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Spin Coherence and Betatron Chromaticity of Deuteron Beam in "Quasi-Frozen" Spin Regime

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Abstract—A distinctive feature of the "quasi-frozen" spin mode in the synchrotron is the installation of special elements with crossed magnetic and electric fields in straight sections that compensate the spin rotation from the MDM components on the arcs. Moreover, because of the presence of the longitudinal length and the momentum spread inside the beam, spin rotation may occur incoherently. In order to suppress this effect, sextupoles are installed, which also affect suppression of chromaticity.

Keywords: spin coherence, betatron chromaticity, quasi-frozen spin, electric dipole moment

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INTRODUCTION

The possibility of spin control for an electric dipole moment (EDM) experiment can be implemented by setting Wien filters in straight bypass sections, which ensure that the spin of particles retains the mean direction in accordance with the "quasi-frozen spin" mode. However, the spin of different particles, owing to their different motion in 3D space, rotates with slightly different frequencies around the invariant axis. Thus, this violates spin coherence. To ensure spin coherence, nonlinear elements, sextupoles, with a special nonzero dispersion placement on arcs must be used. Since sextupoles simultaneously affect the betatron chromaticity, we consider this complicated case.

"QUASI-FROZEN SPIN" REGIME

T-BMT equations describe the evolution of spin vector $\mathbf{S}$ over time in the particle rest frame in electrical $\mathbf{E}$ and magnetic $\mathbf{B}$ fields in the laboratory frame:

$$\begin{aligned} \frac{d\mathbf{S}}{dt} &= \mathbf{S}\times(\mathbf{\Omega}_{\mathrm{MDM}}+\mathbf{\Omega}_{\mathrm{EDM}}),\\ \mathbf{\Omega}_{\mathrm{MDM}} &= \frac{q}{m\gamma}\left\{(\gamma G+1)\mathbf{B}_{\perp}+(G+1)\mathbf{B}_{\parallel} - \left(\gamma G+\frac{\gamma}{\gamma+1}\right)\frac{\boldsymbol{\beta}\times\mathbf{E}}{c}\right\}\\ \mathbf{\Omega}_{\mathrm{EDM}} &= \frac{q\eta}{2m}\left(\boldsymbol{\beta}\times\mathbf{B}+\frac{\mathbf{E}}{c}\right),\quad G=\frac{g-2}{2}. \end{aligned} \tag{1}$$

As can be seen from Eq. (1) for EDM search, it is necessary to lower the impact from magnetic dipole moment (MDM). But NICA has purely magnetic arcs. Thus, the "frozen spin" method cannot be used [1]. Wien filters implemented in the straight section compensate rotation via MDM in the arc and realize a "quasi-frozen spin" condition for deuterons [2]. For this purpose, NICA needs a modernization to operate as a storage ring with alternative straight sections by using bypass channels [3].

SPIN DECOHERENCE

If we follow T-BMT equation (1), spin tunes in electric and magnetic fields are given by the expressions

$$\begin{aligned} \nu_s^B &= \gamma G,\\ \nu_s^E &= \frac{G+1}{\gamma}-G\gamma. \end{aligned} \tag{2}$$

An Equilibrium Level Energy Shift

Different particles have different momentum, and there is a need to use effective energy:

$$\gamma_{\mathrm{eff}} = \gamma_s+\beta_s^2\gamma_s\Delta\delta_{\mathrm{eq}}. \tag{3}$$

The equilibrium momentum spread due to the betatron motion and nonzero second-order momentum compaction factor is based on the synchronous principle [4] and is defined by

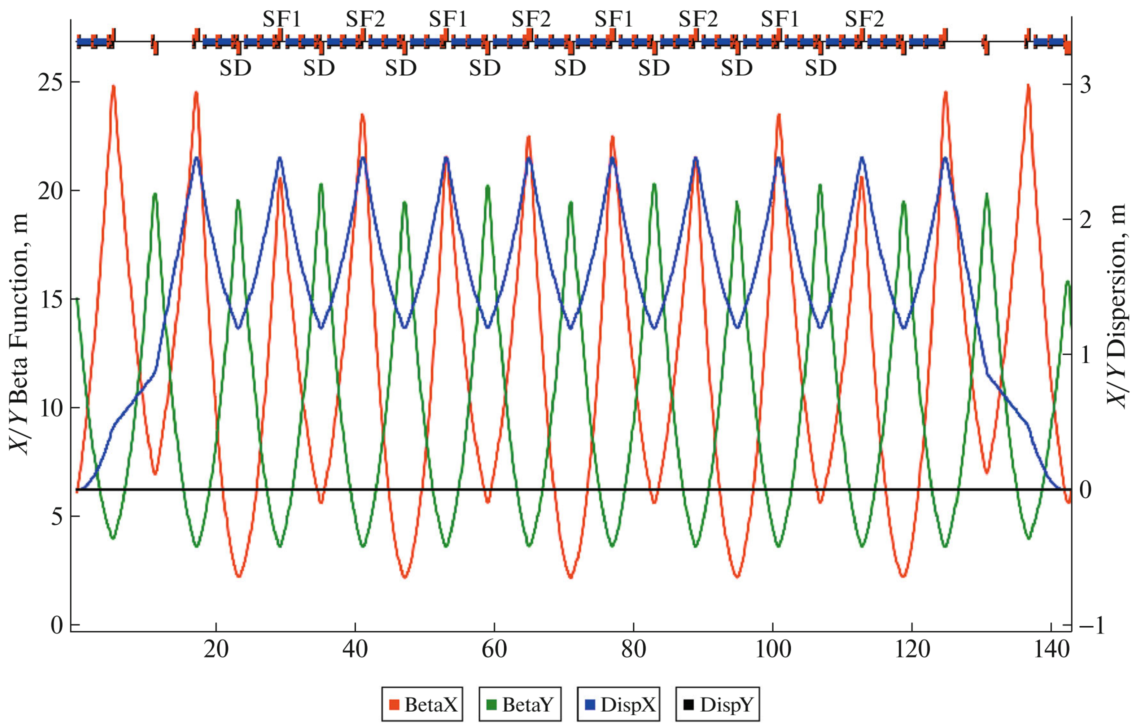

Fig. 1. Bypass NICA Twiss parameters for deuteron mode in OptiM. The arrangement of sextupole families is also shown.

$$\Delta\delta_{eq} = \frac{\gamma_s^2}{\gamma_s^2\alpha_0 - 1}\left[\frac{\delta_0^2}{2}\left(\alpha_1 + \frac{3}{2}\frac{\beta_s^2}{\gamma_s^2} - \frac{\alpha_0}{\gamma_s^2} + \frac{1}{\gamma_s^4}\right) + \left(\frac{\Delta L}{L}\right)_\beta\right], \quad (4)$$

for betatron orbit lengthening term

$$\left(\frac{\Delta L}{L}\right)_\beta = -\frac{\pi}{L_0}[\epsilon_x\nu_x + \epsilon_y\nu_y], \quad (5)$$

where index s means synchronous particle, ϵ_x, ϵ_y are emittances, ν_x, ν_y are tunes, δ_0 is relative deviation of momentum, and α_0, α_1 are two first orders of the momentum compaction factor. Equation (2) together with Eqs. 3–5 shows that spin tune spread depends on the equilibrium energy level of the particle.

Orbit Lengthening and Betatron Chromaticity

More formal theory implies the interaction of an external (sextupole) field. Taking into account the expression for total orbit lengthening from [5],

$$\Delta C_\Sigma = -2\pi(J_x\xi_x + J_y\xi_y) + \delta_0\left(\alpha_0 + \alpha_1\delta_0 + \alpha_2\delta_0^2 + \ldots\right), \quad (6)$$

where ξ_x, ξ_y are chromaticities. If we compare Eq. (6) with (4) and (5), it can be noticed that the orbit length is closely related to the equilibrium energy level.

SEXTUPOLE CORRECTION

As a result, Eqs. (4) and (6) show that using sextupoles can influence spin tune precession ν_s and all things considered do not make it possible to get spin coherence. Such experiments were conducted at COSY (Yulich, Germany) to get SCT at the level of 1000 s [6]. Sextupoles are located in nonzero dispersion regions at arcs. At minima and maxima of dispersion function $D_{x,y}$ and beta function $\beta_{x,y}$, they make the most impact and are physically located next to quadrupole lenses. Twiss functions of NICA arc are regular and can be seen in Fig. 1 [7]. Dispersion is suppressed by the method of "missing magnets" at the edges.

Betatron Chromaticity

For betatron chromaticity correction, only two families of sextupoles are used: one near focusing quadrupoles, the other near defocusing quadrupoles. Natural chromaticity of bypass NICA storage ring is $\nu_{x,y} = -17/-17$. After optimization, one can monitor the spin-tune in Fig. 2: the red line shows natural chromaticity; the blue shows the corrected one suppressed to about zero. For this case, we also achieved spin tracking during 3×10^6 turns for particles with different initial deviation in x, y, d coordinates with initial spin orientation $\mathbf{S}_0$ at an angle of 45 degrees in the y–z plane, shown inn Fig. 3 [8].

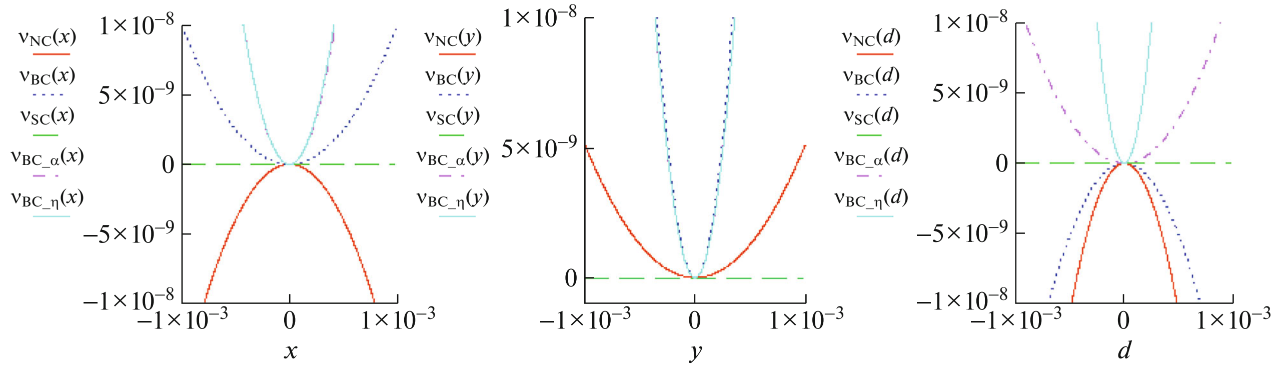

Fig. 2. Dependence of the spin tune precession on the coordinates x, y, d for various optimization cases. NC—natural chromaticity (red line); BC—zero (betatron) chromaticity (blue dotted line); SC—spin coherence (green line); BC_α—zero chromaticity and $\alpha_1 = 0$ (purple line); BC_η—zero chromaticity and zero $\eta_1 = 0$ (light blue line).

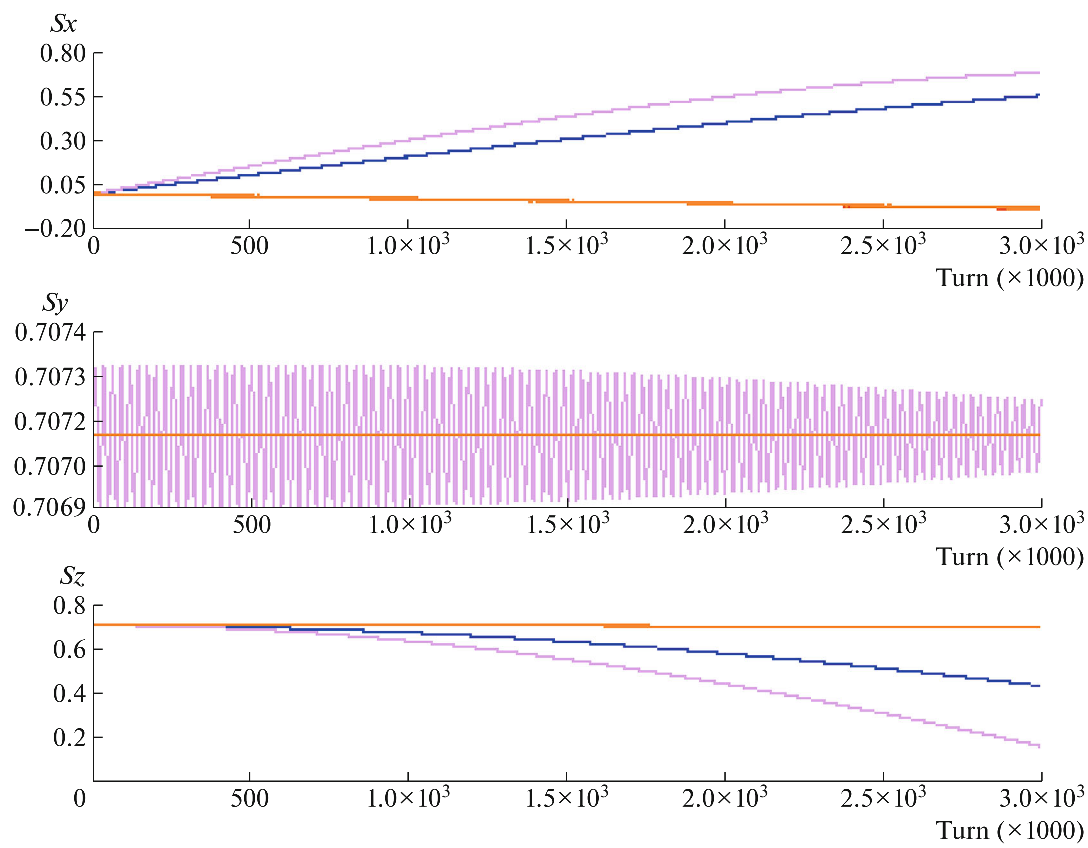

Fig. 3. Spin tracking of particles with different initial deviation in coordinates x, y, d using two families of sextupoles to obtain zero betatronic chromaticity.

Spin Coherence

To get spin coherence, we considered pure spin tune. COSY Infinity [8] cannot operate near zero-value of spin tune. It can cause an error because of resonant denominators, thus we let the spin precess with $\nu_s \sim 10^{-4}$. But the requirement is to do it synchronously—coherently.

The main parameter is the spin tune, which in generally depend on the coordinates and energy. It can be seen that the dominant component is the quadratic term in the expansion of the spin tune in Fig. 2 for both cases: natural and corrected chromaticity. For this reason, sextupoles can be selected in another way just to get spin coherence.

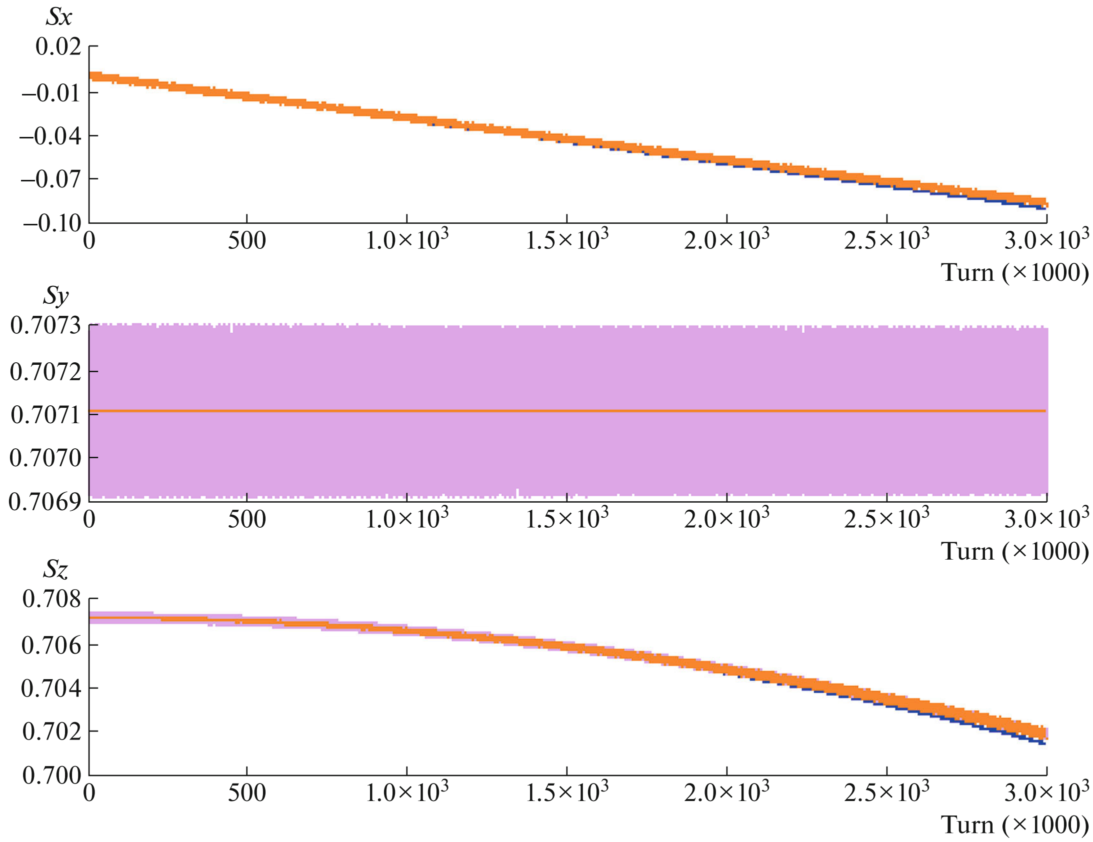

Fig. 4. Spin tracking of particles with different initial deviation in coordinates x, y, d using three families of sextupoles to obtain spin coherence.

As we can see, from Eqs. (4) and (6), it is not sufficient to use two families; thus, a third family is used to influence the energy coordinate. But in regular structures, beta and dispersion functions β and D do not allow using three linear independent families. Figure 1 shows the sextupole arrangement of families: SF1, SF2, SD. In this method, we do not influence chromaticity, but just monitor the main value $\nu_{x,y} = -13/-18$. This is not sufficient for stable orbital motion. For this case, it can be seen that spin coherence is achieved—there is no dependence on coordinates/energy (Fig. 2: green line). Tracking results confirm this. In Fig. 4, the spin tune is switched up to $\nu_s \sim 10^{-7}$, and the number of turns is 3×10^6 or 3 s. Particles with different initial deviation precess with the same spin tune. But in this case, the maximum of the sextupole coefficient is huge and can cause nonlinear effects (Table 1).

α_1, η_1 *Correction*

As we can see, pure betatron chromaticity correction did not allow us to get zero spin tune spread. Simultaneously, getting spin coherence by suppressing the quadratic term of the spin tune expansion did not suppress chromaticity. This brings us back to Eq. (6). The term $\delta_0\alpha_0$ can be averaged using RF for mixing $\langle\delta_0\rangle\alpha_0$. Thus, to ensure a zero orbit lengthening, the chromaticities must be suppressed ξ_x,ξ_y together with α_1 to zero value. It is also possible using three sextupole families. But still, this does not make it possible to get spin coherence. Figure 2 (violet line) shows the nonzero spin tune dependence on the coordinates. The same thing occurs if we follow Eq. (4) and suppress η_1 together with chromaticity correction (Fig. 2). Moreover, the maximum value of the sextupole gradient is too strong and cannot be realized (Table 1).

CONCLUSIONS

In summary, the phenomenon of spin decoherence simultaneously with betatron chromaticity at the bypass NICA storage ring has been considered. It operates in the "quasi-frozen spin" mode and can be used for EDM experiments.

Different cases of optimization of sextupoles were considered. Quadratic terms of spin tune expansion are the most valuable and represent the dependence on coordinates. All the main parameters that were monitored are shown in Table 1. The research shows

Table 1. Main parameters of the structure and experiment

Optimization	Without Optimization	Chromaticity	Spin Coherence	Chromaticity $+\alpha_1$	Chromaticity$+\eta_1$
Betatron tune (working point)	$-17/-17$	0/0	$-13/-18$	0/0	0/0
Coefficient α_1	0.2	-0.4	-0.37×10^{-2}	$\sim -10^{-12}$	-0.85
Coefficient K_x	-0.16×10^{-1}	0.55×10^{-1}	0.27×10^{-13}	0.55×10^{-1}	0.56×10^{-1}
Coefficient K_y	0.51×10^{-2}	0.76×10^{-1}	-0.12×10^{-12}	0.78×10^{-1}	0.78×10^{-1}
Coefficient K_z	-0.43×10^{-1}	0.20×10^{-1}	0.13×10^{-12}	0.13×10^{-1}	1.6×10^{-1}
# sextupole families	without sextupoles	2	3	3	3
Max. sextupole field, m^{-3}	–	2.7	19.4	4.9	104.2

that it is not possible to use three families of sextupoles in the regular structure to achieve both betatron chromaticities and get spin coherence. Moreover, the maximum value of the sextupole coefficient not satisfactory and can cause nonlinear instabilities. It is worth noting that the regular dispersion function on the arc does not make it possible to locate three linear independent families, as they are placed at the same minimum/maximum of β and D functions. But it can be possible to modulate the dispersion function in such a way to get three linear independent sextupole families. Also, one of the possible problem decisions is using the cooled beam at the level of $dp/p \approx 10^{-5}$. This can help to minimize γ-effective and finally get spin coherence simultaneously with corrected betatron chromaticity.

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CONFLICT OF INTEREST

The authors of this work declare that they have no conflicts of interest.

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