BEAM DYNAMICS IN ELECTROSTATIC RINGS

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Abstract

An electrostatic storage ring for atomic physics, ELISA [1], is at present in operation at Aarhus University, and the first results are encouraging. It gives us the possibility to investigate in detail the features of this new class of machines, and in particular the possibilities for future improvements. The advantages of an electrostatic storage ring at low energy as compared to a magnetic ring are obvious, and one of them is the possibility to store different particles without readjustment of the optics. However, the magnetic and electrostatic fields are different. In particular, the particles moving in an electrostatic field can change their kinetic and potential energies, since the electrical force may coincide with the direction of motion. This leads to a coupling between the transverse and longitudinal motion, and this effect may play a very significant role. Additionally, we investigate in this paper the influence of space charge and image charges. Two different types of optics have been considered. The first was based on spherical deflectors, and recently these were changed to cylindrical. The experimental observations are described in [2]. In the paper, the theory of such machines is developed based on motion integrals, and the results of numerical simulations are presented.

1 EQUATIONS OF MOTION

ELISA consists of two arcs and two straight sections. The arc includes two doublets of electrostatic lenses, one 160° electrostatic spherical or cylindrical deflector and two 10° parallel plate deflectors. The most unusual element from the accelerator physics point of view is the spherical and cylindrical deflector. It consists of two segments of two concentric spheres or cylinders with radius R_1 and R_2. The plates are long in the longitudinal direction and narrow in the transverse direction.

1.1 Spherical deflectors

The expression for the electrical field will be derived in the approximation of two not truncated spheres. In accordance with Gauss' law the potential between two spheres varies as

$$\varphi(r) = -\frac{2\varphi_0}{R_2 - R_1} R_2 R_1 \cdot \frac{1}{r} + \frac{\varphi_0}{R_2 - R_1}(R_2 + R_1), \quad (1)$$

where $\pm\varphi_0$ is the potential on the spheres. The usual co-ordinate system in accelerator physics is the cylindrical system r, y, ϑ and we also work in the same system: $s = R_{eq}\vartheta$, $x = r - R_{eq}$, $y = y$, where s is the longitudinal co-ordinate along the trajectory and x is the deviation from the equilibrium radius R_{eq} in the horizontal plane and y is the deviation of the particle in the vertical direction. The Lagrangian of this system is

$$L = \frac{m}{2}\left(\dot{r}^2 + r^2\dot{\vartheta}^2 + \dot{y}^2\right) +$$
$$\frac{2e\varphi_0}{R_2 - R_1} R_2 R_1 \cdot \frac{1}{\sqrt{r^2 + y^2}} - \frac{e\varphi_0}{R_2 - R_1}(R_2 + R_1) \quad (2)$$

Due to the spherical symmetry $\frac{\partial L}{\partial \vartheta} = 0$ we have

$$m\ddot{r} = mr\dot{\vartheta}^2 - eE_x$$
$$mr^2\dot{\vartheta} = p_\vartheta \cdot r = M = const. \quad (3)$$
$$m\ddot{y} = -eE_y$$

Substituting in the first equation the potential (1), we have

$$r'' = \frac{1}{r} - \frac{2em\varphi_0}{M^2}\frac{R_1 R_2}{R_2 - R_1} \cdot \frac{1}{\left(1 + \frac{y^2}{r^2}\right)^{3/2}} \quad (4)$$

Defining the equilibrium radius from $r'' = 0$ and $y = 0$, we have

$$R_{eq} = M^2 \cdot \frac{1}{2em\varphi_0} \cdot \frac{R_2 - R_1}{R_1 R_2}. \quad (5)$$

Retaining terms of order less than two

$$x'' + \frac{1}{R_{eq}^2}x - \frac{1}{R_{eq}^3}x^2 - \frac{3}{2 \cdot R_{eq}^3}y^2 = 0$$
$$y'' + \frac{1}{R_{eq}^2}y - \frac{3}{R_{eq}^3}xy = 0 \quad (6)$$

The spherical deflector focuses the particles in the horizontal and vertical planes with the same force. Both equations in (6) have a sextupolar component, and therefore third order resonances should be avoided.

1.2 Cylindrical deflectors

We consider the cylindrical deflector as a candidate to improve the performance of ELISA. In accordance with Gauss' law the potential from the internal

electrode with radius R_1 to the external electrode with radius R_2 varies as

$$\varphi(r) = -\varphi_0 + \frac{2\varphi_0}{\ln\frac{R_2}{R_1}} \cdot \ln\frac{r}{R_1}, \qquad (7)$$

where $2\varphi_0$ is the potential difference between the electrodes. Using the same technique as in paragraph 1.1, we have for the cylindrical electrodes

$$r'' = \frac{1}{r} - \frac{em}{M^2} \cdot \frac{2\varphi_0}{\ln\frac{R_2}{R_1}} r. \qquad (8)$$

For the stationary case $r = R_{eq}$, when $r'' = 0$

$$R_{eq} = M \cdot \left(\frac{\ln\frac{R_2}{R_1}}{2em\varphi_0}\right)^{1/2}. \qquad (9)$$

In cylindrical co-ordinates

$$\begin{aligned} x'' + \frac{2}{R_{eq}^2} x - \frac{1}{R_{eq}^3} x^2 &= 0 \\ y'' &= 0 \end{aligned} \qquad (10)$$

In the horizontal plane the focusing term is twice as large as in the spherical deflector, and there is no focusing in the vertical plane. Besides, in the ideal case there is no coupling with the vertical plane.

1.3 Electrostatic quadrupoles

If the electrodes have potentials $\pm V_0$, then the electrical gradient is $G_E = \frac{2V_0}{r_B^2}$ and the equation of

motion is $\frac{d^2}{ds^2}\begin{pmatrix} x \\ y \end{pmatrix} \pm \frac{2eV_0}{mv_\vartheta^2 r_B^2}\begin{pmatrix} x \\ y \end{pmatrix} = 0$.

2 ORIGIN OF CURRENT LIMITATION

Now we will briefly analyse possible reasons to current limitations in electrostatic rings.

2.1 Dipole mode due to image charge

Any charged ion has an image charge due to the surrounding metal walls. Due to the different radius of the spheres $R_1 = \rho$ and $R_2 = \rho + 2h$ and the different signs of the curvature relatively to the beam a dipole force arises. It is proportional to the beam current and to the factor

$$\Delta F_{image} \propto \frac{\rho + h + \delta}{(h - \delta) \cdot (2\rho + 3h + \delta)} - \frac{\rho + h + \delta}{(h + \delta) \cdot (2\rho + h + \delta)},$$

where δ is the beam displacement from the center.

The force equals zero, when $\frac{\delta}{h} = \frac{1}{2} \cdot \frac{h}{\rho + h}$. Thus, shifting the equilibrium orbit by δ, we can compensate the dipole force and it cannot represent any serious problem for the current limit.

2.2 Influence from space charge

Knowing how the beam cross section depends on s, $\sigma(s) = \sigma_{av}(1 + \varepsilon_m \cos\omega \cdot s)$, where ε_m is the modulation amplitude, ω is the modulation frequency and using Courant-Snyder formalism, we can write the equation of motion in explicit form

$$\frac{d^2\xi}{ds^2} + \frac{v_0^2}{R^2}\left(1 - 2\frac{\delta v}{v_0} + 2\frac{\delta v}{v_0}\varepsilon_m \cos\omega \cdot s\right)\xi = 0. \qquad (11)$$

The average space charge tune shift δv determines the global current limit, when the particle crosses the half integer resonance. In particular, for ELISA we get the global current limit $I_{G-\max} = 6\mu A$. However, due to the envelope modulation the parametric resonance can be excited as well. For the ELISA lattice with the spherical deflector ($\varepsilon_m = 65$) the parametric resonance occurs, when the current is 70 nA. In reality such a deep density modulation is not achieved due to space charge itself. We have performed simulations of beam dynamics with space charge by the code TrackFMN [3]. It has been shown that the current limit for ELISA with the spherical deflector is about half a µA and with the cylindrical deflectors a few µA because of the smaller envelope modulation.

2.3 Finite electrostatic deflectors

In reality the deflector consists of plates, which are finite segments of spheres or cylinders and the edges distort the field. We have calculated the 3D field by MAFIA and determined the "good-field" region for the beam to ±8 mm for the spherical deflector and more for the cylindrical.

2.4 Longitudinal-transverse coupling

The spherical and cylindrical deflectors have a central field symmetry, namely $E \propto \frac{\alpha}{r^n}$, where

$\frac{\partial L}{\partial \vartheta} = 0$ and $mv_\vartheta r = const$. The angular momentum plays a crucial role for the focusing features of both deflectors. In particular, if the kinetic energy $T = mv_\vartheta^2/2$ is constant and M varies, then in the horizontal plane the spherical deflector de-focuses and the cylindrical deflector is just a drift. But obviously, for both of them energy conservation is fulfilled:

$$W(r,\dot{r}) = \frac{m}{2}\dot{r}^2 + \frac{M^2}{2mr^2} + \varphi(r) = \frac{m}{2}\dot{r}^2 + e\varphi_{eff} . \quad (12)$$

The last expression shows that the radial motion could be considered as one-dimensional motion in the field with the effective potential $\varphi_{eff}(r)$ consisting of the centrifugal potential and the electrical potential. But the centrifugal potential is the kinetic energy along the ϑ direction. Therefore such deflectors lead to exchange of energy between the transverse and longitudinal planes.

Actually, from the expression (5) we can see that each particle is focused around it's own equilibrium radius $R_{eq} \propto M^2$ dependent on the initial angular momentum, in contrast to the magnetic ring, where $R_{eq} \propto p = mv$. Therefore, the linear equation for particles with non-equilibrium angular momentum in the horizontal plane is

$$x'' + \frac{1}{R_0^2}x = \frac{1}{R_0}\cdot 2\frac{\Delta M}{M} , \quad (13)$$

where R_0 is the equilibrium radius for $\Delta M = 0$. The new equilibrium radius is $R_{eq} = R_0(1 + 2\Delta M / M)$. Taking into account $\Delta M / M = \Delta v / v_0 + x_0 / R_0$, where $\Delta v / v_0$ is the initial velocity spread and x_0 is the initial particle deviation, the maximum deviation of the new equilibrium orbit is

$$\Delta R_{max} = 2R_0 \cdot \left[\left(\frac{\Delta v}{v}\right)_{max} + \frac{\sqrt{\varepsilon_x \beta_x}}{R_0}\right], \quad (14)$$

where ε_x, β_x are the emittance and the β-function at the entrance to the deflector. The expression (14) shows how far the potential well is shifted from the deflector centre. Obviously, the particle oscillating around the new orbit has a higher probability to be lost. In particular, stable motion for a parallel monochromatic beam, when all particles have $\dot{r} = 0$ and $\Delta v / v = 0$, is limited by the radius $r = 25 \pm 0.5$ cm in the spherical deflector. For the non-parallel and non-monochromatic beam the stable region even is smaller.

We considered the instantaneous stability in one deflector. But what will happen after many turns of the particles? Using the condition $mv_\vartheta r = const$, we can write the change of velocity after one passage through the deflector $\frac{\Delta v}{v} = -\frac{x - x_0}{R_{eq}} + \frac{\Delta v_0}{v}$. Now let us call that the matrix of the transport channel between two deflectors M_{tr}. Then after n turns

$$\left\{x, x', \frac{\Delta v}{v}\right\}_n = \left(M_{tr} \cdot M_{Def} \cdot M_{tr}\right)^n \cdot \left\{x_o, x_0', \frac{\Delta v_0}{v}\right\}. \quad (15)$$

If the total matrix between two deflectors changes sign of the particle deviation $sign(x) \Rightarrow -sign(x)$, the conversion of the potential energy to kinetic energy will be continued. It is easy to image the opposite situation, when the particle will lose kinetic energy and increase the potential energy. The process of energy conversion has an oscillating character, since the tune of the ring is not an integer.

However, the conversion process is suppressed by the edge field, which changes the angular momentum at the entrance and the exit of the deflector and therefore decreases the equilibrium orbit shift. The coefficient of suppression k is calculated numerically and its influence is given from the next expression $\frac{\Delta v}{v} = -\frac{1}{k}\cdot\frac{x - x_0}{R_{eq}} + \frac{\Delta v_0}{v}$. We have performed multiplication of a few thousands matrixes and found that the momentum spread grows, what obviously causes losses. Due to the different coefficient k the particles increase their amplitude with different increment.

We believe that this mechanism is the main mechanism of the reduction in stable motion.

To reduce the influence of this mechanism we should decrease the dependence of the equilibrium radius on the initial parameters. In particular, for the cylindrical deflector the equilibrium deviation is half that for the spherical deflector $\Delta R_{eq} = R_0 \cdot \Delta M / M$. Numerical simulations of the beam in the real field by the 3D code SIMION have confirmed this result and have shown the obvious advantages of the cylindrical deflectors.

3 CONCLUSIONS

We have studied the beam dynamics in electrostatic rings. Potential candidates to limit the beam intensity are the parametric resonance due to modulation of the space charge tune shift and coupling between the longitudinal and transverse directions. We considered two lattices of ELISA and have shown that the lattice with cylindrical electrodes is more stable and has a higher current limit with respect to both effects.

4 REFERENCES

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