmetric bipolar* γ_t jump providing the same jump amplitude. At the PS the quadrupole doublets are for this reason separated in two families, each connected to a separate power supply [6]. If family number 1 has strengths $+K_1$ or $-K_1$, and family number 2 strengths $+K_2$ or $-K_2$, equation (23) may be rewritten as a sum of three terms:

$$\Delta(\gamma_t^{-2}) = aK_1^2 \; + \; bK_1K_2 \; + \; cK_2^2 \tag{24}$$

The phases μ_i of the quadrupole locations have to be chosen in such a way that coefficient b is large, and larger than a or c. A polarity change of either K_1 or K_2 then changes the sign of $\Delta(\gamma_t^{-2})$. At the PS both families are slowly pulsed with rise and fall times of about 20 ms. At the maximum of the current pulses the polarity of family 1 is reversed as fast as possible, which yields the γ_t curve shown in figure 2.

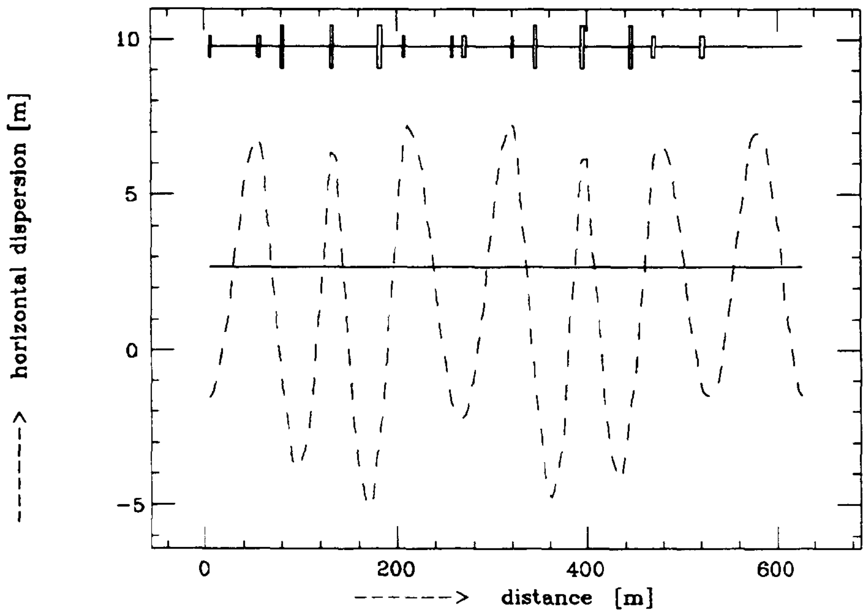

Figure 4: Unperturbed dispersion (solid) of the PS and dispersion just before the polarity change of the gamma jump (dashed), sampled at locations with equal β. Family 1 is shown as the smaller boxes, family 2 as the larger boxes.

The basic brick of the present PS scheme is

$$(+D_1) \quad \phi \quad (+D_2) \tag{25}$$

where $+D_1$ denotes a doublet consisting of two quadrupoles at π phase advance, with strengths $+K_1$ and $-K_1$. The 45^o phase advance per cell in both planes makes it easy in this machine to locate the doublet quadrupoles at the required phase advance. D_1 is separated from the second doublet D_2 (strengths $+K_2$ and $-K_2$) by a space with ϕ betatron phase advance, not containing jump quadrupoles. In order to keep D^*_{max} low it is advantageous to make the quadrupole layout symmetric. The simplest symmetric configuration is:

$$(+D_1) \quad \phi \quad (+D_2) \quad (-D_2) \quad \phi \quad (-D_1) \tag{26}$$

where -D has reversed polarities with respect to +D. The two central doublets together form a triplet with strengths $+K_2$, $-2K_2$, $+K_2$. This will have the same effect on γ_t^{-2} as a single doublet with strengths $+2K_2$ and $-2K_2$, but the local maximum β value will be smaller, according to (22). Doublets or groups of doublets may be displaced by π phase advance if their polarities are reversed, without changing the $\Delta(\gamma_t^{-2})$ value, since the β and dispersion values are identical at all considered quadrupole locations. The γ_t jump produced by the scheme depends thus on ϕ *modulo* π.

For $0 < \phi < \pi$ this configuration stretches over a phase advance between 4π and 6π, and with a tune of 6.25 there is space for installing two such configurations around the ring. A given jump amplitude may thus be obtained with smaller quadrupole strengths, resulting once more in smaller D^*_{max} and β^*_{max} values. The value of ϕ and the phase advance between the two configurations has to be chosen with care in order to optimize a, b and c. With the PS cell phase advance of 45° only 16 independent cases have to be taken into consideration. We will here limit the discussion to two solutions which we call '*symmetric*' and '*antisymmetric*' respectively. At this stage the manipulations may more easily be understood by talking in terms of superperiods. With the above mentioned phase advances the machine may be divided either into two identical superperiods containing jump quadrupoles, or into two identical superperiods containing quadrupoles plus one transparent betatron wavelength without quadrupoles.

The symmetric scheme corresponds to two exactly identical superperiods, each having a tune of Q/2. The coefficients a, b and c may be evaluated using equation (23) putting $Q = 6.25/2$. A large b value is obtained for $\phi = \pi/2$, but c is equally large (values in m^2):

$$a = 0 \qquad b = -7.2 \qquad c = +8.7 \tag{27}$$

The jump is thus not bipolar, and the performance of this scheme is poor:

$$\Delta\gamma_t = 1.0 \qquad D^*_{max} = 21.0 \text{ m} \tag{28}$$

However, with $K_1 = -K_2$ the scheme provides a very low γ_t value of 4.3, which may be useful for other purposes [6].

The scheme may be made antisymmetric by either reversing the polarity of one of the two quadrupole configurations, or displacing one configuration over a π phase advance. The antisymmetric case thus corresponds to two identical superperiods with a tune of $(Q-1)/2$, plus one 'transparent' betatron period without quadrupoles (figure 4) and yields with $\phi = \pi/2$ (values in m^2):

$$a = 0 \qquad b = -7.2 \qquad c = -1.5 \tag{29}$$

The last solution was adopted for the jump scheme of the PS as it provides a large bipolar γ_t jump. The non-zero c coefficient makes $\Delta(\gamma_t^{-2})$ slightly asymmetric: γ_t increases first slowly from 6.1 to 8.2, jumps down to 5.6, and rises back slowly to 6.1 again (figure 2). The maximum dispersion and β values just before or just after the jump are 9 m and 54 m respectively, to be compared with the nominal values of 3 m and 22 m (figure 4). The corresponding quadrupole strengths are

$$K_1 = 0.023 \text{ m}^{-1} \qquad K_2 = 0.050 \text{ m}^{-1} \tag{30}$$

The maximum current in the quadrupoles is 450 and 250 A respectively, and the polarity of K_1 is reversed in 0.4 ms at transition, using special low inductance quadrupoles. This yields a transition crossing rate of $-d\gamma_t/dt = 6000$ s^{-1}, which is 150 times larger than $d\gamma/dt$.

4.2 Example of mismatched normalized dispersion: the CERN SPS

Not all machines have equal $D/\sqrt{\beta}$ values at all quadrupoles. As a counter-example we will briefly discuss a possible γ_t jump scheme for the SPS ($Q = 26.6$, $E_{min} =$ 14 GeV, $E_{max} = 450$ GeV, $\gamma_t = 23.2$, $d\gamma/dt = 130$ s^{-1}) where presently high intensity proton beams cross transition without a γ_t jump. This machine has a regular lattice

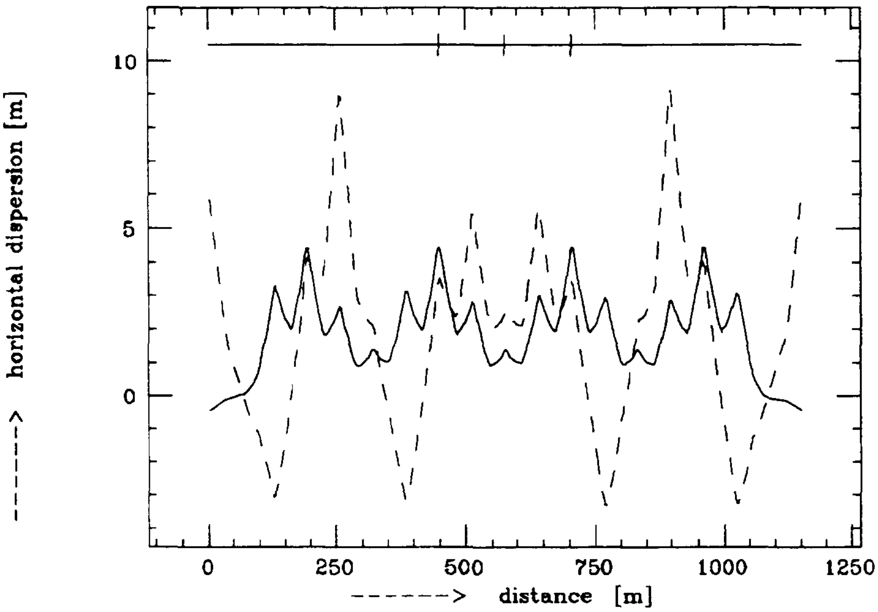

Figure 5: A possible γ_t jump scheme for the SPS: unperturbed (solid) and perturbed (dashed) dispersion in one superperiod (SPS/6), with proposed jump quadrupole locations.

structure in the arcs, but the matching of the dispersion function is destroyed by the presence of 6 long straight sections with zero dispersion: the β values are equal at all F quadrupoles, while the dispersion oscillates between 1 and 4.5 m (figure 5). Since the cell phase advance is nearly 90° it is easy to form doublets yielding $\Delta Q \approx 0$.

With such a dispersion modulation the expression (21) for $\Delta(\gamma_t^{-2})$ may contain a large linear term $C_o^{-1} \sum K_i D_i^2$ and thus one quadrupole family is sufficient to produce a bipolar γ jump. In the regular SPS arc lattice D_{max} and D_{min} are separated by a π betatron phase advance, and doublet quadrupoles located there produce a large $\Delta\gamma_t$. With one triplet (i.e. two contiguous doublets) per superperiod, centered in the arc, K values of only + and - 0.010 m^{-1} are required to obtain a $\Delta\gamma_t$ of 3.6. D_{max} increases from 4.5 to 9 m, and β_{max} from 104 to 226 m.

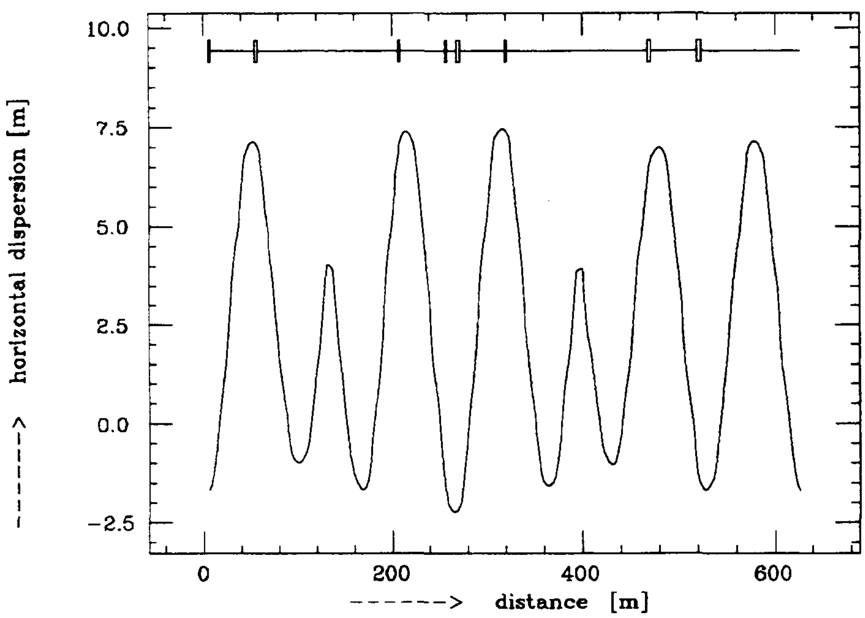

Figure 6: Dispersion in the PS machine perturbed by family 2 only (sampled at locations with equal β), and optimum locations for family 1

5 CONCLUSION

Two simple expressions, (11) and (15), have been derived relating $\Delta(\gamma_t^{-2})$ to the integrated normalized gradients K_i of the quadrupole scheme, using only the nominal unperturbed β and dispersion values at the quadrupoles and their phase advances. At this stage no assumptions have been made about the layout of the quadrupole configuration, and the formalism may be used for the design of any γ_t modification scheme. Pulsed jump schemes have to meet the additional requirement of zero tune shifts, preferably obtained by the use of doublets, each consisting of two quadrupoles with opposite polarity at π (or 3π, 5π, ..) phase advance and $\beta_1 K_1 + \beta_2 K_2 = 0$ in both transverse planes. In this case terms of order higher than 2 in K cancel in the expression for $\Delta(\gamma_t^{-2})$.

In machines where the horizontal normalized dispersion $D/\sqrt{\beta}$ has the same value at all γ_t modification quadrupoles, the linear terms in K cancel for zero tune shifts. The remaining quadratic terms may easily be computed to allow the performance evaluation of the different design options, like the number of families and the superperiodicity. A bipolar γ_t jump provides a large jump amplitude while keeping the maximum β and D excursions from the nominal values at a reasonable level. This requires two quadrupole families, as is shown in the example of the CERN PS scheme. The distribution of the doublets around the ring, and more particularly the superperiodicity, has to be chosen with care in order to maximize the mixing term bK_1K_2 in equation (24) which produces the bipolar jump.

On the other hand, in machines where the $D/\sqrt{\beta}$ modulation is important a scheme may be found where $\Delta(\gamma_t^{-2})$ has non-zero linear terms in K even for zero tune shifts, in which case a single quadrupole family is sufficient to produce a bipolar jump as is shown in the example of the CERN SPS.

The difference between these two cases may more easily be understood by the following. Any machine with a *matched* $D/\sqrt{\beta}$ function may be converted into a *mismatched* machine by powering one quadrupole family. A second family will then produce a $\Delta(\gamma_t^{-2})$ which is linear in its strength value. Figure 6 shows that, as may be expected from the SPS case, in the PS the optimum locations for family 1 correspond to the minima and maxima of the dispersion in the presence of family 2.

The superperiodicity plays an important role in matched $D/\sqrt{\beta}$ lattices. A machine may be divided into n identical superperiods, each having a tune Q/n which determines the m_{ij} values in machines where the quadratic terms in K dominate $\Delta(\gamma_t^{-2})$. Transparent betatron wavelengths without jump quadrupoles may be inserted between these superperiods, thus offering an additional choice of superperiod tunes like $(Q-1)/n$, $(Q-2)/n$ etc., as shown in the example of the CERN PS. To increase the efficiency of a scheme n may be chosen such that the superperiod tune is close to an integer in the quadratic case. Unfortunately D^*_{max} will generally also be larger in this case. On the other hand, machines where $D/\sqrt{\beta}$ is not matched are dominated by the first term of (21) and do not show this resonant behaviour of $\Delta(\gamma_t^{-2})$.

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