

# CAS – Introduction to Accelerator Physics

## Collective effects

### Part IV: Coherent beam instabilities





In the last lecture, we have seen some examples of analytically expressible wake fields and impedances, namely **resonator and resistive wall wakes and impedances**. We have learned that impedances can have a **detrimental impact** on both the **machine environment (beam induces heating)** as well as **the beam itself (coherent beam instabilities)**.

A careful design of machine elements to **minimize the impedance** is therefore necessary.

We have also discussed the mechanism of coherent instabilities on the instability loop. In this lecture, we will be looking **at examples of different types of instabilities** and some of their **phenomenology**.

- Part IV: Coherent beam instabilities
  - Examples of coherent beam instabilities at CERN
  - Slow headtail instability at finite chromaticity
  - Fast headtail or transverse mode coupling instability
  - Longitudinal instabilities



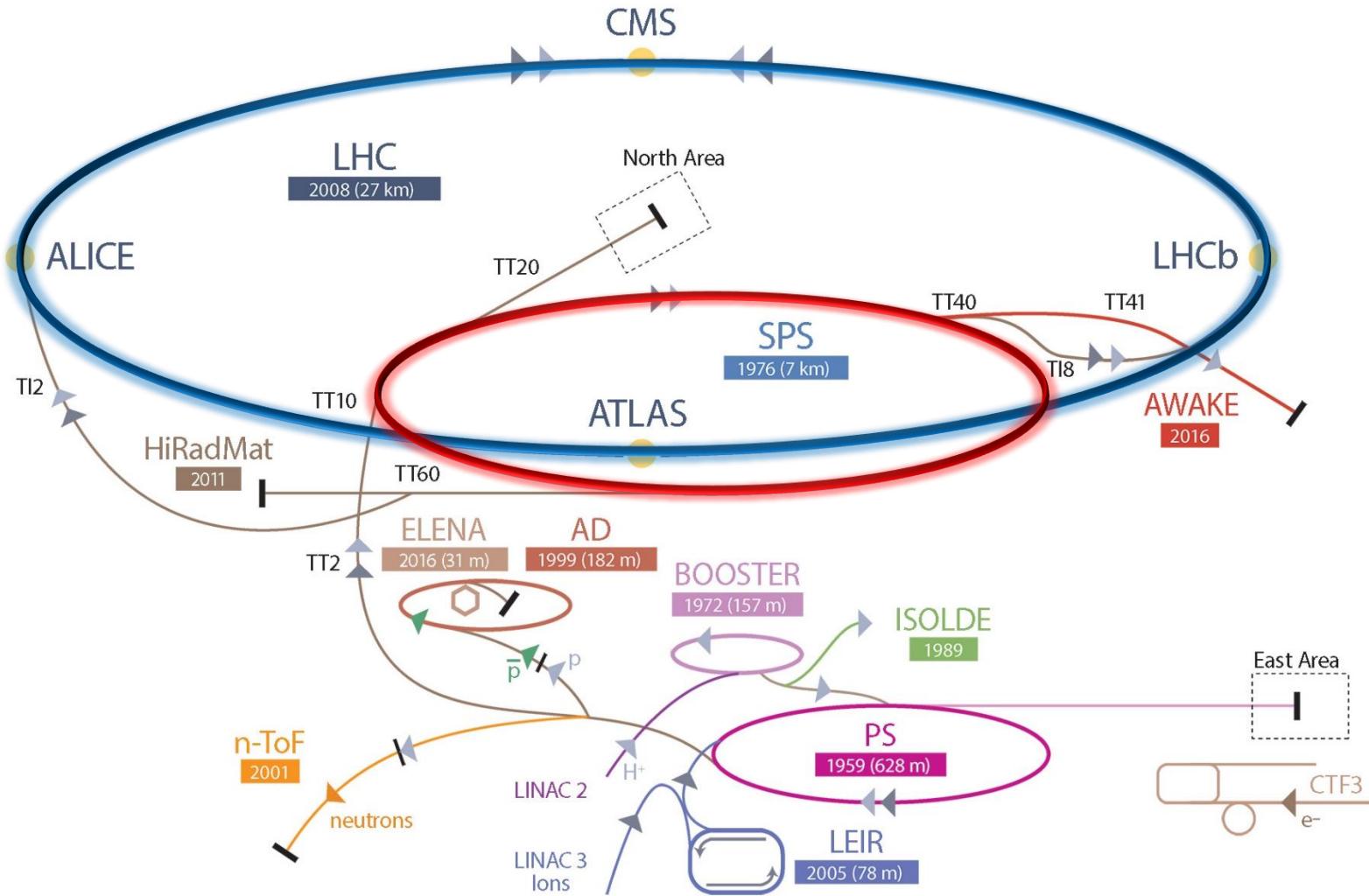
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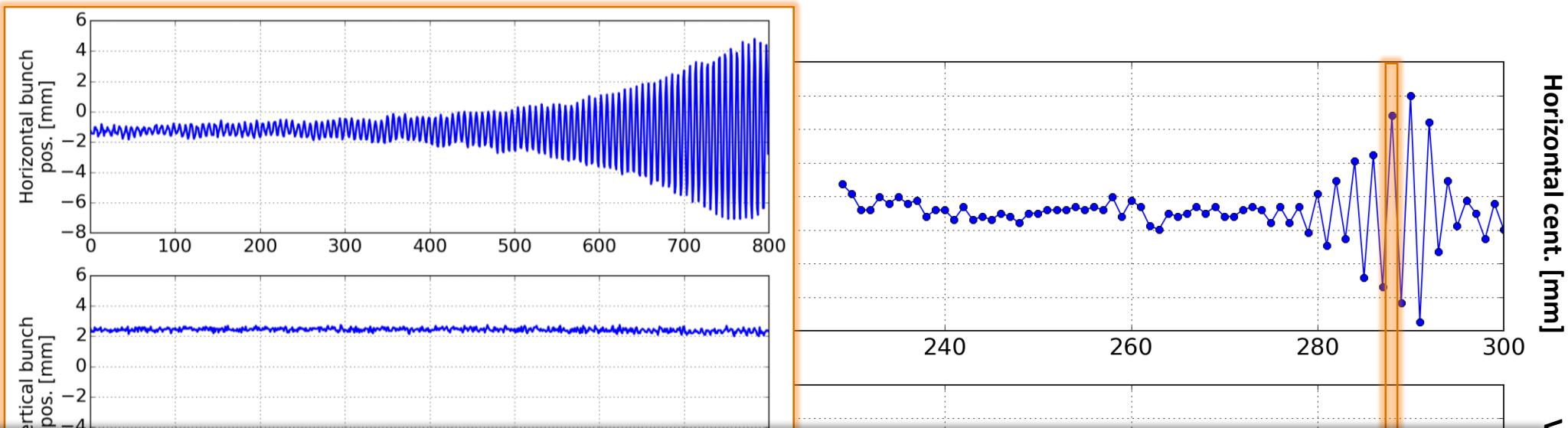
# The CERN accelerator complex



► p (proton)   ► ion   ► neutrons   ►  $\bar{p}$  (antiproton)   ► electron   ► +► proton/antiproton conversion

# Example: coupled bunch instability

- Coupled bunch instabilities vs. single bunch instabilities
  - Depends on: long range vs. short range wakefields
  - Mitigated by: transverse damper systems and Landau damping



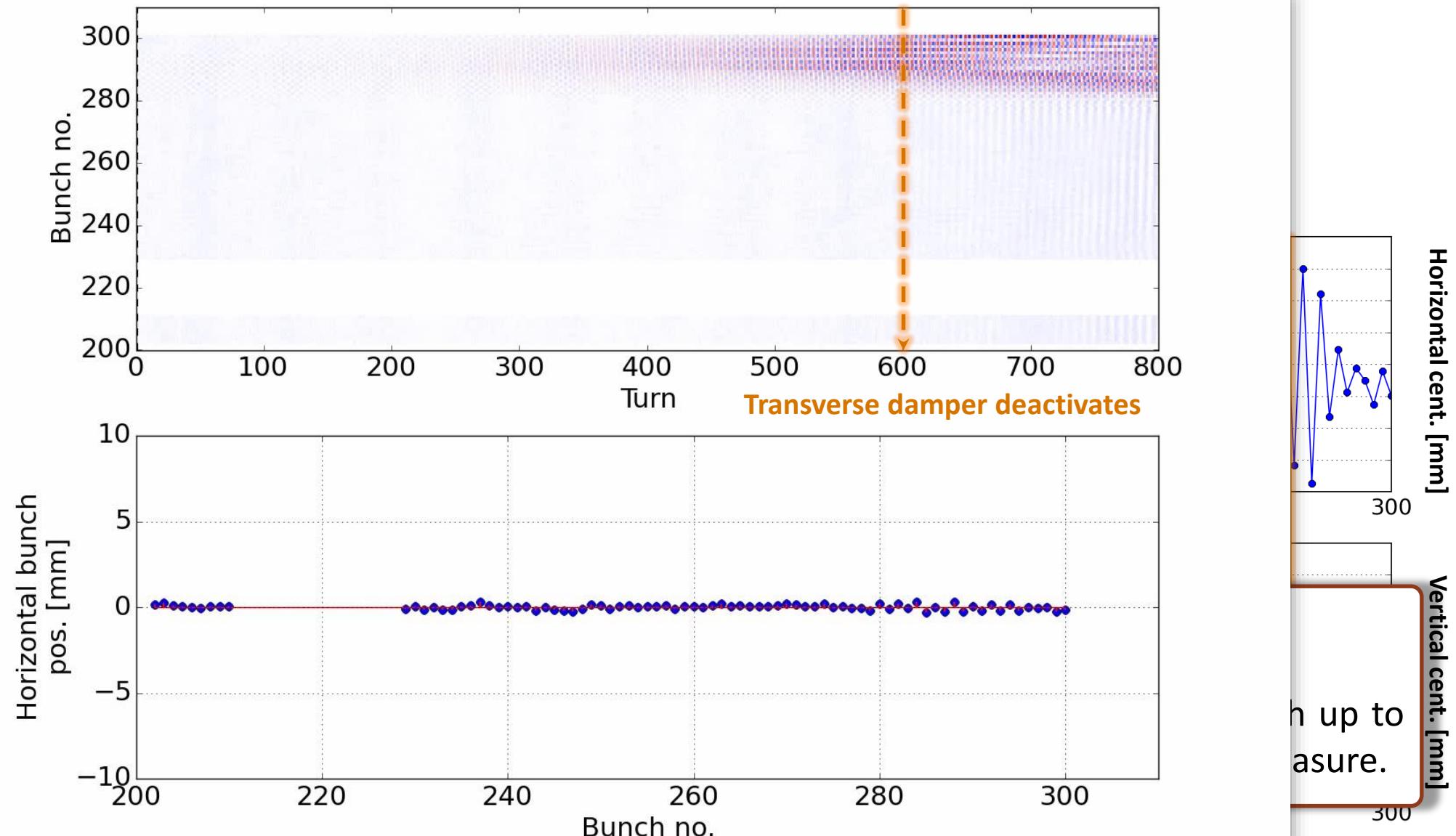
Horizontal **coupled bunch instability** observed during the 2015 scrubbing run in the SPS.

The transverse damper was set up to damp low frequency oscillations. It would not reach up to 20MHz. It would turn off, once a certain oscillation amplitude was exceeded, as a safety measure.

# Example: coupled bunch instability

- Coupled bunch
  - Dependence
  - Mitigation

Horizontal  
The trans-  
20MHz



# Example: coupled bunch instability

- Coupled bunch instabilities vs. single bunch instabilities
  - Depends on: long range vs. short range wakefields
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As a consequence, bunches blow up and intensity or even the full beam are lost during the cycle.

During some adjustments in 2017 the **bandwidth of the transverse damper up to 20MHz** could be fully exploited, successfully suppressing this instability.

Instead, **a new type of instability emerged...**

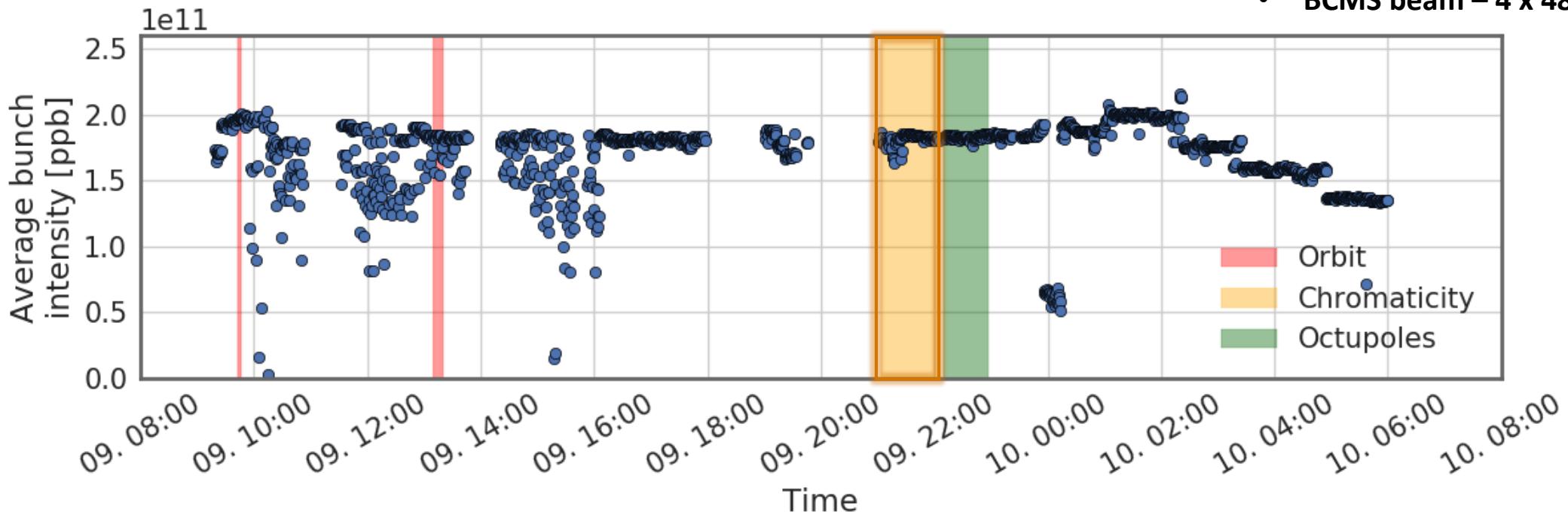
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# Example: single bunch instabilities

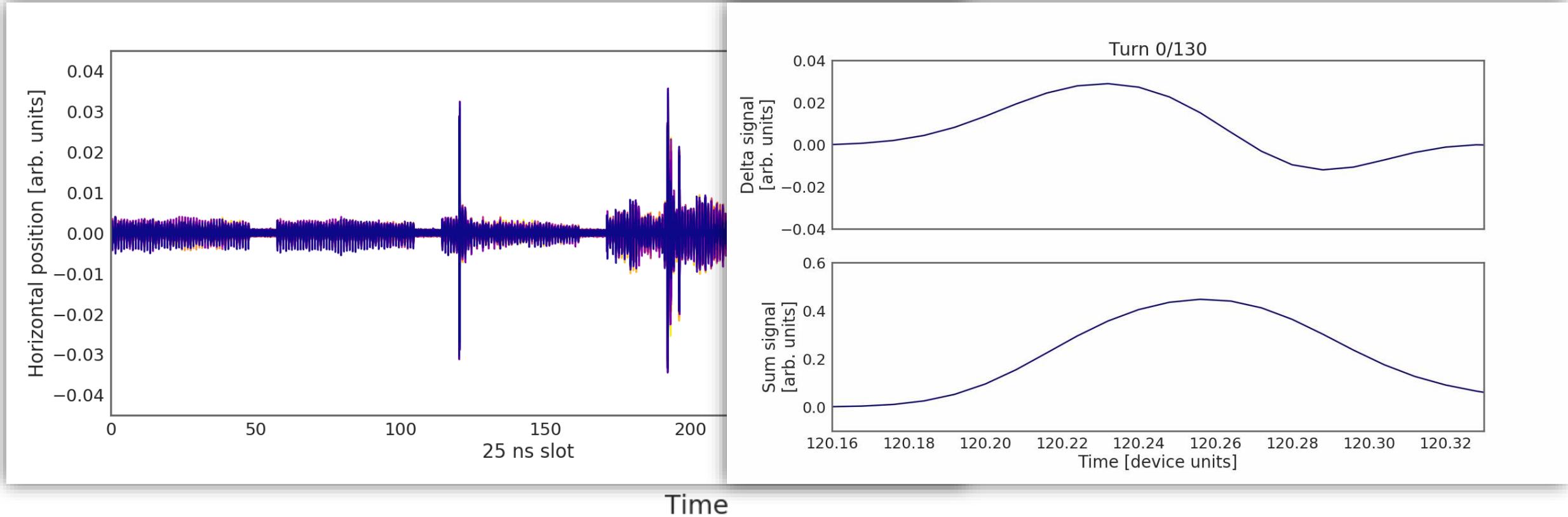
- Pure centroid vs. intra-bunch motion (slow headtail instability)
  - Relation between bunch length and impedance spectrum
- Investigation beam stability and incoherent losses as a function of chromaticity for high intensity beams.
- BCMS beam – 4 x 48 bunches



The horizontal coupled bunch instability at 20 MHz did no longer appear with the transverse damper now having been fine adjusted resulting in finite gain at these frequencies.

# Example: single bunch instabilities

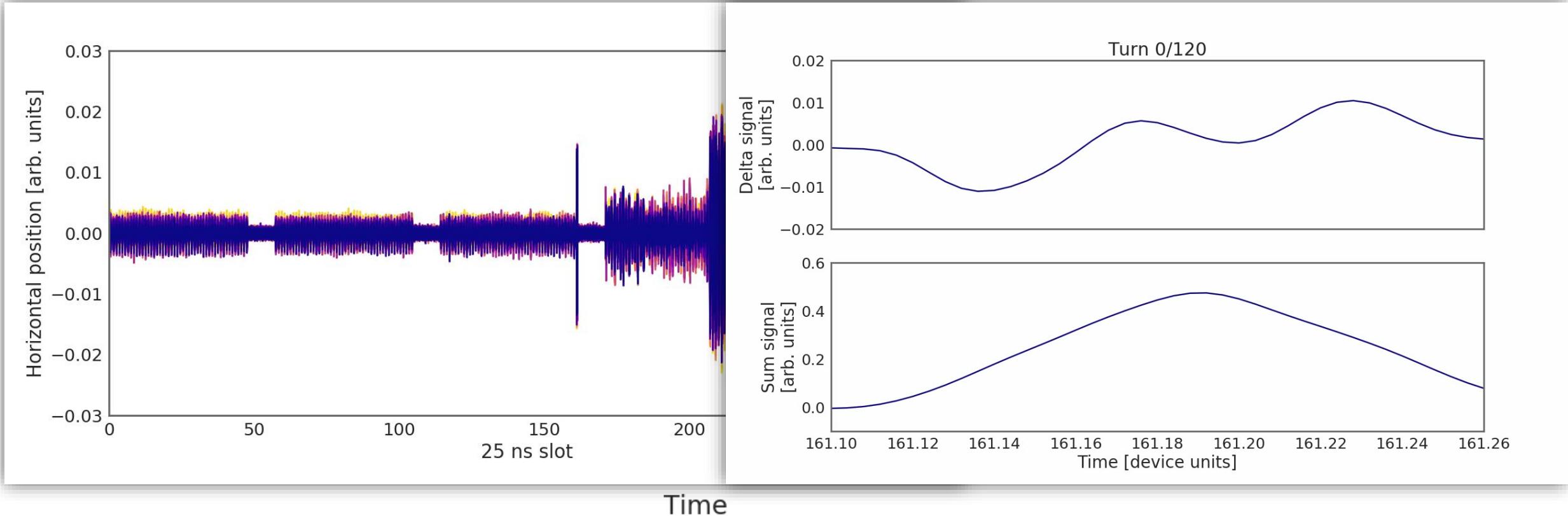
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Setting the **chromaticity to 0.2** (normalized units) yielded **instabilities that were not mitigated by the transverse damper**.

# Example: single bunch instabilities

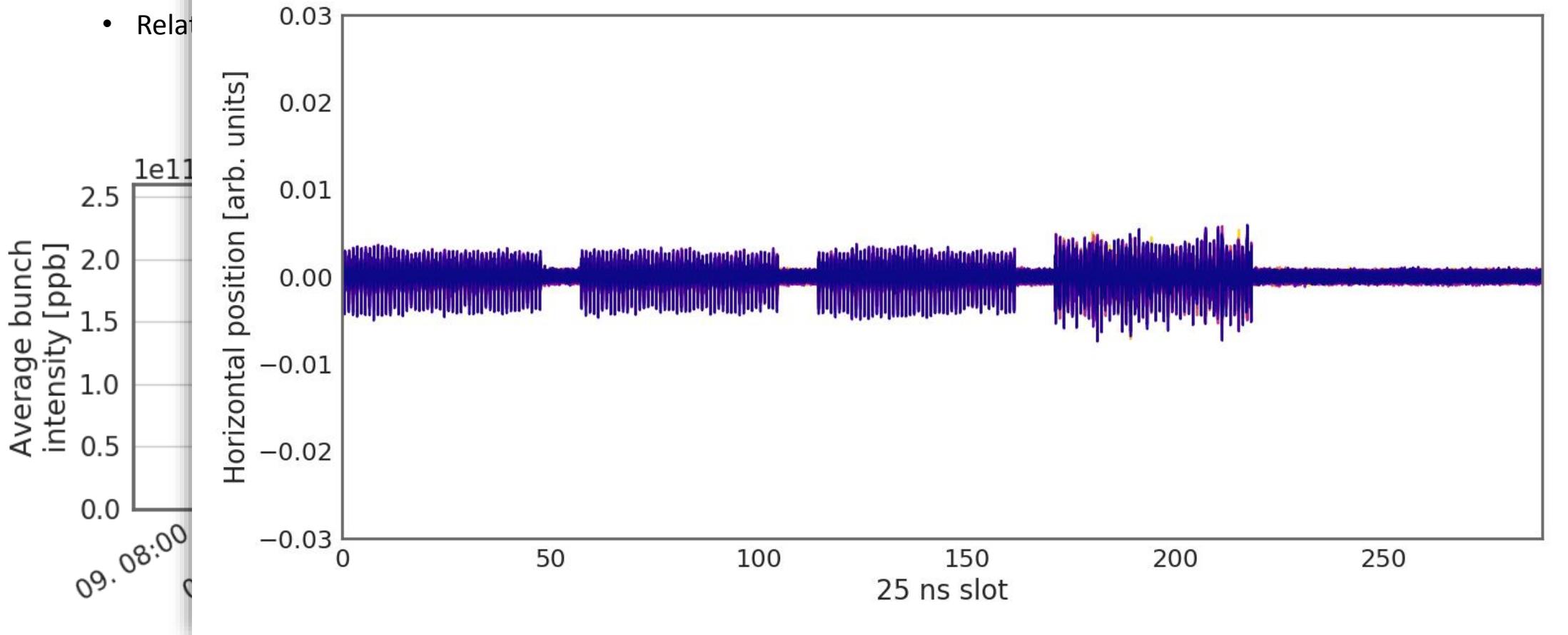
- Pure centroid vs. intra-bunch motion (slow headtail instability)
  - Relation between bunch length and impedance spectrum
- Investigation beam stability and incoherent losses as a function of



Setting the **chromaticity to 0.4** (normalized units) yielded **instabilities that were not mitigated by the transverse damper**.

# Example: single bunch instabilities

- Pure central bunches
  - Relative horizontal position



Setting the **chromaticity to 0.6** (normalized units) the **instabilities were suppressed**.



We have seen some **examples of coherent beam instabilities** observed in recent years at the CERN Super Proton Synchrotron (SPS). We were able to observe a **coupled bunch instability** and have seen that these types of instabilities can be mitigated by a **transverse feedback system**.

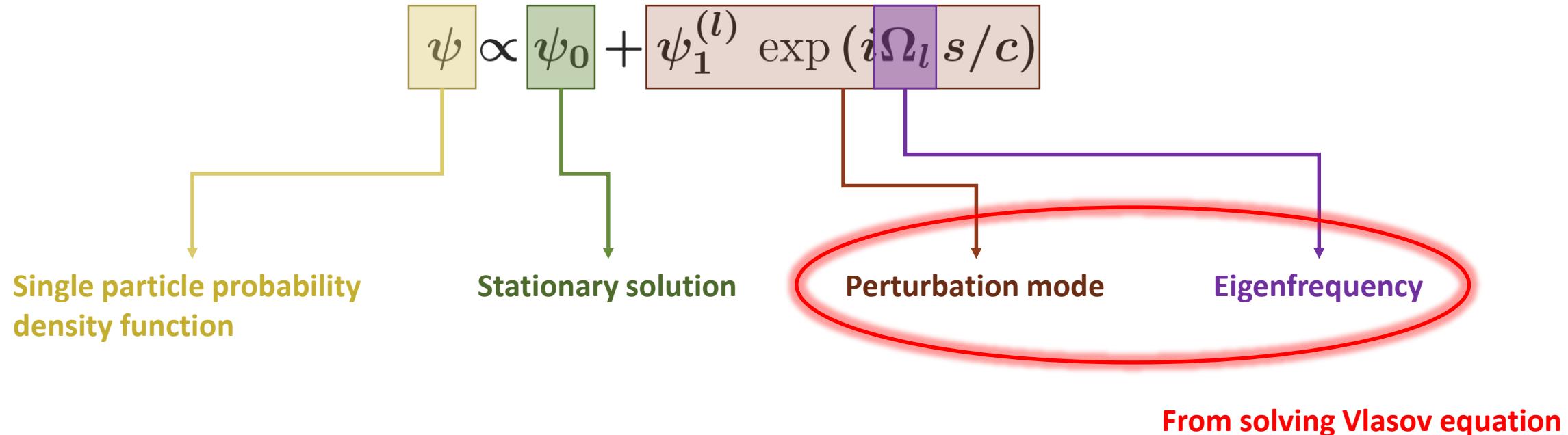
We have then seen another type of instability arising at finite chromaticities. This instability features higher frequency intra-bunch motion and is therefore called a **(slow) headtail instability**.

We will now have a brief look at a **more formal description** of this type of instability.

- Part IV: Coherent beam instabilities
  - Examples of coherent beam instabilities at CERN
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  - Fast headtail or transverse mode coupling instability
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# Headtail modes

- Headtail instabilities can be **derived from the Vlasov equation** in the presence of impedances. They evolve as **eigenmodes of the coupled accelerator-beam system**. The eigenvalue of these modes is **the complex tune  $\Omega$**  – this number fully characterizes a mode and thus an instability:



\* F. Sacherer: *Methods for computing bunched-beam instabilities*, CERN-SI-BR-72-5. - 1972. - 41 p.

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- Headtail instabilities can be derived from the Vlasov equation in the presence of impedances. They evolve as eigenmodes of the coupled accelerator-beam system. The eigenvalue of these modes is the complex tune  $\Omega_l$  – this number fully characterizes a mode and thus an instability.
- Ultimately, we express the characteristics of a given perturbation mode via its **complex tune shift**

$$\Delta\psi_l \propto \exp(i\Omega_l s/c)$$

perturbation  $\Delta\psi_l$  on the single particle probability density function  $\psi$

- The real part of this number gives **the coherent tune shift** of the respective unperturbed mode

$$\Delta Q_l \propto \text{Re}(\Delta\Omega_l)$$

Single particle probability

Stationary solution

Perturbation mode

Eigenfrequency

- The imaginary part **gives the growth rate** of the perturbation mode

$$\tau_l^{-1} \propto \text{Im}(\Delta\Omega_l)$$

From solving Vlasov equation

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# Headtail modes

- Writing down the **full Vlasov equation** with the beam coupling impedance and **finding a solution** in terms of eigenmodes and eigenfrequencies is non-trivial.
- In 1972, Frank Sacherer wrote down **an approximate solution** for the headtail modes, which very well matched observations in the CERN Proton Synchrotron (PS) (\*):

$$p_l(z) = \begin{cases} \cos\left((l+1)\pi \frac{z}{\hat{z}}\right), & l = 0, 2, 4, \dots \\ \sin\left((l+1)\pi \frac{z}{\hat{z}}\right), & l = 1, 3, 5, \dots \end{cases}$$

- Thus, Sacherer found that the observed signal at a (wideband) pickup for a given mode  $l$  can be described as:

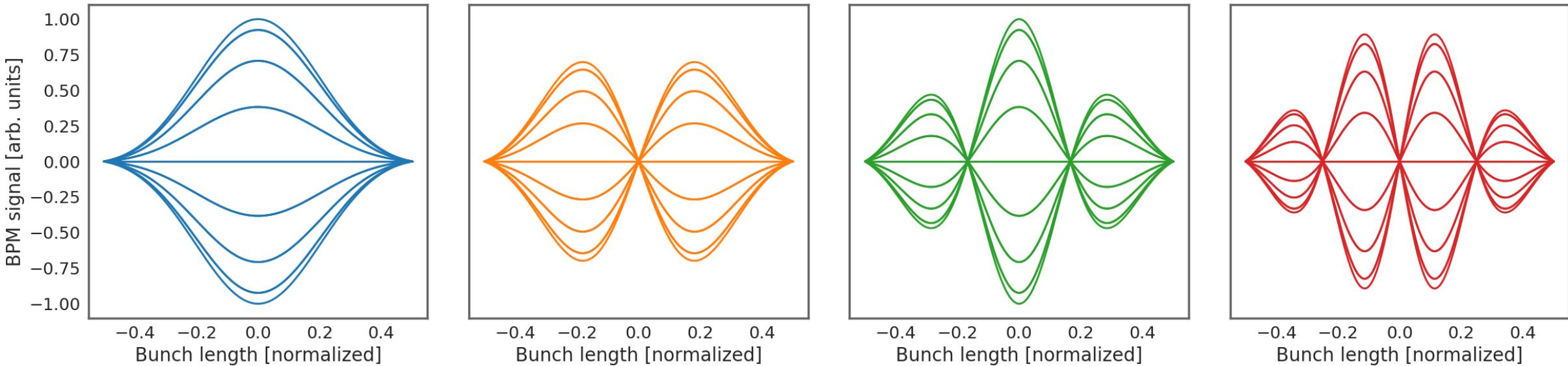
$$S \propto p_l(z) \cdot \exp\left(-2\pi i \left(k Q_x + \frac{\xi Q_x \omega_0}{2\pi \eta \beta c} z\right)\right)$$

$k$ : turn number  
 $\xi$ : chromaticity  
 $\eta$ : slippage factor  
 $\omega_0$ : revolution frequency

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# Headtail modes

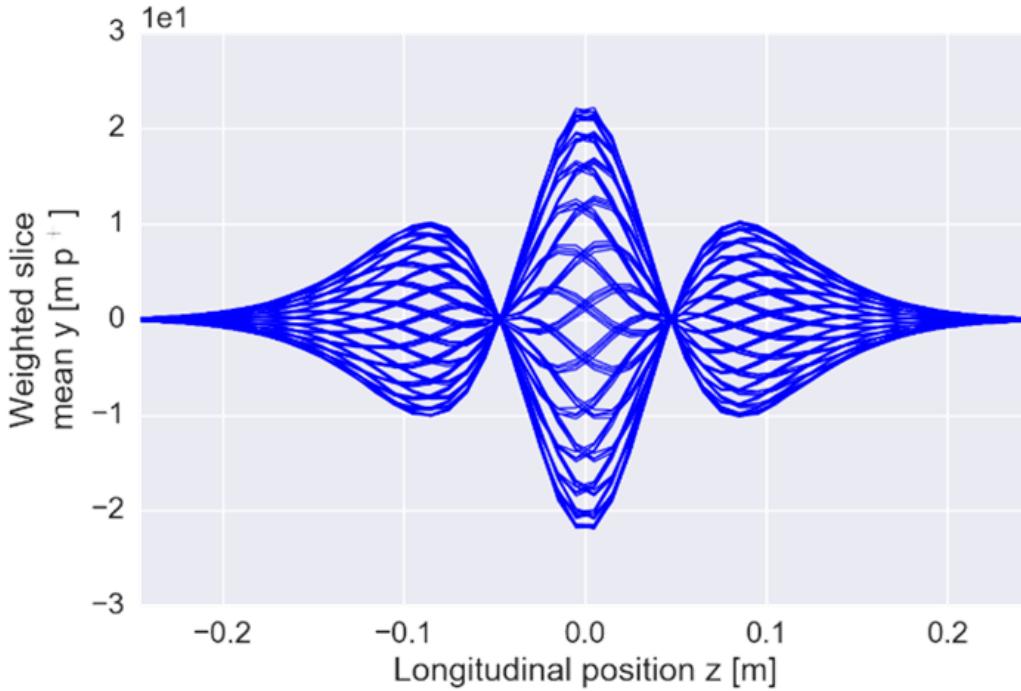
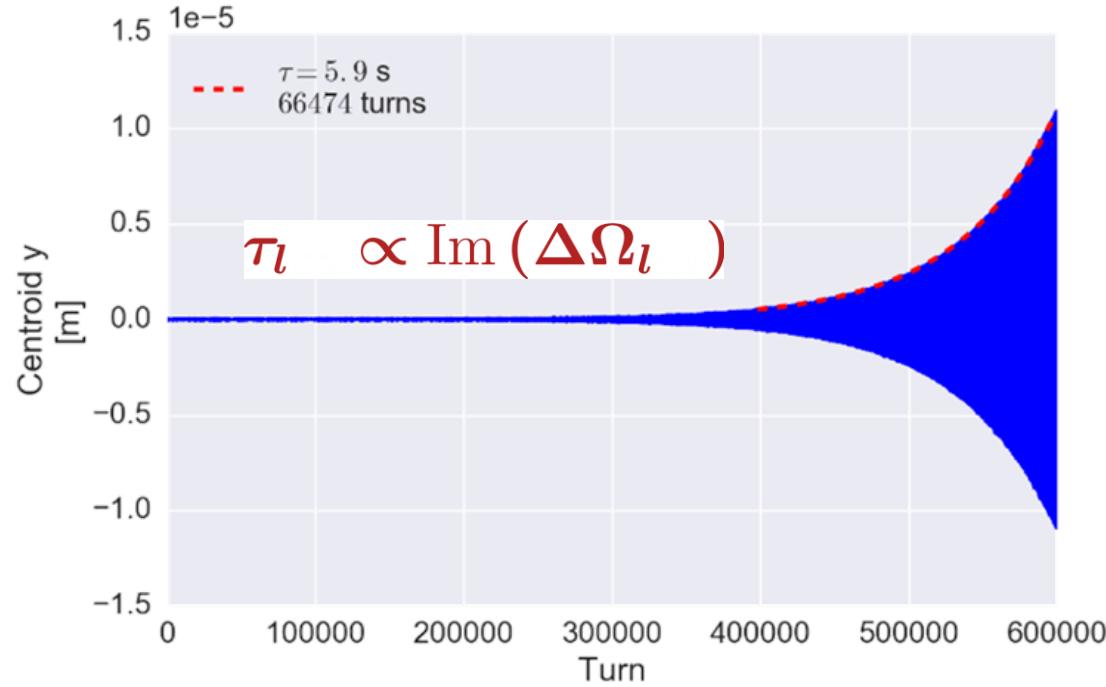


$$\xi = 0$$

It is to be noted, that these bunch modes are always latently present, but not usually excited. It requires an **impedance as source of energy** together with **chromaticity to generate a synchronization** of the bunch motion with the wake fields kicks in order to **drive a given bunch mode into resonance**.

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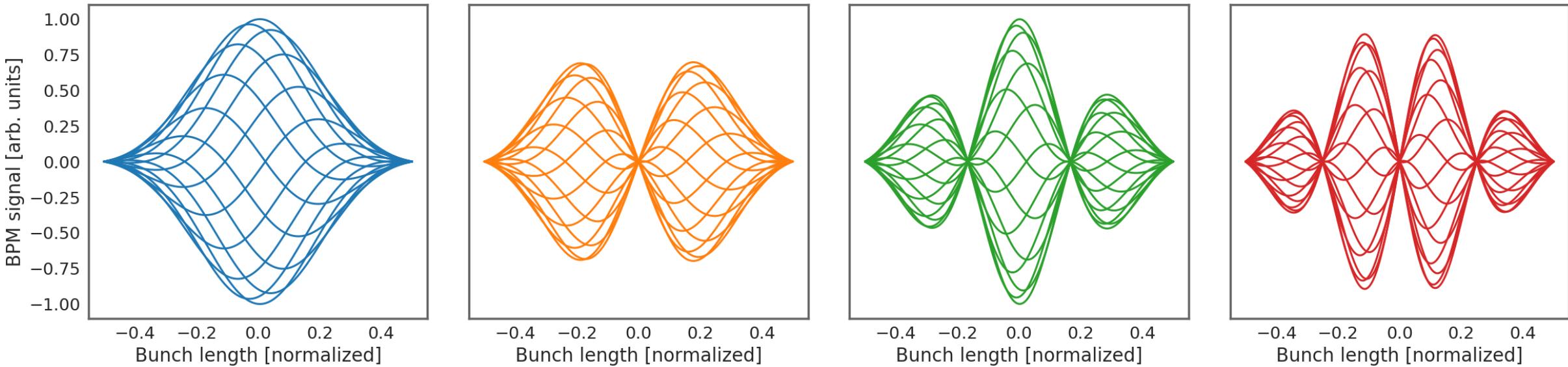
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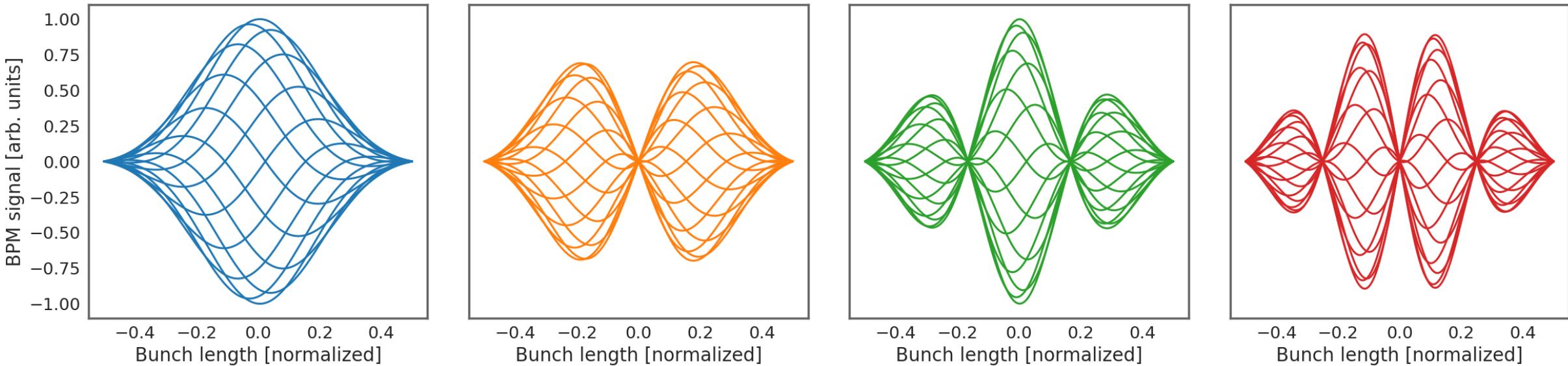


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# Headtail modes



## Below transition

## Above transition

Chromaticity negative

$l = 0$  stable;  
 $l > 0$  unstable

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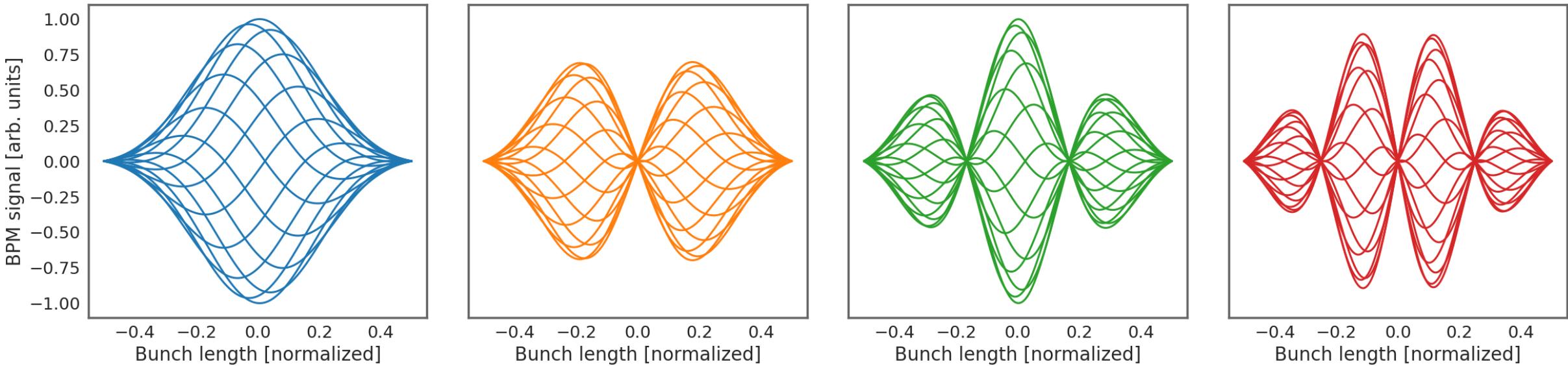
Chromaticity positive

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It turns out that  $1/\tau \propto (1 + l)^{-1}$  i.e.,  
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# Headtail modes



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We have seen how we can describe headtail modes as **Sacherer's sinusoidal modes**. We have learned that it requires impedances and chromaticity for these modes to actually **be resonantly excited via a synchronization** between the transverse and longitudinal **particle motion and the wake field kicks**.

We have also seen how we usually operate **machines in certain chromaticity regimes**, depending on the state of transition, in order to suppress the mode 0.

Another very violent type of instability is the **transverse mode coupling instability (TMCI)** which occurs also for vanishing chromaticity and provides a hard limit on the intensity reach of many machines.

- Part IV: Coherent beam instabilities

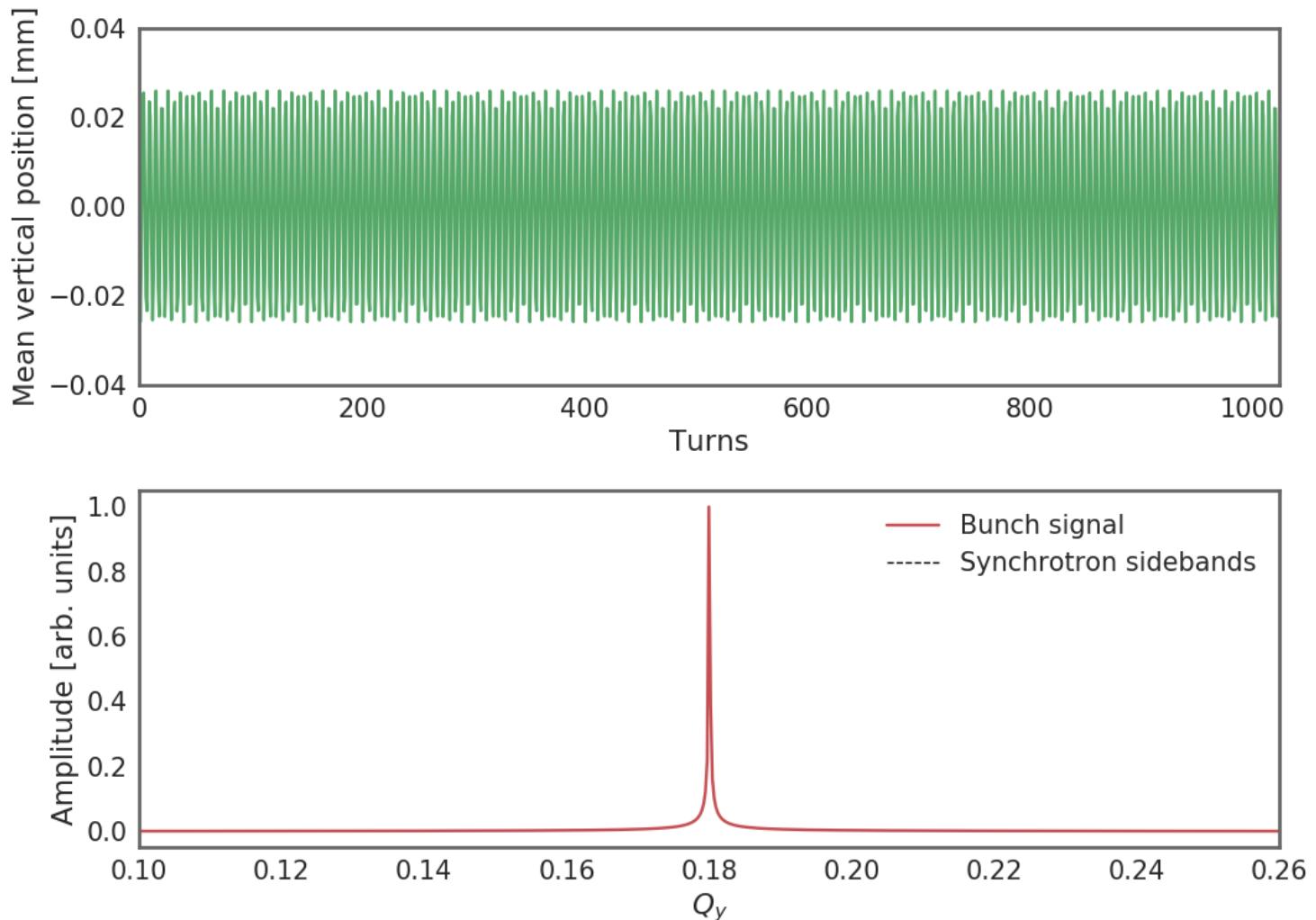
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# Azimuthal bunch modes

- The pure betatron motion follows from Hill's equation to

$$y(s) = \sqrt{2J_y \beta_y(s)} \cos\left(\frac{Q_y}{R} s\right)$$

- A spectral analysis of the betatron motion yield a single distinct line at the betatron tune

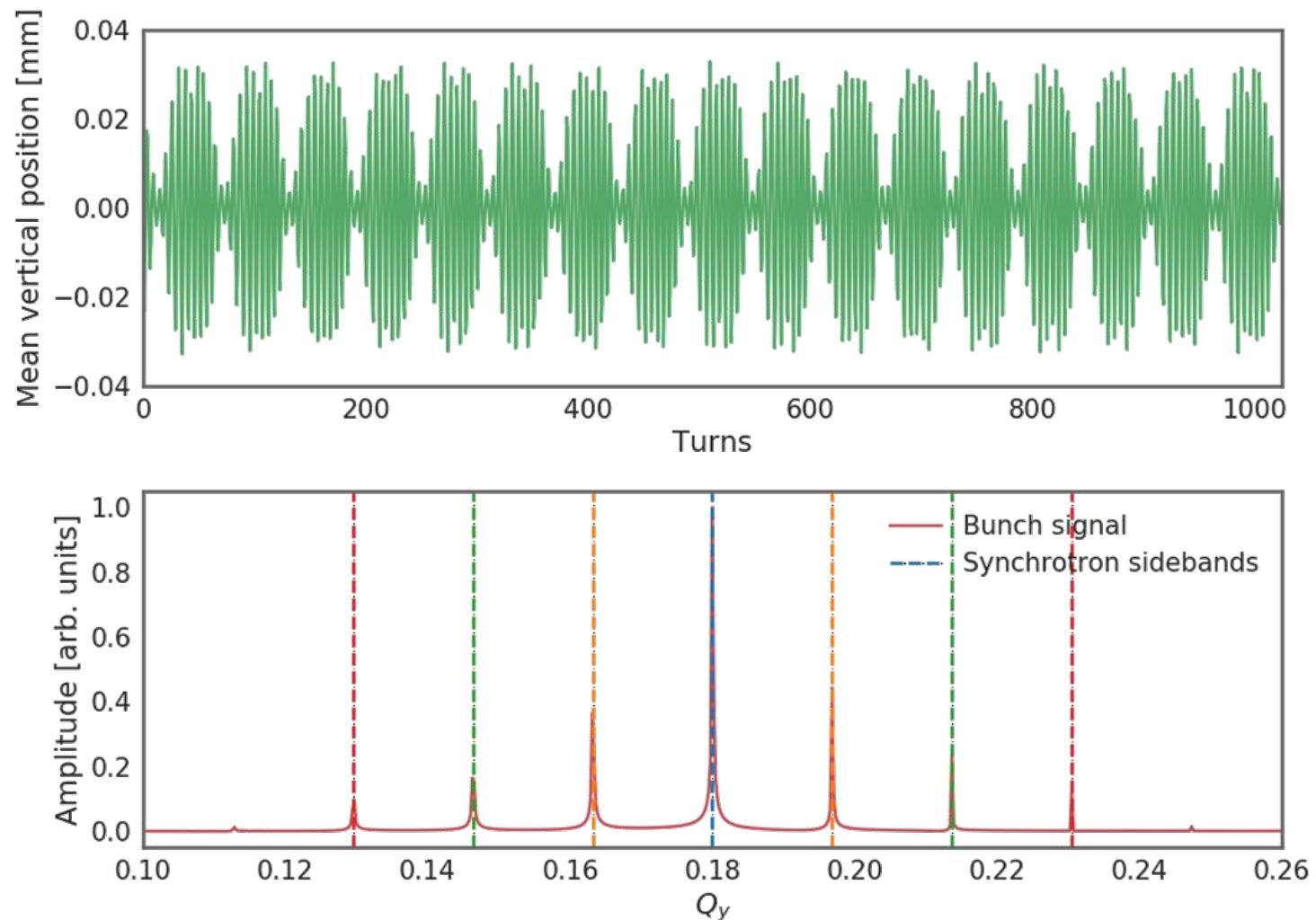


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- Synchrotron sidebands develop in the betatron spectrum – we call each sideband **an azimuthal mode of order I**.



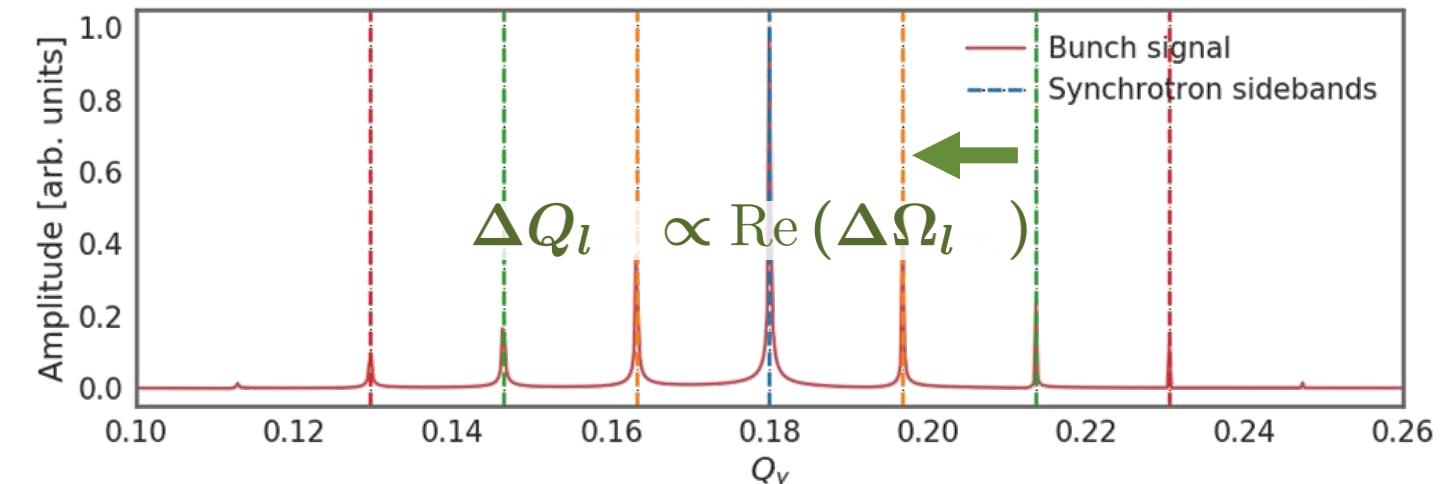
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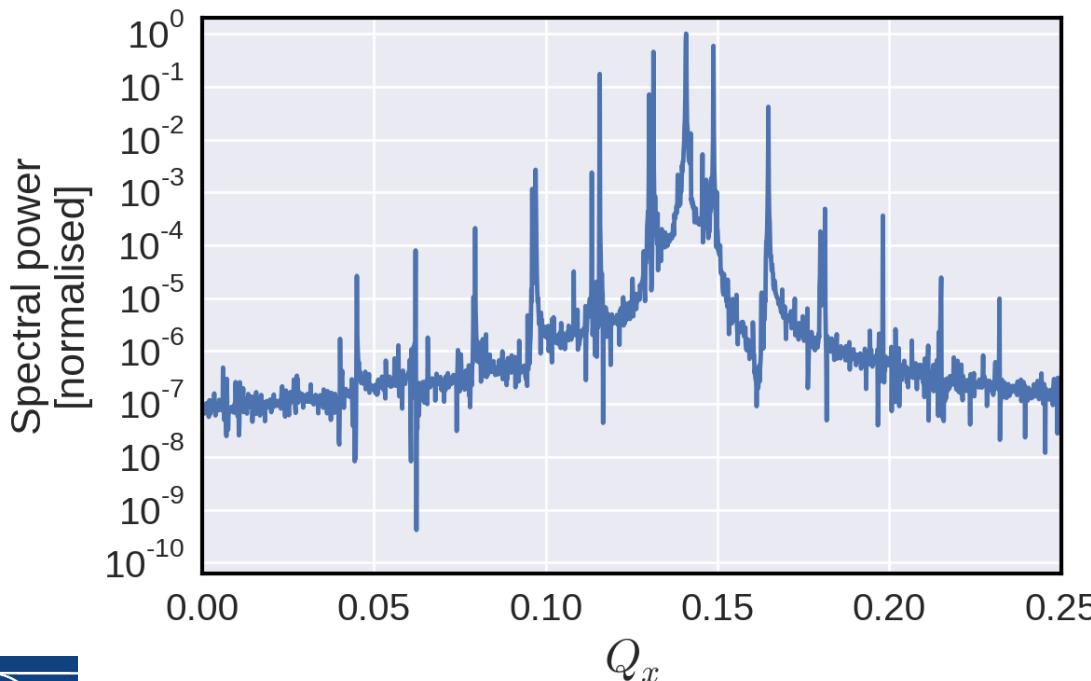
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- Impedances generate **a complex tune shift  $\Delta Q_l$  for every mode**
- The **real part of this shift** indicates the shifting of the corresponding mode
- The **imaginary part of this shift** indicates the growth rate of the corresponding mode



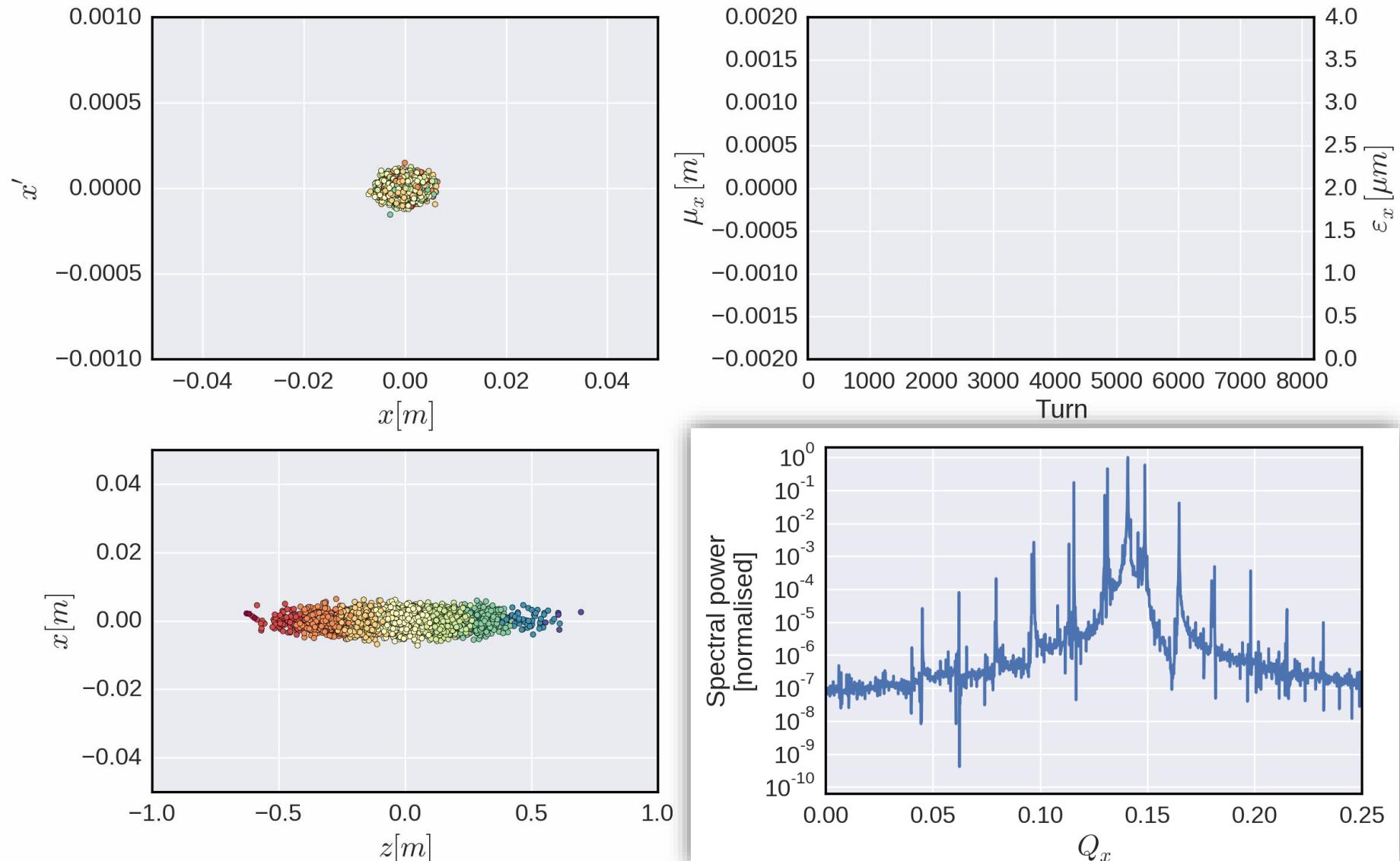
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- At **zero chromaticity**, all headtail modes are usually symmetrically at the same time damped and excited. None of the modes begins to grow and the **beam is inherently stable**.



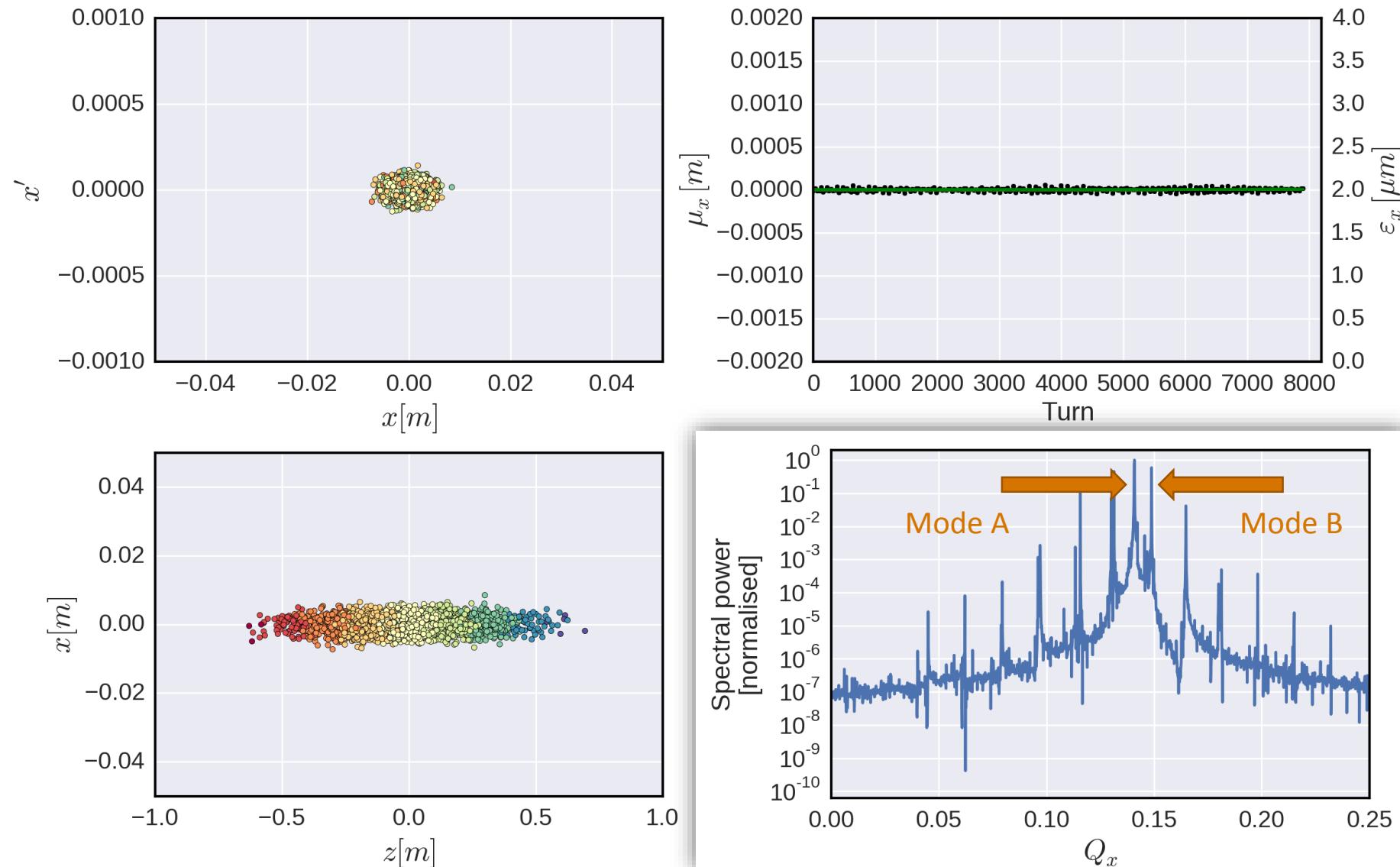
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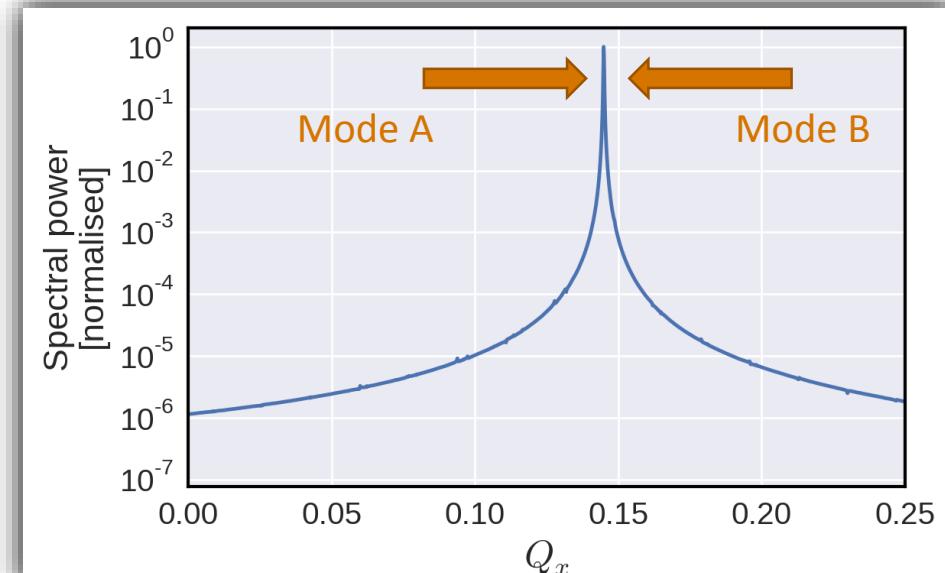
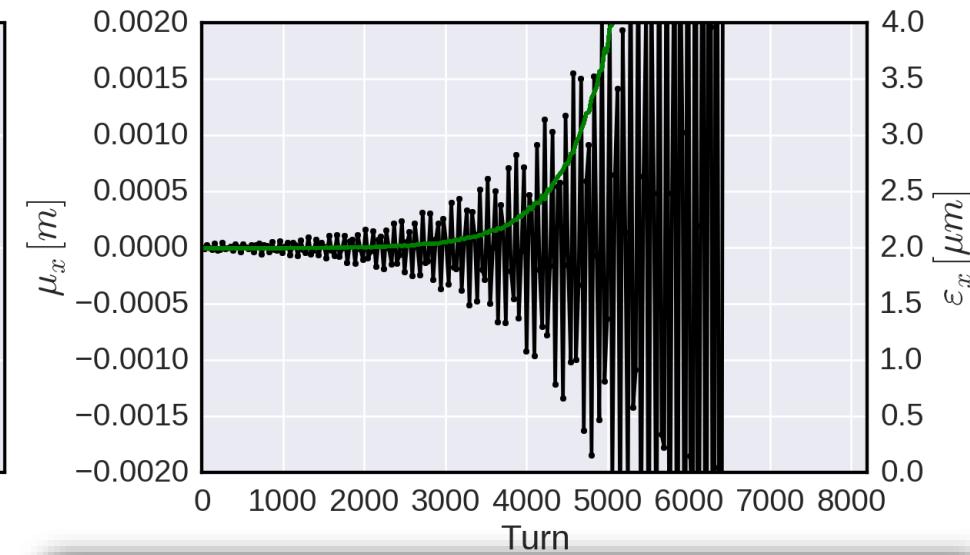
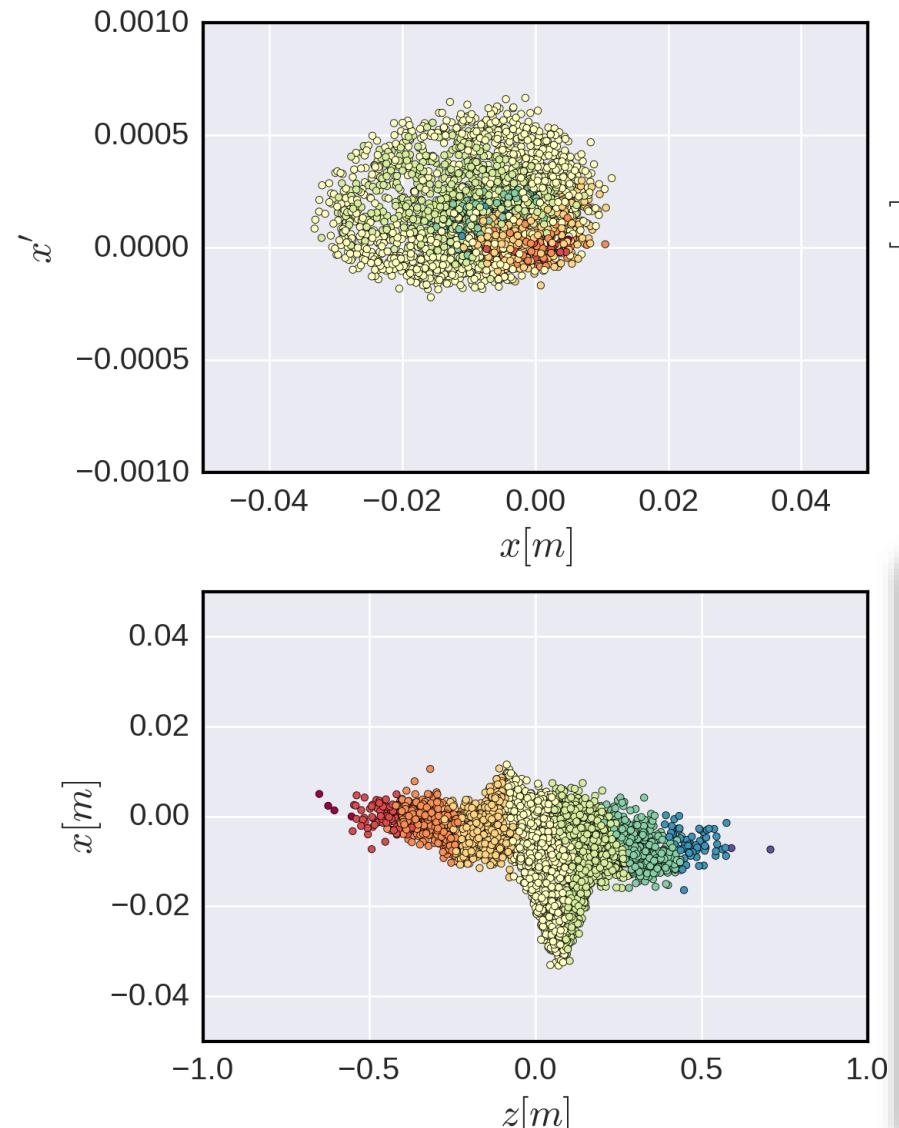
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- Azimuthal mode lines can shift in opposite directions...



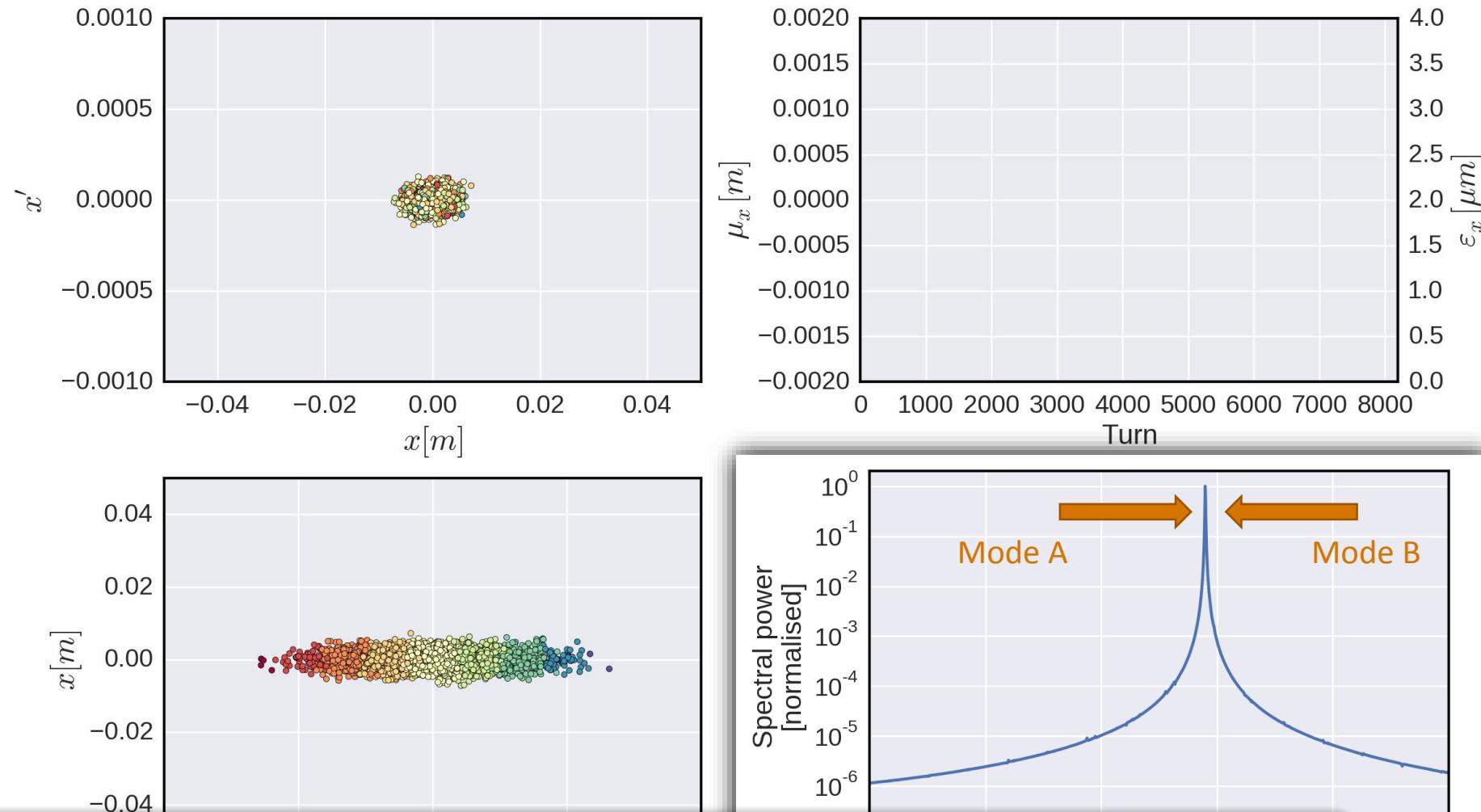
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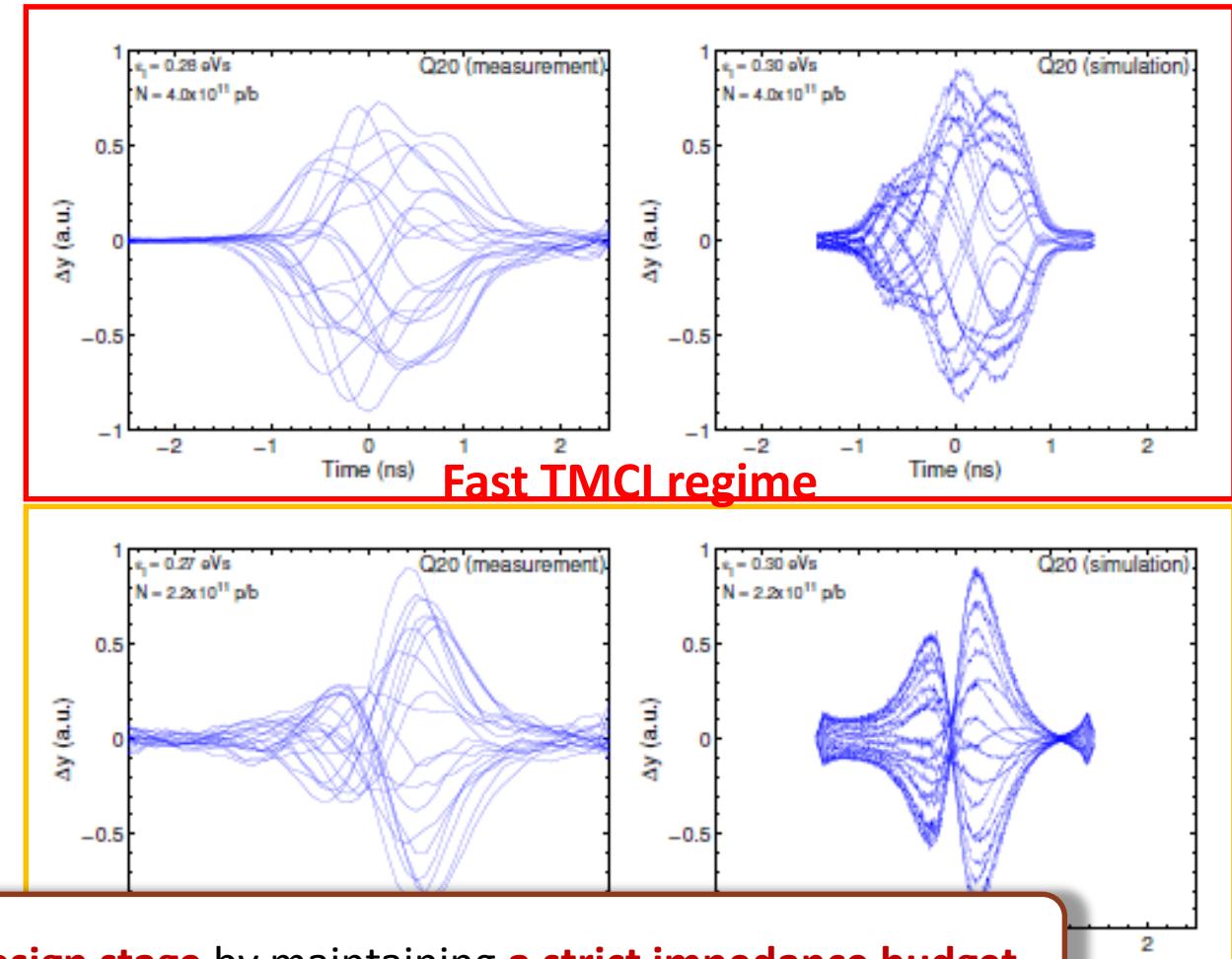
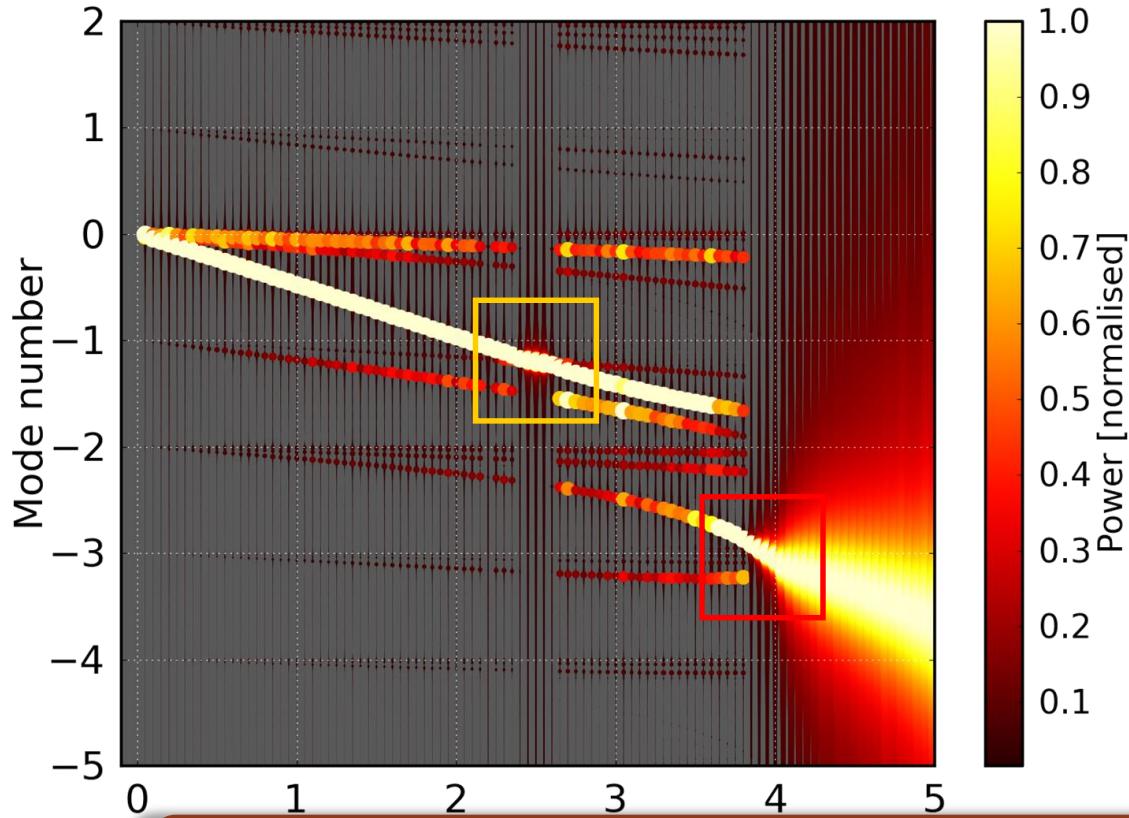
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- Until they couple and generate the **highly violent transverse mode coupling instability**



The TMCI threshold is often a hard limit on the reachable intensity in many machines.

# Transverse mode coupling instability in the SPS

- TMCI in the SPS (modeled using a broadband resonator impedance at around 1.3GHz)

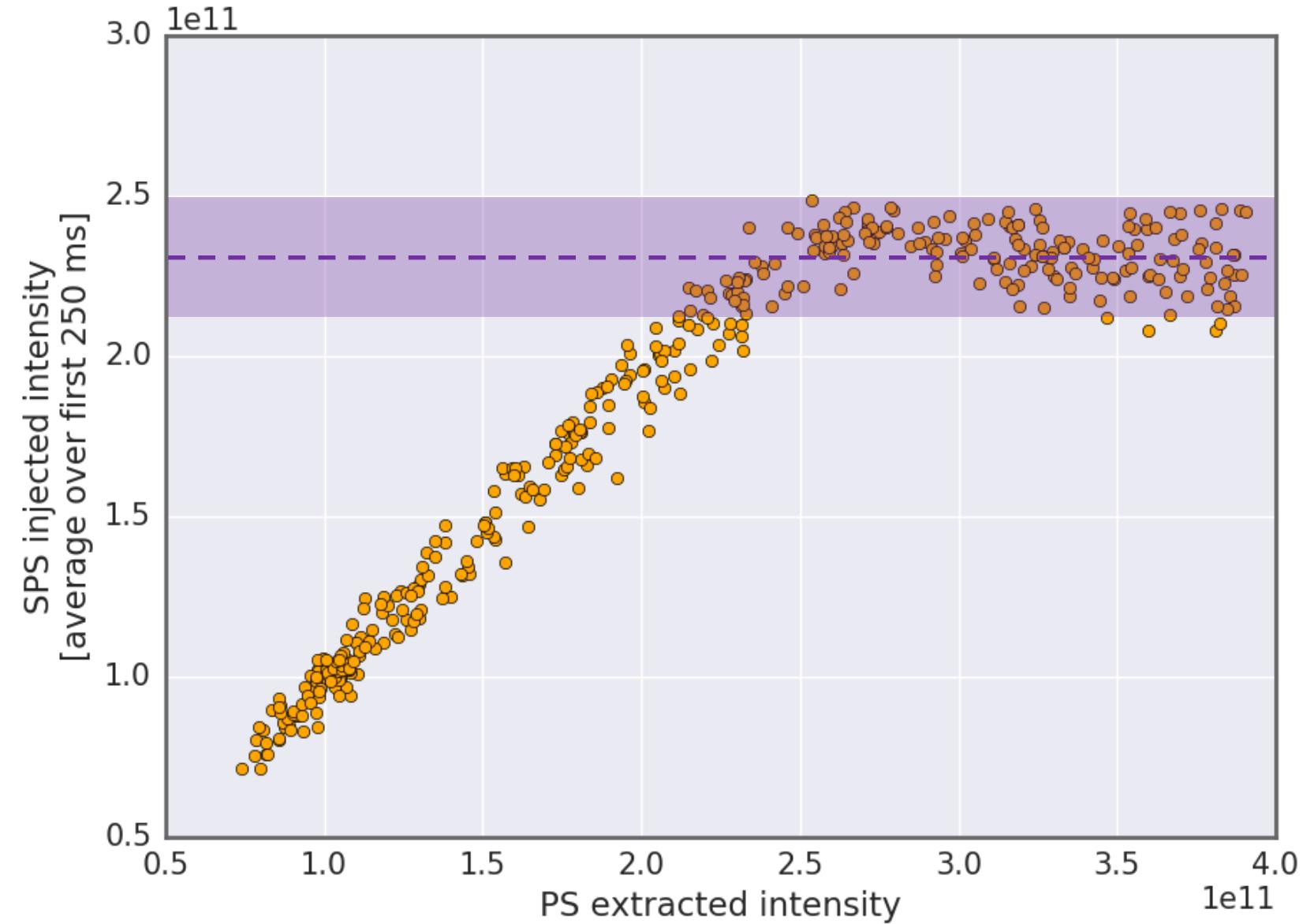


Mitigation of TMCI is usually done **already at the design stage** by maintaining **a strict impedance budget**.

H. Bartosik

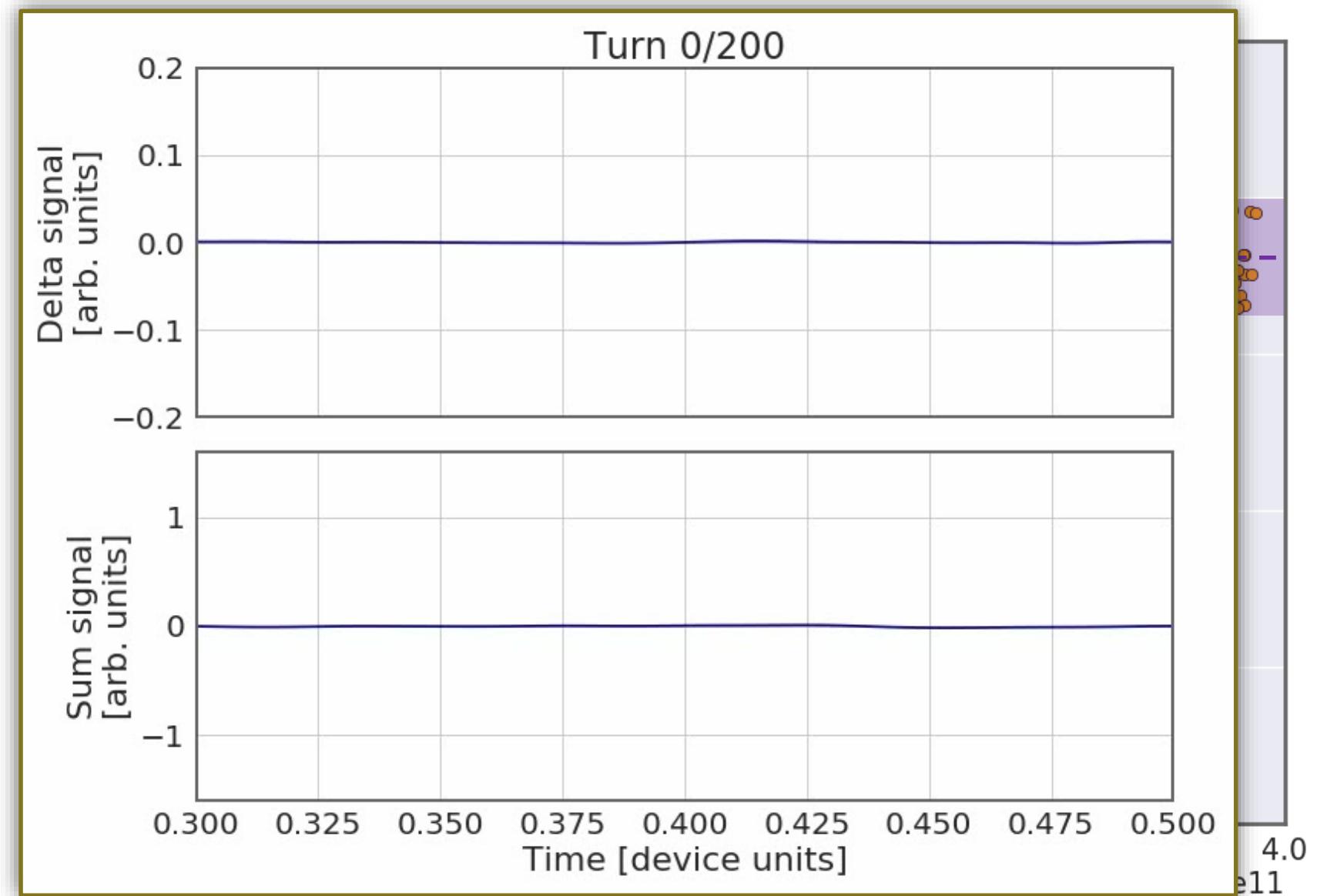
# An intensity scan in the SPS from 2017

- Intensity scans clearly reveal an **intensity limitation at around  $2.4 \times 10^{11}$  ppb**
- This limit manifests itself via a strong instability accompanied by **very fast losses**. This is the transverse mode coupling instability (**TMCI**).

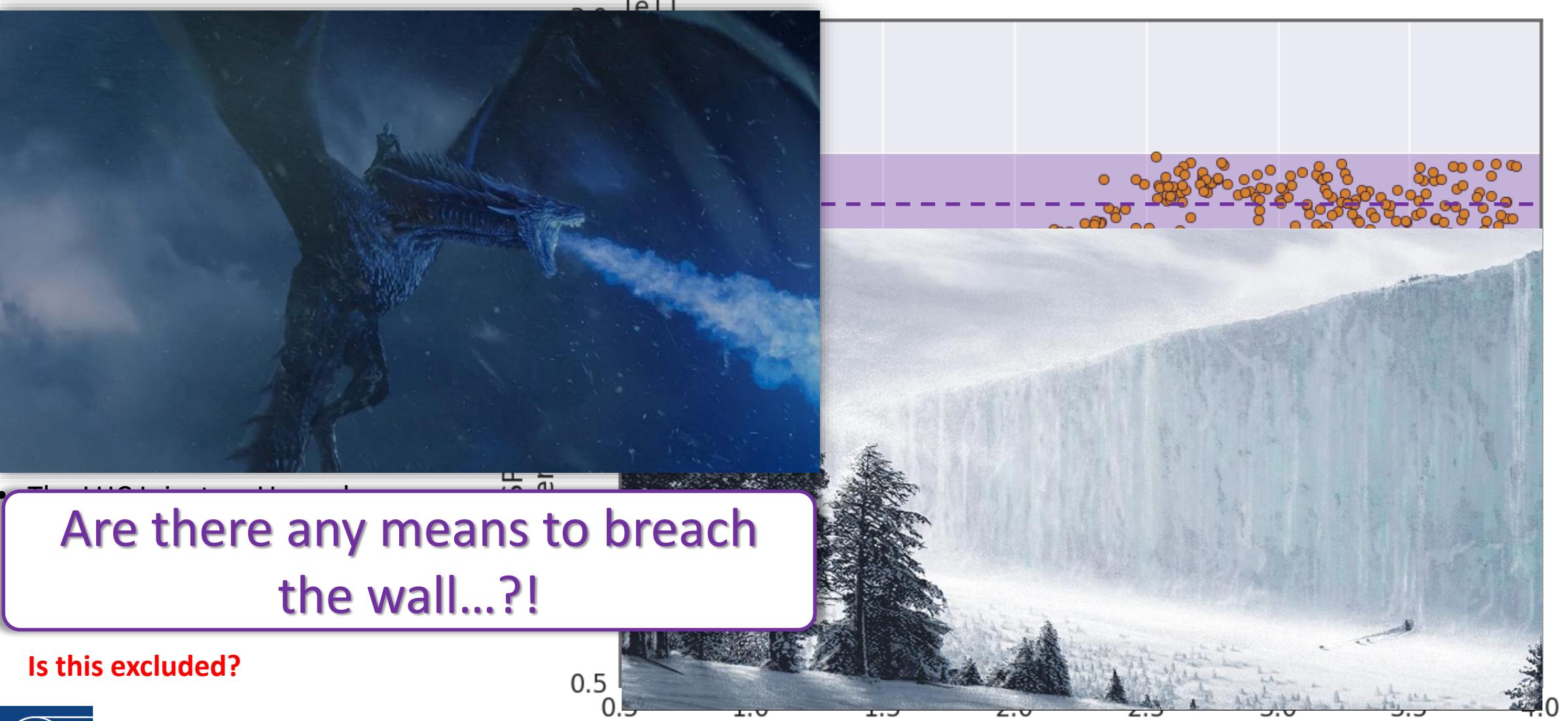


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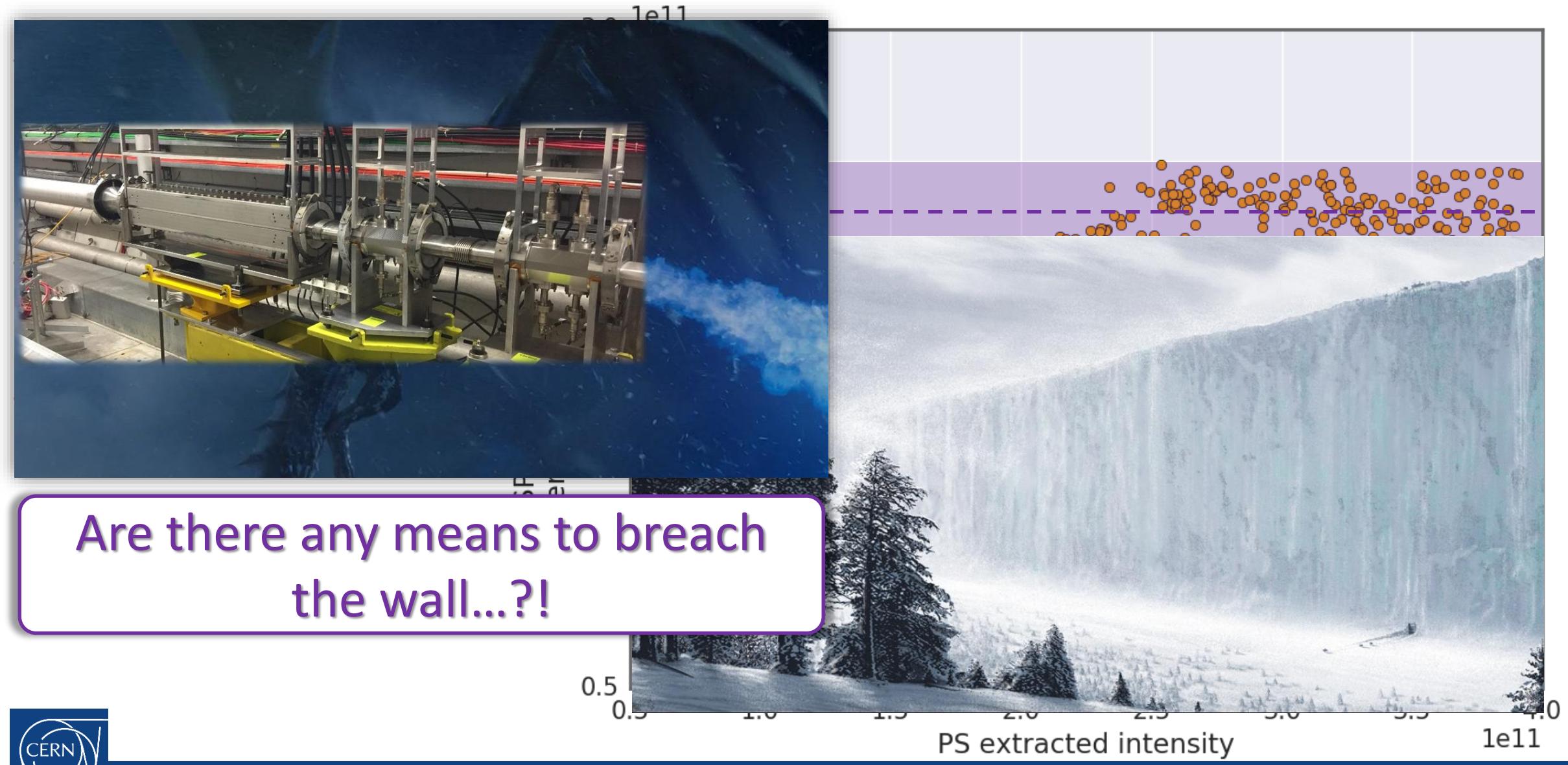
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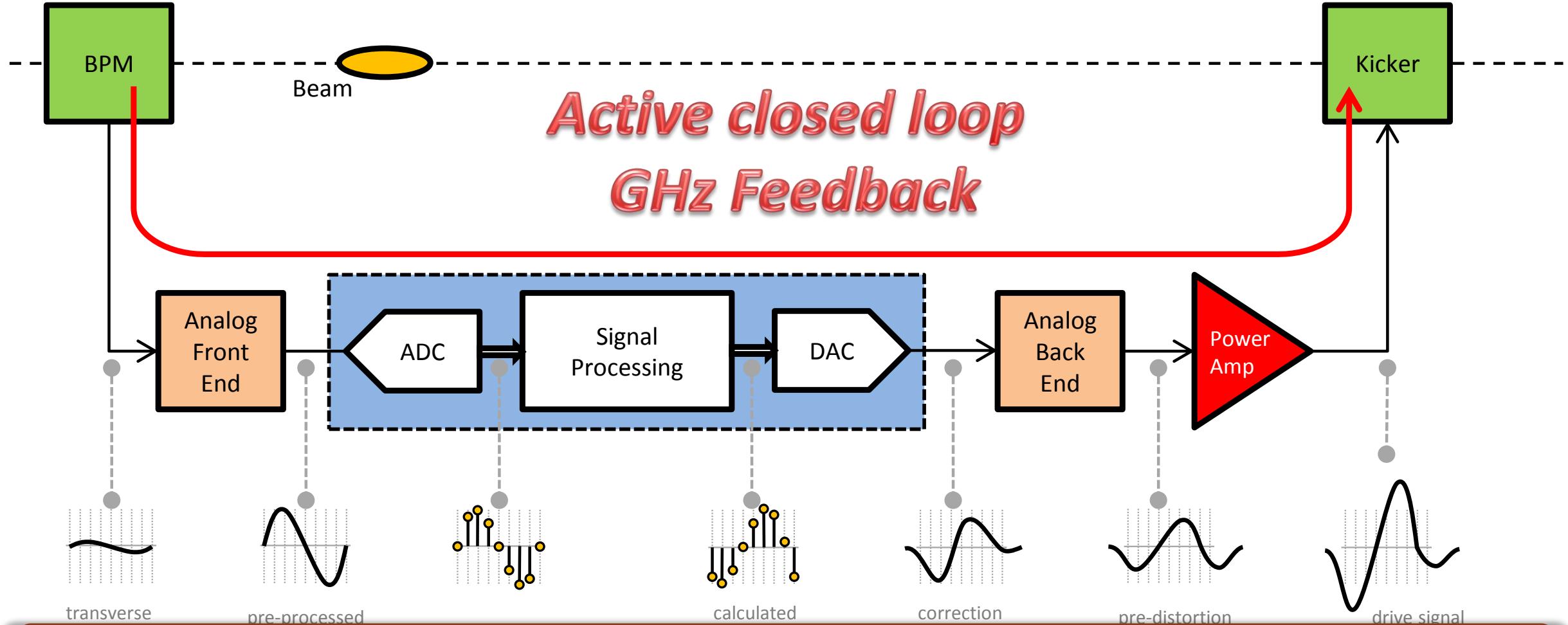
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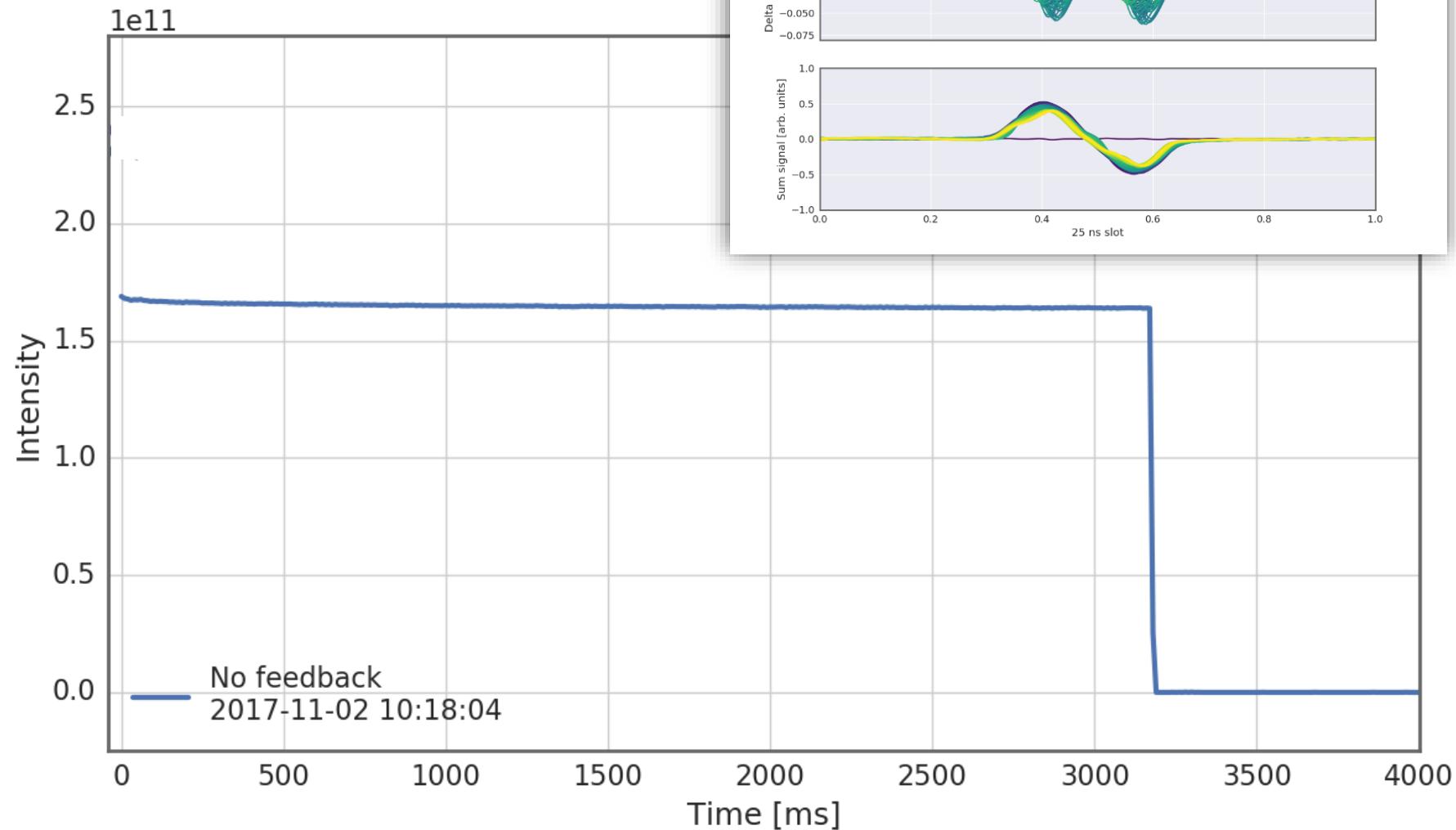
# Wideband feedback systems



Simulations show that a wideband feedback system in principle **can mitigate** both TMCI and e-cloud driven instabilities.

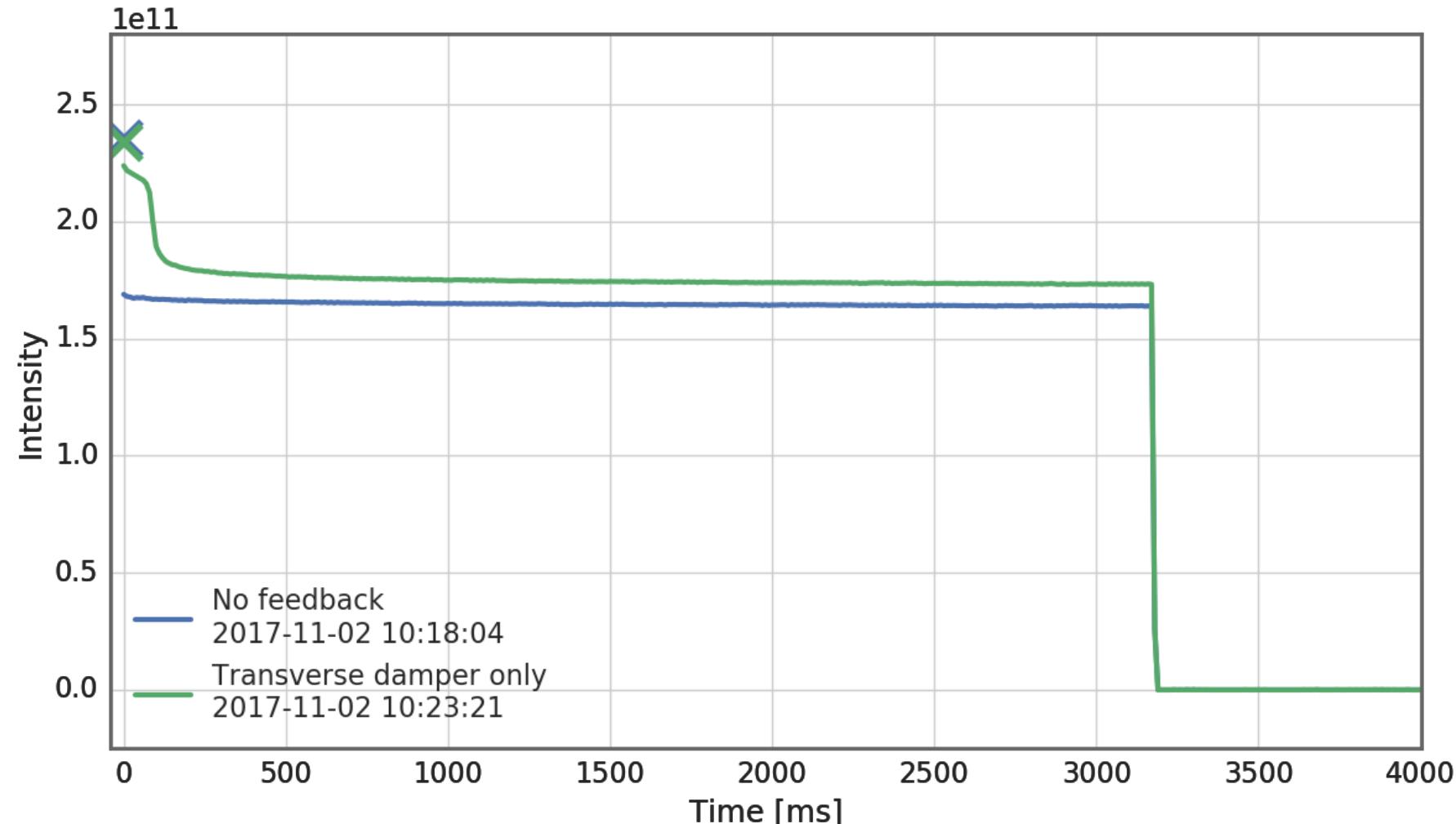
# Intensity scan

- First test... looking at the BCT, **a constant signal** is observed all along the cycle.
- To be noted that we are **injecting high intensity beams** ( $\sim 2.5 \text{e}11$  ppb) → TMCI induced losses occur before the first BCT sampling point!
- A look into the HEADTAIL monitor **just after injection reveals the TMCI**.



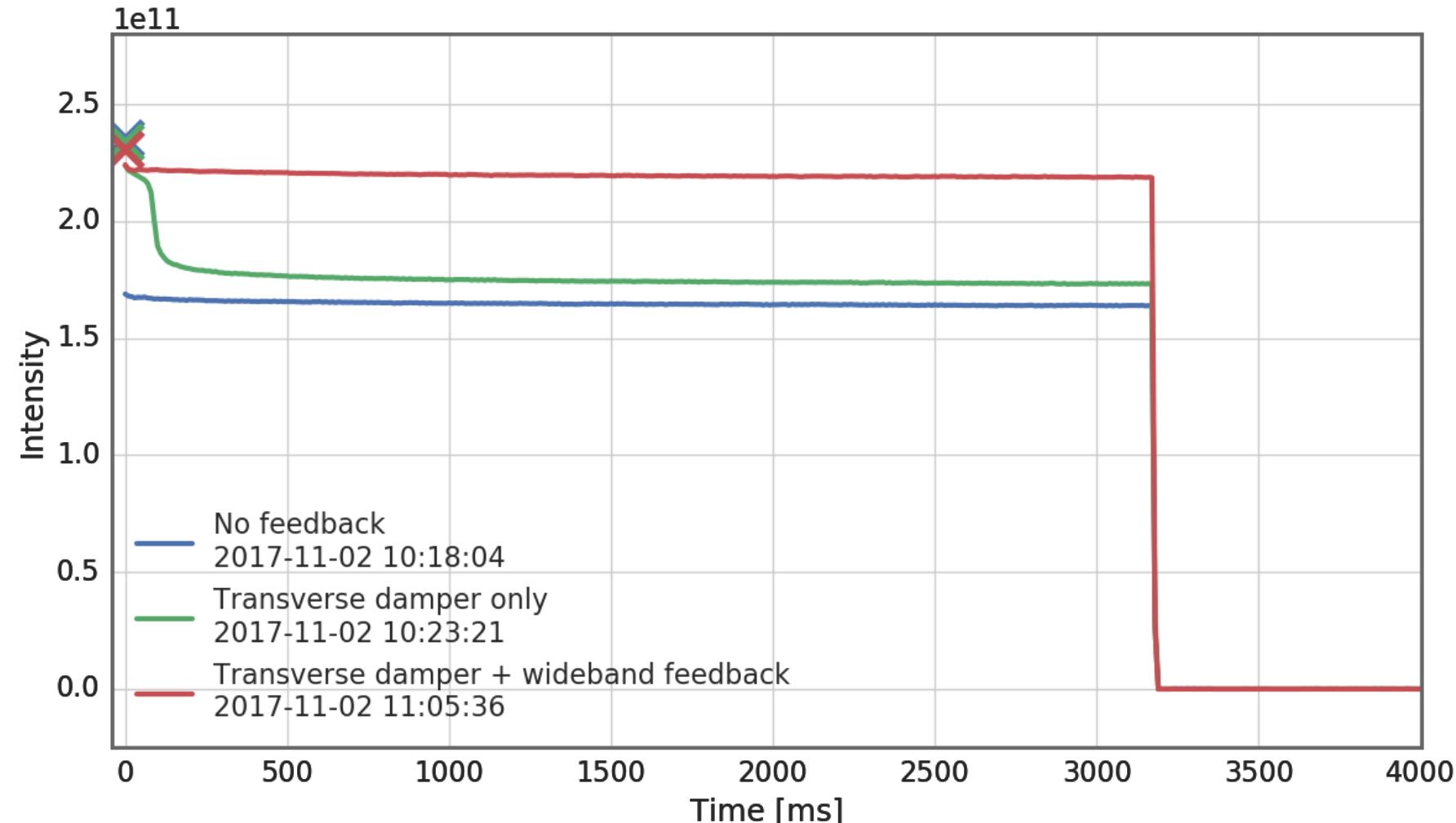
# Intensity scan

- The **standard transverse damper** was set up and put into operation in an attempt to mitigate the instability.
- The fast growth was reduced but **could not be stopped**. The losses are ultimately comparable to running without the transverse damper.
- This is expected **due to the bandwidth limitations** of the transverse damper... the high frequency content of the instability remains unaffected.



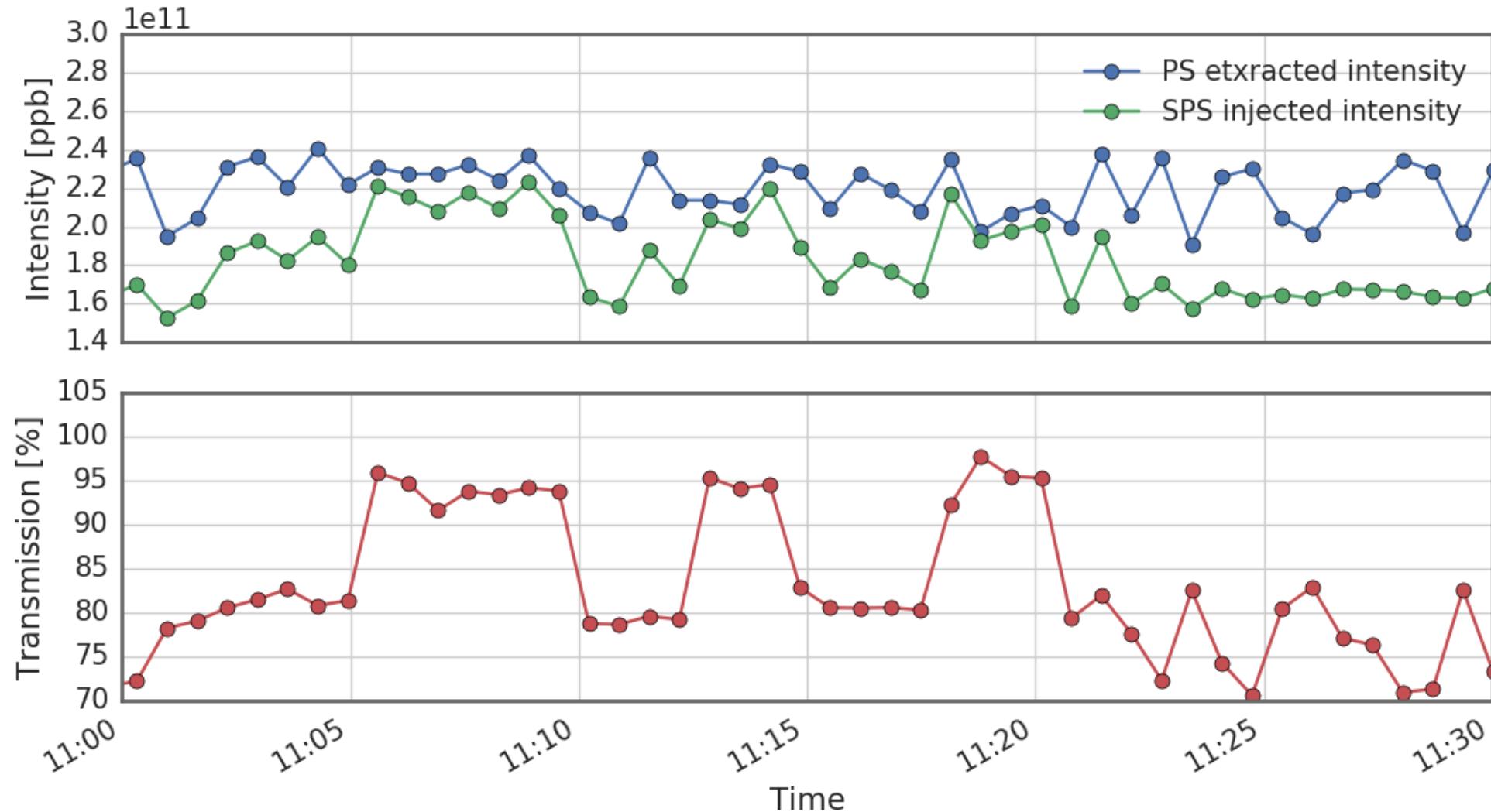
# Intensity scan

- Finally, the **wideband feedback system** was time aligned, configured and activated by closing the loop over the observed instability.
- The **transverse damper was kept active** to control the large amplitude low frequency motion to prevent saturation of the ADCs which would otherwise render the wideband feedback system ineffective.
- With the **two systems active**, the **losses are significantly reduced** and comparable to what is observed in absence of TMCI.



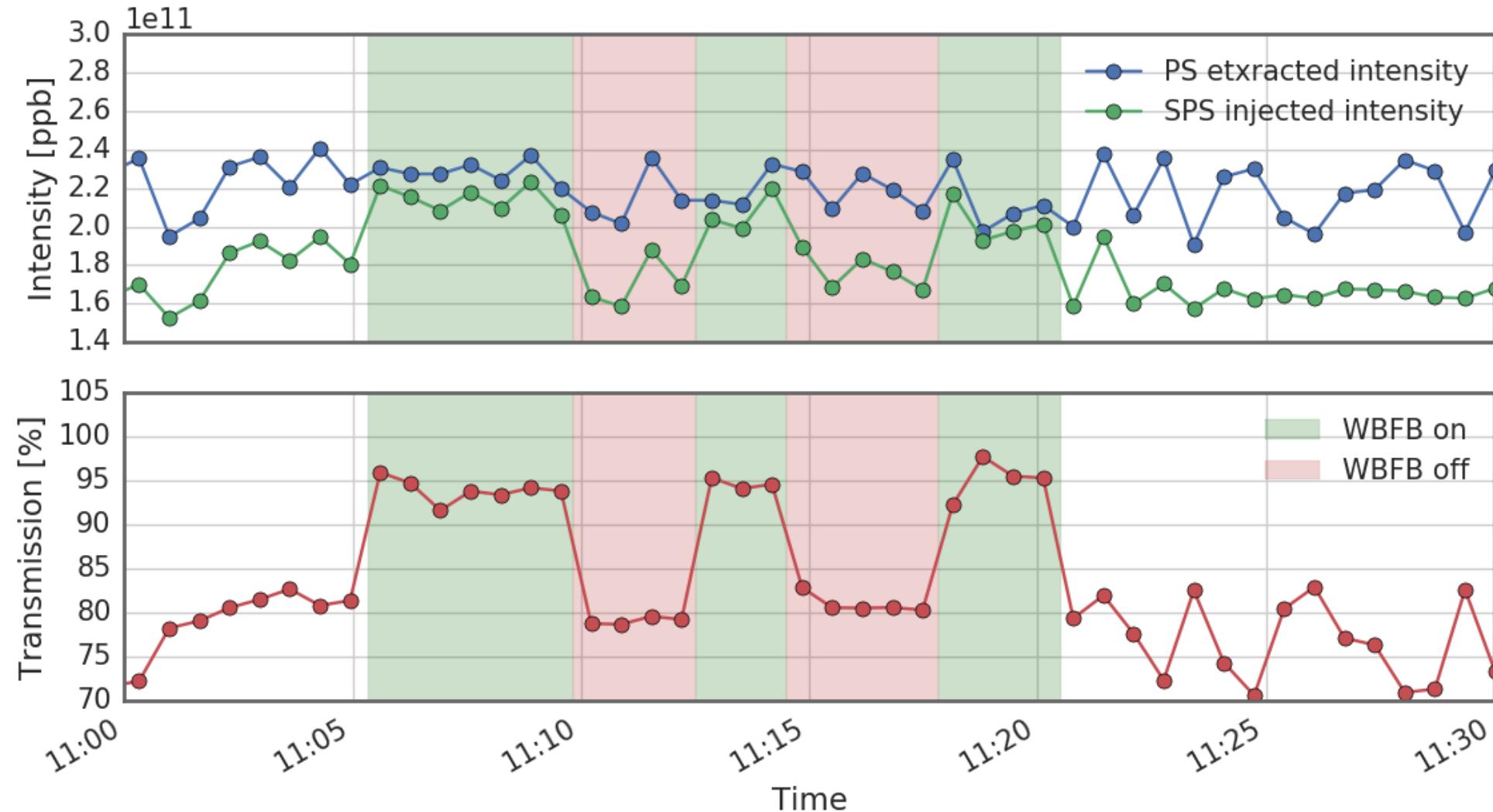
# Intensity scan

- The wideband feedback loop **was closed and opened several times** over a period of half an hour **to ensure reproducibility** of both the TMCI and the stabilization of the latter.



# Intensity scan

- The wideband feedback loop **was closed and opened several times** over a period of half an hour **to ensure reproducibility** of both the TMCI and the stabilization of the latter.
- There is a **clear correlation** between transmission and open/closed loop configuration.





We seen another type of particularly violent instability – **the fast headtail instability**. This occurs at zero chromaticity if the intensity is high enough, such that different sidebands in the bunch spectrum couple. For this reason, it is also called **the transverse mode coupling instability (TMCI)**. As opposed to the slow headtail instabilities, TMCI has **a distinct intensity threshold**.

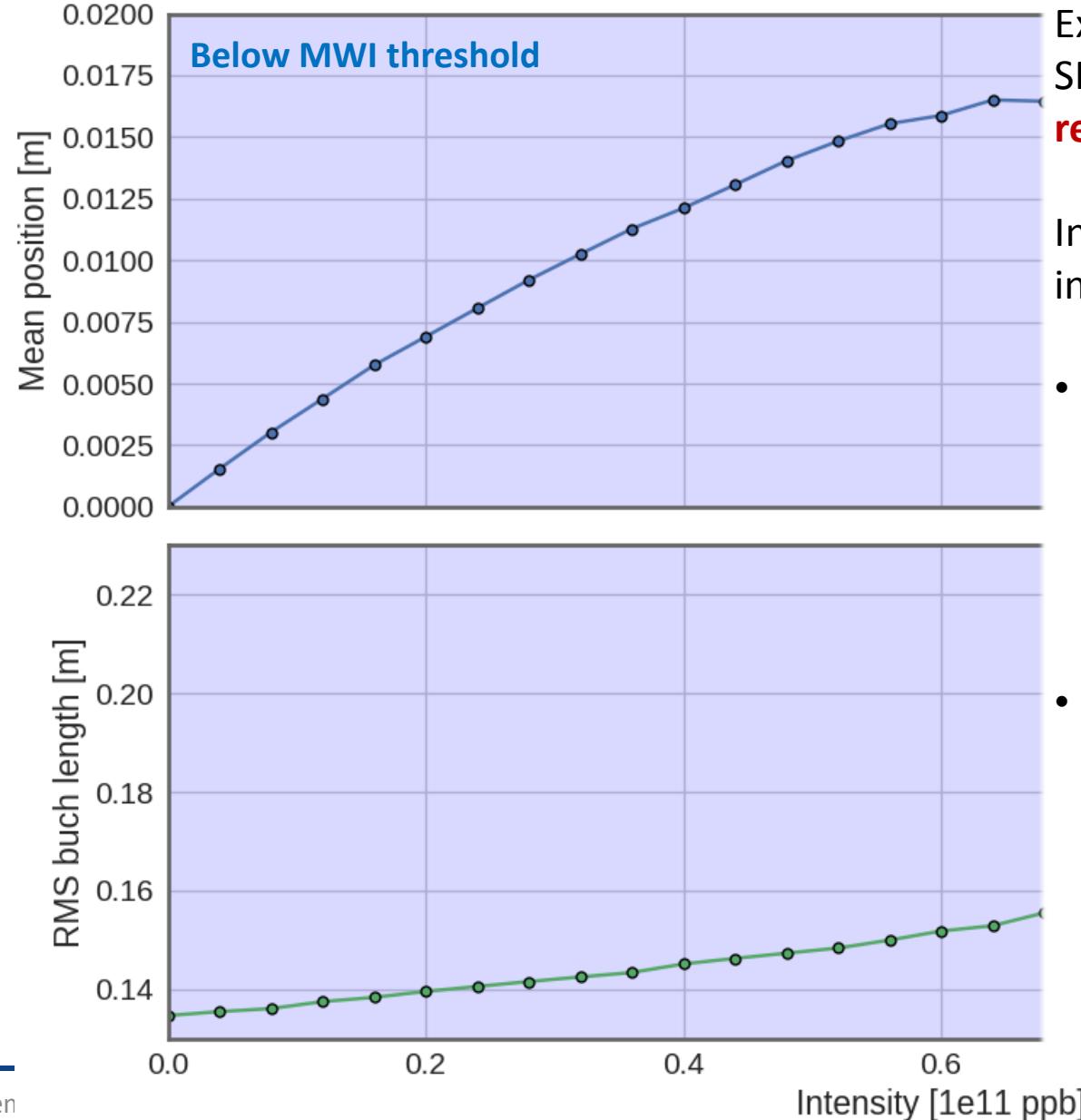
TMCI usually poses **a hard limit** on the machines reachable intensity and is usually avoided during the design phase of the machine. Recent developments of wideband feedback systems indicate, however, that these limits could probably also be breached by means of active mitigation.

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# Longitudinal instabilities

- For the case of longitudinal wake fields, two regimes can be found:
  - Regime of potential well distortion (i.e. perturbations to **equilibrium solutions are damped**)
    - Stable phase shift
    - Synchrotron frequency shift
    - Different matching ( $\rightarrow$  bunch lengthening for lepton machines)
  - Regime of longitudinal instability (i.e. perturbations to **equilibrium solutions grow exponentially**):
    - Dipole mode instabilities
    - Coupled bunch instabilities
    - Microwave instability (longitudinal mode coupling)

# Example: bunch lengthening and MW instability

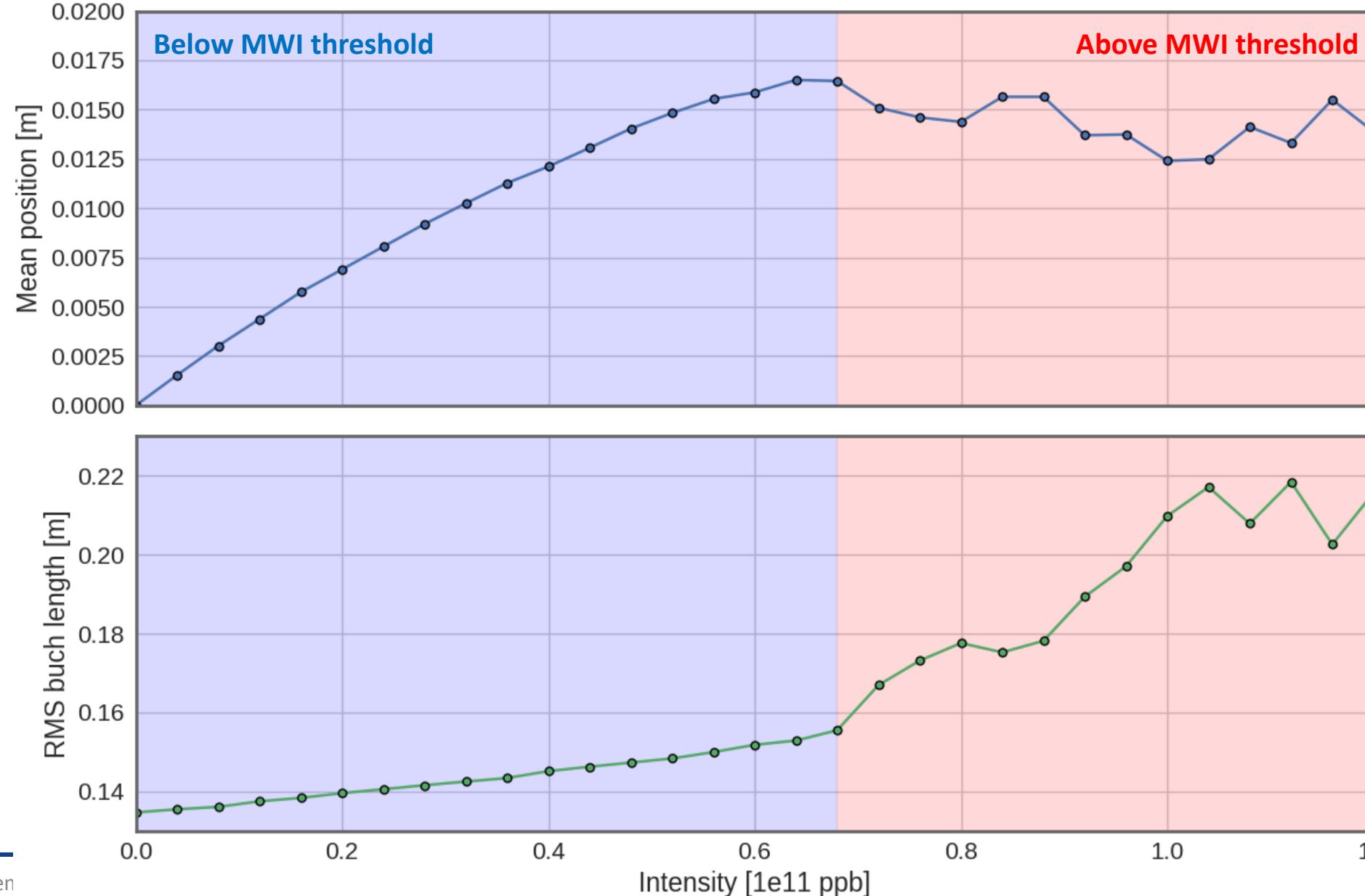


Examples of numerical simulations – SPS bunch with **single broad-band resonator** wake:

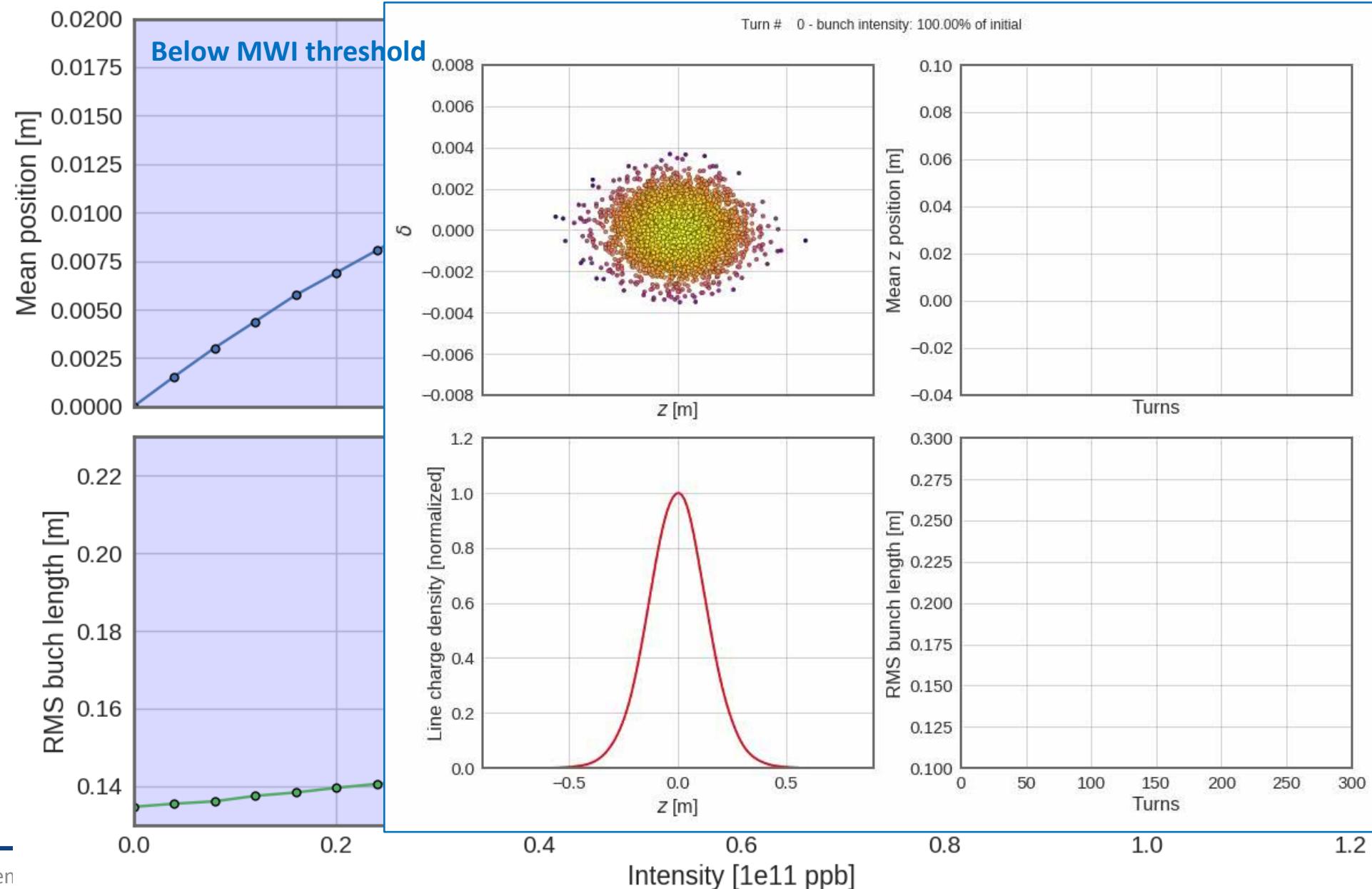
Initializing a matched bunch at low intensity, **two regimes are found**:

- Bunch lengthening/emittance blow up regime with roughly linear increase of the **synchronous phase** and **bunch length** with intensity
- Unstable regime (**turbulent bunch lengthening**)

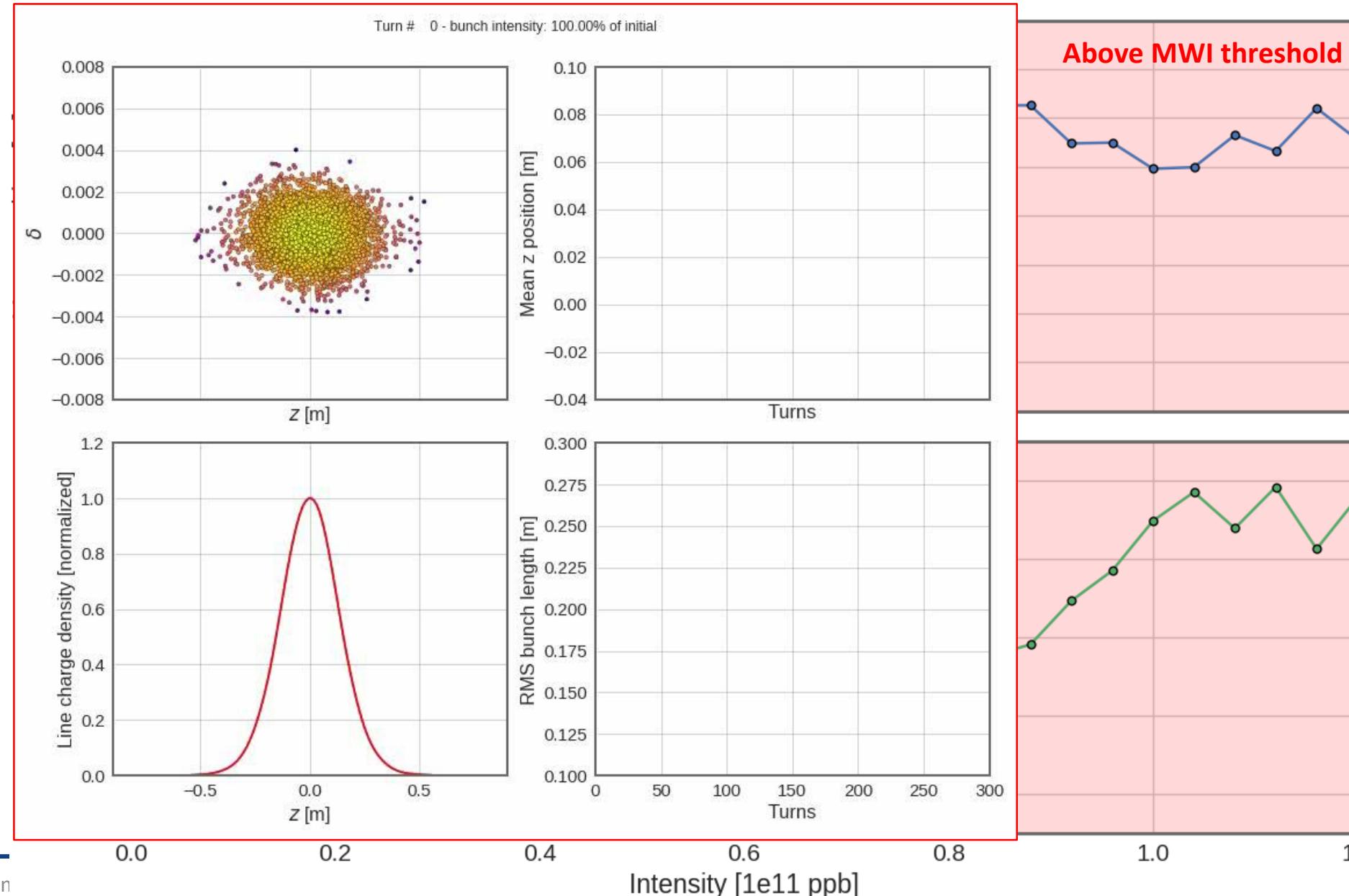
# Example: bunch lengthening and MW instability



# Example: bunch lengthening and MW instability

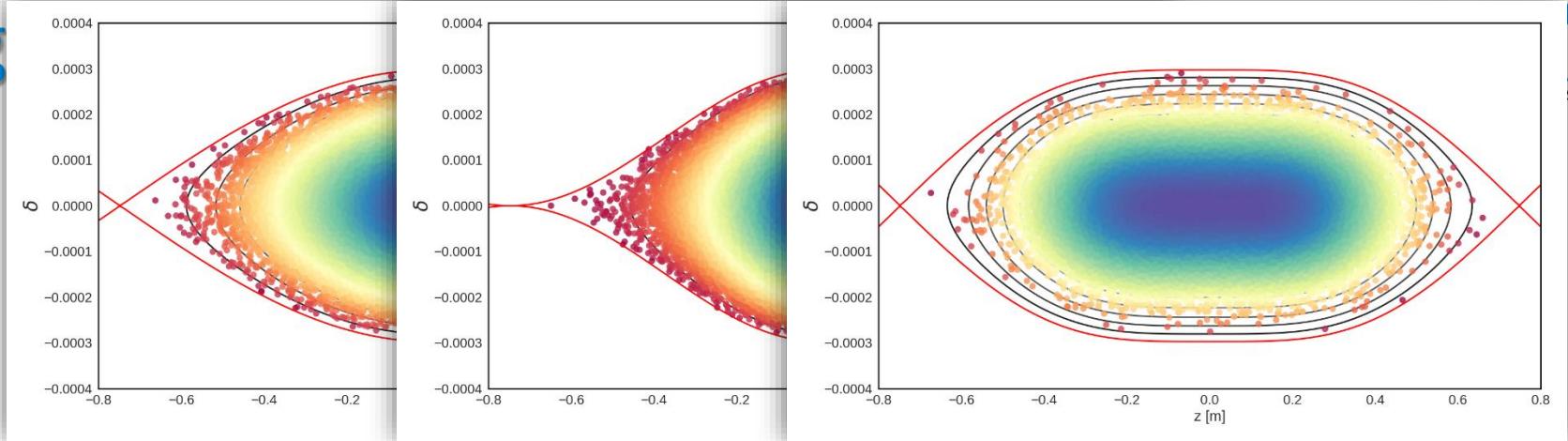


# Example: bunch lengthening and MW instability



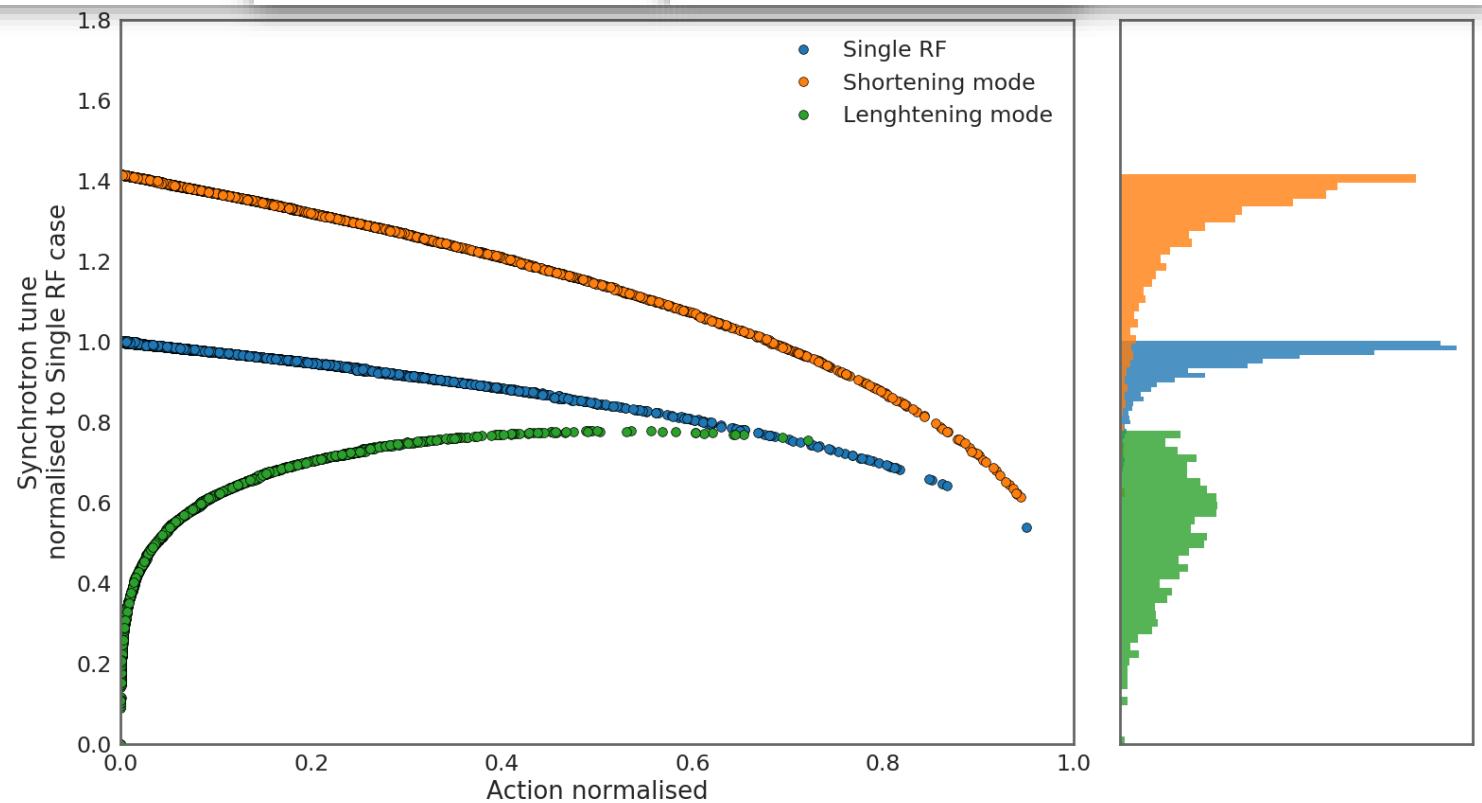
# Other types of longitudinal instabilities

- Other types of longitudinal instabilities:
  - Coupled bunch instabilities
  - Negative mass instability
  - Robinson instability



- Mitigation:
  - Longitudinal feedbacks
  - Enhancing the tune spread

Like in the transverse plane, one can introduce a **synchrotron tune spread** to introduce Landau damping. This is done using higher harmonic cavities.



# Closing remarks

In the last four lectures we have briefly touched the topic of **collective effects in accelerator physics and beam dynamics**. We have treated most items phenomenologically to gain **an intuitive understanding** of the involved mechanisms.

By now we should be able to identify the main **differences between the dynamics** of single particles vs. multi-particle systems.

We understand the features of **collective effects such as space charge** and how instead of being constant forces, instead, they **depend on the particle distribution function** itself.

We know how we can use the concept of **wake fields and impedances** to model the impact of more complex elements.

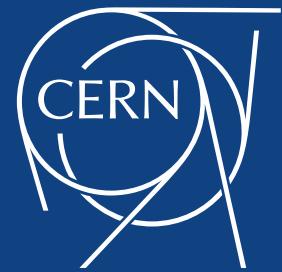
And we know how to identify an instability, with a rough overview over the **different type of instabilities** typically observed in synchrotrons along with possible mitigations.

More complex analysis involve the Vlasov formalism to analytically model simplified cases or full macroparticle models to simulate the beam dynamics of collective effects.



# End part IV





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