The nature of spin decoherence of a polarized beam of light nuclei in a storage ring for EDM search

A. A. Melnikov[+△1)], *Yu. V. Senichev*[+*], *A. E. Aksentyev*[+*⊕], *S. D. Kolokolchikov*[+*]

[+]*Institute for Nuclear Research RAS, 117312 Moscow, Russia*

[△]*L.D. Landau Institute for Theoretical Physics, 142432 Chernogolovka, Russia*

[*]*Moscow Institute of Physics and Tichnology, 141701 Dolgoprudny, Russia*

[⊕]*National Research Nuclear University MEPhI, 115409 Moscow, Russia*

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Coherent spin motion is a necessary condition to search for the electric dipole moment of charged particles in a storage ring. The first step to increase the spin coherence time is to turn on an RF cavity. The next step is to equalize equilibrium energy levels with sextupoles. Spin resonances act as annitional sources of decoherence especially for a proton beam.

INTRODUCTION

One of the essential problems of modern physics is the baryon asymmetry of the universe, which indicates the prevalence of matter over antimatter. In addition, cosmic detectors PAMELA and AMS, whose purpose is to search for antimatter, are yet to have found a signicant amount of it in the universe [1]. The emergence of a new idea claiming that one of the reasons for baryon asymmetry is the breaking of CP invariance began soon after its discovery. A. Sakharov established the necessary conditions for baryogenesis (initial creation of baryons) in 1967 [2]. Many theories beyond the Standard Model (SM) have been proposed and all of them of so-called "New Physics" are able to remove the difficulties that one meets in the SM, but their experimental confirmation has yet to be found. One of the possible arguments for the breaking of CP invariance is the existence of non-vanishing Electric Dipole Moments (EDM) of elementary particles.

The idea of searching for the EDM of the proton and the deuteron using polarized beams in a storage ring was originally proposed at Brookhaven National Laboratory (BNL), USA [3, 4]. Currently, the Institute for Nuclear Physics in the Jülich Research Center (FZJ, Germany) is developing the design of a ProtoType storage Ring (PTR) specifically for testing the equipment for searching for the deuteron and proton EDM [5]. Working in Jülich at the COSY accelerator to search for the EDM, we concluded that this problem includes the following fundamental problems that need to be solved: the maximum Spin Coherence Time (SCT) of particles in a bunch has to be over 1000 seconds, the measurement accuracy of the spin precession frequency and systematic errors have to be smaller than the EDM signal in the contribution to the total frequency of spin precession.

This article discusses the nature of spin decoherence of a polarized beam of protons and deuterons in a storage ring for searching for the EDM.

THE MAIN CAUSES OF POLARIZED BEAM DECOHERENCE

The phenomenon of spin decoherence is related to the unequal frequency of spin precession for different particles in the beam. In polarized beam experiments to search for an electric dipole moment, the preservation of the coherence of particle spin oscillations in a beam during a time sufficient for data collection plays a decisive role.

The first studies on decoherence were carried out at the COSY accelerator in Jülich with a deuteron beam [6]. The ensemble of particle spins was located in the horizontal plane and oscillated around the vertical axis. The polarization was determined using a polarimeter. The change in the amplitude of the total polarization signal was used to determine the spin coherence time.

Today, three main causes of spin decoherence are known. The first of these is the dependence of the spin precession frequency on the particle energy. By analogy with the betatron motion, the dependence of the spin precession frequency on the particle energy

1) e-mail:alexei.a.melnikov@gmail.com

will be called spin chromaticity. The second reason that causes spin decoherence is change in the length of the particle trajectory in 3D space, which causes a change in the particle spin precession frequency. We call this phenomenon the orbit lengthening. The third reason is related to the influence of spin resonances on the spread of the spin frequency of particles in the beam. The first two reasons are related to each other, since both are determined by the dependence of the spin precession frequency on the particle parameters. Nevertheless, their nature formally is different, and they should be considered separately.

For studying the behavior of the particles spin in a beam ensemble we assume the validity of the Ehrenfest theorem, which states that the average values of a quantum mechanical operator, as well as the forces acting on it, are subject to laws of classical mechanics.

Yu. Uzikov (JINR) specially substantiated this transition for the spin, according to which in the representation of classical physics, "spin" means the average value of the quantum mechanical spin operator [7].

The basic principle for measuring EDM in a ring comes from the "Thomas-Bargmann, Michel, Telegdy" equation. In accordance with the Ehrenfest theorem, they describe the classical behavior of a particle spin in a bunch, taking into account the assumed EDM:

$$\frac{d\mathbf{S}}{dt} = \mathbf{S}\times(\mathbf{\Omega}_{MDM}+\mathbf{\Omega}_{EDM}), \tag{1a}$$

$$\mathbf{\Omega}_{MDM} = \frac{q}{m\gamma}\Big[(\gamma G+1)\,\mathbf{B}_\perp + (1+G)\,\mathbf{B}_\parallel - -\gamma\left(G+\frac{1}{\gamma+1}\right)\frac{\boldsymbol{\beta}\times\mathbf{E}}{c}\Big], \tag{1b}$$

$$\mathbf{\Omega}_{EDM} = \frac{q\eta}{2m}\left[\boldsymbol{\beta}\times\mathbf{B}+\frac{\mathbf{E}}{c}\right]. \tag{1c}$$

Here Ω_{MDM} is the spin precession frequency due to the magnetic dipole moment (hereinafter referred to as MDM precession), Ω_{EDM} is the spin precession frequency due to the electric dipole moment (hereinafter referred to as EDM precession). m, q — mass and charge of a particle; $\beta = v/c$, $\gamma = 1/\sqrt{1-\beta^2}$ — relativistic beta and Lorentz factor. $\mathbf{B} = \mathbf{B}_\perp + \mathbf{B}_\parallel$, $\mathbf{B}_\parallel = (\mathbf{v}\cdot\mathbf{B})\mathbf{v}/v^2$. Anomalous magnetic moment $G = \frac{g-2}{2}$, g — gyromagnetic ratio. Dimensionless η factor is connected to the EDM value d and spin s of a particle: $d = \frac{\eta g}{2mc}s$. In the study of decoherence, we will assume $B_\parallel = 0$, since we will not use elements with a longitudinal magnetic field. Longitudinal electric field $E_\parallel$ is also not considered in the study of spin decoherence.

PROTON AND DEUTERON RINGS FOR EDM SEARCH

In this paper, we consider spin decoherence in experiments to search for the EDM of the proton and deuteron. From the point of view of the magneto-optical structure, proton and deuteron accelerators have profound differences. In particular, accelerators for studying the proton EDM have a purely electrostatic structure, while deuteron accelerators require a mixed structure with electrostatic and magnetostatic elements. In each structure, the phenomenon of decoherence has its own peculiarities. But first, we will consider a general approach to the study of spin-coherence time.

The mixed structure uses elements with magnetic and electric fields, which are interconnected by a relationship following from the Lorentz force that keeps the particle on orbit with a radius R: $e\left(c\boldsymbol{\beta}\times\mathbf{B}+\mathbf{E}\right) = \frac{mv^2}{R}\boldsymbol{e}_x$. The angular frequency of revolution of a particle on the orbit of the accelerator determines the speed of rotation of the momentum $\Omega^p_{E,B} = \frac{eE}{m\gamma\beta c}+\frac{eB}{m\gamma}$.

Subtracting the frequency $\Omega^p_{E,B}$ from the frequency Ω_{MDM} (1b) we obtain the frequency of the spin precession with respect to the momentum Ω^p_{MDM}:

$$\mathbf{\Omega}^p_{MDM} = \boldsymbol{\omega}^p_E + \boldsymbol{\omega}^p_B, \tag{2}$$

where

$$\boldsymbol{\omega}^p_E = \frac{e}{m}\left(G-\frac{1}{\gamma^2-1}\right)\frac{\mathbf{E}\times\boldsymbol{\beta}}{c};\quad \boldsymbol{\omega}^p_B = \frac{e}{m}G\mathbf{B}_\perp. \tag{3}$$

To study the electric dipole moment of the proton, it is proposed to use a pure electrostatic ring. The advantage of purely electrostatic machines (magnetic field $B = 0$) is especially evident at the "magic" energy, when the spin vector initially oriented in the longitudinal direction, rotates in the horizontal plane with a frequency ω^p_E.

At "magic" energy

$$G-\frac{1}{\gamma^2_{mag}-1} = 0, \tag{4}$$

thus for the reference particle both $\omega^p_{E,B} = 0$ (3) and the spin orientation with respect to momentum is determined by the EDM signal only $\frac{d\mathbf{S}}{dt} = \mathbf{S}\times\mathbf{\Omega}_{EDM}$. This remarkable property of the lattice was proposed to be used to detect the EDM, and such a lattice was called "frozen spin" structure [3].

We will adopt the following indexing convention for spin projections $\mathbf{S} = (S_x, S_y, S_z)$ and other variables: z is oriented along the momentum, x and y are horizontal and vertical directions respectively.

Now, let's introduce the defenition of a spin tune. The spin tune in an electrostatic ring ν_s^E, defined as the normalized spin precession frequency per particle orbital frequency $\nu_s^E = \omega_E^p/\Omega_E^p$, is given by

$$\nu_s^E = \left(G - \frac{1}{\gamma^2 - 1}\right)\gamma\beta^2. \tag{5}$$

In the case of deuterons with $G = -0.142$ the only possible method is a storage ring with both electric and magnetic fields [4]. This can be done by applying a radial electric field $\mathbf{E}_x$ to balance the vertical magnetic field $\mathbf{B}_y$'s contribution to Ω_{MDM}^p, as shown in Eq. (2):

$$\mathbf{E}_x = \frac{G\mathbf{B}_y c\beta\gamma^2}{1 - G\beta^2\gamma^2} \approx G\mathbf{B}_y c\beta\gamma^2. \tag{6}$$

The frequency of the momentum precession in the B-field is $\Omega_B^p = \frac{eB}{m\gamma}$. Similarly, we find the spin tune $\nu_s^B = \omega_B^p/\Omega_B^p$. In a magnetic field relative to the momentum:

$$\nu_s^B = \gamma G. \tag{7}$$

Thus, for both protons and deuterons there is a general idea of how to construct a ring, but this is realized by means of different types of deflectors: E or $E + B$.

SPIN CHROMATICITY IN THE PROTON AND DEUTERON RINGS FOR EDM SEARCH

The main problem in measuring the EDM is to make the spins of all particles oscillate coherently to observe the polarization build-up caused by the EDM. Here we examine the spin motion of a particle ensemble in the accelerator plane for the fully electrostatic or comnined $E + B$ ring.

In a purely electrostatic proton ring [8] a particle with an energy different from the "magic" value $p \neq p_{mag}$ violates condition (4): $G - \frac{1}{\gamma^2-1} \neq 0$. Expanding $G - \frac{1}{\gamma^2-1}$ in a Taylor series in the vicinity of the magic value of momentum $p = p_{mag}$ we have:

$$\left(G - \frac{1}{\gamma^2 - 1}\right)_{p=p_{mag}+\Delta p} = 0 + 2G\frac{\Delta p}{p_{mag}} - \\ - 3G\left(\frac{\Delta p}{p_{mag}}\right)^2 + o\left(\frac{\Delta p}{p_{mag}}\right)^2. \tag{8}$$

For a deuteron beam, the "frozen spin" condition means that the total spin frequency in the magnetic and electric fields is equal to zero $\omega_E^p + \omega_B^p = 0$. Expanding $\omega_E^p + \omega_B^p$ in the Taylor series in the vicinity of p_0, under which condition (6) is satisfied, we have:

$$(\boldsymbol{\omega}_E^p + \boldsymbol{\omega}_B^p)_{p_0+\Delta p} = \frac{e}{mc}(\mathbf{E} \times \boldsymbol{\beta}) \cdot \\ \cdot\left[0 + \frac{2}{\gamma^2 - 1}\frac{\Delta p}{p_0} - \frac{1 + 3\gamma^2}{\gamma^2(\gamma^2 - 1)}\left(\frac{\Delta p}{p_0}\right)^2 + o\left(\frac{\Delta p}{p_0}\right)^2\right]. \tag{9}$$

The difference between conditions (8) and (9) for the proton and deuteron rings is only that in the proton ring the expression for the magic energy (4) is used, while for the deuteron ring the condition of conservation of the spin direction in the $E+B$ deflector (6) is used together with the condition of stable motion with Lorentz force on the orbit: $e\,(c\boldsymbol{\beta} \times \mathbf{B} + \mathbf{E}) = \frac{mv^2}{R}\boldsymbol{e}_x$.

Taking into account that the vertical electric field component is expected to be small and $\beta_{x,y} << \beta_z$ let us simplify equation (1) to understand the behavior of the spin qualitatively:

$$\begin{aligned} \frac{dS_x}{c\beta_z dt} &= -\frac{e}{m\gamma c^2}\frac{2}{\gamma^2 - 1}\frac{\Delta p}{p_0} \cdot E_x S_z, \\ \frac{dS_z}{c\beta_z dt} &= +\frac{e}{m\gamma c^2}\frac{2}{\gamma^2 - 1}\frac{\Delta p}{p_0} \cdot E_x S_x. \end{aligned} \tag{10}$$

In terms of the normalized coordinate $d\phi = 2\pi \cdot dn = 2\pi \cdot \frac{c\beta_z}{L_{cir}}dt$, where L_{cir} is orbit circumference, we have:

$$\frac{d^2 S_z}{d\phi^2} + \left(\frac{e\bar{E}_x L_{cir}}{2\pi mc^2\gamma} \cdot \frac{2}{\gamma^2 - 1}\frac{\Delta p}{p_0}\right)^2 \cdot S_z = 0. \tag{11}$$

That is the spin oscillates in the horizontal plane with tune $\nu_{sz} = \frac{e\bar{E}_x L_{cir}}{\pi mc^2\gamma} \cdot \frac{1}{\gamma^2-1}\frac{\Delta p}{p_0}$ and $S_z = S_z^{max}\cos(2\pi\nu_{sz}\phi)$, where $\bar{E}_x$ is the average component of the deflecting electric field related to the magnetic component in the $E + B$ deflector by the relation (6).

Thus, due to the spin chromaticity and the momentum spread $\frac{\Delta p}{p_0}$ we find that the spin of each particle oscillates with its own frequency, which clearly leads to the spin decoherence and to a complete depolarization of the beam in a determinate period called the spin coherence time.

For example, assuming the maximum spread of the momentum equal to 10^{-4}, we have determined the spin tune spread $\Delta\nu_{sz} = 1.588 \cdot 10^{-4}$, or SCT = 6300 turns, which is approximately 1 millisecond.

RF CAVITY AS A METHOD TO INCREASE SCT

First, we consider the spin precession (see eq. 9) in linear approximation versus the momentum:

$$(\boldsymbol{\omega}_E^p + \boldsymbol{\omega}_B^p)_{p_0+\Delta p} = \frac{e}{mc}\left(\mathbf{E}\times\boldsymbol{\beta}\right)\cdot\frac{2}{\gamma^2-1}\frac{\Delta p}{p_0}. \quad (12)$$

The idea of using the RF cavity to reduce the precession of the spin of a particle having an energy different from the magic values was proposed by other authors some time ago, for instance [9].

Obviously, a particle oscillating around the level of magic energy with a synchrotron frequency ν_{syn} also changes the behavior of its spin. It is easy to see from equation (11) with $\frac{\Delta p}{p} = \left(\frac{\Delta p}{p}\right)_{max}\cdot\cos\left(\nu_{syn}\phi\right)$ describing the oscillation of the spin in the presence of RF fields:

$$\frac{d^2S_z}{d\phi^2} + \left(\frac{e\bar{E}_x L_{cir}}{\pi mc^2\gamma}\cdot\frac{1}{\gamma^2-1}\left(\frac{\Delta p}{p}\right)_{max}\cdot\cdot\cos\left(\nu_{syn}\phi\right)\right)^2\cdot S_z = 0. \quad (13)$$

This equation describes the pendulum swing in a rapidly oscillating field. Now, instead of the oscillations with a frequency ν_{sz} the spin vibrates within a very narrow angle Φ_{max} with RF frequency $\Phi\sim\Phi_{max}\sin(\nu_{syn}\phi)$. The value $\Phi\sim\left(\nu_{sz}/\Omega_{syn}\right)^2$ depends on the frequency ratio. At our ratio $\nu_{sz}/\Omega_{syn}\sim 1:150$ the spin aberration is determined by $\Phi_{max}\sim 10^{-4}$, which is negligible.

SECOND ORDER APPROXIMATION OF SPIN TUNE VERSUS ΔP/P

From expression (9) one can easily find the frequency of the oscillations of the spin in the second order approximation with respect to the momentum:

$$\frac{d^2S_x}{d\phi^2} + \frac{e\bar{E}_x L_{cir}}{2\pi mc^2\gamma}\cdot\left[\frac{2}{\gamma^2-1}\left(\frac{\Delta p}{p}\right) - \frac{1+3\gamma^2}{\gamma^2}\frac{1}{\gamma^2-1}\left(\frac{\Delta p}{p}\right)^2\right]^2\cdot S_z = 0, \quad (14)$$

and with RF the spin tune has non-zero average value:

$$\bar{\nu}_{sz} = \frac{e\bar{E}_x L_{cir}}{2\pi mc^2\gamma}\cdot\frac{1+3\gamma^2}{\gamma^2}\frac{1}{2}\frac{1}{\gamma^2-1}\left(\frac{\Delta p}{p}\right)^2_{max}. \quad (15)$$

So, in the second order approximation spin does perform rapid oscillations with a synchrotron frequency relative to the average position, which in turn oscillates with a very low frequency $\bar{\nu}_{sz}\sim(\Delta p/p)^2$ determined by the average value (15).

But even with a small momentum spread $(\Delta p/p)\sim 10^{-4}$, simple estimates show that the spin coherence time will be SCT$\sim$ 200 s. We tested this result using the COSY Infinity code [10] and found good agreement with our analytical estimate. Analyzing expressions (5) and (7), we can conclude that the nonlinear spin chromaticity terms exist only in electrostatic deflectors. This means that the spin coherence time is mainly determined by the electrical deflectors. In both proton and deuteron rings, the source of spin chromaticity is only the electrostatic field E, while the magnetic field B does not affect the spin chromaticity.

ORBIT LENGTHENING AS SOURCE OF SPIN DECOHERENCE

The first works concerning the influence of the orbit length on the spin frequency were dedicated to the study of this effect in order to achieve fine tuning of the particle energy in electron-positron colliders [11]. In paper [12] an analysis was made of the effect of betatron chromaticity on the decoherence of the spin.

Now we will discuss the second source of spin decoherence in accelerators, which is related to the influence of different particle orbit lengths on the so-called effective energy, which in turn determines the spin frequency [13-15].

The basic "synchronous acceleration principle" of Veksler and McMillan is formulated in a simple system of equations:

$$\begin{cases} \frac{d\phi}{dt} = -\omega_{rf}\eta\delta, \\ \frac{d\delta}{dt} = \frac{eV_{rf}\omega_{rf}}{2\pi h\beta^2 E}sin\phi. \end{cases} \quad (16)$$

where $\delta = \Delta p/p$ is the momentum deviation from the equilibrium synchronous level of momentum $\Delta p = p - p_s$, ϕ is phase deviation from synchronous phase (no acceleration in storage ring), η is the slip-factor, E is full energy, β is relative velocity, eV_{rf} is energy gain per turn with V_{rf} voltage gap, $\omega_{rf} = 2\pi hf_{rev}$ is angular frequency of RF field, h is the harmonic number, $f_{rev} = 1/T_{rev}$ is revolution frequency.

The first equation of system (16) comes directly from the "synchronous acceleration principle": a particle having shorter revolution time arrives earlier and gets into an earlier phase of the RF field:

$$\frac{\Delta\phi}{\phi} = -\frac{\Delta T_{rev}}{T_{rev}}. \quad (17)$$

In the first order approximation change of orbit length C versus momentum deviation is defined by $\Delta C/C = \alpha_0 \cdot \delta$, and we can write:

$$\frac{\Delta T_{rev}}{T_{rev}} = \frac{\Delta(C/v)}{C/v} = \frac{\Delta C}{C} - \frac{\Delta v}{v} = \left(\alpha_0 - \frac{1}{\gamma^2}\right). \quad (18)$$

The slip factor $\eta = \alpha_0 - 1/\gamma^2$ is introduced as a ratio between the revolution time and momentum deviations:

$$\frac{\Delta T_{rev}}{T_{rev}} = \eta \cdot \delta. \quad (19)$$

Obviously, in the case of a bunched beam the revolution time variation averaged over one synchrotron oscillation is zero. In the first order approximation it follows from the solution of (16). However, using a higher power expansion of the momentum compaction factor $\alpha = \alpha_0 + \alpha_1 \cdot \delta$ and velocity $(v_s + \Delta v)^{-1} = v_s\left[1 - \Delta v/v_s + (\Delta v/v_s)^2 - \ldots\right]$ the deviation of time revolution can be represented in the form:

$$\frac{\Delta T_{rev}}{T_{rev}} = \frac{\Delta C}{C} - \frac{\Delta v}{v_s} - \frac{\Delta C}{C} \cdot \frac{\Delta v}{v_s} + \left(\frac{\Delta v}{v_s}\right)^2 = \\ = \left(\alpha_0 - \frac{1}{\gamma^2}\right) \cdot \delta + \left(\alpha_1 + \frac{3}{2}\frac{\beta^2}{\gamma^2} - \frac{\alpha_0}{\gamma^2} + \frac{1}{\gamma^4}\right) \cdot \delta^2. \quad (20)$$

In addition, we must include the term representing the orbit lengthening $(\Delta L/L)_\beta$ due to the betatron motion:

$$\frac{\Delta T_{rev}}{T_{rev}} = \left(\alpha_0 - \frac{1}{\gamma^2}\right) \cdot \delta + \\ + \left(\alpha_1 + \frac{3}{2}\frac{\beta^2}{\gamma^2} - \frac{\alpha_0}{\gamma^2} + \frac{1}{\gamma^4}\right) \cdot \delta^2 + \left(\frac{\Delta L}{L}\right)_\beta. \quad (21)$$

Then the longitudinal motion equations can be written as:

$$\begin{cases} \frac{d\phi}{dt} = -\omega_{rf}\left[\left(\alpha_0 - \frac{1}{\gamma^2}\right)\cdot\delta + \right. \\ \left. + \left(\alpha_1 + \frac{3}{2}\frac{\beta^2}{\gamma^2} - \frac{\alpha_0}{\gamma^2} + \frac{1}{\gamma^4}\right)\cdot\delta^2 + \left(\frac{\Delta L}{L}\right)_\beta\right], \\ \frac{d\delta}{dt} = \frac{eV_{rf}\omega_{rf}}{2\pi h\beta^2 E} sin\phi. \end{cases} \quad (22)$$

Let us assume that $\phi << 1$, that is $\cos\phi \sim 1$, and write the equation for the momentum deviation δ as:

$$\frac{d^2\delta}{dt^2} + \frac{eV_{rf}\omega_{rf}}{2\pi h\beta^2 E}\left(\alpha_0 - \frac{1}{\gamma^2}\right)\cdot\delta = -\frac{eV_{rf}\omega_{rf}}{2\pi h\beta^2 E}\cdot \\ \cdot\left[\left(\alpha_1 + \frac{3}{2}\frac{\beta^2}{\gamma^2} - \frac{\alpha_0}{\gamma^2} + \frac{1}{\gamma^4}\right)\cdot\delta^2 + \left(\frac{\Delta L}{L}\right)_\beta\right]. \quad (23)$$

From (21) we see that the average value $\overline{\Delta T_{rev}/T_{rev}} \neq 0$ is not zero, and it is defined by $\alpha_0, \alpha_1, \gamma$ and $\left(\frac{\Delta L}{L}\right)_\beta$:

$$\frac{\overline{\Delta T_{rev}}}{T_{rev}} = \left(\alpha_1 + \frac{3}{2}\frac{\beta^2}{\gamma^2} - \frac{\alpha_0}{\gamma^2} + \frac{1}{\gamma^4}\right)\cdot\overline{\delta^2} + \overline{\left(\frac{\Delta L}{L}\right)_\beta}. \quad (24)$$

As follows from equation (23) the orbit lengthening must be compensated by an equilibrium momentum level rising to be consistent with the basic "synchronous acceleration principle". Solving (23) using asymptotic methods [16] we can define the influences of the betatron oscillation, the square term of the momentum compaction factor α_1 and the slip factor η onto the equilibrium level energy shift $\Delta\delta_{eq}$:

$$\Delta\delta_{eq} = \frac{\gamma_s^2}{\gamma_s^2\alpha_0 - 1}\left[\frac{\delta_m^2}{2}\left(\alpha_1 + \frac{3}{2}\frac{\beta^2}{\gamma^2} - \frac{\alpha_0}{\gamma^2} + \frac{1}{\gamma^4}\right) + \right. \\ \left. + \left(\frac{\Delta L}{L}\right)_\beta\right]. \quad (25)$$

Expression (25) means that the equilibrium momentum is different for every particle in the bunch. This energy level, relative to which the particle oscillates, we call the effective energy.

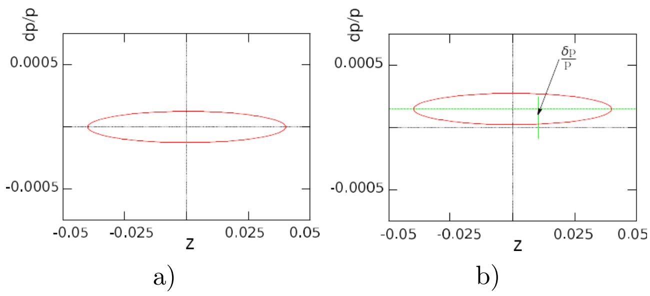

Fig.1. Phase trajectory in longitudinal plane for initial coordinates x=0, y=0 (a) and x=3 mm, y=0 (b)

As example, the results of the COSY Infinity [10] calculation in an electrostatic ring are presented (cf. fig.1), where the equilibrium momentum level rises due to betatron motion. In the same time due to the non-zero second order momentum compaction factor $\alpha_1 \neq 0$ phase trajectories lose symmetry in the longitudinal plane in the direction of the momentum, and thus lead to a shift of the equilibrium momentum value as well.

BETATRON MOTION

Now we define parameters α_0, α_1 and $\left(\frac{\Delta L}{L}\right)_\beta$ on the basis of simple geometric considerations. Obviously, the orbit lengthening can be due to the momentum spread δ and the betatron oscillation $\left(\frac{\Delta L}{L}\right)_\beta$. Let us begin from the case when the orbit lengthening arises due to the betatron oscillation. Assume that the particle has parameters (x_β, x'_β) at the time. Due to a larger radius $\rho + x_\beta$ the orbit is longer by a factor $(\rho + x_\beta)/\rho$ and due to x'_β is longer by a factor $1/\cos(x_\beta)$. Together with the vertical motion the factor of lengthening is $1/\cos\theta$, where $\theta = \sqrt{x'^2_\beta + y'^2_\beta}$. Then the orbit lengthening due to betatron oscillation is:

$$\left(\frac{\Delta L}{L}\right)_\beta = \frac{1}{L}\oint\left(\frac{\rho + x_\beta}{\rho\cos\theta} - 1\right) ds = = \frac{1}{L}\oint\left(\frac{x_\beta}{\rho} + \frac{x'^2_\beta + y'^2_\beta}{2}\right) ds. \quad (26)$$

Since $\langle\frac{x_\beta}{\rho}\rangle = 0$, $\langle x'^2_\beta\rangle = \frac{1}{2}\langle\frac{\epsilon_x}{\beta_x}\rangle$, $\langle y'^2_\beta\rangle = \frac{1}{2}\langle\frac{\epsilon_y}{\beta_y}\rangle$ and $\langle 1/\beta_{x,y}\rangle = \nu_{x,y}/\bar{R}$. Hence, the orbit lengthening due to the betatron motion is:

$$\left(\frac{\Delta L}{L}\right)_\beta = \frac{\pi}{2L}\left[\epsilon_x\nu_x + \epsilon_y\nu_y\right]. \quad (27)$$

MOMENTUM DEVIATION

Now let us go back to the orbit lengthening due to the momentum deviation. First of all, we define the linear and angular dispersion:

$$\begin{aligned} D(s,\delta) &= D_0(s) + D_1(s)\cdot\delta; \\ D'(s,\delta) &= D'_0(s) + D'_1(s)\cdot\delta. \end{aligned} \quad (28)$$

At an arbitrary point along $ds = \rho d\theta$:

$$dl_1 = \left(\rho + D_0\cdot\delta + D_1\cdot\delta^2\right) d\theta = = \left(1 + \frac{D_0}{\rho}\cdot\delta + \frac{D_1}{\rho}\cdot\delta^2\right) ds, \quad (29)$$

$$dl_2 = dl_1\sqrt{1 + (D'_0\delta)^2} = \left(1 + \frac{D_0}{\rho}\cdot\delta + \frac{D_1}{\rho}\cdot\delta^2\right)\cdot \cdot\left(1 + \frac{1}{2}(D'_0\delta)^2\right) ds. \quad (30)$$

As a result, we have:

$$l_2 = \oint\left[1 + \frac{D_0}{\rho}\cdot\delta + \left(\frac{D_1}{\rho} + \frac{1}{2}D'^2_0\right)\cdot\delta^2\right] ds. \quad (31)$$

Thus, the orbit lengthening due to the momentum deviation is:

$$\begin{aligned} \frac{\Delta C}{C} &= \frac{l_2 - C}{C} = \alpha_0\delta + \alpha_1\delta^2 + \ldots; \\ \alpha_0 &= \left\langle\frac{D_0}{\rho}\right\rangle;\ \alpha_1 = \left\langle\frac{D_1}{\rho}\right\rangle + \frac{1}{2}\langle D'^2_0\rangle. \end{aligned} \quad (32)$$

In result we have the equilibrium momentum spread due to the betatron motion and non-zero second order momentum compaction factor:

$$\Delta\delta_{eq} = \frac{\gamma_s^2}{\gamma_s^2\alpha_0 - 1}\left[\frac{\delta_m^2}{2}\left(\alpha_1 + \frac{3}{2}\frac{\beta^2}{\gamma^2} - \frac{\alpha_0}{\gamma^2} + \frac{1}{\gamma^4}\right) + + \frac{\pi}{2L}\left(\epsilon_x\nu_x + \epsilon_y\nu_y\right)\right]. \quad (33)$$

Let's use the example of a magnetic ring to consider how the lengthening of the orbit affects the decoherence of the spin in an accelerator. As we know the spin tune $\nu_s = \gamma G$ in a magnetic ring. If the equilibrium energy deviation $\Delta\gamma_{eq}$ depends on the particle parameters the spin tune spread for N_t turns has incoherent spread:

$$2\pi\langle\Delta\nu_s\rangle = 2\pi G\langle\Delta\gamma_{eq}\rangle N_t. \quad (34)$$

It reduces the spin coherence time. For example, let us consider the case with the spin coherence time limited by 1000 seconds ($\sim 10^9$ turns) and $\langle\Delta\gamma_{eq}/\gamma\rangle < 1\text{rad}/2\pi\gamma G N_t = 7\cdot 10^{-11}$. Then using expression (32) we can define limit for momentum spread:

$$\langle\delta_m^2\rangle < \left\langle\frac{\Delta\gamma_{eq}}{\gamma}\right\rangle\cdot\frac{2}{\beta^2}\cdot\frac{\gamma_s^2\cdot(\gamma_s^2\alpha_0 - 1)}{\gamma_s^4\alpha_1 - \gamma_s^2\alpha_0 + 1}. \quad (35)$$

For example in the COSY ring $\alpha_0 = 0.2$, $\gamma_s = 1.248$, $\alpha_1 = 2$. Taking zero contribution from betatron motion ($\epsilon_{x,y} \sim 0$) RMS momentum spread should not exceed the value $\langle\delta_m\rangle < 10^{-5}$. Reducing the second order of MCF up to $\alpha_1 = 0.01$ we get $\langle\delta_m\rangle < 2\cdot 10^{-5}$. In order to exclude completely the momentum spread influence on the orbit lengthening the expression for $\Delta\delta_{eq}$ in eq. (25) has to be zero:

$$\alpha_1 + \frac{3}{2}\frac{\beta^2}{\gamma^2} - \frac{\alpha_0}{\gamma^2} + \frac{1}{\gamma^4} = 0. \quad (36)$$

Now let us estimate the restriction for the emittance value:

$$\epsilon_{x,y}^{rms} < \left\langle\frac{\Delta\gamma_{eq}}{\gamma}\right\rangle\cdot\frac{1}{\beta^2}\cdot\frac{\gamma_s^2\alpha_0 - 1}{\gamma_s^2}\cdot\frac{L}{\pi\nu_{x,y}}. \quad (37)$$

In the COSY ring, taking $\langle\delta_m\rangle << 10^{-5}$, the emittances should be $\epsilon_{x,y}^{rms} < 1.4$ mm·mrad. Thus, we can conclude that the contribution to the spin tune decoherence is the same for $\epsilon_{x,y}^{rms} \approx 1$ mm·mrad and $\delta_{rms} \approx 10^{-5}$.

THE INFLUENCE OF SPIN RESONANCES ON THE SPREAD OF THE SPIN FREQUENCY

In the forementioned results on optimizing the SCT it has been shown that in the absence of spin resonances spin decoherence can be fully explained by the equilibrium energy shift. For particles with a large value of the magnetic anomaly G vertical motion is subject to the influence of spin resonances [14]. The latter lead to additional phase space dependent/coherent spin-kicks and finally influence the SCT as well. Both these effects need to be considered to optimize the SCT of a beam.

Intrinsic resonances are present even in a perfectly aligned machine with periodicity P for $\gamma G = KP \pm Q_y$, $K \in \mathbb{Z}$. They are caused by spin perturbation due to the vertical betatron motion. Imperfection resonances occur at integer spin-tunes due to the spin perturbation on an imperfect closed orbit.

Comparing polarized proton machines with deuteron ones one can note that a high ratio of the anomalous magnetic moments of the proton and deuteron $G_p/G_d \sim 13$ means a fast proton spin decoherence and high sensitivity to spin resonances. The latter is also explained by a reduced distance between resonances with an energy step $\Delta\gamma \sim 1/G$.

One way to predict the influence of spin resonances on the SCT is to calculate their strengths first and use an analytical model for spin-tune deviations $\Delta\nu_s$. Here the overall effect from spin resonances Q_i can be added up for known values of strengths ϵ_i:

$$\Delta\nu_s(\nu_0) = \sum_i \Big[(Q_i - \nu_0) - - sign(Q_i - \nu_0) \cdot \sqrt{(Q_i - \nu_0)^2 + \epsilon_i^2}\Big]. \quad (38)$$

Taking an estimate of $\epsilon_i \sim 10^{-3}$ for a GeV machine with a revolution frequency f_{rev} ~1MHz we can predict the spread of spin precession frequencies $\Delta f_s = \Delta\nu_s \cdot f_{rev}$. For the assumed distance from resonance $|Q_i - \nu_0| = 0.1$ we get $\Delta\nu_s \sim 5 \cdot 10^{-6}$. Taking the SCT as $\tau \sim 1/\Delta f_s$ we get $\tau \sim 0.2$ s. This estimate corresponds to the preliminary experimental results obtained during accelerator runs with protons at COSY for $\gamma G = 2.06$ and is by far not enough for the purposes of the EDM experiment requiring $\tau \sim 1000$ s.

To obtain high SCT one has to examine the overall picture of spin resonances. From the predicted results for COSY (cf. fig.2) [15] it is evident that the point where particles with different betatron amplitudes have the same spin-tune is the most suitable for the EDM experiment. The location of this "crossing point" depends on beam energy and for COSY it corresponds to $\gamma G \sim 3$. Similar resonance diagrams can be obtained for other storage rings to predict the necessary machine parametrs in terms of spin coherence.

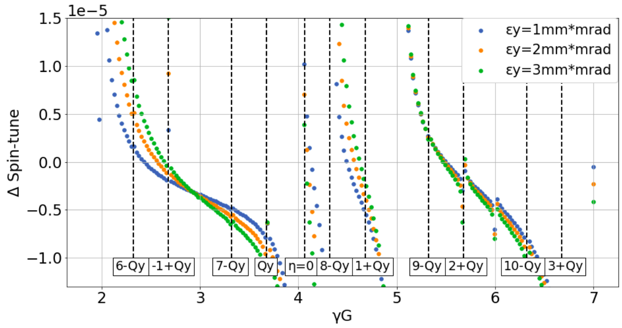

Fig.2. Spin-tune deviations for non-reference particles with different vertical betatron amplitudes for intrinsic + integer resonances; vertical chromaticity $\xi_y = 0$

CONCLUSION

Spin coherence is a key property of the polarized beam to be used for the EDM measurement purposes. High SCT can be obtained by first bunching the beam with an RF cavity and then turning on sextupoles to manipulate the equilibrium energy level. This property is a scalar characteristic of the spin motion of a beam with a distribution in a 6D phase space. The effective equilibrium energy level needs to be the same for all particles in the beam to achieve a high SCT. It is evident from the solution of nonlinear equations of motion in the longitudinal plane.

Spin resonances act as another source of spin-decoherence. Their influence needs to be considered especially for the proton beam in the entire energy range of the machine.

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1. M. Aguilar, G. Alberti, B. Alpat et al., Phys. Rev. Lett. **110**, 141102 (2013).
2. A. Sakharov, JETP Letters, 5, 24-26 (1967) [Pis'ma v ZhETF, 5, 24-26 (1967)].
3. F. J. M. Farley et al., Phys. Rev. Lett. **93**, 052001 (2004).
4. D. Anastassopoulos et al., BNL Report, 2008.
5. F. Abusaif et al., CERN Yellow Reports: Monographs, 2021-003 (CERN, Geneva, 2021).
6. G. Guidoboni et al., Phys. Rev. Lett. **117**, 054801 (2016).
7. Yu. Uzikov, private communication.
8. Yu. Senichev, A. Lehrach, R. Maier, D. Zyuzin, in Proc. IPAC2011, San Sebastián, Spain, pp. 2175-2177.
9. A.P. Lysenko et al., Part.Accel. 18, 215 (1986).

10. COSY Infinity, http://cosyinfinity.org
11. J. Wenninger, preprint CERN SL-note 97-06 OP.
12. I.A. Koop, Ju. M. Shatunov, in Proc. EPAC 1988, Rome, Italy, p.738.
13. Yu.Senichev, R. Maier, D. Zyuzin, in Proc. IPAC2013, Shanghai, China, pp. 2579-2581.
14. A. A. Melnikov, A. E. Aksentyev, Yu. V. Senichev, and E. Syresin, in Proc. IPAC'22, Bangkok, Thailand, 2022, pp. 1832–1834.
15. A. A. Melnikov, A. E. Aksentyev, Y. Senichev, and S. Kolokolchikov, in Proc. IPAC'23, Venezia, Italy, 2023., pp. 2350–2353.
16. N. Bogolyubov, Yu. Mitropolsky, Hindustan Publishing Corpn. Delhi, 1961.