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PHYSICS AND TECHNIQUE OF ACCELERATORS

Collective Effects in the NICA Collider

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Abstract—Collective effects in the NICA collider are considered taking into account the results of calculations of induced electromagnetic fields (wakes) and coupling impedances of vacuum-chamber elements [1]. Coulomb shifts of longitudinal and transverse oscillations are considered and the stability of coherent oscillations in the presence of these shifts is analyzed; their influence is significant, since it can lead to the Loss of Landau Damping (LLD). Special attention is given to transverse head–tail instabilities and longitudinal instabilities caused by High-Order Modes (HOM) of the cavities. Recommendations on the suppression of instabilities are formulated.

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1. INTRODUCTION

In the NICA collider, ^{197}Au+^{79} gold ions with an energy of 1.5–4.5 GeV/aem are supposed to be accumulated [1]. The parameters of the facility are given in Table 1.

The concept of the facility and the parameters of the collider changed more than once, and, for each of the options, the limits for collective effects were estimated. For the last version, stability estimates were made in [4], but the impedances of the chamber elements were unknown. Currently, most of the elements of the NICA collider have been designed. The beam coupling impedances for these elements were calculated using the Microwave Studio code; the results of these calculations are described in report [1], presented at this seminar. These calculations were completed in May 2019, so the analysis of collective instabilities of the NICA collider in this report corresponds to that date. We emphasize that, in this report, we omitted some effects arising from the finite conductivity of the chamber walls, since they were considered earlier in [4].

The theory of coherent instabilities is considered in a number of textbooks and monographs [5–9]. However, these books describe the "classical" theory that does not take into account the Coulomb shifts of the betatron and synchrotron frequencies. Only relatively recently have a number of theoretical works been published that have pushed this issue along [11–14]. The modern review [14] emphasizes that these works do

Table 1. Parameters of the NICA collider

Ring length, m	503
Ion energy, GeV/n	1–4.5
γ_{tr}	7.091
Ion species	Au^{+79}_{192}
Betatron frequencies, Q_x/Q_y	9.44/9.42
Chromaticity, ξ_x/ξ_y	–33/–28
RMS momentum spread σ_p^{max} ($E = 4.5$ GeV/n)	1.65×10^{-3}
Multiplicity of RF field, q	72
Number of bunches	22
RMS bunch size, l_b, m	0.6
RMS emittance, m rad	~ 10^{-6}

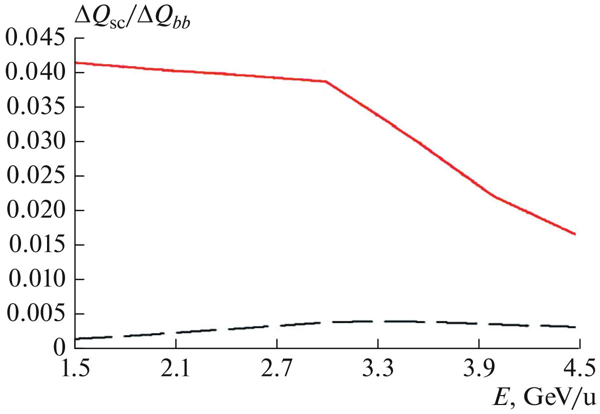

Fig. 1. Coulomb shift of betatron frequencies vs. ion energy E.

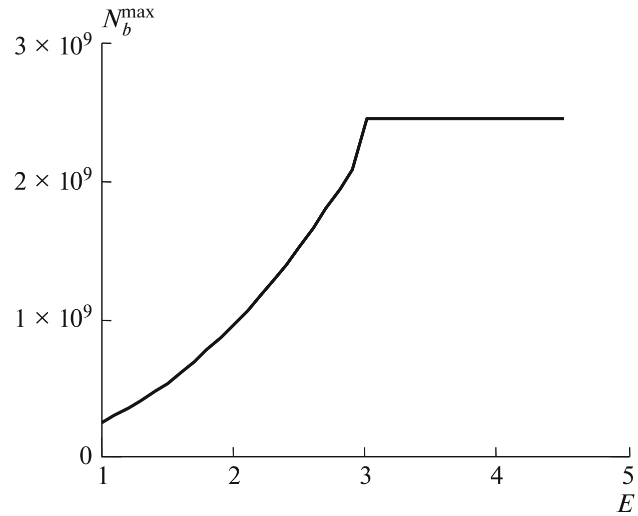

Fig. 2. Maximum number of particles in a bunch vs. ion energy E.

not solve the problem for moderate Coulomb shifts typical of the NICA collider.

Currently, another similar project with a large Coulomb shift of the betatron frequency is being developed: the SIS100 synchrotron project (Darmstadt, Germany, FAIR). The synchrotron is supposed to accelerate U^{28+} ions; the energy range is $E = 0.2$–1.5 GeV/n. FAIR collaborators performed a large cycle of work on coherent instabilities at the SIS-100 synchrotron [15–21]. The results are used in this work.

2. TRANSVERSE EFFECTS

2.1. Coulomb Shift of the Betatron Frequency

The shift of betatron frequencies, ΔQ_0 (which are assumed to be equal), due to the Coulomb forces is defined as

$$\Delta Q_0 = \Delta Q_{sc} + 2\xi, \tag{1}$$

where ξ is the shift of the betatron frequency at one meeting point due to interbeam interaction and ΔQ_{sc} is the shift of betatron oscillations due to single-beam Coulomb disruptive forces. When designing the collider, it was assumed that $|\Delta Q_0| \le 0.05$ (note that, when designing the SIS100, it was assumed that, for vertical oscillations, $|\Delta Q_0| \le 0.13$). According to the scenario chosen, the collection mode is divided into two regions: (1) Coulomb (energy 1.5–3.2 GeV/n) and (2) the region with the determining effect of intra-beam scattering (energy 3.2–4.5 GeV/n). In the Coulomb region, the number of particles in the bunch, N_b, is limited by the Coulomb shift of betatron frequencies, while the number of particles remains constant in the intra-beam scattering (IBS) region ($N_b = 2.45 \times 10^9$).

The dependences of ΔQ_{sc} and $\Delta Q_{bb} = 2\xi$ on ion energy E is shown in Fig. 1. Note that, over the entire range of energies, $\dfrac{\Delta Q_{bb}}{|\Delta Q_{sc}|} \ll 1$. Therefore, we neglected the coupling of coherent oscillations due to interbeam interaction and limited the consideration to coherent oscillations of the beam in one of the rings.

The energy dependence of the limiting number of particles in a bunch, corresponding to this scenario, is constructed below.

The analysis of stability, basically, should be conducted for the entire range of E. Note that, for a large part of the instabilities, the most dangerous one is the inflection point in Fig. 2, corresponding to $N_b = 2.45 \times 10^9$ and $E = 3.2$ GeV/aem.

2.2. Microwave Transverse Instability

Longitudinal and transverse microwave instabilities are similar to the instabilities of an unbunched beam. They can occur in a bunch under the following conditions: (1) the perturbation wavelength is much smaller than the bunch length, i.e., $\dfrac{R}{nl_b} \ll 1$; (2) the instability development time is much shorter than the synchrotron period. With such a rapid development of instabilities, the particles do not have time to change their position and, therefore, these instabilities are similar to the unbunched beam instabilities and are suppressed by the Landau damping, caused by the momentum spread. A sufficient condition for stability (the Zotter criterion) can be written as follows:

$$K_{st} = F_t \frac{|\Delta Q_0|}{\Delta Q_n} \sqrt{1 + \left(\frac{Z_\perp^n}{Z_\perp^{sc}}\right)^2} \le 1. \tag{2}$$

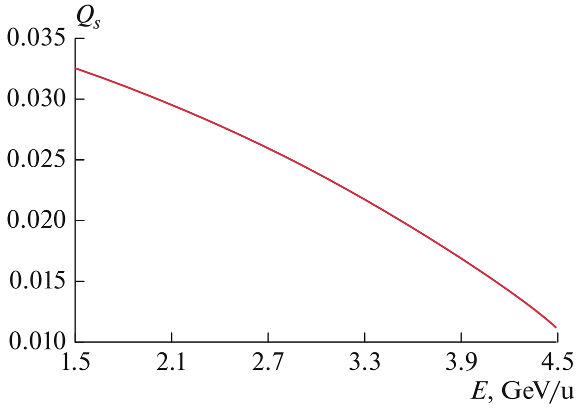

Fig. 3. Synchrotron frequency Q_s vs. ion energy E.

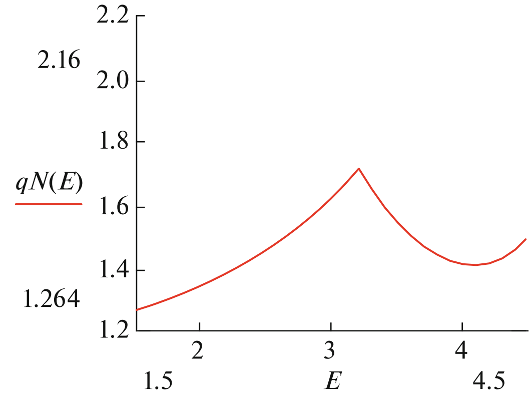

Fig. 4. Dependence $q(E)$ for the NICA collider.

Here, F_t is a coefficient depending on the type of momentum distribution, $\Delta Q_n = n|\xi|\sigma_p$ (ξ is chromaticity, σ_p is the mean-square momentum spread in the bunch, and n is the azimuthal harmonic number), $Z_\perp^n$ is the impedance of the chamber, and $Z_\perp^{\text{sc}}$ is transverse impedance of space charge:

$$Z_\perp^{\text{sc}} = \frac{Z_0 R}{\beta\gamma^2}\left(\frac{1}{a^2} - \frac{1}{b^2}\right). \tag{3}$$

In (3), β and γ are the relativistic parameters, $Z_0 = 377\ \Omega$ is the impedance of free space, R is the radius of the chamber, a is the beam radius, and b is the radius of the vacuum chamber.

To suppress IBS, the momentum spread σ_p should be near the state of thermodynamic equilibrium [24]. Under this condition, $\sigma_p = \gamma\sigma_p^0$, $\sigma_p^0 = \dfrac{\sigma_p^{\max}}{\gamma_{\max}}$, and $\sigma_p^{\max} = 1.8\times 10^{-3}$.

The necessary stability condition was derived in [25]; it has the following form (Burov–Lebedev criterion):

$$K_{\text{st}}^{\text{BL}} = \frac{|\Delta Q_{\text{sc}}|}{\Delta Q_n 1.7\ln\left(\dfrac{Z_\perp^{\text{sc}}}{Z_\perp^{\text{res}}}\right)} \le 1. \tag{4}$$

An analysis of high-frequency resonances shows that, at a chromaticity $\xi = -1.5$, they all satisfy criterion (3); i.e., since this is a sufficient criterion, there is some margin. An analysis showed that all these resonances arise for two reasons: (1) the absence of shielding of the bellows in the arches; (2) a jump in the diameter of the chamber near the quadrupole lenses. Currently, these defects of the chamber have already been removed.

2.3. Weak Head-Tail Transverse Instability

This instability arises due to the chromaticity of the accelerator. For mode with number l, the barycenter varies proportionally to $\exp(il\varphi_s)$, where φ_s is the phase of the synchrotron oscillations of the particle. An exact theory with allowance for the space charge is still missing. The character of the oscillations depends on parameter $q(E) = \dfrac{\Delta Q_0}{Q_s^0}$, where Q_s^0 is the synchrotron oscillation frequency, which is determined by the formula

$$Q_s^0 = q\gamma\left(\frac{\sigma_0^p}{\varphi_0}\right)\eta(\gamma), \tag{5}$$

where the orbital slip factor is $\eta(\gamma) = \dfrac{1}{\gamma^2} - \dfrac{1}{\gamma_{\text{tr}}^2}$ and $\varphi_0 = q\dfrac{l_{\text{b}}}{R}$ (R is the average radius of the facility). The dependence of Q_s^0 on the ion energy E is shown in Fig. 3 and the dependence $q(E)$ is given in Fig. 4.

It can be seen from the figure that the values of $q(E)$ in the collider are not very large ($1.3 \le q(E) \le 1.7$). In this situation, to study instabilities, we can use the phenomenological model of V. Kornilov [18], according to which

$$\Delta Q = \Delta Q_{\text{Re}} + i\gamma_{\text{dr}} - i\gamma_{\text{damp}}. \tag{6}$$

In this formula, γ_{dr} is the growth rate excited by the impedances of the ring at zero beam intensity and γ_{damp} includes Landau damping due to the nonlinearity of transverse oscillations and the Landau damping caused by the spread of the Coulomb shift of incoherent beam frequencies. The spread of betatron frequencies occuring in the bunch is due to their dependence on the longitudinal and transverse coordinates of the bunch. This spread leads to the rise of Landau damping, which was found in numerical simulation using the PATRICK code [18]. According to these calculations, the damping of the dipole mode (with the number $l = 0$) is absent; the attenuation rate for the mode with $l = 1$ is shown in Fig. 5.

For higher order modes, the rate rapidly increases.

The instability growth rate for the zero intensity, $\gamma_{\rm dr}$, can be calculated using the concept of the effective impedance $(Z_\perp)_{\rm eff}$ [A.Chao, 6-205). The results of calculating the nominal operating point of the NICA collider by this theory are as follows:

Mode number l	0	1
Instability growth rate	0.219×10^{-4}	0.21×10^{-5}

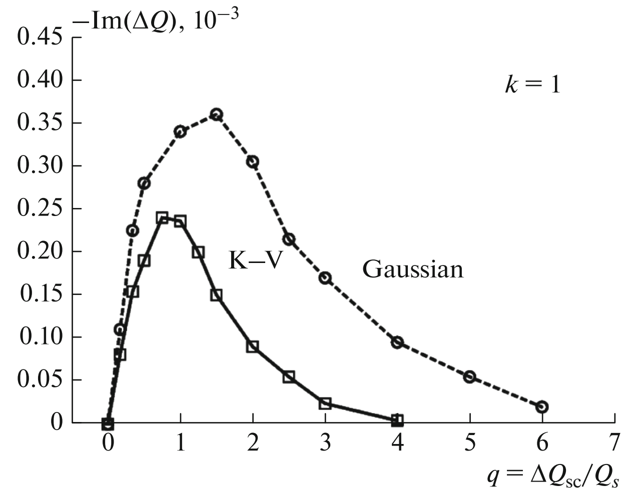

Fig. 5. Damping ratio for the first mode; (upper curve) Gaussian distribution in the transverse space and (lower curve) K–V distribution.

3. LONGITUDINAL INSTABILITIES

3.1. Coulomb Shift of the Synchrotron Frequency and the Loss of Landau Damping

The effect of the longitudinal forces from the space charge leads to a shift in the frequency of synchrotron oscillations, the value of which is determined by the Coulomb parameter [15]:

$$\Sigma = F_{\rm dis}\frac{|\eta| r_i N_b R_C^2}{\beta^2\gamma^3 L_b^3 Q_{s0}^2}, \tag{8}$$

where η is the slip factor ($\eta = \frac{1}{\gamma_{\rm cr}^2} - \frac{1}{\gamma^2}$, $\gamma_{\rm cr}$ corresponds to the critical energy), r_i is the classical radius of the ion, N_b is the number of ions in the bunch, R_C is the radius of the facility, β and γ are relativistic parameters, $2L_b$ is the rms length of the bunch, Q_{s0} is the synchrotron frequency, and $F_{\rm dis}$ is a coefficient depending on the shape of the distribution in space (for a Gaussian distribution, $F_{\rm dis} = \frac{1}{\sqrt{2\pi}}$.). The synchrotron frequency shift is

$$R(E) = \frac{\Delta Q_s^{\rm sc}}{Q_{s0}} = -\frac{\Sigma}{2}. \tag{9}$$

The calculated values of $R(E)$ are given in Table 2.

At zero Coulomb parameter, free dipole oscillations decay due to the spread of the synchrotron frequencies, caused by the nonlinearity of the RF field. With a sinusoidal accelerating field, the magnitude of this spread is determined by the formula (4, 9.51):

$$\frac{\delta Q_s}{Q_s} = \frac{\pi^2}{16}(h\tau_L f_0)^2, \tag{10}$$

where h is the harmonic number of the accelerating field, f_0 is the revolution frequency, and τ_L is the total length of the bunch in seconds. Given that $\tau_L = 2L_{\rm b}/c\beta$ and $f_0 = \frac{c\beta}{L_C}$ (L_C is the length of the ring), we write this formula in the following form:

$$\frac{\delta Q_s}{Q_s} = \frac{\pi^2}{16}\left(h\frac{2L_{\rm b}}{L_C}\right)^2. \tag{11}$$

Substituting the values of the NICA collider parameters, we find that $\frac{\delta Q_s}{Q_s} = 0.01529$. With an increase in the Coulomb parameter, the Landau damping decreases. At a certain value $\left(\frac{\Delta Q_s^{\rm sc}}{Q_{s0}}\right)_{\rm lim}$ (depending on the type of distribution over the longitudinal invariants), it becomes equal to zero. According to [4], for a Gaussian distribution, $\left(\frac{\Delta Q_s^{\rm sc}}{Q_{s0}}\right)_{\rm lim} = 0.025$. Thus, compared with the calculated values of the Coulomb shift, we may hope that the Landau damping does not drop too much when compared with the case of $\frac{\Delta Q_s^{\rm sc}}{Q_{s0}} = 0$, so stability estimates by the classical theory can be used.

3.1.1. Longitudinal microwave instability. This instability can arise in bunched beams under the same conditions as a similar transverse instability (see Sec-

Table 2. Dependence of the Coulomb shift of the synchrotron frequency (multiplied by 10^3) on the ion energy

E, GeV	1.5	2	2.5	3.0	3.5	4.0	4.5
$R(E)$	10.9	8.01	6.34	5.39	4.03	2.98	2.6

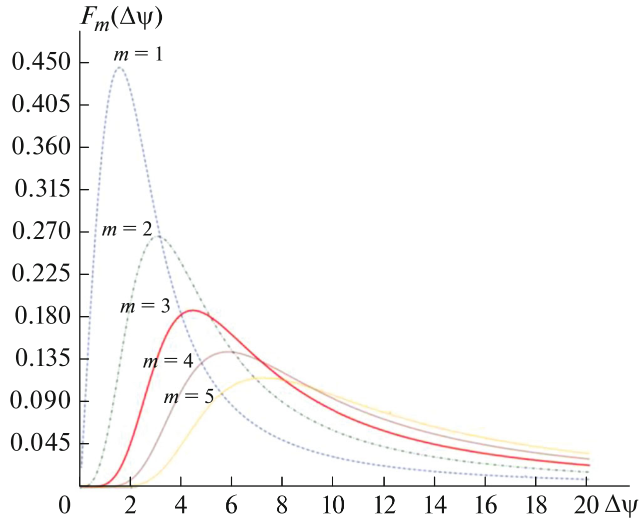

Fig. 6. Form factor vs. $\Delta\psi$ cavity phase change during passage of a bunch.

tion 2.1). The criterion for suppressing the instability by the Landau damping caused by the momentum spread is written in the following form (Gaussian distribution, $I_{\max}^{\text{long}}$ is the maximum current in the bunch):

$$I_{\max}^{\text{long}} \le 2\pi \frac{U_p}{(|Z_L|/n)} \frac{A_i}{Z_i} \gamma\beta^2 |\eta| \sigma_p^2, \tag{12}$$

where the longitudinal impedance is

$$\frac{Z_L}{n} = i\frac{Z_{\text{sc}}}{n} + \operatorname{Re}\left(\frac{Z_{\text{long}}}{n}\right). \tag{13}$$

In (13), the space charge impedance is

$$\frac{Z_{\text{sc}}}{n} = \frac{Z_0}{2\beta\gamma^2}[1 + 2\ln\left(\frac{b}{a}\right). \tag{14}$$

For the most dangerous point, $E = 3.2$ GeV/n and $\frac{Z_{\text{sc}}}{n} = 49.7\ \Omega$. The role of the source of the active part of the impedance can be played by various elements, the most dangerous of which are the cavities. Consider the most dangerous high-frequency oscillation, mode 2.2.

f, MHz	1187.9
$\text{Re}(Z_\parallel)$, Ω	4578
Q	2846

The harmonic number of the revolution frequency for this mode is $h_{\text{mode}} = 2045$. Accordingly, we find that $\operatorname{Re}\left(\frac{Z_{\text{long}}}{n}\right) = 8.95\ \Omega$ and $\frac{|Z_L|}{n} = 50.5\ \Omega$. The limiting number of particles in a bunch is related to the current as $N_{\max}^{\text{b}} = \sqrt{2\pi}\frac{L_{\text{b}}}{c\beta e Z_i}$. Substituting the numbers, we find that, for the most dangerous energy $E =$ 3.2 GeV/n, we have $N_{\max}^{\text{long}} = 3.45 \times 10^{10}$, which significantly exceeds the planned number of particles in the bunch, $N_{\text{b}} = 6 \times 10^9$.

3.2. Coherent Longitudinal Instabilities due to the High-Order Parasitic Modes of the Cavities

In the beam, a set of modes with different azimuthal number m can be excited. The most dangerous of them are multibunch instabilities; the instability growth rate of the mode number m is determined as (5, Eq. (9.69))

$$\Omega - m\Omega_s = \left(\frac{1}{\tau_m}\right)_{\max} = \frac{i r_i n_{\text{b}} N_{\text{b}} |\eta| R_s}{4\pi\beta^2\gamma T_0 Q_s \hat{\tau}} D(\alpha\tau_{\text{sep}}) F_m(\Delta\psi). \tag{16}$$

In this formula, Ω is the frequency at which the instability develops, $\hat{\tau}$ is the bunch duration ($\hat{\tau} = \frac{L_{\text{b}}}{c\beta}$), $\Delta\psi = 2\omega_r\hat{\tau}$ is the phase change of the cavity during the passage of the bunch, R_s is the impedance of the cavity at a resonance frequency), and $D(\alpha\tau_{\text{sep}})$ is the so-called separation factor (henceforward, we will assume that $D(\alpha\tau_{\text{sep}}) = 1$). The dependence on the mode number determines the dimensionless form factor

$$F_m(\Delta\psi) = -\frac{4\pi m\hat{\tau}}{\omega_r}\int_0^\infty dr \frac{dg_0}{dr} J_m(\omega_r r)^2, \tag{17}$$

where $g_0(r)$ is the stationary distribution function of the amplitude of the synchrotron oscillations and J_m is the Bessel function of order m. For a Gaussian distribution, $g_0(r) = \frac{1}{2\pi\hat{\tau}}\exp\left(-\frac{r^2}{2\hat{\tau}^2}\right)$. Substituting this into the expression for the form factor, we obtain

$$F_m(\Delta\psi) = \frac{4m}{\Delta\psi}\int_0^\infty \exp\left(-\frac{x^2}{2}\right) J_m\left(\frac{1}{2}x\Delta\psi\right)^2 x dx. \tag{18}$$

The dependence of the form factor on $\Delta\psi$ for various m is shown in Fig. 6.

The stability of the oscillations is determined by the Landau damping associated with the spread over the incoherent synchrotron frequencies δQ_s. The mode is stable under the condition [2]

$$\frac{1}{\tau_m} \le \frac{1}{4}\frac{\delta Q_s}{Q_s} Q_s \Omega. \tag{18}$$

We substitute $\frac{1}{\tau_m}$ into this expression in the form $\frac{1}{\tau_m} = A n_{\text{b}} F_m(\Delta\psi)$, where $A = \frac{i r_i N_{\text{b}} |\eta| R_s}{4\pi\beta^2\gamma T_0 Q_s \hat{\tau}} D(\alpha\tau_{\text{sep}})$.

Table 3. Parameters of high-order modes

Mode no.	1.1	1.2	1.3	2.1	3.1	3.2	3.4
f, MHz	118	273	466	450	30.5	546.2	642.5
$Z_{\|}$, Ω	480	880	149	11340	3224	206	2320
m_{dang}	1	2	4	4	1	5	6
λ_1, s^{-1}	0.860	0.451	0.0688	5.518	6.697	1.311	0.803
λ_{mb}, s^{-1}	18.9	9.926	1.513	121.4	147.3	28.85	17.7
λ_d, s^{-1}	324	459	648	648	324	726	795

Separating the terms depending on the mode number, we obtain

$$\frac{F_m(\Delta\psi)}{\sqrt{m}} \leq \frac{1}{4}\frac{\delta Q_s}{Q_s} Q_s \Omega \frac{1}{A n_b}. \quad (19)$$

Thus, the most dangerous mode is that for which the function $\frac{F_m(\Delta\psi)}{\sqrt{m}}$ is maximal. The results of calculations according to this theory for the most dangerous high-order cavity modes are presented in Table 3. The first line gives the mode number (the first number is the number of the RF system and the second is the mode number in increasing frequency), and the fourth column gives the number of the most dangerous mode. Rows 5 and 6 give the calculated grown rates. The case of $n_{\mathrm{b}} = 1$ describes single-bunch oscillations (row 5), and $n_{\mathrm{b}} = 22$ describes coupled multibunch oscillations (row 6). Row 7 gives the damping ratio of this mode due to the Landau damping for the most dangerous kinetic energy of ions.

We see that, within this approximation, the higher order cavity modes in the NICA collider do not excite coherent synchrotron oscillations. The most dangerous, due to the relatively large beam coupling impedance, is the first mode of the second accelerating system, but there is more than fivefold margin for it.

CONCLUSIONS

The above analysis made in this work shows that the situation with collective effects in the NICA collider is relatively favorable. A high synchrotron frequency, a large momentum spread, and a moderate Coulomb shift make the beam stable to most instabilities. The main results of this work are as follows:

• two elements of the vacuum chamber that have abnormally high impedances have been found: bellows and jumps in the diameter of the vacuum chamber in the arches. Currently, the design of these elements has been corrected;

• single-bunch lateral instabilities can be suppressed by the right choice of chromaticity. For low positive chromaticity $\xi \sim +(1-3)$), the dipole instability is absent and high-order head–tail modes are suppressed by the Landau damping, which arises due to the nonlinearity of the Coulomb field of the bunch. Additional Landau damping is created by the nonlinear dependence of the betatron frequency on the amplitude due to the nonlinearity of the magnets;

• longitudinal instabilities do not arise due to the small form factor for high-frequency high-order modes;

• the transverse multibunch instability caused by the finite conductivity of the chamber walls should be suppressed using a feedback system.

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