Geometric phase effect in a cpEDM ring

Offset of bends and longitudinal magnetic fields

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Outline

- Hybrid Ring Concept: symmetric hybrid ring design
- Study of systematic effects: geometric phase effect and analytical derivations
- Simulation results for the cw and the ccw beams
  - BMAD implementation
  - Spin tracking results and comparison with analytical estimates-longitudinally polarized beam
  - Spin tracking results: the case of a radially polarized beam
- Study of a more realistic case: all bendings with radial misalignments and longitudinal magnetic fields at many locations
  - Spin tracking results for a longitudinally polarized beam
  - Spin tracking results for a radially polarized beam
- Conclusions

Hybrid Ring Concept: symmetric proposal

Z. Omarov and Y. K. Semertzidis, PRAB 105, 032001 (2022)

- Ring operated at "magic energy" and focusing using magnetic quadrupoles
  - Gradients of electric bendings must be avoided

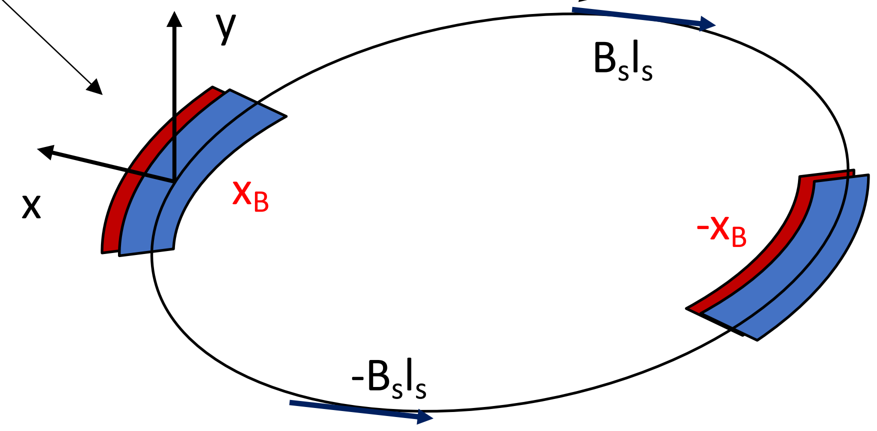
- Features and consequences
  - No spin rotation proportional to average radial magnetic field
  - Lattice different for counter-rotating beams - tuning more delicate (tunes, closed orbit, spin coherence)
  - Systematic effects to be evaluated with care → unwanted electric gradients, higher magnetic fields, etc.

Quantity	Value
Bending radius R_0	95.49 m
Number of periods	24
Electrode spacing	4 cm
Electrode height	20 cm
Deflector shape	Cylindrical
Radial bending E field	4.4 MV/m
Straight section length	4.16 m
Quadrupole length	0.4 m
Quadrupole strength	± 0.21 T/m
Bending section length	12.5 m
Bending section circumference	600 m
Total circumference	800 m
Cyclotron frequency	224 kHz
Revolution time	4.46 μs
$\beta_x^{\max}$, $\beta_y^{\max}$	64.54 m, 77.39 m
Dispersion, $D_x^{\max}$	33.81 m
Tunes, Q_x, Q_y	2.699, 2.245
Slip factor, $\frac{dt}{t}/\frac{dp}{p}$	-0.253
Momentum acceptance, (dp/p)	5.2×10^{-4}
Horizontal acceptance (mm · mrad)	4.8
RMS emittance (mm · mrad), ϵ_x, ϵ_y	0.214, 0.250
RMS momentum spread	1.177×10^{-4}
Particles per bunch	1.17×10^{8}
RF voltage	1.89 kV
Harmonic number, h	80
Synchrotron tune, Q_s	3.81×10^{-3}
Bucket height, $\Delta p/p_{\text{bucket}}$	3.77×10^{-4}
Bucket length	10 m
RMS bunch length, σ_s	0.994 m

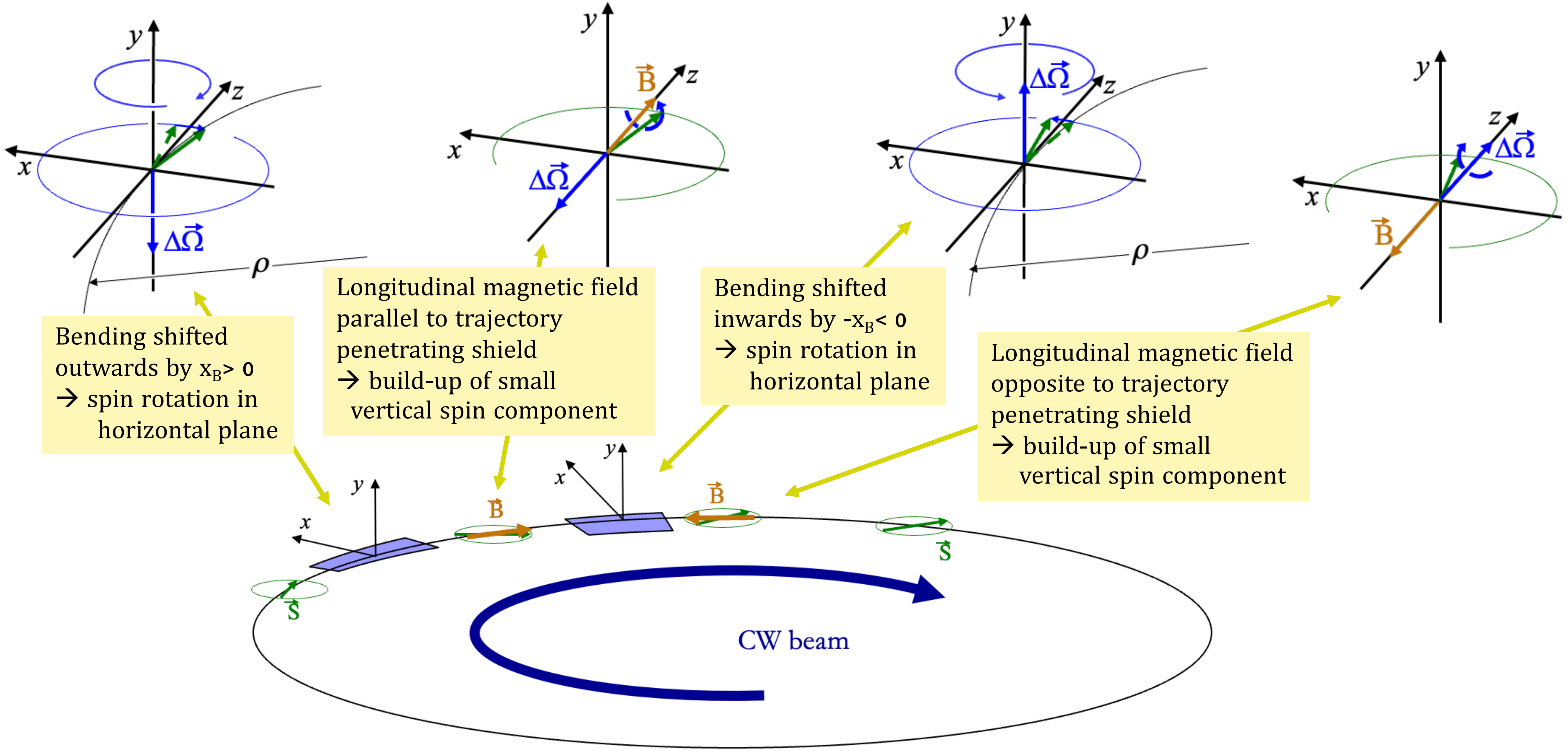
Study of systematic effects: Geometric phase effect

- This study includes the BMAD implementation of the symmetric hybrid ring lattice proposal and the study of 2nd order systematic effects:
  - Geometric phase effect → offset of bends and longitudinal magnetic fields
  - The offset of the bending magnets considered is $\pm$ 10 mm with no net deflection
    → Simpler to understand (analytical derivation)
    → Electrode spacing error compensating offset (in practice impossible to disentangle deflection from offset and electrode spacing)
  - The longitudinal magnetic fields $B_s l_s$ considered are: 0 nTm, 1 nTm, 10 nTm and 100 nTm to see how the spin components behave in all the cases
  - The case with the radially polarized beam has been considered as well, with the same offsets and the same magnetic fields

Same effect for the hybrid and the fully electrostatic scheme



Study of systematic effects: Geometric phase effect



Study of systematic effects: Geometric phase effect

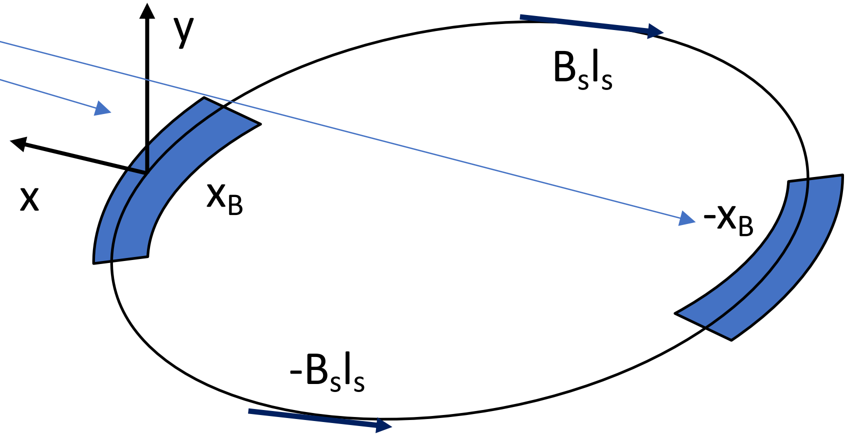
- Derivations for the analytical estimates

→ A horizontal offset of bend induces spin rotations around the vertical axis

- Assume no deflection due to offset to simplify (deflection due to electrode spacing cannot be disentangled from offset)
- Change of relativistic gamma by a bending offset of x_B $\Delta\gamma = -\frac{qE_x}{mc^2}x_B = \beta^2\gamma\frac{\alpha_B}{l_B}x_B$
- Angular frequency of spin rotation with respect to direction

$$\Delta\omega_y = \frac{q}{m}\left(G - \frac{1}{(\gamma+\Delta\gamma)^2 - 1}\right)\frac{\beta E_x}{c} = \frac{q}{m}\frac{2\gamma\Delta\gamma}{(\gamma^2-1)^2}\frac{\beta E_x}{c} = -2\frac{\beta}{\gamma}c\left(\frac{\alpha_B}{l_B}\right)^2 x_B$$

- Change of radial spin $\Delta s_x = \frac{l_B}{\beta c}\Delta\omega_y = -\frac{2}{\gamma}\frac{\alpha_B^2}{l_B}x_B$



→ An integrated magnetic field generates a rotation around the longitudinal axis

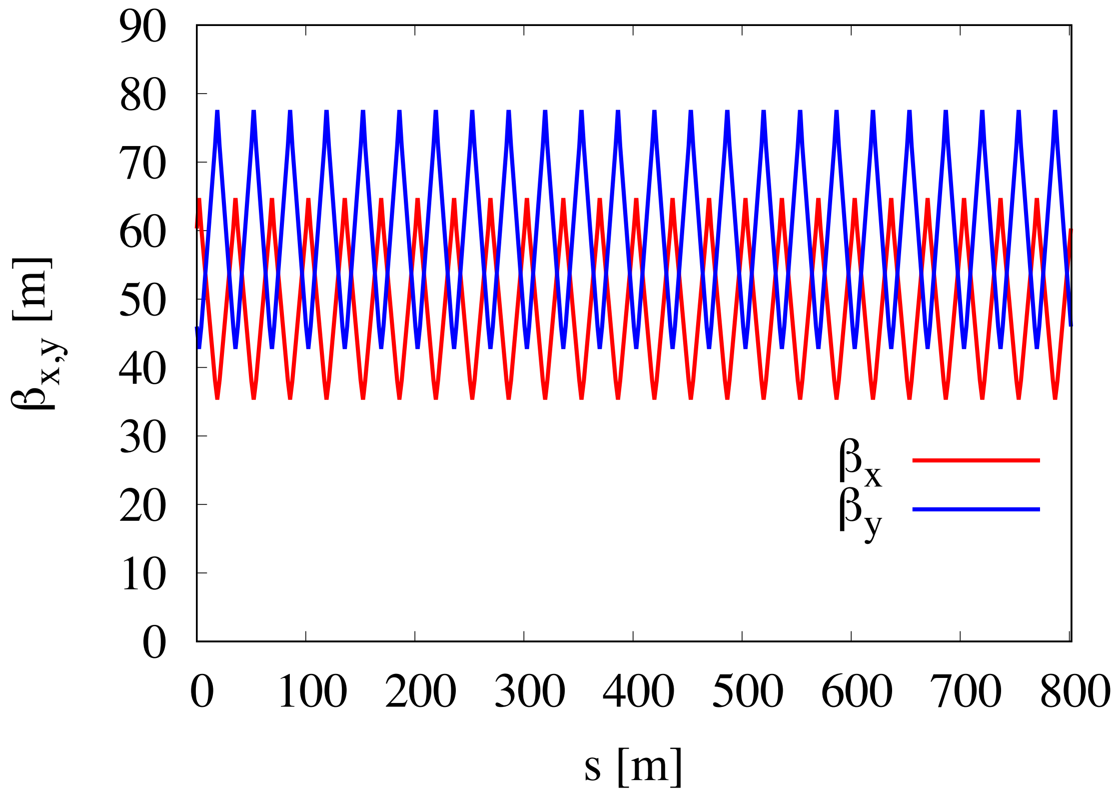
- Vertical spin per turn $\Delta s_y = 2\frac{l_B}{\beta c}\Delta\omega_y s_x = 2\frac{q}{m}\frac{G+1}{\beta\gamma c}B_s l_s s_x$
- Build up rate $\frac{d\,s_y}{dt} = \frac{\Delta s_y}{C/\beta c}$

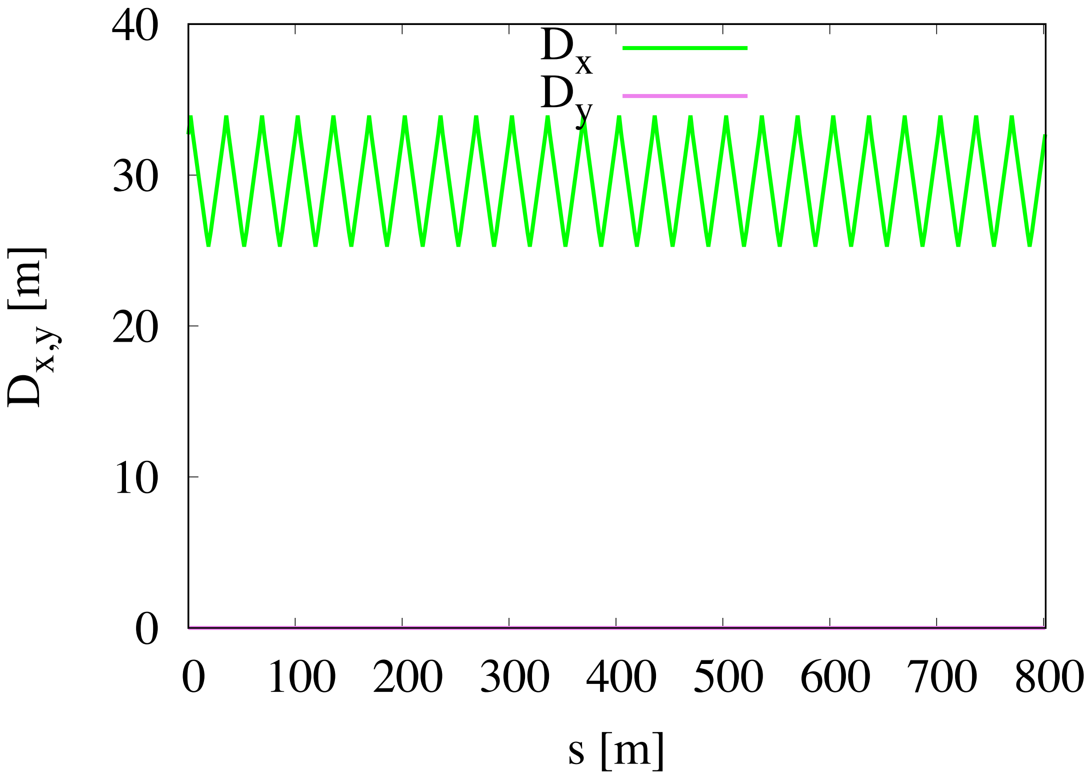
Simulation results

Simulation results for the cw beam

- BMAD Implementation

  → Lattice implementation and evaluation of the Twiss parameters, horizontal dispersion and closed obit

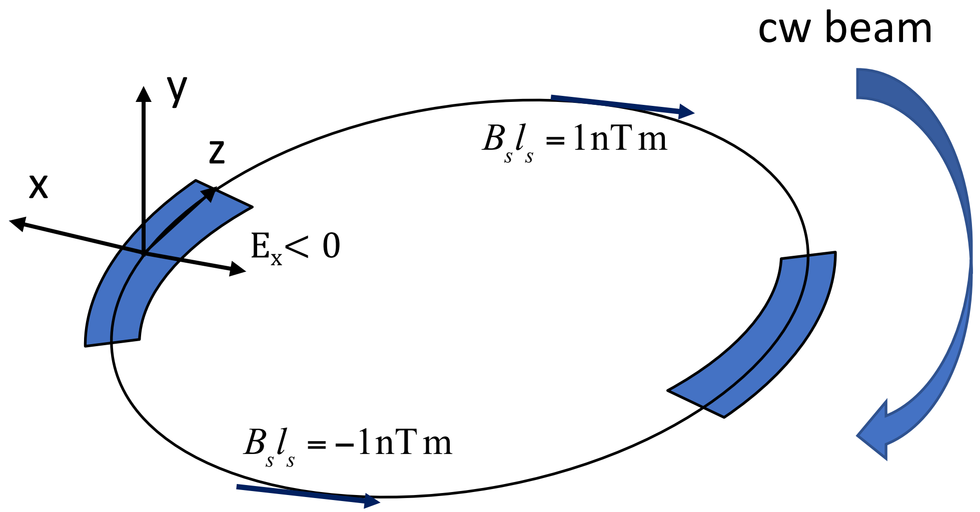




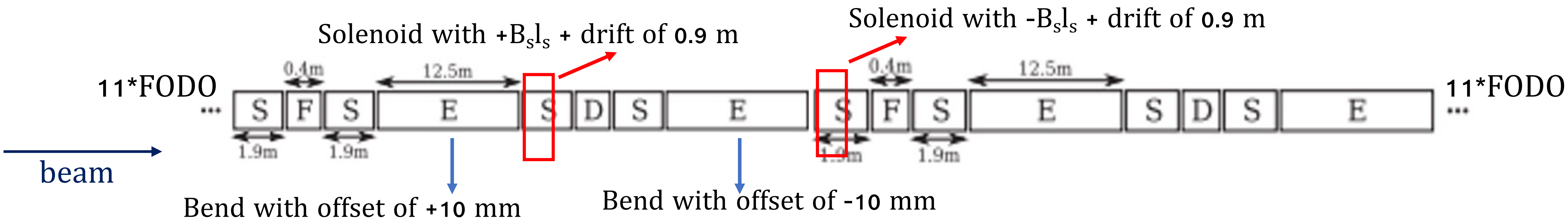
Simulation Results for the cw beam

- BMAD Implementation

  → Horizontal offset of the bends of $x_B = \pm 10$ mm
  → Correction of the horizontal offsets (by modifying the bending strenghts)
  → Adding a longitudinal magnetic field (different values for comparison of the results) → 1,10,100 nTm cases
  → Correction of the energy to reduce the horizontal spin build up



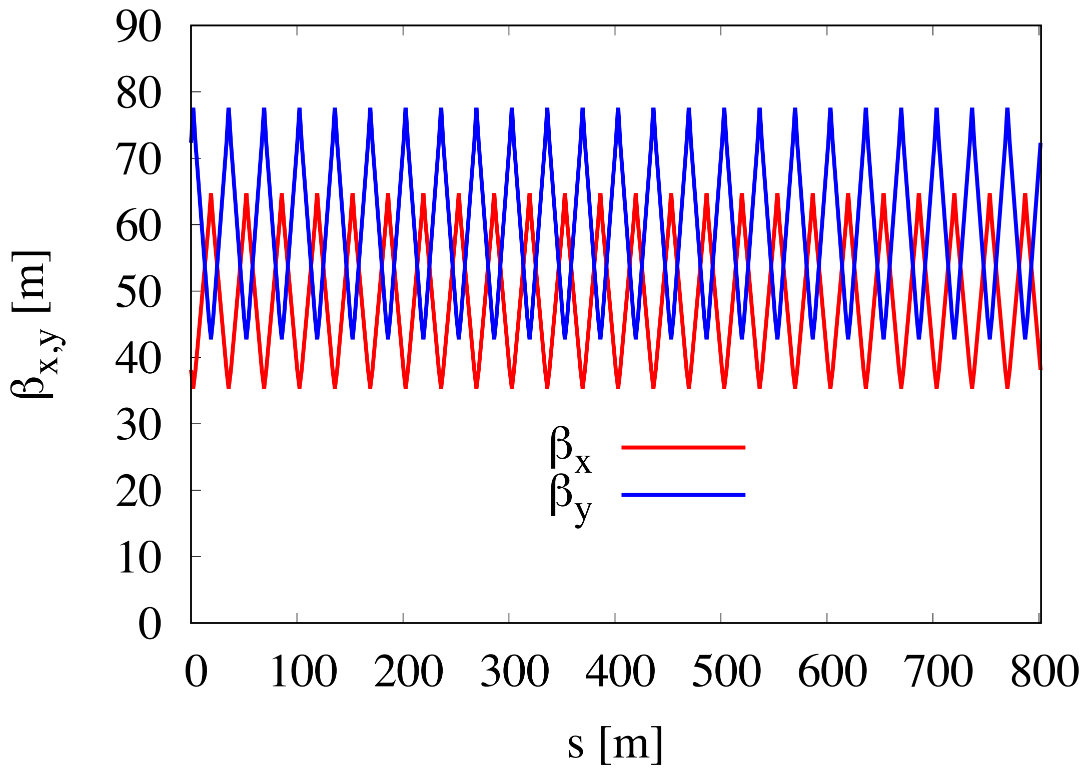
- In BMAD lattice file:

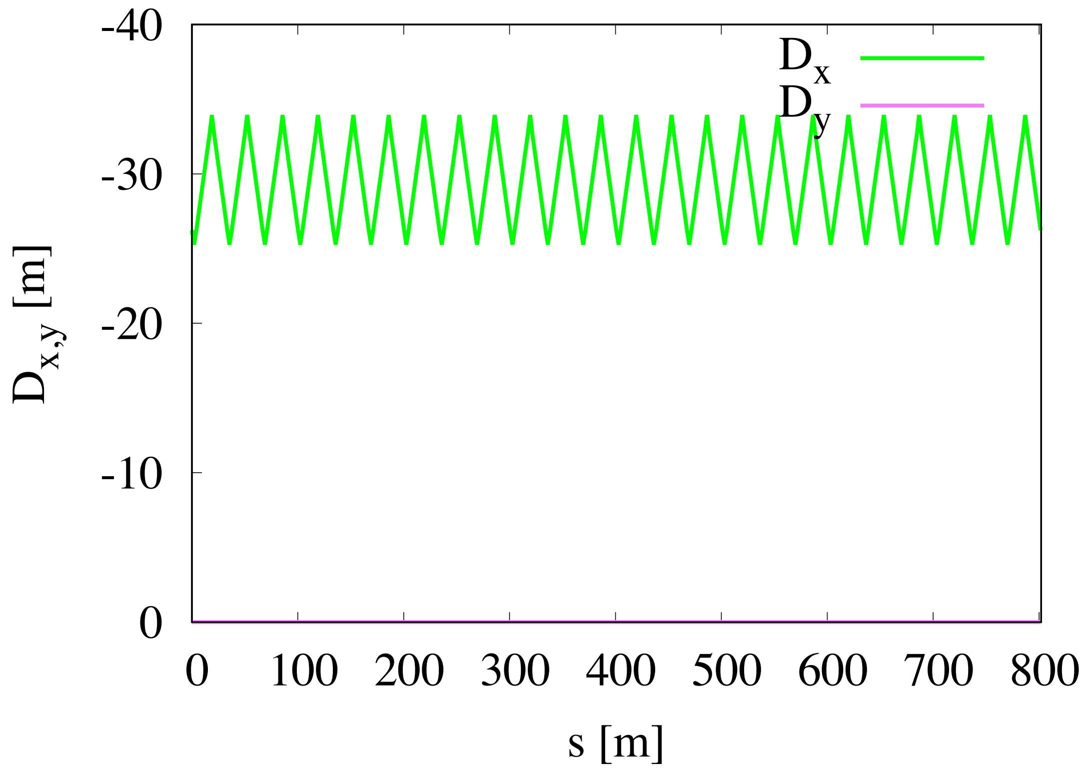


Simulation results for the ccw beam

- BMAD Implementation

  → Lattice implementation and evaluation of the Twiss parameters, horizontal dispersion and closed orbit

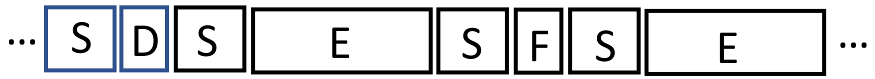


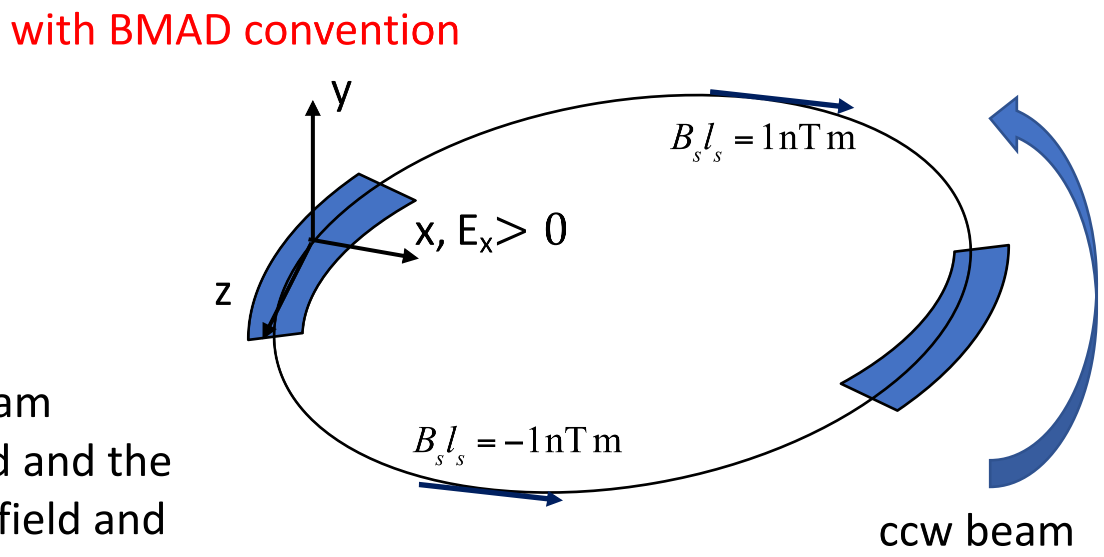


Simulation results for the ccw beam

- BMAD Implementation

→ The FODO cell has been inverted to produce the ccw beam:

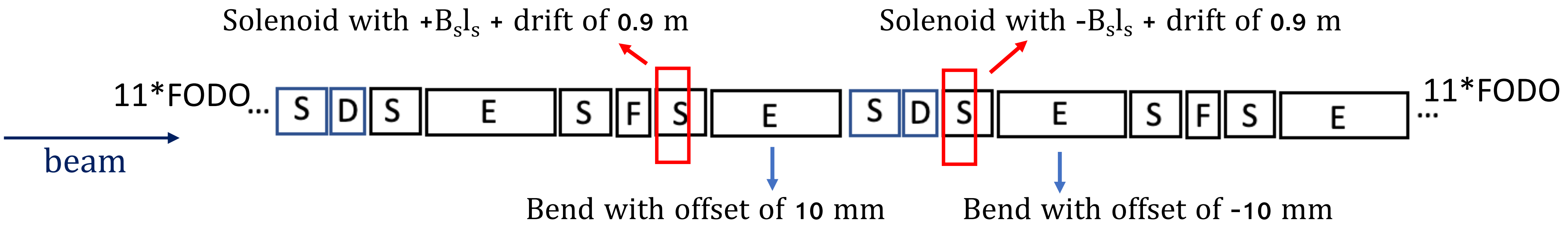




→ The sign of the electric and the magnetic field has changed

→ The same systematic effects has been studied but with the beam passing before through the positive longitudinal magnetic field and the positive offset and after going towards the negative magnetic field and the negative offset

- In BMAD lattice file:



Simulation Results

- Comparison with analytical estimates (formulas seen before)

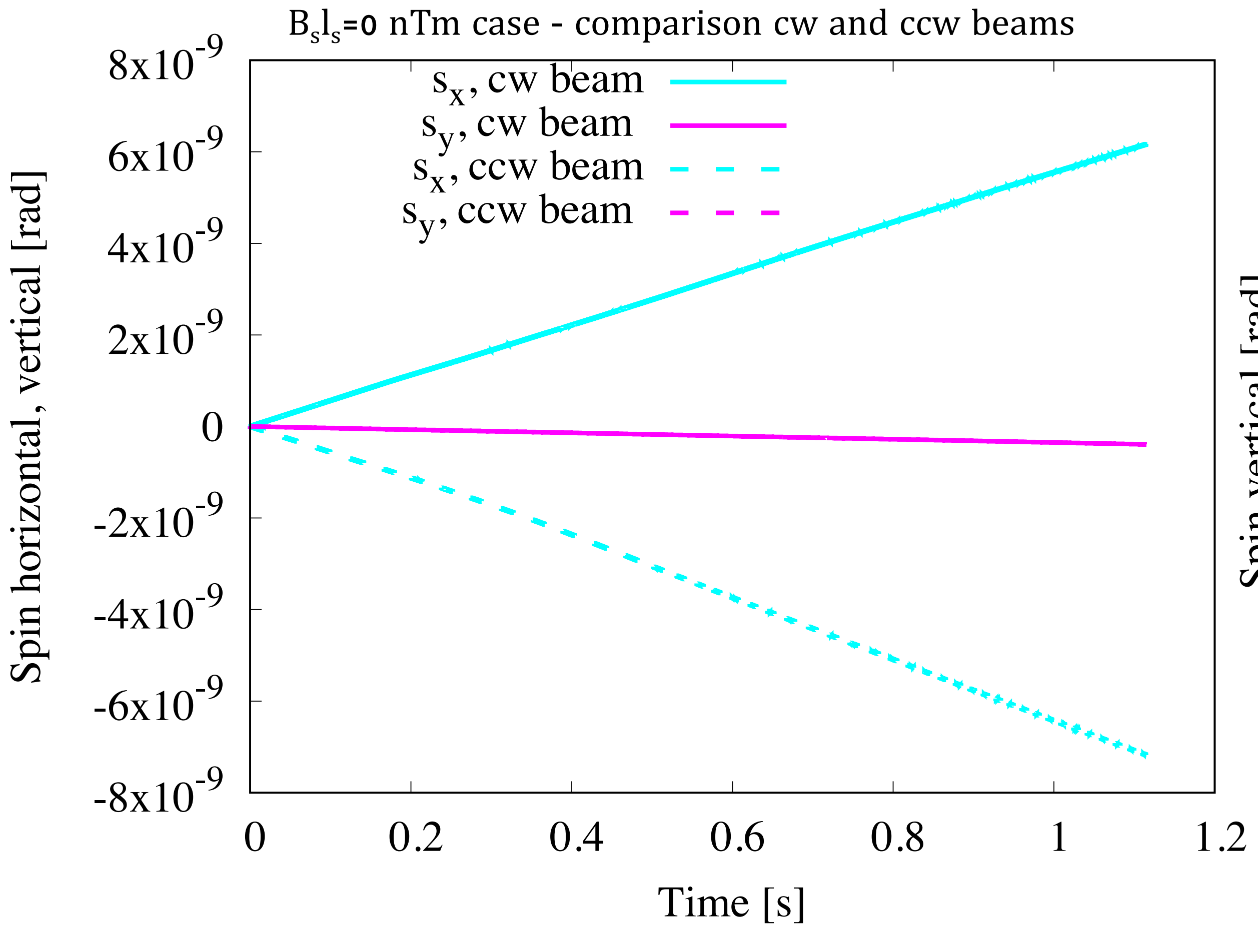
→ Horizontal offset $x_B = \pm 10$ mm of bends induces
  - Change of radial spin $\Delta s_x = -22\ \mu$rad
    and spin $\pm s_y = \pm 11\ \mu$rad at the bending exit and entrance
  - with the bending length of $l_B = 12.5\,\mathrm{m}$ and the deflection of $\alpha_B = \pi/24$

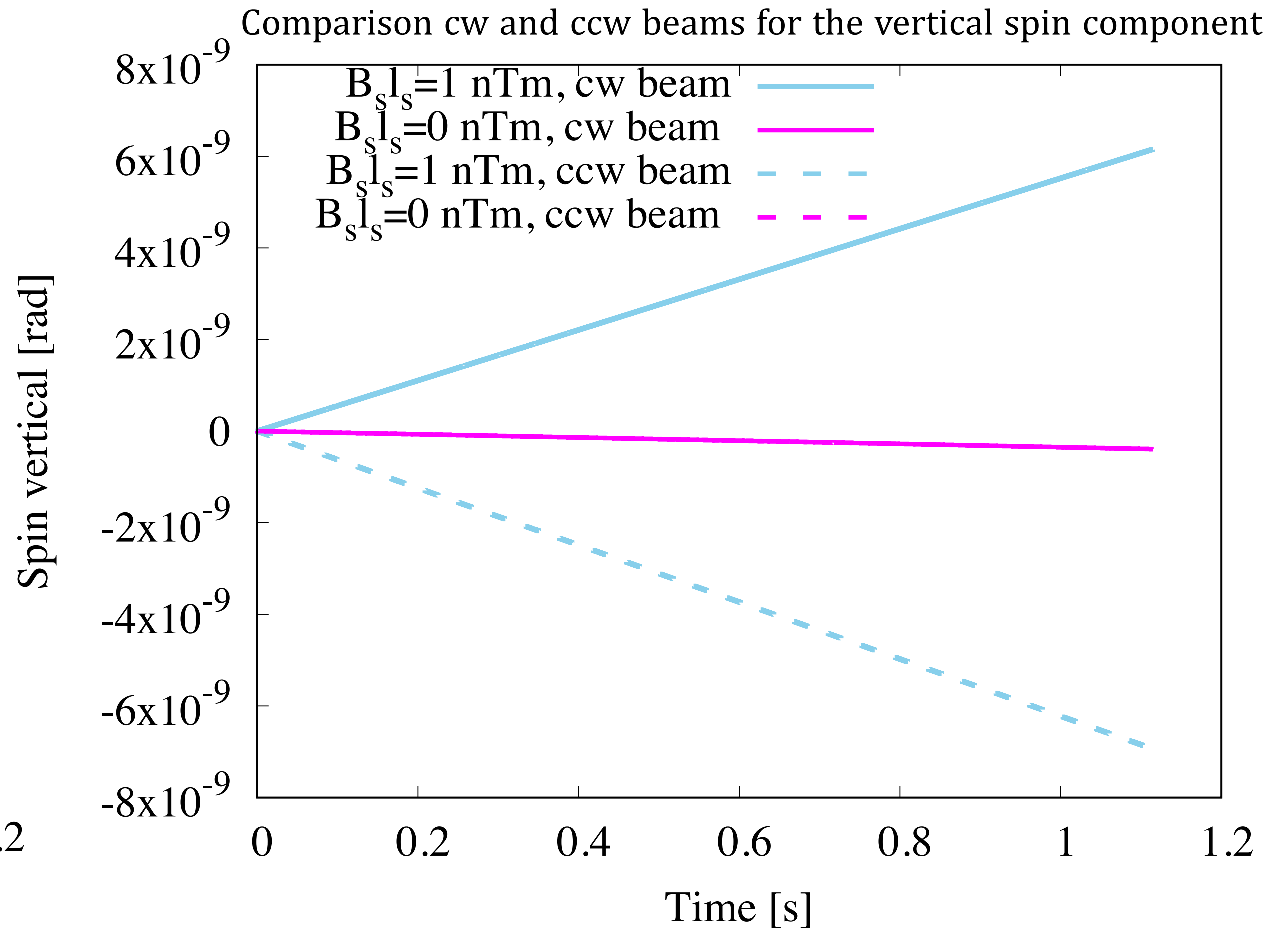
$B_s l_s$	S_y per turn (analytical)	S_y per turn (simulation)	Build-up rate (analytical)
1 nT m (cw)	$2.6 \cdot 10^{-14}$ rad	$2.46 \cdot 10^{-14}$ rad	5.9 nrad s^{-1}
10 nT m (cw)	$2.6 \cdot 10^{-13}$ rad	$2.60 \cdot 10^{-13}$ rad	59 nrad s^{-1}
100 nT m (cw)	$2.6 \cdot 10^{-12}$ rad	$2.62 \cdot 10^{-12}$ rad	590 nrad s^{-1}
1 nT m (ccw)	$-2.6 \cdot 10^{-14}$ rad	$-2.77 \cdot 10^{-14}$ rad	-5.9 nrad s^{-1}
10 nT m (ccw)	$-2.6 \cdot 10^{-13}$ rad	$-2.63 \cdot 10^{-13}$ rad	-59 nrad s^{-1}
100 nT m (ccw)	$-2.6 \cdot 10^{-12}$ rad	$-2.62 \cdot 10^{-12}$ rad	-590 nrad s^{-1}

showing good agreement with simulations…

Simulation results for the cw and the ccw beam

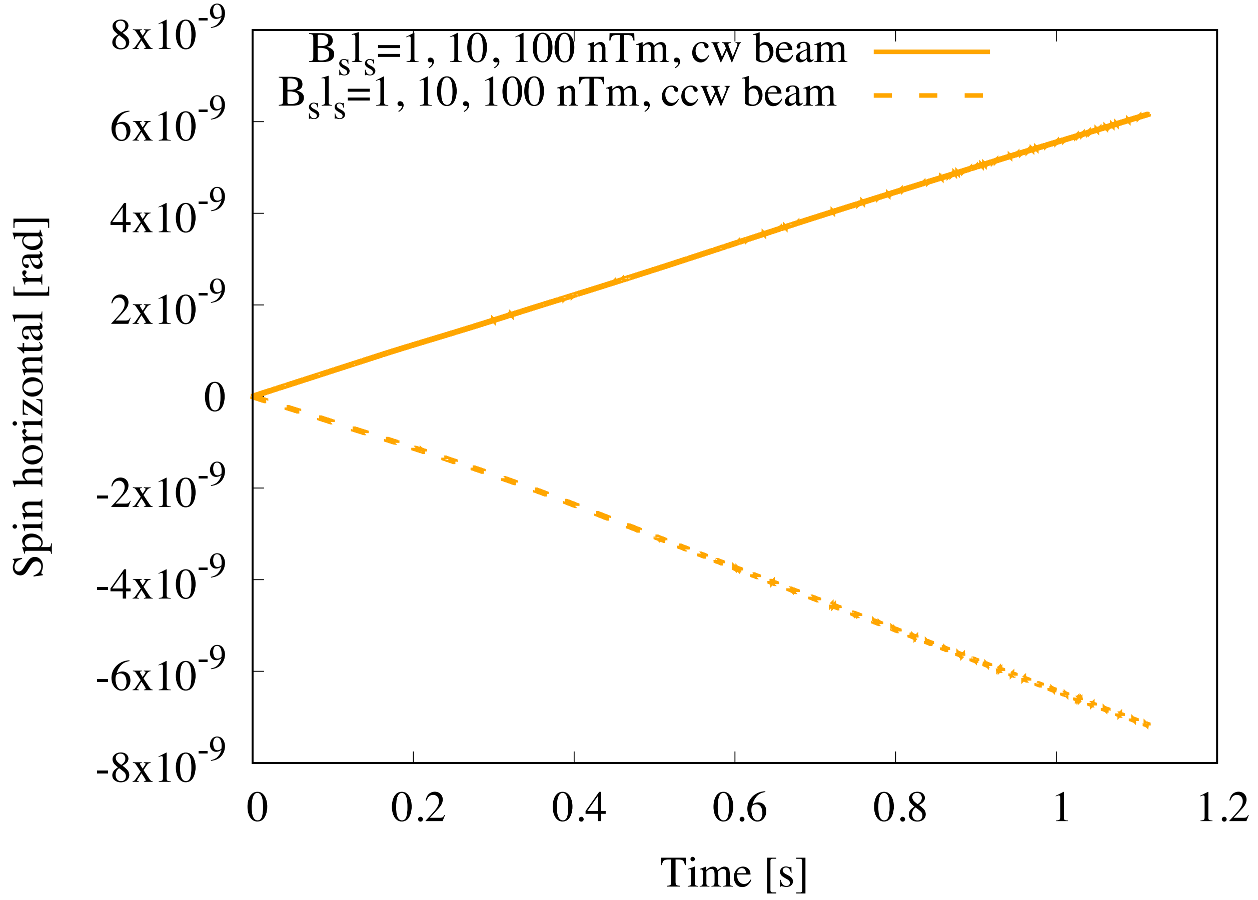
- Spin tracking results - longitudinally polarized beam (parallel to the particle direction)

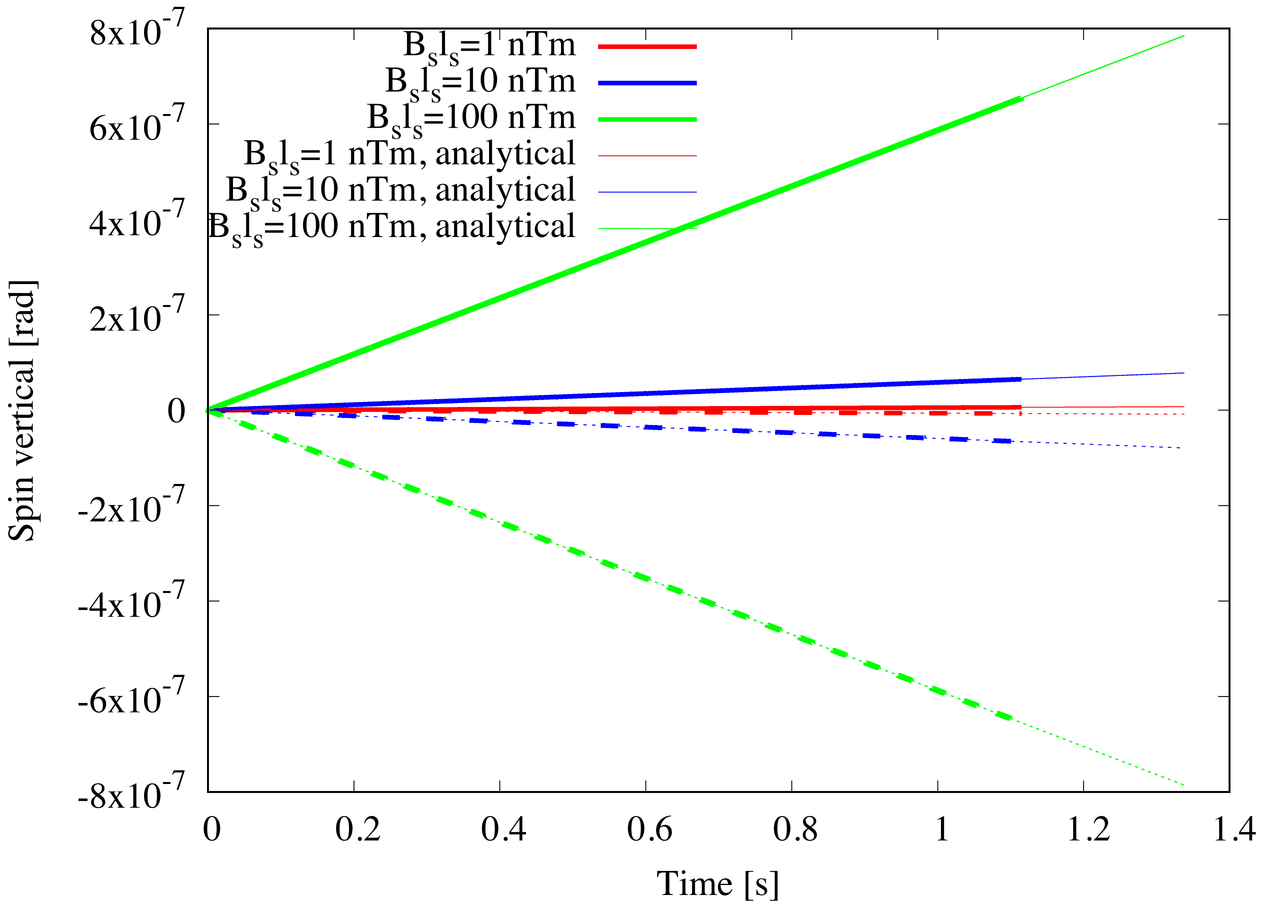




Simulation results for the cw and the ccw beam

- Spin tracking results - cases $B_s l_s$= 1,10, 100 nTm - longitudinally polarized beam (parallel to the particle direction)

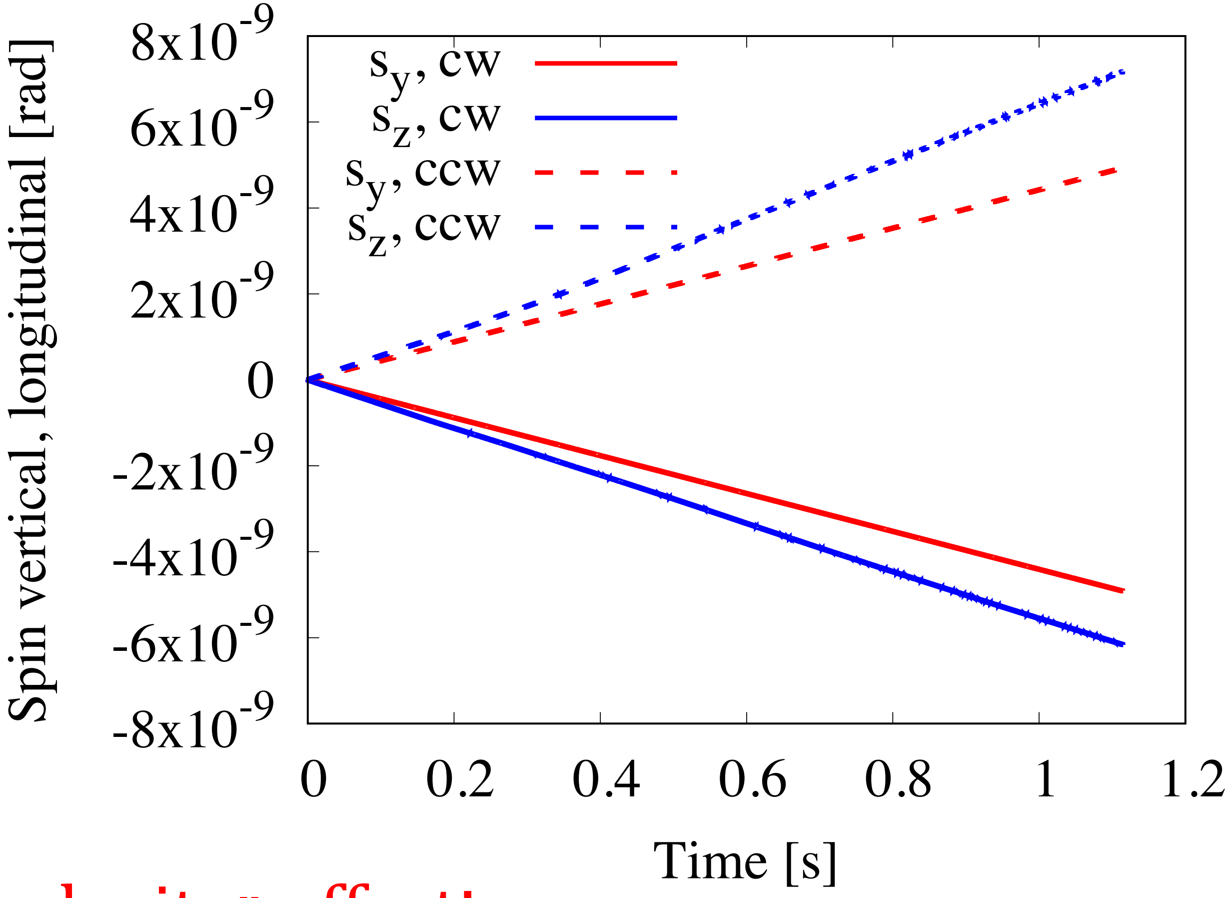




→ Effect mimics EDM!

Simulation results for a radially polarized beam

- Spin tracking results - comparison for cw and ccw beams all the magnetic fileds 0, 1, 10, 100 nTm



→ No “vertical velocity” effect!

Simulation results for a more realistic case

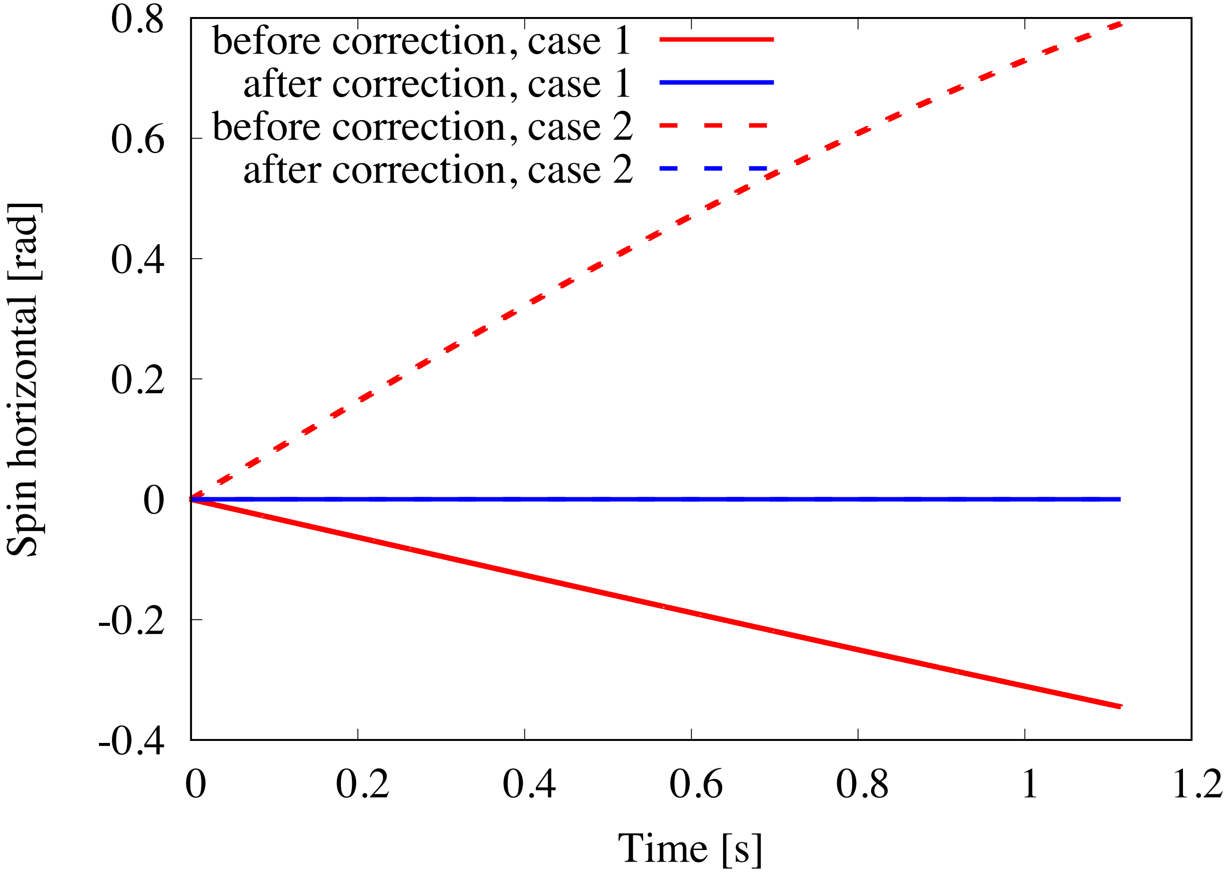
Study of geometric phase effect: more realist case

- More realistic case has been studied that involves:
  - Random offset of all the bending magnets
    → reasonable radial misalignement (order of 0.1 mm rms)
  - Field of bendings adjusted (electrode spacing) to have no (or negligible) orbit deformation
  - Correction of the energy to bring the horizontal spin close to zero:
    $$\Delta\gamma = \frac{\gamma^2}{(\gamma^2+1)G+1}\Delta Q_s$$
  - Reasonable longitudinal magnetic fields added with 48 solenoids with a $l = 1\ m$ and a random k_s value of 10^{-9} rms → $k_s = \frac{B_s}{B\rho}\left[\frac{1}{m}\right]$
  - The integral over one turn has been set to zero:
    $$(B_s l)_i \rightarrow (B_s l)_i - \frac{1}{48}\sum_{i=1}^{48}(B_s l)_i$$
  - Two cases have been studied → longitudinally and horizontally polarized beam
  - Studied for two different random seed

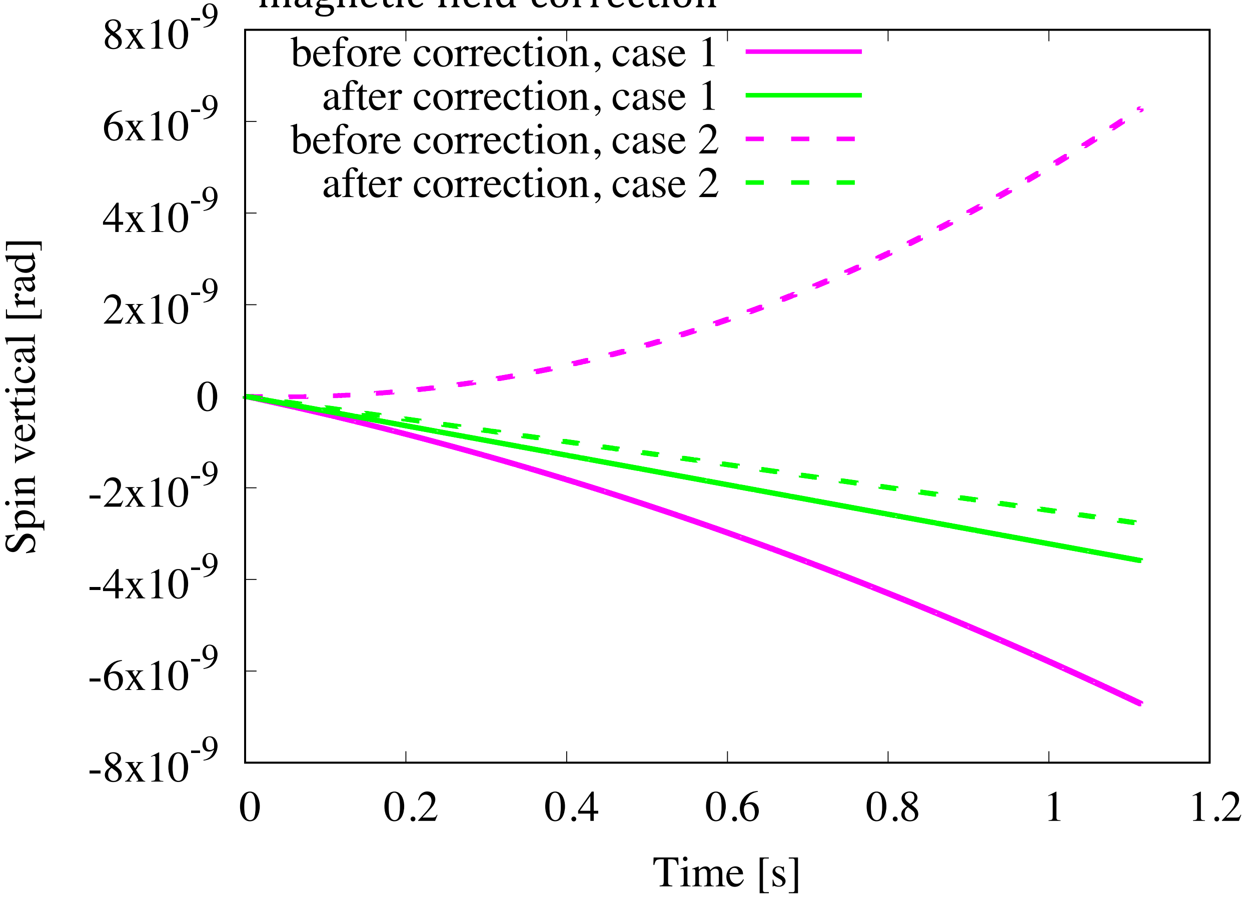
Simulation Results

- Longitudinally polarized beam (s_z=1) - different random seed

s_x before and after applying the energy correction



s_y before and after applying the longitudinal magnetic field correction



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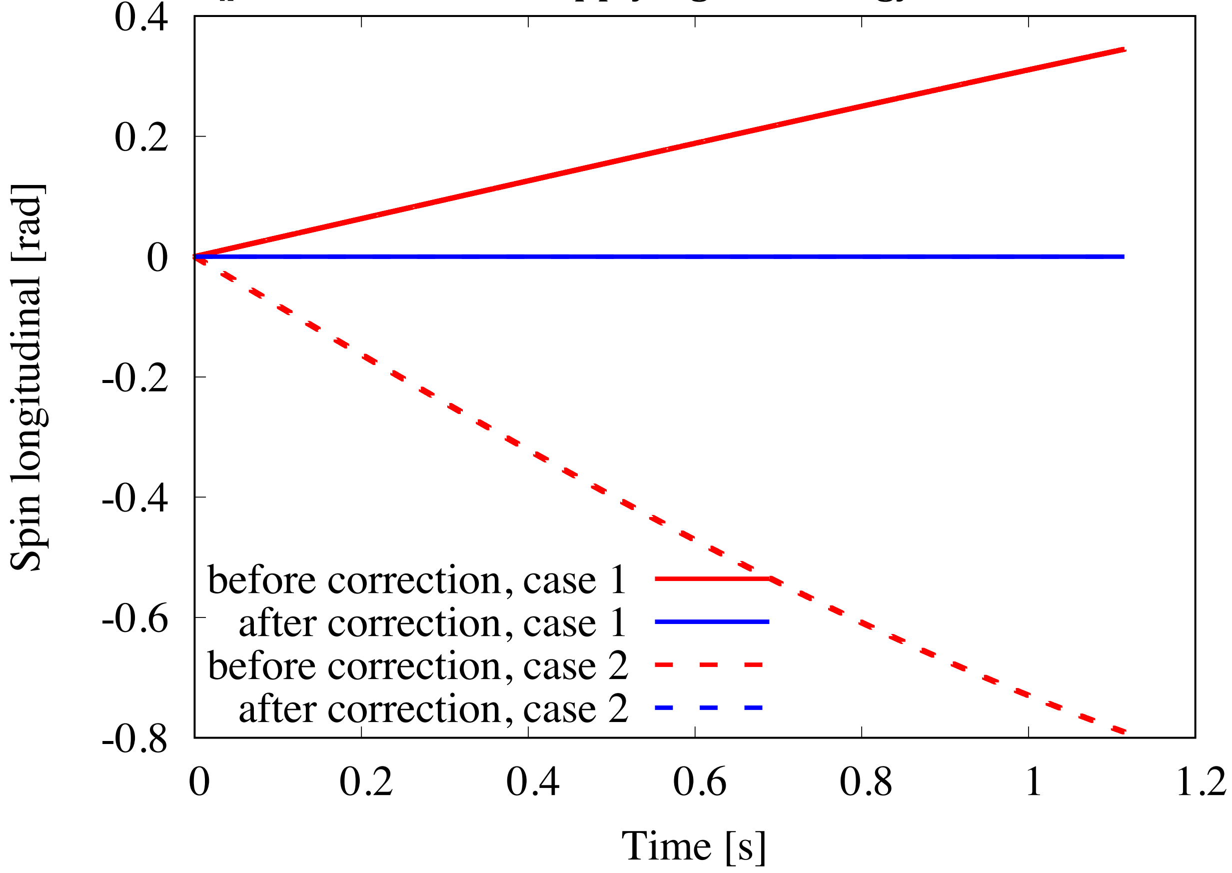
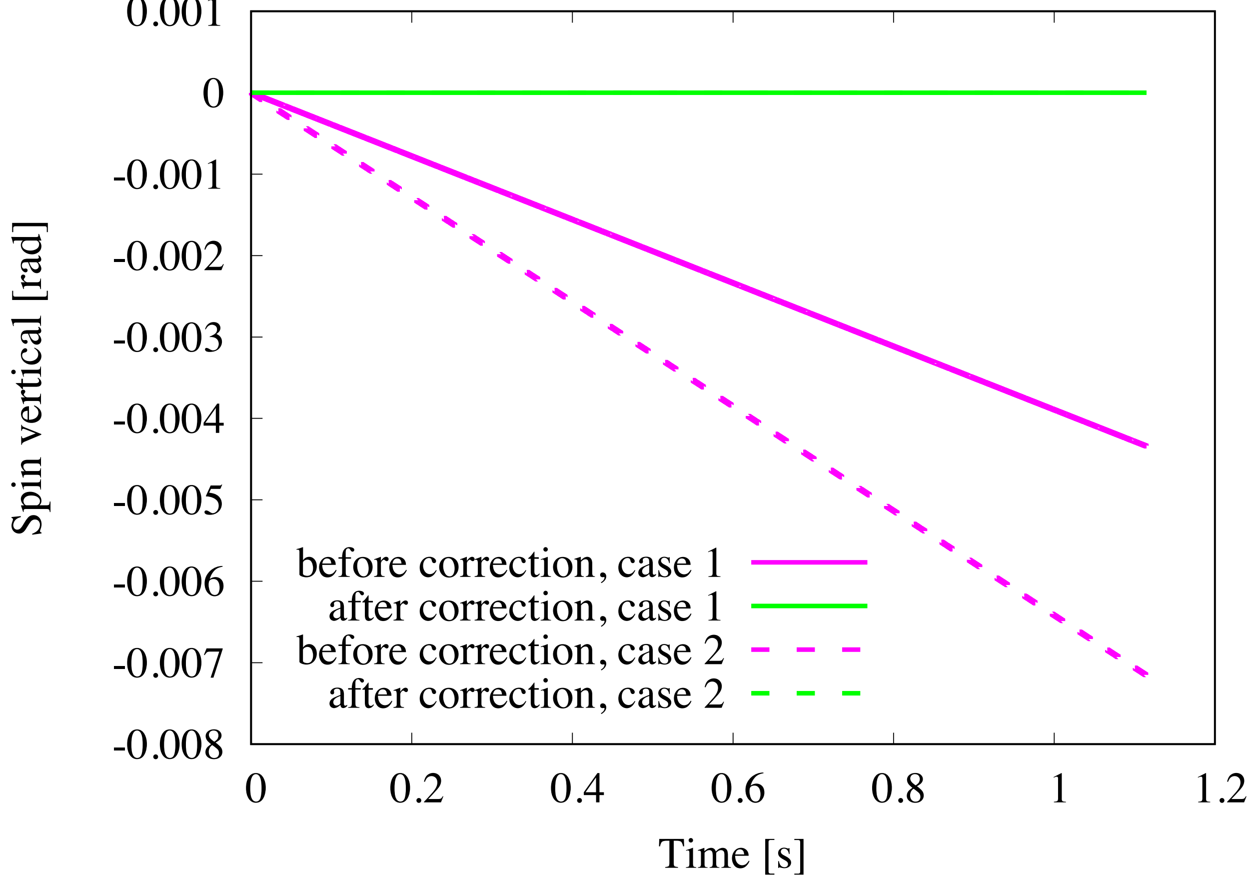


s_y before and after applying the longitudinal magnetic field correction



Comparison between analytical estimates and simulation results

- Horizontally polarized beam (s_x=1) - case of no longitudinal magnetic field correction

  - Solenoid rms k_s value of 10^{-9} 1/m

$$\int k_s ds = 6.2 \cdot 10^{-9} \frac{1}{m}$$

  - The longitudinal magnetic field is $\bar{B}_{\parallel} = \frac{B\rho \cdot \int k_s ds}{C} = 1.81 \cdot 10^{-11} T$

  - The $\bar{\omega}_s^{\parallel} = -\frac{e}{m}(G+1)\frac{\bar{B}_{\parallel}}{\gamma} = -0.00388\, s^{-1}$

  - From simulations: $\dot{s}_y = \frac{-1.736 \cdot 10^{-8}}{T_{rev}} = -0.00389\, s^{-1}$
    with $T_{rev} = 4.46 \mu s$

  - Case of longitudinal magnetic field correction:

$$\dot{s}_y = \frac{-2.3461 \cdot 10^{-16}}{T_{rev}} = -5.2603 \cdot 10^{-11} s^{-1}$$

showing good agreement with simulations…

Conclusions

- The effects mimics a finite EDM (cannot be disentangled from EDM combining observations with CW and CCW beam)
- No net rotation around the longitudinal axis rotating radial spin component into the vertical direction (not a ″vertical velocity″ effect)
- In the case of a longitudinally polarized beam there is a geomteric phase effect of approx 3 nrad/s
- In the case of a horizontally polarized beam after the correction we can only see rotation in the radial axis
- Are these simulations really realisitic?
  - Larger or more imperfections to be taken into account?
- Further studies
  - Combination of different magnetic fields
  - Superposition of the bends with the longitudinal magnetic fields
  - Possible mitigation strategies?

Thank you for the attention!