

ACCELERATION AND CROSSING OF TRANSITION ENERGY INVESTIGATION USING AN RF STRUCTURE OF THE BARRIER BUCKET TYPE IN THE NICA ACCELERATOR COMPLEX

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Abstract

During particle acceleration to the energy of experiment $E_{exp} = 12.6 \text{ GeV}$ ($\gamma_{exp} = 14.4$) in NICA collider there is a need for the crossing of transition energy $E_{tr} = 5.709 \text{ GeV}$ ($\gamma_{tr} = 7.087$). The slip-factor $\eta = \frac{1}{\gamma_{tr}^2} - \frac{1}{\gamma^2}$, which included in the equation of longitudinal motion, changes the sign, that leads to a loss of stability in the longitudinal plane when γ approaching to γ_{tr} . In order to minimize the reduction of the beam parameters, a rapid jump of critical energy is assumed during the time $t_{jump} \approx 10 \text{ ms}$ [1] with a simultaneous change of the polarity of the RF system to ensure the stability of the beam after the jump. RF structure based on “Barrier Bucket” a feature of the NICA collider accelerating system. A non-zero value of the field between the barriers, ensures the acceleration of the beam. This feature makes the system original.

In this paper, the main features of the dynamics of the longitudinal motion of the beam crossing through the transition energy are considered, taking into account its jump in the RF structure of the “Barrier Bucket” type. Due to the rapid jump of transition energy the time at which the particles are near the zero value of the first order slip-factor is significantly reduced. Obviously, in this case, the second order of the slip-factor begins to play a decisive role in the behaviour of particles inside the barrier bucket and completely determines the stability region near transition. In this case, when energy of particles crosses transition energy, focusing of the beam in the longitudinal plane disappears and the influence of the space charge becomes essential.

RAPID JUMP OF THE TRANSITION ENERGY

Jump Schemes

When the particle energy approaches the transition energy, it is assumed to make a rapid jump in the critical energy. This can be achieved by changing the field gradient in the focusing quadrupoles of the arcs, since for periodic structures $1/\gamma_{tr}^2 = \alpha_0 = 1/\nu_x^2$, where α_0 – first order of momentum compaction factor, ν_x – normalized frequency of betatron oscillations (tune) in the horizontal plane. The proposed fast jump in transition energy is proposed to be equal to $2 \cdot \Delta\gamma_{tr} = 0.09$ (from $\gamma_{tr} = 7.087$ to $\gamma_{tr} - 2 \cdot \Delta\gamma_{tr} = 6.997$) [1]. With such a jump, the working point shifts the values from $\nu_x = 9.44$ и $\nu_y = 9.44$ to $\nu_x = 9.365$ и $\nu_y = 9.465$. The maximum change rate of

transition energy is limited by the parameters of quadrupoles and their power systems $\dot{\gamma}_{tr} = d\gamma_{tr}/dt = 8.5 \text{ s}^{-1}$. Such a jump can be made during the time:

$$t_{jump} = T_0 \Delta n_{jump} = \frac{2 \cdot \Delta\gamma_{tr}}{\dot{\gamma}_{tr}} = 10.5 \text{ ms}, \quad (1)$$

where $T_0 = 1.7 \mu\text{s}$ – the time of one revolution period in the ring, $\Delta n_{jump} = 6226$ – the number of revolution periods of the particles on orbit during which a transition energy jump occurs.

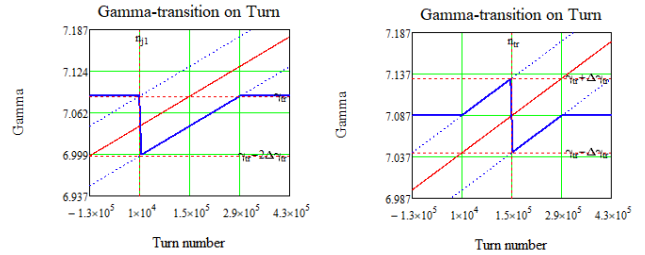


Figure 1: On the left – a schematic diagrams of the transition energy change γ_{tr} during the jump and its recovery. The blue solid line shows the transition energy γ_{tr} , the solid red line is the energy of particles γ . Blue dashed lines – γ of particles shifted by $\pm\Delta\gamma_{tr}$.

Figure 1 shows the change of transition energy γ_{tr} together with the beam energy γ at the moment of the jump and after it, at a different scale.

Dynamic Aperture

A change of the transition energy is possible due to a change in the tune of betatron oscillations in the x-plane, which is achieved by changing the gradient of the focusing lenses in the collider arcs. With the changed parameters of quadrupole lenses, the dynamic aperture was evaluated, it determines the stable area for movement of particles in the transverse plane. The corresponding calculations were carried out using OptiM and MADX programs.

The results shown on Figure 2a indicate a working point at $\nu_x = 9.44$ и $\nu_y = 9.44$. It is not suboptimal, since the dynamic aperture in the vertical plane is already initially small at these operating values (for this reason octupole correctors are used). If we assume the fact that it is necessary to reach the low frequency in the horizontal plane, with $\nu_x = 9.3627$ and in the vertical plane $\nu_y = 9.4541$ (see Figure 2 a). So thus, the transition energy is achieved $\gamma_{tr} - 2 \cdot \Delta\gamma_{tr} = 6.997$, the dynamic aperture in the horizontal plane disappears completely at these values of betatron tunes.

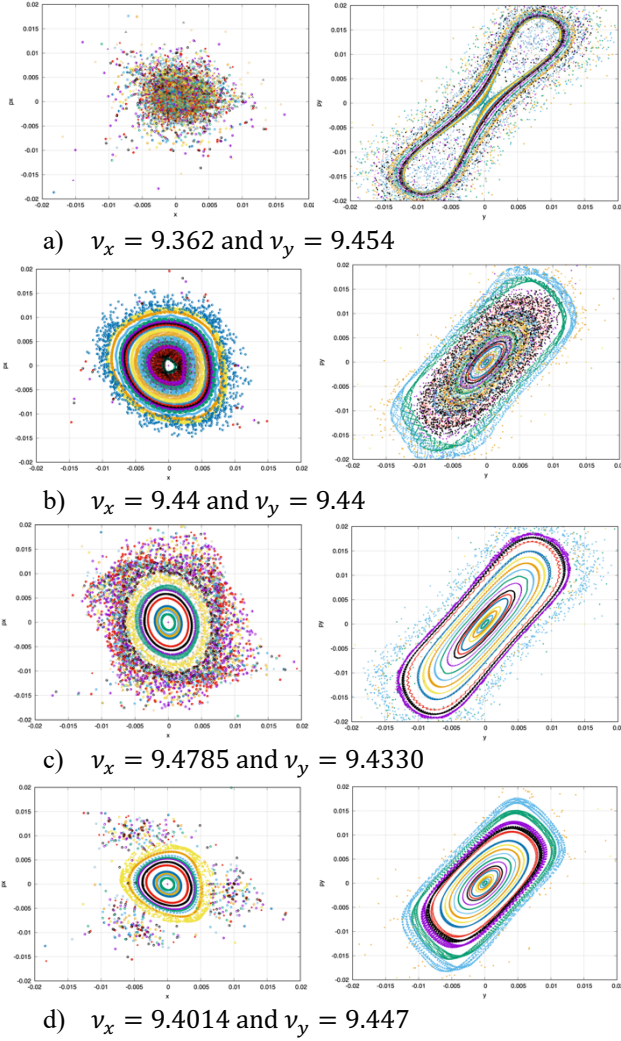


Figure 2: Dynamic apertures (x-plane on the left, y-plane on the right) for the various (a-d) working point with $dp/p = 0$. On the abscissa axis is the coordinate in meters, on the ordinate axis is the relative momentum in radians.

Hence, we considered another option (see Figure 1 right): first, we gradually raise the transition energy to $\gamma_{tr} + \Delta\gamma_{tr} \approx 7.13$, then we make a rapid jump down to $2 \cdot \Delta\gamma_{tr}$ up to $\gamma_{tr} - \Delta\gamma_{tr} \approx 7.04$. In this case, the working point changes from $v_x = 9.44$ и $v_y = 9.44$ up to the value before the jump $v_x = 9.4769$ и $v_y = 9.43$ (see Figure 2 c) and after the jump down to $v_x = 9.4015$ и $v_y = 9.447$ (see Figure 2 d). In this case, the dynamic aperture is achieved a sufficient value.

BARRIER BUCKET RF SYSTEM

The RF-1 system is used to retain, accumulate and accelerate particles to the experimental energy in the collider rings. Each collider ring has one RF-1 system. During retention and accumulation, 2 pairs of rectangular pulses with opposite signs are generated with the amplitude of each barrier $V_{bb} = \pm 5$ kV (see Figure 3). The time duration of a single pulse can vary from $T_{bb} = 10$ till 80 ns. The accumulated particles enclosed between

2 pulses will be inductively accelerated by a constant potential $V_{acc} = 300$ V, which is additionally created also by the RF-1 system [1].

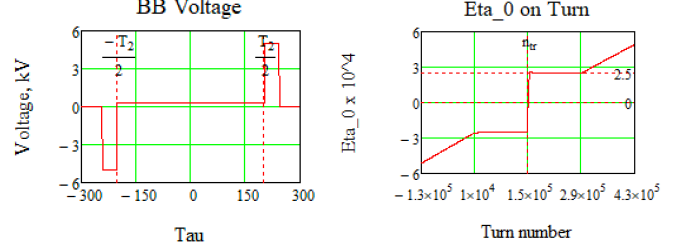


Figure 3: Left – shows the Barrier Bucket RF type; right – scheme of η_0 change during jump.

When the energy approaches the transition value, the RF barriers turn off and, after the proton energy becomes greater than the transition energy, the RF barriers turn on and a polarity changes. This is necessary because the slip-factor value changes its sign after transition energy crossing.

SLIP-FACTOR NEAR TRANSITION ENERGY

On the one hand, when the slip-factor is purely zero $\eta = 0$ the system is isochronous and with any spread of momentum, the bunch does not increase the length. On the other hand, the following second order of slip-factor begins to play an essential role $\eta = \eta_1 \delta$, which distorts the movement and can lead to an increase in momentum spread. And finally, in the absence of focusing in the longitudinal plane, the space charge can introduce large distortions into the phase portrait of the bunch.

During acceleration and retention of particles far away from the transition energy, the value of the slip-factor is calculated as (see Figure 3 right)

$$\eta = \eta_0 = \alpha_0 - \frac{1}{\gamma_0^2}, \quad (2)$$

however, such an expression takes into account only the first order, and in general, the slip-factor depends on $\delta = dp/p$ as [2]:

$$\eta = \eta_0 + \eta_1 \delta + \dots, \quad (3)$$

$$\text{where } \eta_1 = \alpha_1 - \frac{\eta_0}{\gamma_0^2} + \frac{3}{2} \frac{\beta^2}{\gamma^2}.$$

Modeling the process of transition energy crossing, the first two orders of decomposition are taken into account. The corresponding momentum compaction factor (MCF) coefficients depend only on the collider magneto-optical structure.

Making a rapid jump of transition energy, parameters $\alpha_0, \alpha_1, \dots$ also undergo a jump. Value α_0 , as we have already said, it is determined by $\alpha_0 = 1/\gamma_{tr}^2$. Coefficient α_1 for different values γ_{tr} is calculated using the program MADX/PTC [3].

LONGITUDINAL DYNAMIC

Equations of longitudinal motion in Barrier Bucket in coordinates $(\tau, \Delta E)$ [4]:

$$\frac{d\tau}{dt} = \frac{\eta h}{\beta^2 E_0} \Delta E \quad \text{and} \quad \frac{d\Delta E}{dt} = \frac{Ze \omega_0}{A 2\pi} U g(\tau), \quad (4)$$

where E_0 – synchronous particle energy, $U g(\tau)$ – voltage generated by RF-barriers, $\omega_0 = 2\pi/T_0$, h – harmonic number.

For modelling, in equations (4) it is convenient to switch from the time derivative to the turnover derivative $t = nT_0$: (also note that for protons $Z/A = 1$)

$$\frac{d\tau}{dn} = \eta \frac{T_0 h}{\beta^2 E_0} \Delta E \quad \text{and} \quad \frac{d\Delta E}{dn} = V(\tau), \quad (5)$$

The influence of the second-order slip-factor η_1 on the longitudinal dynamics during the jump

First, let's consider a jump taking into account only the first-order of the slip factor $\eta = \eta_0$. Let's evaluate the value η_0 before the jump itself, due to the symmetry with respect to zero, it will be equal to the value after:

$$|-\eta_{0\ tr}| = |+\eta_{0\ tr}| = |\eta_0(\gamma_{tr} - \Delta\gamma_{tr})| = 2.5 \cdot 10^{-4}$$

As it can be seen on Figure 4 (c,d) there is a jump of slip-factor in a different time for different particles because of dependence of slip-factor on δ . Obviously, after the jump, particles with a negative value of the slip-factor will not be in a stable region, since the polarity of the retaining barriers changes and will tend to leave the phase plane, as it can be seen on Figure 4 (a, c, d). Also, due to the momentum spread, there is an asymmetry of the phase portrait relate to the zero value of the momentum spread $\frac{dp}{p}$.

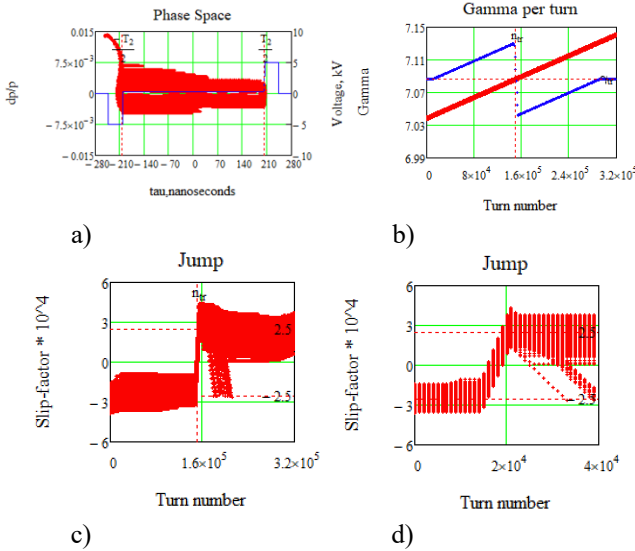


Figure 4: Crossing of the transition energy by a jump taking into account the second order of the slip-factor: a) Blurring the phase space in barrier bucket RF; b) jump of transition energy γ_{tr} (blue dots), γ – gamma particles (red dots); c) Jump of the slip-factor $\eta = \eta_0 + \eta_1 \delta$; d) other scale of jump.

It can be seen that after the jump itself, not all particles are retained inside the separatrix due to the change of the polarity of the RF voltage of retaining barriers. Let's review Figure 4(a), it represents the blurring of phase

space. Before the jump ($\eta_0 < 0$) particles with negative $\delta < 0$ have a slip-factor value η closer to zero than particles with positive $\delta > 0$. Because of this, the phase plane behaves asymmetrically, since the total value of the slip-factor affects the dynamics and change coordinate τ . Thus, the particles accumulate in the area of the left barrier. During the jump, the RF is turned off and does not affect the dynamics. After the jump ($\eta_0 > 0$) the particle distribution is shifted to the left edge and now the opposite is true – for particles with positive $\delta > 0$ the value of the slip-factor η is closer to zero, than for particles with $\delta < 0$. Those particles that turned out to be on the left in the area of the RF barrier (with reversed polarity) due to the proximity to the zero value of the slip-factor, do not have time to stay in the separatrix.

CONCLUSION

In this paper, the dynamics of the longitudinal motion of particles in an RF accelerating system of the Barrier Bucket type at a transition energy jump near the transition is considered. The influence of the second order slip-factor and space charge during the transition is considered. When the critical energy jumps, the working points changes, dynamic apertures are considered for them.

During modelling and calculations, it was found that with a transition energy jump, the determining factor, which influence on the stability of the particles inside the Barrier Bucket RF system is the second order of slip-factor. It depends on momentum spread and can determine the maximum momentum spread for the transition energy crossing in a jump-scheme without loss. Also, it is necessary to take into account the space charge and impedances. These problems require further research.

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