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IMPROVED WAVEFORMS FOR BARRIER-BUCKET SYSTEMS

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Abstract

Barrier-Bucket (BB) systems provide a method to apply a short gap to a coasting beam. This is utilized for different applications, like ion cleaning, or to compensate the medium energy loss caused by internal experiments. BB-cavities are broadband cavities, and the applied signal is commonly a short sine burst, followed by a flat section at zero voltage. Since the transfer function of the BB-system is usually neither flat nor linear, it is common to predistort the signal to obtain the desired shape at the rf gap. Nevertheless, the resulting waveform still has a ripple in the flat section. This is due to the lowpass characteristic of the amplifier and the sharp edges at the ends of the sine, which lead to an infinite number of harmonics. This paper provides better suited BB-waveforms, which are designed with a finite number of harmonics from the beginning. It is shown that a much better flatness can be achieved than for a conventional BB-waveform, without sacrificing any performance. These advanced waveforms are currently used at the hadron synchrotron COSY at Forschungszentrum Jülich, leading to improved BB-bunch shapes, in particular for electron-cooled beams.

INTRODUCTION

Barrier bucket (BB) cavities offer a way to gain more control over the bunch shape in a synchrotron. They periodically apply short pulses to the beam, each shaped like one single sinusoidal period. The length of the pulse is only a fraction (e.g. 20 %) of the revolution time, so most phases do not experience any accelerating or decelerating fields. This results in a very long bucket. In addition, most of the bucket is of one potential, so beams of low momentum spread homogeneously along the bucket.

At COSY Jülich, a BB system is used to compensate the energy loss caused by the internal experiments [1]. A constant event rate is desired for the experiments. A bunched beam counteracts this need. The BB is a good compromise because the beam intensity is almost constant for a large fraction of each revolution.

For HESR at FAIR, two BB systems are planned to provide efficient longitudinal stacking [2]. They are shifted against each other to open new buckets in longitudinal phase space. Furthermore, the large gap allows for ion cleaning of residual gases in the beam pipe.

Other applications for barrier buckets include pre-compression of the beam [3], reduction of beam loss during extraction [4], or aid for transition energy crossing [5].

BB systems typically consist of a signal source such as an arbitrary waveform generator, amplifiers, a loaded cavity, and long cables. Unlike acceleration cavities, such systems must have good broadband performance. The barrier signal can be described by its Fourier decomposition. Since short pulses result in high harmonics, the system must be able to handle frequencies in the order of at least 12 times the rotational frequency.

The exact shape of the barrier signal is not clearly defined. Rather, it must meet several characteristics. The barrier part of the signal should be symmetrical. This is to achieve symmetrical bunches, but most importantly to be free of a DC component. The barrier width must not exceed a certain maximum value. In addition, the bucket section should be as flat as possible. A ripple in the the bucket can lead to micro bunches, as was observed at COSY during beam cooling.

In general, the system response is neither flat nor linear in phase. This results in completely distorted voltage shapes at the gap of the cavity. Various strategies have been evaluated to predistort the source signal in such a way that a good shape is obtained at the gap. Furthermore, it is possible to compensate the nonlinear behavior of the amplifiers in the high power regime [6]. All strategies have in common that the system response is measured, so that each harmonic in the signal can be corrected in amplitude and phase.

However, one source of distortion that cannot be addressed by system response compensation is the low-pass nature of the system. Frequency components that are attenuated to zero by the system cannot be restored by increasing their spectral power at the input. Thus, a sinusoidal pulse signal will always result in a broadened pulse and a well-defined ripple in the bucket after low-pass filtering, no matter how much the signal is predistorted.

In this paper, this problem is circumvented by optimizing the undistorted barrier signal itself. Different construction strategies for band-limited BB signals are presented and compared. Furthermore, an attempt is made to find the best possible signal by optimizing the individual Fourier coefficients.

SIGNAL SHAPES

To find an appropriate band-limited signal shape, one must first define some quality attributes. Figure 1 shows two signals. The first is a perfect one-period sinusoidal burst,

$$s_1(t) = \sin\left(0.2 \cdot 2\pi \frac{t}{T}\right) \Pi_{0.2}\left(\frac{t}{T}\right) \quad \text{for} \quad |t| \le \frac{T}{2}.$$

The signals are periodic in T. The displayed interval from 0 to T was chosen for a bucket-centered visualization.

Since s_1 is not differentiable at the ends of the barrier, the signal has infinite harmonics. The Fourier sinusoidal coefficients,

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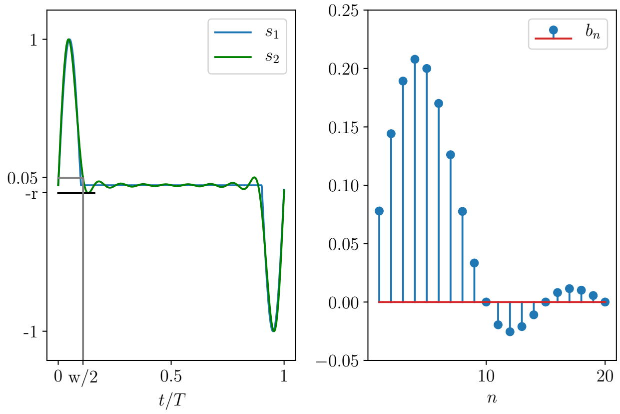

Figure 1: Definition of width and ripple of BB signals. Left: Signal shape of a perfect sinusoidal burst signal s_1, and the same burst but low-pass filtered after the ninth harmonic and renormalized to [-1, 1], s_2. Right: Fourier sinusoidal coefficients b_n of s_1.

$$b_n = \frac{\alpha}{\pi}\left(\frac{\sin\left(\frac{\pi}{T}(\alpha n - 1)\right)}{(\alpha n - 1)} - \frac{\sin\left(\frac{\pi}{T}(\alpha n + 1)\right)}{(\alpha n + 1)}\right)$$

are shown in the right plot for $\alpha = 0.2$. Due to its odd symmetry, the spectrum consists only of sinusoidal components.

Truncating the spectrum after the ninth harmonic results in the signal s_2, also shown in the left diagram. Due to its band-limited nature, it has a smooth transition between barrier and bucket part, and the barrier has become wider. To define the width w of the barrier in a very general way, the point was chosen where the value drops to 5% of the peak value. The unfiltered signal s_1 has a width of 19.7%, s_2 of 22.0%. Furthermore, the bucket is no longer flat. The resulting ripple is quantified by the maximum of its absolute value, r. For s_2, it is 5.4%.

Obviously, truncation of sinusoidal-bust signals is not a reasonable way to generate band-limited BB signals. A sharp edge at the end of the barrier cannot be produced with typical bandwidth, nor is it really needed for pulse shaping purposes. A flat bucket, on the other hand, is critical for homogeneous particle distribution in beams with low momentum spread. Therefore signal shapes with a small ripple r must be found.

Here, three different soft-edge barrier strategies are compared: A raised cosine windowed sinusoidal burst, two Gaussian pulses, and the best solution found by an optimizer. Their shapes are illustrated in Figure 2.

For comparison, the unfiltered sinusoidal burst

$$s_s(t) = -\sin\left(\frac{1}{\alpha}\cdot 2\pi\,\frac{t}{T}\right)\,\Pi_\alpha\left(\frac{t}{T}\right) \quad \text{for} \quad |t| \le \frac{T}{2}.$$

is used. The α parameter specifies the width of the window. This is not exactly the same as the 5%-width w.

The raised-cosine window is a common filter function in digital signal processing. It has a strong harmonic rejection. Applied to the sinusoidal burst,

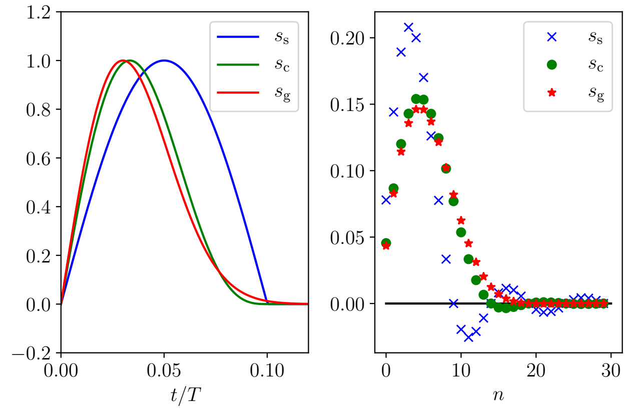

Figure 2: Comparison of the unfiltered shapes of sinusoidal burst s_s, raised-cosine windowed sine s_c, and Gaussian pulses s_g. The s_s and s_c signals have a parameter of $\alpha = 0.2$. The Gaussian pulses in s_g have a standard deviation of $\sigma = \pi/(4 \cdot 0.2)$.

$$s_c(t) = s_s(t)\ \cdot\ \frac{1}{2}\left(1 + \cos\left(\frac{1}{2\alpha}\cdot 2\pi\,\frac{t}{T}\right)\right),$$

results in the desired barrier shape, but will reduce the width considerably.

The third shape is constructed from two Gaussian pulses, one with a positive sign and one with a negative sign:

$$s_g(t) = \exp\left(-\frac{1}{2}\left(\frac{t/T - \sigma}{\sigma}\right)^2\right) - \exp\left(-\frac{1}{2}\left(\frac{t/T + \sigma}{\sigma}\right)^2\right).$$

The standard deviation σ defines the width of the barrier. The choice of Gaussian pulses is motivated by the uncertainty principle of the Fourier transform. Short pulses in the time domain lead to wide pulses in the frequency domain and vice versa. The minimum of the product of the two widths is only achieved by normal distributions [7]. Therefore, it seems reasonable that a train of periodic Gaussian pulses will have fast decaying harmonics, and consequently a truncation of the harmonics will have little effect on the signal shape.

As can be seen in the figure, a major difference between the two approaches is the behavior in the bucket. While s_c is exactly zero, the Gaussian pulses have a soft transition. The fourier coefficients of s_c oscillate around zero, but the coefficients of s_g again have a Gaussian shape and decay monotonously.

APPROACH

The quality of the signal is limited by two parameters. The first is the number of harmonics h. It is determined by the frequency limit of the system divided by the revolution frequency of the beam. Second, the desired width w of the barrier. Narrower barriers require more harmonics, or lead to more ripple if h is kept constant.

Low-pass filtering of the ansatz functions leads to variable pulse widths. Thus, for each pair of (h, w_0), a root finding

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method is used to find the correct parameters α or σ that lead to the desired width $w(\{\alpha, \sigma\}) - w_0 = 0$.

To find the best solution, an optimization problem was defined. For a given desired barrier width w_0 and a number of harmonics h, the flattest bucket is sought. While the h coefficients b_n are the variables of the optimization problem, w is a result of the computation. Thus, a cost function must be defined that minimizes r, but $|w - w_0|$ at the same time and with a higher priority:

$$f_{\text{cost}}(r, w) = r + c_1 \begin{cases} c_2(w - w_0) & \text{if } w < w_0 \\ w_0 - w & \text{else.} \end{cases}$$

The constant c_2 allows to weight slimmer barriers less than thicker ones. With $c_2 = 0$ all barriers that are thinner than the threshold are of equal quality. The other constant, c_1, defines the different weights of ripple and width. Since the width of the barrier is of higher priority than the ripple, c_1 should be chosen sufficiently large. Good results have been obtained with c_1=10 and c_2=1. Smaller values of c_2 didn't give satisfactory results.

The cost function has local minima. For example, the solution for a sinusoidal burst with alternating Fourier coefficients and the solution for a double Gaussian pulse with its monotonically decaying coefficients do not converge to the same optimum solution with a gradient method. Therefore, it is difficult to estimate whether the global minimum has been found. For this work, the basin-hopping algorithm of the SciPy optimization module was used [8].

RESULTS AND CONCLUSION

Figure 3 shows the ripple as a function of the number of harmonics, for barrier widths of 15%, 20%, and 25%.

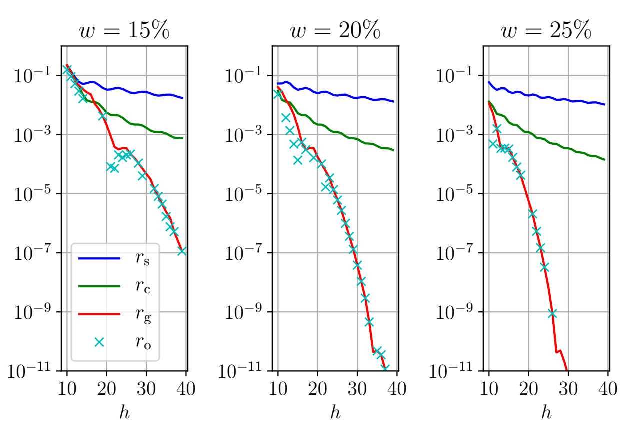

Figure 3: Ripple of different signal shapes with respect to the number of harmonics.

As expected, the ripple of all shapes decreases as the signal contains higher frequency components. The low-pass filtered sinusoidal burst always has the largest ripple, s_c, s_g and s_o perform significantly better. E.g., for w=20% and h=12, r_s is about 6%, r_c and r_g are less than 2% and r_o is less than 0.4%. For large values of h, the ripple decreases faster for the raised-cosine windowed signals than for the sine-burst signals. The ripple of the double-Gaussian pulse signals decays even faster. The optimizer was not able to find better solutions, but this is not of technical interest for such small values of r.

For realistic systems, a ripple of less than 1% is desirable. Much smaller values are generally difficult to achieve due to the limitations of pre-distortion. But such systems are also often severely limited in bandwidth.

For small harmonic numbers, neither the raised cosine nor the double Gaussian is always better for all h. But they are both much better than the sine burst. So if a strong minimization of the ripple is not very important, using one of the two smooth functions is already a huge improvement.

On the other hand, if minimizing the ripple is critical, optimizing the coefficients may be beneficial. For the given plots, the optimizer sometimes found no better solution at all, or failed completely. But in some regimes, such as w=20%, h=13, a signal with a ripple about three times smaller than the best analytical approach was found.

In conclusion, a soft-edged barrier shape should always be preferred. Further effort can be put into minimizing the ripple if it is worthwhile for the given application. For COSY, a new Gaussian or raised-cosine signal is manually generated for each experiment using the BB system, depending on the energy, barrier width and cooling requirements. For HESR, the bandwidth of the system is sufficiently large with respect to the rotation frequency, so a Gaussian-shaped signal will give satisfactory results.

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